$\beta = \eta$ is not Enough: Reexamining the Efficiency of the Labour Market in a Matching Model with Differentiation of Skills

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Abstract

In order to study the efficiency of the labour market, we build a matching model where the differentiation of workers and jobs, as well as the searching behaviour of the unemployed are made explicit. Concerning the differentiation of agents, along the same line as Marimon and Zilibotti (1999), we follow the approach of Salop (1979). Workers and firms lie on the same circle and the distance between two points of this circle measures the mismatch between a firm and worker type. Regarding the technology of contacts, we retain an adaptation of the urn-ball model. This framework allows us to state some interesting insights about the efficiency of a decentralized labour market with skills differentiation

Key words: Matching, Differentiation of skills, Efficiency of the labour market.

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Introduction

For the sake of simplicity, the standard search matching model (Pissarides (2000)) formalize the notion of *frictions* in the labour market by assuming that the hiring flow per period rises continuously with the stocks of unemployed workers and vacancies.

Using this formalization, Hosios (1990) and Pissarides (1990) have shown that the efficiency of the labour market only requires that the bargaining strength of workers (denoted by β) be equal to the elasticity of the matching function with respect to unemployment (denoted by η). Under this condition, the entry and exit of firms as well as the searching efforts and the job acceptance behaviour of agents (in the stochastic matching model) arise to be optimal.

By relaxing the assumptions of homogeneity and transparence of the labour market, matching models have substantially improved the understanding of unemployment and numerous empirical studies have exhibited the important role of *mismatches* (see for instance the influence of the *turbulence index* in Layard,Nickell and Jackman (1991)). However, although it is a simple and useful tool, the matching function is a short cut whith no microecomic foundations. In this paper, we show that when throwing some light in the blackbox (along the same line as Petrongolo and Pissarides (2001)), the robustness of the $\beta = \eta$ condition is seriously damaged.

To this aim, we build a matching model where the differentiation of workers and jobs as well as the searching behaviour of unemployed are made explicit. The matching function is then derived from first principles.

Concerning the differentiation of agents, we follow the approach of Salop (1979). Workers and jobs lie on the same circle and the distance between two points of this circle measures the mismatch between the requirements of a firm and the skill of a worker. From this point of view, our model is similar to Marimon and Zilibotti (1999). It is also similar to Pissarides (1984) where the productivity of a match is random. However, in our framework, the randommness of productivity is endogenous. It proceeds from the (horizontal) differentiation of jobs and workers

Regarding job search, we retain an adaptation of the urn-ball model (Pissarides (1979), Petrongolo and Pissarides (2001)) where workers have some *a priori* information about firms. For each period, unemployed draw one vacancy among the subsets of the firms whose types are not further than a given distance from their own. Search is then partially oriented and this distance measures the *transparence* of the labour market.

Using this model, we state two main insights. First, concerning the entry and exit of firms, the famous "$\beta = \eta$" efficiency condition still holds with horizontal differentiation. Second, unlike stochastic job matchings (Pissarides (1984), Hosios (1990), Pissarides (1990) and Pissarides (2000)) where productivity randomness is exogenous, the "$\beta = \eta$" equality does not ensure the efficiency of job acceptance any longer. To the contrary, we prove that the efficiency of job acceptance is not compatible with

the efficiency of job creation. In short, this means that the decentralized equilibrium is unavoidably inefficient.
The paper is organized as follows. The second section presents the model and its equibrium. In section 3, we study the efficiency of the decentralized equilibrium and establish the main results. These results are illustrated by a numerical analysis of an increase in unemployment benefits. The last section collects some final comments.

1 The decentralized equilibrium

1.1 The model

The economy consists of two sets of very numerous risk neutral agents: workers and firms. Workers are heterogenous. The jobs which the firms offer are heterogenous too.
All workers are infinitely lived. To the contrary, the firms, which produce the same good, can die. Namely, we assume that, each period, the firms face a constant risk of breakdown with probability s. However, the free entry of new firms on the market stabilizes their number. All agents have the same discount rate, r, and $R = (1 + r)$.
In order to describe the differentiation of workers and jobs, we use the tool of analysis of Salop (1979).

