# Spatial Search and Commuting with

# Asymmetric Changes of the Wage Distribution

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## Abstract

Job search theory predicts that reservation wages increase with the mean and mean-preserving spread of the wage distribution. However, changing dispersion holding the mean constant implies symmetric stretching or compression of the wage distribution in both tails, which is not likely to be the case when confronted with the real data. The presented model predicts that the commuter stream and the reservation wage increase with the median-preserving spread in the right tail and decrease with the median-preserving spread in the left tail in the destination. The empirical part based on German data confirms predictions of the theory.

# I. Introduction

The theory of job search has already established itself as an important actor in labor economics. The theory has been successfully applied to explain interregional mobility. Traditional search models (see among others McCall (1970), Pissarides (1990), Mortensen (1986) have been successfully applied in a locational context by Burda and Profit (1996), Molho (2001), van Ommeren et al. (1997), van Ommeren and van der Straaten (2005), Damm and Rosholm (2003), and Arntz (2005) among others. Several studies on interregional mobility analyze the commuting patterns in search model context (see for example van Ommeren and van der Straaten (2005), van Ommeren et al. (1997), van Ommeren et al. (1998)). It should be noted, however, that search theory is not the only candidate to explain interregional mobility. An interesting example of the efficiency wage theory in a locational context can be found in Zenou (2002).

The search models in the tradition of McCall (1970) and Mortensen (1986) predict increase in the reservation wage with the mean and the mean-preserving spread of the wage offer distribution (see for details Mortensen (1986) and Rogerson et al. (2005)). However, the mean-preserving spread, i.e. changing the spread holding the mean constant implies a symmetric stretching or compression of the wage distribution which is problematic in the empirical context. If the wage distribution is not symmetric and variances in the left tail and in the right tail are allowed to change independently then the mean-preserving spread is not an adequate measure anymore (asymmetric changes of the dispersion in the left and right tail will change the mean as well). A good solution to this problem could be using the median as a location parameter of the distribution and the median-preserving spread in the left tail as a scale parameter for the left tail and the median-preserving spread in the right tail as a scale parameter for the right tail of the wage distribution (see Möller and Aldashev (2007)) who first point to that problem). It will be shown that if the wage distribution is not symmetric and variance in two tails of the wages distribution can change independently of one another, the implications of the search theory change. Namely, the dispersion in the left tail of the wage distribution reduces reservation wage and search intensity, and the dispersion in the right tail increases reservation wage and search intensity. It will be shown that the commuter flows also increase with the median-preserving spread in the right tail and decrease with the median-preserving spread in the left tail in the destination. The effect of the parameters of the wage distribution in the origin on commuter streams is ambiguous. As a consequence, in my empirical model I include dispersion in the left tail and the right tail of the wage distribution as separate additional regressors.

The estimation results based on commuter stream data between German regions fully support implications of the theoretical model. Hence, the paper suggests that empirical models on commuting (can be easily extended to account for migration) should take into account the inappropriateness of using the mean wage and mean-preserving spread and advocates using the median wage and the median-preserving spreads in the left and right tail of the wage distribution as regressors.

## II. Reservation Wages

Assume for simplicity that there are only two locations in the economy: place of residence and a distant region.[1] The model presented here is a simple locational search model where agents have an option to search in a distant region and, if successful, commute. Throughout the paper I will use the terms region A to denote the local labor

[1] Extension to a multilocational model is straightforward.

market or home region and region B to denote the distant labor market. I consider here only job search decisions of a resident of region A as decisions of residents of B are derived in a likewise manner. Offers to work in region A and B arrive to the searcher according to a stationary Poisson process with the arrival rate $\lambda_A(\theta_A)$ and $\lambda_B(\theta_B)$, where $\theta_A$ is the intensity with which a resident of region A searches for jobs in the local labor market and $\theta_B$ is the search intensity of a resident of region A in distant labor market, which are to be determined endogenously. The arrival rates satisfies the following properties: $\lambda_A'(\theta_A) > 0;\ \lambda_A''(\theta_A) < 0$ and $\lambda_B'(\theta_B) > 0;\ \lambda_B''(\theta_B) < 0$. Searching in each region involves a search cost which is a function of search intensity. The cost functions satisfy the following properties: $c_A'(\theta_A) > 0;\ c_A''(\theta_A) > 0$ and $c_B'(\theta_B) > 0;\ c_B''(\theta_B) > 0$. The utility of being unemployed is equal to *b*; the discount rate is *r*; the travel (commuting) cost between the regions is denoted by $\delta$. In continuous time the reservation wage equation can be written as::

$$w_A^R = b - c_A(\theta_A) - c_B(\theta_B) + \frac{\lambda_A(\theta_A)}{r}\int_{w_A^R}^{\infty}\left(w - w_A^R\right)\mathrm{d}F_A(w) + \frac{\lambda_B(\theta_B)}{r}\int_{w_B^R}^{\infty}\left(w - w_B^R\right)\mathrm{d}F_B(w), \quad (1)$$

with $w_B^R = w_A^R + \delta$ and where $w_A^R$ denotes the reservation wage in region A of the resident of region A and $w_B^R$ denotes the reservation wage in region B of the resident of region A.

Reservation wages in A (B) are decreasing (increasing) in the travel cost between A and B, moreover, the elasticities are less than unity in absolute value. The interpretation is straightforward: higher travel cost reduces the value of search so the reservation wage at location A goes down, i.e. agents become less picky and are ready to accept jobs they would not have taken before. Reservation wage in B cannot go down with commuting cost as part of the wage would have to be sacrificed to cover the travel costs. However, since the elasticities are less than unity in absolute value, this implies that although the wage aspirations become higher at location B, the net demanded wage is lower than before, so, indeed, agents are less picky. These results are rather standard in the literature and are presented without a proof here.

