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Choosing recruitment channels
to fill high job positions

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Abstract
This paper analyzes the recruitment strategies of firms aiming to fill high job positions. For that, we consider three recruitment channels actually used to hire managers: employee referrals, private employment agency and internal promotion. This last channel leads to consider also the recruitment of lower job positions (executives) mediated through traditional channels or through the TNU channel. The model highlights the connection between executives hired through TNU who hold particular strong network and managers who will be hired through the employee Referrals channel. The efficiency of this channel is then made dependant to the size of the TNU network in the labor market. When several firms are considered in a stationary equilibrium setting, the combination of channels generates multiple rational expectation equilibria. If firms are homogeneous, the optimal choices of firms can involve coordination failures. If they are heterogeneous by their size and technologies, conflicts of interest can also emerge among firms and between firms and Grandes Ecoles.

JEL Classification: D21; J23; M51
Keywords: recruitment channels, firm behavior, heterogeneity.

1 Introduction and Related Works
In the labor market, the lack of information leads both workers and employers to invest in search strategies to find the best possible partner. The way workers find jobs and firms fill their vacancies plays an important role in the quality of this matching process. Hence, a strand of economic literature has emerged to analyze the job seekers and employers strategies through the recruitment channels they used.

From the seminal work of Rees (1966) and Rees and Shultz (1970), most of the papers focusing on employer’s behavior distinguish formal and informal channels and point out a trade off between the quality and the size of the pool of applicants (see for instance Ropper 1988, Montgomery 1991, Simon and Warner 1992, Gorter and Van Ommeren 1999, De Varo 2005). On the one hand, formal channels such as advertisement in newspapers or

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placement agencies provide extensive information, *i.e.* an important pool of applicants which reduce the time to fill the vacancy. On the other hand, informal channels such as employee referrals generate intensive information, *i.e.* better information about applicants characteristics which reduce the risk of adverse selection.

According to the conditions of the labor market (Gorter et al. 2001), the size of the firm (Barron et al. 1987), the number of workers to hire (Bessy and Marchal 2007), the sector or the type of position to fill, the number of activated channels change (Gorter et al. 2000) and the best suited channels vary. The conclusions may differ on the relevant channel to use: while Gorter and Van Ommeren (1999) emphasize the relevant role of informal channels for high job positions, Pelizzari (2004, 2005) highlights that formal methods are better suited for this same kind of jobs, if firms invest enough in the recruitment process. Beyond these results, these works point out that recruitment policy varies in relation to the job positions in the firm.

In fact, no many attempts have been made on the question dealing with how firms use recruitment channels. Economic literature only addresses this issue through empirical studies. When a theoretical approach is chosen, papers focus on the strategic choice of firms between formal and informal channels but disregarding hiring channels strategies. In light of this, our paper provides accurately a theoretical model on the strategic use and combination of hiring channels. Focusing on high skilled workers recruitment, we argue that the choice of hiring channels to fill high job positions is a long term strategy. Firms choose hiring channels not just to get the most possible productive employees during the current recruitment but also to get the most possible productive ones in the future. This long term strategy is obviously relevant if promotion process occurs in firms and if workers hired today interfere tomorrow (once they have been promoted) on subsequent recruitments. Following empirical observations on recruitment trends, we consider that firms hire high skilled workers through three channels, private employment agencies (external formal channel), employee referrals (external informal channel) and promotion (internal channel). Promotion channel leads obviously to consider also the recruitment of “juniors” (high skilled but less experienced workers) and studying employee referrals channel means that professional networks matter. For these reasons, we focus on a particular hiring channel that we call the Top-Notch Universities channel (TNU hereafter). This channel offers a suitable case of study since new graduates are likely to start their professional career in high skilled jobs and because they are embedded within strong “old boy network”.\(^1\) The pool of TNU graduates being limited, firms compete to hire them if the demand for this type of workers increases. In this context, the individual hiring choices matter and the recruitment strategies of firms may differ when they face other firms.

Finally, through these multiple possible combinations of channels, we analyze how firms choose their hiring strategies and interact with other firms and TNU to compose optimally their top executive workforce.

The rest of the paper is organized as follows. Section 2 provides some empirical evidence on recruitment practices actually used to hire high skilled workers. Section 3 introduces the structure of the basic model specifying the recruitment of a single firm. Section 4 presents and discusses recruitment strategies when firms face other homogenous firms.

\(^1\)An “old boy network” is an association of former student who maintain links in their professional career.
or heterogeneous firms. Section ?? concludes this theoretical framework.

2 Empirical background

According to a French study on recruitment channels (“Offre d’emploi et recrutment” 2005), firms use essentially personal referrals and intermediaries (including school placement officers, private agencies and agencies specialized in temporary work) to look for top executives and recruit effectively them also via these two channels (35.7% of top juniors are hired through personal referrals and 32.1% through intermediaries). Another report in the UK consulting sector (“the Top-Consultant.com Recruitment Channel Report” 2008) confirms this trend on the supply side. The two first hiring channels used by consulting candidates are recruitment agencies and referrals (respectively 70% and more than 65% of candidates use these channels). In the same way, a French survey on the executives’ mobility (“Enquête sur la mobilité des cadres” 2007) reports that 35% of executives have been hired through referrals and 19% through private employment agencies. This survey explores also internal channels such as promotion and reveals that 13% of executives have been promoted in 2006.

Top-notch Universities channel is the third external hiring channels. This channel is generally not considered as a recruitment channel. Yet, introducing TNU in a framework devoted to hiring channels is relevant because relationships with these TNU often induce hiring opportunities for firms. In the higher education system, this TNU are prestigious institutions providing both high level of education and a strong sense of solidarity among students and former students who regularly keep in touch during all their professional career. By hiring TNU new graduates, firms obtain the immediate benefit of their good level of skill. Moreover, these workers can also improve the access of the firms to the TNU graduates networks for the subsequent external recruitment of high skilled workers. Approaching this promising workforce is then complex but possible if firms build ties with TNU. For that, firms can enter into research contracts with them or can respond to the famous Universities fundraising by giving donations. The “Council for Aid to Education” reports from the council’s annual “Voluntary Support of Education” survey that fundraising yield $29.75B for US colleges and universities in 2007. The twenty TNU, which represent just 2% of the survey respondents, raised more than a quarter of all the contributions. For example Stanford and Harvard Universities raised respectively $832M and $614M in 2007. Even if firms are not the first contributors ($4.8B or around 16% of the total gifts$^{2}$), these donations make easier future contacts with new graduates by giving a good corporate image to the generous firms.

Finally, these empirical observations on recruitment practices to fill high job positions emphasize the relevance of the recruitment channels analyzed in the current paper, employment agencies and employee referrals. Besides, given funds that firms provide in donations to TNU, we can consider that these “gifts” are in fact a strategic investment aiming to use TNU as a recruitment channel.

$^{2}$But note that this amount does not take into account the other ways firms support universities like partnerships or sponsorships.
3  A single firm model

We first consider a single firm in which the high job positions correspond to junior executives and top executives positions. Juniors are hired on the external market while top executives can either be promoted or recruited on external markets. By juniors, we mean that firms search for non experienced workers, i.e. agents who have just left the high education system after completing their Master degree or PhD. Conversely, to fill top executives positions, firms search for experienced workers only.

The recruitment of juniors could be done either through the TNU (denoted $U$) channel or through other channels\(^3\) (denoted $O$). By “other channels”, we unify artificially both informal and formal channels like employee/personal referrals, newspapers, agencies specialized in temporary works or Internet job boards. All these recruitment channels involve economies in term of wages and do not require any previous investment in social or professional links.

