Advances in Complex Systems

A SEARCH MODEL OF UNEMPLOYMENT AND FIRM DYNAMICS

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An urn-ball probabilistic model of the labor market is developed. Agents can be employed, (voluntary or involuntary) unemployed or entrepreneurs. The analytical long run equilibrium probabilities for each state and the matching function are derived. In equilibrium, a higher reservation wage increases the number of start-ups, but has an overall negative impact on the unemployment rate. A more buoyant economy (higher average growth rate and higher average wages) is shown to be associated with a lower unemployment rate. Higher start-up costs discourage entrepreneurship and increase unemployment. More active search behavior leads first to a decrease in the unemployment rate, and then to a small increase, due to increased coordination failure induced by the higher number of applications sent by job seekers. The out-of-equilibrium dynamics are investigated through an agent-based simulation, which also provides results on firm demography. Important empirical regularities such as the Beveridge and the Okun curve are recovered. Finally, the simulation model is used to investigate departures from maximizing individual behavior and the effects of more realistic assumptions about profits and the business cycle.

Keywords: Unemployment; start-up; search; simulation.

1. Introduction

The purpose of this paper is to incorporate the analysis of entrepreneurship and firm dynamics within a search-theoretic framework.

Search models have become the standard reference in the economic literature for the analysis of unemployment [a]. However, they still fail to take into account many features of real labor markets. In particular, firms are hardly considered, and are replaced by vacancies, *i.e.* by single-job entities. Old firms never die; new firms are never born: instead, jobs appear and disappear. Job creation is endogenous, but job destruction is generally exogenously given. A first attempt to provide a more realistic description of layoffs is found in [4], where each job offer is characterized by two variables – a wage and a constant probability of the position being closed

[a] [13, 16, 15] provide extensive reviews of search models for the labor market.

down. Clearly, this is still a rather simplistic way of treating job destruction. In order to improve the model, two mechanisms have been introduced. The first considers (stochastic) shocks to the productivity of each job. The job is then closed down if its productivity falls below a minimum threshold [12]. The alternative is to consider job obsolescence over time. Old jobs offer smaller wages. Thus, they will become increasingly less attractive to workers, and will eventually be closed down [1, 5]. Note that in neither cases does job destruction depend on unemployment. Because of their oversimplified description of firms, these models never allow job creation and job destruction to depend on variables such as the number of firms in the market, or the size of the firm. Moreover, job destruction is generally not modeled separately from firing decisions, making it impossible to distinguish between worker and job turnover. The only other way to provide for such a distinction without modeling layoffs is through consideration of on-the-job searches. The number of vacancies must then be updated accordingly, and should increase with a greater number of job-to-job changes. [3] provides the first attempt to model on-the-job search. However, in his model there is no determination of the number of vacancies. Thus, job quits may be indifferently interpreted as job destruction. On-the-job search intensity may depend on experienced wage shocks, or on learning about the utility deriving from that work [10]. In particular, since learning increases with tenure, models like Jovanovic's imply that workers with longer tenures are less likely to quit, and are more likely to be gaining higher wages.

Job creation is obviously linked to entrepreneurship. In particular, the decision to create a new business rather than to look for an existing vacancy plays a central role. Entrepreneurship, *i.e.* the option of creating one's own firm, has been added to a standard search model by [9]. They show that higher start-up costs discourage entrepreneurs (who seek employment instead) and increase the unemployment rate. However, this result depends on the assumption of constant return to scale in an aggregate matching function.

Moreover, the realism of optimizing behavior could be called into question. Strong empirical evidence in the literature shows that labor market choices are often made on the basis of rules of thumbs [18, 11]. Only when learning is allowed can such rules lead to near-optimal choices [6]. However, the systemic (dynamical) consequences of their out-of-equilibrium properties are generally unknown.

In order to remain more closely related to the existing literature, I provide a simple reference analytical model in which individuals can be employees, employers or unemployed. The matching function is not exogenously assumed, but recovered from the searching behavior of individual workers. An agent-based simulation of the model is then developed, in order to explore the out-of-equilibrium dynamics and the effects of some variations in the main assumptions. In particular, a more behaviorist version of the model, with agents following rules of thumb, will be presented. Finally, more structured hypotheses on the value of some relevant parameters about the profitability of firms will be introduced. The model is set up in section 2. State transition probabilities are derived in section 3. Section 4 characterizes the long-run equilibrium of the system. Section 5 presents an agent-based implementation of the model, and investigates firm dynamics. Section 6 shows that the model is capable of reproducing some well-known aggregate labor market regularities, namely the Beveridge and the Okun curve. Section 7 deals with the above mentioned extensions of the model, while section 8 concludes.

2. The Model

The model belongs to the class of urn-ball search models, with private information and single offer. However, the modeling approach considered is rather different from the typical search literature. Optimal individual choice rules are outlined, given a two-step decision process where workers are characterized by inertia and change job only when their satisfaction level falls below a threshold, no matter what the utility deriving from other choices is. Next, the *a-priori* probability of each choice being made, in equilibrium, is computed. This allows filling a transition matrix, for each state of the system (unemployment, employment, self-employment), defining a regular Markov chain. The long-run probabilities for each state are then computed, using the global balance equations implied by the Markov chain. The approach is similar to that of [7].

2.1. *Labor Supply*

Individuals can be self-employed, employed or unemployed, and in each period they face the choices described in table 1:

Table 1. Individual Choices.

Stay(*)	Remain in the present organization (firm)
Join	Apply for another job
Start	Found a new start-up
Quit	Withdraw from the labor market

*Only if currently employed

Considering that actions may not lead to the desired intentions (and thus can be either successful or unsuccessful), the state transition matrix looks like table 2:

Table 2. State Transition Matrix.

