Labour Market Performance and the Quality of Industrial Relations

Giulio Piccirilli
giulio.piccirilli@unicatt.it

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Abstract

We present a model with search and matching imperfections where wages are set by a monopoly union. The union commits to future wages but the commitment technology is loose. Firms, in turn, have an imperfect knowledge on the commitment technology and may overestimate the chance of rejection for the announced plan.

This model proves useful to analyse the notion of trust in the context of industrial relations and to derive comparative statics for the performance of the labour market. In particular, we show that market tightness increases with respect to firms trust on the compliance of the union with the announced plan.

A basic empirical analysis conducted on OECD data does not contradict this theoretical result.

1 Introduction

In this paper we provide a theoretical and empirical analysis on the link between the quality of industrial relations and the performance of the labour market.

Good quality industrial relations are typical of bargaining environments where conflicts of interests and hold up problems are resolved by mutual trust and cooperative behaviour. For this reason, we build a model where an hold up problem arises through the interplay between a union and a firm sector. Firms make an irreversible investment to create jobs and become naturally exposed to ex-post rent appropriation by the union in the form of high wage claims. The union, in turn, has an incentive to announce low future wages to boost job creation at current time and to renege on the announcement once jobs have been created.

It is well known that, in this type of situation, the risk for the vulnerable party of being exploited leads to a suboptimal outcome and that efficiency is restored only if the risk is made ineffective. This, in turn, typically requires some sort of economic incentive against opportunistic decisions. Alternatively, the vulnerable party must trust its partner in the sense that it must expect that the partner will not exploit its vulnerability. The last solution, however, raises a conflict with the notion of rationality since the party that is trusted has an economic incentive to behave opportunistically and the party that trusts should not do so in anticipation of being exploited (James, 2002). In other words, interactions based on mutual trust and, more specifically, cooperative industrial relations are difficult to rationalise by means of traditional economic concepts.

Due to the difficulty of endogenising the concept of trust, papers concerned with the issue are forced to adopt some *ad hoc* assumption. In Blanchard and Philippon (2006), for instance, vulnerable agents trust their opponents if these are lucky and extract the label for being trustworthy within an exogenous lottery. In this paper, instead, we take a different route and model trust as the exogenous opinion of the vulnerable party regarding the strength of a commitment made by the other party. More specifically, we assume that the union has access to some general commitment technology that may allow, with some exogenous probability, to renege previous announcements. Firms, in turn, have an imperfect knowledge on the commitment technology and hold beliefs that may overestimate the true probability of deviation from the announced plan. Indeed, it is precisely this belief that conveys the notion of trust, the lower the expectation of future deviations the higher the trust of firms towards the union.

To build a model with these features we resort to a very standard description of the labour market, which is the one popularised by D. Mortensen and C. Pissarides (1999). To create a job and hire a worker, firms spend resources which are lost in case the match is dissolved. The union can then announce compliance with a plan of moderate wages to stimulate entry and renege on the announcement once matching has taken place. So, it is the irreversible nature of searching expenses that makes firms vulnerable.

To emphasize the mechanism of trust we abstract from the complications related to the formation of the announced wage plan and, for simplicity, assume that the union has monopoly power in setting this plan. However, none of our key results hinges on the assumption of monopoly since what is really relevant is that the union has *some* bargaining power in setting wages and an *ex post incentive* to renege on the announcement. In a companion paper, we allow for firms bargaining power and show that results are robust to this extension.

The main findings of our theoretical investigation are the following. First, we show that in our setting the hold-up problem is so disruptive that commitment and trust are necessary for positive steady state employment. Second, we show that the performance of the labour market improves with trust. More specifically, equilibrium market tightness increases with the belief of firms concerning union compliance.

This happens for two distinct reasons. Firms have a larger incentive to post vacancies for given announced wages whilst the union has a stronger incentive to moderate wage claims. Third, for given firms beliefs, an improvement in the commitment technology produces higher wages but reduces market tightness.

The applied part of the paper concerns the empirical consistency of the link between trust and labour market performance. We use a panel of 20 OECD countries observed for 15 years, from 1990 to 2004. Performance is measured with the rate of unemployment (source: Oecd) while, for trust, we use the overlapping notion of "cooperative attitudes" and resort to interview evidence produced by the World Economic Forum. We run a basic regression on a modified phillips curve and find that estimations do not contradict our theoretical finding on the impact of trust.

The paper is organised as follows. In section 2 we present the economy. In section 3 we solve for the equilibrium and study the comparative statics. In section 4 we check the empirical consistency of our predictions while, in section 5, we offer some concluding remarks.

2 The Economy

2.1 Search and Matching

The economy is a search and matching economy as in Mortensen and Pissarides (1994) with the exception that wages are determined in a centralised manner instead of being bargained at the level of single worker-firm pairs.

There is a unit mass of workers with linear utility that can be either employed or in search of employment. The employed produces an output flow p and receives a wage w_t while the unemployed receives a benefits flow b ($p > b$). There is a large mass of single-job firms but only some of them participate to market activity. Those that do not participate can freely access the market by posting vacancies and by searching for suitable workers. Holding a vacancy entails a constant per-period search cost c. Time is discrete, worker-firm matches that initiate at time t become productive at time $t+1$. However, from $t+2$ onwards, productivity may be hit by an irreversible negative shock, this happens with a per-period probability $\rho(< 1)$.

Labour market frictions are described according to an urn-ball matching process as in Pissarides (1979), Blanchard and Diamond (2004), Burdett et. al (2001) and Smith and Zenou (2003) among many others. Thus, let v_t and n_t represent the current number of vacancies and employed workers respectively, $\theta_t \equiv v_t/(1-n_t)$ gives the measure of labour market tightness while $q(\theta_t)$ and $p(\theta_t)$ give respectively the probability for a searching firm and a searching worker to match with a partner during period t. We assume that a) workers and firms come in touch thanks to signals sent by firms, b) a signal arrives only to one worker while firms can send only one signal in every period, c) firms do not coordinate so that some workers receive more than one signal whereas some others receive no signal at all, d) workers respond only to current signals and, in case of many signals, choose randomly, e) after a worker responds to a signal the pair matches and extract productivity and f) productivity turns out to be p with probability $\pi(<1)$ and nil otherwise. Under these assumptions, the functions $q(\theta)$ and $p(\theta)$ are given by the formulas

$$\begin{aligned} p(\theta) &= 1 - e^{-\pi\theta} \qquad q(\theta) = p(\theta)/\theta \\ q(\theta),\, p(\theta) &\in [0,1] \qquad p'(\theta) > 0,\, q'(\theta) < 0 \end{aligned} \tag{1}$$

We do not give any proof of these formulas but refer to Smith and Zenou (2003). In fact, equation 1 turns out to be an extension to large populations of matching probabilities arising in their small population setting. A property of these matching probabilities that will be used in the paper concerns the shape of $\eta(\theta)$, which represents the (absolute value of the) elasticity of $q(\theta)$:

$$\eta(\theta) \in [0,1] \qquad \eta'(\theta) > 0 \tag{2}$$

In words, the elasticity of the matching probability of firms is smaller than one and increasing with respect to market tightness. Since $p(\theta) = \theta p(\theta)$, this also implies that the elasticity of the matching probability of workers $1 - \eta(\theta)$ is smaller than one and decreasing.