We limit the external recruitment channel of top executives to private Agencies (denoted $A$) and employee Referrals (denoted $R$)\(^4\). Private Agencies provide an externalization of the search and screening services for the recruitment of executives endowed by the general and specific competences needed by the firm. The employee Referrals channel consists in using the address book of the current employees of the firm. Its performance depends essentially on the proportion of TNU graduates in the top executive staff of the firm and on the size of the TNU network. Complementarily, Promotion (denoted $P$) has the advantage to provide an accurate selection process and to increase the performance of the employee Referrals channel, given the proportion of its juniors originally from the TNU channel.

3.1 Juniors and top executives

The size of the firm $i$ is defined by the pair $(\bar{J}_i, \bar{E}_i)$ where $\bar{J}_i$ and $\bar{E}_i$ figure respectively the number of the junior and top executive positions ($\bar{J}_i < \bar{E}_i$). This size is constant and determined by general considerations relative to the level of activity of the firm. The turnover generated by individual choices and by the normal activity of the firm determines the given annual exit rates, respectively $\bar{k}$ and $\bar{k}'$ for juniors and top executives (with $\bar{k} > 0$ and $\bar{k}' > 0$). The firm has to determine on the one hand the number of juniors $J^U_i$ and $J^O_i$ originally from TNU and other channels, and on the other hand the number of executives $E^A_i$, $E^R_i$ and $E^P_i$ respectively originally from private Agencies, employee Referrals and Promotion. If we limit the analysis to stationary values of $J^U_i$, $J^O_i$, $E^A_i$, $E^R_i$ and $E^P_i$, the determination of these five populations inside the firm amounts choosing the flows of entry $j^U_i$, $j^O_i$, $e^A_i$, $e^R_i$ and $e^P_i$ for each category. We introduce at this point two working assumptions: i) promotions are effective at the beginning of the year and the decisions of exit at the end of the year, ii) promotions are uniformly distributed among juniors and they are independent on their origins and on their firm tenure.

\(^3\)We do not describe these others hiring channels since the goal of the paper is to analyze the recruitment process of top executives.

\(^4\)TNU could not be a hiring channel to recruit top executives since new graduates do not have enough professional experience at the end of their education.
If \( k'' \) represents the endogenous annual promotion rate of juniors (with \( k'' > 0 \)), the dynamics of populations are represented by Figure 1.

\[
\begin{align*}
\text{Juniors } \bar{J}_i & \quad \text{Executives } \bar{E}_i \\
\end{align*}
\]

![Diagram of dynamics of populations](image)

Figure 1: Dynamics of populations

When applied to the components of the firm high job positions, the stationary conditions are then the following:

\[
\begin{align*}
j_U^i &= \bar{k} J_U^i + k''(1 - \bar{k}) J_U^i \quad (1) \\
j_O^i &= \bar{k} J_O^i + k''(1 - \bar{k}) J_O^i \quad (2) \\
e_A^i &= \bar{k} E_A^i \quad (3) \\
e_R^i &= \bar{k} E_R^i \quad (4) \\
e_P^i &= \bar{k} E_P^i \quad (5) \\
e_P^i &= k''(1 - \bar{k}) \bar{J}_i \quad (6)
\end{align*}
\]

These conditions are completed by the identities (7) and (8):

\[
\begin{align*}
J_O^i &= \bar{J}_i - J_U^i \quad (7) \\
E_R^i &= \bar{E}_i - E_A^i - E_P^i \quad (8)
\end{align*}
\]

### 3.2 Heterogeneity of recruitment channels

The five recruitment channels analyzed in this paper are differentiated by three main characteristics. More precisely, the productivity of workers, their wage and the hiring costs supported by firms will differ from one channel to another.

**Productivity of juniors.** Given their position, juniors are less productive than top executives; otherwise they would have been promoted. The productivity of juniors is
given by $\bar{q}^U$ and $q^O$ (with $\bar{q}^U \geq \bar{q}^O > 0$), according to the channel they have been hired (TNU or Other channels).

**Productivity of executives.** The productivity of executives averages the individual productivities and integrates additionally the synergies between them. These synergies actually arise and improve the productivity of the firm when the components of a team are sufficiently different and complementary. The objective of the firm is then to combine hiring channels to search for both the best productive workers and the most complementary ones, so finally to search for the most productive and consistent workforce.\(^5\)

**Individual Productivity.** The individual average productivity of executives who have been recruited through private Agencies or who have been promoted are given and respectively described by $\bar{q}^A$ and $q^P$. The individual average productivity of executives who have been recruited through employee Referrals is expressed by a function denoted $q^R(.)$. This function associates an exogenous component $\bar{q}^R$ and an endogenous component related to the TNU network. We assume that the TNU network helps to recruit productive workers through employee Referrals if (i) the size of this network is sufficiently wide (since it becomes easier to recruit a worker originally from TNU and therefore a productive one) and (ii) if capacity of the firm to access to the TNU channel is sufficiently high.

(i) The size of the network is expressed by the total number $N^U$ of juniors on the job originally from TNU employed in firms (and not the pool of graduates from TPU). Only these employees are supposed to give a professional network to the firm because they are those who have been hired as a result of the partnership between firms and TNU. We assume that the higher is the total number $N^U$ of juniors on the job originally from TNU and the higher is the productivity of agents hired through employee Referrals but less than proportionally such that $\delta q^R(.)/\delta N^U > 0$ and $\delta^2 q^R(.)/\delta N^U^2 < 0$.

(ii) The idiosyncratic capacity of the firm to access to this network is an increasing function of the number $E^P_i$ of promoted executives and the amount $J^U_i$ of the juniors originally from TNU in the firm $i$. This last relation is less than proportional, due to the overlaps of individual contributions to the extension of the firm points of access to the TNU network. The number $E^R_i$ of executives hired through employee Referrals impacts also positively on the firm capacity to access to the TNU network since they have been referred by promoted executives originally from TNU. We capture here a self reinforcing effect: if the firm recruits juniors through TNU and promotes them, the more employee Referrals channel is used, the greater is the firm capacity to access to the TNU network and the higher is the productivity of executives hired through this channel.

**Synergies.** Executives recruited through private Agencies are likely to develop synergies with all other members of the executive team, given that employment agencies are especially used by firms to find complementary individualities linked to the heterogeneity of their professional experience or their education. We assume that executives hired through employee referrals also develop synergies with the promoted ones, but only with those who do not come from the TNU channel. The additional productivity term associated with these synergies can be evaluated to the function $f(E^A_i, E^P_i, E^R_i, J^U_i)$.

\(^5\)We suppose that given their job position, juniors do not interact each other as much as top executives and then do not develop synergies with their peers. Adding this kind of synergies will have no other effects than reducing the proportion of corner solutions in $J^U_i$ and $J^O_i$. 

