Integration of Unemployment Insurance with Pension Through Individual Savings Account

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ABSTRACT

This paper analyzes a social insurance system that integrates unemployment insurance with pension program through an individual savings account, allowing workers to borrow against their future wage income and thus improves their search incentives while reducing risks. Taking into account the advantage of tax-funded insurance, this paper identifies factors on which the optimal degree of pension-funded self-insurance through an individual savings account depends. We show that when unemployment duration is very short compared to the period of employment or retirement, the optimal system involves an exclusive reliance on pension-funded self-insurance, which yields a negligible risk burden for workers without attenuating their search incentives. We also consider a case of multiple risks - unemployment and disability - in which the integration of both unemployment and disability insurance with a pension program through a joint savings account is desirable unless the two risks are perfectly correlated to each other.

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1. Introduction

The East Asia crisis brought home to much of the developing world a lesson that the Great Depression brought home to more advanced countries seventy years ago – the importance of a safety net. But as countries like Korea go about constructing their safety nets, they are cognizant of the complaints that have been raised against unemployment insurance systems: they attenuate incentives. To be sure, there are adverse incentive effects (or, as they are today generally referred to, moral hazard effects) in all insurance programs. What worries critics is that the risk reduction benefits might, on the face of it, be outweighed by the adverse incentive effects. For most individuals, a typical spell of unemployment is less than six months (and that spell would presumably be shorter, possibly much shorter) in the absence of unemployment insurance. Over a working time of, say, forty five years, an individual with three such spells would lose perhaps 4% of his lifetime income – a risk which presumably the individual could easily absorb if he had sufficient savings or could borrow against future earnings. With the bulk of savings used for retirement, and mostly dedicated to social security programs, the amount individuals have to buffer themselves against these income shocks is limited; and well documented limitations in capital markets make it difficult for individuals to borrow much against future earnings. Compulsory old age public pension programs, while they help resolve one problem -the tendency of individuals not to save enough for their old age, because of the proclivity of public "bail-outs"- exacerbate another.

This naturally leads to the suggestion of an integrated unemployment and pension program. Such integration makes particular sense with the systems of individual accounts, which are increasingly forming the basis of even public pension programs. In such programs, benefits are related to contributions by simple formulae; in the simplest, there is no redistribution. Such programs are like defined contribution pension programs, though some of the contributions can be used to "purchase" insurance (e.g. against inflation or interest fluctuations) which is not available on the market. But it is easy to impose redistributions on top. For simplicity, in this paper, we will ignore the redistributive components.

Under the integrated unemployment-pension system through an individual savings account, an individual who is unemployed can have his unemployment payments taken out of his individual account. Thus, the individual obtains the liquidity to maintain his standard of living; the compulsory and universal nature of the contributions provide, in effect, perfect collateral, i.e., early on in his life, his balance could go negative. The fact that normally the risk is small means that the individual can bear this risk – when it is spread out over his entire life; and since the individual bears the risk, there is no attenuation of search incentives.

If, however, the loss from unemployment is large enough, it is optimal to have some tax-funded unemployment insurance – the individual should not bear the cost even over his lifetime. In general, individuals should not rely exclusively on the savings-funded self-insurance under the integrated system. The system should be supplemented by a tax-funded program. Taking this into account, we characterize in this paper the optimal benefit structure of the integrated system, that is, the optimal combination of the two types of benefits – tax-funded and savings-funded. The lower the risk-aversion, or the greater the elasticity of search with respect to the insurance benefit, the less reliance should be placed on the tax-funded insurance as opposed to (what might be viewed as implicit) savings-funded self-insurance. In an extreme case, if a worker is risk-neutral, then there should be no need for tax-funded unemployment insurance, and if there is no incentive problem associated with unemployment insurance, there is no need to rely on savings-funded self-insurance. Not surprisingly, the larger the risk, which in turn is related to the length of the period of unemployment relative to the working period, the greater the need for tax-funded insurance. In the limit, if the period of unemployment is vanishingly small, then the individual can bear all the risk through savings-funded self-insurance with no welfare loss.

The more interesting problem arises, however, when individuals face multiple risks. We consider a simplified case where there are two risks, one early in life (unemployment) and one late in life (disability), where the second risk is large enough that if the individual pays for his early bout of unemployment out of his lifetime savings, he cannot afford to pay for the later. The integrated lifetime insurance system, which integrates both unemployment insurance and disability insurance with pension program under a joint savings account, has two notable aspects. First, the savings-funded unemployment benefit can generate adverse disincentives due to the possibility of a government bailout in the case of a negative balance. The individual will know that if he does not search, the net payment from the government should he become disabled will be greater. Thus, even under the fully integrated system funded solely by savings , where individuals always pay for the initial bout of unemployment seemingly out of their individual accounts, there is some attenuation of incentives. This problem will never arise if the two risks are perfectly negatively correlated, but it is much more likely to occur when the two risks are highly correlated.

Second, the integrated system through joint savings account enables the disability insurance to respond differently to different balances in the individual savings account, and thus is welfare-improving. This benefit of integration – having a common pool from which to draw upon – gets larger as the correlation gets smaller. When the two risks are perfectly correlated, a single fund will clearly do as well as two funds, simply because there is, in effect, a single risk. We are able to show that so long as the correlation between the two risks is not perfect, then it always pays to have some degree of integration, that is, some of the unemployment and/or disability benefits should come out of individual savings accounts. Not surprisingly, the greater the degree of correlation, the less reliance on individual accounts under the integrated system.

In the next section we present the basic model for the integrated system to characterize its optimal benefit structure and to show how it varies with the relative size of unemployment risk and other parameters. Section 3 also presents a simple model for an integrated lifetime insurance system that integrates both unemployment and disability insurances with pension savings through a joint savings account, and examines how its optimal structure changes with the correlation between different risks. Some concluding remarks are given in Section 4.

2. The Model

Consider an infinitely-lived worker who spend (M+2) periods of working and retires thereafter. In period 2 a worker becomes unemployed with probability q, and the length of unemployment would depend upon his search decision. In this paper we assume that a worker with unemployment shock can choose either 'no search' or 'search', which leads him either to be unemployed for one period or to be unemployed for zero period (i.e., to be reemployed immediately after unemployment shock), respectively. Thus, depending upon his search decision, a worker with unemployment shock can either be unemployed in period 2 or avoid remaining unemployed. The cost of search e is a random variable, which is distributed with distribution function F( ). The search decision by a worker is made through his choice of the threshold search cost e′, such that he chooses to search (or not to search) if e < (or >) e′. Thus the probability of being unemployed in period 1 would be q(1-F(e′)).

The income support system for the unemployed in the model is the one that integrates unemployment insurance with pension through a retirement savings account (RSA). The unemployment benefit provided by the integrated system comes from two sources: unemployment tax, and the past and prospective savings of a worker that is mandated to be made in his retirement savings account (RSA). A critical point of this system is that a worker can borrow against his future retirement savings to finance part of his unemployment benefit. Any negative balance in one's RSA, a case which will be dealt with in the next section, will be bailed out by the government.

In addition to the minimum level of savings mandated by the government, an employed worker may make voluntary savings to his RSA. It is assumed in this section that the retirement savings of a worker is greater than the mandated level, so that a worker may always end up with a positive balance in his RSA at the time of retirement. This would be reasonable because, in this section, the unemployment is the only risk a worker has in his lifetime.[2]

A worker may also contribute to his private savings account from which he can withdraw at anytime. In period 1 a worker may want to save some money in his private savings account to finance any additional consumption during unemployment that is in excess of the unemployment benefit provided by the government.[3] Note, however, that a worker would not need a private savings account after period 1 because any savings he accumulates after period 1 would be for retirement in this model.[4]

Let s_1, s_u , and s_n be the pre-unemployment savings rate in period 1, the post-unemployment savings rate for those with unemployment experience, and the post-unemployment savings rate for those without unemployment experience, respectively. We will make a couple of simplifying assumptions in the model. First, UI tax is paid in period 1 only before unemployment shock. Second, there is no discounting during the first (M+2) periods of one's career, while both income and utility during the retirement period are discounted at the same rate r.

The expected utility of a worker with wage w, $V(r_1, r_2; w, q)$, would then be

[2] In the next section, a worker faces multiple risks including unemployment and therefore he may end up with a negative balance in his RSA due to a reduction in savings caused by another subsequent shock.

[3] It will be shown later by Proposition 1, however, that a worker would not need a private savings account even in period 1.

[4] Note that in this model there is no risk a worker faces in his lifetime other than unemployment.

$$V(r_1, r_2; w, q)$$

$$= \underset{s_1, s_n, s_u, e'}{Max}\ v(s_1, s_n, s_u, e'; r_1, r_2, w, q)$$

$$= \underset{s_1, e'}{Max}\ U(w - s_1 w - tw) + (1-\overline{q}) I(M, r, s_1) + \overline{q} J(r_1, r_2; M, r, s_1) - q\int_0^{e'} edF(e)$$

where

$$I(M, r, s_1) = \underset{s_n}{Max}\ (M+1)U((1-s_n)w) + \int_0^{\infty} U\{((M+1)s_n + s_1^{*})rw\} e^{-rt} dt$$

$$J(r_1, r_2; M, r, s_1) = \underset{s_u}{Max}\ U((r_1 + r_2)w) + MU((1-s_u)w) + \int_0^{\infty} U(\{Ms_u + s_1^{*} - r_2\} rw) e^{-rt} dt$$

while r is the discount rate during retirement period, $\overline{q} \equiv q(1 - F(e'))$, and

$$t = \overline{q} r_1 . \qquad (1)$$

Note that I(.) or J(.) indicates the payoff that is expected at period 2 from not being unemployed or from being unemployed in period 2, respectively. Hereafter, for simplicity, we will suppress the wage notation w in the argument of utility function U( ) except when it is necessary. One thing that should be mentioned here is that the payoff J(.) presumes that an unemployed worker consumes just the amount of the socially optimal consumption level provided by the government. This will be proved later by Proposition 1. Thus, unless the government provides a worker with an unemployment benefit less than the socially optimal amount of consumption, he would not need to make precautionary savings (in private savings account) in period 1 against possible unemployment in period 2.