2.2 Entry

We assume that entry is free. Thus, if entry entails positive net returns, free entry drives these returns to zero. By contrast, if entry entails zero or negative returns, the number of vacancies is nil. Let V_t represent the market value of an empty firm, the free entry condition is

$$\text{a) } V_t \leq 0 \qquad \text{b) } \theta_t \geq 0 \qquad \text{c) } V_t\theta_t = 0 \tag{3}$$

In addition, let J_t represent the market value of a filled firm. Assets V_t and J_t solve the following bellman equations:

$$V_t = -c + \beta q(\theta_t) E_t J_{t+1} \tag{4}$$

$$J_t = (p - w_t) + \beta(1-\rho) E_t J_{t+1} \tag{5}$$

The first bellman clarifies that the value of an empty firm is due to the chance of matching in the current period and becoming productive from the next period onward, $\beta(< 1)$ represents the discount factor of firms. The second bellman implies that the fundamental value of a filled firm is the discounted stream of profits. Both bellmans embeds the free entry condition. If an empty firm does not match in the current period it has the chance, in the next period, of exiting or searching again. Thus, with probability $1-q(\theta_t)$, the continuation value of an empty firm is $\max(0, V_{t+1})$, which is clearly nil in the light of equation 3. Analogously, if a filled firm is hit by a negative shock in the current period, it has the chance, in the next period, of exiting or searching again. Thus, with probability $1-\rho$, the continuation value of a filled firm is also nil.

Notice that, in principle, the stream of profits for a filled firm can be interrupted for two reasons. First, wages are set at a level so high that the firm prefers to exit the market. Second, productivity is hit by an exogenous negative shock. Once one considers both causes, it becomes clear that equation 5 contains the implicit restriction that termination can only be due to exogenous reasons, i.e. that wages never induce a closure:

$$(p - w_t) + \beta E_t(1-\rho) J_{t+1} \geq 0 \tag{6}$$

We refer to equation 5 as the *no-exit condition* and hold it as an assumption. We discuss its rationale in section 3.1.

2.3 Wage Setting and Commitment

We assume that wages are set by an utilitarian union that weights equally the welfare of all workers. The union uses the same discount factor of firms, its objective is represented by the discounted stream of workers utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[n_t w_t + (1 - n_t) b \right] \tag{7}$$

In this paper we assume that the union is monopolistic in the sense that the unique constraint is represented by firms labour demand. In a companion paper we show that the results we obtain under monopoly are robust to an extension that allows for firms bargaining power.

Since entry is costly, the Mortensen-Pissarides economy provides scope for an hold-up problem. The union has an incentive to announce low future wages to enhance current job creation and to renege on the announcement later on. We assume that to circumvent the hold-up problem the union has access to a commitment technology. However, this technology is not perfect in the sense that a reoptimisation may take place with a per-period probability $\gamma(< 1)$. In addition, we assume that the union does not possess full knowledge as for the commitment technology and holds the belief that reoptimisations take place with a per-period probability $\alpha(< 1)$. Finally, we assume that α is not larger than γ so that the difference $\gamma - \alpha$ is a measure of firms distrust on union compliance with the announced plan:

$$\gamma \geq \alpha \tag{8}$$

3 Equilibrium

3.1 The Union Problem

The objective of this section is to set up the problem facing the monopoly union. For this purpose, let $P(t)$ represent the probability that a commitment that starts at time 0 terminates at time t:

$$P(t) = \gamma^{t-1}(1-\gamma) \qquad t \geq 1$$

In addition, define $\widetilde{W}(n_t)$ as the optimal discounted stream of utility accruing to the union from time t onward if the union makes a commitment at time t and current employment is n_t. $\widetilde{W}$ is made conditional on n_t since the latter, due to the search imperfections, represents the only state variable of the economy.

Use P and $\widetilde{W}$ to eliminate the expectation operator from equation 7**:**

$$nw_0 + (1-n)b + \sum_{t=1}^{\infty} P(t) \left[\sum_{j=1}^{t-1} \beta^j \left[n_j w_j + (1-n_j)b \right] + \beta^t \widetilde{W}(n_t) \right] \tag{9}$$

The general term in the square brackets represents the discounted stream of utility if the commitment taken at time 0 terminates at time t. This term is multiplied through the corresponding probability of duration. Thus, the expectation operator in 7 is transformed into a sum of infinite terms corresponding to all possible commitment durations. A few mathematical steps allow to rewrite equation 9 in the following simpler form (see the appendix for details):

$$\sum_{t=0}^{\infty} (\gamma\beta)^t \left\{ \left[n_t w_t + (1-n_t)b \right] + (1-\gamma)\beta \widetilde{W}(n_{t+1}) \right\} \tag{10}$$

Use $\omega_t \equiv E_t J_{t+1}$ to indicate the *next period promised* firm value along the announced wage policy. Running forward the expression 5 and using the law of iterated expectation, ω_t can be expressed as follows:

$$\omega_t = E_t \sum_{j=1}^{\infty} [\beta(1-\rho)]^{j-1} (p - w_{t+j}) \tag{11}$$

The promised value is given by the expected discounted profit flow from $t+1$ onwards. Discounting accounts for time preference as well as for the exogenous probability of firm destruction. As we have done with the objective function of the union, the purpose is to eliminate the expectation operator from the formula. For this reason, use $\widetilde{\omega}(n_t)$ and $\widetilde{w}(n_t)$ to represent the promised value and the wage at the time of reoptimisation respectively and restate the expression for ω_t in a recursive manner (see the appendix for details):

$$\omega_t = \alpha \left[(p - w_{t+1}) + \beta(1-\rho)\omega_{t+1}\right] + (1-\alpha)\left[(p - \widetilde{w}_{t+1}) + \beta(1-\rho)\widetilde{\omega}_{t+1}\right] \tag{12}$$

Having defined the functions $\widetilde{W}(n_t)$, $\widetilde{w}(n_t)$ and $\widetilde{\omega}(n_t)$, we now proceed *as if* these functions were known. In this case, the problem facing the union is the following:

$$\begin{array}{rcl}
\widetilde{W}(n_0) & = & \max\limits_{\{w_t\}} \sum\limits_{t=0}^{\infty} (\gamma\beta)^t \left\{[n_t w_t + (1-n_t)b] + (1-\gamma)\beta\widetilde{W}(n_{t+1})\right\} \\
\text{free entry} & : & \text{a) } \dfrac{c}{q(\theta_t)} \geq \beta\omega_t \qquad \text{c) } \theta_t \geq 0 \qquad \text{c) } \theta_t \left[\dfrac{c}{q(\theta_t)} - \beta\omega_t\right] = 0 \\
\text{No-exit} & : & (p - w_t) + \beta(1-\rho)\omega_t \geq 0 \\
\text{Employment dynamics} & : & n_{t+1} = (1-\rho)n_t + p(\theta_t)(1-n_t) \qquad n_0 = \overline{n} \\
\text{Promised value} & : & \omega_t \;\; \text{as in 12}
\end{array} \tag{13}$$

At time 0, the union inherits employment n_0 and announces a wage policy $w(n_0, t)$ $t \geq 0$ with the purpose of maximising the expected flow of utility. This flow, in accordance with the above definition of $\widetilde{W}$, turns out to be equal to $\widetilde{W}(n_0)$. In setting the optimal policy, the union needs to take account of two dynamic constraints. The first is backward looking and concerns the evolution of employment. The second is forward looking and concerns the determination of the promised value ω_t. If one runs forward equation 12, it becomes clear that ω_t depends on wages from $t+1$ onwards. In addition, as it is clear from the free entry condition, the promised value ω_t represents the driver for current job creation. Thus, to enhance current and future employment the union needs to set future wages at a moderate level.