7
Wages. The wages $\bar{w}^A$, $\bar{w}^R$, $\bar{w}^P$ and $\bar{w}^O$ relative to the different recruitment channels are considered as given and such that $\bar{w}^O \leq \inf\{\bar{w}^A, \bar{w}^R, \bar{w}^P\}$ to account for the two types of positions in the firm. The wage paid to TNU graduates is rather different since it should be linked to the size of the TPU network $N^U$. This relation between $w^U$ and $N^U$ is then expressed by an endogenous function denoted $w^U(N^U)$ which is common knowledge. More precisely, a high value of $N^U$ means that TNU graduates are searched by firms, the competition of which making an upward pressure of wages. Formally, $w^U(N^U)$ is a two times continuous function and increases at a growing rate with the number of the juniors in activity originally from the TNU channel such that $\frac{\delta w^U(N^U)}{\delta N^U} > 0$ and $\frac{\delta^2 w^U(N^U)}{\delta N^U^2} > 0$. In other words, the more TNU graduates are employed in firms and the higher is the wage offered by firms to hire the remaining ones. Besides, we suppose that given their top-notch level of education, new graduates always get a better wage that other workers, i.e. that $w^U(N^U=0) > \bar{w}^O$. This difference of wages matters and involves that juniors from TNU channel could have a lower "net productivity" than those from other channels if $\bar{q}^U - w^U < \bar{q}^O - \bar{w}^O$. In this first case, firms recruit new graduates only to enter in the TNU network which may improve the efficiency of future executives hiring. In the other case i.e. if $\bar{q}^U - w^U \geq \bar{q}^O - \bar{w}^O$, firms always invest in TNU channel given the high productivity of juniors and not necessary for a long term strategy on the executives recruitment. To exclude trivial results, we thus suppose that at least for some firms $\bar{q}^U - w^U < \bar{q}^O - \bar{w}^O$ holds.

Hiring costs. The access to the TNU new graduates job market, firms have to invest annually in “social links” incurring donations but also opportunity costs (including for instance time spent to give some courses in the TNU or to supervise students in firms). In the model, firms support an annual fixed cost $\bar{c}^U$ which is assumed to be the average cost supported by firms, to enter in this job market.

The services of Agencies are charged on the basis of a variable cost denoted $c^A(e^A_i)$, proportional to the use of the private Agencies channel $e^A_i$ recruitment by the firm. We have supposed these services proportional to the recruitment wage of the executives hired by this channel. This cost given by $a \bar{w}^A$ where $a$ is a positive parameter.

Promotion, employee Referrals and Other channels are supposed free from both fixed and variable hiring costs\(^6\).

3.3 Profit maximization

The objective of the firm is to maximize its intertemporal profit given by:

$$\lim_{T \to +\infty} \left[ \pi^0_i + \sum_{t=1}^{T} \pi_t^i (1 + r)^{-t} \right]$$

where $\pi^i_t$ figures the annual profit of firm $i$ at time $t$.

\(^6\)This assumption could be less obvious in the case of Other channels. In fact, we made this simplification even if hiring costs exist for this channel since they are too low compared to those of TNU and private Agencies channels.
At stationary equilibria, i.e., the undischon terms of the intertemporal profits are all identical and the stationary solutions for the intertemporal profit are also the solutions of the current profit maximization. From the equality conditions (5) to (8), the continuous control variables of the firm \( \{ J^U_i, J^O_i, E^A_i, E^R_i, E^P_i, k^m \} \) are reduced to \( \{ J^U_i, E^A_i, E^P_i \} \). Under the assumption of stationarity, the optimal profit is then represented by:

\[
\pi^*_i = \sup \left[ \max_{J^U_i, E^A_i, E^P_i} \pi_i(\phi = 0), \max_{J^U_i, E^A_i, E^P_i} \pi_i(\phi = 1) \right] \tag{9}
\]

where \( \phi = 0 \) expresses that the firm does not invest in TNU (\( \bar{c}^U_i = 0 \)) and then could not hired juniors from TNU (\( J^U_i^* = 0 \)); \( \pi_i \) is the profit of the firm \( i \) with,

\[
\pi_i = \ E^A_i \left[ \bar{q}^A - \bar{w}^A \right] - c^A(e^A_i) + E^P_i \left[ \bar{q}^P - \bar{w}^P \right] + E^R_i \left[ q^R(J^U_i, E^R_i, E^P_i, N^U) - \bar{w}^R \right] + f(E^A_i, E^P_i, E^R_i, J^U_i) + \phi \left[ J^U_i \left[ \bar{q}^U - w^U(N^U) \right] - \bar{c}^U \right] + J^O_i \left[ \bar{q}^O - \bar{w}^O \right] \tag{10}
\]

The structure of the objective function (9), the form of the equality and inequality constraints and the properties of function satisfy the continuity and compactness conditions sufficient to obtain for each admissible values of the parameters and \( N^U \), a set of solution variables \( J^U_i^*, J^O_i^*, E^A_i^*, E^R_i^*, E^P_i^*, k^m \) and \( \phi^* \) associated with \( \pi_i^* \). We exclud in the rest of this paper the singular cases of vanishing measure where this set is not unique. We then deduce from this set and from conditions (1) to (8) the optimal annual levels of recruitment or promotion \( J^U_i^*, J^O_i^*, e^A_i^*, e^R_i^* \) and \( e^P_i^* \) of the firm \( i \).

### 3.4 A specification of the model

The profit function presented above can be specified in order to analyze the optimal choices of the firm. We propose to use particular functions for \( q^R(\cdot), w^U(\cdot), c^A(\cdot) \) and \( f(\cdot) \), consistent with the derivative properties defined above.

Let the individual average productivity of executives who have been recruited through employee Referrals be described by:

\[
q^R(J^U_i, E^R_i, E^P_i, N^U) = \bar{q}^R + b N^{U(\frac{1}{2})} \left( \frac{E^P_i + E^R_i E^P_i}{E^R_i} \right) \left( \frac{2 \bar{J}_i - (J^U_i)^2}{(J^U_i)^2} \right)
\]

where \( b \) is the index of quality of the TNU network such that \( b > 0 \). The last factor of the second right hand term expresses that the more the firm hires juniors from TNU and the higher is the efficiency of the employee Referrals channel. This factor is related to the quality of the TNU network \( b N^{U(\frac{1}{2})} \), to the number of promoted workers in the firm \( E^P_i \) and to the number of executives from TNU hired through employee Referrals thanks to the promoted executives from TNU \( E^P_i E^R_i / E_i \).
A simple possible specification of the recruitment wage of the new graduates from TNU is depicted by:

\[ w^U(N^U) = \bar{w}^U + d(N^U)^2 \]

where \( d \) is the strength of the endogenous part of wages for the TNU channel such that \( d > 0 \).

The hiring cost of the private Agencies channel can be simply expressed as follows:

\[ c^A(e^A) = a \bar{w}^A e^A \]

where \( 0 < a < 0.5 \) since evidence shows that firms pay between 15% and 25% of the gross annual wage to private agencies and around 40% to head hunters.

Finally, the synergies developed by top executive teams can be expressed as:

\[ f(E^A_i, E^P_i, E^R_i, J^U_i) = \alpha E^A_i(\bar{q}^A_i) + \beta E^A_i E^P_i(\bar{q}^P_i) + \gamma E^A_i E^R_i(\bar{q}^R_i) \]

\[ + \theta \left( \frac{\bar{J}^U_i - \phi J^U_i}{\bar{J}_i} \right) E^R_i E^P_i(\bar{q}^P_i) \]

where \( \alpha, \beta, \gamma \) and \( \theta \) are positive parameters representing the strength of synergies between executives. The three first terms depict the synergies among executives hired through private Agencies and the other types of workers (other executives hired through private Agencies, promoted workers and those hired through employee Referrals). The fourth term expresses the synergies between promoted workers and executives hired through employee Referrals. But if all juniors have been recruited through TNU, there cannot be synergies anymore since the referrals of the promoted executives would be also executives from TNU. This is captured by the factor \((\bar{J}_i - \phi J^U_i)/\bar{J}_i\).