(1) Characterization of Optimal Benefit Structure of Integrated System

In characterizing the optimal benefit structure of the integrated system we will assume for the moment that the government provides a worker with the full amount of the optimal savings-funded benefit, $r_2^{*}w$, from the RSA. This implies that, unless $r_2^{*}w$ is less than what a worker would like to consume while unemployed, he would put all his pre-unemployment savings s_1 in his RSA for his retirement consumption. We will change this assumption later, however, to show how much government provision of savings-funded benefit is needed when a worker withdraws pre-unemployment savings s_1 from his private savings account to finance his unemployment consumption.

Let us start with checking the choices of savings and the threshold search cost, $(s_1^{*}, s_n^{*}, s_u^{*}, e^{*})$, by an individual worker. First, the decisions on post-unemployment savings rates, which will be made to maximize the payoffs I(.) and J(.), yield the following results:.

$$s_n^* = \frac{1}{1+(M+1)r}(1-s_1^* r) \qquad (2)$$

$$s_u^* = \frac{1}{1+Mr}(1-s_1^* r + rr_2), \qquad (2')$$

or

$$1-s_n^* = \{(M+1)s_n^* + s_1^*\}r = \frac{(M+1+s_1^*)r}{1+(M+1)r} \qquad (3)$$

$$1-s_u^* = \{Ms_u^* + s_1^* - r_2\}r = \frac{(M+s_1^* - r_2)r}{1+Mr}. \qquad (3')$$

Next the savings decision in period 1 will be made as follows:

$$-U'(1-s_1^* - t) + (1-\overline{q})U'\{((M+1)s_n^* + s_1^*)r\} + \overline{q}U'\{(Ms_u^* + s_1^* - r_2)r\} = 0 \qquad (4)$$

Note from (4) that the savings s_1^* in period 1 is determined as a part of retirement savings, not as a precautionary savings against unemployment risk. In collecting comparative statics of the pre-unemployment savings s_1^* we will suppose that q is so small relative to M and $\frac{1}{r}$ that $\frac{\overline{q}}{M} \approx 0$, $\overline{q}r \approx 0$. Then, as the following Lemma shows, the effects of some parameters upon pre-unemployment savings s_1^* are not significant.

Lemma 1

(i) $\frac{\partial s_1^*}{\partial e^*} \approx 0$[5]

(ii) $\frac{\partial s_1^*}{\partial r_1} = -\overline{q}$, $\frac{\partial s_1^*}{\partial r_2} \approx 0.$

(iii) $\frac{\partial s_1^*}{\partial M} \in (-1,0)$, $\left.\frac{\partial s_1^*}{\partial r}\right|_{Mr=a} \in (-1, 0)$, $\frac{\partial s_1^*}{\partial q} \approx -r_1$.

The proof of this Lemma is in the Appendix. First, the tax-funded benefit would reduce pre-unemployment savings because of the tax burden, while the savings-funded benefit would not affect the savings.[6] Second, the pre-unemployment savings s_1 may also depend upon other parameters such as M, r, and q. As the post-unemployment shock period

[5] This result enables us to ignore the indirect effect of any parameter change upon savings s_1 through its effect upon the search decision e^*.

[6] The savings-funded benefit could affect the savings in period 1 because a worker might increase his savings in response to the reduction in RSA balance. However, the magnitude of this effect would be very small because pre-unemployment savings is just a small portion of lifetime savings and because the probability q of unemployment shock is in fact small.

lengthens, pre-unemployment savings would decrease for it would act as a substitute for post-unemployment savings in contributing to retirement savings. The longer retirement period (or the smaller r) would surely increase the pre-unemployment savings even if its ratio to the post-unemployment working period is kept constant as a. The higher probability of unemployment risk would reduce the savings due to the higher tax burden.[7]

A worker also makes his decision on the threshold search cost e′ to maximize V( ), taking the tax t as given.

$$e^* = I(M, r, s_1) - J(r_1, r_2; M, r, s_1). \qquad (5)$$

We can collect some comparative statics of the threshold cost of search effort.

Lemma 2

(i) $\dfrac{\partial e^*}{\partial s_1} = w\{U'(1-s_n{}^*) - U'(1-s_u{}^*)\} < 0$.

(iv) $\dfrac{\partial e^*}{\partial r_1} = -wU'(r_1 + r_2) < 0$

$\dfrac{\partial e^*}{\partial r_2} = w\{-U'(r_1 + r_2) + U'(1-s_u{}^*)\}$

(v) $\dfrac{\partial e^*}{\partial M} < 0$.

The proofs are in the Appendix. First, the pre-unemployment savings would affect search effort decision negatively, because a worker with greater private savings would not take as serious the reduction in his RSA balance.[8] The individual worker's search decision will also be affected by the parameters of the integrated system, (r_1, r_2), which is the source of welfare cost associated with the unemployment insurance system. The tax-funded benefit r_1 will adversely affect search decision, because it increases the consumption under unemployment by that amount. The savings-funded benefit r_2, however, improves the search incentive of a worker relative to the tax-funded benefit to the extent that it is charged to the retirement income. That is,

$$\frac{\partial e^*}{\partial r_2} > \frac{\partial e^*}{\partial r_1}.$$

[7] The probability of unemployment shock may have the opposite effect on $s_1{}^*$ because it increases the savings for the unemployment benefit. This effect is small in this model, however, because the savings in period 1 is small relative to the whole retirement savings that can be used in the case of unemployment.

[8] Since the effect of pre-unemployment savings upon search effort is not negligible, the indirect effect of a parameter change upon search effort through its effect upon savings should be considered, which we do in the model.

The search decision also depends upon the career structure parameterized by M. A long post-unemployment working period would adversely affect the search incentive of a worker by enabling him to mitigate the burden of the reduction in RSA balance through the adjustment of his post-unemployment savings.[9]

Now let us characterize the parameters (r_1^*, r_2^*) of the optimal system. If we differentiate V( ) with respect to r_1, we have

$$\bar{q}[U'(r_1^* + r_2) - U'(1 - s_1^* - t) - U'(1 - s_1^* - t)Hr_1^* wU'(r_1^* + r_2)] \\ = 0 \tag{6}$$

where $H \equiv \frac{f(.)}{1 - F(.)}$, indicating search elasticity or the sensitiveness of reemployment probability to the increase in threshold search cost e'. Here we assume that H is constant for all e'.

Similarly, if we differentiate the expected utility function V( ) with respect to r_2, taking into account the above individual decisions, we have

$$\bar{q}[U'(r_1 + r_2^*) - U'(\{Ms_u^* + s_1^* - r_2^*\}r)\}[1 - \{(1-\bar{q})U'(1 - s_n^*) + \bar{q}U'(1 - s_u^*)\}Hr_1 w] \\ = 0$$

or,

$$r_1 + r_2^* = (Ms_u^* + s_1^* - r_2^*)r = 1 - s_u^* \tag{7}$$

which yields the following by (3'):

$$r_2^* = \frac{1}{1 + (M+1)r}(Mr + s_1^* r) - \frac{1 + Mr}{1 + (M+1)r} r_1$$

Note from (7) that the savings-funded benefit r_2^* is determined solely by consumption smoothing and that it is not affected by incentive consideration.

Then we can prove the following.

Lemma 3

The solution (r_1^*, r_2^*) for (6) and (7) is unique if it exists.

Although the proof can be found in the Appendix, the following Figure 1 gives us an informal explanation of the unique solution.

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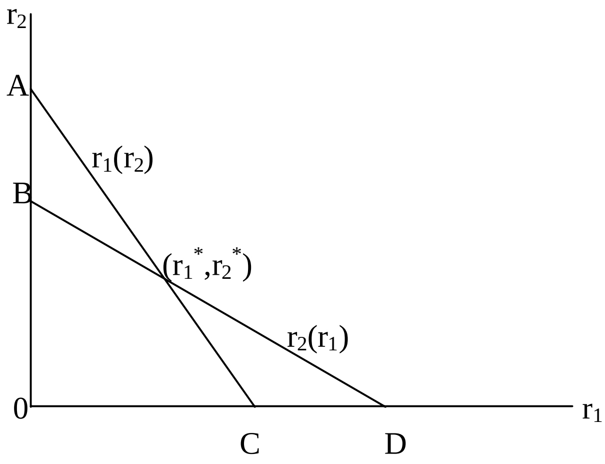
[9] The effect of the retirement duration upon search incentive is ambiguous, because the longer retirement duration would increase both I(.) and J(.) through more effective consumption smoothing.



<Figure 1>

We can see from (6) and (7) that the interior solution (r_1^*, r_2^*) will satisfy the following conditions:

$$X\delta(1-\overline{q}) - U'(wY)Hr_1^* = 0 \ , \qquad (9)$$

$$\begin{aligned} r_2^* &= \frac{1}{1+(M+1)r}(Mr + s^* r) - \frac{1+Mr}{1+(M+1)r} r_1^* \\ &= \frac{1+Mr}{1+(M+1)r}(1-r_1^*) - \frac{1-s_1^* r}{1+(M+1)r} \end{aligned} \qquad (10)$$

where $\delta \equiv -\frac{U''}{U'}$,

$$\begin{aligned} X &\equiv s_u^* - s_n^* \\ &= \frac{r}{1+(M+1)r}(1-r_1) \end{aligned} \qquad (11)$$

and

$$\begin{aligned} Y &\equiv r_1^* + r_2^* \\ &= \frac{r}{1+(M+1)r}(M + s_1^* + r_1^*), \end{aligned} \qquad (12)$$

We can see from (9) that the tax-funded benefit r_1^* is affected by risk-aversion δ and search elasticity H. Also, (10) implies that, for a given tax-funded benefit r_1^*, the savings-funded benefit r_2^* is determined by the consumption smoothing after being unemployed. Note that Y indicates the usual replacement ratio defined as the ratio of the unemployment benefit to the wage. The variable X represents the increase in savings, which a worker would like to have after experiencing unemployment in period 2 to recover from the reduction in RSA balance. As X can be rewritten as

$$X \equiv s_u^* - s_n^* = (1 - s_n^*) - (1 - s_u^*)$$
$$= (1 - s_n^*) - Y,$$

X reflects the difference between the total amount of unemployment benefits and the amount of consumption by a worker if he were not unemployed. That is, X represents the degree of the incompleteness of unemployment insurance. If X = 0, for example, it implies that a worker is fully insured against unemployment risk. In general, however, X is greater than zero due to the incentive problem associated with the unemployment insurance system. In this sense X also indicates the welfare cost minimized by the optimal integrated system.