It is interesting to see the effects of changes in the moments of the wage offer distribution on reservation wages. It is common in the search literature to use the mean and the mean-preserving spread to characterize the wage distribution due Rotschild and Stiglitz (1970). However, as it was already mentioned, the mean-preserving spread is not an appropriate measure of dispersion in case of asymmetric changes in the wage offer distribution. In order to control for asymmetric changes in the wage dispersion one has to abandon the concept of the mean-preserving spread as one cannot change the spread in the tails of the distribution separately without affecting the mean. In the following I therefore utilize the notion of a median-preserving spread.

Define the median of the wage offer distribution as $\bar{w}$, the median-preserving spread in the right tail as $\sigma_R$, and the median-preserving spread in the left tail as $\sigma_L$. The spread is median-preserving if for any arbitrary $\sigma_{R1} < \sigma_{R2}$ and $\sigma_{L1} < \sigma_{L2}$:

$$F(\bar{w};\sigma_{R1}) = F(\bar{w};\sigma_{R2}) = F(\bar{w};\sigma_{L1}) = F(\bar{w};\sigma_{L2}) = 1/2\,. \tag{2}$$

Moreover,

$$\int_0^{\bar{w}} F\left(w;\sigma_{L1}\right)\mathrm{d}w < \int_0^{\bar{w}} F\left(w;\sigma_{L2}\right)\mathrm{d}w;\ \int_{\bar{w}}^{\infty} F\left(w;\sigma_{R1}\right)\mathrm{d}w > \int_{\bar{w}}^{\infty} F\left(w;\sigma_{R2}\right)\mathrm{d}w \tag{3}$$

**Proposition 1.** *The reservation wage for region A increases with the median wage of region A and region B (but the elasticity is less than unity) and the median-preserving* *spread in the right tail of the wage distribution of region A and region B. It decreases with the median-preserving spread in the left tail of the wage distribution of region A and region B. The same applies for the reservation wage for region B.*

**Proof**. See Appendix.

**Corollary:** Reservation wage set by a searcher for any location does not only depend on wages in this location, but also on wages in all other locations.

Some ideas in Proposition 1 are not new. For example in classical search models the effect of the elasticity of the mean wage is also positive and less than unity. However, in standard search models reservation wage increases with the variance. This is because “the worker has the option of waiting for an offer in the upper tail of the wage distribution” (Mortensen (1986): 865). The effect of the *median-preserving spread in the right tail* has the same interpretation. However, increasing the spread in the lower tail of the wage distribution allocates more probability mass to the jobs with lower wages and less probability mass to the jobs with higher wages. Moreover, some of the probability mass has gone to jobs which pay wages below the reservation wage. To compensate for this loss of probability mass, reservation wage declines with the *median-preserving spread in the left tail*.

## III. Search Intensities

It is assumed that agents optimize their search effort to maximize the returns to search. This implies that search intensity in the region A solves:

$$c_A'\left(\theta_A\right)=\frac{\lambda_A'\left(\theta_A\right)}{r}\int_{w_A^R}^{\infty}\left(w-w_A^R\right)\mathrm{d}F_A(w)\,, \tag{4}$$

and likewise:

$$c_B'\left(\theta_B\right)=\frac{\lambda_B'\left(\theta_B\right)}{r}\int_{w_B^R}^{\infty}\left(w-w_B^R\right)\mathrm{d}F_B(w) \tag{5}$$

**Proposition 2.** *Agents search harder in the local labor market and less intensively in the distant labor market if median wage or the median-preserving spread in the right tail of the wage distribution increase in the home region or when the median-preserving spread in the left tail of the wage distribution decrease in the home region. If the median wage or the median-preserving spread in the right tail of the wage distribution increase both in the distant and local labor markets by the same amount, agents search harder in both regions; and if the median-preserving spread in the left tail of the wage distribution increase both in the distant and local labor markets by the same amount, agents search less intensively in both regions.*

**Proof**. See Appendix.

The result of Proposition 2 is important that it establishes interdependency of search intensities, i.e. reallocation of search intensity to regions where expected wage increases. An important result of Proposition 2 is also that if the median wage in both regions increases by the same amount, search intensities increase in both regions.

## IV. Maximal Acceptable Travel Cost

The next important issue which I would like to address in this paper is the maximal acceptable commuting cost. The necessary condition that a resident of region A searches in region B is that the returns to search in a distant labor market cover the search costs. It is then possible that after some critical level of commuting cost the returns to search do not cover the search costs anymore. The condition that the returns to search are fully offset by the search costs is called here a *zero search condition* meaning that at this point, a resident of A is indifferent between searching in both regions or in the local labor market only. It is possible to determine the value of the travel cost making the searcher indifferent between investing in search in region B, or search in A only. In order to find the travel cost which makes an agent indifferent between searching in A and B and searching in A only, the following condition has to be imposed:

$$c_B\left(\theta_B\right) \leq \frac{\lambda_B\left(\theta_B\right)}{r} \int_{w_B^R}^{\infty} \left(w - w_B^R\right) \mathrm{d}F_B(w) \tag{6}$$

Restriction in (6) sets the condition that the returns to search in region B should not be less than the costs of the search in B (the Kuhn-Tucker condition). Solving Equations 1, 4, 5, and 6 simultaneously in five endogenous variables: $w_A^R,\ w_B^R,\ \theta_A,\ \theta_B$, and $\delta$ yields the value of the maximal acceptable commuting cost. If the travel costs exceed this critical level, then the returns to search in a distant region do not cover search costs. Hence, a resident of A will invest in search only in those regions which lie within the acceptable travel cost.

