INVOLUNTARY UNEMPLOYMENT AND EFFICIENCY-WAGE COMPETITION*

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Abstract

This paper introduces a model of efficiency-wage competition along the lines put forward by Hahn (1987). Specifically, I analyze a two-firm economy in which employers screen their workforce by means of increasing wage offers competing one another for high-quality employees. The main results are the following. First, using a specification of effort such that the problem of firms is well-behaved, optimal wage offers are strategic complements. Second, a symmetric Nash equilibrium can be locally stable under the assumption that firms adjust their wage offers in the direction of increasing profits by conjecturing that any wage offer above (below) equilibrium will lead competitors to underbid (overbid) such an offer. Finally, the exploration of possible labor market equilibria reveals that effort is counter-cyclical.

Keywords: Efficiency-Wages; Wage Competition; Nash Equilibria; Effort.
JEL Classification: C72, E12, E24, J41.

1. Introduction

Discussing the actual possibility of involuntary unemployment equilibria, Hahn (1987) sketches a model economy in which a finite set of firms is engaged in a wage competition

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process within an efficiency-wage setting. In that paper, resuming some arguments of Cournot's (1838) game, Hahn (1987) describes a situation in which under a persistent excess of labor supply, firms do not cut wages not only because this would lower their profitability, but also because wage cuts would enhance the productivity of their competitors. Building on this strategic framework, Hahn (1987) argues that involuntary unemployment is well defined, compatible with rationality and not inconsistent with an equilibrium of the model economy.

The main goal of Hahn's (1987) model is to show that firms might find unprofitable to voluntary agree on a generalized wage reduction in order to reduce equilibrium unemployment.[1] However, important aspects of the efficiency-wage competition process in which firms are assumed to be engaged are left unexplored. For instance, although reaction functions are explicitly derived, nothing is said about the strategic relation among the optimal wage offers put forward by competing firms. Moreover, the achievement of a Nash equilibrium in the efficiency-wage competition process is taken for granted without specifying which kind of out-of-equilibrium adjustment might lead to the mutual consistency among firms' wage offers. Finally, on a genuine macroeconomic perspective, there is no discussion about the cyclical behavior of effort.

After its publication, Hahn's (1987) model has been revisited (inter-alia) by van de Klundert (1988) and, more recently, by Jellal and Wolff (2002). In the context of segmented markets, both contributions derive a Stackelberg version of Hahn's (1987) framework by assuming that the primary sector acts as a leader by setting efficiency-wages while the secondary sector acts as a follower by paying competitive wages. However, to the best of my knowledge, the gaps of the seminal Cournot version reviewed above had never been filled.[2] As a consequence, the present contribution aims at carrying out this task. Specifically, I build a two-firm efficiency-wage model in which each competitor tries to overbid the wage offer of the other employer aiming at maximizing its profits. Consistently with Akerlof (1984) and Hahn (1987), I assume that for each firm the efficiency of the employed labor force is positively correlated to its own wage offer but negatively correlated to the offer put forward by the other firm.

[1] By contrast, macroeconomic interventions such as expansionary monetary policies could be more effective in this direction.

[2] A more general Stackelberg version of Hahn's (1987) model in which wages are endogenously is derived in Appendix.

Within this framework, I discuss the shape of the strategic relation among optimal wage offers and their link with the corresponding iso-profit curves. Thereafter, considering the most recurrent adjustment mechanisms exploited in similar game-theoretic contexts (e.g. Kopel 1996 and Varian 1992), I consider the way in which the wage strategy prevailing in a symmetric Nash equilibrium can actually be achieved. Furthermore, taking into account possible labor market equilibria, I discuss effort cyclicality.