Before analyzing how the benefit structure (r_1^*, r_2^*) of the optimal system changes with some parameters, we must make an important point. The amount r_2' that an unemployed worker would like to withdraw from his account for his consumption given tax-funded benefit r_1 is the same amount as the socially optimal savings-funded benefit r_2^* that the government would provide to him. That is,

Proposition 1

$r_2' = r_2^*$ for any given tax-funded benefit r_1.

<Proof>

The privately optimal savings-funded benefit r_2' of an unemployed individual will be the one that maximizes his expected utility taking UI tax t as given:

$$\overline{q}[U'(r_1 + r_2') - U'(\{Ms_u^* + s_1^* - r_2'\}r)] = 0,$$

which is the same condition (7) for r_2^*.

The private marginal benefit of the savings-funded benefit is in general not equal to its social marginal benefit because an individual worker would not take into consideration the incentive effect of his savings-funded benefit. Since the private marginal benefit is proportional to the social marginal benefit, however, the two optimal choices, r_2' and r_2^* would be the same. In other words, both r_2^* and r_2' are determined solely by the same consumption smoothing. This point is important, because unless the government provides a worker with less than the optimal savings-funded benefit r_2^*, a worker could use his savings s_1 in period 1 just for his retirement.[10]

(2) Comparative Statics

Assuming that an interior solution exists, we will examine some important factors that determine the optimal benefit structure of the integrated system. First, we will start with the typical factors affecting the reward structure under moral hazard – risk-aversion δ and search elasticity H.

Proposition 2

(i) $\frac{\partial r_1^*}{\partial H} < 0, \quad \frac{\partial r_2^*}{\partial H} > 0, \quad \frac{\partial r_1^*}{\partial \delta} > 0, \quad \frac{\partial r_2^*}{\partial \delta} < 0$.

(ii) $\frac{\partial Y}{\partial H} < 0, \quad \frac{\partial Y}{\partial \delta} > 0$.

The proof of Proposition 2 is included in the Appendix. As the search elasticity (indicated by H) increases, the potential incentive cost of unemployment insurance grows also large, which tends to favor savings-funded benefit more than tax-financed benefit. Since higher search elasticity implies higher efficiency cost associated with unemployment insurance, however, it would reduce the total amount of unemployment benefit. The greater risk-aversion of a worker, on the other hand, which implies the greater need for insurance against unemployment risk, is likely to favor tax-financed benefit more than savings-financed benefit and an increase in the total unemployment benefit.

An interior solution may not exist in the model, however. We can present the following corner solutions, which may be informative.

Corollary 1

If δ = 0, $r_1^* = 0$

If H = 0, $r_2^* = 0$

The above results are clear from conditions (9) and (10). When a worker is risk-neutral, there is no need for unemployment insurance. If there is not an incentive problem associated with unemployment insurance, savings-funded benefit is unnecessary. Corollary 1 thus implies that the search incentive issue is the very reason the integrated system is introduced.

Turning back to the interior solution (r_1^*, r_2^*), we can identify a couple of other factors that can affect the optimal benefit structure of the integrated system.

[10] In other words, a worker would not need his private savings account to make any precautionary savings against possible unemployment in period 2.

Proposition 3

(i) $\frac{\partial r_1^*}{\partial q} < 0,\ \frac{\partial r_2^*}{\partial q} > 0,\ \frac{\partial Y}{\partial q} < 0$.

(ii) $\frac{\partial r_1^*}{\partial w} \le (or \ge) 0,\ \frac{\partial r_2^*}{\partial w} \ge (or \le) 0,\ \frac{\partial Y}{\partial w} \le (or \ge) 0$ as δw ≤ (or ≥) 1.

The proof is also delegated to the Appendix. Proposition 3 (i) implies that a high unemployment risk leads to high (expected) incentive costs, resulting in low tax-financed benefit and a low level of total unemployment benefit. A high unemployment risk, however, increases savings-funded unemployment benefit due to the reduction in tax-funded benefit and a decrease in pre-unemployment savings. Proposition 3 (ii) suggests that, provided that a worker ends up with positive RSA balance at the time of retirement, the relationship between a worker's tax-funded benefit and his wage is dependent upon his relative risk-aversion.[11]

Now we will move on to more important comparative statics. The optimal benefit structure of the integrated system changes with the individual career structure, such as the duration of employment and retirement relative to unemployment duration, which are indicated by M and r in the model. In response to a reduction in his RSA balance a worker with unemployment experience would increase his savings over the post-unemployment working period to partially offset the change in retirement income. In other words, a worker would optimally adjust to the reduction in RSA balance by reducing consumption not only over the retirement period but also over the post-unemployment working period. Thus, the change in the career structure, which affects the worker's pattern of consumption smoothing, will affect the optimal benefit structure of integrated system.

Since the length of employment or retirement period would also affect consumption per period of a worker, the absolute amounts of benefits would not be relevant in identifying effects of the career structure upon the optimal benefit combination. Let

$$\gamma_i^* \equiv \frac{r_i^*}{r_1^* + r_2^*}, \quad i = 1,2$$

which indicates the share of tax-funded or of savings-funded benefit to the total benefit consumed by a worker. Let also

[11] Technically the relative risk-aversion of a worker could affect some other comparative statics through the change of {U′(X)X} in X. Whenever the relative risk-aversion is greater than, or equal to, or less than 1, {U′(X)X} is decreasing, or constant, or increasing in X, respectively. In order to avoid the technical complications caused by this, however, we will hereafter assume that the relative risk-aversion of a worker is 1, i.e., that δw = 1.

$$y \equiv \frac{Y}{1-s_n^*}.$$

The variable y indicates the replacement ratio, which is defined as the ratio of unemployment benefit to what a worker could consume if not unemployed.

Rewriting condition (10) as follows:

$$X(1-\bar{q}) - Z(wY)H\gamma_1^* = 0 \qquad (13)$$

we can collect the following results.

Proposition 4

Suppose that δw =1. Then,

(i) $\frac{\partial \gamma_1^*}{\partial M} < 0, \quad \frac{\partial \gamma_2^*}{\partial M} > 0, \quad \frac{\partial y}{\partial M} > 0,$

(ii) $\left.\frac{\partial \gamma_1^*}{\partial r}\right|_{Mr=a} > 0, \quad \left.\frac{\partial \gamma_2^*}{\partial r}\right|_{Mr=a} < 0, \quad \left.\frac{\partial y}{\partial r}\right|_{Mr=a} < 0$

The proof is in the Appendix. Proposition 4 (i) describes the effects of the post-unemployment working period upon the benefit structure. As the post-unemployment working period gets longer or as unemployment occurs in the earlier stage of one's career, the optimal system entails a higher share of savings-funded benefit to tax-funded benefit and a greater replacement rate. Also, as Proposition 4 (ii) demonstrates, the optimal system entails the same result as the retirement period lengthens while keeping its ratio to duration of employment constant as a. These results come from the fact that a longer post-unemployment or retirement period can reduce the adverse risk effect of savings-funded benefit. First, a worker with unemployment experience can ease the burden of a reduction in RSA balance by increasing his savings during the post-unemployment period, mitigating the welfare burden by spreading it out more effectively over the longer post-unemployment period. Second, a worker can mitigate the welfare burden of a reduction in RSA balance more effectively by spreading it out over the longer retirement period. Thus the integrated system would bring to us more welfare gain as the post-unemployment or retirement period lengthens relative to unemployment duration.

(3) Welfare Performance of Integrated System: Possibility of The First-Best State

In this subsection we will discuss how the integrated system could possibly reduce welfare distortion associated with the insurance system as the post-unemployment or retirement period gets longer. We will first characterize the first-best state as a benchmark. The first-best state can be represented by two elements. One is the equal marginal utility of income (i.e., the equal amount of consumption) for every state in each period, which is realized through full unemployment insurance. The second element is the efficient choice of the threshold search effort.

The first-best threshold search cost, e^o, will be determined as the level which maximizes V given the parameter values:

$$e^o = U'(z^o)w \tag{14}$$

where z^o is the first-best consumption per period. Since the expected total wage income W would be

$$W = (M + 2 - \bar{q}^o)w$$

where $\bar{q}^o \equiv q(1 - F(e^o)))$, the first-best per-period consumption z^o would be

$$z^o = \frac{M + 2 - \bar{q}^o}{M + 2 + \frac{1}{r}} w = \frac{(M + 2 - \bar{q}^o)r}{(M + 2)r + 1} w. \tag{15}$$

Thus the first-best payoff for a worker, denoted by V^o, will be

$$\begin{aligned} V^o &= (M + 2 + \frac{1}{r})U(z^o) - \int_0^{e^o} edF \\ &= (M + 2 + \frac{1}{r})U(z' - \frac{\bar{q}^o}{M + 2 + \frac{1}{r}} w) - \int_0^{e^o} edF \\ &\approx (M + 2 + \frac{1}{r})U(z') - \bar{q}^o wU'(z') - \frac{(\bar{q}^o w)^2}{2(M + 2 + \frac{1}{r})} U'' - \int_0^{e^o} edF \end{aligned} \tag{16}$$

where $z' \equiv \frac{M + 2}{M + 2 + \frac{1}{r}} w$.