The dynamics of employment is consistent with the no-exit condition since it contains the restriction that the employed may become unemployed only for exogenous reasons. In a more general formulation, one should omit the no-exit condition and allow for employment to drop to zero if the current value of firms becomes negative. However, the mere fact that employment may drop discontinuously to zero legitimates the conjecture that the value of firms is never negative along the optimal wage sequence. The reason is the following. For any wage sequence that leads to the exit of firms at some date τ, there exists a better sequence which is similar to the original up to $\tau - 1$ and that, from τ onwards, has wages equal to p. The alternative sequence is better since firms do not exit at τ while workers receive a wage p instead of the subsidy b until the match dissolves. It follows that a sequence which implies negative discounted profits can never be optimal. Henceforth, imposing the no-exit condition *jointly with* the restricted employment dynamics turns out to be immaterial for the set of optimal policies.

3.2 Recursive Formulation

Due to the forward dynamic constraint 12, the above union problem is non-recursive. As a consequence, the optimal sequence $w(n_0, t)$ does not take the form $w(n_t)$. In the appendix, we adopt the Lagrangean method of Marcet and Marimon (2011) and transform the problem into a recursive one by introducing the fictitious state variable A_t. This variable has a negative value and represents the marginal costs in terms of lower employment from a small change in $w(n_0, t)$. In fact, if the announced wage $w(n_0, t)$ increases, the value of firms prior to time t decreases and this reduces job creation from time 0 to time t. As a consequence, A_t has a cumulative dynamics in the sense its (absolute) value increases as planning moves further into the future. If a reoptimisation occurs in some future period τ, A_τ is reset to zero since the new wage $\widetilde{w}(n_\tau)$ does not affect previous job creation.

The bellman for the recursive problem is the following:

$$W(n,A) = \min_{\mu,\zeta\geq 0,\sigma\geq 0,\lambda} \max_{w,\omega,\theta\geq 0} \Big\{ [nw + (1-n)b] + (1-\gamma)\beta\widetilde{W}(n') + (\zeta - \mu\theta)\left[c/q(\theta) - \beta\omega\right] +$$

$$+\sigma\left[(p-w) + (1-\rho)\beta\omega\right] - \lambda\omega - A(p-w) - A\frac{1-\alpha}{\alpha}\left[(p - \widetilde{w}(n)) + (1-\rho)\beta\widetilde{\omega}(n)\right]\Big\} + \gamma\beta W(n', A') \tag{14}$$

$$n' = (1-\rho)n + p(\theta)(1-n) \qquad A' = \frac{\alpha}{\gamma}\left[(1-\rho)A - \lambda/\beta\right] \tag{15}$$

The vector $[\mu, \zeta, \sigma, \lambda]$ represents the (current value) lagrangean multipliers for the free entry condition (μ, ζ), the no-exit condition (σ) and the forward looking constraint 12 (λ). Once one solves this program, the vector of policy functions takes the form $\mathbf{x}_t = \mathbf{x}(n_t, A_t)$ $\mathbf{x} = [w, \omega, \theta, \mu, \zeta, \sigma, \lambda]$. In turn, the policy vector $\mathbf{x}$ and state dynamics 15 induce a correspondence from the current state the the future state: $n' = F(n, A)$ and $A' = G(n, A)$. Thus, starting from the initial state $(\overline{n}, 0)$ one may compute the whole sequence (n_t, A_t) $t \geq 0$. Finally, once the state sequence is known, applying the policy function $w(n_t, A_t)$ to the sequence gives the wage profile $w(n_0, t)$ which is announced by the union at time 0.

3.3 Solution

In the appendix we list the first order conditions and the Euler's conditions for the above bellman problem. In this section we discuss the main features of the solution.

Result 1: The no-entry condition is binding in the first period of the plan.

Result 2: If $\theta > 0$ and the no-entry-condition is not binding the solution is characterised by:

$$\beta\left[(1-\gamma)\widetilde{W}_n(n') + \gamma W_n(n', A')\right]p'(\theta) = \frac{\lambda}{\beta}\frac{-cq'(\theta)}{q^2(\theta)}\frac{1}{1-n} \tag{16}$$

$$c = q(\theta)\beta\omega \tag{17}$$

Result 3 : $F_A = G_A = 0$

Result 1 is important to characterise the behaviour of the union since it clarifies that the latter exploits

its power of full profit appropriation only at the beginning of a plan, i.e. when full appropriation is of no consequence for job creation. In fact, job creation only depends on profits from the second period onwards.

Result 1 is also important from a technical perspective. In setting up the union problem we have proceeded *as if* $\widetilde{w}(n)$ and $\widetilde{\omega}(n)$ were known functions. Yet, $\widetilde{w}(n)$ and $\widetilde{\omega}(n)$ are in fact unknown since they are part of the solution: $\widetilde{w}(n) = w(n, 0)$ and $\widetilde{\omega}(n) = \omega(n, 0)$. These expressions make it clear that, for consistency, the wage and the promised value at the time of re-optimisation must coincide with the optimal wage and the optimal promised value once A is reset to zero. In other words, here we face a circularity. To be able to compute a solution we need to know the solution. In this respect, Result 1 is remarkable because it allows to ignore this circularity. In fact, the result says that the no-entry condition is binding at times of re-optimisation:

$$(p - \widetilde{w}(n) + (1 - \rho)\beta\widetilde{\omega}(n) = 0 \tag{18}$$

In turn, this implies that we may drop from the bellman the unique term that contains the two unknown functions.

Result 2 provides conditions for efficient job creation. Equation 17 represents the behaviour of firms as it replicates the free entry condition. In equilibrium, firms enter the market up to the point holding an extra vacancy entails a per-period expected benefit equal to the per-period cost. Equation 16 represents the behaviour of the union once firm entry is properly taken into account. The equation balances the marginal cost and the marginal benefit for the union from inducing the creation of an extra vacancy. The LHS represents the marginal benefit. An extra vacancy at current time increases next period employment by $p'(\theta)$. The value of an extra employed worker turns out to be equal to $W_n(n', A')$ if the plan continues and to $\widetilde{W}_n(n')$ if the plan is re-optimised. The RHS represents the marginal cost. From 17, to induce the creation of an extra vacancy at current time the union needs to increase the promised value by $-\frac{cq'(\theta)}{q^2(\theta)}\frac{1}{1-n}\frac{1}{\beta}$. This variation is multiplied through λ as the latter represents the cost in terms of current utility from a marginal increase in the promised value.