The main results of this theoretical exploration are the following. First, using a specification of effort such that the problem of the representative firm is well-behaved in the sense that it does not deliver corner solutions, optimal wage offers are strategic complements, i.e., whenever the competitor increases (decreases) its wage offer, the optimal response for each firm is to rise (decrease) its wage offer as well. Second, a symmetric Nash equilibrium exists but is unstable under the traditional cobweb adjustment. In other words, when the game is played by means of alternate wage offers there is no way to achieve the Nash equilibrium. Instead, such an allocation can be locally stable under the assumption that each firm continuously adjusts its optimal wage offer in the direction of increasing profits by conjecturing that any wage offer above (below) equilibrium will lead the competitor to underbid (overbid) such an offer. Moreover, the exploration of possible labor market equilibria reveals that effort is counter-cyclical, i.e., consistently with efficiency-wage models in which unemployment acts as a worker discipline device (e.g. Uhlig and Xu 1996 and Guerrazzi 2008), equilibria with higher (lower) unemployment are characterized by higher (lower) effort levels.

This paper is arranged as follows. Section 2 describes the model. Section 3 derives the symmetric Nash equilibrium. Section 4 investigates its local dynamics. Section 5 discusses possible labour market outcomes and the cyclicality of effort. Finally, section 6 concludes.

2. The Model

The model economy is populated by two identical firms indexed by $i = 1,2$ and a mass L^S of identical workers that inelastically supply their labor services. As in Solow (1979), each firm seeks to maximize its profit (π_i) by taking into account that it can simultaneously set employment (L_i) and the real wage (w_i). Furthermore, as in Akerlof (1984) and Hahn (1987), the efficiency of employed labor force (e_i) is assumed to positively depends on the wage offer carried out by the firm that actually provides the job but negatively correlated to the wage offer put forward by the other firm. Therefore, the problem of each firm is given by

$$\max_{L_i, w_i} \pi_i = F_i\left(e_i\left(w_i, w_j\right)L_i\right) - w_i L_i \qquad i, j = 1,2 \tag{1}$$

where $F_i(\cdot)$ is the production function of firm i while $\partial e_i(\cdot)\partial w_i > 0$ and $\partial e_i(\cdot)\partial w_j < 0$.

The first-order conditions (FOCs) for the problem in eq. (1) are the following:

$$L_i : F_i^{'}\left(e_i\left(w_i, w_j\right)L_i\right)e_i\left(w_i, w_j\right) = w_i \qquad i, j = 1,2 \tag{2}$$

$$w_i : F_i^{'}\left(e_i\left(w_i, w_j\right)L_i\right)\frac{\partial e_i\left(w_i, w_j\right)}{\partial w_i} = 1 \qquad i, j = 1,2$$

(3)

Exploiting the FOCs in eq.s (2) and (3), the Solow (1979) condition can be conveyed as

$$\frac{\dfrac{\partial e_i\left(w_i, w_j\right)}{\partial w_i} w_i}{e_i\left(w_i, w_j\right)} = 1 \qquad i, j = 1,2 \tag{4}$$

The expression in eq. (4) suggests that in order to maximize profits, each firm has to set a real wage such that the effort-wage elasticity is equal to one no matter the shape of the production function.[3] From a mathematical perspective, the Solow (1979) condition is both necessary and sufficient if and only if – in addition to eq. (4) – even second order conditions are met and this happens when the effort function is concave; indeed, effort convexity would lead competing firms to settle in a corner solution by pushing employment towards the full employment allocation by questioning the possibility of involuntary unemployment. In this case, provided that the individual wage offers imply a positive level of effort, firms – just like in a competitive environment – will always prefer lower wages.[4] Taking into account those arguments, in the remainder of the paper I will exploit a concave effort function by considering employment adjustments that occur along the (decreasing) labor demand schedules of each firm.

[3] An equivalent reading of the Solow (1979) condition provides that firms set the wage-employment pair in order to minimize the cost of labor in terms of efficiency, i.e., in order to minimize the wage-effort ratio (e.g. Lindbeck and Snower 1987).

[4] Formally speaking, when the effort function is convex, the Solow (1979) selects an allocation in which profits are at their minimum level; indeed, under effort convexity, the Hessian matrix of the maximum problem in eq. (1) evaluated in the wage pair conveyed by eq. (4) is positive definite (e.g. Guerrazzi 2012).

