Education and Income Mobility

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Abstract

We study income mobility in a two-period overlapping generation model. Mothers choose whether invest in offspring education or not. Children are hetereogenous both with respect to inborn ability and parental income. The model shows that in the long run a steady state is achieved; along the transition path we can have both upward and downward mobility; in the first case children coming from poor families tend to invest in education over time while the opposite in the second. Hence, the number of educated worker in the economy is increasing or deacresing over time according to low educated individuals behaviour.

1 Introduction

This paper focusses on the relationship between investment in schooling and social mobility. Literature on human capital stressed this role of education since pioneering contributions by Becker and Schultz. Individual investment in education provides a powerful channel for social mobility; nevertheless when education is costly and credit market are imperfect or, even worse, missing, richer individuals are the only who can afford this type of investment. In this case social mobility is very low, in sense that richer are better off over time while the opposite happens for poor. In other words, inequality persists and gets worse over time.

In this point of view, there is enough room for policy actions, either via redistributive scheme or via schooling reforms supporting luckless agents. Our goal is the latter. In this paper we analyze a dynamic economy with intergenerational transfers and heterogeneity of agents with respect both to ability and to starting income. Liquidity constraints are at work, inducing a strong form of persistent inequality. However poor but talented individuals find beneficial to afford the education cost, thanks to the higher wage that skilled workers get

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in the economy. The model is a general equilibrium scheme, so exogenous parameters are related only to technology; this is particularly beneficial because the model can be analyzed only by numerical schemes and the low number of parameters allows us to exploit a large number of possible cases.

In general, the numerical exercises show a low degree of social mobility and rich tends to get richer and richer over time and conversely for poor people. As said, this opens to a policy evaluation of alternative measure for fighting poverty traps and under-investment in education. This second part of the paper is still in progress and it does not appear in this preliminary version.

2 The Model

The economy consists of two sectors (firms): the first sector produces a final commodity by two inputs: unskilled labour L_u and technology G. The latter is produced by the second sector; this one is represented by a multi-product firm supplying both new technology G and schooling slots n. The technology frontier is traditionally convex in the final production G and n, i.e.:

$$L_s = G^{\delta} + n^{\alpha}$$

where L_s is the only production input (skilled labour) and $\delta, \alpha \geq 2$.

Agents are heterogeneous both with respect to inborn ability θ and parental income (low and high, respectively Y_H, Y_L). At each instant t two generations are alive: mothers and daughters. There is a daughter per each mother and populations is stationary. At the beginning of the world, mothers are exogenously distributed in skilled and unskilled. Mothers take care of offspring by leaving it their life income. As we shall see, the schooling choice can be either performed by mothers or daughters; final results are not very different in the two assumptions but the model is.

Section I: Positive Analysis

3 Sector 1: Final commodity

This sector produces the only consumption good (numeraire) according to a Cobb-Douglas production function:

$$Q_1 = G^{1-\beta} L_u^{\beta}$$

hence the profit function is:

$$\pi_1 = G^{1-\beta} L_u^{\beta} - w_u L_u - pG$$

By first order conditions we obtain:

$$
\begin{aligned}
p &= (1-\beta)\left(\frac{Lu}{G}\right)^{\beta} \\
w_u &= \beta\left(\frac{G}{L_u}\right)^{1-\beta}
\end{aligned}
$$

4 Sector 2: joint production

This sector produces two commodities, G and n, by means of skilled labour; the convex technology frontier is given as usual by:

$$L_s = G^{\delta} + n^{\alpha}$$

The supply of graduates slots, n, is charged by a tuitition fee k per student; for such a reason the profit function for this firm is:

$$\pi_2 = kn + pG - w_s(G^{\delta} + n^{\alpha})$$

First order conditions lead to:

$$
\begin{aligned}
p &= \delta w_s G^{\delta-1} \\
k &= \alpha w_s n^{\alpha-1}
\end{aligned}
$$

By matching demand and supply of G, the equilibrium value for the hi-tech input is:

$$G^* = \left(\frac{\delta}{1-\beta} w_s\right)^{\frac{1}{1-\delta-\beta}} (1-L_s)^{\frac{\beta}{\delta+\beta-1}}$$

5 Household Behaviour

We can think to economy as populated by two-period lived agents; in the first period agents are young and they do not produce income. Either they attend school or they consume leisure, giving up to accumulate human capital; this depends on their parents choice. In the second one, agents are old, produce labour income according to their accumulated human capital and then die. Old agents can decide to give up to second period consumption by investing in offspring education. Hence the choice of investing in human capital is entirely in the hands of the mothers generation.