This specification provides an analytical solution to the model. We have selected numerical examples to illustrate possible outcomes associated to the optimal choices of the firm.

**Example 1.**

Agents’ productivity: \( \bar{q}^A = 4, \bar{q}^P = 3.5, \bar{q}^R = 2, \bar{q}^U = 2, \bar{q}^O = 2 \)

Wages: \( \bar{w}^A = \bar{w}^P = \bar{w}^R = 2, \bar{w}^U = 1, d = 0.00002, \bar{w}^O = 0.95 \)

Hiring costs: \( c^U = 2, a = 0.4 \)

Synergies: \( \alpha = 2, \beta = 2.5, \gamma = 2.5, \theta = 2 \)

Size: \( \bar{J}_i = 10, E^A_i = 40 \)

Exit rate: \( \bar{k} = 0.3, \bar{k}' = 0.25 \)

The index of quality of TNU network: \( b = 0.023 \)

The number of juniors from TNU on the job: \( N^U = 50 \)

According to these parameters, the optimal values of the level of recruitment or promotion maximizing the firm profits are resumed in Table 1.

With these parameters, the firm does not invest in TNU since the firm profit associated is lower than the one without it, \( i.e. \pi^*_i(\phi = 0) > \pi^*_i(\phi = 1) \). This choice has obvious
Table 1: Optimal values of the level of recruitment and promotion

<table>
<thead>
<tr>
<th>Annual amount of recruitment employed in the firm through the channel X</th>
<th>The number of insiders employed in the firm through the channel X</th>
</tr>
</thead>
<tbody>
<tr>
<td>private Agencies</td>
<td>$e^{A^*} = 7.86$</td>
</tr>
<tr>
<td>Promotion</td>
<td>$e^{P^*} = 1.65$</td>
</tr>
<tr>
<td>employee Referrals</td>
<td>$e^{R^*} = 0.49$</td>
</tr>
<tr>
<td>TNU</td>
<td>$j^{U^*} = 0$</td>
</tr>
<tr>
<td>Others</td>
<td>$j^{O^*} = 4.65$</td>
</tr>
</tbody>
</table>

an impact on the recruitment of juniors but also on the recruitment of executives. Since there is no agent originally from TNU among juniors, those who will be promoted do not hold the TNU network. Then, the firm hires only few executives from the employee Referrals channel which is less efficient in such case. The optimal strategy of the firm returns to activate mainly the private Agencies channel which allows important synergies among executives and high level of individual productivity.

This result may be explained by the low the number of juniors from TNU on the job ($N^U$). Since the juniors from TNU channels are costlier than those from other channels (here we have $q^U - w^U < q^O - w^O$), the firm chooses to invest in this channel only if the improvement on the recruitment of executives is rather significant. Another explanation can be that the size of the firm ($\bar{J}_i$ and $\bar{E}_i$) is too small in relation to the fixed cost of the TNU channel $c^U$.

In order to verify these intuitions, let us consider the same values of the parameters, first associated with a higher value of the number of juniors from TNU on the job, then with a larger firm size.

**Example 2.**

The number of juniors from TNU on the job: $N^U = 250$

The impact of this change on the optimal values of the levels of recruitment and promotion is exposed in Table 2. Table 2 emphasizes the relevance of the TNU channel (here

<table>
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<tr>
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</tr>
<tr>
<td>employee Referrals</td>
<td>$e^{R^*} = 4.58$</td>
</tr>
<tr>
<td>TNU</td>
<td>$j^{U^*} = 6.45$</td>
</tr>
<tr>
<td>Others</td>
<td>$j^{O^*} = 1.42$</td>
</tr>
</tbody>
</table>

$\pi_i^*(\phi = 1) > \pi_i^*(\phi = 0)$). With a higher number of juniors from TNU on the job, the
efficiency of the TNU network arises and involves a higher expected profit if the firm invests in this channel. As a result, the firm massively recruits through TNU and increases for executives the part of recruitment through employee Referrals. However, given the role of synergies between executives from heterogeneous origins, the employee Referrals channel does not become the only hiring channel for the firm. Private Agencies are still activated but less used. Note also that even if a higher number $N^U$ encourage firms to invest in TNU (via the improvement of the productivity of executives who will be hired through employee Referrals), a too large success of TNU discourages the firm to invest because of the upward pressure it creates on wages. Without any changes of the other values of the parameters of example 2, one can verify that if $N^U = 500$, firms do not invest and the optimal values of the control variables return to those in Example 1.

**Example 3.**
The number of juniors from TNU on the job: $N^U = 50$

Size: $\bar{J}_i = 20$, $\bar{E}_i = 80$

The optimal values of the levels of recruitment and promotion of a larger firm are resumed in Table 3. According to the Table 3, the size of the firms has a relevant effect on the composition of the juniors and executives teams: larger is the firm, smaller is proportionally the weight of fixed recruitment costs of access to the TNU job market. All things equal, large firms have then more potentiality to optimize the use of employee Referrals.

![Table 3: Optimal values of the level of recruitment and promotion with a larger firm](image)

<table>
<thead>
<tr>
<th>Channel</th>
<th>Annual amount of recruitment through the channel $X$</th>
<th>The number of insiders employed in the firm through the channel $X$</th>
</tr>
</thead>
<tbody>
<tr>
<td>private Agencies</td>
<td>$e^{A*} = 0.47$</td>
<td>$E^{A*} = 1.9$</td>
</tr>
<tr>
<td>Promotion</td>
<td>$e^{P*} = 9.66$</td>
<td>$E^{P*} = 38.6$</td>
</tr>
<tr>
<td>employee Referrals</td>
<td>$e^{R*} = 9.87$</td>
<td>$E^{R*} = 39.5$</td>
</tr>
<tr>
<td>TNU</td>
<td>$j^{U*} = 14.64$</td>
<td>$J^{U*} = 18.7$</td>
</tr>
<tr>
<td>Others</td>
<td>$j^{O*} = 1.01$</td>
<td>$J^{O*} = 1.3$</td>
</tr>
</tbody>
</table>

Other factors have also a great influence on the recruitment decisions and the profit of a single firm: the net productivity of the employees related to channel they have been hired has a crucial role and the synergies explain why corner solutions are generally not optimal.

The last examples of this section have illustrated how much the size of the TNU network is important to determine the nature of the recruitment of a given firm, the origin if its executives and its investment in the education system. The size of the TNU network that we have considered as given is however the result of interacting decisions made by the firms. Given the general recruitment policy, each single firm determines its own recruitment policy whereas the general recruitment policy is the consequence of the aggregation of individual ones.
4 Equilibrium and coordination with several firms

When the labor market is considered as a whole, the number of employed juniors $N_U$ from TNU becomes endogenous. This number depends on the firms’ individual decisions of recruitment $J_i^U$ (with $i = 1, 2, \ldots, n$), and those last depend conversely on the expectation by individual firms of the TPU network size $\tilde{N}_U$ (since $\tilde{N}_U$ appears in the determination of $J_i^U$ through functions $q_i^R(\tilde{N}_U)$ and $w_i^U(\tilde{N}_U)$). The stationary “rational expectation” equilibria are then determined by the coincidence between the expectations $\tilde{N}_U$ used by firms concerning the extent and efficiency of the network and the actual size $N_U$ of this network, determined by the optimal choices of the firms. To analyze these equilibria, we first consider the firms’ expectations, choices and profits when firms are homogeneous; we then extend the analysis to some cases of heterogeneity.