## V. Participation and Commuting

The probability that a resident of region A commutes to B can be given by the hazard rate $\lambda_B\left(\theta_B\right)\left(1 - F_B(w_B^R)\right)$ (probability that he is offered a job in B and the offered wage exceeds his reservation wage). The commuter flow from A to B is $S_A \cdot \lambda_B\left(\theta_B\right)\left(1 - F_B(w_B^R)\right)$, where $S_A$ is the number of active searchers who live in region A. If workers are homogenous the number of active searchers is constant (either all or none participate in the labor market).

Now suppose that workers differ in the value they attach to leisure (see also Albrecht and Axell (1984), Möller and Aldashev (2007)). I allow for three states:

employment, unemployment, and nonparticipation. Individuals do not participate in the labor market if their returns to search are less than the value of not participating in the labor market. Suppose that individuals can stay inactive thereby earning pure leisure which is worth *b*. If they participate, their reservation wage as previously defined is:

$$w_A^R = b - c_A\left(\theta_A\right) - c_B\left(\theta_B\right) + \frac{\lambda_A\left(\theta_A\right)}{r}\int_{w_A^R}^{\infty}\left(w - w_A^R\right)\mathrm{d}F_A(w) + \frac{\lambda_B\left(\theta_B\right)}{r}\int_{w_B^R}^{\infty}\left(w - w_B^R\right)\mathrm{d}F_B(w)\,. \quad (7)$$

The participation condition is thus: $w_A^R \geq b$. Ruling out corner solutions, there exists a "marginal individual" for whom $w_A^R = b$. Suppose the values of leisure are distributed across individuals with the distribution function $G(b)$. The participation rate is then $G(\bar{b})$, where $\bar{b}$ is defined as $\bar{b} \equiv w_A^R\left(\bar{b}\right)$.

**Proposition 3.** *The participation rate increases with the median wage (both in origin and destination), median-preserving spread in the right tail (both in origin and destination), but decreases with the median-preserving spread in the left tail (both in origin and destination).*

**Proof**. See Appendix.

Proposition 3 enables us to make predictions about the sign of the effect of the parameters of the distribution on commuter flows, which will be tested in the next section.

Assume for notational convenience a normalization so that the worker with the lowest value of leisure has $b = 0$ and the population of the region A is 1. The number of commuters from region A to B can be given as:

$$\int_0^{\bar{b}} \lambda_B(\theta_B)\left(1-F_B(w_B^R)\right)\mathrm{d}G(b) \qquad (8)$$

**Proposition 4.** *The commuter flow increases with the median wage in the destination, median-preserving spread in the right tail in the destination, but decreases with the median-preserving spread in the left tail in the destination. The effects of the parameters of the wage distribution in the origin are ambiguous.*

**Proof**. See Appendix.

## VI. Data

The commuter stream data used in this paper are produced by the Institute for Labor Research (IAB) from the employment register of the Federal Labor Office with regional information. The data contain the flows of commuters in 1997 between 440 NUTS-3 regions, which makes 193 160 observations. Unfortunately, the data do not differentiate commuters with respect to gender. The dependent variable is then the commuter flow from region *i* to *j*. Wage quantiles of the wage distribution were calculated using the IABS-REG microdataset for 1997 (see description of the data in the Appendix and Bender et al. (2000)). Data on population of NUTS-3 regions were taken from the INKAR database of the Federal Office for Building and Regional Planning. The information on travel time was taken from the data of the Institute for Regional Planning of the University of Dortmund (see description of the data in the Appendix and Spiekermann et al. (2000)).

The exogenous parameters used for estimation are:

- $\overline{w}_i$ - the median log wage in region *i*,
- $\overline{w}_j$ - the median log wage in region *j*,

- $POP_i$ - log population of region *i* (as an indicator of the size of the labor market in *i*),
- $POP_j$ - log population of region *j*,
- $D8/D5_i$ - the log difference of the 8th to 5th decile of the wage distribution in region *i* (as an indicator of the median-preserving spread in the right tail),
- $D8/D5_j$ - the log difference of the 8th to 5th decile of the wage distribution in region j,
- $D5/D2_i$ - the log difference of the 5th to 2nd decile of the wage distribution in region *i* (as an indicator of the median-preserving spread in the left tail),
- $D5/D2_j$ - the log difference of the 5th to 2nd decile of the wage distribution in region *j*,
- $t_{ij}$ - log travel time between regions *i* and *j*.

Since in a multi-region model commuting streams depend on exogenous parameters of all other regions, I also include average wage in other regions (except *i* and *j*) with inverse travel time as weights and average population in other regions (except *i* and *j*) with inverse travel time as weights. These spatially weighted variables are calculated as:

$$\tilde{W}_{ij} = \sum_{s \neq i, s \neq j} \frac{\overline{w}_s}{t_{is}}, \qquad \tilde{P}_{ij} = \sum_{s \neq i, s \neq j} \frac{POP_s}{t_{is}}. \qquad (9)$$

## VII. Estimation

If we look at the distribution of commuter flows we see that about one third are zeros (see for example Figures A.1). This should not be surprising. In the theoretical model the zero search intensity condition implies that if a region lies beyond the circle of acceptable commuting destinations, agents do not search there. This is plausible as the travel time between some regions is about 8-10 hours, making commuting virtually impossible. We could split the decision making process of a resident of A into two stages. At stage one, he solves for the maximal acceptable travel cost (given exogenous variables of all regions) and if a region lies within the acceptable travel cost he allocates his effort into search in this region, i.e. enters stage two. At stage two, he searches for jobs at this location and, if successful, becomes consequently employed. This implies zero and nonzero commuting streams are generated by two different processes. Zeros could mean that agents do not search in these regions - decision taken by workers only, a *choice* outcome. Positive outcomes are generated through a different process - matching, a *random* outcome. To handle models like this a class of hurdle modes has been developed, for example the zero-inflated negative binomial or zero-inflated Poisson approach (see details in the Appendix).