Mothers are divided into "rich" and "poor", or high H and low L income. Each parent, indifferently of her income, must decide whether enroll the only daughter to school or not. This decision relies on the comparison of expected income accruing to offspring in the two future states: working as skilled or unskilled. Education is costly and success in schooling depends on offspring inborn ability which is known to parents.

Mother i is indifferent between the two future states when:

$$U(Y^i-k)+[\theta E(U(w_s)|I_t)+(1-\theta)E(U(w_u)|I_t)] = U(Y^i)+E(U(w_u)|I_t) \qquad i=H,L \tag{1}$$

The left hand side shows the mother welfare in case she enrolls daughter to school; the first term on L.H.S. is the mother utility loss due to fee payment and the second one is the utility gain she receives by the higher expected wage earned by daughter when she is employed as skilled rather than unskilled worker. This second term depends on the probability θ in succeeding at school, i.e. on daughter ability. The right hand side is simply the mother's utility in case of no schooling.

Equation 1 can be rearranged in:

$$U(Y^i)-U(Y^i-k)=\theta\left[U(E(w_s|I_t))-(U(E(w_u|I_t))\right] \tag{2}$$

It is worth stressing that parental income comes only from the labour market, hence a parent is rich when $Y^H=w_s$ and poor when $Y^L=w_u$. If we use the time index t, then equation 2 can be rewritten as:

$$U(w_{i_t})-U(w_{i_t}-k)=\theta\left[U(E(w_{s_{t+1}}|I_t))-(U(E(w_{u_{t+1}}|I_t))\right] \qquad i=s,u$$

Generally speaking, equation 2 provides two cut-off value, θ_s and θ_u; rich (poor) families whose daughter ability is higher than θ_s (θ_u) strictly prefer schooling. It is obviously expected that $\theta_s<\theta_u$ since the relative cost of schooling is lower for rich families.

6 Equilibrium

6.1 Case I: Static Expectations

In order to find the cut-off value, we have to specify how expectations about future daughter wage are modelled; in this first case we assume static expectations (random walk), namely:

$$E(w_{i_{t+1}}|I_t)=w_{i_t} \qquad i=s,u \tag{3}$$

In order to close the model, we have to account for two flow constraints, viz.:

$$n = (1-\theta_s(n))L_s+(1-\theta_u(n))L_u \tag{4}$$
$$L_u = 1-L_s \tag{5}$$

Constraint 4 show the equilibrium between available school slots (supply), n, and number of people enrolled in education (demand), where it is implicitly assumed that inborn ability is distributed according to a uniform distribution on $[0,1]$. So, $(1-\theta_s)L_s$ is the number of children from rich (skilled) households enrolled in schooling and, likewise for $(1-\theta_u)L_u$. The condition 5 guarantees for full employment; agents are working either as skilled or as unskilled but nobody is unemployed.

From the equilibrium conditions we obtain:

$$w_s(n) = \frac{1-\beta}{\delta}(1-L)^{\beta}\left[L-n^{\alpha}\right]^{\frac{1-\delta-\beta}{\delta}} \tag{6}$$

$$w_u(n) = \beta\left(\frac{\delta}{1-\beta}w_s\right)^{\frac{1-\beta}{1-\delta-\beta}}(1-L)^{(1-\beta)\frac{1-\delta}{\delta+\beta-1}} \tag{7}$$

$$\theta_s(n) = \frac{\ln(w_s)-\ln(w_s-\alpha n^{\alpha-1}w_s)}{\ln(E(w_{s_{t+1}}|I_t))-(\ln(E(w_{u_{t+1}}|I_t))} \tag{8}$$

$$\theta_u(n) = \frac{\ln(w_u)-\ln(w_u-\alpha n^{\alpha-1}w_s)}{\ln(E(w_{s_{t+1}}|I_t))-(\ln(E(w_{u_{t+1}}|I_t))} \tag{9}$$

By substituting 8 and 9 in 4 the model can be solved for n^*, the number of daughters enrolled at school.