From a game-theoretical point of view, the intriguing feature of the framework outlined in eq.s. (1) – (4) is that the Solow (197) condition does not only depend on the wage offer of the individual firm but also on the wage offer put forward by its competitor. As a consequence, similarly to the situation described by Cournot (1838) in the context of output-quantity competition, the two firms are in a situation of strategic interaction regarding wages. Specifically, the optimal wage offer of firm 1 depends on the offer put forward by firm 2 and vice-versa.

In order to derive explicit results, it is obviously necessary to define production and effort functions. First, along the lines put forward by Akerlof (1982) and, more recently, by Alexopoulos (2004), for each firm, the production function is assumed to be the following:

$$F_i\left(e_i\left(w_i, w_j\right)L_i\right) = \left(e_i\left(w_i, w_j\right)L_i\right)^{\alpha} \qquad 0 < \alpha < 1 \quad i, j = 1,2 \tag{5}$$

where α measures the curvature of the (convex) production possibility set.

Furthermore, for each firm, the effort function is assumed to be given by

$$e_i\left(w_i, w_j\right) = \left(\kappa + w_i - w_j\right)^{\beta} \qquad \kappa > 0\,,\ 0 < \beta < 1\ ,\ i, j = 1,2 \tag{6}$$

where κ conveys productivity shocks while β is the curvature of the effort function.[5]

The expression in eq. (6) suggests that the efficiency of the employed labor force is an exponential concave function that encloses an erratic positive term. Moreover, such a function increases (decreases) as the wage differential between the two firms becomes wider (tighter). On the one hand, anecdotal evidence and empirical tests of efficiency-wage theories are consistent with this formulation (e.g. Raff and Summers 1987, Krueger and Summers 1988 and Huang et al. 1998). On the other hand, a flavor of micro-foundation for the exploited effort function grounded on fairness in given in Appendix. An illustration is given in Fig. 1.

[5] Akerlof (1984) and Hahn (1987) consider a similar effort function that also positively depends on unemployment. In a subsequent part of the paper, I will show that this disciplining effect of unemployment endogenously emerges from the simplest formulation in eq. (6).

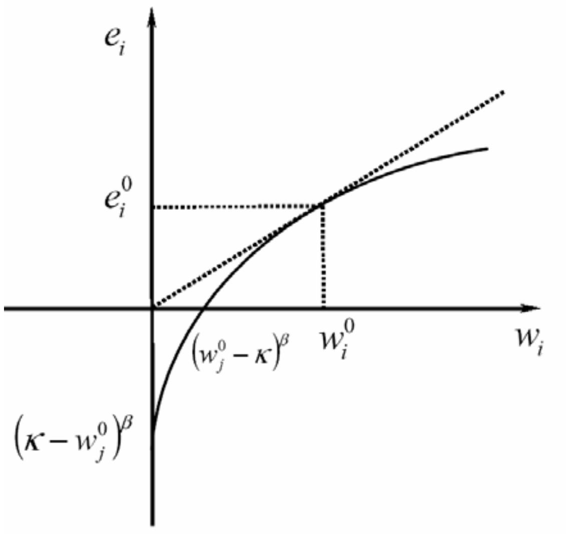

Fig. 1: Effort function.

It is worth noting that under concavity the existence of an interior solution that fulfils the Solow (1979) condition implies that the vertical intercept of eq. (6) has to be negative. As a consequence, for each firm, the wage offer of its competitor cannot be lower than κ.

3. Nash equilibrium

Combining eq.s (4) and (6) it becomes possible to derive the reaction functions (f_i) of the two firms; indeed, straightforward algebra leads to following linear expression:

$$w_i = -\frac{\kappa}{1-\beta} + \frac{1}{1-\beta} w_j \qquad i,\, j = 1,2 \tag{7}$$

The positive slope of the function in eq. (7) shows that the optimal wage offers of the two firms are strategic complements, i.e., whenever the competitor increases (decreases) its wage offer, the optimal response for each firm is to rise (decrease) its wage offer as well. Technically speaking, the rationale for such behavior is straightforward. Everything else being equal, assuming the concavity of eq. (6), an increase (decrease) of the wage offer carried out by the competitor reduces (increases) workers' effort provision by leading the u-shaped wage-effort ratio to shift right (left). As a consequence, in order to restore efficiency, each firm has to increase (decrease) its offer as well.