4.1 Homogeneous firms

If there exist $n$ homogeneous firms, the size of the network $N_U$ is obtained by a multiple of the optimal level of juniors from TNU employed by one single firm:

$$N_U^* = \sum_{i=1}^{n} J_i^U(\tilde{N}_U) = n J_i^U(\tilde{N}_U)$$

(11)

The convexity of function $w_i^U(\tilde{N}_U)$ and the concavity of function $q_i^R(\tilde{N}_U)$ are not sufficient to determine the sense of the relation between $\tilde{N}_U$ and $N_U$ expressed by equation (11). Properties of functions $w_i^U(\tilde{N}_U)$ and $q_i^R(\tilde{N}_U)$ give however information on the relation between $\tilde{N}_U$ and $N_U$. A small expected value $\tilde{N}_U$ means that firms expect a small TNU network size. Then, the productivity of the executives recruited through employee Referrals $q_i^R(\tilde{N}_U)$ remains low. Therefore, recruiting TNU graduates to benefit from their ‘small’ network is not an efficient strategy and the effective TNU network size $N_U^*$ is also small. For higher values of $\tilde{N}_U$, the positive effect on $q_i^R(\tilde{N}_U)$ leads firms to recruit TNU juniors in order to use their network in the subsequent recruitment of executives. Then, the effective TNU network size $N_U^*$ increases. But, when firms expect a too high value of $\tilde{N}_U$, they also expect an upward pressure of wages resulting on the tightness of the TNU new graduates job market. Indeed, the convexity of $w_i^U(\tilde{N}_U)$ and the concavity of function $q_i^R(\tilde{N}_U)$ involve that the negative effect on $w_i^U(\tilde{N}_U)$ dominates the positive effect on $q_i^R(\tilde{N}_U)$. Then, firms could not recruit these “expensive” TNU graduates and the effective TNU network size $N_U^*$ is small.

According to the values of the parameters, four possible cases of stationary equilibria associated with different recruitment channels choices may exist. Figure ?? depicts these cases (the values of the parameters chosen in Figure ?? are the same than those used in the above numerical examples except for $b$, $d$, $q^U$ and $q^O$ which differ from a case to another).

Figure 2a and 2b depict cases where the function $N_U(\tilde{N}_U)$ is increasing\(^7\) in $\tilde{N}_U$. With Figure 2a there exist one single stable equilibrium and with Figure 2b three equilibria (but only two stable ones). The case with two equilibria is not depicted since it is a limit case. In these two cases, the equilibrium $\tilde{N}_U = N_U = 0$ is obtained: when firms expect that

\(^7\)The relation between $\tilde{N}_U$ and $N_U$ could be increasing only if $q^U - w^U < q^O - \bar{w}^O$. 

13
no juniors will be recruited through TNU channel, they do not use this channel. At this
equilibrium, firms recruit juniors only through Other channels and essentially use private Agencies to recruit executives. In Figure 2b, the increasing relation between $\hat{N}^U$ and $N^U$ leads to the occurrence of a second stable equilibrium where optimal recruitment choices are not the same anymore. At this second equilibrium, firms activate the TNU channel and essentially recruit juniors through this channel while the recruitment executives is done through Promotion and employee Referrals.

In figures 2c and 2d, we have chosen values of $\bar{q}^U$ and $\bar{q}^O$ such that $\bar{q}^U > \bar{q}^O$ involving that $N^U(\hat{N}^U)$ is a non increasing function anymore. Figure 2c represents a decreasing function where firms invest when they expect $\hat{N}^U = 0$ but the pressure on wages tends to decrease the number of juniors hired through TNU for high values of $\hat{N}^U$. Finally, figure 2d depicts a particular case where firms hire all their juniors through TNU channel whatever the value of $\hat{N}^U$. In these two later cases, the equilibrium is unique and associated with the same recruitment policy: firms recruit essentially juniors through TNU one the one hand and executives through Promotion and employee Referrals on the other hand.

As a shortcut of the analysis of these different cases, the following proposition can be proved:

**Proposition 1** When firms are homogeneous and when there exist multiple rational expectation stationary equilibria, multiplicity generates possible coordination failures but no conflicts of interest among firms.

*Proof.* see Appendix 1.

The coordination failures correspond to cases where multiple equilibria are Pareto rankable. In our case, from one equilibrium to another, firms level of profits increase with the number of juniors originally from TNU employed in the market $N^U$.

When the net productivity of juniors recruited through TNU is smaller than the one of those recruited through Other channels, and when firms expect a low value of $\hat{N}^U$, they choose to recruit no agents from TNU. Indeed, firms recruit this kind of workers only if they could help to recruit high productive executives more easily, which is not the case when firms expect a small TNU network size. This decision induces low levels of profit for these firms.

Conversely, when firms expect a high value of $\hat{N}^U$, they finally decide to hire a large amount of graduates from TNU. This creates higher levels of profit due to the positive impact of the workers from TNU on future recruitments. This multiplicity can cause problems of coordination (firms can separately expect different stationary equilibria). However, whatever the equilibrium on which they coordinate, all firms rank in the same way the two possible equilibria and should coordinate on the “high equilibrium”. In this sense, there is no conflict of interest among firms.

### 4.2 Heterogeneous firms

Different forms of heterogeneity can be considered among firms. The technological differentiation creates a source of heterogeneity with consequences on productivity parameters

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8The profit is systematically higher when firm choose to use TNU channel since this decision comes from an optimal choice (see equation (9)).
(parameters associated with the strength of synergies or with the individual average productivity). Other sources of heterogeneity are the size of the firms \((\bar{J}_i, \bar{E}_i)\) and the employee turnover rate. In this paper, we will only focus on two relevant sources of heterogeneity: the size of the firm and the productivity of TNU graduates, this last being captured by the level of the parameter \(\bar{q}_U\). We have however tested other and more complete sources of heterogeneity. These tests do not contradict the main result of this point.

**Firms heterogeneous by their size.** Consider first the case where only the size of the firms differs but not the internal distribution or their employees between juniors and executives. Equations (1) to (10) have to be written differently for the small firms the size of which is given by \((\bar{J}_i, \bar{E}_i) = (\bar{J}_S, \bar{E}_S)\), \(\forall i (i = 1, 2, \ldots, n_S)\), and for the large firms the size of which is given by \((\bar{J}_l, \bar{E}_l) = (\bar{J}_L, \bar{E}_L)\), \(\forall l (l = 1, 2, \ldots, n_L)\) with \(\bar{J}_L/\bar{J}_S = \bar{E}_L/\bar{E}_S = \mu\) where \(\mu > 1\). The equivalent to the equilibrium condition (11) is then (12):

\[
N^U = n_S J_i^U(\tilde{N}^U) + n_L J_l^U(\tilde{N}^U)
\]

As with homogeneous firms, there exist in this case unique or multiple equilibria. Two cases can however occur: (i) if hiring through TNU channel is profitable for juniors \((\bar{q}_U - w_U > \bar{q}_O - \bar{w}_O)\), the equilibrium is always unique (as in Figure 2c and 2d); (ii) if not \((\bar{q}_U - w_U < \bar{q}_O - \bar{w}_O)\), the equilibrium is unique or not (as in Figure 2a and 2b).

In case (i), both small and large firms invest in TNU channel and hire graduates for the same particular level of \(\tilde{N}^U\), i.e. \(\tilde{N}^U = 0\) if the cost incurred the TNU channel \(\bar{c}_U\) is quite low, otherwise only large or no firms use the TNU channel.