<table>
<tr><td></td><td colspan="3">Ending state</td></tr>
<tr><td>Starting state</td><td>Unemployed</td><td>Employed</td><td>Self-employed</td></tr>
<tr><td>Unemployed</td><td>Unsuccessful Join
Quit</td><td>Successful Join</td><td>Quit</td></tr>
<tr><td>Employed</td><td rowspan="2">Unsuccessful Stay
Unsuccessful Join
Quit</td><td rowspan="2">Successful Stay
Successful Join</td><td rowspan="2">Start</td></tr>
<tr><td>Self-employed</td></tr>
</table>

Let P_{stay}, P_{join}, P_{start} and P_{quit} be the equilibrium probabilities of making either a Stay decision or, when having decided not to Stay, of making a Join, Start or Quit decision. Each individual has a reservation wage, r. Individuals are risk neutral. They first compute their expected wage in their present state, and compare it with their reservation wage. They only try to change their status if their expected wage falls below r. Thus, like many real people, they are characterized by inertia, and prefer not to change unless they are forced to. When they *are* forced to, their decision either to look for a new job, or to become entrepreneurs, or to remain idle, is based on a comparison of the expected payoffs of the different choices. On-the-job searching is not allowed. Therefore, employed individuals must quit their present job if they decide to apply for other jobs. Should all applications fail, they thus fall into unemployment.

There is a fixed number of N individuals. Let e and $u = 1-e$ be the equilibrium employment and unemployment rate. The expected number of workers willing to stay in equilibrium is $N_{stay} = NeP_{stay}$ and the expected number of applicants is $N_{join} = N(u+e(1-P_{stay}))P_{join} = N(1-eP_{stay})P_{join}$. Finally, the expected number of new start-ups is $N_{start} = Ne(1-P_{stay})P_{start} + NuP_{start} = N(1-eP_{stay})P_{start}$. In addition, there is a (variable) number of F_t firms. Individuals and firms are not located in space: every worker can contact any firm.

2.2. *Labor Demand*

In every period, each individual has a business idea, whose exploitation requires a new start-up that can employ up to J_i units of labor, with J_i randomly extracted from a distribution D_J with mean $\overline{J}$. These business opportunities are valid only for one period. Once a firm is set up, job opportunities grow at the rate g_t, with g_t randomly extracted each period for all firms from a distribution D_g. g_t can be interpreted as a business cycle parameter, and is thought to be purely stochastic. [b] This means that a W_f employee firm at time t will try to become a $W_f(1+g_t)$ employee firm at time $t+1$, thus opening (or destroying) g_tW_f positions. The number of available vacancies will be equal to the number of new positions, plus the number of old positions left vacant by employees who have decided to leave the firm. Workers make their decisions before the rate g_t is revealed. Note that a positive value of g_t does not automatically imply the expansion of a particular firm or the economy as a whole, since jobs could remain vacant.

2.3. *Wages*

All firms in the market in each period receive a market return of $W_f(1+s_{f,t})$, where $(1+s_{f,t})$ is an *a priori* unknown firm- and time-specific multiplier, with $s_{f,t}$ randomly extracted from a distribution D_s, with mean $\overline{s}$. All employees receive equal

[b] Autocorrelation of g was introduced in the simulation experiments, but failed to result in significant or interesting changes to the dynamics of the system.

pay. Wages are thus equal to $(1 + s_{f,t})$. The wage shock $s_{f,t+1}$ is made known to employees before they make any decisions about whether to leave the firm, but it is unknown to applicants. The intuition behind this hypothesis is that this payoff accounts both for monetary and non-monetary rewards, which could well be assumed to be an experience good. Of course it would be reasonable to consider $s_{f,t}$ as being correlated over time, or across firms, or as being related to the business cycle parameter g_t in some way. Section 7 will discuss this parameter in more detail. Here, for the sake of simplicity, it is considered to be purely idiosyncratic.

Start-ups cause an additional cost of αJ_i for the entrepreneur, which is proportional to the size of the business opportunity, and accounts for all the set-up costs. After the first period, all the differences between employer and employees disappear.

Thus, each employee receives $w_{f,t} = (1 + s_{f,t})$, while the founder receives $(1 + s_{f,t}) - \alpha J_i$. Workers are aware of the uncertainty over s in the aggregate. Consequently, their expectations are:

$$\begin{aligned} w^e_{stay,f} &= (1 + s_{f,t+1})\, P^{succ}_{stay} \\ w^e_{join} &= (1 + \overline{s})\, P^{succ}_{join} \\ w^e_{start} &= (1 + \overline{s}) - \alpha J_i \end{aligned} \tag{1}$$

where P^{succ} is the probability of being confirmed in the present firm or being hired by another firm, once a Stay or Join decision is made.

2.4. *Stay*

As explained above, firms decide how many jobs they can sustain during each period. Jobs are first given to old employees, by means of a tournament. Only when the number of jobs exceeds the number of employees willing to stay are new vacancies opened. In the simplest case with no heterogeneity among workers (*i.e.* $r_i = r$), all the workers in the same firm make the same decision regarding whether to stay or to leave. If they all decide to stay, the probability of being confirmed depends on the business cycle parameter g_t. For positive realizations of g_t this probability is 1, while for negative realizations it is $1 + E[g_t|g_t < 0]$. Suppose g is uniformly distributed between $g_L > -1$ and $g_H > g_L$, then:

$$P^{succ}_{stay} = \frac{g_H}{g_H - g_L} - \frac{g_L}{g_H - g_L}\left(1 + \frac{g_L}{2}\right) = 1 - \frac{g_L^2}{2(g_H - g_L)} \tag{2}$$