Table A.1 presents estimation results of the zero inflated negative binomial regression. The Vuong test shows that the hurdle model is clearly preferred to the standard negative binomial regression. The value of $\alpha$ significantly different from one suggests that the zero-inflated negative binomial is preferred over the zero-inflated Poisson.

Some of the estimation results are not new: the travel time between the regions reduces the commuter flows as in standard gravity models. The negative effect of the spatially weighted population size and spatially weighted median wage imply that regions of larger size and with higher wages are more likely to distract potential commuters from other destinations.

Most importantly, the results support the prediction of the theoretical model. The empirical evidence suggests that commuter streams decrease with the median-preserving spread in the left tail in the destination and increase with the median wage and the median-preserving spread in the right tail of the wage distribution in the destination.

## VIII. Conclusion

The search models in the tradition of McCall (1970) and Mortensen (1986) predict increase in the reservation wage with the mean and the mean-preserving spread of the wage offer distribution. However, the mean-preserving spread, i.e. changing the spread holding the mean constant implies a symmetric stretching or compression of the wage distribution which is not likely to be the case in the empirical application. If the wage distribution is not symmetric and variances in the left tail and in the right tail are allowed to change independently then the mean-preserving spread is not an adequate measure anymore (asymmetric changes of the dispersion in the left and right tail will change the mean as well).

In this paper I present a bilocational search model where individuals have an option to commute if offered a job in a region other than their place of residence. I show that if the wage distribution is not symmetric and variance in two tails of the wages distribution can change independently of one another, the implications of the search theory change. Namely, the dispersion in the left tail of the wage distribution reduces reservation wage and search intensity, and the dispersion in the right tail increases reservation wage and search intensity. Moreover, the commuter flows also increase with the median-preserving spread in the right tail and decrease with the median-preserving spread in the left tail in the destination.

The estimation results based on commuter stream data between German regions fully support implications of the theoretical model. Hence, the paper suggests that empirical models on commuting should take into account the inappropriateness of using the mean wage and mean-preserving spread and advocates using the median wage and the median-preserving spreads in the left and right tail of the wage distribution as regressors.

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## Appendix

### A.1 Proof of Proposition 1

To derive the effect of the change of the median on the reservation wage one needs to introduce the notion of the translation of the distribution. Changing the median holding the shape of the distribution constant is simply a parallel shift of the distribution. If we increase the median of the distribution $F(x)$ by the value $\mu$, the resulting distribution would be a translation of the original c.d.f. $F(x)$. The distribution $G(x)$ is a translation of $F(x)$ if $G(x+\mu)=F(x)$ and, hence, $G(x)=F(x-\mu)$.

The reservation wage in the region A is given as:

$$w_A^R = b - c_A(\theta_A) - c_B(\theta_B) + \frac{\lambda_A(\theta_A)}{r}\int_{w_A^R}^{\infty}\left(w - w_A^R\right)\mathrm{d}F_A(w) + \frac{\lambda_B(\theta_B)}{r}\int_{w_B^R}^{\infty}\left(w - w_B^R\right)\mathrm{d}F_B(w) \quad \text{(A.1)}$$

Increase of the median in region A by $\mu$ (given that $w_B^R = w_A^R + \delta$) would result in a new reservation wage:

$$w_A^R(\mu) = b - c_A(\theta_A) - c_B(\theta_B) + \frac{\lambda_A(\theta_A)}{r}\int_{w_A^R(\mu)}^{\infty}\left(w - w_A^R(\mu)\right)\mathrm{d}F_A(w-\mu) + \frac{\lambda_B(\theta_B)}{r}\int_{w_A^R+\delta}^{\infty}\left(w - w_A^R - \delta\right)\mathrm{d}F_B(w)$$

(A.2)

Subtracting A.2 from A.1 we obtain:

$$\begin{aligned}\left(w_A^R(\mu) - w_A^R\right) = {} & \frac{\lambda_A(\theta_A)}{r}\left[\int_{w_A^R(\mu)}^{\infty}(w - w_A^R(\mu))\mathrm{d}F_A(w-\mu) - \int_{w_A^R}^{\infty}(w - w_A^R)\mathrm{d}F_A(w)\right] + \\ & \frac{\lambda_B(\theta_B)}{r}\left[\int_{w_A^R(\mu)+\delta}^{\infty}(w - w_A^R(\mu) - \delta)\mathrm{d}F_B(w) - \int_{w_A^R+\delta}^{\infty}(w - w_A^R - \delta)\mathrm{d}F_B(w)\right]\end{aligned} \quad . \text{ (A.3)}$$

By integration by parts one obtains:

$$\int_{w_A^R(\mu)}^{\infty} (w - w_A^R(\mu))\mathrm{d}F_A(w-\mu) = E(w) + \mu - w_A^R(\mu) + \int_0^{w_A^R(\mu)} F_A(w-\mu)\mathrm{d}w\,, \qquad \text{(A.4)}$$

and

$$\int_{w_A^R}^{\infty} (w - w_A^R)\mathrm{d}F_A(w) = E(w) - w_A^R(\mu) + \int_0^{w_A^R} F_A(w-\mu)\mathrm{d}w\,. \qquad \text{(A.5)}$$

