Workforce Adjustment Costs and Employment

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Abstract

We study a strategic interaction between a single monopoly union and a mass of competitive firms. Firms are subject to costly workforce adjustments and to changes in the economic environment in the form of a stochastic cycle between good and bad business conditions. The game is solved both under the assumption that the union can commit to a given wage sequence and under the assumption that such a commitment is not feasible.

We find that the equilibrium under commitment and the one without commitment produce the same employment and wage outcome during a bad business spell. By contrast, the two equilibria exhibits in general different outcomes during a good business spell. However, when individual objective functions are linear and the union is utilitarian, the two equilibria are similar also in good times.

We argue that these findings shed some light on the robustness of a corollary of the insider-outsider theory whereby the inability to commit is detrimental for the employment level in a world of high workforce replacement costs.

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1 Introduction

Lindbeck and Snower (1988) show that replacement costs allow insiders to charge wages above the reservation level of outsiders without facing the risk of being fired. This leads to lower employment levels and to involuntary unemployment. For, at the time of hiring, firms anticipate the opportunistic behaviour of new recruits once they become insiders and refrain from hiring too much. In short, low employment levels turn out to be the result of a particular hold-up problem. By the same token, if outsiders were able to commit not to raise future wages once they become insiders, the employment level would not be decreased by replacement costs through their effect on future wage claims. Thus, replacement costs such as mandated firing costs reduce employment only to the extent workers are unable to pre-commit to a given wage sequence.

This argument has been very influential in the labour literature and has prompted a whole stream of research. In particular, a number of authors have tried to assess its robustness from a pure theoretical perspective (Bertola, 1990, for instance). No effort, however, has been made to tackle the issue of robustness in a model featuring workers unionisation and a volatile economic environment.

In this paper we present a bargaining game between a monopoly union and a mass of small competitive firms that are subject to linear workforce adjustment costs. The model is stochastic as productivity and demand conditions are modelled through a two-state Markov process while the strategic interaction is analysed both with and without a union commitment on wages.

We find that, in the absence of a commitment, the union increases the wage by the full amount of turnover costs after new workers have been hired in a business upturn. This accords with the basic insider-outsider model of Lindbeck and Snower. However, in sharp contrast with this model, we find that the employment level without commitment is not necessarily lower than the one under commitment. This happens because the union compensates future wage increments by charging low wages at the time of hiring. In particular, if the union is utilitarian and the utility functions of individuals linear, the ability to commit does not bring any gain to workers in terms of higher employment or higher wages. This neutrality result represents the main finding of this paper.

The plan of the paper is as follows. In section 2 we present the economic environment. In section 3 and 4 we study the union-firms interaction respectively with and without a commitment on wages and under fairly general functional forms. In section 5 we compare the two equilibria and establish their equivalence in case the utility function of individuals is linear. Section 6 concludes.

2 The strategic environment

A single union faces a unit mass of identical competitive firms that belong to the same industry. Firms are subject to the same demand and productivity perturbations and maximize the present discounted value of the following cash flow

$$cash\ flow_t = R(\alpha_t, l_t) - w_t l_t - I_{l_t \leq l_{t-1}} F\left(l_{t-1} - l_t\right)$$

Since production is realised through a labour-only technology, the revenue function $R(\alpha_t, l_t)$ depends on the level of firm's employment l_t and on the shifter α_t which summarizes business conditions at sector level. The shifter cycles stochastically between two values, α_g in good times and α_b ($< \alpha_g$) in bad time: if $\alpha_t = \alpha_g$ ($\alpha_t = \alpha_b$) then α_{t+1} is equal to α_b (α_g) with probability q and to α_g (α_b) with probability $1-q$. Current costs are given by the wage bill $w_t l_t$ plus mandated firing costs, F represents the total cost from firing a single worker and $I_{l_t \leq l_{t-1}} \left(l_{t-1} - l_t\right)$ the total number of fired workers. Since the indicator $I_{l_t < l_{t-1}}$ switches from 1 to 0 if current employment becomes strictly higher than past employment, the formula correctly implies that the number of firings is zero at the time of workforce increases.

Finally, we assume that the marginal productivity of labour is decreasing while, for any amount of labour, revenues and marginal revenues increase with respect to business conditions: $R_l > 0$, $R_{ll} < 0$, $R_\alpha > 0$ and $R_{\alpha l} > 0$.

The union maximizes a discounted flow whose current component depends on the current wage and on aggregate sectorial employment L_t: $U(w_t, L_t)$. We assume that the function U satisfies the following properties:

$$\begin{aligned} U_w, U_L &> 0 \\ U_{wL} &> 0 \\ U_{ww}, U_{LL} &\leq 0 \end{aligned} \tag{1}$$

Thus, both *goods* - wages and employment - enter the pay-off with decreasing marginal utility and with a positive cross derivative. These restrictions appear rather loose. The function U, for instance, nests as particular cases both the utilitarian objective function and the expected utility function.

In the next two sections we study the union-firms strategic interaction with and without a union commitment on wages. In both cases the union behaves as a monopolist, it fixes the wage unilaterally at the beginning of each period whereas firms observe the wage and set unilaterally the employment level for the period.

3 Wages and Employment under Commitment

3.1 The optimal hiring and firing policy

The dynamics of the forcing variable α define a stochastic business cycle at the level of any single sector. Spells of good business conditions alternate with spells of bad conditions. In this section we study the firms-union interaction under the assumption that the union can commit to a particular wage sequence. For simplicity we focus on time stationary equilibria and assume that wage and employment levels are not made conditional on calendar time but only on the type of business conditions - good or bad - and on the elapsed duration of any business spell. In other words, we allow wages and employment to change across good and bad spells and, within each spell, from one period to the other. Of course, stationarity may need a transition period to set in or it may not arise at all depending on the initial conditions . For this reason, later in the section we devote some effort in identifying under what initial conditions stationarity arises right from the outset of the game.

Thus, let $w_{j,\tau}$ and $l_{j,\tau}$ be the wage and the employment level chosen respectively by the union and by any single firm in state (j, τ), that is for the τ-*th* period of a business spell of type j, $j = g, b$ and $\tau = 1, 2...$

Given the wage policy, the optimal employment sequence - or, equivalently, the optimal hiring and firing sequence - solves the following Bellman problem:

$$\begin{aligned} V_{j,\tau}(l_{t-1}) &= \max_{\{l_{j\tau}\}} R(\alpha_j, l_{j,\tau}) - w_{j,\tau} l_{j,\tau} - I_{l_{j,\tau} \le l_{t-1}} F\left(l_{t-1} - l_{j,\tau}\right) + \\ &\quad + \frac{1}{1+r}\left[q V_{j_{-1},1}(l_{j,\tau}) + (1-q) V_{j,\tau+1}(l_{j,\tau})\right] \end{aligned}$$

$V_{j,\tau}(l_{t-1})$ represents the value of the firm given by the sum of the current cash flow plus the expected discounted continuation value. Next period business conditions either change (with probability q) or remain constant (with probability $1-q$). In the first case the vector that describes the state becomes $(j_{-1}, 1)$ with j_{-1} representing business conditions opposite to j, in the second case the state vector becomes $(j, \tau+1)$ as the only variable that changes is the elapsed duration of the current spell.