Result 3 implies that the state in the next period does not depend on the current value of A, i.e. it is irrelevant whether the plan continues or it is re-optimised at current time. It follows that the dynamics of employment is not affected by the actual sequence of re-optimisations. The basic intuition for this result is the following. Vacancies posting depends on future wages not on current wages. Thus, if the plan interrupts at current time, the union exploits the chance of pushing firms onto the no-exit boundary uniquely by increasing current wages. Future optimal wages are not revised at all and, as a consequence, vacancy posting is not affected.

The relevant implication of result 3 is that convergence to the steady state is not perturbed by events that trigger a re-optimisation. Thus, the steady state is reached asymptotically with a speed that does not depend on actual history of reoptimisations. Thanks to this result, we may proceed as it is customary in the search and matching literature and focus on the properties of the steady state.

3.4 Steady State

In this section we focus on the determination of $[n^*, A^*, w^*, \theta^*, \omega^*, \lambda^*]$, which represents the vector of steady state levels for the corresponding variables. In addition, we only consider a steady state with positive employment. Below we report 5 of the 6 equations that are required to solve for these levels:

$$A^* = n^* \tag{19}$$

$$n^* = p(\theta)/(\rho + p(\theta)) \tag{20}$$

$$\omega^* = \frac{\alpha(p - w^*)}{1 - \alpha\beta(1-\rho)} \tag{21}$$

$$c = q(\theta^*)\beta\omega^* \tag{22}$$

$$\lambda^*/\beta = \left[(1-\rho) - \frac{\gamma}{\alpha}\right] A^* \tag{23}$$

Equation 19 replicates the first order condition for optimal wage setting if the no-exit conditions is non-binding. In fact, this condition can not be binding in the steady state as firms would not post vacancies and steady state employment would not be positive. The condition equates the marginal benefit from a small increase in wages (n^*) with the corresponding marginal cost (A^*). Equation 20 derives from employment dynamics. Equation 21 uses the definition of the promised value 12 plus the corollary of Result 1 spelled in equation 18. Equation 22 replicates equation 17 while equation 23 derives from the dynamics of A.

To close the system, we need the steady state version of equation 16. The difficulty with this equation, however, is that we have proceeded so far as if the function $\widetilde{W}(n)$ were known, which is in fact not true. To circumvent this problem we observe that, for consistency, $\widetilde{W}(n)$ must be equal to $W(n, A)$ once A is set to nil:

$$\widetilde{W}(n) = W(n, 0)$$

Thus, setting $A = 0$ in the the above bellman 14, we obtain a recursive expression for $\widetilde{W}(n)$. In turn, the recursive expressions for $W(n, A)$ and $\widetilde{W}(n)$ can be used to compute the steady state derivatives $W_n(n^*, A^*)$ and $\widetilde{W}_n(n^*)$ that appear in equation 16 (see the appendix for details). Once these steps have been take, equation 16 becomes

$$\beta \frac{\gamma (w^* - b) + (1 - \gamma) [p - b + (1 - \rho) \beta \omega^*]}{1 - \beta [1 - \rho - p(\theta^*)]} p'(\theta^*) = \frac{\lambda^*}{\beta} \frac{-cq'(\theta^*)}{q^2(\theta^*)} \frac{1}{1 - n^*} \tag{24}$$

The latter closes the system that solves for the steady state. In the next subsection we deal with the comparative statics of the economy at the steady state.

3.5 Comparative Statics

We now focus on the comparative statics of the economy described by equations 19-24. Since the system is non-linear we collapse the economy in two equations for θ^* and w^* and use graphical analysis:

$$
\begin{aligned}
\frac{c}{q(\theta^*)} &= \beta\frac{\alpha(p-w^*)}{1-\alpha\beta(1-\rho)} && (25)\\
\frac{w^*-b}{p-w^*} &= \Gamma(\gamma,\alpha,\theta^*) \qquad \Gamma(\gamma,\alpha,\theta^*) \equiv \frac{1-\beta\left[1-\rho-p(\theta^*)\right]}{1-\beta\alpha\left(1-\rho\right)}\frac{\gamma-\alpha\left(1-\rho\right)}{\rho}\frac{\eta(\theta^*)}{1-\eta(\theta^*)}-\frac{1-\gamma}{1-\beta\alpha\left(1-\rho\right)} && (26)
\end{aligned}
$$

The first is often referred as the job-creation condition. This condition is derived from combining equations 21 and 22, it is meant to determine market tightness for given wages. The second is similar to a traditional wage-setting schedule and may be interpreted as a condition that determines wages for given market tightness. It is obtained by substituting equations 19-23 in 24 and by using the expressions for the elasticity of $q(\theta)$ and $p(\theta)$.

In the remainder of this section we study the economy under some special cases.

3.5.1 No Commitment

Under no commitment - i.e. $\gamma = 0$ - equations 25 and 26 can not be used. In fact, these equations are derived upon the assumption of positive steady state employment while we will shortly see that, under no commitment, one obtains $n^* = 0$.

To understand this result compute the foc for w from the belmann 14:

$$n = \sigma - A$$

The absence of a commitment technology implies that A is set to nil in every periods and, as a consequence, that the no-exit condition is binding ($\sigma > 0$) unless current employment is zero. In turn, if the no-exit condition is binding at all times, the promised value ω is always nil and vacancy posting is zero. This implies that the union sets a wage equal to p at all times and that employment declines at rate ρ until it reaches asymptotically the level $n^* = 0$.

3.5.2 Full Commitment

Assume that $\gamma = 1$ and that the firms know that the announced plan is irreversible, i.e. $\alpha = 1$. In this case, the job creation condition is similar to the textbook specification while the wage setting condition simplifies as follows:

$$\frac{w^* - b}{1 - \beta\left[1 - \rho - p(\theta^*)\right]} \Big/ \frac{p - w^*}{1 - \beta\left(1 - \rho\right)} = \frac{\eta(\theta^*)}{1 - \eta(\theta^*)}$$

The LHS represents the ratio between the value of employment and the value of a matched firm while the RHS gives the ratio between the job filling and the job finding elasticity. Thus, the expression reproduces the so called Hosios condition for efficient wage setting. Since the utility of the employed and of the unemployed are weighted equally, the union is concerned with maximising the flow of total net production. For this reason, it commits to a wage policy that induces the same allocation commanded by a benevolent planner.