Under the assumption 3 equation 8 can be rewritten as:

$$\theta_s(n) = \frac{-\ln(1-\alpha n^{\alpha-1})}{\Delta-\ln(L-n^{\alpha})}$$

where Δ is a positive parameter depending on L. It is straightforward to show that:

$$\theta_s(0)=0 \qquad \theta_s(\hat{n})=\infty \rightarrow \frac{\partial\theta_s}{\partial n}>0$$

where $\hat{n}=\frac{1}{\alpha}^{\frac{1}{\alpha-1}}$.

Lemma 1 *Likewise it is possible to show that* $\frac{\partial\theta_u}{\partial n}>0$.

In fact $\frac{\partial w_s}{\partial n}<0$, being $1-\delta-\beta<0$, and consequently $\frac{\partial w_u}{\partial n}>0$; by starting from a positive value for the numerator of 9, i.e. $w_u-\alpha n^{\alpha-1}w_s>0$, and increasing n we let tend $w_u-\alpha n^{\alpha-1}w_s$ to zero e consequently $-\ln(w_u-\alpha n^{\alpha-1}w_s)$ to infinity.

Summing up:

Proposition 2 *Equation 4 has a unique fixed point* n^*.

Proof. The proof comes from looking at the right hand side of equation 4. When $n=0$ this function starts from 1, in base to lemma1 and lemma2. By increasing n we have both $(1-\theta_s)$ and $(1-\theta_u)$ converging to zero, even if with different velocity. Hence the right hand side of 4 is monotonically decreasing in n, This means that there is only one crossing point where both sides of the equation match. ■

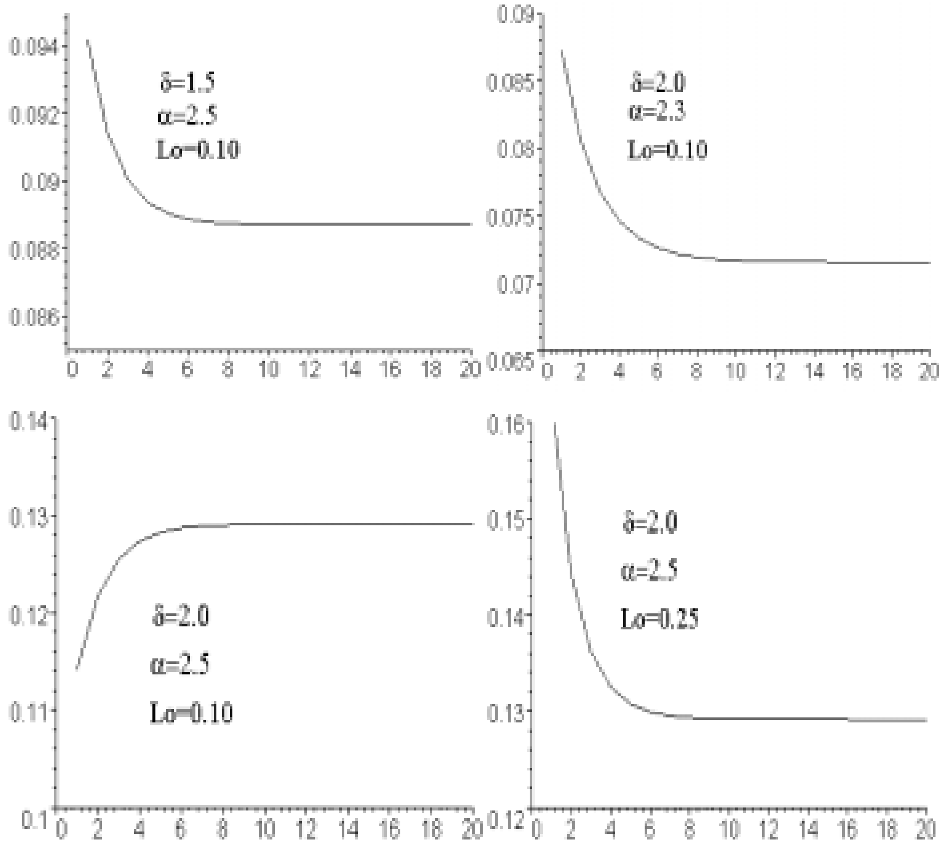
Given $n^*(L_s)$ we can calculate the number of children who succeed in schooling; these provide the new vintage of skilled worker, $L_{s_{t+1}}$ and the economy starts again with a new parents generation:

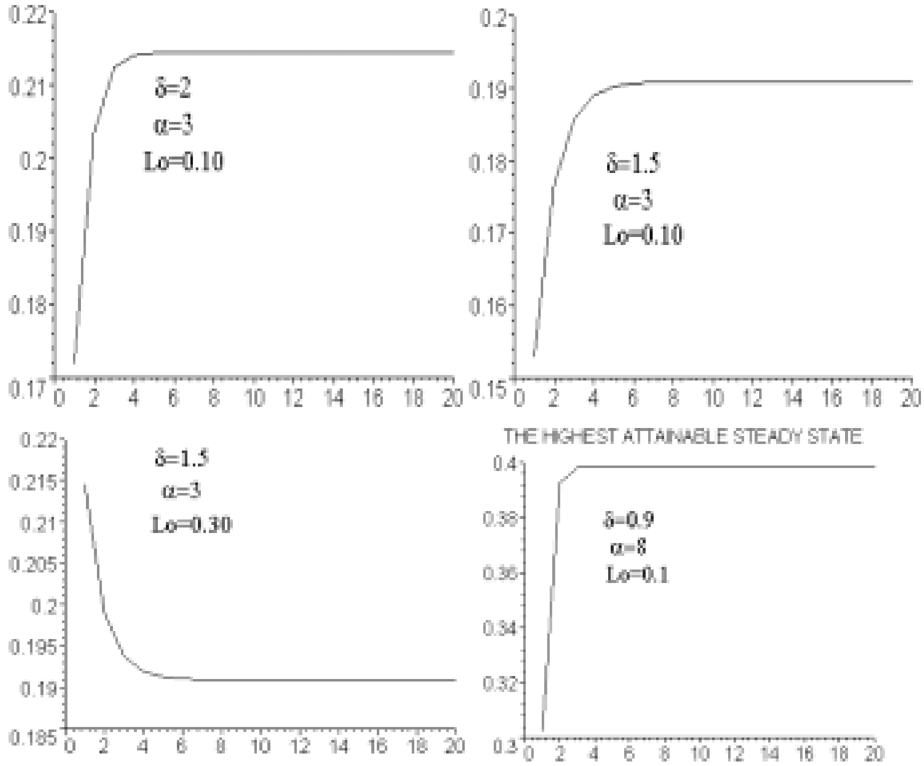
$$L_{s_{t+1}} = L_{s_t} \int_{\theta_s(L_{s_t})}^{1} \theta d\theta + (1 - L_{s_t}) \int_{\theta_u(L_{s_t})}^{1} \theta d\theta \qquad (10)$$

with L_{s_t} given.

Equation 10 provides the human capital (skilled workers) evolution over time. It owns a steady state; in fact we have previously shown that there exists n^* such that both θ are bounded from zero and one. This means that we can consider θ be a constant in 10 and it is easy to show that for $L_s = 0$ the intercept is positive and that the slope is less than one.

In the following we perform several numerical simulation supporting this conclusion.





Numerical results show that there exists a long run steady state which is monotonically approached either from above or below; when approached from above, the number of skilled workers is going down over time. This is due to the progressive abandon of schooling from children coming from unskilled, or poor, mothers. In fact θ_u increases steadily until its steady state and conversely θ_s decreases. The net effect is a progressive reduction in L. In other words, we have a downward mobility for unskilled and upward for rich; inequality gets worse over time. Opposite conclusions hold when L is approaching the steady state from below.