The Nash equilibrium is found where the two reaction functions intersect each other. Therefore, the symmetric optimal wage strategy is given by

$$w_i^* = \frac{\kappa}{\beta} \qquad i,\, j = 1,2 \tag{8}$$

Plugging the result in eq. (8) into eq. (6) shows that in equilibrium workers are paid more than their individual efficiency. From a formal point of view, this result comes from the fact that when the effort function is concave, $\kappa^{\beta} < \kappa / \beta$, $\forall \kappa > 0$. An illustration of the Nash equilibrium is given in Fig. 2.

The diagram in Fig. 2 shows the reaction functions of the two firms together with equilibrium iso-profit curves, i.e., the iso-profit curves associated to the wage strategy in eq. (8). In general, for each firm, those curves are non-linear functions such as

$$w_j = \kappa + w_i - \left(\left(\frac{\overline{\pi}_i}{\Phi} \right)^{\frac{1-\alpha}{\alpha}} w_i \right)^{\frac{1}{\beta}} \qquad i, j = 1,2 \tag{9}$$

where $\Phi \equiv (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}$ while $\overline{\pi}_i$ is a constant level of profit.

The set of non-linear functions conveyed by eq. (9) is represented by reverse-u-shaped curves with a vertical intercept equal to κ which reach their maximum in the point when they intersect the relevant reaction function. In other words, consistently with the textbook definition of a Nash equilibrium (e.g. Varian 1992), when firm 2 decides to pay κ / β it is in the best interest of firm 1 to pay κ / β as well and vice-versa, so that none of the two players will have incentives to deviate from the such a wage strategy. Moreover, for each firm, higher (lower) iso-profit curves, are associated with lower (higher) levels of profit. Furthermore, it is worth noting that in Fig. 2 the equilibrium iso-profit curves of the two firms intersect each other. As in Cournot's (1838) output-quantity competition, this geometrical feature conveys the non-cooperative feature of the Nash wage equilibrium derived in this strategic context.

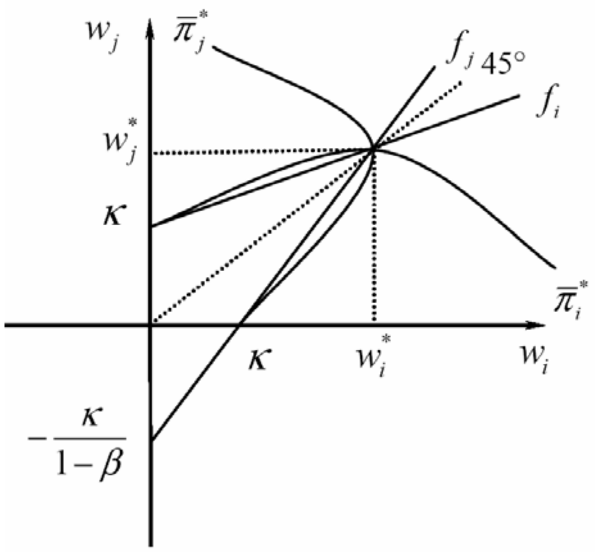

Fig. 2: Nash equilibrium.

4. Local dynamics

Before discussing possible labor market outcomes, it is necessary to say something about the way in which the wage strategy in eq. (8) can actually be reached; indeed, if starting from a different allocation there was no way to achieved it, then such a symmetric wage distribution, together with its labor market implications, would lose a great deal of its practical significance.