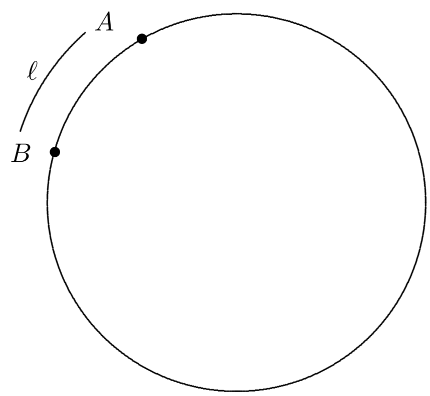
1.1.1 The circle of skills

We assume that the set of the workers is uniformly distributed on a circle whose circumference is equal to two (see Figure 1). This distribution is exogenous. The position of a worker on this circle represents the "**type**" of his skill. The distribution of the firms on the circle is also uniform. Likewise, the position of a firm on the circle represents its "**type**", that is the skill which perfectly suits its needs.

Consider two points A and B on the circle of skills: l, the distance between A and B ($0 \leq l \leq 1$), measures the match between the "type" of a worker located at A (respectively a firm) and the "type" of a firm located at B (respectively a worker). Thus, *the match is perfect when the distance l equals zero*. On the opposite, *the mismatch reaches a maximum when l is unity.*

The productivity of a worker, $y(l)$, is a decreasing function in the distance l between his skill and the needs of the firm which employs him. This "*mismatch function*" $y(l)$ is concave ($y'(l) < 0$, $y''(l) \leq 0$). Each firm employs only one worker. The productivity of this worker determines then the production of the firm.

Figure 1: The circle of the skills



1.1.2 Intertemporal utilities and profits

The workers

When holding a job, their productivity, and hence their wage $w(l)$ depend on the distance l between the type of their job and their own type. The same goes for their (expected) intertemporal utility $W_E(l)$.

The intertemporal utility of unemployed, W_U, depends mainly on the distance below which the formation of an employer/employee pair generates a positive surplus. This *mismatch limit* is a measure of job acceptance which is denoted by λ. At this stage of the analysis, the limit λ is an exogenous variable which affects the expected intertemporal utility of a worker who gets a job, denoted by $\overline{W}_E$.

Because the distribution of vacant jobs is uniform, the conditional expectation $\overline{W}_E$ satisfies:

$$\overline{W}_E = \frac{1}{\lambda} \int_0^\lambda W_E(l) dl \tag{1}$$

In stationary state, the intertemporal utilities $W_E(l)$ and W_U satisfy:

$$W_E(l) = w(l) + R^{-1}[sW_U + (1-s)W_E(l)] \tag{2}$$

$$W_U = d + R^{-1}[p\overline{W}_E + (1-p)W_U] \tag{3}$$

where d denotes domestic production and p the hiring rate.

The firms

The jobs offered are either vacant or filled. The value of a filled job, $J_F(l)$, depends also on the distance l between the types of the employer and the employee:

$$J_F(l) = y(l) - w(l) + R^{-1}[sJ_V + (1-s)J_F(l)] \tag{4}$$

where J_V is the value of a vacant job.

The value of a vacant job is a function of the limit λ. Indeed, this limit affects the probability, q, of filling this job as well as the expected value of a job which is filled, $\overline{J}_F$, which satisfies:

$$\overline{J}_F = \frac{1}{\lambda}\int_0^\lambda J_F(l)dl \tag{5}$$

The value of a vacancy can then be written as:

$$J_V = -c + R^{-1}[q\overline{J}_F + (1-q)J_V] \tag{6}$$

where c denotes the cost of a vacancy.

Like the standard search matching model (Pissarides (2000)), we assume free entry of firms, that is:

$$J_V = 0 \tag{7}$$

Let $\overline{y}$ and $\overline{w}$ denote respectively the average output and wage:

$$\overline{y} = \frac{1}{\lambda}\int_0^\lambda y(l)dl \tag{8}$$

$$\overline{w} = \frac{1}{\lambda}\int_0^\lambda w(l)dl \tag{9}$$

Given equations (4), (5) and (6), free entry (7) implies:

$$\overline{J}_F = \frac{R(\overline{y} - \overline{w})}{r+s} \tag{10}$$

and

$$\overline{J}_F = \frac{Rc}{q} \tag{11}$$

1.2 Surplus sharing and mismatch limit

Following the generalized Nash rule, the surplus of a firm/worker match is divided between the two parties according to their respective bargaining power. Let β $(0 < \beta < 1)$ be the bargaining strength of workers. Hence, the expected rent of workers is:

$$W_E(l) - W_U = \beta[W_E(l) + J_F(l) - W_U - J_V] \tag{12}$$

The match between an employer and an employee will be viable if (and only if) it creates a positive total surplus. Thus, the limit λ satisfies:

$$W_E(\lambda) + J_F(\lambda) - W_U - J_V = 0 \tag{13}$$

It is interesting to notice that, because of the Nash solution, both parties agree on the same mismatch limit λ. Now, given equation (13) and the Nash rule, we deduce:

$$W_E(\lambda) = W_U \iff J_F(\lambda) = J_V \tag{14}$$

Then, we can define a reservation wage and a reservation profit. The free entry assumption sets the reservation profit $(y(\lambda) - w(\lambda))$ to zero. As a consequence, the reservation wage of unemployed equals the minimal output:

$$w(\lambda) = y(\lambda) \tag{15}$$

1.2.1 The hiring process

We assume that the unemployed issue just one application per period. However, unemployed have some "*a priori*" information about job offers. Namely, we assume that the hiring firm they meet is drawn at random among the subset of the firms whose types are located at a distance lower than x from their own type. The variable x is exogenous and it represents *the transparence of the labour market* $(x \leq 1)$. The lower the variable x, the more transparent the labour market is. When x is zero, search is fully directed and there is no mismatch.
Let U be the number of unemployed and V, the number of vacancies. The tightness of the labour market is then given by the ratio $\theta = V/U$. Hence, one can show that, for $\lambda \leq x$, the probability of filling a vacancy, denoted by q, is determined by:

$$q = 1 - e^{-\frac{\lambda}{x\theta}} \tag{16}$$

Indeed, to fill its vacancy, a hiring firm only needs to meet just one employable worker, that is a worker whose type is not further located than the limit λ. For obvious reasons, more choosiness (*i.e.* a decrease in λ) and either more vacancies or less unemployment (*i.e.* an increase in θ) reduce the

probability of filling a job. In addition, for a given level of variables λ and θ, more transparence in the labour market (*i.e.* a decrease in x) raises probability q.
Total hiring of workers, H, derives then from the following function:

$$H = \left(1 - e^{-\frac{\lambda}{x\theta}}\right) V \tag{17}$$

The hiring probability of workers obtains by dividing total hiring H by unemployment U:

$$p = \theta \left(1 - e^{-\frac{\lambda}{x\theta}}\right) \tag{18}$$

Clearly, probability p is increasing in the mismatch limit λ and a decreasing function of distance x. p is also an increasing function in the labour market tightness θ ((see Appendix 2).

1.2.2 Flow equilibrium in the labour market

In stationary equilibrium, the flow of workers who get a job is equal to the flow of workers who loose their job. Let u denote the rate of unemployment. We then have:

$$pu = s(1 - u) \tag{19}$$

1.3 Solving the model

1.3.1 Choosiness and tightness of the labour market

Given equations (14) and (15), equation (2) yields:

$$rW_U = Ry(\lambda) \tag{20}$$

From equations (3) and (20), we deduce:

$$y(\lambda) = d + R^{-1} p(\overline{W}_E - W_U) \tag{21}$$

Using the surplus sharing relation (12), we get the difference $(\overline{W}_E - W_U)$ as a function of tightness θ. Taking into account (11) yields:

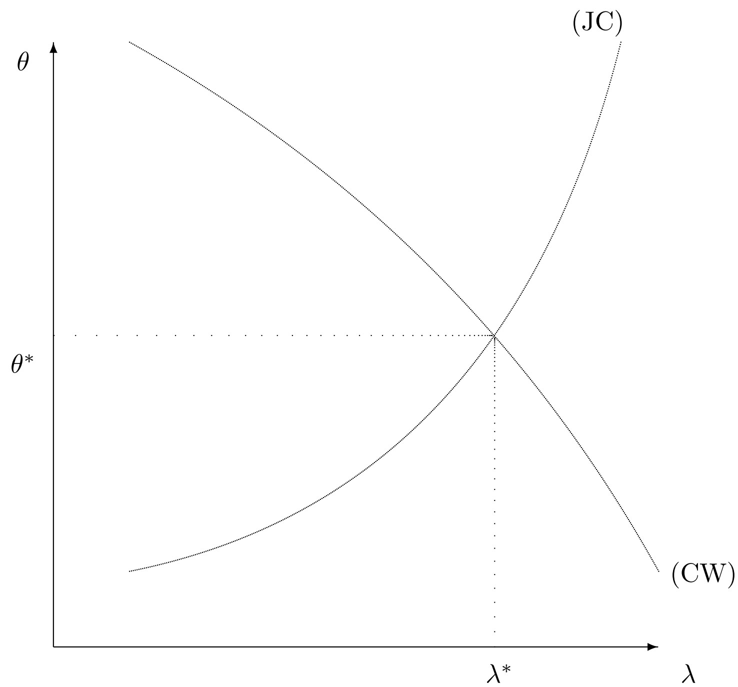
$$\overline{W}_E - W_U = \frac{\beta}{1-\beta} \overline{J}_F = \frac{\beta}{1-\beta} \frac{Rc}{q} \tag{22}$$

Combining equations (21) and (22), knowing that $p = \theta q$, we obtain one first relationship between variables λ and θ:

$$y(\lambda) = d + \frac{\beta c}{1-\beta} \theta \tag{23}$$

In (λ, θ) space, equation (23) is depicted by the concave curve (CW) (see Figure 2). (CW) curve slopes down because of the properties of the hiring technology. When tightness θ increases, unemployed find jobs more easily. Others things being equal, the utility of unemployed increases and, the total surplus of some "bad" matches becomes then negative. As a consequence, the agents become more choosy leading then to a decrease in the limit λ.

Figure 2: Stationary equilibrium



1.3.2 Job creation and equilibrium

In order to set another relationship between variables λ and θ, we use equations (12) and (10), it obtains:

$$\overline{W}_E - W_U = \frac{\beta}{1-\beta}\overline{J}_F = \frac{\beta}{1-\beta}\frac{R(\overline{y} - \overline{w})}{r+s} \tag{24}$$

Using the relations (2) and (3), one can easily show that:

$$\overline{W}_E - W_U = \frac{R(\overline{w} - d)}{r + s + p} \tag{25}$$

In the case of differentiaton of skills, the rent of an employee is written in terms of the expectations $\overline{W}_E$ and $\overline{w}$.

By combining (24) and (25), taking into account (22), we get the following equation:

$$\overline{w} = \beta\overline{y} + (1 - \beta)d + \beta c\theta \tag{26}$$

Written in terms of the average variables, equation (26) is a standard wage setting relationship. Taking into account (23), the wage setting equation (equation (26)) can be rewritten as:

$$\overline{w} = \overline{y} - (1 - \beta)(\overline{y} - y(\lambda)) \tag{27}$$

Finally, in order for the wage setting (equation (27)) to be coherent with the opening of new vacancies, depicted by equations (22) and (24), the pair (λ, θ) must satisfy:

$$(1 - \beta)q[\overline{y} - y(\lambda)] = (r + s)c \tag{28}$$

Using the concavity assumption of $y(l)$, it follows that the difference $(\overline{y} - y(\lambda))$ is an increasing function in the mismatch limit λ. In addition, an increase in λ (θ) raises (reduces) probability q. Consequently, in space (λ, θ) equation (28) is depicted by an upward sloping curve (JC) (see Figure 2).

The intuition behind this result is quite simple. Other things equal, an increase in the labour market tightness leads to a decrease in the probability of filling a vacancy. Therefore, the coherence with the wage setting behaviour (equation (27)) is achieved by an increase in the mismatch limit λ which raises both the probability q and profits per period $(1 - \beta)(\overline{y} - y(\lambda))$.

From this, we deduce the:

Definition 1. *A decentralized equilibrium of the labour market is a pair (λ, θ) which jointly satisfies equations (23) and (28).*

The Figure 2 depicts an equlibrium of the market. From the pair (λ^*, θ^*), one can deduce the equilibrium values of the other variables.

2 The efficiency of the labour market

In order to establish the conditions which ensure the efficiency of the labour market, we compare the decentralized stationary equilibrium with the optimality conditions of a social planner who maximizes the intertemporal collective surplus subject to the constraint that the change in unemployment derives from the hiring function.