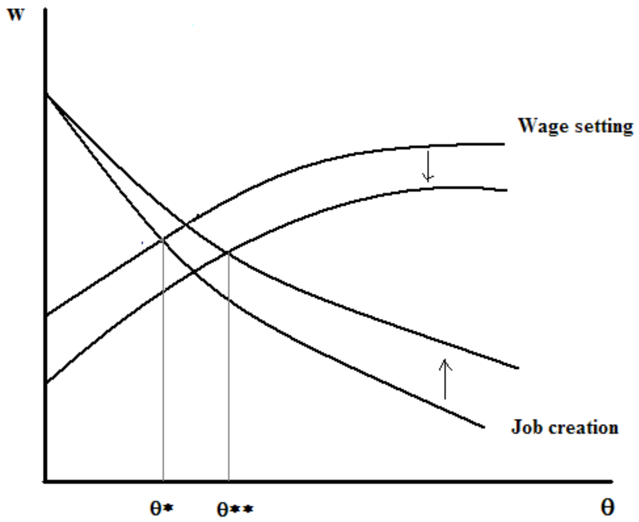
3.5.3 Distrust

Assume that $\gamma = 1$ but firms distrust the union and underestimate its ability to comply with the announced plan: $\alpha < 1$. In this case, the wage setting line becomes

$$\frac{w^* - b}{1 - \beta\left[1 - \rho - p(\theta^*)\right]} \Big/ \frac{p - w^*}{1 - \beta\alpha\left(1 - \rho\right)} \frac{\rho + \left(1 - \alpha\right)\left(1 - \rho\right)}{\rho} = \frac{\eta(\theta^*)}{1 - \eta(\theta^*)}$$

We obtain an Hosios condition similar to the case of full commitment with the exception that the value of the firm is multiplied through the factor $\frac{\rho+(1-\alpha)(1-\rho)}{\rho}$. We interpret this term as a wedge between the private and the social value (union value) of a matched firm. This wedge is induced by the difference in expected destruction. The union expects destruction at rate ρ while entrepreneurs expect destruction at rate $\rho + (1 - \alpha)(1 - \rho)$. More in detail, they expect genuine destruction at rate ρ and, conditional on survival, expect re-optimisation at rate $(1 - \alpha)$. The latter event, however, turns out to be similar to genuine destruction as in both cases firm value is driven to zero.

Figure 1 depicts the job creation condition and the wage setting line for $\Gamma = \Gamma(1, \alpha, \theta^*)$. In addition, the figure illustrates the effects of an increase in trust (i.e. an increase in α):



The impact of an increase in α.

The job creation condition is downward sloping since an higher wage discourages entry and reduces market tightness. By contrast, the wage setting schedule is upward sloping ($\Gamma_\theta > 0$) since an higher market tightness increases the probability of re-matching after destruction and, for given wages, reduces the net gain from being employed instead of unemployed[1]. As a consequence, to restate the efficient sharing of match surplus the wage has to rise.

As α increases the job creation condition moves upward. This is due to the fact that firms attach a lower probability to re-optimisations. Thus, the same level of market tightness can be attained through higher announced wages. By contrast, as α increases the wage setting condition moves downward. Technically, an increase in α increases the private value of firms for given wages but reduces the gap between the private and the social value. The net effect is a reduction in thesocial value (union value) of firms so that, to restate efficiency, wages must decrease. More intuitively, the gain in trust raises the incentive for the union to announce low future wages. When trust is weak, firms discount future profits at a very high rate along the announced wage profile. Thus, low wages are not so effective in stimulating current job creation. By contrast, when trust is strong, future profits are discounted at a lower rate and wages

[1] In a decentralised equilibrium the wage setting line is upward sloping because higher market tightness increases the fallback option of workers in wage bargaining. The wage increases as a consequence of the improvement in the bargaining position of workers.

are more effective for current job creation. From this perspective, the downward movement of the wage setting condition is due to a stronger union incentive towards wage moderation. The picture illustrates how the two effects of trust interact and move equilibrium market tightness to the right.

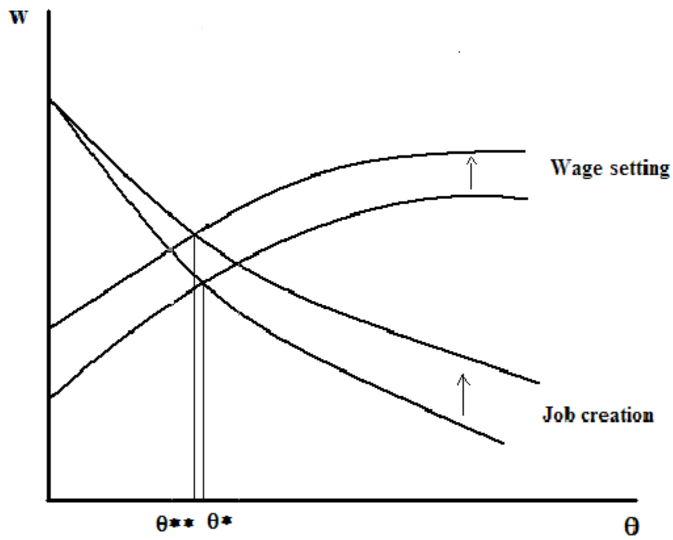
3.5.4 Imperfect Commitment

Assume that distrust is absent but that the commitment technology is not perfect: $\alpha = \gamma = \psi$. We term this situation as imperfect commitment. In this case, ψ replaces α in the job creation condition while the wage setting schedule reads as follows:

$$\frac{w^* - b}{p - w^*} = \Gamma(\psi, \psi, \theta^*) = \frac{1 - \beta\left[1 - \rho - p(\theta^*)\right]}{1 - \beta\psi\left(1 - \rho\right)}\psi\frac{\eta(\theta^*)}{1 - \eta(\theta^*)} - \frac{1 - \psi}{1 - \beta\psi\left(1 - \rho\right)}$$

Figure 2 depicts the two schedules and illustrates the impact of an improvement in technology, i.e. an increase in ψ. Both schedules move upwards, this leads to an increase in wages and to an ambiguous impact in market tightness. In any case, it can be proved that the value of the plan $\widetilde{W}(n)$ increases with respect to ψ.[2]

[2] Debortoli and Nunes (2010) study the behaviour of a fiscal authority that has access to an imperfect committment technology and prove that welfare increases if the technology improves. Their proof can be adapted to our economy with no major change.



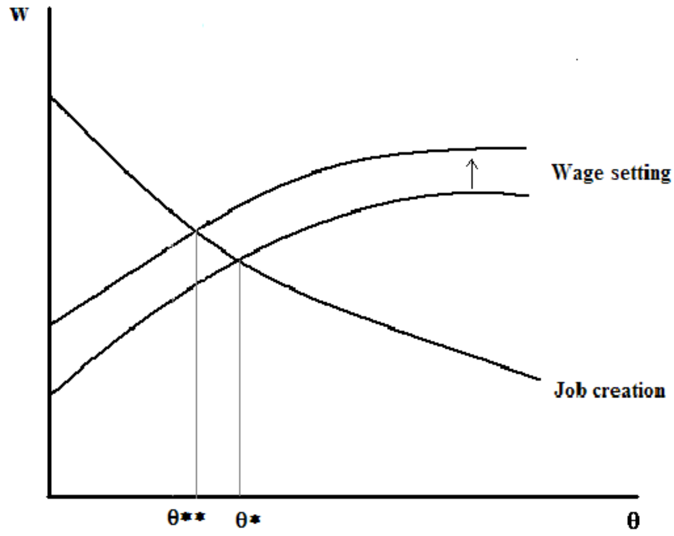
The impact of an increase in ψ.