Dividing Eq. A.2 by $\mu$ and taking the limit at $\mu = 0$ with the help of the results obtained in A.4 and A.5 one gets:

$$\begin{aligned} \frac{\partial w_A^R}{\partial \mu} &= \frac{\lambda_A(\theta_A)}{r}\left[1 - \frac{\partial w_A^R(\mu)}{\partial \mu}\left(1 - F_A(w_A^R)\right) - F_A(w_A^R)\right] + \\ &\frac{\lambda_B(\theta_B)}{r}\left[-\frac{\partial w_A^R(\mu)}{\partial \mu}\left(1 - F_B(w_A^R + \delta)\right)\right] \end{aligned}. \qquad \text{(A.6)}$$

Therefore:

$$\frac{\partial w_A^R}{\partial \mu} = \frac{\lambda_A(\theta_A)\left(1 - F_A(w_A^R)\right)}{r + \lambda_A(\theta_A)\left(1 - F_A(w_A^R)\right) + \lambda_B(\theta_B)\left(1 - F_B(w_A^R + \delta)\right)}\,. \qquad \text{(A.7)}$$

Obviously, $0 \le \frac{\partial w_A^R}{\partial \mu} \le 1$. Knowing that $w_A^R + \delta = w_B^R$, one obtains $\frac{\partial w_A^R}{\partial \mu} = \frac{\partial w_B^R}{\partial \mu}$. In the same fashion, increasing the median in B by $\mu$:

$$\frac{\partial w_B^R}{\partial \mu} = \frac{\lambda_B(\theta_B)\left(1 - F_B(w_B^R)\right)}{r + \lambda_B(\theta_B)\left(1 - F_B(w_B^R)\right) + \lambda_A(\theta_A)\left(1 - F_A(w_A^R)\right)}\,. \qquad \text{(A.8)}$$

Again, $0 \le \frac{\partial w_B^R}{\partial \mu} \le 1$.

For the effect of the spreads, assume that the reservation wage is below the median (reservation wages above the median would be unaffected by the spread below the median). Denote $\Lambda = \int_{w^R}^{\infty} (w - w^R)\mathrm{d}F(w)$. One could rewrite:

$$\begin{aligned}\Lambda &= \int_{w^R}^{\overline{w}} (w - w^R)\mathrm{d}F(w) + \int_{\overline{w}}^{\infty} (w - w^R)\mathrm{d}F(w) = \\ &= \frac{\overline{w}}{2} - \int_{w^R}^{\overline{w}} F(w)\mathrm{d}w + \int_{\overline{w}}^{\infty} w\mathrm{d}F(w) - w^R\end{aligned} \tag{A.9}$$

Note that $\frac{\partial}{\partial \overline{w}} \int_{\overline{w}}^{\infty} w\mathrm{d}F(w) > 0$. This result is intuitively clear -- truncated mean increases if you move the truncation point to the right. Moreover, $\frac{\partial}{\partial \sigma_R} \int_{\overline{w}}^{\infty} w\mathrm{d}F(w) > 0$ - truncated mean increases if you increase the variance to the right of the truncation point. Hence, $\frac{\partial \Lambda}{\partial \sigma_R} > 0$. The effect of the spread in the left tail is: $\frac{\partial \Lambda}{\partial \sigma_L} = -\int_{w^R}^{\overline{w}} \frac{\partial}{\partial \sigma_L} F(w)\mathrm{d}w < 0$, because $\frac{\partial F(w)}{\partial \sigma_L} > 0$. The logic here is straightforward – increasing the spread in the left tail moves some of the probability mass away to the left of the reservation wage (fewer jobs become attractive). As a result, the reservation wage declines to compensate for the loss of the probability mass. Hence, $\frac{\partial w_A^R}{\partial \sigma_{AR}} > 0$ and $\frac{\partial w_A^R}{\partial \sigma_{AL}} < 0$. In the same fashion one obtains $\frac{\partial w_B^R}{\partial \sigma_{BR}} > 0$ and $\frac{\partial w_B^R}{\partial \sigma_{BL}} < 0$. Given the relationship $w_A^R + \delta = w_B^R$, one also obtains: $\frac{\partial w_A^R}{\partial \sigma_{BR}} > 0$ and $\frac{\partial w_A^R}{\partial \sigma_{BL}} < 0$, and $\frac{\partial w_B^R}{\partial \sigma_{AR}} > 0$ and $\frac{\partial w_B^R}{\partial \sigma_{AL}} < 0$.

**A.2 Proof of Proposition 2**

Differentiate $c_A'\left(\theta_A\right)=\frac{\lambda_A'\left(\theta_A\right)}{r}\int_{w_A^R}^{\infty}\left(w-w_A^R\right)\mathrm{d}F_A(w)$ with respect of $\Lambda_A$:

$$c_A''(\theta_A)\frac{\partial\theta_A}{\partial\Lambda_A}=\frac{\lambda_A''(\theta_A)}{r}\frac{\partial\theta_A}{\partial\Lambda_A}\Lambda_A+\frac{\lambda_A'(\theta_A)}{r}, \tag{A.10}$$

and

$$\frac{\partial\theta_A}{\partial\Lambda_A}=\frac{\lambda_A(\theta_A)}{rc_A''(\theta_A)-\lambda_A''(\theta_A)\Lambda_A}>0. \tag{A.11}$$

From $\frac{\partial\theta_A}{\partial\overline{w}_A}=\frac{\partial\theta_A}{\partial\Lambda_A}\cdot\frac{\partial\Lambda_A}{\partial\overline{w}}$ it immediately follows that $\frac{\partial\theta_A}{\partial\overline{w}_A}>0,\ \frac{\partial\theta_A}{\partial\sigma_{AR}}>0,$ and $\frac{\partial\theta_A}{\partial\sigma_{AL}}<0$.