In case (ii), the values of \(\tilde{N}^U\) from which firms decide to hire TNU graduates is different according to their size. Small firms never invest in the TNU channel from lower values of \(\tilde{N}^U\) than large ones do, since they get lower returns on the fixed cost incurred by this channel. In the same way, small firms never invest for higher values of \(\tilde{N}^U\) than large ones since the negative effect of the upward pressure of wages is more costly for them. In other words, the heterogeneity of firm’s size impacts on the range of the values of \(\tilde{N}^U\) for which recruiting graduates is profitable. The range of the values of \(\tilde{N}^U\) for which small firms invest in TNU is never broader than the range of investment of large firms.

When we consider the ranking of the equilibria in the case of multiplicity, the analysis is the same that with homogeneous firms. Whatever their size, firms always decide to hire through TNU channel when their profits increase with \(\tilde{N}^U\). This decision is then a consequence of the positive effect associated with \(q_i^R(\tilde{N}^U)\) on the recruitment of executives. When the profit decreases, it is the consequence of the wages of juniors via \(w_U(\tilde{N}^U)\) which ends up over-compensating the positive effects of productivity. When this decrease occurs, firms do no longer activate the TNU network. In this context, small and large firms do rank equilibria rather in the same way. Figure ?? depicts the shape of the curve \(N_i^U(\tilde{N}^U)\) where multiple stationary equilibria occur, and the shape of the profit’s curve for small firms \(\pi_i^U(\tilde{N}^U)\) and large firms \(\pi_l^U(\tilde{N}^U)\) when parameters are such that both all firms decide to hire through TNU channel. It is then easy to verify that there can be at most two or three stable equilibria according to the values of the parameters. Let us describe these two situations:
When the value of the parameters involve two stable equilibria, the first is always a “low” equilibrium such that $\tilde{N}^U = 0$ where both small and large firms do not recruit through TNU channel. The other is a “high” equilibrium such that $\tilde{N}^U > 0$ where either both small and large firms recruit new graduates (this equilibrium is then associated with a larger profit compared to the low equilibrium for all firms) or only large firms (this equilibrium gives then the same profit for small firms that the low equilibrium but a highest profit for the large ones) or no firms at all (the two equilibria give the same profit for all firms).

When the value of the parameters involve three stable equilibria, the first is always a “low” equilibrium where both small and large firms do not recruit through TNU channel. The second is a “medium” equilibrium where only large firms invest in the TNU channel and the third is a “high” equilibrium where either both small and large firms or only large firms recruit new graduates. Suppose that the medium equilibrium is associated with the highest profit for large firms. Such case only occurs if small firms invest in TNU channel for values of $\tilde{N}^U$ corresponding to decreasing profits for large firms due to the negative effect of $w^U(\tilde{N}^U)$. Yet, if the level of $w^U(\tilde{N}^U)$ decreases the profit of large firms, it obviously decreases the profit of small ones that contradicts our assumption. We then conclude that the “high” equilibrium is also greater than the second one for large firms. For small firms, either all equilibria provide the same profit or the “high” equilibrium gives a greater profit. Once again there cannot be any conflict of interest among firms.

Firms heterogeneous by their size and technology. A more conclusive case appears when an heterogeneity on $\bar{q}^U$ is added to the heterogeneity on the populations sizes (see Figure ??). Suppose for instance that $\bar{q}^U_l > \bar{q}^U_i$, i.e. that the individual average productivity of TNU graduates is higher for small firms than for large ones. This case could correspond to the coexistence between large “industrial” firms with a quite low level of productivity of high skilled labor and rather small firms of the sectors of services with a higher level of productivity of the same competences. Another interpretation of such heterogeneity on technologies could be that the education instilled by TNU is more well-adapted to small firms than large ones for instance. In such a setting we can then prove the following proposition:

**Proposition 2** In case of multiplicity, when small firms technology is more adapted to top-notch Universities initial training than large firms technology, there may exist conflicts of interest between small and large firms.

**Proof.** see Appendix 2.

Figure ?? illustrates one possible case with 3 stationary equilibria (with 2 stable ones). This figure could be divided into three parts according to the level of $\tilde{N}^U$. From $\tilde{N}^U = 0$ to $\tilde{N}^U = 78$, small firms recruit all graduates from TNU to fill junior positions due to their high level of productivity. From $\tilde{N}^U = 78$ to $\tilde{N}^U = 118$, small firms still hire only graduates from TNU and large firms hire a significant proportion of graduates from TNU given their positive impact on future executive hiring. From $\tilde{N}^U = 118$ to $\tilde{N}^U = 210$...
(this latter proportion being the total number of junior positions to fill), only large still recruit through TNU channel.

The first stable equilibrium occurs in the first part of the figure, i.e. when only small firms invest in TNU channel. For these small values of $N^U$, small firms have higher profit than in the reservation case whereas large firms get the reservation one. The second stable equilibrium occurring in the third part of the figure gives opposite results on profits. While small firms do not hire through TNU channel and get a reservation profit, large firms which recruit graduates from TNU, get a greater profit. This illustrates how and why there can exist conflicts of interest between small and large firms.

4.3 Welfare analysis

The possible occurrence of conflicts of interest among firms motivates a welfare comparison of equilibria and an exploration of the adapted second bests solutions. In this setting, the relevant welfare index is the sum of the profits of firms increased by a term relative to TNU. The index of utility $u$ of TNU has two components. The first one is a positive multiple of the total annual fixed cost $\bar{c}^U$ supported by the $n$ firms to access to these Universities. The second component is related to the firms payroll. It could be natural to suppose that tuition fees depend on the employment rate and starting wage of graduates after having completed their education. Then, the utility of TNU could be written as follows:

$$u = \lambda (n_S + n_L)\bar{c}^U + g(N^U, w^U)$$  \hspace{1cm} (13)

where $g$ is a continuous function such that $\delta g/\delta N^U > 0$ and $\delta g/\delta w^U > 0$ since the TNU interest is to have the largest possible level of recruitment at the higher wage.

Then, we are able to describe the economic welfare of the recruitment system:

$$W = n_S \pi_i + n_L \pi_l + u$$  \hspace{1cm} (14)

When firms are heterogeneous by their size and technologies, there can be at most two stable equilibria (see Appendix 2) that we denote equilibrium 1 and equilibrium 2 (hereafter $W_1$ and $W_2$). We compare the profit of small firms at $W_1$ and $W_2$. We do the same for the large firms and the TNU.

For small firms, $W_1$ (associated with a small TNU network size) always involves high level of profit since the new graduates hired are endowed of a high productivity and paid at a rather small starting wage. At the second stable $W_2$(associated with a higher TNU network size), their profit is smaller, due to the negative effects of the upward pressure on wages. We express the losses from $W_2$ to $W_1$ by $\Delta \pi_i = (\pi_i)_2 - (\pi_i)_1$.

For large firms, $W_1$ and $W_2$ are ranked in an opposite order. $W_2$ is always more profitable than $W_1$ since increasing the network size makes more efficient the recruitment of executives. The extra-profit from $W_2$ to $W_1$ is then such that $\Delta \pi_l = (\pi_l)_2 - (\pi_l)_1$.