Assuming adjustments to lagged quantity signals, i.e., adjustments grounded on alternate wage offers, the Nash equilibrium is stable if and only if firm 1's reaction function is steeper than firm 2's reaction function (e.g. Kopel 1996). Taking the result in eq. (7) into account, this happens whenever

$$\frac{1}{(1-\beta)^2} < 1 \tag{10}$$

The inequality in (10) could be hypothetically verified by assuming the convexity of the effort function in eq. (6). This stability requirement, taking into account the slope of eq. (7), would also lead to overturn the result on complementarity derived in the previous section by conveying to the substitutability of optimal wage offers. However, as stated in section 2, under effort convexity the Solow condition in eq. (4) is totally inconsistent with firms' maximum profit problem so that eq. (8) would fail to identify the optimal response for each competitor's wage offer; indeed, exploiting a convex effort function, the reaction function in eq. (8) would actually detect the *worse* wage reply, i.e., the wage offer that leads to minimum profits. Considering those arguments, it becomes possible to state that as far as effort concavity is concerned – together with its well-behaved solutions – the inequality in (10) cannot be verified so that under the traditional cobweb adjustment the symmetric Nash equilibrium is unstable. Specifically, unless the starting wage strategy coincides with the one in eq. (8), optimal wage offers explode or implode depending on whether their initial values are above or below κ / β.

The badly-behaved dynamic patterns conveyed by effort concavity raises the issue of finding another possible mechanism able to describe how the Nash equilibrium might be actually reached. In this regard, a different type of micro-founded (or behavioral) adjustment can be derived by assuming that each firm adjusts its wage offer in the direction of increasing profits (e.g. Varian 1992). In this case, adjustments are simultaneous and the out-of-equilibrium dynamics of real wages is described by

$$\dot{w}_i = \gamma \left(\frac{\partial \pi_i \left(w_i, \hat{w}_j \left(w_i \right) \right)}{\partial w_i} \right) \qquad \gamma > 0, \quad i, j = 1,2 \tag{11}$$

where $\hat{w}_j(w_i)$ is the conjecture of firm i about the wage behavior of firm j while γ is a constant that conveys the speed of out-of-equilibrium adjustments.

Considering the properties of mutual consistency of a Nash equilibrium stressed above, I assume that each firm conjectures the wage behavior of its competitor by means of the following conjectural or 'learning' rule:

$$\hat{w}_j(w_i) = \frac{\kappa}{\beta} + \lambda_i \left(w_i - \frac{\kappa}{\beta} \right) \qquad i, j = 1,2 \tag{12}$$

where λ_i is a constant that conveys the so-called conjectural variation, i.e., the 'expected' variation of the wage offer put forward by firm j when firm i marginally changes its own proposal.

For each firm, eq. (12) can be interpreted as a Stackelberg leadership rule that approximates competitor's reaction function; indeed, the nearer λ_i to the shape of the optimal response function, the closer eq. (12) to eq. (7). Formally speaking,

$$\lim_{\lambda_i \to \frac{1}{1-\beta}} \hat{w}_j(w_i) = -\frac{\kappa}{1-\beta} + \frac{1}{1-\beta} w_i \qquad i, j = 1,2 \tag{13}$$

In addition to the asymptotic result in eq. (13), the main implications of eq. (12) can be summarized as follows. First, consistently with the static case developed by Hahn (1987) and warmly supported by a number of game theorists that question the rationality of adjustments occurring outside a Nash equilibrium (e.g. Bacharach 1976), the suggested learning rule implies that when firm i decides to bid the equilibrium wage offer it conjectures that its competitor will do the same by warding off any out-of-equilibrium dynamics and confirming κ/β as the barycentre of the stable wage strategy.[6] However, the range of possibility covered by eq. (12) is wider; indeed, depending on the sign and the magnitude of λ_i, eq. (12) also defines the conjectures of firm i about the proposal of firm j outside the Nash equilibrium. Specifically, if λ_i is equal to zero, then each firm neglects the strategic interaction between its own behavior and the behavior of its competitor. In other words, in this case, each firm thinks that for any given wage offer the competitor will leave its proposal unaltered by playing the equilibrium wage strategy. Furthermore, when λ_i is positive (negative), then firm i conjectures that any wage offer above equilibrium will lead firm j to overbid (underbid) such an offer.

Taking into consideration eq. (12), the Jacobian matrix (J) of the dynamic system in eq. (11) evaluated in eq. (8) is given by

$$J \equiv \begin{bmatrix} \Omega(\beta - 1 + \lambda_1) & \Omega \\ \Omega(\beta - 1 + \lambda_2) & \Omega \end{bmatrix} \tag{14}$$

where $\Omega \equiv \gamma\alpha^{\frac{1}{1-\alpha}}\beta^2\left(\beta\kappa^{\beta-1}\right)^{\frac{\alpha}{1-\alpha}-1}\kappa^{\beta-3} > 0$.