6.2 Perfect Foresight

The hypothesis of static expectation can be relaxed in favour of a rational expectation scheme (perfect foresight). In this case the previous theoretical scheme does not allow us to find either an explicit or numerical solution. This is due to the strong non-linearity of the model. However a perfect foresight assumption can be still useful by a slight change in the model basic setup.

Unlike the previous section, we are going to assume that the educational choice is burdened on daughters rather than on mothers. Young individuals are endowed with a given mothers income Y_H and Y_L and the number of rich (poor) children at time zero is equal to the number of the mothers L_0, $(1 - L_0)$. By so doing, at the beginning of the world, agents are partitioned in two separate classes; the richest (educated) are in number of L_0 and characterized by income Y_H whilst the remaining ones are in number of $1 - L_0$ with income Y_L.

In the first period of life, youngest must decide whether attending school or not; as in the previous section, education is costly but the new hypothesis here is that youngest must refund the education cost in the second period, i.e. during the working life. In other words, at time t the young generation decide how much "knowledge units" n_t must be consumed (human capital demand). These units must be produced in the second period, n_{t+1} (human capital supply), by the same individuals; equilibrium condition requires $n_t = n_{t+1}$. The key assumption here is the way schooling is produced and consumed. Education is a scarce resource; children consume it when young but they must reconstitute the amount of consumed units by producing them in the second period.

With this new assumption, the schooling technology is in the hands of the current generation instead of the previous one, and each generation is responsible by in its turn for the accumulation of knowledge. Mothers have only a bequest reason but they do not affect the educational choice.

The timing is as follows: at the starting time zero there are L_0 skilled mothers and consequently $1 - L_0$ unskilled mothers. Each mother has a daughter. Offspring of skilled mothers receive Y_H and Y_L for the remaining ones. Daughters must decide how much units of schooling they find optimal to buy by knowing that this educational demand must be filled by themselves in the next period, $t = 1$, at the production price k_1.

Summing up, daughters, at time zero, maximize equation 2 under a perfect foresight assumption, i.e. $E(w_{i_{t+1}}|I_t) = w_{i_{t+1}}, i = s, u$ and with $k_0 = \beta k_1$ where β is the discount factor. Wages w_{s1} and w_{u_1} come from 6 and 7 respectively; they depend on L_1 the number of daughters who succeed at schooling and L_1 comes from 10, which is determined by solving equations 2 and 4 under the equilibrium constraint $n_0 = n_1$. The model is now closed and L_{s1} and n_1 are jointly carried out; by them wages are calculated and the model can now restart with $Y_H = w_{s1}$ and $Y_U = w_{u_1}$. The following diagram shows the logical algorithm:

$$\boxed{\textit{Initial Conditions}:\ L_0, 1 - L_0, Y_H, Y_L}$$
$$\Downarrow$$
$$\boxed{w_{u1}(L_1), w_{s1}(L_1), G_1(L_1)}$$
$$\Downarrow$$
$$\boxed{\theta_{s0}(L_1), \theta_{u0}(L_1)}$$
$$\Downarrow$$
$$\boxed{n_0 = n_1(L_1), L_1(n_1)}$$
$$\Downarrow$$
$$\boxed{\text{Given the I.C. we solve for } L_1, w_{u,1}, w_{s1}}$$
$$\Downarrow$$
$$\boxed{\text{New I.C.: } L_1, Y_L = w_{u,1}, Y_H = w_{s1}}$$
$$\Downarrow$$
$$\boxed{\text{New Generation...}}$$

6.2.1 Numerical Results

The numerical simulations show that, likewise the previous model, the dynamics is characterized by a long run steady state; the transition path converges monotonically, either from below or from above, to the long run equilibrium. Similarly to the previous section, parameter α and δ are responsible for the long run value, while initial conditions do not seem play a particular role, since the stability in large of the steady state.

Conclusions about social mobility are the same of the previous case; when L is increasing in approaching the long run value, θ_s increases and θ_u decreases; we have upward mobility, in sense that the number of children enrolled successfully at school is increasing over time and conversely for rich individual. Opposite conclusions hold when the steady state is approached from above.

Section II - Normative Analysis

To be continued.

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