3.5.5 Imperfect commitment and distrust

In this section we analyse the general case with imperfect commitment and distrust: $1 > \gamma \geq \alpha$.

The impact from an increase in α is qualitatively similar to the one depicted in Figure 1, the job creation condition moves upwards while the wage setting schedule downwards ($\Gamma_\alpha < 0$). Thus, an improvement in trust increases market tightness even if the union holds doubts on the robustness of the commitment.

Picture 3 illustrates the impact from an increase in γ for a given α. The job creation condition is not affected while the wage setting schedule moves upwards ($\Gamma_\gamma > 0$). As a consequence, equilibrium wages increase and market tightness decrease. The intuition for this result is the following. From the point of view of the union, re-optmisations are similar to a lump-sum non-distorsive taxation on firms. The larger the number of firms the larger the revenue from the tax. Thus, as γ increases, the frequency of taxation decreases so that the benefit of having a large tax base is lower. For this reason, the union has a lower incentive to moderate wages and to sustain job creation.



The impact of an increase in γ.

4 Empirical Analysis

4.1 Empirical strategy and data

In the sections above we have presented a model that allows to predict a negative relationship between unemployment and the credibility of wage announcements by the union. In this section we would like to check whether this prediction is consistent with some basic cross-country evidence.

To accomplish this task we need primarily to measure the sentiment of trust or credibility that surrounds the announcements and, more generally, the actions of unions. For this purpose, we observe that, in the real world, the notions of trust and credibility overlap with that of cooperation. Indeed, James (2002) explains that trust represents a way to obtain a pareto-efficient cooperative solution in a prisoner dilemma context. Thus, from the perspective of our model, we conjecture that high credibility tends to be associated with cooperative industrial relations whereas low credibility with adversarial relations.

We test the prediction of the model by exploiting time-series and cross-country variability in unemployment and quality of industrial relations. In particular, we augment a standard phillips-type equation by adding a measure of industrial relations quality to the vector of regressors.

We use a panel that includes 20 OECD countries observed for 15 years, from 1990 to 2004; annual data are averaged over 5-years periods in order to clear for short run movements. Information regarding the rate of unemployment and its main determinants - inflation, unemployment benefits, labour taxation, employment protection, bargain institutions - is the one provided by the OECD and largely used in the macro-labour empirical literature (Nuziata 2003, for instance). The OECD, however, does not provide systematic information on the climate of industrial relations for member countries. To fill the gap, we resort to the index of "perceived" cooperation in industrial relations computed by the World Economic Forum (WEF). This index is constructed by asking a panel of qualified operators to quantify over a given scale the degree of cooperation in their country. For instance, in 1997 respondents were asked to express their opinion on the sentence "Labor-employer relations are generally cooperative" (answers: 1=strongly disagree, 7=strongly agree).

Due to the subjective nature of these answers doubts may arise regarding the reliability of the index. This issue, however, has been already addressed by Blanchard and Philippon (2006), who conclude that the index is a good approximation for an ideal objective measure in light of the high correlation with lagged measures of strike activity. The WEF index is available annually for a large number of countries since 1985. However, the wording of the question asked by the interviewers has changed over time, especially in early years. Thus, to preserve a certain degree of uniformity, we have decided to drop observations for years 1985-1989. This explains the reason for our dataset to begin with the year 1990. Finally, for the purpose of estimation, a weakness of the WEF indicator is the small degree of variability. To get around this problem we have re-scaled the index over a 4-points array (0,1,2,3) by using quartile thresholds.

4.2 Evidence

In table 1 we summarise results from OLS estimation. In model 1, we condition unemployment on the index of industrial relations quality (I.R. Quality), on the change in inflation and, finally, on a set of institutional determinants. In model 2, we add four union variables that contribute to the description of the bargaining environment (union centralisation, union coordination, union coverage and union density).

Observe that, consistent with our prediction, the quality of industrial relations has a negative impact on unemployment. In particular, this variable turns out to be the one with the highest statistical significance. Furthermore, the size of the coefficient decreases only marginally when other union variables are added to the conditioning vector.

Dependend variable: unemployment		
Model	I	II
I.R. Quality	-1.590***	-1.516***
	(.288)	(.307)
Inflation(t) - Inflation(t-1)	-1.026*	-1.031*
	(.608)	(.619)
Replacement Rate	-.009	-.034
	(.021)	(.027)
Labour Taxation	.117***	.121**
	(.042)	(.050)
Epl	-.319	-.562
	(.464)	(.573)
Other Union Controls	No	Yes
Rsq.	0.56	.56
Nr. Obs.	60	60

Table 1: Robust standard errors in parentesis; *** 1% significance, ** 5% significance, * 10% significance.

We are aware that unobserved country heterogeneity could bias estimation by affecting both the rate of unemployment and the cooperative climate of the bargaining environment. Yet, by its own nature, I.R. quality does not exhibit much time variability so we can not disentangle the impact of heterogeneity by using country dummies. In spite of this warning, however, we regard the evidence in the table as basically consistent with our model.

5 Concluding Remarks

Informal and formal evidence show that countries with cooperative industrial relations exhibit a good performance in the labour market. In this paper, we show that this evidence can be explained through a very intuitive mechanism and by means of a standard labour market description. In the model at the core of the paper, firms hold up from creating jobs since they attach some probability to the fact that the union may renege on pre-announced low wages. The reason from holding up relates to the irreversible nature of searching expenses. Firms can not recoup these expenses if they wish to dissolve the match face to high wage claims. In this context, job creation gains momentum if firms trust the union or, more in general, if industrial relations are characterised by cooperative attitudes.

This explanation for the link between unemployment and industrial relations is quite general as it only requires union power and search imperfections. The first ingredient is common to many European economies. The second ingredient is thought to be a feature common to all labour markets.

The paper offers an alternative to the explanation advanced by Blanchard and Philippon (2004). In their framework, bad industrial relations lead to higher unemployment only in presence of recessionary shocks. This is due to the fact that bad relationships slow down the adjustment of wages face to these shocks. By the same token, however, bad industrial relationships should also lead to *lower* unemployment if the economy is hit by expansionary shocks. Or, more in general, their model does not imply any long run relationship between labour market performance and the quality of industrial relations.

In sharp contrast with this conclusion, our model *does imply* a long-run positive link between performance and quality. Thus, a necessary follow up of this research is to check the consistency of our prediction over long time intervals.