We can rewrite condition (5) for the threshold level of search cost as follows:

$$\begin{aligned} e^* &= \{I(.) - J(.)\} \\ &= U(1 - s_n^*) - U(r_1^* + r_2^*) + (M + \frac{1}{r})\{U(1 - s_n^*) - U(1 - s_u^*)\} \\ &= (M + 1 + \frac{1}{r})\{U(1 - s_n^*) - U(1 - s_u^*)\} \\ &\approx (M + 1 + \frac{1}{r})\{U'(1 - s_u^*)Xw + U'' \frac{X^2 w^2}{2}\} \\ &= (1 - r_1^*)wU'(1 - s_u^*) + \frac{U''}{2}(M + 1 + \frac{1}{r})X^2 w^2 \end{aligned} \tag{17}$$

The payoff for a worker then under the optimal system (r_1^*, r_2^*), denoted by V*, would be

$$
\begin{aligned}
V^* &= U(1-s_1^*-t)+(1-\bar{q})I(.)+\bar{q}J(.)-q\int_0^{e^*} edF \\
&= U(1-s_1^*-t)+(M+1+\frac{1}{r})U(1-s_n^*) \qquad (18)\\
&\quad -\bar{q}\{(1-r_1^*)wU'(1-s_u^*)+\frac{U''}{2}(M+1+\frac{1}{r})X^2w^2\}-q\int_0^{e^*} edF
\end{aligned}
$$

We can now establish the following important proposition:

Proposition 5

(i) As M $\rightarrow \infty$, $r_1^* \rightarrow 0,\quad r_2^* \rightarrow 1$, which approximates the first-best state where $z^o = w,\quad e^o = U'(w)w$.

(ii) As r $\rightarrow$ 0 while Mr = a, $r_1^* \rightarrow 0,\quad r_2^* \rightarrow \frac{a}{1+a}$ $(or\ \gamma_2^* \rightarrow 1)$, which approximates the first-best state where $z^o = \frac{a}{1+a},\quad e^o = U'(\frac{a}{1+a})w.$

<Proof>

(i) X $\rightarrow$ 0 as M $\rightarrow \infty$ by (11), which implies that $r_1^* \rightarrow 0$ by (9) and thus that $r_2^* \rightarrow 1$ by (10). By (2) and (2') we have $s_n^*, s_u^* \rightarrow 0$ as M goes to infinity. Since $(M+1+\frac{1}{r})X^2 \rightarrow 0$ as M $\rightarrow \infty$, we have from (17)

$$e^* = \{I(.)-J(.)\} \rightarrow U'(w)w,$$

which implies the first-best search decision by (14). As M $\rightarrow \infty$, $s_1^*, s_u^* \rightarrow s_n^*$ by (2), (2') and (4). Also, by (3) and (3'), $(1-s_u^*),(1-s_1^*) \rightarrow z'$ and $z' \rightarrow 1$. Thus, by (16) and (18), we have, as M $\rightarrow \infty$,

$$
\begin{aligned}
V^o - V^* &\rightarrow (M+1+\frac{1}{r})(U(z')-U(1-s_n^*)) \\
&\rightarrow (\frac{1+(M+1)r}{r})\frac{(s_1^*-s_n^*)r}{1+(M+1)r}U'(z') = (s_1^*-s_n^*)U'(z') \\
&\rightarrow 0
\end{aligned}
$$

because $s_1^* \rightarrow s_n^*$ as M $\rightarrow \infty$.

(ii) X $\to 0$ as r $\to 0$ by (11), which implies that $r_1^* \to 0$ by (9), and $r_2^* \to \frac{a}{1+a}$ by (10). Note that this implies that $\gamma_2^* = 1$. By (2) and (2') we have $s_n^*, s_u^* \to \frac{1}{1+a}$ as r goes to zero. Since $(M + 1 + \frac{1}{r})X^2 \to 0$ as r $\to 0$, we have from (17)

$$e^* = \{I(.) - J(.)\} \to U'(\frac{1}{1+a})w,$$

which implies the first-best search decision by (14). As r $\to 0$, $s_1^*, s_u^* \to s_n^* \to \frac{1}{1+a}$ by (2), (2') and (4). Also, by (3) and (3'), $(1-s_u^*), (1-s_1^*) \to z'$ and $z' \to \frac{a}{1+a}$. Thus, by the same reasoning as above, $V^* = V^o$ as r $\to 0$. *Q. E. D.*

Proposition 5 highlights one of the important aspects of the integrated system. As the period of post-unemployment or retirement gets longer compared to the period of unemployment, the integrated system makes the amount of welfare distortion associated with savings-funded benefit arbitrarily small. This is because the system makes its adverse risk effect as small as possible, while maintaining the desired level of search incentive through the reduction in RSA balance. In the limiting case, this will lead to the complete replacement of tax-funded benefit by savings-funded benefit and will lead to the first-best outcome.

When the retirement period and post-unemployment working period increase proportionally, the first-best consumption per period is determined by the ratio a of the working period to the retirement period. Since the consumption costs equal less than the wage, the replacement ratio, defined as the consumption (under unemployment) over wage, should be less than one in the first-best state. Note, however, that all unemployment benefits are financed by RSA savings, not by taxes, under the first-best integrated system.

(4) The Role of Government Provision of Savings-funded Benefit

The role of the government in the integrated system is critical to the extent that it allows a worker to borrow against his future savings in order to finance his unemployment benefit. Without the government provision of savings-funded benefit,[12] a worker would have to withdraw his pre-unemployment savings to maintain the privately optimal consumption level during unemployment. He then needs to make some precautionary

[12] In fact, a full provision of savings-funded benefits by the government may have some problems. One of them would be the cost of the mandatory savings needed to cover the benefit r_2. This cost is due to the fact that a worker cannot withdraw his savings until the time of retirement.

savings in his private savings account to supplement the tax-funded benefit r_1 for his consumption during unemployment. This would be an exact case for the pure UI system.

We will first briefly examine the pure UI system, and compare it with the integrated system. Let $s_1(r_1)$ be the pre-unemployment savings a worker makes under the integrated system, where he is offered the optimal savings-funded benefit if unemployed. To focus on the case when the government provision of savings-funded benefit is necessary, we will suppose that $s_1(r_1) < r_2(r_1)$ for any tax-funded benefit r_1.[13] Then, as the following Proposition shows, the consumption level of an unemployed worker is lower under the pure UI system than under the integrated system.

Proposition 6

Suppose that $s_1(r_1) < r_2(r_1)$. Then,

$$r_1^* + r_2^* > \hat{r}_1 + \hat{s}_1,$$

where $\hat{r}_1$, $\hat{s}_1$ are tax-funded UI benefit, a worker's precautionary savings under the pure UI system, respectively.

The proof of the Proposition 6 is delegated to the Appendix. Under the pure UI system, a worker whose retirement savings in period 1 is not enough to replace the optimal savings-funded benefit $r_2(r_1)$ would have to make additional savings to prepare for unemployment risk. Since this precautionary savings involves some efficiency costs, however, it would still be in short of the optimal savings-funded benefit for the unemployed. Although the tax-funded benefit will increase to fill up the gap to some extent, it would not be enough to secure the optimal consumption for the unemployed because of its incentive cost.

Proposition 6 implies that the pure UI system is inferior to the integrated system in that the UI system does not allow an unemployed worker to borrow against his future savings. To see how much the government needs to allow the unemployed worker to borrow in order to secure optimal consumption, let us suppose that the government provides the individual with the necessary minimum savings-funded benefit.

Note by Proposition 1 that the privately optimal savings-funded benefit r_2' is equal to the socially optimal one r_2^*. Taking this into account, we can present the desired government provision of savings-funded benefit, $\bar{r}_2^*$, as

$$\bar{r}_2^* = r_2^* - s_1^*,$$

which will be called the desired RSA-funded benefit. Once the government offers an unemployed worker $\bar{r}_2^*$, then he will save s_1^* in period 1 and use it together with $\bar{r}_2^*$ to consume r_2^* during unemployment. Thus there will be no change in s_1^*, r_1^* or r_2^* under

---

[13] If $s_1(r_1) > r_2(r_1)$, there would be no role for the government to provide savings-funded benefit. In this case the traditional UI system would achieve the same result as the integrated system.

this new regime. The amount $\bar{r}_2^{\ *}$ of desired RSA-funded benefit may in fact represent the size of welfare gain that the government brings to a consumption-constrained unemployed worker.

By (10) the desired RSA-funded benefit provided by the government would be

$$\bar{r}_2^{\ *} \equiv r_2^{\ *} - s_1^{\ *} = \frac{Mr}{1+(M+1)r} - \frac{1+Mr}{1+(M+1)r}(s_1^{\ *} + r_1^{\ *})$$

The following proposition demonstrates how the desired RSA-funded benefit would change with search elasticity, risk-aversion, and the probability of unemployment shock, with the proof being delegated to the Appendix.

Proposition 7

$$\frac{\partial \bar{r}_2^{\ *}}{\partial H} > 0, \quad \frac{\partial \bar{r}_2^{\ *}}{\partial \delta} < 0, \quad \frac{\partial \bar{r}_2^{\ *}}{\partial q} > 0.$$

The desired amount of RSA-funded benefit for consumption-constrained unemployed workers increases under certain circumstances. For example, when the economy is subject to serious incentive distortion, tax-funded benefit decreases while the need for savings-funded benefit grows. Also, an economy with high unemployment risk tends to have large desired RSA-funded benefit because the optimal system requires a low level of tax-funded benefit and a high level of savings-funded benefit. On the other hand, the desired RSA-funded benefit would decrease as workers become more risk-averse, because larger tax-funded benefit is provided while the optimal savings-funded benefit gets smaller.

Now let us turn to the effects of the career structure – the lengths of employment and retirement period, timing of unemployment – upon the desired RSA-funded benefit. Let

$$\bar{\gamma}_2^{\ *} \equiv \frac{\bar{r}_2^{\ *}}{r_1^{\ *} + r_2^{\ *}}$$

which indicates the share of the desired RSA-funded benefit to the total consumption under unemployment. Let also

$$\bar{y} \equiv \frac{Y - s_1^{\ *}}{1 - s_n^{\ *}} ,$$

which indicates the share of the desired RSA-funded benefit to what a worker would have consumed if not unemployed. Then we can state the following.

Proposition 8

(i) $\frac{\partial \bar{\gamma}_2^{\,*}}{\partial M} > 0, \quad \frac{\partial \bar{y}}{\partial M} > 0$ if δw = 1.

(ii) $\bar{r}_2^{\,*} \to 1$ as $M \to \infty$

(iii) $\bar{r}_2^{\,*} \to \frac{a-1}{1+a}$ as $r \to 0$ while keeping Mr = a.