To characterise the optimal hiring and firing policy in intuitive terms it appears convenient to introduce the notion of the shadow value of labour. We define the shadow value $S_{j,\tau}$ as the variation in the value of the firm $V_{j,\tau}(l_{t-1})$ following a marginal upward shift in the employment path from time $t-1$ onwards. This marginal shift is computed along the optimal hiring and firing policy so that, by the envelope theorem, $S_{j,\tau}$ also coincides with the derivative of $V_{j,\tau}(l_{t-1})$ with respect to l_{t-1} and, as a consequence, can be interpreted as the evaluation of an extra unit of labour *permanently* added to the workforce at the beginning of period t.

Thus, bearing in mind that a marginal increase in l_{t-1} is accompanied by an equal increase in $l_{j,\tau}$, differentiate $V_{j,\tau}(l_{t-1})$ with respect to l_{t-1} and express $S_{j,\tau}$ in recursive form:

$$S_{j,\tau} = R_l(\alpha_j, l_{j,\tau}) - w_{j,\tau} + \frac{1}{1+r}\left[q S_{j_{-1},1} + (1-q) S_{j,\tau+1}\right] \tag{2}$$

The current shadow value is given by the current net marginal revenue of labour - that is the difference between the marginal revenue and the wage - plus the expected discounted next period shadow value.

We are now ready to characterise the optimal policy. Notice that the derivative of $V_{j,\tau}(l_{t-1})$ with respect to $l_{j\tau}$ jumps from $S_{j\tau}$ to $S_{j\tau} + F$ if $l_{j\tau}$ moves from a value strictly above l_{t-1} to a value equal or below l_{t-1}. Thus, $S_{j\tau}$ represents the gain from hiring an extra unit of labour while $-(S_{j\tau}+F)$ the gain from firing. As usual when dealing with discontinuous derivatives, inaction - i.e. $l_{j,\tau}=l_{t-1}$- may turn out to be the optimal decision for a non-degenerate subset of values in the forcing variables. In the present case, this happens when the shadow value $S_{j\tau}$ is negative but greater than $-F$. In fact, in these circumstances hiring and firing both entail a loss for the firm. Intuitively, the shadow value is negative and, if firing were costless, firms would certainly fire not hire. With costly dismissals, however, the shadow value must be lower than the F to trigger a firing decision.

Positive workforce adjustments occurs only when the shadow value falls outside the inaction interval $[-F,0]$. When the shadow value is positive, optimality requires recruiting new workers up to the point an extra hiring becomes valueless or, more formally, up to point the shadow value decreases to zero. By contrast, when the shadow value is below $-F$, firms fire until the value increases to $-F$.[1] Formally, the policy just described is expressed by the following couple of f.o.cs:

$$S_{j,\tau}\leq 0 \qquad l_{j\tau}\geq l_{t-1} \qquad S_{j,\tau}\left(l_{j\tau}-l_{t-1}\right)=0 \tag{3}$$

$$S_{j,\tau}\geq -F \qquad l_{j\tau}\leq l_{t-1} \qquad \left(F-S_{j,\tau}\right)\left(l_{t-1}-l_{j\tau}\right)=0 \tag{4}$$

For future reference, we end this sub-section by presenting a different formula for $S_{j,\tau}$ whereby the shadow value is expressed as a discounted sum of net marginal revenues across all possible future states. This amounts to running forward equation 2. To accomplish this task, however, we need to bring some more structure to the model. Thus, we first introduce the function $p[(j,\tau);(j',\tau'),s]$ which describes the transition probability after an interval of exactly s periods from the current state (j,τ) to state (j',τ'). Second, we represent with $T[(j,\tau),(j',\tau')]$ the value in state (j,τ) of a perpetual asset that pays one euro when state (j',τ') occurs. Borrowing from the general equilibrium theory, T can be regarded as a pricing function for state-contingent Arrow-Debreu securities. Using transition probabilities, T can be

[1] The shadow value is clearly decreasing with respect to the employment level, see equation 5 below.

expressed as follows:

$$T[(j,\tau),(j',\tau')] = \sum_{s=0}^{\infty} p[(j,\tau);(j',\tau'),s]\left(\frac{1}{1+r}\right)^{s}$$

By using T one can express the shadow value of labour as a weighted sum of net marginal values for all possible states:

$$S_{j\tau} = \sum_{j'}\sum_{\tau'} T[(j,\tau),(j',\tau')]\left[R_l(\alpha_{j'}, l_{j',\tau'}) - w_{j',\tau'}\right] \tag{5}$$

3.2 The optimal wage policy

For simplicity - and without loss of generality - suppose the union sets up the optimal wage policy in the first period of a spell of type $\widehat{j}$ and let $W_{\widehat{j},1}$ represent the expected discounted flow of union payoffs:

$$W_{\widehat{j},1} = \sum_{j'}\sum_{\tau'} T[(\widehat{j},1),(j',\tau')]U(w_{j'\tau'}, L_{j'\tau'})$$

Notice that, in the spirit of equation 5, $W_{\widehat{j},1}$ has not been written in recursive form but as a weighted sum of flow utilities across all possible future states. The union maximises $W_{\widehat{j},1}$ by choosing the whole wage sequence $\{w_{j,\tau}\}$ and, since aggregate employment $L_{j,\tau}$ coincides with $l_{j,\tau}$, by taking account of the optimal employment policy of firms in the form of equations 3 and 4. This leads us to write the union problem by adopting the lagrangean approach:

$$\max_{\{w_{j,\tau}\}} W_{\widehat{j},1} - \sum_{\widetilde{j}}\sum_{\widetilde{\tau}}\left[\mu_{\widetilde{j}\widetilde{\tau}} S_{\widetilde{j}\widetilde{\tau}} + \lambda_{\widetilde{j}\widetilde{\tau}}\left(-F - S_{\widetilde{j}\widetilde{\tau}}\right)\right]$$