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Technical Appendix

From equation 9 to equation 10

Use $u_t = n_j w_j + (1 - n_j)b$ and substitute $P(t)$ in 9:

$$u_0 + \sum_{t=1}^{\infty}(\gamma\beta)^{t-1}\beta(1-\gamma)\widetilde{W}(n_t) + \Omega \qquad \Omega \equiv \sum_{t=1}^{\infty}\gamma^{t-1}(1-\gamma)\sum_{j=1}^{t-1}\beta^j u_j \tag{27}$$

Expand Ω as follows:

$$\Omega = (1-\gamma)\sum_{j=1}^{0}\beta^j u_j + \gamma(1-\gamma)\sum_{j=1}^{1}\beta^j u_j + \gamma^2(1-\gamma)\sum_{j=1}^{2}\beta^j u_j + \gamma^3(1-\gamma)\sum_{j=1}^{3}\beta^j u_j + ...$$

Observe that $\sum_{j=1}^{0}\beta^j u_j = 0$ and expand summations over j:

$$\Omega = (1-\gamma)\cdot 0 + \gamma(1-\gamma)\cdot[\beta u_1] + \gamma^2(1-\gamma)\cdot\left[\beta u_1 + \beta^2 u_2\right] + \gamma^3(1-\gamma)\cdot\left[\beta u_1 + \beta^2 u_2 + \beta^3 u_3\right] + ...$$

Collect u-terms with the same time index:

$$\Omega = \beta u_1 \cdot \sum_{t=1}^{\infty}\gamma^t(1-\gamma) + \beta^2 u_2\cdot\gamma\cdot\sum_{t=1}^{\infty}\gamma^t(1-\gamma) + \beta^3 u_3\cdot\gamma^2\cdot\sum_{t=1}^{\infty}\gamma^t(1-\gamma) + ...$$

Observe that $\sum_{t=1}^{\infty}\gamma^t(1-\gamma) = \gamma$ and rearrange:

$$\Omega = \sum_{t=1}^{\infty}(\beta\gamma)^t u_t$$

Substitute Ω in 27 and simplify to obtain equation 9 in the main text

$$\sum_{t=0}^{\infty}(\beta\gamma)^t\left[u_t + \beta(1-\gamma)\widetilde{W}(n_{s+1})\right]$$

From equation 11 to equation 12

Define the indicator I_t as follows:

$$I_t = \begin{matrix} 1 & \text{if a plan continues at time } t \\ 0 & \text{if a new plan starts at time } t \end{matrix}$$

Write 11 as follows:

$$\begin{aligned} \omega_t &= \alpha \left[E_t \left((p - w_{t+1}) \middle| I_{t+1} = 1 \right) + E_t \left(\sum_{j=2}^{\infty} [\beta(1-\rho)]^{j-1} (p - w_{t+j}) \middle| I_{t+1} = 1 \right) \right] + \\ &\quad +(1-\alpha) \left[E_t \left((p - w_{t+1}) \middle| I_{t+1} = 0 \right) + E_t \left(\sum_{j=2}^{\infty} [\beta(1-\rho)]^{j-1} (p - w_{t+j}) \middle| I_{t+1} = 0 \right) \right] \end{aligned}$$

Use the law of iterated expectations and change the index of summations ($s = j - 1$):

$$\begin{aligned} \omega_t &= \alpha \left[(p - w_{t+1}) + \beta(1-\rho) E_t \left(E_{t+1} \left(\sum_{s=1}^{\infty} [\beta(1-\rho)]^{s-1} (p - w_{t+1+s}) \middle| I_{t+1} = 1 \right) \right) \right] + \\ &\quad +(1-\alpha) \left[(p - \widetilde{w}(n_{t+1})) + \beta(1-\rho) E_t \left(E_{t+1} \left(\sum_{s=1}^{\infty} [\beta(1-\rho)]^{s-1} (p - w_{t+1+s}) \middle| I_{t+1} = 0 \right) \right) \right] \end{aligned}$$

Use the definition of ω_t and $\widetilde{\omega}_t$:

$$\omega_t = \alpha \left[(p - w_{t+1}) + \beta(1-\rho) E_t (\omega_{t+1}) \right] + (1-\alpha) \left[(p - \widetilde{w}(n_{t+1})) + \beta(1-\rho) E_t (\widetilde{\omega}(n_{t+1})) \right]$$

Notice that ω_{t+1} and $\widetilde{\omega}(n_{t+1})$ are both non-stochastic at time t. Thus, the latter is similar to equation 12 in the main text.

Recursive Formulation

Express the forward looking constraint as a sum of infinite terms:

$$\omega_t = \sum_{j=0}^{\infty} [\alpha\beta(1-\rho)]^j \{\alpha(p - w_{j+1}) + (1-\alpha)[(p - \widetilde{w}_{j+1}) + \beta(1-\rho)\widetilde{\omega}_{j+1}]\}$$

Use $(\gamma\beta)^s \zeta_s$, $(\gamma\beta)^s \mu_s$ and $(\gamma\beta)^s \sigma_s$ for the static constraints and $(\gamma\beta)^s \lambda_s$ for the forward looking dynamic constraint and write the lagrangean of problem 13:

$$\mathcal{L} = \sum_{t=0}^{\infty} (\gamma\beta)^t \left\{ \Phi_t + \lambda_t \sum_{j=0}^{\infty} [\alpha\beta(1-\rho)]^j g_{t+1+j} \right\}$$

$$\begin{aligned} g_s &= \alpha(p - w_s) + (1-\alpha)[(p - \widetilde{w}(n_s)) + (1-\rho)\widetilde{v}(n_s)] \\ \Phi_t &: [n_t w_t + (1-n_t)b] + (1-\gamma)\beta\widetilde{W}(n_{t+1}) + (\zeta_t - \mu_t\theta_t)\left[\frac{c}{q(\theta_t)} - \beta\omega_t\right] + \sigma_t[(p - w_t) + (1-\rho)\beta\omega_t] - \lambda_t\omega_t \end{aligned}$$

Expand the summations and collect Φ and g terms with the same time index:

$$\begin{aligned} &\Phi_0 \\ &(\beta\gamma)\left\{\Phi_1 + \lambda_0 \frac{g_1}{\beta\gamma}\right\} \\ &(\gamma\beta)^2\left\{\Phi_2 + \left[\lambda_0\frac{\alpha}{\gamma}(1-\rho) + \lambda_1\right]\frac{g_2}{\beta\gamma}\right\} \\ &(\gamma\beta)^3\left\{\Phi_3 + \left[\lambda_0\left[\frac{\alpha}{\gamma}(1-\rho)\right]^2 + \lambda_1\frac{\alpha}{\gamma}(1-\rho) + \lambda_2\right]\frac{g_3}{\beta\gamma}\right\} \\ &\ldots \end{aligned}$$

Set

$$A_t = \frac{\alpha}{\gamma}[(1-\rho)A_{t-1} - \lambda_{t-1}/\beta] \qquad A_0 = 0 \tag{28}$$

Use $A_t$and rearrange the lagrangean:

$$\mathcal{L} = \sum_{t=0}^{\infty} (\gamma\beta)^t \left\{\Phi_t - \frac{1}{\alpha}A_t g_t\right\}$$

The recursive formulation 14 in the main text follows from the latter.