2.5. *Join*

As in standard search models, workers apply for vacancies rather than to firms. When looking for a new job, each worker has a fixed number of applications A to send. Vacancies select randomly a prospective worker from all the applications received (if any). The worker accepts the first offer received. As already mentioned, the firm specific wage is revealed only after the employee has been hired. With homogeneous workers, all W employees in the same firm f make the same decision regarding whether to stay or to leave. Thus, the expected number of vacancies in any one existing firm, given that no information about $s_{f,t+1}$ is publicly available [c], is given by:

$$\begin{array}{l} g_{t+1} \leq 0: \\ \quad V^e_{f,t+1} = \begin{cases} W_{f,t}\left(1+g_{t+1}\right) & \text{with prob. } 1-P_{stay} \\ 0 & \text{with prob. } P_{stay} \end{cases} \\ g_{t+1} \geq 0: \\ \quad V^e_{f,t+1} = \begin{cases} W_{f,t}\left(1+g_{t+1}\right) & \text{with prob. } 1-P_{stay} \\ W_{f,t}g_{t+1} & \text{with prob. } P_{stay} \end{cases} \end{array} \tag{3}$$

The expected number of vacancies in new start-ups is:

$$V^e_s = N\left(1-eP_{stay}\right)P_{start}\overline{J} \tag{4}$$

and the total expected number of vacancies is $V^e = V^e_f + V^e_s$.

As in [2], the probability that any one applicant has applied to a particular vacancy is A/V^e, so the number of applications for a particular vacancy is $N_v = bin\left(N_{join}, A/V^e\right)$. The probability that the vacancy has at least one application to consider, assuming $V^e \geq A$, is:

$$p = 1-\left(1-A/V^e\right)^{N_{join}} \tag{5}$$

From the individual's perspective, the probability of selection of any single application sent out is 1 over the number of applications received for that vacancy. On average, this number is equal to the number of applications sent out (AN_{join}) over the number of vacancies that receive applications (pV^e). Consequently, the probability for an application to be selected for a vacancy, given $r_i = r$, is $q = \frac{pV^e}{AN_{join}}$. Note that in considering what happens to a particular vacancy we are considering the case $N_{join} \geq 1$. The probability of being selected for at least one vacancy is $1-(1-q)^A$. Therefore, the *a priori* probability of a successful Join, given a Join decision, is:

$$P^{succ}_{join} = 1-\left(1-\frac{pV^e}{AN_{join}}\right)^A \tag{6}$$

At this point the matching function can also be specified:

$$M(u,e,V,A) = N_{join}P_{join}P^{succ}_{join} = N\left(1-eP_{stay}\right)P_{join}P^{succ}_{join} \tag{7}$$

Note that, unlike the exogenous matching function literature [14], all u, e and V are endogenous here.

[c] although it is, as explained, privately available to individual employees

2.6. *Start*

To successfully form a start-up, no particular requirements are necessary, and at least one vacancy is automatically filled (the founder). The recruiting mechanism involves first choosing an applicant and then asking if he is still on the market. The probability therefore that the selected applicant has not been recruited yet for other vacancies is proportional to the number of the selected worker's applications receiving positive answers, $(A-1)q+1$. The probability of filling any one vacancy is thus:

$$z = \frac{p}{(A-1)q+1} \tag{8}$$

Hence, the average number of vacancies a J_i start-up will be able to fill is:

$$W_i^e = (J_i - 1)z + 1 \tag{9}$$

3. Choices

Suppose s is uniformly distributed between $s_L > -1$ and $s_H > s_L$. Substituting into eq. (1) yields:

$$\begin{aligned} P_{stay} &= \Pr(w_{stay}^e \geq r) = \Pr\left(s_f \geq \frac{2\left(g_H - g_L\right)(r-1) + g_L^2}{2\left(g_H - g_L\right) - g_L^2}\right) = \\ &= \begin{cases} 0 & \text{for } a > s_H \\ \dfrac{s_H - a}{s_H - s_L} & \text{for } a \in [s_L, s_H] \\ 1 & \text{for } a < s_L \end{cases} \\ a &= \frac{2\left(g_H - g_L\right)(r-1) + g_L^2}{2\left(g_H - g_L\right) - g_L^2} \end{aligned} \tag{10}$$

Note that the lower threshold for a, when $r = 0$, is $a = -1$

Now, suppose J is uniformly distributed between $J_L > 0$ and $J_H > J_L$. In order to obtain P_{join}, P_{start} and P_{quit} we must distinguish between the following cases :

$$\begin{array}{ll} \text{(I)} & \dfrac{1+\bar{s}-r}{\alpha} \leq J_L \quad \text{and} \quad P_{join}^{succ} \leq \dfrac{r}{1+\bar{s}} \\ \text{(II)} & P_{join}^{succ} \geq \max(1+\bar{s}-\alpha J_L, r) \\ \text{(III)} & \text{otherwise} \end{array} \tag{11}$$

We then obtain the results in table 3:

Table 3. Choice probabilities.

Case	P_{join}	P_{start}	P_{quit}	Sum
I	0	0	1	1
II	1	0	0	1
III	$\dfrac{\alpha J_H - (1+\bar{s})(1-P_{join}^{succ})}{\alpha(J_H - J_L)}$	$\dfrac{(1+\bar{s})(1-P_{join}^{succ}) - \alpha J_L}{\alpha(J_H - J_L)}$	0	1

Note that when $J_L = 0$, $\frac{1+\bar{s}-\alpha J_L}{1+\bar{s}} = 1$ and case II becomes very unlikely. Note also that the reservation wage does not directly affect individual choices, once a leave decision is made.