Differentiate $c_B'\left(\theta_B\right)=\frac{\lambda_B'\left(\theta_B\right)}{r}\int_{w_B^R}^{\infty}\left(w-w_B^R\right)\mathrm{d}F_B(w)$ with respect to $\overline{w}_A$:

$$c_B''(\theta_B)\frac{\partial\theta_B}{\partial\overline{w}_A}=\frac{\lambda_B''(\theta_B)}{r}\frac{\partial\theta_B}{\partial\overline{w}_A}\Lambda_B-\frac{\partial w_B^R}{\partial\overline{w}_A}\left(1-F_B(w_B^R)\right)\frac{\lambda_B'(\theta_B)}{r}, \tag{A.12}$$

and hence, $\frac{\partial\theta_B}{\partial\overline{w}_A}=\frac{\frac{\partial w_B^R}{\partial\overline{w}_A}\left(1-F_B(w_B^R)\right)\lambda_B'(\theta_B)}{rc_B''(\theta_B)-\lambda_B''(\theta_B)\Lambda_B}<0$. In the same fashion:

$$\frac{\partial\theta_B}{\partial\sigma_{AR}}=\frac{\frac{\partial w_B^R}{\partial\sigma_{AR}}\left(1-F_B(w_B^R)\right)\lambda_B'(\theta_B)}{rc_B''(\theta_B)-\lambda_B''(\theta_B)\Lambda_B}<0 \text{ and } \frac{\partial\theta_B}{\partial\sigma_{AL}}=-\frac{\frac{\partial w_B^R}{\partial\sigma_{AL}}\left(1-F_B(w_B^R)\right)\lambda_B'(\theta_B)}{rc_B''(\theta_B)-\lambda_B''(\theta_B)\Lambda_B}>0.$$

If we want to see how search intensities react to changes in wages in both regions simultaneously, we simply let the wage distributions in both regions be identical. Then increase in wages in region A would mean the same increase in region B. Then, $c_A'\left(\theta_A\right)=\frac{\lambda_A'\left(\theta_A\right)}{r}\int_{w_A^R}^{\infty}\left(w-w_A^R\right)\mathrm{d}F\,(w)$ and $c_B'\left(\theta_B\right)=\frac{\lambda_B'\left(\theta_B\right)}{r}\int_{w_B^R}^{\infty}\left(w-w_B^R\right)\mathrm{d}F\,(w)$. It is then easy to show that $\frac{\partial\theta_A}{\partial\overline{w}}>0,\ \frac{\partial\theta_A}{\partial\sigma_R}>0,\ \frac{\partial\theta_A}{\partial\sigma_L}<0$ and $\frac{\partial\theta_B}{\partial\overline{w}}>0,\ \frac{\partial\theta_B}{\partial\sigma_R}>0,\ \frac{\partial\theta_B}{\partial\sigma_L}<0$.

**A.3 Proof of Proposition 3**

The participation rate is given as $G(\overline{b})$. Hence, the participation rate is increasing in $\overline{b}$.

Since:

$$w_A^R(\overline{b}) = \overline{b} - c_A(\theta_A) - c_B(\theta_B) + \frac{\lambda_A(\theta_A)}{r}\int_{w_A^R}^{\infty}\left(w - w_A^R\right)\mathrm{d}F_A(w) + \frac{\lambda_B(\theta_B)}{r}\int_{w_B^R}^{\infty}\left(w - w_B^R\right)\mathrm{d}F_B(w) \text{ ,(A.13)}$$

and $w_A^R(\overline{b}) \equiv \overline{b}$, it is easy to show that $w_A^R(\overline{b})$ and hence $\overline{b}$ increase with the median wage (both in the origin and destination) and the median-preserving spread in the right tail (both in the origin and destination) and decreases with the median-preserving spread in the left tail of the wage distribution (both in the origin and destination) (see also proof of Proposition 1).

**A.4 Proof of Proposition 4**

Increase in the median wage in the destination would increase $\overline{b}$. Moreover, increase in the median in region B would make agents reallocate their intensity to region B (see Proposition 2) and therefore $\lambda_B(\theta_B)$ also increases. The reservation wage in B increases with the median wage in B but the elasticity is less than unity, hence, $(1 - F_B(w_B^R))$ also increases with the median wage in B. As a consequence, the commuter flow from A to B unambiguously increases with the median wage in the destination. If the median wage in the origin increases, the participation rate also goes up ($\overline{b}$ increases). However, agents would reallocate their search intensity to region A and hence, $\lambda_B(\theta_B)$ declines. This implies that when wages increase in the origin, less commuting is possible because agents start searching harder in the origin and less harder in the destination, but on the other hand, more commuting is possible because overall number of searchers in the origin increases. Hence, the overall effect is ambiguous.