Using state contingent prices converts the dynamic problem into a static one. The optimal employment policy is accounted by lagrangean multipliers μ and λ. This policy is binding from the point of view of the union in all states where the shadow value of labour is reset to the extremes of the interval $[-F, 0]$ through positive workforce adjustments. For, in these states, a change in the wage sequence leads to a change in the employment level. More specifically, an increase in the wage level charged in some state induces a reduction in the shadow value S in all possible states - see equation 5 - and triggers a reduction in the employment level in all those states where firms fire or hire at positive rates. In the first case firms fire more, in the second case firms hire less. Thus, lagrangean multipliers $\mu_{\widetilde{j},\widetilde{\tau}}$ and $\lambda_{\widetilde{j},\widetilde{\tau}}$ represent the impact in terms of the union welfare in state $(\widehat{j},1)$ from a marginal relaxation of the constraints in state $(\widetilde{j},\widetilde{\tau})$

Below, we derive the f.o.c.s for the wage and the employment levels in state (j,τ) - $w_{j,\tau}$ and $l_{j,\tau}$ - together with complementary slackness:

$$U_w(w_{j,\tau},L_{j,\tau})T[(\widehat{j},1),(j,\tau)]+\sum_{\widetilde{j}}\sum_{\widetilde{\tau}}\left(\mu_{\widetilde{j}\widetilde{\tau}}-\lambda_{\widetilde{j}\widetilde{\tau}}\right)T[(\widetilde{j},\widetilde{\tau}),(j,\tau)]=0 \tag{6}$$

$$U_l(w_{j,\tau},L_{j,\tau})T[(\widehat{j},1),(j,\tau)]-R_{ll}(\alpha_j,l_{j,\tau})\sum_{\widetilde{j}}\sum_{\widetilde{\tau}}\left(\mu_{\widetilde{j}\widetilde{\tau}}-\lambda_{\widetilde{j}\widetilde{\tau}}\right)T[(\widetilde{j},\widetilde{\tau}),(j,\tau)]=0 \tag{7}$$

$$with\ L_{j,\tau}=l_{j,\tau}$$

$$\mu_{j,\tau}\geq 0 \qquad S_{j,\tau}\leq 0 \qquad \mu_{j,\tau}S_{j,\tau}=0$$

$$\lambda_{j,\tau}\geq 0 \qquad -F-S_{j,\tau}\leq 0 \qquad \lambda_{j,\tau}\left(-F-S_{j,\tau}\right)=0$$

By manipulating equations 6 and 7 we derive the following two expressions. The first governs the "dynamics" of wages within each spell, the second the size of the wage in any period of the spell:

$$\begin{aligned}\frac{U_w(w_{j,\tau},L_{j,\tau})}{\frac{1-q}{1+r}U_w(w_{j,\tau+1},L_{j,\tau+1})} &= \frac{A}{-\left(\mu_{j,\tau+1}-\lambda_{j,\tau+1}\right)+\frac{1-q}{1+r}A} \\ \text{where} \qquad A &= -\sum_{\widetilde{j}}\sum_{\widetilde{\tau}}\left(\mu_{\widetilde{j}\widetilde{\tau}}-\lambda_{\widetilde{j}\widetilde{\tau}}\right)T[(\widetilde{j},\widetilde{\tau}),(j,\tau)]>0\end{aligned} \tag{8}$$

$$U_l(w_{j,\tau},L_{j,\tau})=-R_{ll}(\alpha_j,l_{j,\tau})U_w(w_{j,\tau},L_{j,\tau}) \qquad with\ L_{j,\tau}=l_{j,\tau} \tag{9}$$

Equation 8 is obtained by dividing equation 6 through itself after advancing the τ index (see the appendix for details), equation 9 results from combining 6 and 7.

Intuitively, equation 8 equates the marginal rate of substitution between the two wage rates $w_{j,\tau}$ and $w_{j,\tau+1}$ [on the LHS of the equation] to their marginal rate of transformation [on the RHS]. The marginal utility of $w_{j,\tau+1}$ turns out to be weighted by $\frac{1-q}{1+r}$ since state $(j, \tau+1)$ occurs *after* state (j, τ) and, conditional on the occurrence of the latter, only with probability $(1-q)$. The amount A represents the cost of a marginal increase in $w_{j,\tau}$ in terms of lower employment levels in all states. The cost of a marginal increase in $w_{j,\tau+1}$ can be higher or lower than A depending on whether firms adjust employment in state $(j, \tau+1)$. If no adjustment takes place, then $\left(\mu_{j,\tau+1} - \lambda_{j,\tau+1}\right) = 0$ and the marginal cost of an increase in $w_{j,\tau+1}$ is given by $\frac{1-q}{1+r}A$. which is obviously lower than A. In this case $w_{j,\tau}$ is more effective than $w_{j,\tau+1}$ in reducing employment in all states. Again, this is due to the fact that $(j, \tau+1)$ occurs with a $(1-q)$ probability only *after* state (j, τ) has already occurred.

Equation 9 implies that the optimal wage policy is such that the union equates in all states the marginal rate of substitution between employment and wages to the slope of labour demand. In graphical terms, this solution coincides with the tangency between the demand schedule and the highest indifference curve. However, in contrast with the textbook analysis, the position of labour demand is not exogenous, rather it depends on the entire sequence of wages as implied by the employment policy and by equation 2.

We are now ready to establish two lemmas and a proposition.

Lemma 1: $\mu_{j,\tau+1} = 0$, $j = b, g$ and $\tau = 1, 2....$

Proof:

The proof is conducted by contradiction. Assume the opposite to be true, that is $\mu_{j,\tau+1} > 0$. By the complementary slackness, it must also be true that $S_{j,\tau+1} = 0$ and $L_{j,\tau+1} > L_{j,\tau}$ while equation 8 requires that $U_w(w_{j,\tau}, L_{j,\tau}) > U_w(w_{j,\tau+1}, L_{j,\tau+1})$. Since $U_{wL} > 0$, if $U_{ww} = 0$ the last inequality can never be true. If instead $U_{ww} < 0$ the inequality can be true only in case $w_{j,\tau+1} > w_{j,\tau}$. That is, *both* the wage and the employment level *increase* from τ to $\tau+1$. Since the wage and the employment level are both normal goods for the union, the two variables increase together only if the labour demand moves *upward* in the wage employment space. Below we report the demand schedules for the two periods, τ and $\tau+1$, after imposing $S_{j,\tau+1}=0$:

$$w_{j,\tau} = -S_{j,\tau} + \frac{q}{1+r} S_{j_{-1},1} + R_l(\alpha_j, l_{j,\tau})$$

$$w_{j,\tau+1} = \frac{1}{1+r}\left[qS_{j_{-1},1} + (1-q)S_{j,\tau+2}\right] + R_l(\alpha_j, l_{j,\tau+1})$$

We observe that $S_{j,\tau}$ and $S_{j,\tau+2}$ belong to the closed interval $[-F, 0]$. Accordingly, the highest labour demand in period $\tau+1$ obtains when $S_{j,\tau+2}=0$ while the lowest labour demand in period τ obtains when $S_{j,\tau}=0$. Yet, even in this case, demand in period $\tau+1$ does not represent an upward shift with respect to demand in period τ.$\diamond$

Lemma 2: $\lambda_{j,\tau+1}=0$, $j=b,g$ and $\tau=1,2....$

Proof:

Similar to lemma 1.$\diamond$

Proposition 1: Employment and wages do not change within a spell of constant business conditions.