From TNU side, it is less immediate to rank equilibria because from one equilibrium to the other since the two components of the utility function can move in opposite directions. The first component is related to the number of firms making donations. All things equal,

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9The whole annual fixed costs is not received by TNU since these costs include donations but also opportunity costs.
if there are more small firms than large firms, this first component is greater with \( W_1 \) than with \( W_2 \). Conversely otherwise (i.e. if large firms are the most numerous, this first component is greater with \( W_2 \) than with \( W_1 \)). The second component depends positively on the size of the TNU network and on the level of wages (which depends also positively on the size of the TNU network). \( W_2 \) is then always associated with the highest value of the second component of the TNU network. In summary, TNU do not always rank equilibria in the same way. According to the relative number and weight of the small and large firms, TNU may have conflicts of interest either with small firms or with large firms. A sensible case emerges when the small firms use TNU channel only at \( W_1 \).

**Proposition 3** In case of conflicts of interest and when small firms use TNU channel only at \( W_1 \), if there are more small than large firms \( n_S > n_L \) and if the spread of income from firms donations is higher than the one of tuition fees \( \lambda \bar{c}^U (n_S - n_L) > g_2(N^U, w^U) - g_1(N^U, w^U) \), there are conflicts of interest between large firms and top-notch Universities. If these two conditions are not fulfilled, conflicts of interest emerge between small firms and top-notch Universities.

*Proof.* see Appendix 3.

Figure ?? depicts conflicts of interest between firms and TNU. To describe the utility of TNU, we have chosen the following specification: \( u = \lambda (n_S + n_L)c^U + \mu N^U w^U \). On the one hand, we have used the same parameters than in Figure ???. Adding TNU utility function to Figure ??, gives Figure 5a. For these values of parameters, we have \( n_S > n_L \) and \( \lambda \bar{c}^U (n_S - n_L) < g_2(N^U, w^U) - g_1(N^U, w^U) \). Figure 5a then emphasizes conflicts of interest between small firms and TNU. On the other hand, we have chosen in the example of Figure 5b values of parameters such that conflicts of interest emerge between large firms and TNU, i.e. \( n_S > n_L \) and \( \lambda \bar{c}^U (n_S - n_L) > g_2(N^U, w^U) - g_1(N^U, w^U) \).

The difference of utility from \( W_2 \) to \( W_1 \) is such that \( \Delta u = (u)_2 - (u)_1 \) expressing extra-utility when \( \Delta u > 0 \) and losses of utility when \( \Delta u < 0 \).

Finally, four scenarios occur:

**When \( \Delta u > 0 \)**
- **scenario i**: \( \Delta \pi_l + \Delta u < \Delta \pi_i \implies W_1 \) is the “high equilibrium”
- **scenario ii**: \( \Delta \pi_l + \Delta u > \Delta \pi_i \implies W_2 \) is the “high equilibrium”

**When \( \Delta u < 0 \)**
- **scenario iii**: \( \Delta \pi_l < \Delta \pi_i + \Delta u \implies W_1 \) is the “high equilibrium”
- **scenario iv**: \( \Delta \pi_l > \Delta \pi_i + \Delta u \implies W_2 \) is the “high equilibrium”

The government could subsidize the recruitment of TNU graduates in such a way that the “high” equilibrium is reached.

Consider the **scenario i**. \( W_1 \) is the “high equilibrium” since the extra-profit of small firms is higher than the losses of large firms and TNU. To move the recruitment system...
towards this equilibrium, the government should compensate large firms which are not able to recruit TNU students due to the small size of the network but also TNU which do not receive sufficiently high tuition fees. For that, a part of the small firms extra-profit $\delta_S$ could be redistributed to large firms and TNU such that: $\delta_S = \Delta \pi_l + \Delta u$.

Consider now the scenario ii. $W_2$ is the “high equilibrium”. Here, small firms need compensation since they are not able to recruit TNU students anymore (who are yet very productive in these firms) due to the high level of wages. Then, a part of the large firms extra-profit and TNU extra-utility $\delta_L$ and $\delta_{TNU}$ could be redistributed to small firms such that $\delta_L + \delta_{TNU} = \Delta \pi_i$.

The same conclusions could be done with two last scenarios. For scenario iii, a part of the small firms extra-profit and TNU extra-utility should be redistributed to large ones. Finally, if scenario iv occurs, a part of the large firms extra-profit could be redistributed to small firms and TNU.

5 Conclusion

The model we present in this paper analyzes in a microeconomic setting the choice of recruitment channels for high job positions. We study three recruitment channels actually used to hire high skilled workers: employee referrals, private employment agencies and internal promotion. This last channel leads to consider also the recruitment of “juniors” (high skilled workers less experienced) which can be made through traditional channels or directly through top-notch Universities. In the higher education system, these TNU are defined as prestigious institutions providing both high level of education and strong professional networks among graduates. In the model, this particularity involves connection between juniors hired through TNU and executives who will be hired through the employee Referrals channel. The efficiency of this channel is then made dependent to the size of the TNU network in the labor market.

When the recruitment choice is considered within an isolated firm, the determination of the optimal hiring decisions essentially depends on the firm’s technology, the size of the firm and the size of the top-notch Universities network. The model emphasizes two optimal recruitment policies. If the size of the firm and the size of the network are small, firms tend to activate essentially private Agencies to hire top executives and do not invest in TNU channel for juniors. If they are large, firms activate the TNU channel and hire essentially top executives through employee Referrals and Promotion.

When several firms are considered in a stationary equilibrium setting, the same recruitment policies are observed but the combination of channels generates multiple rational expectation equilibria. We point out the potential existence of coordination failures when firms are homogeneous or when the heterogeneity is quite limited. When heterogeneity of firms is more pronounced, conflicts of interest can emerge. Those conflicts may involve small and large firms but also top-notch Universities. Second best policies can then be implemented in order to improve the welfare properties of the market equilibria.

An extension of this paper could be to generalize the analysis of informal methods in the recruitment process. Different kinds of networks could indeed be considered in their coordinating contribution of hiring activities.
Appendix

Appendix 1: Proof of Proposition 1

Consider first the optimal choices of firms when $\hat{N}^U = 0$. Two cases can be distinguished: i) $N^U = \sum_{i=1}^n \hat{J}_i U^*_i(0) = 0$ and ii) $N^U = \sum_{i=1}^n \hat{J}_i U^*_i(0) = n \bar{J}_i > 0$.

In the case i) $N^U = 0$ is a stationary equilibrium with $\forall i, J_i^U = 0$ and then $\tilde{c}^U = 0$. Since firms are homogeneous, they all decide to invest or not in TNU channel for the same value of $\tilde{w}(\hat{N}^U)$. In all cases, the profit associated with the second stable equilibrium is always greater than the profit associated with the first one, i.e. $\pi_i^*(N^U) > \pi_i^*(0)$.