4. Long-run Equlibrium

It is straightforward to see that in case I unemployment is the absorbing state. More generally, the transition matrix of table 2 defines a regular Markov chain with stationary transition probabilities. Its limiting distribution, *i.e.* the long-run probability of finding the process in each state, regardless of the initial state (which is also the long run mean fraction of time that the process is in each state) is given by:

$$\begin{aligned}
\pi_e &= \frac{P_{join}P_{join}^{succ} + P_{stay}P_{stay}^{succ}P_{start}}{1 + P_{stay}\left(P_{start} + P_{join}P_{join}^{succ}\right) - P_{stay}P_{stay}^{succ}} \\
\pi_s &= \frac{P_{start}\left(1 - P_{stay}P_{stay}^{succ}\right)}{1 + P_{stay}\left(P_{start} + P_{join}P_{join}^{succ}\right) - P_{stay}P_{stay}^{succ}} \\
\pi_u &= 1 - \frac{P_{start} + P_{join}P_{join}^{succ}}{1 + P_{stay}\left(P_{start} + P_{join}P_{join}^{succ}\right) - P_{stay}P_{stay}^{succ}}
\end{aligned} \tag{12}$$

where π_e, π_s, π_u are the long run probabilities of being employed, self-employed and unemployed, and thus $e = \pi_e + \pi_s, \quad u = \pi_u$. The system is solved numerically. The figures below report the effects of the various parameters on individual choices, starting from a reference case with:

$$\begin{array}{lllll}
g_L = -.5 & g_H = .5 & s_L = -.5 & s_H = .5 & A = 10 \\
J_L = 0 & J_H = 20 & r \;\; = .75 & \alpha \;\; = .01 & N = 1000
\end{array} \tag{13}$$

The effect of the reservation wage is linear (fig. 1a). Above a certain threshold, it starts lowering the probability of making a Stay decision; then, as it approaches 1 it decreases the probability of starting a new business to 0. An increasing average growth rate (fig. 1b) increases the probability of making a Stay decision (P_{stay}^{succ} gets higher, *i.e.* there are more chances of being confirmed, once this decision is made), and also - once the worker has left - the probability of applying for a new job (there are more vacancies: hence the probability of getting a new job P_{join}^{succ} is higher). Higher average wages (fig. 1c) increase the chances of staying, but - above a certain threshold - do not influence the other probabilities. A greater value of start-up sunk costs α (fig. 1d) has a positive effect on the probability of making a Stay decision and, of course, it has a negative effect on the probability of making a Start decision. The number of applications that can be simultaneously sent out by workers has very little impact on their choices (fig. 1e).

The effects of the parameters on the final outcome, *i.e.* on (a) π_u, (b) π_s and (c) on the number of matches M are shown in figures 2,3,4,5,6, with reference to the benchmark case outlined above.

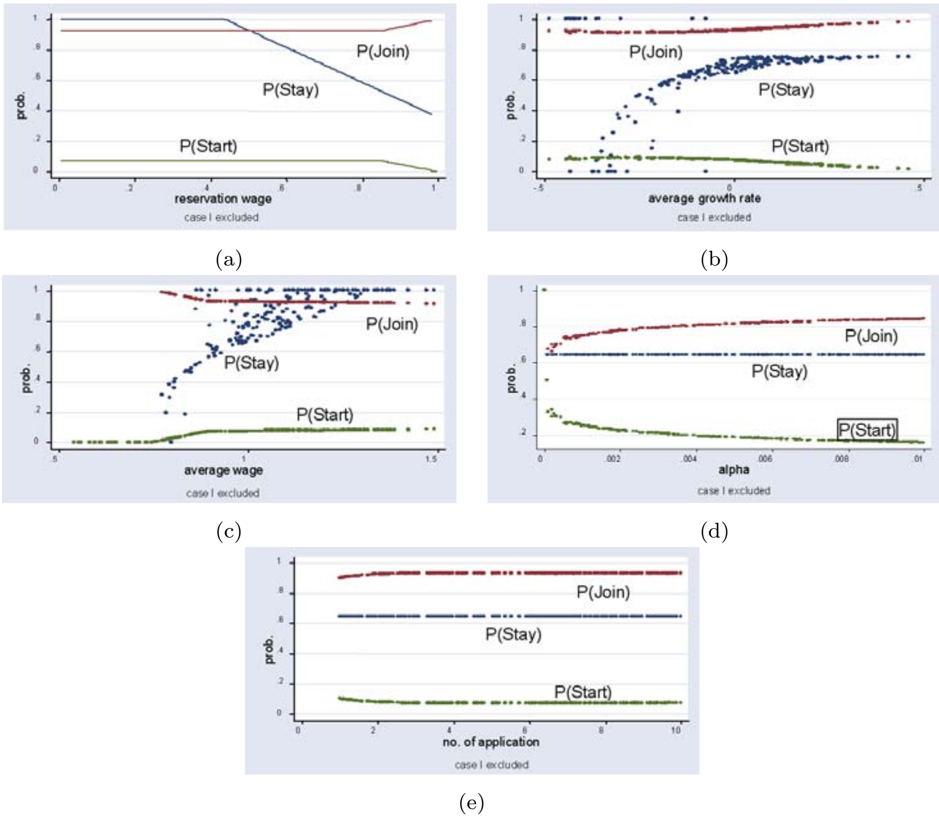
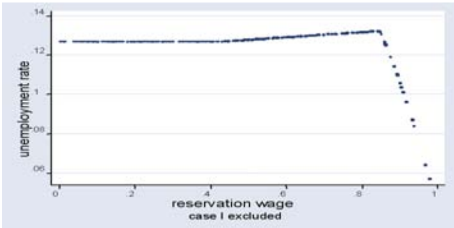


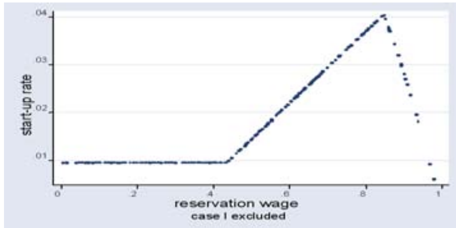
Fig. 1. Choice probabilities.