Proof:

Lemmas 1 and 2 show that constraints imposed by the hiring and firing policy are not strictly binding in all periods of a given spell with the exception of the first. This means that employment does not change within the spell to force the shadow value inside the closed interval $[-F, 0]$. If employment is constant, equation 9 implies that wages are also constant.$\diamond$

In principle, these results are consistent with a number of actual wage and employment sequences. Equilibrium, for instance, may exhibit constant employment not only within each business spell but also across spells. Alternatively, employment during a good spell may be higher than employment during a bad spell[2]. We refer to the latter case as the equilibrium with positive workforce adjustments and, due to

[2]For obvious reasons, we omit to consider the equilibrium with the employment level in good times below the one prevailing in bad times.

its empirical relevance, devote to it some more attention in the next subsection. An equilibrium with positive adjustments, however, does not exist in all circumstances. Constant employment in all states, for instance, can be an equilibrium if firing costs turns out to be particularly high. Thus, in the next sub-section we also need to check what parameters restrictions guarantee the existence of an equilibrium with positive workforce adjustments.

3.3 Positive workforce adjustments

Let us indicate with $l_{g,c}$, $w_{g,c}$, $l_{b,c}$, $w_{b,c}$ [c: commitment] the employment and the wage levels respectively in the good and bad spells in an equilibrium with positive adjustments. In such an equilibrium, it must be true that $l_{g,c} > l_{b,c}$. Firms hire in the first period of a good spell and fire in the first period of a bad spell, inaction prevails at all other times. Accordingly, the shadow value is equal to 0 in the first period of a good spell and to $-F$ in the first period of a bad spell. In addition, constant wages and employment levels coupled with the memoryless property of Markov transitions lead to a constant shadow value within each business spell: $S_{g,\tau} = S_g = 0$ and $S_{b,\tau} = S_b = -F$.

In each business state, employment and wages must be consistent with optimal behaviour on the part of firms and the union. Formally, this requires consistence with equations 3 and 4 for the firm and with equation 9 for the union policy. We notice that these equations do not depend on the initial state, that is on the state at the time the union decides the wage sequence. Thus, the equilibrium is state-consistent in the sense that the wage sequence does not become sub-optimal after the initial period of the game.

Solving for the couple ($l_{g,c}$, $w_{g,c}$) amounts to solving the system composed by equations 2 and 9 after substituting the relevant values for S in the two business states:

$$w_{g,c} = -\frac{q}{1+r}F + R_l(\alpha_g, l_{g,c}) \tag{10}$$

$$U_l(w_{g,c}, l_{g,c}) = -R_{ll}(\alpha_g, l_{g,c})U_w(w_{g,c}, l_{g,c}) \tag{11}$$

The first equation is similar to the standard labour demand schedule with linear workforce adjustment costs usually derived in a context of exogenous wages (Bertola, 1990). Here we show that it holds unchanged if wages are set by a union that can commit to the whole wage sequence. The second is even more familiar as it coincides with the optimality condition in a static union monopoly model. Thus, the union chooses a point on the labour demand where the latter is tangent to the highest indifference curve.

The system that solves for the couple $(l_{b,c},\, w_{b,c})$ parallels the one above:

$$w_{b,c} = F - \frac{1-q}{1+r}F + R_l(\alpha_b, l_{b,c}) \tag{12}$$

$$U_w(w_{b,c}, l_{b,c}) = -R_{ll}(\alpha_b, l_{b,c})U_l(w_{b,c}, l_{b,c}) \tag{13}$$

Although these equations close the solution of the game under commitment, we still need to tackle the issues of existence and stationarity. More formally, we need to find under what parameter restrictions a) the equilibrium with positive adjustments exists and b) the equilibrium is stationary.

Existence requires employment in good times $l_{g,c}$ to be greater than employment in bad times $l_{b,c}$. If this were not the case, firms would not hire at business upturns and fire at downturns. An equilibrium with positive adjustments would not exists. By inspecting the two systems of equations that provide the solution, we conclude that a necessary and sufficient condition for existence is that the labour demand in the good state lie above the labour demand in the bad state in the wage-employment space:

$$R_l(\alpha_g, l) - R_l(\alpha_b, l) > \frac{2q+r}{1+r}F \tag{14}$$

We assume that this restriction holds. Intuitively, we assume that firing costs are sufficiently low and/or the variation in marginal revenues at business changes sufficiently large. Notice also that firing costs enter the inequality in combination with the transition rate q. An higher transition probability makes business spells less durable and subtract incentives to workforce adjustments. For this reason, for given firing costs the above inequality tend to be true for low values of q.

We now turn to the issue of stationarity. We observe that stationarity requires that at the outset of the game the triple (S, w, l) be equal to $(0, w_{g,c}, l_{g,c})$ if business conditions are good and to $(-F, w_{b,c}, l_{b,c})$ if these are bad. Is is rather easy to show that this is the case if and only if:

$$l_{b,c} < l_{t-1} < l_{g,c} \tag{15}$$

This restriction guarantees that in the initial period the shadow value S lies on the hiring line $[S = 0]$ if the state is good. Lagged employment, in fact, is lower than $l_{g,c}$ so that firms need to hire in order to reach $l_{g,c}$. For the same reason, this restriction guarantees that the shadow value lies on the firing line $[S = -F]$ if the state is bad. This means that the systems of equations that solve for $(l_{g,c},\ w_{g,c})$ and $(l_{b,c},\ w_{b,c})$ hold in the first period too.

4 Wages and employment without commitment

In this section we analyse the union-firms interaction under the assumption that the union can not commit to a state-dependent wage policy. In this case, union and firms play a game where decisions are optimal at any point in time conditional on the current business state, on the opponent decisions and on the expected future outcomes of the game. Since the dynamics is governed by a Markov process, if each player conjectures that all players adopt Markov strategies, that is their moves are made conditional only on the current state, the current state also encapsulates all relevant information for expectations held at current time. This implies that optimal decisions become ultimately only functions of the current state and that conjectures are self-fulfilling. Thus, let the current state from the point of view of the union be summarised by the vector (j, L_{t-1}) and let $w(j, L_{t-1})$ represent the optimal wage strategy. From the point of view of each single firm, due to the presence of adjustment costs, the state of the game is also comprehensive of its own level of lagged employment so it should be represented by the triple (j, l_{t-1}, L_{t-1}). We observe, however, that the strategy adopted by the union implies that, for given business conditions, the wage represents a sufficient statistics for lagged aggregate employment L_{t-1}. It follows that the state of the game for each firm can be equivalently represented by the vector (j, l_{t-1}, w_t) while the optimal employment strategy takes the form $l\,[j, l_{t-1}, w_t]$.