In case ii) given that firms are identical and since there is no special gain to expect from the promotion of juniors graduated from TNU when $\hat{N}^U = 0$ ($q^R(0) = q$), if TNU graduates are however recruited, this recruitment is only justified by the net productivity of these employees as juniors. Since there exist no synergies between juniors, the only possible solution are then $\forall i, J_i^U = 0$ or $\forall i, J_i^U = J_i^O$. Only the second remains in this case, i.e. $N^U = \sum_{i=1}^n J_i U^*_i(0) = n \bar{J}_i$. All firms then choose to recruit the greater possible number of TNU new graduates. In this stage, two possibilities exist. The first is that $\forall \tilde{N}^U, N^U = \sum_{i=1}^n J_i U^*_i(\tilde{N}^U) = n \bar{J}_i$ (see Figure 2d). The only stationary equilibrium is then $N^U = n \bar{J}_i$. The second is that $\exists \tilde{N}^U/N^U = \sum_{i=1}^n J_i U^*_i(\tilde{N}^U) < n \bar{J}_i$ (see Figure 2c). This possibility implies that the continuous function $N^U = \sum_{i=1}^n J_i U^*_i(\hat{N}^U)$ has a decreasing portion between $\tilde{N}^U = 0$ and $\hat{N}^U = n \bar{J}_i$. Given the form of $w^U(\tilde{N}^U)$, it is excluded that after a decreasing portion of the relation between $J_i U^*_i$ and $\hat{N}^U$, we observe an increasing one when $\tilde{N}^U$ increases. We conclude that the case ii) is a case of uniqueness

Appendix 2: Proof of Proposition 2

Suppose that the following conditions hold: c1) The profit of the small firms is a non increasing function of $N^U$ (i.e. $\tilde{q}^U - w^U \geq \tilde{q}^O - \tilde{w}(0)$), c2) when the large firms are excluded, there exists one single stationary equilibrium $N^U*$ such that $N^U* > 0$, c3) when the large firms are added, $N^U*$ is still an equilibrium and corresponds to a case where only the small firms invest in the TNU channel. If c2) holds, small firms invest in the TNU channel when $\hat{N}^U = 0$. This investment is then only motivated by the productivity $\tilde{q}_i^U$ of the juniors recruited by this channel: it can correspond or not to a corner solution, according to the weight of $\delta$ (i.e. the influence of the synergies between the promoted and recruited by the referral channel). However, since c1) also holds, when $\hat{N}^U$ increases, the positive effect of $\hat{N}^U$ on $\pi_i$ via $E_i^F$ is always dominated by its negative effect via $J_i^U$. With c3), we suppose that the technology of the large firms is not really adapted to the
recruitment of TNU initial training and that $n_LJ^U_1(0) = 0$. Since there exist multiple equilibria, the positive effects of $\tilde{N}^U$ on $\pi_i$ however ends up dominating the negative one for a value $\tilde{N}^U_1$ of $\tilde{N}^U$ larger than $N^U_1$. At this level, the large firms invest in the TNU, then increase (or not) their recruitment of juniors from TNU with subsequent increases of $\tilde{N}^U$. Still when $\tilde{N}^U$ increases over $N^U$, the profit of the small firms continues to decrease (and this decrease ends involving for a given value $\tilde{N}^U_2$ of $\tilde{N}^U$ larger than $N^U_2$, optimal recruitment levels of small firms such that $J^U_2(\tilde{N}^U_2) = 0$). Last, given the asymptotic behavior of $q^R(\tilde{N}^U)$ and $w^U(\tilde{N}^U)$, the profit $\pi_i$ begins to decrease for a value $\tilde{N}^U_3$ of $\tilde{N}^U$ larger than $\tilde{N}^U_1$ without however falling under $\pi_i(N^U_3)$ since, whatever $\tilde{N}^U$ the large firms have always the possibility to choose not investing in the TNU channel and to obtain $\pi_i(N^U_3)$. Despite there exist different possible ranking orders between $\tilde{N}^U_2$ and ($\tilde{N}^U_1$ and $\tilde{N}^U_3$), the consequence is that, for large values of $\tilde{N}^U$, $N^U(\tilde{N}^U)$ ends up increasing at a decreasing rate, then decreasing. Since there we have not supposed an upper limit for $\tilde{N}^U$, the last equilibrium is then always such that $N^U(\tilde{N}^U)$ crosses the 45° curve from upside to downside.

In case of multiplicity, i) since the first equilibrium $N^U_1$ is also such that $N^U(\tilde{N}^U)$ crosses the 45° curve from upside to downside, and ii) since $N^U(\tilde{N}^U)$ never increases at an increasing rate (except if there exist another categories of firms than small and large), there are always three equilibria with only two stable ones. Suppose that there exist three equilibria such that $N^U_1$, $N^U_2$ and $N^U_3$. The stable equilibria $N^U_1$ and $N^U_3$ are then such that $\pi_i(N^U_1) > \pi_i(N^U_3)$ and $\pi_i(N^U_1) < \pi_i(N^U_3)$ exhibiting a conflict of interest between small and large firms.

### Appendix 3: Proof of Proposition 3

Consider the case where firms are heterogeneous by their size and technologies. Appendix 2 shows that conflicts of interest between firms occur when there are two stable equilibria. More precisely, remind that this situation involves the following properties of equilibria. At equilibrium 1, only small firms recruit through TNU channel. At equilibrium 2, there are two possible cases: either only large firms recruit through TNU (case 1) or both small and large firms (case 2). Figure ?? depicts these two cases. Let us analyze these two cases from the TNU side.

If at equilibrium 2, both small and large firms invest in TNU channel, the utility associated with each equilibrium is such that $u_1 = \lambda(n_s)c^U + g_1(N^U, w^U)$ and $u_2 = \lambda(n_s + n_L)c^U + g_2(N^U, w^U)$ where $u_2 > u_1$ since $g(N^U, w^U)$ is an increasing function of $N^U$ and $w^U$ (involving $g_2(N^U, w^U) - g_1(N^U, w^U)$). Then, in this case there are always conflicts of interest between small firms and TNU.

If at equilibrium 2, only large firms invest in TNU channel, the levels of utility become $u_1 = \lambda(n_s)c^U + g_1(N^U, w^U)$ and $u_2 = \lambda(n_L)c^U + g_2(N^U, w^U)$. At this stage, one can see that if $n_L > n_s$, then $u_2 > u_1$ involving that once again conflicts of interest between small firms and TNU emerge. Conversely, if $n_L < n_s$, the equilibrium associated with the highest utility depends on the comparison between the spread of income from firms donations and the one of tuition fees. Thus, if $\lambda c^U(n_s - n_L) < g_2(N^U, w^U) - g_1(N^U, w^U)$, then there are conflicts of interest between small firms and TNU. Finally, it becomes obvious that conflicts of interest between large firms and TNU emerge only if $n_L < n_s$ and $\lambda c^U(n_s - n_L) < g_2(N^U, w^U) - g_1(N^U, w^U)$.
References


APEC (2007), La mobilité professionnelle des cadres, presse.apec.fr
Figure 2: Examples of stationary equilibria with homogeneous firms

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Figure 3: Example of stationary equilibria with firms heterogeneous by their size
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Figure 4: Conflicts of interest with firms heterogeneous by their size and technology
Figure 5a. Conflicts of interest between small firms and top-notch Universities

$b = 0.04; d = 12.10^{-6}; \bar{q}^U, q^O = 1.7; \bar{J}_S = 5; \bar{E}_S = 20; \bar{J}_L = 40; \bar{E}_L = 160; n_S = 10; n_L = 4; \lambda = 0.9; \mu = 0.5$

Figure 5b. Conflicts of interest between large firms and top-notch Universities

$b = 0.0027; d = 55.10^{-6}; \bar{q}^U, q^O = 1.5; \bar{J}_S = 4; \bar{E}_S = 16; \bar{J}_L = 50; \bar{E}_L = 200; n_S = 27; n_L = 3; \lambda = 0.9; \mu = 0.5$

Figure 5: Conflicts of interest between firms and top-notch Universities
Only large firms recruit through TNU at equilibrium 2

Both small and large firms recruit through TNU at equilibrium 2

Figure 6: Properties of equilibria when there are conflicts of interest between firms