A sufficient requirement for the local stability of the system in eq. (11) is the negativity (positivity) of the trace (determinant) of J. Straightforward calculations suggest that the trace (Tr(J)) and the determinant (Det(J)) are equal to

$$\mathrm{Tr}(J) = \Omega(\beta + \lambda_1) \tag{15}$$

$$\mathrm{Det}(J) = \Omega(\lambda_1 - \lambda_2) \tag{16}$$

The results in eq.s (15) and (16) show that local stability requires λ_1 (λ_2) to be negative and higher than β (λ_1) in modulus.[7] Obviously, this means that the symmetric Nash equilibrium can be locally stable when each firm adjusts its wage offers in the direction of

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[6] The dynamic system in eq. (11) has the nice feature to verify Nash stationarity, i.e., its steady-state coincides with the Nash equilibrium of the game at hand (e.g. Sandholm 2005).

[7] Under reasonable calibrations, e.g., $\alpha = 2/3$, $\beta = 1/2$, $\kappa = \gamma = 1$, $\lambda_1 = -0.7$ and $\lambda_2 = -0.8$, J displays two complex-conjugate eigenvalues with negative real part. In this case, convergence towards the Nash equilibrium occurs though convergent oscillations.

increasing profits by conjecturing that any wage offer above (below) equilibrium will lead its competitor to underbid (overbid) such an offer.[8]

From an economic point of view, those dynamic findings imply that convergence towards the symmetric wage strategy in eq. (8) requires that each firm myopically perceives a certain degree of substitution among the optimal wage offers put forward by its competitor. In this strategic framework, such a misperception could be achieved by assuming that κ is subject to idiosyncratic shocks that – for each firm – systematically fades the correct perception of actual competitor's reaction function.[9] Along this way, avoiding the issue of (unrealistic) corner solutions and inconsistent reactions functions, the model economy recovers the stability requirement of the game of alternate wage offers.

5. Labor market outcomes

Plugging eq. (8) into eq. (6) and then substituting in eq. (2) allows to derive the equilibrium aggregate demand for labor. Specifically, in the symmetric Nash equilibrium the quantity of labor services demanded by the two competing firms amount to

$$L^D = n\beta^{\frac{1}{1-\alpha}}(\kappa)^{-\left(\frac{1-\alpha\beta}{1-\alpha}\right)} \tag{17}$$

where $n = 2$.

The result in eq. (17) allows to characterize labor market tightness in a precise manner. In details,

- if $L^D < L^S$, then the model economy experiences an involuntary unemployment rate equal to $\left(L^S - L^D\right)/ L^S$ as in the seminal Hahn's (1987) contribution;
- if $L^D = L^S$, then there prevails full employment which would coincides with the Nash wage equilibrium;
- if $L^D > L^S$, then firms are rationed in the labor market so that actual employment is equal to L^S and each firm would have $1/n\left(L^D - L^S\right)$ vacant positions. However, as suggested by

[8] It is worth noting that without any conjectural variations, i.e., $\lambda_i = 0$, the dynamic system would display a saddle-note bifurcation without any guide for dynamics. Moreover, when each firm conjectures that any wage offer above (below) equilibrium will lead each competitor to overbid (underbid) such an offer, i.e., $\lambda_i > 0$, the Nash equilibrium is locally unstable.

[9] In the context of exchange rate dynamics, Gourinchas and Torell (2001) argue that idiosyncratic shocks might lead to systematic biases in individual forecasts.