The proof can be found in the Appendix. As the post-unemployment working period gets longer, pre-unemployment savings decreases because both the unemployment savings and other savings act as substitutes for each other. The reduction in pre-unemployment savings, coupled with the increased need for savings-funded benefit as shown in Proposition 4, would further increase the share of desired RSA-funded benefit to total benefits. This result implies that the welfare gain, through the intertemporal consumption smoothing under the integrated system, increases for those who are unemployed earlier in their careers.

In the limiting case where the post-unemployment period is very long, pre-unemployment savings in period 1 will go to zero and thus the share of the desired RSA-funded benefit would go up to1. By Proposition 5, this is the case where the first-best state is approximated. Thus, it is through government provision of the desired RSA-funded benefit that the first-best efficiency can be achieved when the post-unemployment working period is very long. This case can be contrasted with the case when the retirement period and the post-unemployment working period are very long compared to the unemployment period. In this case pre-unemployment savings is quite large, so that government provision of RSA-funded benefit may not be as needed as in the previous case. When a ≤ 1, i.e., when the retirement duration is long compared to the post-unemployment working period, RSA-funded benefit may not be needed to achieve the first-best efficiency. In other words, the worker's private savings take care of any adverse risk effect, so that government intervention may be unnecessary.

The timing of unemployment in one's career is critical for the relevance of the integrated system. If a worker tends to frequently become unemployed later in his career, then the desired RSA-funded benefit would become small or nil. This is because the worker's pre-unemployment savings may be large enough and because the need for savings-funded benefit itself would be small.

Another factor that affects the relevance of the integrated system is the possibility that a worker will end up with a negative RSA balance at the time of retirement,[14] which affects the incentive-effectiveness of the integrated system. We will examine this issue in the next section.

[14] See Feldstein and Altman (1998) for a simulation study on this possibility.

3. Integrated Lifetime Insurance Through Joint Savings Account

So far we have considered the case where there is no possibility of a negative balance in one's RSA and where the RSA only covers unemployment risk. Since, in reality, the amount of consumption during unemployment is a small portion of one's retirement savings, the chances of a negative RSA balance are slim if RSA only covers unemployment risk. In this section we will consider a multi-risk case in which a worker may have more than one shock – unemployment and disability, for example - in his career. We will analyze the system that integrates both unemployment and disability insurance with a pension program through a joint RSA, which will be called the integrated lifetime insurance (LI) system. In a multi-risk case, a worker may end up with a negative balance in his RSA from the two shocks in his career. In this section we will examine how the possibility of a negative balance, and consequently the government bailout of it, affects the optimality of the integrated LI system and its benefit structure.

Consider a worker who lives for three periods. He works for the first two periods and retires in the last period to consume his retirement savings. The worker is subject to two risks during his career – unemployment and disability risks. Suppose a worker has unemployment shock with probability q in period 1, and disability shock with probability p in period 2. Once a shock occurs, he could either choose or not choose to exert some effort to prevent unemployment. A worker under unemployment shock, for example, can choose to search to avoid being unemployed in period 1, while a worker under disability shock can choose to make some extra work efforts to work in period 2.

Thus the probability of unemployment in each period depends not only upon the probability q or p of shock, but also upon the effort decision by a worker. As in the previous section it is assumed that the cost of search effort e or of work effort c is distributed with distribution function F( ) or with G( ), respectively, and that a worker chooses the threshold effort cost e′ or c′, respectively. The probability then of remaining unemployed would be $q(1-F(e'))(\equiv \bar{q})$ in period 1 and $p(1-F(c'))(\equiv \bar{p})$ in period 2.

In addition to a certain amount b of initial savings, a worker can accumulate savings at any period of employment. These savings are to be accumulated in his RSA. Instead of introducing a specific amount of mandatory savings, we assume that the RSA is the only savings account available to a worker. We will mainly focus on the circumstance where the initial savings b alone is not sufficient enough to cover savings-funded benefit for the unemployed. When a worker experiences the two shocks and b is his only retirement savings, therefore, he may end up with a negative balance in his RSA.[15] The negative balance is covered by the tax revenue, and thus it may reduce his search and work incentives. In other words, to the extent that the savings-funded unemployment benefit a worker has received is not charged to him when he is unemployed again in period 2, a worker would lose incentives for search and work.

A worker pays tax t_1 in period 1 and tax t_2 or T_2 in period 2 to cover the tax-funded benefits and the negative balance. In this model the tax for the expected amount of non-

[15] As will be seen later, the optimal savings-funded benefit for the unemployed gets smaller as the two risks are more positively correlated to each other. When they are strongly positively correlated, therefore, the savings-funded benefit would be so small that it may be less than b, and therefore no negative balance may occur. This case will be mentioned later in this section.

chargeable (savings-funded) benefit ($r_2 - b$) is paid in period 2 rather than in period 1.[16] On the other hand, the government pays a certain amount of benefits to the unemployed or the disabled. In period 1 the government provides the unemployed with tax-funded benefit r_1 and savings-funded benefit r_2. In period 2 it offers the disabled different combinations of tax and savings-funded benefits, depending upon whether they have previously experienced unemployment. The disabled with no unemployment history are given the tax-funded disability benefit d_1 and the savings-funded benefit d_2, while those with previous unemployment experience will be offered only the tax-funded benefit D_1 since they do not have a positive balance in their RSA's.

Let I(.) and J(.) be the expected payoff of a worker from being employed and unemployed in period 1, respectively. The expected utility V of a worker can be defined as follows:

$$V(r_1, r_2, d_1, d_2, D_1; q, p_q, p_{-q})$$
$$= \underset{e'}{Max}\ (1-\bar{q})I(b, p_{-q}, q; d_1, d_2) + \bar{q}J(b, p_q; r_1, r_2, D_1) - q\int_0^{e'} edF$$

where

$$I(b, p_{-q}, q; d_1, d_2) = \underset{c', s_1, s_2}{Max}\ U(w - s_1 - t_1) + \bar{p}_{-q}\{U(d_1 + d_2) + U(b + s_1 - d_2)\}$$
$$+ (1 - \bar{p}_{-q})\{U(w - s_2 - t_2) + U(b + s_1 + s_2)\} - p_{-q}\int_0^{c'} cdG$$
$$J(b, p_q; r_1, r_2, D_1) = \underset{C', S_2}{Max}\ U(r_1 + r_2) + \bar{p}_q\{U(D_1) + U(0)\}$$
$$+ (1 - \bar{p}_q)\{U(w - S_2 - T_2) + U(b + S_2 - r_2)\} - p_q\int_0^{C'} cdG$$

and p_q or p_{-q} is the probability of disability shock occurring conditional upon unemployment or no unemployment in period 1, respectively, while

$$\bar{q} \equiv q(1 - F(e')),\quad \bar{p}_{-q} \equiv p_{-q}(1 - G(c')),\quad \bar{p}_q \equiv p_q(1 - G(C'))$$

and

$$(1-\bar{q})t_1 = \bar{q}r_1,\quad (1 - \bar{p}_{-q})t_2 = \bar{p}_{-q}d_1\ ,\quad (1 - \bar{p}_q)T_2 = \bar{p}_q(D_1 + r_2 - b) \qquad (19)$$

---

[16] This tax system improves a worker's search incentive given the possibility of negative balance, at the expense of further disincentive for work in period 2. For the analytical simplicity the tax system modeled in this section leaves all the adverse incentive effects (associated with negative balance) to period 2, while maintaining the same search incentive in period 1. If a worker pays the additional tax in period 1, search incentive in period 1 as well as work incentive in period 2 will be affected, which complicates the model. Note, however, that the total amount of disincentives would remain the same given any tax system introduced in the model.

Higher p_q or lower p_{-q} is associated with higher correlation ρ between unemployment and disability shocks. There are two things to be noted here. First, the tax T_2 in period 2 includes the expected value of the non-chargeable savings-funded benefit (r_2-b), $\bar{p}_q(r_2 - b)$. Second, it is assumed for the moment that ($b+s_1$) > d_2 and that ($b+s_2$) > r_2, which is confirmed later by Lemma 5 as individual savings and benefit structures are optimally chosen. This assumption implies that in the hypothetical single-risk case when the probability of unemployment or disability shock is zero, there is no possibility of negative balance.[17]

Let us first examine an individual worker's choices for savings. He chooses the three levels of savings s_1^*, s_2^* and S_2^* as follows.

$$s_1^* : -U'(w - s_1^* - t_1) + \bar{p}_{-q}U'(b + s_1^* - d_2) + (1 - \bar{p}_{-q})U'(b + s_1^* + s_2) = 0 \qquad (20)$$

$$s_2^* : -U'(w - s_2^* - t_2) + U'(b + s_1 + s_2^*) = 0 \qquad (21)$$

$$S_2^* : -U'(w - S_2^* - T_2) + U'(b + S_2^* - r_2) = 0 \qquad (22)$$

Some comparative statics of individual savings decisions are presented in Lemma 4 in the Appendix.

A worker also determines the threshold levels of his search and work efforts, e^*, c^* and C^* in the following way:

$$e^* = I(b, p_{-q}, q; d_1, d_2) - J(b, p_q; r_1, r_2, D_1) \qquad (23)$$

and

$$c^* = U(w - s_2 - t_2) + U(b + s_1 + s_2) - \{U(d_1 + d_2) + U(b + s_1 - d_2)\} \qquad (24)$$

$$C^* = U(w - S_2 - T_2) + U(b + S_2 - r_2) - \{U(D_1) + U(0)\} \qquad (24')$$

Using the envelope theorem, we can collect some comparative statics of individual search and work decisions. Let us first check how a worker's search decision is affected by the system.

$$\frac{\partial e^*}{\partial d_1} = \frac{\partial e^*}{\partial d_2} = \frac{\partial e^*}{\partial D_1} = 0\,.$$

$$\frac{\partial e^*}{\partial r_1} = -U'(w - s_1 - t_1)\frac{\bar{q}}{1 - \bar{q}} - U'(r_1 + r_2) \qquad (25)$$

$$\frac{\partial e^*}{\partial r_2} = -U'(r_1 + r_2) + U'(b + S_2 - r_2) \qquad (25')$$

[17] Otherwise there would be no incentive effect of savings-funded benefit.

$$\frac{\partial e^*}{\partial \overline{p}_{-q}} > 0 \qquad (26)$$

Given that different sets of optimal disability benefits are offered for workers with and without unemployment history, the parameters (d_1, d_2, D_1) of the benefit structure for the disabled would not affect the search decision of a worker.[18]

By (25) and (25') we can say that

$$\frac{\partial e^*}{\partial r_1} < \frac{\partial e^*}{\partial r_2}$$

which justifies the savings-funded benefit for the unemployed in the integrated LI system.