2.1 The social optimum

The collective surplus per head of period t is given by:

$$S_t = (1 - u_t)\overline{y}_t + u_t d - \theta_t u_t c \tag{29}$$

So, the problem of the social planner can be written as follows:

$$Max \quad CS = \Sigma_0^\infty R^{-t} S_t$$

$$s.c : u_{t+1} = u_t + s(1 - u_t) - \theta_t q_t u_t$$

Let μ_t denote the Lagrange's multiplier. So, in a steady state, the social optimum must satisfy the following first order conditions:

$$\frac{\partial CS}{\partial \theta_t} = -cR^{-t} - \mu_t q(1 + \frac{\theta}{q}\frac{\partial q}{\partial \theta}) = 0 \tag{30}$$

$$\frac{\partial CS}{\partial u_t} = -R^{-t}(\overline{y} - d + \theta c) + \mu_t(1 - s - \theta q) - \mu_{t-1} = 0 \tag{31}$$

$$\frac{\partial CS}{\partial \lambda_t} = -R^{-t}(1 - u)[\overline{y} - y(\lambda)] - \mu_t u\theta \frac{\partial q}{\partial \lambda}\lambda = 0 \tag{32}$$

Let $\eta(z)$ denote the elasticity of probability q with respect to the ratio $z = \frac{\lambda}{x\theta}$. Hence, we have:

$$\frac{\partial q}{\theta}\frac{\theta}{q} = -\eta(z)$$

and,

$$\frac{\partial q}{\lambda}\frac{\lambda}{q} = \eta(z)$$

In other terms, the elasticity of q with respect to z (which is positive) is equal to its elasticity with respect to λ and is the opposite of its elasticity with respect to θ. Notice that $\eta(z)$ is also equal to the elasticity of the hiring function with respect to unemployment.
Now, let's consider equation (30). In a steady state, multiplier μ_t must satisfy:

$$\mu_t = -R^{-t}\frac{c}{(1 - \eta(z))q} \tag{33}$$

Combining (31) and (33) gives a first optimality condition in terms of the pair (z, λ). This condition can be written as:

$$(1-\eta(z))q(z)(\overline{y}-d)-\eta(z)\frac{\lambda}{xz}q(z)c=(r+s)c \tag{34}$$

This latter equation ensures that the ratio z (hence the tightness θ) is optimal for a given level of the mismatch limit λ. With no differentiation of skills, *i.e.* $y(l) = y(0)$, it is equivalent to the condition which determines the optimal value of the tightness of the labour market in Hosios (1990) and Pissarides (2000). So, its meaning is the same. On the one hand, an increase in tightness θ raises the employment $(1-u)$ and the production per head $(1-u)\overline{y}$ tending then to increase the collective surplus. On the other hand, raising θ reduces domestic production and imposes the opening of more vacancies whose cost tends to reduce the collective surplus.

In our framework, we have to exhibit a second optimality condition in terms of the pair (λ, z). This is achieved by combining equations (32) and (33). Taking into account the constraint of the optimization problem (in stationary state), it follows that:

$$\overline{y}(\lambda)-y(\lambda)=\frac{\eta(z)sc}{(1-\eta(z))q(z)} \tag{35}$$

Condition (35) ensures that the mismatch limit λ is optimal for a given level of the tightness θ. When deciding of job acceptance, the social planner faces a trade-off between production per firm (which falls with λ) and hiring (which rises with λ).

It turns out that a social optimum can be defined as follows:

Definition 2. *A social optimum of the labour market is a pair* (λ, z) *which jointly satisties equations (34) and (35).*

2.2 The efficiency of the decentralized equilibrium

With no differentiation of skills, it is well known that the efficiency of the decentralized equilibrium imposes that the elasticty of the hiring function with respect to unemployment $(\eta(z))$ be equal to the bargaining strength of workers (β) (Hosios (1990), Pissarides (1990)). One can show that this famous result extends to our formalization of skills differentiation.