4.1. *Reservation Wage*

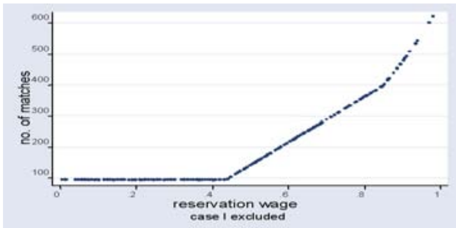
Unemployment is affected by the reservation wage when it is above a certain threshold, but the relationship may appear counterintuitive: a greater reservation wage lowers the unemployment rate. In explanation, note first that the values of the parameters do not allow for case I situations, *i.e.* the probability of staying out of the labor market (making a Quit decision) is null. Hence, only two ways of becoming unemployed remain: the first is by making a Stay decision, and not being reconfirmed in the same job due to adverse business conditions; the second by making a Join decision, and not being selected for any of the A applications sent out. However, P_{stay}^{succ} does not depend on r, while P_{join}^{succ} is decreasing in r, as the number of vacancies increases (the number of matches also increases, as depicted in fig. 2c). Since the probability of making a Stay decision is also decreasing in r, the resulting relationship between the reservation wage and the unemployment rate must be negative. By allowing all the parameters to change randomly, it becomes clear that the



(a)

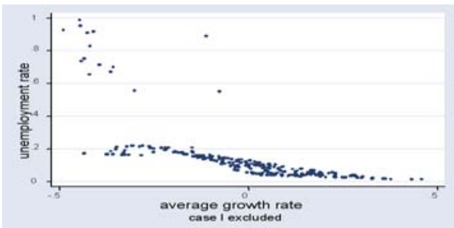


(b) % of non-stayers

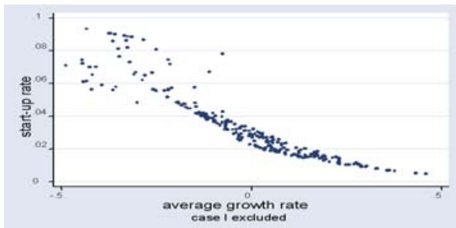


(c)

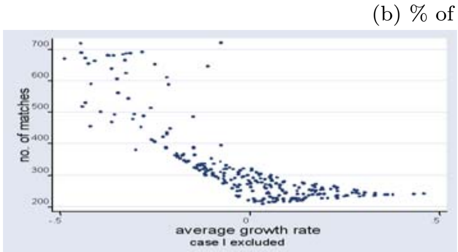
Fig. 2. Long-run equilibrium probabilities. Effect of reservation wage



(a)



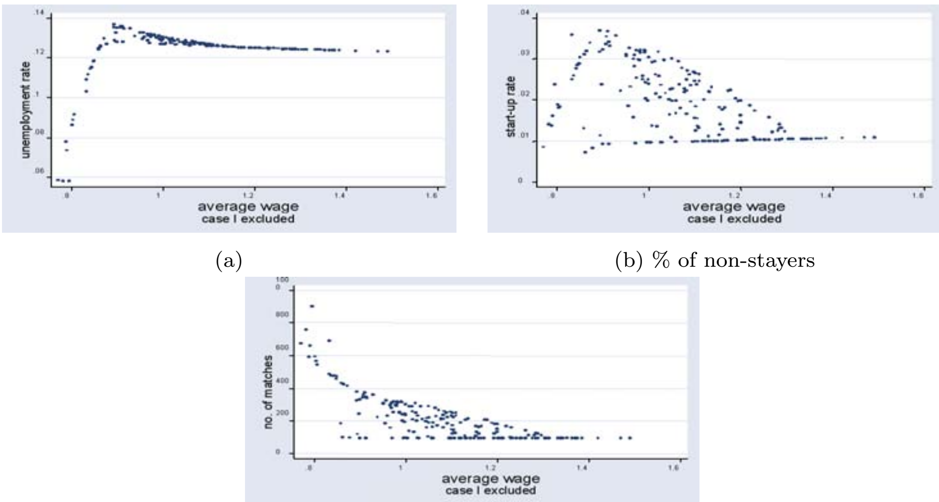
(b) % of non-stayers



(c)

Fig. 3. Long-run equilibrium probabilities. Effect of average growth rate

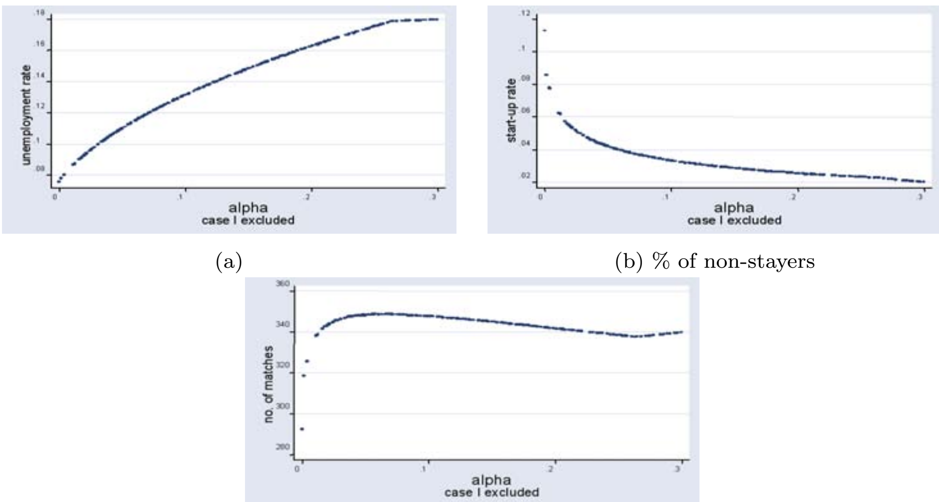
probability of having a case I situation is increasing in r (table 4), thus leading to the expected positive correlation between unemployment and the reservation wage.