Weiss (1991, p. 21), such an allocation cannot be a proper equilibrium; indeed, the shortage of labor would lead firms to increase their wage offers until L^D and L^S become equal.[10] In such a situation, the dynamic adjustments described in the previous section would fail to hold because firms would have to compete not only for the quality of workers, but also for their (scant) services. As a consequence, in addition to eq. (11), the analysis of this scarcity scenario would require the definition of the out-of-equilibrium dynamics for the employment level in the two firms.[11]

Since the paper focuses on the properties of the wage strategy in eq. (8), I will discuss the cyclicality of effort under the first two points. Within those scenarios, taking into account movements in κ, the result in eq. (17) can be exploited to discuss how equilibrium employment react to effort movements. Specifically, plain differencing suggests that effort is counter-cyclical, i.e., equilibria with higher (lower) unemployment are characterized by higher (lower) effort levels. Such an effort pattern is perfectly consistent with the idea underlying efficiency-wage models in which involuntary unemployment acts a worker discipline device. In this class of models popularized by Shapiro and Stiglitz (1984), involuntary unemployment is the threat that prevents workers from shirking. As a consequence, an increase (decrease) in unemployment should lead workers with jobs to work harder (slowly), making them more (less) efficient (e.g. Uhlig and Xu 1996 and Guerrazzi 2008).

Although in the efficiency-wage competition model developed in section 2 the payment of an efficiency-wage is not related to the shirking motivation, effort is counter-cyclical as well. However, there is an important difference between this model and the efficiency-wage models with shirking workers; indeed, in those models the counter-cyclicality of effort emerges as the result of a Marxian (or Ricardian) endogeneity of labor supply (e.g. Bowles 1985 and Drago 1989-1990). By contrast, in the model economy developed in section 2 such a counter-cyclicality is the upshot of a wage competition process

[10] It is worth noting that in this case the value of the marginal productivity of labor is higher than the level satisfying the Solow (1979) condition. Specifically, when firms are rationed in the labor market the effort-wage elasticity is lower than one. The same possibility is contemplated in dynamic efficiency-wage models developed inter alia by Faria (2000) and Guerrazzi (2008).

[11] More technically, when the Nash equilibrium depicts a situation in which the two firms are rationed in the labor market it becomes necessary to study a 4×4 dynamic system in w_1, w_2, L_1 and L_2 whose resting point is the full employment allocation without any room for involuntary unemployment.

engaged by firms in the attempt to hire workers of higher quality in a technology scenario with decreasing returns with respect to labor.

6. Concluding remarks

This paper provides a model of efficiency-wage competition along the lines put forward by Hahn (1987). Specifically, I build a two-firm efficiency-wage model in which the effort attainable by the representative firm is an increasing function of its own wage offer but declining in the offer put forward by its competitor. As a consequence, employers screen their workforce by means of increasing wage offers competing one another for high-quality employees.

The main results achieved in this paper can be summarized as follows. First, using a specification of effort such that the maximum profit problem of the representative firm is well-behaved in the sense that it does not deliver corner solutions, optimal wage offers are strategic complements, i.e., whenever the competitor increases (decreases) its wage offer, the optimal response for each firm is to rise (decrease) its wage offer as well. Second, a symmetric Nash equilibrium can be locally stable under the assumption that each firm adjusts its optimal wage offer in the direction of increasing profits by conjecturing that any wage offer above (below) equilibrium will lead the competitor to underbid (overbid) such an offer. Finally, the exploration of possible labor market equilibria reveals that effort is counter-cyclical, i.e., equilibria with higher (lower) unemployment are characterized by higher (lower) effort levels.

A. Appendix: Stackelberg equilibria

In this section I derive the Stackelberg equilibrium of the model economy described in section 2. This exercise is relegated in Appendix because the effort function in eq. (6) delivers meaningful equilibria of this kind if and only its curvature is quite strong, i.e., whenever β is close to zero.