Equilibrium requires that each strategy maximises the discounted payoff flow for the corresponding player given the strategy adopted by the opponent. In short, strategies must be mutual best responses. This notion of equilibrium is often referred as Markov equilibrium (Maskin e Tirole, 1988).

4.1 The employment strategy

Let us indicate with $S\,[j,, l_{t-1}, w_t]$ the shadow value of labour in the Markov equilibrium:

$$\begin{aligned} S\,[j,, l_{t-1}, w_t] &= R_l(\alpha_j, l) - w(j, L_{t-1}) + \\ &+ \frac{1}{1+r}\left[qS\,[j_{-1}, l, w(j_{-1}, L)] + (1-q)S\,[j_{-1}, l, w(j, L)]\right] \end{aligned}$$

L and l represent current employment levels, at sector and firm level respectively. Albeit conceptually distinct, however, the two levels are in fact equal as firms sum up to a unit mass.

The optimal policy is similar to the one that arises under commitment. If, in the absence of workforce adjustments, S falls within the closed interval $[-F, 0]$, then inaction is optimal. If instead the shadow cost lies above 0, then firms hire and the new employment level l is such that the shadow cost decreases to 0. Finally, if the shadow cost lies below $-F$, then firms fire and the new employment level l is such that the shadow cost increases to $-F$.

4.2 The wage strategy

In this section we characterise the wage strategy and, more generally, the whole equilibrium by means of a set of formal results and propositions.

Given the above employment strategy the union chooses the wage strategy that solves the following Bellmann problem where, to save on notation, we use w' instead of the fully specified policy variable $w(j, L_{t-1})$:

$$W(j, L_{t-1}) = \max_{w'} U(w', L') + \frac{q}{1+r} W(j_{-1}, L') + \frac{1-q}{1+r} W(j, L')$$

$$s.t. \quad L' \equiv l(j, l_{t-1}, w')$$

The welfare of the union is given by the current payoff plus the expected discounted continuation value. Alternatively, $W(j, l_{t-1})$ can be interpreted as the expected discounted sum of flow payoffs U computed along the equilibrium path. Since the employment path from time $t-1$ onwards is non-decreasing with respect to l_{t-1} such an interpretation of $W(j, l_{t-1})$ allows us to state $W_l(j, l_{t-1}) \geq 0$ and $W_{ll}(j, l_{t-1}) \leq 0$.[3] In short, the value function inherits, albeit in a weak form, some of the properties of the function U. This fact is relevant as it implies that neither the current wage w' nor the current employment level L' are inferior goods from the point of view of the union.

Result 1: *In equilibrium, neither the wage nor the employment are inferior goods for the union.*

A further relevant feature of the equilibrium is that the shadow value S must be equal to $-F$ in all states where employment does not change from the previous period. In fact, if this were not the case the union would not optimise. More specifically, if employment does not change and the shadow value remains above the firing boundary the union gives up the opportunity to increase the wage - at least up to the point S is pushed downwards to the value $-F$ - without paying any cost in terms of lower employment. In addition, such a wage increase is of no consequence not only in terms of current employment but also for the future employment and wage levels since the latter, in a Markov equilibrium, depend uniquely on current

[3] According to the optimal employment policy, an increase in l_{t-1} either leaves l_t unaffected or leads to an increase of the latter. This case arises when firms choose inaction at time t.

Thus, an increase in l_{t-1} moves upwards or leaves unaffected the whole employment path.

employment and business conditions. The argument also implies the following obvious corollary. In equilibrium, in all states of the game a marginal increase in the wage *must* produce a reduction in the employment level. Again, if this were not the case the union would not fully exploit its monopoly position. Thus, in equilibrium, the shadow value either lie on the firing boundary or on the hiring boundary.

Result 2

a) $S[j, l_{t-1}, w'] = -F$ *if* $l[j, l_{t-1}, w'] = l_{t-1}$

b) S *is either equal to* 0 *or to* $-F$.

We are now ready to establish two propositions.

Proposition 3

In a Markov equilibrium, the shadow value moves from the hiring to the firing barrier if business conditions remain constant.

Proof:

By contradiction. Suppose the shadow value lies on the hiring barrier both in period t and $t+1$ while business conditions are indexed by j in both periods, formally:

$$S[j, l_{t-1}, w_t] = S[j, l_t, w_{t+1}] = 0$$

In this case, the relevant labour demand schedules facing the union in the two periods are:

$$w_t = \frac{q}{1+r} S[j_{-1}, l_t, w(j_{-1}, L_t)] + R_l(\alpha_j, l_t)$$

$$w_{t+1} = \frac{1}{1+r}\{qS[j_{-1}, l_t, w(j_{-1}, L_{t+1})] +$$

$$+(1-q)S[j, l_t, w(j, L_{t+1})]\} + R_l(\alpha_j, l_{t+1})$$

Observe that 1) both schedules are downward sloping in the wage employment space and 2) the second schedule coincides with the first if $S[j, l_t, w(j, L_{t+1})] = 0$

or lies below if $S[j, l_t, w(j, L_{t+1})] = -F$. Thus, since neither the wage nor the employment are inferior goods, employment in period $t+1$ can not be higher than employment in period t. That is, either employment decreases - but this contradicts the assumption that the shadow value lies on the hiring boundary in period $t+1$ - or employment does not change. Result 2, however, dictates that when employment does not change the shadow value must lie on the firing, not on the hiring, barrier. Again, a contradiction arises.◇

Proposition 4

In a Markov equilibrium, the shadow value does not move from the firing to the hiring barrier if business conditions remain constant.

Proof:

By contradiction. Suppose the shadow value lies on the firing barrier in period t and on the hiring barrier in period $t+1$ while business conditions are indexed by j in both periods. Formally:

$$S[j, l_{t-1}, w_t] = -F \quad \text{and} \quad S[j, l_t, w_{t+1}] = 0$$

In this case, the relevant labour demand schedules facing the union in the two periods are:

$$w_t = F + \frac{1}{1+r} qS[j_{-1}, l_t, w(j_{-1}, L_t)] + R_l(\alpha_j, l_t)$$

$$\begin{aligned} w_{t+1} = & \frac{1}{1+r}\{qS[j_{-1}, l_{t+1}, w(j_{-1}, L_{t+1})] + \\ & +(1-q)S[j, l_{t+1}, w(j, L_{t+1})]\} + R_l(\alpha_j, l_{t+1}) \end{aligned}$$

Observe that since the highest value for $S[j, l_{t+1}, w(j, l_{t+1})]$ is 0 the schedule in period $t+1$ is always below the schedule in period t in the wage employment space. Since neither the wage nor the employment are inferior goods, in period $t+1$ the union chooses a point where the wage and the employment levels can not be higher than previous values. Thus, in period $t+1$ firms either fire or keep the previous employment level, this contradicts the assumption that the shadow value lies on the hiring boundary at $t+1$.$\diamond$

Proposition 3 and 4 imply that firms are for most of the times on the firing boundary and that they can be on the hiring boundary only in the first period of a spell of constant business conditions. Thus, only two types of equilibrium may arise. The first type is an equilibrium where the shadow value stays on the firing barrier at all times. This means that employment is constant in all states and that wages change at business turns in order to peg the shadow value on the firing boundary. The second type is an equilibrium characterised by the shadow value on the hiring boundary in the first period of a good spell and on the firing boundary at all other times. Such an equilibrium features positive workforce adjustments, firms hire when conditions turn good from bad and fire in the opposite case. Again, due to its empirical relevance, in the reminder of this section we focus on this type of equilibrium.