(a) (b) % of non-stayers

(c)

Fig. 4. Long-run equilibrium probabilities. Effect of average wage



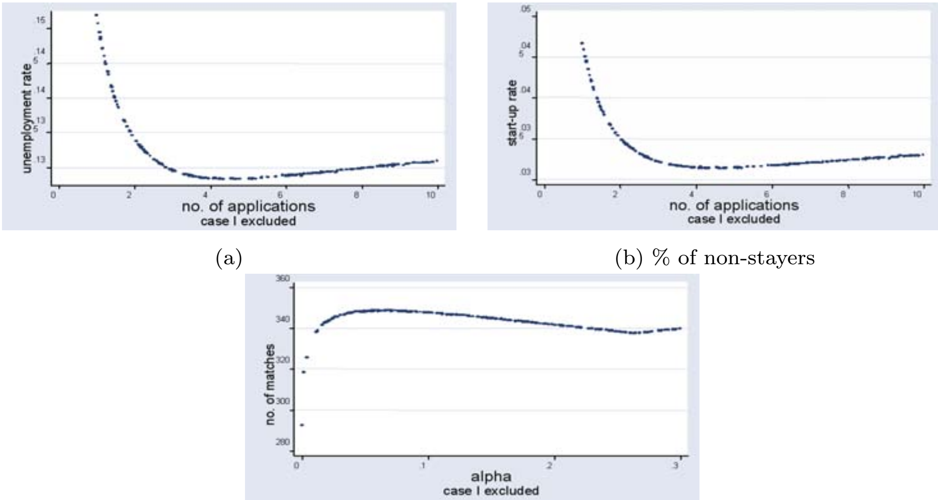
(a) (b) % of non-stayers

(c)

Fig. 5. Long-run equilibrium probabilities. Effect of start-up costs

4.2. *Average Growth Rate*

Higher expected growth rates increase the probability of being confirmed in the present job, thus increasing the probability of making a Stay decision. This simul-



(a) (b) % of non-stayers

(c)

Fig. 6. Long-run equilibrium probabilities. Effect of number of application

taneously lowers the unemployment rate, the number of matches and the number of new start-ups (fig. 3a,b,c).

4.3. *Average Wage*

A similar story holds for average wage (fig. 4a,b,c). Here, however, the correlation between the start-up rate and the average wage is somehow tent-shaped. High average wages increase the probability of workers' satisfaction with their present job, and thus reduce the incentive for starting their own business. However, low average wages increase the significance of the α_J sunk cost, and thus also reduce the likelihood of starting a new business.

4.4. *Start-up Sunk Costs*

An increase in sunk costs reduces incentives to start a new business, increasing the probability of applying for other jobs. Since the probability of making a Stay decision is unaffected, the total number of vacancies decreases. Hence the positive correlation with the unemployment rate (fig. 5a,b,c). This is in line with the predictions of [9].

4.5. *Number of Contemporary Applications*

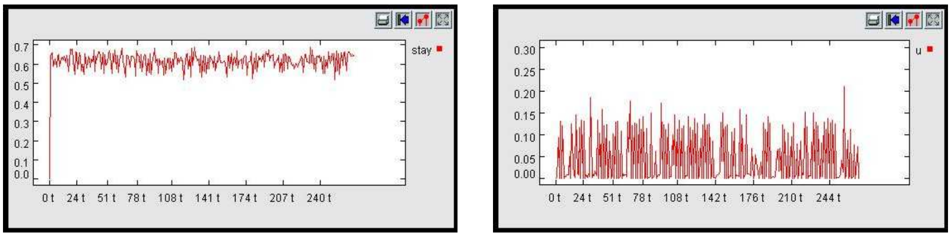
The effect of the number of applications A on the number of matches (fig. 6c) is the same as in [2]. The model presented here, however, also allows its effects on total unemployment and new businesses to be studied. A higher A increases the probability of making a Join decision (by increasing the probability that at least one application is selected), while decreasing the probability of making a Start decision. The overall effect on the total number of vacancies is decreasing in A, although this effect is slightly reversed above a certain threshold. The unemployment rate follows this trend (since the number of people holding their jobs remains constant), while the figure for the number of vacancies is reversed (fig. 6a,b).

5. Firm Demography

In contrast to standard search models, here vacancies are linked to firms, thus allowing for the analysis of firm demography. While the birth rate of new firms is given by the start-up probability derived above, firm size distribution and firm number (which is obviously given by the interaction of the birth and death rates) are explored by means of an agent-based simulation. Agent-based models are computer programs that simulate the behavior of the basic entities in the system (*i.e.* workers, vacancies and firms), given specific interaction rules. Aggregate behavior is thus reconstructed "from the bottom up".[d] The simulation is written in Java code, using JAS libraries (http://sourceforge.net/projects/jaslibrary/). The decision to write a simulation has some consequences, regardless of the model specified. Generally, an analytical model is not immediately operational, *i.e.* the imaginary manual for playing the search game described by the model has to be worked out. This may induce some change in the model itself. In particular, due to the non-parallel discrete processing characteristics of most PCs, the model must be sequential and cast in discrete time, in contrast to the analytical reference model. In addition to time, some other variables that are continuous in the analytical model (such as the number of employees) have to be treated in units. Equilibrium relations cannot be used directly; rather, they have to be derived through non-equilibrium steps. For instance, the number of people expected to make a Join decision, which in the analytical model is the solution of an equilibrium equation involving rational expectations, is considered to be the same as the number of people making a Join decision in the last period (adaptive expectations). Similarly, the expected number of vacancies is the number of vacancies observed in the last period. Therefore, the question naturally arises as to whether this adaptive expectations version of the model converges toward any equilibrium at all, and whether this equilibrium is the same as the rational expectations version. However, with the exception of some noise, the simulation model proves successful in recovering the equilibrium relations, as depicted in fig. 7.