The comparative statics of individual work decisions with respect to the parameters of the benefit structure for the disabled is

$$\frac{\partial c^*}{\partial d_1} = -U'(w - s_2 - t_2)\frac{\overline{p}_{-q}}{1 - \overline{p}_{-q}} - U'(d_1 + d_2) \qquad (27)$$

$$\frac{\partial c^*}{\partial d_2} = U'(b + s_1 - d_2) - U'(d_1 + d_2) \qquad (27')$$

and

$$\frac{\partial C^*}{\partial D} = -U'(w - S_2 - T_2)\frac{\overline{p}_q}{1 - \overline{p}_q} - U'(D_1)$$

$$\frac{\partial C^*}{\partial r_2} = -U'(b + S_2 - r_2) \qquad (28)$$

A couple of points deserve to be mentioned. First, as has been the case, the savings-funded benefit is more incentive-effective than the tax-funded benefit (i.e., $\frac{\partial c^*}{\partial d_1} < \frac{\partial c^*}{\partial d_2}$). Secondly, and more importantly, as (28) shows, the savings-funded unemployment benefit adversely affects individual work incentive under disability shock. This is because a worker with unemployment history is not charged for the savings-funded UI benefit if he becomes unemployed again in period 2, and the tax for the non-chargeable

[18] As (25') shows, search effort is not affected by the possibility of a negative balance. This is due to the tax system in the model, in which the cost of non-chargeable savings-funded benefit r_2 is paid in period 2 (through tax as much as $\overline{p}_q(r_2 - b)$) rather than in period 1. Thus the impact of the cost burden of the negative balance will adversely fall upon the work incentive of a worker in period 2, which will be explained later.

UI benefit is paid by those who are employed in period 2. Because of this problem associated with the government bailout of a negative balance, some disincentives on the part of a worker will always be present even in a fully integrated LI system that is soley funded by the savings.

In characterizing the optimal benefit structure of the integrated LI system and examining the optimality of the system in the multi-risk case, we can assume that the integrated system is optimal in the case of hypothetical single-risk. In other words, when the probability of unemployment shock or the probability of disability shock is zero, the optimal benefit structure entails a positive amount of savings-funded benefit. This assumption will enable us to figure out how the multiple risks and the possibility of a negative balance would change the benefit structure and the optimality of the integrated system.

(1) Optimal Benefit Structure of Integrated Lifetime Insurance System

One important feature of the integrated LI system is that it offers different combinations of tax- and savings-funded benefits for the disabled with different histories of employment and benefit payments. To the extent that employment history affects RSA balance, the disability benefit should be conditional upon employment history.

First we will analyze the case for workers who have been unemployed in period 1. The optimal tax-funded benefit D_1^* should satisfy

$$(w - S_2 - T_2 - D_1^*)\delta - \frac{Q_2}{(1-Q_2)(1-\bar{p}_{-q})} = 0 \qquad (29)$$

where δ and $H'(\equiv \frac{g(c)}{1-G(c)}$, which is assumed to be constant for all c) represent the worker's risk aversion and the elasticity of work effort, respectively, and $Q_2 \equiv \frac{U'(w-S_2^* - T_2)}{1-\bar{p}_q} D_1^* H'$. Taking into account the savings decision (22), we can rewrite (29) as follows:

$$\frac{w - T_2 - D_1^*}{2}\delta - \frac{Q_2}{(1-Q_2)(1-\bar{p}_{-q})} = 0 \qquad (29')$$

Note from (29') that, as usual, the tax-funded benefit D_1^* is increasing and decreasing in the risk-aversion and the elasticity of work effort, respectively. Since there is no positive RSA balance, the optimal savings-funded disability benefit D_2^* is zero:

$$D_2^* = 0 \qquad (29'')$$

Let us turn to the optimal benefit structure for the disabled who have not been unemployed in period 1. Differentiating the expected utility function V with respect to d_1, d_2, we have

$$(w - s_2 - t_2 - d_1^* - d_2)\delta - \frac{Q_1}{(1-Q_1)(1-\bar{p}_{-q})} = 0 \qquad (30)$$

$$(d_1 + d_2^*) - (b + s_1 - d_2^*) \ = 0, \qquad (31)$$

where $Q_1 \equiv \frac{U'(b + s_1^* + s_2^*)}{1-\bar{p}_{-q}} d_1^* H'$. Taking individual savings decisions (20) and (21) into consideration, we can present the conditions for (d_1^*, d_2^*) as follows:

$$\frac{w - t_2 - d_1^*}{2}\delta - \frac{Q_1}{(1-Q_1)(1-\bar{p}_{-q})} = 0 \qquad (30')$$

$$d_2^* = \frac{b + s_1 - d_1^*}{2} \qquad (31')$$

Note that the optimal benefit structure for the disabled without unemployment history in period 1 is the same as that in the hypothetical single-risk case when the probability of unemployment shock is zero, because their RSA balances are $(b+s_1)$. Since the optimal benefit structure is assumed to entail a positive savings-funded benefit in the hypothetical single-risk case, the savings-funded disability benefit for these workers under the integrated LI system will also be positive, i.e.,

$$d_2^* > 0. \qquad (31'')$$

Now let us turn to the benefit structure for the unemployed under the integrated LI system. One important factor that determines the benefit structure for the unemployed is the disincentive effect associated with the possibility of a negative balance. As we recognized in (28), the possibility of negative RSA balance and the subsequent government bailout of it can aggravate a worker's incentives. Note that this disincentive problem affects the benefit structure for the unemployed, not that for the disabled.

Differentiating the expected utility function V with respect to r_1, we have

$$\bar{q}[\{w - s_1 - t_1 - (r_1^* + r_2)\}\delta - \frac{P}{(1-\bar{q})(1-P)}] = 0 \qquad (32)$$

where $P \equiv \frac{r_1^* H}{1-\bar{q}} U'(w - s_1 - t_1)$. Similarly, if we differentiate V with respect to r_2, taking into account the individual decisions (25), (25') and (28), we have

$$\bar{q}[\{w - S_2 - T_2 - (r_1 + r_2^*)\}\delta - \frac{Q'\bar{p}_q}{(1-\bar{p}_q)(1-P)}] = 0, \qquad (33)$$

where $Q' \equiv \frac{(D_1^* + r_2 - b)H'}{1-\bar{p}_q} U'(w - S_2^* - T_2)$. The last term of (33) indicates the disincentive effect associated with the negative RSA balance, which reduces the savings-funded benefit r_2^*. Note that the disincentive effect is positively related to p_q or to the correlation between the two risks. Since the optimal savings-funded benefit is assumed to be positive in the single-risk case when $p_q = p_{-q} = 0$, we have

$$r_2^* > 0$$

when $p_q = 0$ and $p_{-q} > 0$ by Proposition 9 presented below.

Finally, we can prove that, as s_1, S_2, d_2, and r_2 are optimally chosen, there will not be a negative RSA balance when only one shock occurs, i.e.,

Lemma 5

$$b + s_1^* > d_2^*, \quad b + S_2^* > r_2^*.$$

The proof is delegated to the Appendix.

(2) Correlation between Risks, Optimality and Benefit Structure in the Integrated LI System

One of the most important factors that determines the benefit structure of the integrated LI system is the correlation between the two risks. Suppose as before that the optimal savings-funded unemployment benefit r_2^* is greater than b so that a negative RSA balance is possible. As the following Proposition shows, a higher correlation leads to a smaller savings-funded benefit and a greater tax-funded benefit.

Proposition 9

(i) $\frac{\partial d_1^*}{\partial \rho} > 0, \quad \frac{\partial d_2^*}{\partial \rho} < 0$

(ii) $\frac{\partial r_1^*}{\partial \rho} > 0, \quad \frac{\partial r_2^*}{\partial \rho} < 0$

Proposition 9 demonstrates that the optimal savings-funded benefit for the unemployed or for the disabled under the integrated LI system decreases as the correlation between the risks increases. Note that the savings-funded disability benefit d_2^* is not affected by the disincentive caused by the possibility of the government bailout of a negative balance. Higher correlation affects d_2^* through the lower probability of disability shock for those without unemployment history, reducing d_2^*. The results on savings-funded benefit r_2^* (in (ii)) for the unemployed, however, results from the disincentive effects associated with the possibility of a negative balance. As was seen before, the possibility of the government bailout of a negative RSA balance, which increases in the correlation between the two risks, would reduce a worker's incentive under disability shock.[19]

There are additional factors that are responsible for the effects of the correlation on the savings-funded benefit for the unemployed. As the probability of disability shock for those with previous unemployment history increases, for example, the expected payoff of being unemployed would decrease, reducing the savings-funded benefit.

Since the amount of savings-funded benefit in the integrated LI system decreases in the correlation ρ, one may wonder if the integration of unemployment and/or disability insurance with a pension program would still be necessary when the two risks are highly correlated to each other. There are two points to be considered on this issue. If the savings-funded benefit r_2 for the unemployed decreases below the initial savings b as a result of strong correlation, there is no possibility of a negative balance and thus no disincentive problem. If the correlation is very high, however, the unemployed would have a high chance of ending up with the retirement savings b with no additional RSA balance. This would lead to a low or even zero savings-funded benefit r_2 for the unemployed when b is low.[20] Thus the integration of unemployment insurance alone with pension may not be optimal in a multi-risk case where the risks are highly correlated. The following argument, however, secures the optimality of the integration of unemployment *and* disability insurance with pension through a joint RSA.

A critical point to be emphasized is that the integrated LI system allows the disability benefit structure to respond optimally to different unemployment histories of a worker. To the extent that different unemployment histories in period 1 lead to different balances in the joint RSA, they can also affect the optimal benefit structure for the disabled. In other words, as we can see from (29'), (29"), (30'), (31') and (31"),

$$d_1^* \neq D_1^*$$
$$d_2^* > 0 = D_2^*$$

Since a DI system that is integrated with pension but not with an UI system offers the same benefit for the disabled regardless of their previous unemployment history, it is inferior to the integrated LI system.