4.3 Positive workforce adjustments

Since the shadow value stays permanently on the firing barrier during a bad spell, the labour demand in bad times does not move from one period to the other. This means that, along a bad spell, the same wage and employment levels, $w_{b,nc}$ and $l_{b,nc}$ [nc: non commitment], are chosen in all periods. By contrast, when the state is good, the analysis becomes slightly more complicated as the shadow value stays on the hiring boundary in the first period and on the firing boundary in all other subsequent periods [proposition 3]. We deal with this case in proposition 5

Proposition 5

In a Markov equilibrium with positive workforce adjustments, during a spell of good business conditions:

a) employment does not change from the first to the second period of the spell;

b) the wage increases by F from the first to the second period of the spell;

c) employment and wages remain constant from the second period onwards.

proof:

Let $l_{g,nc}$ and $w_{g,nc}$ represent the employment and the wage levels in the first period of a good spell while $l'_{g,nc}$ and $w'_{g,nc}$ represent the same variables in the second period. Notice that $l'_{g,nc} > l_{g,nc}$ is ruled out by the fact that in the second period the shadow value is on the firing boundary. Thus, contradict assertion a) and suppose $l'_{g,nc} < l_{g,nc}$, that is suppose in the second period firms fire even if conditions remain good. In this case, the relevant labour demand schedules in the first and second period are respectively:

$$w_{g,nc} = -\frac{F}{1+r} + R_l(\alpha_g, l_{g,nc}) \tag{16}$$

$$w'_{g,nc} = F - \frac{F}{1+r} + R_l(\alpha_g, l'_{g,nc}) \tag{17}$$

Both schedules embed the result that the shadow value becomes equal to $-F$ in the second period no matter whether business conditions remain good or turn bad (proposition 3). Observe that the second schedule lies above the first in the wage employment space. Thus, since neither the wage nor the employment are inferior goods, it follows that $l'_{g,nc} \geq l_{g,nc}$. This contradicts the assumption $l'_{g,nc} < l_{g,nc}$ and proves part a) of the proposition. More precisely, it proves that the only possible case is $l'_{g,nc} = l_{g,nc}$.

If the employment level does not change from the first to the second period, it follows that the shift in labor demand only affects wages. Subtract equation 17 from equation 16 and find:

$$w'_{g,nc} = w_{g,nc} + F$$

This ends the proof of point b).

Finally, notice that in the third period of a good spell the union inherits the employment level $l_{g,nc}$ and is faced with the same schedule arising in the second period. Thus, the wage and employment levels of the second period are replicated in the third period and, by induction, in all other periods. This ends the proof of point c).$\square$

We conclude this section by stating the conditions that allow the computation of the four relevant equilibrium variables: $w_{g,nc}$, $l_{g,nc}$, $w_{b,nc}$ and $l_{b,nc}$.

We start with the employment and wage levels in good times and observe that the wage $w_{g,nc}$ is determined so as to maximise the present discounted payoff flow along the spell upon taking account of the F increase in wages from the second period onwards:[4]

$$\max_{w_{g,nc}} U(w_{g,nc}, L_{g,nc}) + \frac{1-q}{r+q} U(w_{g,nc} + F, L_{g,nc}) \tag{18}$$

$$s.t.\ w_{g,nc} = -\frac{F}{1+r} + R_l(\alpha_g, l_{g,nc}) \quad \text{and} \quad L_{g,nc} = l_{g,nc} \tag{19}$$

The f.o.c. for this problem is:

$$\begin{aligned} &U_l(w_{g,nc}, l_{g,nc}) + \frac{1-q}{r+q} U_l(w_{g,nc} + F, l_{g,nc}) = \\ &= -R_{ll}(\alpha_g, l_g) \left[U_w(w_{g,nc}, l_{g,nc}) + \frac{1-q}{r+q} U_w(w_{g,nc} + F, l_{g,nc}) \right] \end{aligned} \tag{20}$$

this needs to be solved together with the constraint 19 to find $w_{g,nc}$ and $l_{g,nc}$.

Since both wages and employment are constant along a bad spell, $w_{b,nc}$ and $l_{b,nc}$ may be computed as a solution of a simple static problem:

$$\begin{aligned} &\max_{w_{b,nc}} U(w_{b,nc}, l_{b,nc}) \\ &s.t.\ w_{b,nc} = F - \frac{1-q}{1+r} F + R_l(\alpha_j, l_{b,nc}) \end{aligned}$$

Straightforward differentiation produces the same conditions that solve for $w_{b,c}$ and $l_{b,c}$ in the equilibrium under commitment, i.e. equations 12 and 13. Thus, the wage and the employment in the bad state are the same no matter whether the union is able or not to commit to a wage sequence. By contrast, in the good state the first

[4] With positive workforce adjustments the level of employment in good times does not affect the welfare of the union during the following bad spell. Thus, the choice of $w_{g,nc}$ needs to maximise the payoff flow only along the good spell (equation 18).

order condition and the demand schedule for the commitment case are different in comparison with those for the no-commitment case. This implies that employment and wages in good times depend in general on whether the union is capable to make a commitment. The ability to commit is relevant only in good times.

5 The utilitarian-expected utility case

In general, the ability to commit produces different equilibrium outcomes only when an hold-up problem occurs. In the present context this problem arises only at hiring times or, more specifically, during a spell of good business conditions. In this section we study whether the ability to commit increases the employment level in the good state when the payoff function of the union takes the following form:

$$U(w,l) = lv(w) + (m-l)v(\overline{w})$$

This expression is usually referred as the utilitarian objective function, m gives union membership and $(m-l)$ the number of unemployed members. The utility of each member is given by the function v whose argument is represented by the union wage w for those who happen to be employed and by the "alternative" wage $\overline{w}$ for the unemployed. With a constant membership m the utilitarian function is isomorphic to the expected utility function. In this case the objective of the union coincides with the expected utility of a randomly chosen member under the assumption that all members are equal and, more importantly, face an equal probability of unemployment:

$$U(w,l) = \frac{l}{m}v(w) + \frac{(m-l)}{m}v(\overline{w})$$

Thus, albeit conceptually different, the two objective functions produce in fact the same outcome in terms of the ordering over different wage-employment baskets. For this reason, in the remainder of this section we only deal with the utilitarian function.