FOCs (complementary slackness omitted)

$$
\begin{aligned}
n-\sigma+A &= 0 && (29)\\
-(\zeta-\mu\theta)\frac{cq'(\theta)}{q^2(\theta)}+\mu\left[\beta\omega-\frac{c}{q(\theta)}\right]+\beta\left[(1-\gamma)\widetilde{W}_n(n')+\gamma W_n(n',A')\right]p'(\theta)(1-n) &\leq 0 \quad \theta\geq 0 && (30)\\
-(\zeta+\mu\theta)+\sigma(1-\rho) &= \lambda/\beta && (31)\\
\frac{c}{q(\theta)}-\beta\omega &\geq 0 \quad \zeta\geq 0 && (32)\\
\theta\left[\frac{c}{q(\theta)}-\beta\omega\right] &= 0 && (33)\\
(p-w)+(1-\rho)\beta\omega &\geq 0 \quad \sigma\geq 0 && (34)\\
-\omega-\alpha W_A(n',A') &= 0 && (35)
\end{aligned}
$$

EULERs

$$W_n(n,A) = (w-b)+\beta\left[(1-\gamma)\widetilde{W}_n(n')+\gamma W_n(n',A')\right][1-\rho-p(\theta)] \quad (36)$$

$$W_A(n,A) = -(p-w)+\beta\alpha(1-\rho)W_A(n',A') \quad (37)$$

Proof of Result 1

If a plan starts at time τ, A_τ is set to zero. As a consequence, 29 implies

$$n_\tau=\sigma_\tau>0$$

Since $\sigma_\tau>0$, the no-exit constraint is binding.$\diamond$

Proof of Result 2

The first equation follows from combining 30 and 31. If $\theta>0$, the expression holds as an equality while the term $\beta\omega-c/q(\theta)$ drops. The second equation is implied by 33.$\diamond$

Proof of Result 3

Use the policy functions $\mathbf{x}(n, A)$ and describe the evolution of state variables as follows:

$$\begin{aligned} n' &= F(n, A) = (1 - \rho)\, n + p\, [\theta(n, A)]\, (1 - n) \\ A' &= G(n, A) = \frac{\alpha}{\gamma}\, [(1 - \rho)\, A - \lambda(n, A)/\beta] \end{aligned}$$

Observe that equation 35 clarifies that the promised value ω depends only on the next period state (n', A'). Thus, the policy function $\omega(n, A)$ can be written as

$$\omega(n, A) = \widehat{\omega}(F(n, A), G(n, A))$$

Further, observe that a) equation 33 implies that θ depends only upon ω, b) equation 32 implies that ζ depends only upon ω and θ, c) equation 30 implies that μ depends only upon ω, θ, ζ and n. These observations boil down to the following expressions for the policy functions $\theta(n, A)$, $\zeta(n, A)$ and $\mu(n, A)$:

$$\begin{aligned} \theta(n, A) &= \widehat{\theta}(F(n, A), G(n, A)) \\ \zeta(n, A) &= \widehat{\zeta}(F(n, A), G(n, A)) \\ \mu(n, A) &= \widehat{\mu}(F(n, A), G(n, A), n) \end{aligned}$$

Finally, observe that 29 implies $\sigma(n, A) = n + A$ while equation 31 gives

$$\lambda(n, A)/\beta = -\widehat{\zeta}(F(n, A), G(n, A)) + \widehat{\mu}(F(n, A), G(n, A), n)\widehat{\theta}(F(n, A), G(n, A)) + \sigma(n, A)\, (1 - \rho)$$

We are now ready to differentiate F and G with respect to A:

$$F_A(n,A) = p'(\theta)(1-n)\frac{d\widehat{\theta}(F(n,A),G(n,A))}{dA}$$

$$G_A(n,A) = \frac{\alpha}{\gamma}(1-\rho) - \frac{\alpha}{\gamma}\frac{d\lambda(n,A)/\beta}{dA}$$

Differentiate $\widehat{\theta}(F(n,A),G(n,A))$ and $\lambda(n,A)/\beta$, substitute and rearrange:

$$\begin{aligned}
F_A &= \frac{p'(\theta)(1-n)\frac{d\widehat{\theta}}{dG}}{1-p'(\theta)(1-n)\frac{d\widehat{\theta}}{dF}}G_A \\
G_M &= \frac{\alpha}{\gamma}(1-\rho) - \frac{\alpha}{\gamma}\left[-\widehat{\zeta}_F F_A - \widehat{\zeta}_G G_A + \widehat{\mu}\widehat{\theta}_F F_A + \widehat{\mu}\widehat{\theta}_G G_A + \widehat{\mu}_F\widehat{\theta} F_A + \widehat{\mu}_G\widehat{\theta} G_A + (1-\rho)\right]
\end{aligned}$$

Observe that in the second equation, the RHS simplifies so that all remaining terms contain either F_A or G_A. The proposition is proved since the only solution of this system is $F_A = G_A = 0$.$\diamond$

Equation 24

This equation is derived from 16 upon substituting the term $(1-\gamma)\widetilde{W}_n(n^*) + \gamma W_n(n^*,A^*)$. To find this term proceed through the following steps.

1. Compute the main bellman 14 at $A = 0$ and recall that, in this case, the no-exit condition is binding:

$$W(n,0) = n\left[p + (1-\rho)\beta\omega\right] + (1-n)b + (1-\gamma)\beta\widetilde{W}(n') + (\zeta - \mu\theta)\left[c/q(\theta) - \beta\omega\right] - \lambda\omega + \gamma\beta W(n',A')$$

2. Use the identity $\widetilde{W}_n(n) = W_n(n,0)$ in the latter and compute the derivative with respect to n (using the envelope theorem):

$$\widetilde{W}_n(n) = (p-b) + (1-\rho)\beta\omega + \left[(1-\gamma)\beta\widetilde{W}(n') + \gamma\beta W_n(n',A')\right]\left[1-\rho-p(\theta)\right]$$

3. Evaluate the latter at the steady state:

$$\widetilde{W}_n(n^*) = (p-b) + (1-\rho)\,\beta\omega^* + \left[(1-\gamma)\beta\widetilde{W}(n^*) + \gamma\beta W_n(n^*, A^*)\right][1-\rho-p(\theta^*)]$$

4. Evaluate the Euler's condition 36 at the steady state:

$$W_n(n^*, A^*) = (w^* - b) + \beta\left[(1-\gamma)\widetilde{W}_n(n^*) + \gamma W_n(n^*, A^*)\right][1-\rho-p(\theta^*)]$$

5. Solve the last two equations with respect to $\widetilde{W}_n(n^*)$ and $W_n(n^*, A^*)$:

$$(1-\gamma)\widetilde{W}_n(n^*) + \gamma W_n(n^*, A^*) = \frac{\gamma\,(w^* - b) + (1-\gamma)\,[p - b + (1-\rho)\,\beta\omega^*]}{1 - \beta\,[1-\rho-p(\theta^*)]}$$