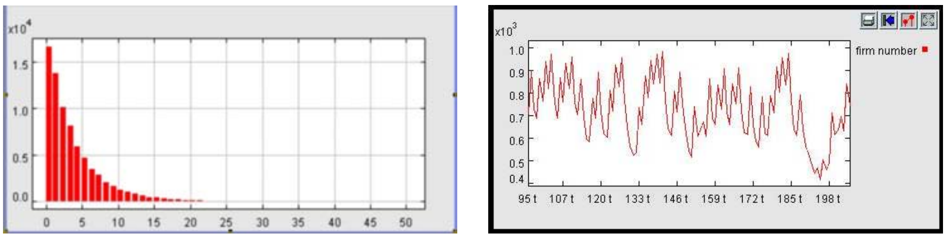
It is thus possible to use the simulation model to analyze firm demography. As an example, the resulting outcome for the parameters values given in (13) are reported in fig. 8:

[d]For a methodological discussion on agent-based computational models, see [17]

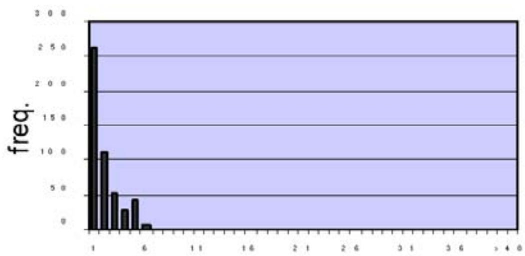


(a) The theoretical probability is $P_{stay} = 0.64$. (b) The theoretical probability is $\pi_u = 0.08$.

Fig. 7. Analytical vs. simulation results. Parameter values given in (13).



(a) Firm age distribution (b) Firm number

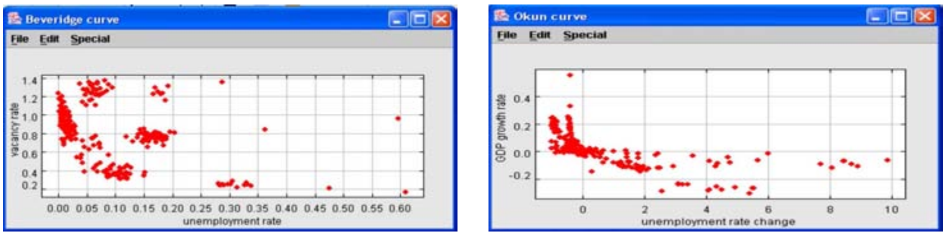


(c) Firm size distribution

Fig. 8. Firm demography. Parameter values given in (13).

6. Aggregate Labor Market Regularities

The model reproduce some aggregate regularities of actual labor markets, namely the Beveridge curve (BC) and the Okun curve (OC). This is shown in fig. 9. The BC postulates a negative relationship between unemployment and vacancy rate, while the OC describes a negative, linear relationship between changes in the unemployment rate and GDP growth rates. For a discussion of the theoretical backgrounds of the BC and the OC, as well as an assessment of their empirical evidence, see [8]. The main point here is to notice that, while many models reproduce these two regularities *separately*, they generally fail to reproduce both of them *jointly*.



(a) Beveridge curve (b) Okun curve

Fig. 9. Beveridge and Okun curves. Parameter values given in (13).

7. Extensions of the Model

This section deals with the relaxation of some assumptions of the model. In particular, variations in the structure of the stochastic wage multiplier $s_{f,t}$ are considered. When s is correlated across firms or in time, or when it is dependent on the business cycle variable g, it becomes difficult to analytically solve for the probabilities of a successful Stay or Join decision, except in simple cases. For instance, when s is firm-specific *i.e.* $s_{f,t} = s_f$, workers will always want to stay, once they are employed in a firm offering a high enough wage (which they will sooner or later find). They will then become unemployed only if (randomly) fired, when the firm is experiencing negative growth. The fact that the probability of a successful choice becomes difficult to compute in more complicated cases has more than analytical consequences. One could question whether real individuals could be thought of acting *as if* they were able to make such complex computations, in order to make the best choice. The realism of the model is thus challenged. When complex feedback is involved, it becomes more sensible to consider simpler individual choice rules, thus abandoning the realm of maximization in favor of a bounded rationality model of individual behavior.

7.1. *Bounded Rationality*

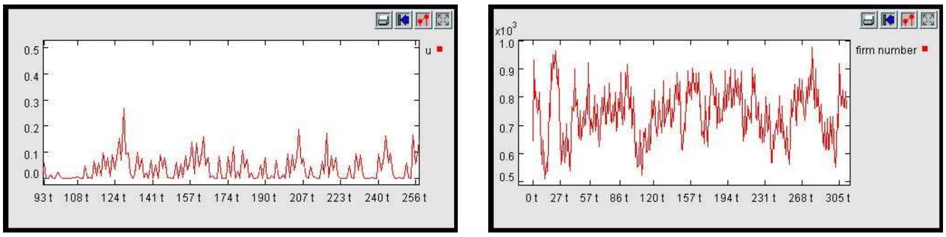
The first step is thus to slightly change the rules of the game:

(1) Workers have adaptive expectations concerning their future wage, and they discount them for a simple proxy of the probability of being fired, *i.e.* the unemployment rate: $w^e_{stay} = w_{f,t}\pi_e$.
(2) As for the expected payoff resulting from applying for other jobs, workers take the average wage of all employees, multiplied by the probability of one of their applications being selected, which remains unvaried: $w^e_{join} = \overline{w}_t P^{succ}_{join}$. Note

that the average wage of all employees may differ from $(1+\overline{s})$, since workers with a low s are more willing to change job. The expected number of vacancies is again thought to be the same as in the last period.