We also assume that the utility function of each worker may be linear or concave with non-negative third derivatives:

$v(w) > 0$, $v'(w) > 0$, $v''(w) \leq 0$ and $v'''(w) \geq 0$

To solve for $l_{g,c}$ substitute the utilitarian objective function in equation 11 and combine with equation 10. Analogously, to solve for $l_{g,nc}$ substitute the function in equation 20 and combine with equation 19. Below, we present the expressions that result from these procedures, notice that to save on notation we have omitted the arguments of the marginal revenue function R_l:

$$\eta_{R_l,l} R_l = \frac{v\left[a\left(-\frac{1}{1+r}F + R_l\right) + (1-a)\left(\frac{r}{1+r}F + R_l\right)\right] - v(\overline{w})}{v'\left[a\left(-\frac{1}{1+r}F + R_l\right) + (1-a)\left(\frac{r}{1+r}F + R_l\right)\right]} \qquad \text{(for } l_{g,c}\text{)}$$

$$\eta_{R_l,l} R_l = \frac{a\ v\left(-\frac{1}{1+r}F + R_l\right) + (1-a)\ v\left(\frac{r}{1+r}F + R_l\right) - v(\overline{w})}{\left[a\ v'\left(-\frac{1}{1+r}F + R_l\right) + (1-a)\ v'\left(\frac{r}{1+r}F + R_l\right)\right]} \qquad \text{(for } l_{g,nc}\text{)}$$

In these expressions, parameter a is equal to $\frac{r+q}{1+r}$ while $\eta_{R_l,l}$ represents the elasticity of the marginal revenue with respect to the employment level:

$$\eta_{R_l,l} = -R_{ll}\frac{l}{R_l}$$

Since the marginal revenue is assumed to be decreasing with respect to employment, this elasticity is positive.

Observe first that when v is linear $[v'' = 0]$, the two conditions coincide and the employment level is the same no matter whether the union is able to commit or not. This result represents a notable exception to the proposition whereby the inability to commit gives scope to an hold up problem and depresses the level of employment. Intuitively, when the utility function is linear, the union is not interested to the actual path of wages but only to the discounted value from the whole wage flow. Thus, the union does not find costly to charge a low wage in the first period in order to overcome the reluctance of firms towards hirings. In fact, the union is able to replicate the commitment outcome by charging a wage in the first period that is low enough to buy the same number of jobs which arise under commitment.

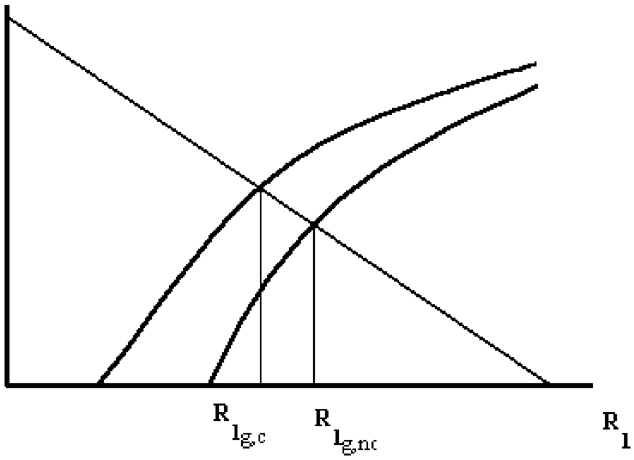

Figure 1: Linear revenue function.

Suppose next that the utility function is concave with a positive third derivative. By the Jensen's inequality, the numerator on the RHS of the expression for $l_{g,c}$ is higher than the numerator of the expression for $l_{g,nc}$. The denumerator instead is lower. This means that the RHS of the first expression, providing it is positive, is always higher than the RHS of the second expression. Further, if one regards the RHSs of the two expressions as functions of R_l, straightforward differentiation shows that the two RHSs increase and become closer as R_l increases.

Characterising the behaviour of the LHS appears trickier as the elasticity $\eta_{R_l,l}$ crucially depends on the type of revenue function at hand. For this reason we focus on the two cases commonly used in the literature, the linear and the log-linear (cobb-douglas) revenue functions. When the revenue function is linear, such as $R_l(\alpha, l) = \alpha - dl$, the elasticity is decreasing with respect to R_l: $\eta_{R_l,l} = (\alpha/R_l) - 1$. When the function is log-linear, such as $R_l(\alpha, l) = \alpha l^{-\beta}$, the elasticity is constant: $\eta_{R_l,l} = \beta$.

Below we draw the RHS and the LHS of the two expressions as functions of R_l.

Notice that the marginal revenue is lower under commitment both in the linear and in the log-linear case. Thus, we conclude that the employment level is higher under commitment, a result which is consistent with the hold up problem envisaged by Lindbeck and Snower. By the same argument, since firms equate the discounted flow of the marginal revenue to the discounted flow of wages, wages are on average

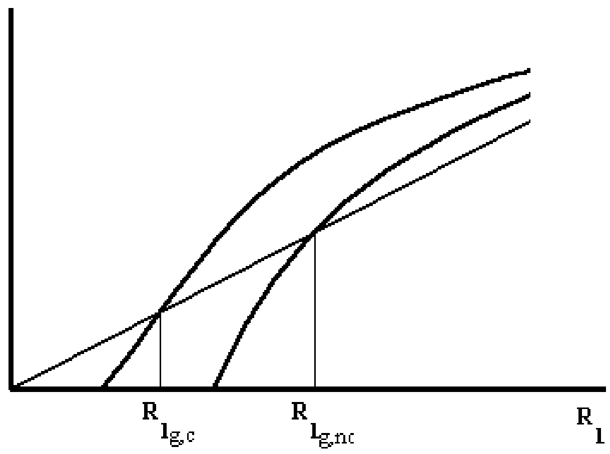

Figure 2: Log-linear revenue function.

lower under commitment.