Indeed, combining equations (23) and (28) gives:

$$(1-\beta)q(z)(\overline{y}(\lambda)-d)-\beta\frac{\lambda}{xz}q(z)c=(r+s)c \tag{36}$$

Hence, the efficiency condition (34) holds in the decentralized equilibrium if and only if:

$$\eta(z^*) = \beta \tag{37}$$

However, with differentiation of skills, the equality $\eta(z) = \beta$ is not sufficient. It only ensures that the tightness of the labour market θ^* maximizes the social surplus for the equilibrium level of the mismatch limit λ^*. To be efficient, the decentralized equilibrium must also satisfy equation (35). Let us consider the equilibrium equation (28). It can be written as:

$$\overline{y}(\lambda) - y(\lambda) = \frac{(r+s)c}{(1-\beta)q(z)} \tag{38}$$

Therefore, the efficiency condition (35) holds in the decentralized equilibrium if and only if:

$$\eta(z^*) = \frac{r+s}{r+s+(1-\beta)s} \tag{39}$$

This second condition ensures that the mismatch limit λ is optimal for the equilibrium value of the tightness of the labour market θ^*.

Now, can the decentralized equilibrium be efficient? In other terms, are conditions (37) and (39) compatible? It is easy to see that the answer is no. Indeed, combining (37) and (39) gives:

$$\eta(z^*) = \beta = \frac{r+s}{s} \tag{40}$$

Clearly, these equalities cannot be jointly satisfied. Indeed, the ratio $\frac{r+s}{s}$ is necessarily greater than the bargaining strength of workers β which is less than one. In addition, the elasticity $\eta(z)$ is also lower than one. It follows that:

Proposition. *With differentiation of skills, the equilibrium of the labour market is unavoidably inefficient.*

It is interesting to compare this result with stochastic job matchings (Pissarides (1984), Hosios (1990), Pissarides (1990)). As is well known, when productivity is an exogenous random variable, the condition $\beta = \eta$ internalizes the externalities that arise when a firm and a worker leave the market for any reason (Pissarides (1990), p.131). To the contrary, when productivity randomness proceeds from skills differentiation, condition $\beta = \eta$ is not sufficient any longer. The intuition behind this is very simple. With (horizontal) skills differentiation, the externalities which result from the exit and entry of jobs are quite different from the externalities of job acceptance. If, say, a firm and a worker decide to form a "bad" job, not only do they remove one participant from each side of the market but they also lower the probability of "better" jobs. In other terms, they deteriorate the assignment of workers to jobs.

2.3 An illustration: Unemployment benefits and welfare

With no differentiation of skills, a fall in employment is unavoidably associated with a fall in total production. Hence, raising unemployment benefits is generally inefficient (as soon as $\beta > \eta$). In our framework, unemployment benefits also affect the assignment of workers to jobs and their effects are therefore not so straightforward any longer (see also Acemoglu and Shimer (2000)).

Using Figure 2, one can see that an increase in unemployment benefits leads to a fall in both variables λ and θ. On the one hand, average production per firm $\overline{y}$ rises. However, on the other hand, employment falls. Hence, the effect of benefits on total production per head $((1-u) = \overline{y})$ is indeterminate. The same holds for the collective surplus

In order to clarify the effects of unemployment benefits on the efficiency of the labour market, we now simulate the model. The question we ask is the following:

Are benefits able to improve the efficiency of the labour market when employment is initialy too low $(\beta > \eta)$? In other terms, we wonder if raising (average) productivity might compensate for lowering employment.

Following Hosios (1990), we assume for simplicity that the actualization rate r is zero. This assumption allows us to compare steady states.

Formally, the simulated equations are slightly different from the analytical ones. In particular, we introduce taxes which finance unemployment benefits. In order not to affect the main analytical outcomes, we impose the neutrality of these taxes (Holmlund (1998)). Specifically, we assume that all incomes bear the same proportional tax.

For simplicity, the match function is assumed to have a linear form: $y(l) = 110 - 130l$.

Other parameters are assumed to take the following values: $\beta = 0.5$ (as assumed in the symmetric Nash bargain solution), $x = 0.8, s = 0.04$, $r = 0$, $d = 10$. Unemployment benefits are denoted by b.