(3) When considering the option to start a new business, workers expect a payoff equivalent to the average wage of all entrepreneurs, net of the start-up costs: $w^e_{start} = \overline{w}_{start,t} - \alpha J_i$.

These rules are simple variations of those of the analytical model, which trade off optimality for computability and simplicity. When combined with firm- and time-specific wage shocks $s_{f,t}$ they typically produce cycles. These cycles are characterized both by periods of sharp decline in the number of active firms and consequent steep rise of the unemployment rate, and by periods in which the number of firms and the unemployment rate "breathe" in and out more regularly (fig. 10).



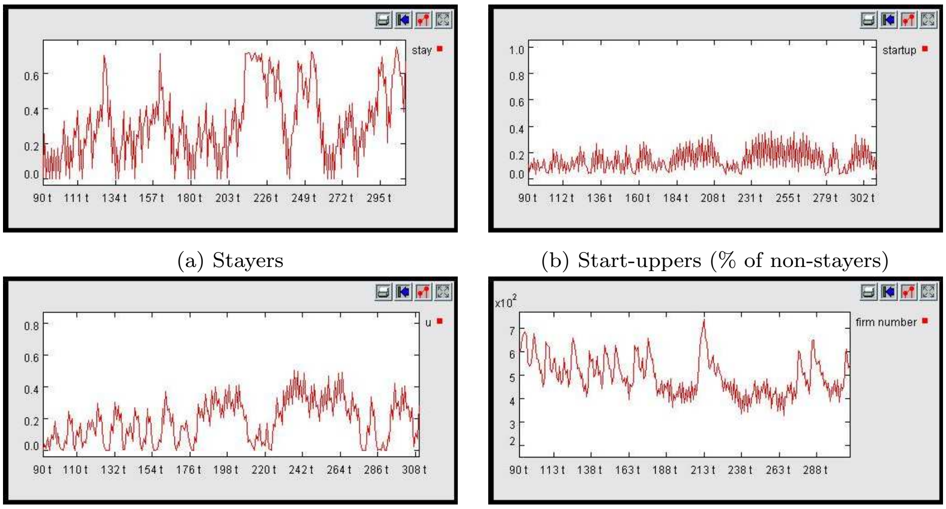
(a) Unemployment rate (b) Firm number

Fig. 10. Outcome of non-optimizing model . Parameter values given in (13)

Overall, these results do not differ substantially from those of the optimizing model, although the dynamics bear somewhat more resemblance to what happens in the real world. The analytical model is thus shown to be robust to its operationalization, and to small departures from optimizing behavior. Having a robust model of individual behavior makes it possible to add some structure to the stochastic wage multiplier $s_{f,t}$. Among the many possible variations, one simple extension of the benchmark model is presented here.

7.2. *Auto-correlation of Start-up Profits*

Suppose start-ups do not get their $s_{f,t}$ from the D_s distribution, but rather from the actual distribution of other start-ups. This may cause a self-sustaining process: following some particularly high extraction of the $s_{f,t}$ among the first start-ups, expectations of start-up profits will rise, hence producing more start-ups, which will also enjoy high profits. However, this will slowly raise the average s, thus leading, in conjunction with a decreased unemployment rate, to a higher probability of Stay decisions. Eventually, the number of start-uppers will decrease, thus making it easier for unlucky low-profit start-ups to impact the average start-up profits. A new period characterized by few low-wage start-ups can begin, and will last until a new generation of lucky new businesses appear. Typical results for this model are reported in figure 11. The dynamics now look more complex, with periods of high unemployment alternating with periods of almost full employment. Moreover, many combinations of the parameter values give rise either to full employment or to full unemployment, which becomes very stable, once established.



(a) Stayers

(b) Start-uppers (% of non-stayers)

(c) Unemployment rate

(d) Firm number

Fig. 11. Outcome of model 7.2. Parameter values given in (13)

8. Conclusions

This paper provides an analytical model of (two-sided) search in the labor market, with optimizing individuals. Building on the previous literature on the topic, this model allows the joint investigation of unemployment and firm dynamics by explicitly considering the vacancy generation process of firms. The model is capable of reproducing a number of stylized facts about industry structure and labor market regularities (such as the Beveridge and the Okun curve). The convergence of the model to the equilibrium is tested through an agent-based simulation, which also shows that a non-optimizing but more realistic version of the model leads to similar results. This bounded rationality version of the model is then used to investigate the effects of different (and more realistic) assumptions about the relevant parameters.

A general consideration drawn from the result of this work is that the equilibrium model shows very uninteresting out-of-equilibrium behavior. While small changes toward more realistic models of individual behavior do not significantly alter the outcome, thus showing the robustness of the benchmark model, small changes in its structure may lead to more complex dynamics, which cannot be investigated analytically but bear more resemblance with those happening in the real world. More detailed investigation of these modified versions of the model, and of how they depart from the analytical benchmark, are left for future research.

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Table 4. Case occurrences.

	Case		
r	I	II	III
	%	%	%
0–.1	-	.88	99.12
.1–.2	-	1.56	98.44
.2–.3	-	-	100.00
.3–.4	-	-	100.00
.4–.5	-	-	100.00
.5–.6	.76	-	99.24
.6–.7	5.15	.74	94.12
.7–.8	11.48	2.46	86.07
.8–.9	26.13	-	73.87
.9–1.0	40.15	-	59.85