To derive the effects of the spreads, consider first the case with exogenous search intensity. Denote $\phi = \lambda \int_0^{\bar{b}} \left(1 - F\left(w^R(b)\right)\right) dG(b) = \lambda G\left(\bar{b}\right) - \lambda \int_0^{\bar{b}} F\left(w^R(b)\right) dG\left(b\right)$. Moreover,

integrating by parts, $\int_0^{\bar{b}} F\left(w^R(b)\right) dG\left(b\right) = G\left(\bar{b}\right) F\left(w^R(\bar{b})\right) - \int_0^{\bar{b}} G\left(b\right) f\left(w^R(b)\right) \frac{\partial w^R(b)}{\partial b} db$.

$$\begin{aligned}\frac{\partial \phi}{\partial \sigma_R} &= \lambda g\left(\bar{b}\right)\frac{\partial \bar{b}}{\partial \sigma_R} - \lambda g\left(\bar{b}\right) F\left(w^R(\bar{b})\right)\frac{\partial \bar{b}}{\partial \sigma_R} - \lambda G\left(\bar{b}\right) f\left(w^R(\bar{b})\right)\frac{\partial w^R(\bar{b})}{\partial \sigma_R} - \\ &- \lambda G\left(\bar{b}\right)\frac{\partial F\left(w^R(\bar{b})\right)}{\partial \sigma_R} + \lambda G\left(\bar{b}\right) f\left(w^R(\bar{b})\right)\frac{\partial w^R(\bar{b})}{\partial b}\frac{\partial \bar{b}}{\partial \sigma_R} + \lambda \int_0^{\bar{b}} G\left(b\right)\frac{\partial f\left(w^R(b)\right)}{\partial \sigma_R}\frac{\partial w^R(b)}{\partial b} db,\end{aligned} \tag{A.14}$$

where $\frac{\partial F\left(w^R(\bar{b})\right)}{\partial \sigma_R} = \frac{\partial F\left(w\right)}{\partial \sigma_R}$ at $w = w^R(\bar{b})$, $\frac{\partial w^R(\bar{b})}{\partial b} = \frac{\partial w^R(b)}{\partial b}$ at $b = \bar{b}$. Clearly,

$\frac{\partial w^R(\bar{b})}{\partial b}\frac{\partial \bar{b}}{\partial \sigma_R} = \frac{\partial w^R(\bar{b})}{\partial \sigma_R}$ with $\frac{\partial w^R(\bar{b})}{\partial \sigma_R} = \frac{\partial w^R(b)}{\partial \sigma_R}$ at $b = \bar{b}$.

Hence,

$$\frac{\partial \phi}{\partial \sigma_R} = \lambda g\left(\bar{b}\right)\frac{\partial \bar{b}}{\partial \sigma_R}\left[1 - F\left(w^R(\bar{b})\right)\right] - \lambda G\left(\bar{b}\right)\frac{\partial F\left(w^R(\bar{b})\right)}{\partial \sigma_R} + \lambda \int_0^{\bar{b}} G\left(b\right)\frac{\partial f\left(w^R(b)\right)}{\partial \sigma_R}\frac{\partial w^R(b)}{\partial b} db. \tag{A.15}$$

As it was shown (see proof of Proposition 3), $\frac{\partial \bar{b}}{\partial \sigma_R} > 0$. Moreover, $\frac{\partial F\left(w^R(\bar{b})\right)}{\partial \sigma_R} < 0$.

Since $\int_{\bar{w}}^{\infty} f(w) dw = 1/2$, there exists some point *y*, such that: $\frac{\partial f\left(w^R(b)\right)}{\partial \sigma_R} < 0$ for

$w^R(b) < y$ and $\frac{\partial f\left(w^R(b)\right)}{\partial \sigma_R} > 0$ for $w^R(b) > y$.

$\frac{\partial w^R(b)}{\partial b} = \frac{1}{1+\lambda / r(1-F(w^R))}$, which implies that $\frac{\partial w^R(b)}{\partial b}$ increases in $w^R(b)$ and hence in *b*. Hence,

$$\lambda \int_0^{\bar{b}} G(b) \frac{\partial f\left(w^R(b)\right)}{\partial \sigma_R} \frac{\partial w^R(b)}{\partial b} db > \lambda G\left(\bar{b}\right) \int_0^{\bar{b}} \frac{\partial f\left(w^R(b)\right)}{\partial \sigma_R} \frac{\partial w^R(b)}{\partial b} db . \qquad \text{(A.16)}$$

The interpretation is the following: in the integral on the left side of A.16 negative values have smaller weights and positive values have larger weights, hence, if we weigh all the values equally, the resulting integral (the right-hand side of A. 16) would be "more negative". Therefore, if we replace the last term in Equation A.15 by the left-hand side of A. 16, the resulting sum would be smaller.

Therefore, if

$$\lambda g\left(\bar{b}\right) \frac{\partial \bar{b}}{\partial \sigma_R}\left[1-F\left(w^R(\bar{b})\right)\right] - \lambda G\left(\bar{b}\right) \frac{\partial F\left(w^R(\bar{b})\right)}{\partial \sigma_R} + \lambda G\left(\bar{b}\right) \int_0^{\bar{b}} \frac{\partial f\left(w^R(b)\right)}{\partial \sigma_R} \frac{\partial w^R(b)}{\partial b} db > 0 ,$$

then also $\lambda g\left(\bar{b}\right) \frac{\partial \bar{b}}{\partial \sigma_R}\left[1-F\left(w^R(\bar{b})\right)\right] - \lambda G\left(\bar{b}\right) \frac{\partial F\left(w^R(\bar{b})\right)}{\partial \sigma_R} + \lambda \int_0^{\bar{b}} G(b) \frac{\partial f\left(w^R(b)\right)}{\partial \sigma_R} db > 0$.