The source of the welfare improvement by the integrated LI system comes from its ability to share the RSA balance between unemployment and disability benefits. For example, for those who have not been unemployed and thus have not been paid unemployment benefits, the system allows the savings b to be used for the disability benefit. In other words, the savings b serves as a common pool of savings to be shared within the system. The system also allows the additional savings s_1, which those workers make in period 1, to be used for disability benefit. With these savings summed up in a joint RSA, the system offers a relatively large amount of savings-funded benefit for the disabled who have not been previously unemployed.

[19] Note also that, depending upon how the tax for the non-chargeable UI benefit is collected, the possibility of the negative balance could also affect search incentive as well as work incentive.

[20] To avoid unnecessary complexities in this model, we will not examine explicitly the multi-risk case with no possibility of negative balance.

For those who are unemployed in period 1, on the other hand, the RSA balance decreases for the two reasons; some of the balance may be withdrawn to finance a part of the unemployment benefit, but an unemployed worker will not be able to make additional savings in his RSA. If the savings-funded unemployment benefit r_2^* is greater than the initial savings b as specified above, the resulting negative RSA balance would lead to a zero savings-funded benefit D_2^* for the disabled. Even if r_2^* is less than b, the relatively small amount of RSA balance would lead to a small or zero savings-funded disability benefit, which would be clearly different from d_2^*.[21]

These arguments have established following Proposition on the optimality of the integrated LI system:

Proposition 10

Unless ρ = 1, that is, unless the two risks are perfectly correlated to each other, it is optimal to integrate unemployment and disability insurance with a pension program through a joint RSA.

Proposition 10 argues that the integrated LI system is optimal despite the disincentives caused by the possibility of a negative balance or despite the possibility of a low RSA balance resulting from the occurrence of two consecutive shocks.[22] When the two risks are perfectly positively correlated to each other, however, only the tax-funded benefit D^* might be provided to the disabled if the initial savings b is small, as illustrated in the previous subsection. In this extreme case, therefore, there would be no welfare gain expected from the integrated LI system compared with the unintegrated system.

4. Conclusion

The failure of markets to provide adequate social insurance has long been recognized. This, combined with the fact that social norms do not allow individuals in their old age to suffer from insufficient income, even when their misfortune arises because they have *chosen* to save sufficiently, provides a rationale for a public, compulsory pension program. This paper has developed a further advantage to the public, mandatory program; it allows for the collateralization of future wage income in a way which is not easily possible otherwise, thus allowing individuals to in effect self insure.

This paper has addressed two related issues. The possibility of savings-funded self-insurance does not eliminate the desirability of some tax-funded insurance, except under extreme circumstances. We have identified the factors on which the optimal degree of savings-funded self-insurance depends. Our analysis is consistent with the suggestion in the introduction of a heavy reliance on savings-funded self-insurance.

[21] For the purpose of simplicity, explicit analysis of the optimal benefit structure in this case is omitted in this paper.

[22] In this respect the Provident Fund in Singapore and Malaysia might be of the desirable form, to the extent that it covers a number of contingencies for a worker by savings he has accumulated in his account.

When there are multiple risks (including the risk of multiple bouts of unemployment), again some reliance on savings-funded self-insurance is in general desirable, unless the risks are perfectly correlated. Although the multi-risk case may involve some adverse disincentives due to the government bailout in the event of a negative balance, the integrated system can always generate welfare gain from allowing a common pool of pension savings to be shared. The general principle naturally leads to the suggestion of a fully integrated lifetime insurance system through a joint account, similar to the Provident Fund of Singapore and Malaysia,[23] where not only unemployment and disability risks, but also health risks are integrated with pensions.

[23] For detailed information on the system, see Asher (1994).

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APPENDIX

Proofs of propositions:

1. Proposition 2

Note that $\frac{\partial e^*}{\partial H} = \frac{\partial e^*}{\partial \delta} = 0,$ and that $\frac{\partial s_1^*}{\partial H} = 0,\ \frac{\partial s_1^*}{\partial \delta} = 0$.

(i) $\frac{\partial r_1^*}{\partial H} < 0$ by (9), which leads to $\frac{\partial r_2^*}{\partial H} > 0$ by (10). Also, we have $\frac{\partial r_1^*}{\partial \delta} > 0$ by (9), and $\frac{\partial r_2^*}{\partial \delta} < 0$ by (10).

(ii) From (12) we have $\frac{\partial Y}{\partial H} = \frac{r}{1+(M+1)r}(\frac{ds_1^*}{dH} + \frac{\partial r_1^*}{\partial H})$, while $\frac{ds_1^*}{dH} = \frac{\partial s^*}{\partial r_1^*}\frac{\partial r_1^*}{\partial H}$ because $\frac{\partial s_1^*}{\partial H} = 0$. Since $\frac{\partial s_1^*}{\partial r_1} = -\overline{q} < 1,$ $\frac{\partial Y}{\partial H} = \frac{r}{1+(M+1)r}(1-\overline{q})\frac{\partial r_1^*}{\partial H} < 0$. Since $\frac{ds_1^*}{d\delta} = \frac{\partial s_1^*}{\partial \delta} + \frac{\partial s_1^*}{\partial r_1^*}\frac{\partial r_1^*}{\partial \delta},$ and $\frac{\partial s_1^*}{\partial \delta} = 0$, we have $\frac{\partial Y}{\partial \delta} = \frac{r}{1+(M+1)r}(\frac{ds_1^*}{d\delta} + \frac{\partial r_1^*}{\partial \delta}) = \frac{r}{1+(M+1)r}(\frac{\partial s_1^*}{\partial \delta} + (1-\overline{q})\frac{\partial r_1^*}{\partial \delta}) > 0$.

2. Proposition 3

(i) $\frac{\partial r_1^*}{\partial q} < 0$ by (9) because $\frac{\partial \overline{q}}{\partial q} > 0$, which leads to the second result of by (10). Since $\frac{\partial s_1^*}{\partial q} < 0$, we have the third result by (12).

(ii) (9) can be rewritten as

$$Xw\delta(1-\overline{q}) - \{1-\delta(1-\overline{q})Xw\}U'(wY)wHr_1^* = 0,$$

where $\frac{\partial\{U'(wY)w\}}{\partial w} \geq (or \leq)0$ as δw ≤ (or ≥) 1. Thus the first two results come from the above condition and (10) and from that $\frac{\partial s^*}{\partial w} \approx 0$, which lead to the last result.

3. Proposition 4

(i) Since $\frac{\partial A}{\partial M} < 0 (\because \frac{\partial X}{\partial M} < 0,\ \frac{\partial e^*}{\partial M} < 0),\ \frac{\partial B}{\partial M} < 0 (\because \frac{\partial Y}{\partial M} > 0 (\because -1 < \frac{\partial s_1}{\partial M} < 0))$, we have the first two results. The first result implies that X decreases as M increases by (11), and $(1-s_n^*)$ increases in M by (3), which lead to the last two results.

(ii) First we will show that $Xq\{\frac{\partial e^*}{\partial r}\Big|_{Mr=a}\} \approx 0$. Rewriting (5) as

$$e^* = I(.) - J(.)$$
$$= (r_2 + \frac{1}{1+(M+1)r}(1 - s_1 r))U'(1-s_n)(1+\frac{\delta}{2}X) .$$

Thus, $\{\frac{\partial e^*}{\partial r}\Big|_{Mr=a}\}$ does not contain the term (1/r), which implies that $Xq\{\frac{\partial e^*}{\partial r}\Big|_{Mr=a}\} \approx 0$ because $qr \approx 0$.

Next, setting $ZH\frac{r_1^*}{Y} = k$ (constant) in (13) and differentiating both sides with respect to r_1 and r, while keeping Mr as a, we have

$$\frac{dr_1}{dr}\Big|_{k,Mr=a} < \frac{r_1(s_1(a+1)+1)}{(a+(1+s_1)r)(1+a+r)}$$

taking into account the condition that $\frac{\partial s_1}{\partial r}\Big|_{Mr=a} < 0$. Using this inequality, we have

$$\frac{dX}{dr}\Big|_{Mr=a} = \frac{\partial X}{\partial r}\Big|_{Mr=a} + \frac{\partial X}{\partial r_1}\frac{\partial r_1}{\partial r}\Big|_{k,Mr=a}$$
$$> \frac{1}{\{1+a+r\}^2}\{(1+a)(1-r_1) - r_1\frac{(1+s_1(1+a))r}{a+(1+s_1)r}\}$$
$$> 0,$$

because $r_1 < \frac{a+rs_1}{1+a}$ by (10). This proves the first result, which implies the second one. By (9), $\frac{\partial r_1^*}{\partial r}\Big|_{Mr=a} > 0$. Also, $\frac{dM}{dr}\Big|_{Mr=a} = \frac{M}{r} > 1$. From the definition of x, then, we have the third result because $\frac{\partial s_1}{\partial r}\Big|_{Mr=a} > -1$ by Lemma 1. The last result comes from the third one and the definition of y.

4. Proposition 6

Note that $s_1(r_1)$ and $r_2(r_1)$ satisfy

$$-U'(1-s_1(r_1)-t) + (1-\bar{q})U'((M+1)s_n^* + s_1) + \bar{q}U'(1-s_u^*) = 0,$$
$$r_1 + r_2(r_1) = 1 - s_u^*$$

$\Omega(s_1(r_1)) > 0$, because $1 - s_u^* = r_1 + r_2(r_1) > r_1 + s_1(r_1)$. Thus, $s_1'(r_1) > s_1(r_1)$.

Let $s_1'(r_1)$ be the precautionary savings of a worker who is not offered any RSA-funded benefit by the government. Then, it satisfies

$$\Omega(s_1') \equiv -U'(1 - s_1'(r_1) - t) + (1 - \bar{q})U'((M+1)s_n^* + s_1') + \bar{q}U'(r_1 + s_1') = 0.$$

Note that $\Omega(r_2(r_1)) < 0$, because $r_2(r_1) > s_1(r_1)$ and by the above condition for $s_1(r_1)$. Thus, $s_1'(r_1) > r_2(r_1)$. Since $-\frac{dr_1^*}{dr_2} < 1$ and $-\frac{dr_2^*}{dr_1} < 1$, we have from Figure 1 the desired result.