Without loss of generality, I assume that firm 1 is the leader while firm 2 is the follower.[12] In this case, firm 1 will try to maximize its profits by taking into account that firm 2 will adhere to its own reaction function. Therefore, firm 1's problem becomes

[12] Identical firms can play those different roles if, for instance, the labour market is segmented and there are relevant mobility costs that workers have to bear in order to switch from one segment to another.

$$\max_{L_1, w_1} \pi_1 = F_1(e_1(w_1, w_2)L_1) - w_1 L_1 \tag{A.1}$$

s.to

$$w_2 = -\frac{1}{1-\beta}\kappa + \frac{1}{1-\beta}w_1 \tag{A.2}$$

Taking into account eq. (6), the solution of the problem in eq.s (A.1) and (A.2) provides the following wage distribution:

$$w_1^S = \frac{\kappa}{\beta}\left(1 + \frac{1}{1-\beta}\right) \tag{A.3}$$

$$w_1^S = \frac{\kappa}{\beta}\left(1 + \frac{1}{(1-\beta)^2}\right) \tag{A.4}$$

The results in eq.s (A.3) and (A.4) show that in the Stackelberg equilibrium that firms 2 pays more than firm 1. As a consequence, firm 2 will be more efficient and will achieve higher profits; indeed, consistently with textbook results derived in the context of output competition (e.g. Varian 1992), under complementarity among optimal wage offers, leadership is never preferred. Furthermore, non-uniform wage and profit distributions, reveals that a Stackelberg equilibrium can provide a theoretical underpinning for segmented (or dual) labor markets (e.g. van de Klundert 1988 and Jellal and Wolff 2002). An illustration is given in Fig. A.1.

The diagram in Fig. A.1 recalls that the Stackelberg equilibrium is found where the highest iso-profit curve of firm 1 is tangent with reaction function of firm 2.[13] Moreover, it is worth noting that w_1^S does not satisfy the Solow (1979) condition; indeed, in the Stackelberg equilibrium the leader effort-wage elasticity is higher than one. This possibility is contemplated by Faria (2005) who develops an inter-temporal model with investment and efficiency-wages.

[13] When the wage offer of firm 1 is lower than the one of firm 2, firm 1's profits are very low. Under those circumstances, the iso-profit curves of firm 1 become convex.

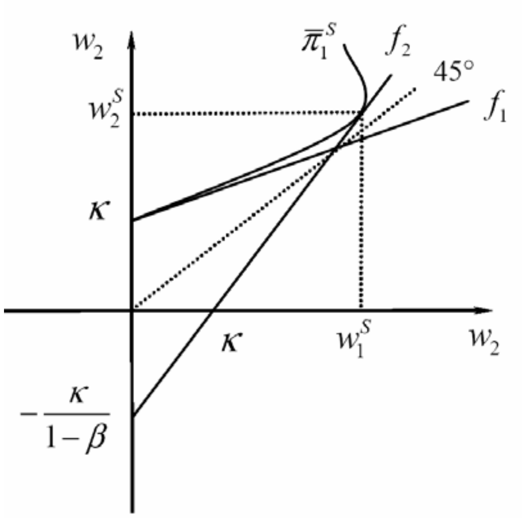

Fig. A.1: Stackelberg equilibrium.

B. Appendix: Effort micro-foundation

In this section I sketch a possible micro-foundation of the effort function in eq. (6). Straightforward integration suggests that the problem of the representative worker called in to provide effort for firm i should be given by

$$\max_{e_i} \; U = \left(\kappa + w_i - w_j\right)^{\beta} e_i - \frac{1}{2} e_i^2 + \varphi \qquad\qquad i, j = 1,2 \qquad\qquad \text{(B.1)}$$

where φ is a constant that without loss of generality can be normalized to zero.

On worker's side, the expression in eq. (B.1) can be interpreted as follows. First, κ conveys the intrinsic motivation of the worker, i.e., the measure of the marginal utility of providing effort which does not depend on wages. This parameter can well follow a stochastic process by mirroring the behavior of erratic productivity shocks. Moreover, along the arguments put forward by Adams (1963) and more recently by Kahneman et al. (1986a-b), the wage differential $\left(w_i - w_j\right)$ can be thought as the (simplest) functional form catching worker's perception of being treated fairly by job offering firms.[14] As a consequence, when a firm decides to pay less than the other such a behavior will be perceived as unfair so that the representative worker will adjust effort provision downward until a mutual fair treatment is psychologically restored. Obviously, a positive wage premium will lead the worker to do the opposite.

[14] From a psychological point of view, the fair wage theory developed by Adams (1963) is an economic implementation of the theory of cognitive dissonance put forward by Festinger (1957).

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