The main results of the simulations are presented in the following table:

	$b = 0$	$b = 4$	$b = 8$	$b = 12$	$b = 16$	$b = 17$
λ	0.49	0.48	0.47	0.46	0.45	0.447
θ	0.40	0.37	0.34	0.31	0.284	0.277
q	0.79	0.80	0.82	0.84	0.862	0.867
p	0.314	0.297	0.281	0.263	0.245	0.240
$\overline{y}$	77.96	78.65	79.35	80.06	80.77	80.95
u	0.113	0.118	0.125	0.132	0.140	0.143
CS	716	719	722	723	724.11	724.02

This table reports some interesting outcomes. First, the increase in the unemployment benefits (b) lowers job acceptance. As a consequence, we note a decrease in the mismatch limit λ which raises

the production per firm $\overline{y}$.
An increase in the unemployment benefits leads to a decrease in the labour market tightness. According to our own analytical outcomes and to standard matching models, the decrease in θ implies a decrease (an increase) in the transition probability of unemployed (vacancies). In this model, the decrease in transition probability p is strengthened by more choosiness.
For low levels of b, despite the increase in the unemployment rate (u), the increase in the average production per firm is high enough to give rise to an increase in the collective surplus (CS)[1]. However, one must be careful in interpreting this result. It could stem from a situation of overemployment and the efficiency of benefits would result from the employment fall. In order to exclude this possibility, we computed the elasticity $\eta(z)$ in the benchmark. This elasticity is smaller than β. This means that the increase in the collective surplus does result from less job acceptance.

3 Final comments

In this paper, we showed that introducing skills differentiation in a search matching model dramatically changes the standard view about the efficiency of the labour market. "$\beta = \eta$" is not enough any longer. In fact, the labour market is unavoidably inefficient. This result makes the effects of public policies more subtle. We illustrated this point by numerical simulations which show that an increase in unemployment benefits may be efficient in a situation of under- employment $(\beta > \eta)$.
Of course, benefits are not the lonely instrument. In a companion paper, we argue that introducing a minimum wage might be a better instrument (Gavrel and Lebon (2002)).

[1] An increase in the slope of $y(l)$ allows to expand the numerical variations

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Appendix 1: The urn-ball model with skills differentiation

The purpose of this appendix is to determine the probability of filling a vacancy in the labour market. A vacant job is filled if and only if it has at least one applicant located at a distance lower than the limit λ.

For $\lambda \leq x$, the probability that not any employable worker (these unemployed are λU) apply for this vacancy, is given by:

$$\left(\frac{xV-1}{xV}\right)^{\lambda U} = exp\left[\lambda U ln\left(1-\frac{1}{xV}\right)\right]$$

Because unemployed and vacancies are "very numerous", it obtains:

$$q = 1 - e^{-\frac{\lambda U}{xV}} = 1 - e^{-\frac{\lambda}{x\theta}}$$

Appendix 2: Behaviour of $p(\theta, \lambda)$

We have:

$$p(\theta,\lambda) = \theta q(\theta,\lambda) = \theta\left(1 - e^{-\frac{\lambda}{x\theta}}\right)$$

The derivative of $p(.)$ with respect to θ is given by:

$$1 - e^{-\frac{\lambda}{x\theta}} - \frac{\lambda}{x\theta}e^{-\frac{\lambda}{x\theta}}$$

It is easy to show that this derivative takes its values within the interval $]0,1[$. Therefore, probability p is an increasing function in tightness θ.

Appendix 3: The simulated equations

In the quantitative analysis, following Marimon and Zilibotti (1999), we assume that the match function is linear. Hence, we have:

$$Y(l) = Y(0) - \alpha l$$

Let ζ denote the sum $(d+b)$.

The λ value is given by solving:

$$(1-\beta)\frac{\alpha\lambda}{2}\left[1 - e^{\frac{-\lambda\beta c}{x(1-\beta)[(Y(0)-\alpha\lambda)-\zeta]}}\right] - (r+s)c = 0$$

θ is then deduced as follows:

$$\theta = \frac{(1-\beta)}{\beta c}[Y(0) - \alpha\lambda - \zeta]$$

and the transition probabilities satisfy:

$$q = 1 - e^{\frac{-\lambda}{x\theta}}$$

$$p = \theta\left[1 - e^{\frac{-\lambda}{x\theta}}\right]$$

The average production and the limit one can be written as:

$$\overline{y} = \left(Y(0) - \frac{\alpha\lambda}{2}\right)$$

$$y(\lambda) = Y(0) - \alpha\lambda$$

The pre-tax average wage takes then the following value:

$$\overline{w} = \frac{\beta(r+s+p)(\overline{y}-c) + (1-\beta)(r+s+q)\zeta}{\beta(r+s+p) + (1-\beta)(r+s+q)}$$

The rate of unemployment satisfies:

$$u = \frac{s}{s+p}$$