4. Proposition 7

These results come from Propositions 2-3 and the fact that $\frac{\partial s_1^*}{\partial H} = 0$, $\frac{\partial s_1^*}{\partial \sigma} = 0$, $\frac{\partial s_1^*}{\partial q} < 0$.

5. Proposition 8

(i) By $\frac{\partial \gamma_1^*}{\partial M} < 0$ (Proposition 4) and by $\frac{\partial s_1^*}{\partial M} < 0$ (Lemma 1), the first result obtains by the definition of $\bar{\gamma}_2^*$. That $\frac{\partial x}{\partial M} < 0$ (by Proposition 6) and Lemma 1 lead to the second result.

(ii) Since $s_1^*, r_1^* \to 0$ as M goes to infinity (Proposition 5), we get the desired results by the definitions of $\bar{\gamma}_2^*$ and y.

(iii) Since $s_1^* \to \frac{1}{1+a}$, $r_1^* \to 0$ as r goes to zero (Proposition 5), we get the desired results by the definitions of $\bar{\gamma}_2^*$ and y.

6. Proposition 9

(i) First we can prove the following lemma.

Lemma A

$$\frac{\partial Q}{\partial \bar{p}_{-q}} > 0$$

<Proof>

$$\frac{\partial Q}{\partial \bar{p}_{-q}} = \frac{1}{(1-\bar{p}_{-q})^2}\left[\frac{\partial(b + s_1 + s_2)}{\partial \bar{p}_{-q}}(1-\bar{p}_{-q})U''(b + s_1 + s_2) + U'(b + s_1 + s_2)\right] > 0$$

because δw = 1 and because $\frac{\partial(b + s_1 + s_2)}{\partial \bar{p}_{-q}} < b + s_1 + s_2$ by Lemma 3. *Q. E. D.*

By (30) we have $\frac{\partial d_1^*}{\partial \bar{p}_{-q}} < 0$ (or $\frac{\partial d_1^*}{\partial \rho} > 0$) for given d_2, because $\frac{\partial Q}{\partial \bar{p}_{-q}} > 0$ by Lemma 4. Since $\frac{\partial s_1}{\partial \bar{p}_{-q}} > 0$, we have $\frac{\partial d_2^*}{\partial \bar{p}_{-q}} > 0$, (or $\frac{\partial d_2^*}{\partial \rho} < 0$) by (31).

(ii) To figure out the effects of the correlation between the two risks upon the benefit structure (r_1^*, r_2^*) of the integrated system, we prove the following technical results .

Lemma B

(a) Suppose r_1^* is chosen optimally for given r_2 by (32). Then,

$$\frac{\partial(1-P)}{\partial \bar{p}_{-q}} > 0$$

(b) Suppose that D^* is chosen optimally by (31). Then,

$$\frac{\partial Q'}{\partial \bar{p}_q} > 0.$$

<Proof>

(a) From (32) we have

$$1 - P = \frac{U'(w - s_1 - t_1)}{(1-\bar{q})U'(r_1 + r_2) + \bar{q}U'(w - s_1 - t_1)} \qquad (33)$$

Then we can see that (1-P) is decreasing in r_1 (note that $\delta w = 1$) and that RHS of the above condition (33) is increasing in r_1 (note that $\frac{\partial \bar{q}}{\partial r_1} > 0$). Since $\frac{\partial s_1}{\partial \bar{p}_{-q}} > 0$ by Lemma 4, it is clear that both (1-P) and RHS of (33) are increasing in $\bar{p}_{-q}$ for any given r_1. Thus we get the desired result.

(b) From (31) we have

$$Q' = \frac{\bar{p}_q}{1 + \frac{1}{(1-\bar{p}_q)(w - S_2 - T_2)}}. \qquad (34)$$

It is clear that Q′ is increasing in D, while RHS of (34) is decreasing in D (Note that $(w - S_2 - T_2)$ is decreasing in D)). Since $(w - S_2 - T_2)$ is decreasing in $\bar{p}_q$, we can see that both Q′ and RHS of (34) are increasing in $\bar{p}_q$ (using $\delta w = 1$). Thus we get the desired result. *Q. E. D.*

Note that

$$\frac{\partial e^*}{\partial \rho} > 0 \;\; (\because \frac{\partial e^*}{\partial p_{-q}} < 0, \; \frac{\partial e^*}{\partial p_q} > 0)$$

$$\frac{\partial s_1^*}{\partial \rho} < 0 \; (\because \frac{\partial s_1^*}{\partial p_{-q}} > 0)$$

which implies that $\frac{\partial r_1^*}{\partial \rho} > 0$ for given r_2 by (32) and Lemma B. This and Lemma B lead to the result that $\frac{\partial r_2^*}{\partial \rho} < 0$ by (33).

Proofs of Lemma

1. Lemma 1

$$X \equiv s_u^* - s_n^* = \frac{r}{1+Mr}\{r_2 + \frac{1}{1+(M+1)r}(1-s_1^* r)\}. \qquad (2'')$$

The condition (4) can be rewritten as

$$-U'(1-s_1^* - t) + U'(1-s_n^*)\{1+\bar{q}\delta w X\} = 0 \qquad (4')$$

or

$$-U'(1-s_1^* - t) + (1-\bar{q})U'\{\frac{(M+1+s_1^*)r}{1+(M+1)r}\} + \bar{q}U'\{\frac{(M+s_1^* - r_2)r}{1+Mr}\} = 0. \qquad (4'')$$

(i) Differentiating (4') with respect to e^* (using (2") and the assumption that $qr \approx 0$) yield the result.

(ii) If we differentiate (4") with respect to r_1 and r_2, using (1) and assuming that q and r are so small that $\frac{q}{1-q} \approx q, \; \frac{r}{1-r} \approx r$ and that $\frac{\bar{q}}{M} \approx 0, \; \bar{q}r \approx 0$, we have the desired results.

(iii) Differentiation of (4') with respect to δ and (2") lead to the first result.
Differentiation of (4") with respect to M, r (keeping Mr constant), and q yield the rest of the results.

2. Lemma 2

(i) Differentiation of (5) with respect to s_1 and the envelope theorem (for s_n and s_u) yield the desired result.

$$\frac{\partial e^*}{\partial r_1} = w[-U'(r_1+r_2) + \frac{\partial s_1{}^*}{\partial r_1}\frac{\partial e^*}{\partial s_1}]$$

(ii) $$= -w[U'(r_1+r_2) - \overline{q}\{U'(1-s_u{}^*) - U'(1-s_n{}^*)\}]$$

$$\approx -wU'(r_1+r_2)$$

by Lemma 1 and 2 and by (2”). Also

$$\frac{\partial e^*}{\partial r_2} = w\{-U'(r_1+r_2) + U'(\{Ms_u{}^* + s^* - r_2\}r)\}$$

$$= w\{-U'(r_1+r_2) + U'(1-s_u{}^*)\}$$

(iii) Differentiating (5) with respect to M and using Lemma 1, 2, we have

$$\frac{de^*}{dM} = \frac{\partial e^*}{\partial M} + \frac{\partial e^*}{\partial s_1}\frac{\partial s_1}{\partial M}$$

$$\approx s_u{}^* w\{U'(1-s_n{}^*) - U'(1-s_u{}^*)\} + \frac{(Xw)^2}{2}U'' < 0$$

4. Lemma 3

Since $\frac{\partial(1-s_1{}^*-t)}{\partial r_1} = 0$ by (1) and Lemma 1 (ii), we have by differentiating (6)

$$-\frac{dr_1{}^*}{dr_2} < 1.$$

Differentiating (7) with respect to r_1 and r_2, we have

$$-\frac{dr_2{}^*}{dr_1} < \frac{1+(M+r)r}{1+(M+1)r} < 1.$$

These two conditions lead to the unique solution $(r_1{}^*, r_2{}^*)$ by Figure 1.

3. Lemma 4

$$\frac{\partial s_1{}^*}{\partial \overline{p}_{-q}} > 0, \quad \frac{\partial s_2{}^*}{\partial \overline{p}_{-q}} < 0, \quad \frac{\partial S_2{}^*}{\partial \overline{p}_q} < 0, \quad \frac{\partial(s_1{}^* + s_2{}^*)}{\partial \overline{p}_{-q}} < b + s_1{}^* + s_2{}^*$$

<Proof>

$$\frac{\partial s_1{}^*}{\partial \overline{p}_{-q}} = \frac{s_2+d_2}{2} + \frac{d_1}{4(1-\overline{p}_{-q})} > 0 \qquad \text{by (19) and (20)}$$

$$\frac{\partial s_2{}^*}{\partial \overline{p}_{-q}} = -\frac{1}{2}\frac{\partial s_1{}^*}{\partial \overline{p}_{-q}} - \frac{1}{2}\frac{\partial t_2}{\partial \overline{p}_{-q}} < 0 \qquad \text{by (19) and (21)}$$

$$\frac{\partial S_2^{\ *}}{\partial \overline{p}_q} = -\frac{1}{2}\frac{(D+r_2)}{(1-\overline{p}_q)^2} < 0 \qquad \text{by (19) and (22)}$$

$$\frac{\partial (s_1^{\ *} + s_2^{\ *})}{\partial \overline{p}_{-q}} = \frac{1}{2}\frac{\partial s_1^{\ *}}{\partial \overline{p}_{-q}} - \frac{1}{2}\frac{\partial t_2}{\partial \overline{p}_{-q}} < s_2 + d_2 < b + s_1 + s_2 \quad \text{by (19) and (20)}$$

4. Lemma 5

From (30), we have that $b + s_1 - d_2^{\ *} = \frac{s_1^{\ *} + d_1^{\ *}}{2} > 0$. Also, from (22) we have $b + S_2^{\ *} - r_2^{\ *} = \frac{w + b - T_2 - r_2^{\ *}}{2}$. Since $r_2^{\ *} < w + b - T_2 - 2r_1^{\ *}$ by (33), we have