What happens when the utility function is concave? Concavity implies two things. First, workers are risk averse, second they dislike sharp variations in wages. Risk aversion, however, plays the same role both under commitment and without a commitment. That is, in both cases it moderates wages in order to reduce the unemployment risk so it does not represent the source of the difference between the two cases. By contrast, aversion towards sharp changes in the wage profile bites for obvious reasons only in the non-commitment case. The inability to commit harms workers in that a constant wage profile with equal discounted value is strictly preferred to the actual one as the latter presents a sharp increase of size F at the end of the first period. This fact, however, does not explain by itself why the union chooses a lower employment level, and higher wages, in the non commitment case. Yet, it is not difficult to see how this outcome results either from a lower return from employment and from an higher return from wages. From the point of view of the union, the wage shift of size F from the first to the second period reduces the utility of each single employed worker and, henceforth, reduces the gain from being employed in comparison to being unemployed. Accordingly, the union faces a lower benefit from having a large number of employed workers, this effect is captured by the numerator in the expressions above. On the other hand, since the shift is fixed in size it becomes relatively less harmful in terms of workers utility when wages are particularly high. It follows that the union experiences an higher return from a wage increase, this effect is captured by the denominator.

6 Concluding remarks

We have presented a union-firms interaction featuring two basic assumptions, costly labour shedding and stochastic business conditions. Changes in business conditions induce hiring and firing, firms adjust the level of employment up to the point the discounted expected flow of marginal productivity equates the discounted expected flow of wages plus the expected future dismissal cost. Thus, the union can obtain high employment levels only if it is able to credibly promise to charge low wages in future periods.

We show that, after an hiring phase has been completed, the union has an incentive to increase the wage and to exploit the insider protection guaranteed by firing costs. Thus, the credibility regarding the promise of future wage moderation becomes an issue only after new workers have been hired. If the union was able to make a commitment over future wages, the ensuing equilibrium would feature constant wage and employment levels all over the spell. More importantly, the wage would be on average lower - and the employment level higher - when one compares this outcome with the one arising in the absence of a commitment. In addition, since the outcome without a commitment can be replicated under commitment, the ability to commit leads to a welfare gain for the union.

We have also explored the reasons for the two different outcomes with and without a commitment and have come to the conclusion that a crucial role is played by the curvature of individual utility functions. This curvature in fact controls for the substitutability of two wage rates received at different points in time. When workers are only concerned with the discounted flow of wages but not with the time profile of this flow, the two outcomes coincide in terms of employment levels and overall union welfare. This happens because the union does not find costly to ask particularly low wage rates during the hiring phase so as to buy the same number of jobs that arise under commitment. By contrast, when workers exhibit aversion towards sharp jumps in the wage path, buying jobs through very low initial wages is costly so that the union opts for an employment level lower than the one arising under commitment.

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Appendix

Derivation of the equation 8.

Write the f.o.c. for $w_{j,\tau}$ and $w_{j,\tau+1}$:

$$U_w(w_{j,\tau}, L_{j,\tau})T[(\widehat{j},1),(j,\tau)] = -\sum_{\widetilde{j}}\sum_{\widetilde{\tau}}\left(\mu_{\widetilde{j}\widetilde{\tau}} - \lambda_{\widetilde{j}\widetilde{\tau}}\right)T[(\widetilde{j},\widetilde{\tau}),(j,\tau)] \equiv A \quad (21)$$

$$U_w(w_{j,\tau+1}, L_{j,\tau+1})T[(\widehat{j},1),(j,\tau+1)] = -\sum_{\widetilde{j}}\sum_{\widetilde{\tau}}\left(\mu_{\widetilde{j}\widetilde{\tau}} - \lambda_{\widetilde{j}\widetilde{\tau}}\right)T[(\widetilde{j},\widetilde{\tau}),(j,\tau+1)] \quad (22)$$

These equations impose a balance between the discounted expected benefit and cost from a marginal increase in $w_{j,\tau}$ and $w_{j,\tau+1}$.

Consider now the following identity:

$$p[(\widetilde{j},\widetilde{\tau});(j,\tau+1),s] = (1-q)\ \ p[(\widetilde{j},\widetilde{\tau});(j,\tau),s-1]$$

and use the latter within the following stream of equations:

$$\begin{aligned}
T[(\widetilde{j},\widetilde{\tau}),(j,\tau+1)] &= \sum_{s=0}^{\infty} p[(\widetilde{j},\widetilde{\tau});(j,\tau+1),s]\left(\frac{1}{1+r}\right)^s = \\
&= p[(\widetilde{j},\widetilde{\tau});(j,\tau+1),0] + \sum_{s=1}^{\infty} p[(\widetilde{j},\widetilde{\tau});(j,\tau+1),s]\left(\frac{1}{1+r}\right)^s = \\
&= p[(\widetilde{j},\widetilde{\tau});(j,\tau+1),0] + \sum_{s=1}^{\infty} p[(\widetilde{j},\widetilde{\tau});(j,\tau),s-1]\,(1-q)\left(\frac{1}{1+r}\right)^{s-1}\frac{1}{1+r} = \\
&= p[(\widetilde{j},\widetilde{\tau});(j,\tau+1),0] + \frac{1-q}{1+r}\sum_{s=0}^{\infty} p[(\widetilde{j},\widetilde{\tau});(j,\tau),s]\left(\frac{1}{1+r}\right)^s = \\
&= p[(\widetilde{j},\widetilde{\tau});(j,\tau+1),0] + \frac{1-q}{1+r}T[(\widetilde{j},\widetilde{\tau}),(j,\tau)] \quad (23)
\end{aligned}$$

By analogy it also holds:

$$T[(\widehat{j},1),(j,\tau+1)] = p[(\widehat{j},1);(j,\tau+1),0] + \frac{1-q}{1+r}T[(\widehat{j},1),(j,\tau)]$$

Notice, however, that since $\tau \geq 1$ the probability of being in state $(j, \tau+1)$ at the outset of the game is zero. Thus, $p[(\widehat{j}, 1); (j, \tau+1), 0] = 0$ so that the last equation becomes:

$$T[(\widehat{j}, 1), (j, \tau+1)] = \frac{1-q}{1+r} T[(\widehat{j}, 1), (j, \tau)] \tag{24}$$

Substitute equations 23 and 24 in 22 and obtain:

$$\begin{aligned}
&U_w(w_{j,\tau+1}, L_{j,\tau+1}) \frac{1-q}{1+r} T[(\widehat{j}, 1), (j, \tau)] = \\
&= -\left(\mu_{j,\tau+1} - \lambda_{j,\tau+1}\right) - \frac{1-q}{1+r} \sum_{\widetilde{j}} \sum_{\widetilde{\tau}} \left(\mu_{\widetilde{j}\widetilde{\tau}} - \lambda_{\widetilde{j}\widetilde{\tau}}\right) T[(\widetilde{j}, \widetilde{\tau}), (j, \tau)] = \\
&= -\left(\mu_{j,\tau+1} - \lambda_{j,\tau+1}\right) + \frac{1-q}{1+r} A
\end{aligned}$$

Finally, divide both the LHS and the RHS of the latter by the corresponding terms of the equation 21 and obtain equation 8 in the main text.