But $\lambda G\left(\bar{b}\right) \int_0^{\bar{b}} \frac{\partial f\left(w^R(b)\right)}{\partial \sigma_R} db = \lambda G\left(\bar{b}\right) \frac{\partial F\left(w^R(\bar{b})\right)}{\partial \sigma_R}$. Hence, $\frac{\partial \phi}{\partial \sigma_R} > 0$. Allowing endogenous search intensity does not change the result qualitatively: $\frac{\partial \phi}{\partial \sigma_{BR}} > 0$ as $\frac{\partial \lambda_B(\theta_B)}{\partial \sigma_{BR}} > 0$, where $\sigma_{BR}$ stands for the spread in the right tail of the wage distribution in the destination. Hence, the commuter flow unambiguously increases with the spread in the right tail of the wage distribution in the destination. However, $\frac{\partial \lambda_B(\theta_B)}{\partial \sigma_{AR}} < 0$, thus the effect of the change in the spread in the right tail in the origin is ambiguous.

In the same fashion it can be established that the commuter flow unambiguously declines with the spread in the left tail of the wage distribution in the destination. The effect of the change in the spread in the left tail in the origin is ambiguous.

**A.5 Negative binomial model**

Let $\lambda_{ij} = \mu_{ij}\nu_{ij}$. If we specify $\mu_{ij} = e^{x\beta}$ and $\nu_{ij}$ have a Gamma distribution with $E\left[\nu_{ij}\right]=1$ and $\operatorname{var}\left[\nu_{ij}\right]=\alpha$, then the distribution of $y_{ij}$ can be written as:

$$h\left(y_{ij},\alpha,\mu\right)=\frac{\Gamma\left(\alpha^{-1}+y_{ij}\right)}{\Gamma\left(\alpha^{-1}\right)\Gamma\left(1+y_{ij}\right)}\left(\frac{\alpha^{-1}}{\alpha^{-1}+\mu_{ij}}\right)^{\alpha^{-1}}\left(\frac{\mu_{ij}}{\alpha^{-1}+\mu_{ij}}\right)^{y_{ij}}. \quad \text{(A.17)}$$

The first two moments of the negative binomial distribution are: $E\left[y_{ij}\right]=\mu_{ij}$ and $\operatorname{var}\left[y_{ij}\right]=\mu_{ij}\left(1+\alpha\mu_{ij}\right)$. If $\alpha$ is zero then $E\left[y_{ij}\right]=\operatorname{var}\left[y_{ij}\right]$ and negative binomial is identical to the Poisson. Hence, testing $\alpha=0$ after estimating the negative binomial is identical to testing the negative binomial specification vs. the Poisson.

**A.6 Zero-inflated models**

In hurdle or zero-inflated models zeros and positive outcomes are generated by different processes. Zeros have the density $h_1(\cdot)$, so $\Pr\left[y_{ij}=0\right]=h_1(0)$. Positive outcomes come from the truncated density $h_2\left(y_{ij} \mid y_{ij}>0\right)=\frac{h_2\left(y_{ij}\right)}{1-h_2(0)}$. The probability that $y_{ij}$ is drawn from this truncated density is $1-h_1(0)$. Hence:

$$q\left(y_{ij}\right)=\begin{cases} h_1(0) & \text{if} \quad y_{ij}=0 \\ \dfrac{h_2\left(y_{ij}\right)}{1-h_2(0)}\left(1-h_1(0)\right) & \text{if} \quad y_{ij}>0 \end{cases} \quad \text{(A.18)}$$

The likelihood function follows immediately from Equation A.18.

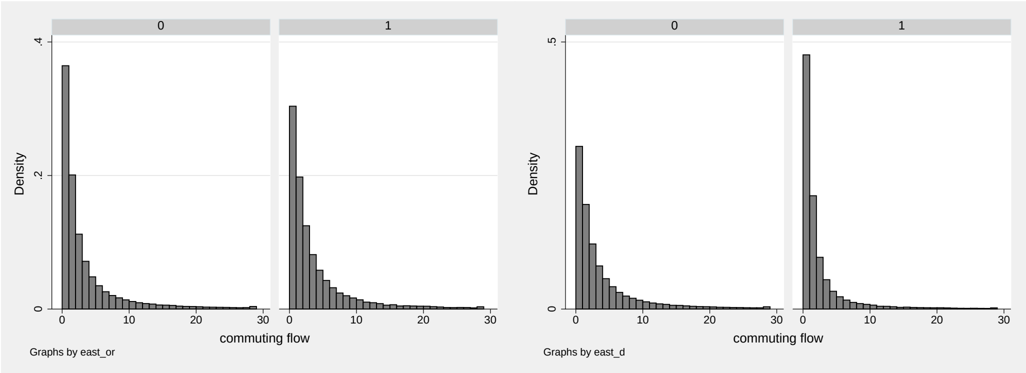
### A.7 Data used

The description of the IABS data set is taken from Möller and Aldashev (2006). The data on wages and wage dispersion were calculated from IABS-REG. IABS-REG is a 2% random sample from the employment register of the Federal Labor Office with regional information. The data set includes all workers, salaried employees and trainees obliged to pay social security contributions and covers more than 80\% of all employment. Excluded are public servants, minor employment and family workers (see Bender et al. (2000) for an extensive description of the data). Because of legal sanctions for misreporting, the earnings information in the data is highly reliable. Among others, IABS-REG contains variables on individual earnings and skills. The regional information is based on the employer. For the empirical analysis the data were restricted to full-time workers of the intermediate skill group (apprenticeship completed without a university-type of education). All male and female workers were selected that were employed on June 30th, 1997. For all regions the median wage and the second and eighth decile of daily earnings were calculated.

INKAR database of the Federal Office for Building and Regional Planning contains basic geographic and demographic indicators on regional level. Regional population used in estimation in Section 6 was taken from the INKAR dataset.

The data on commuting time was produced by the Institute for Regional Planning of the University of Dortmund (IRPUD). Using the data on daily commuters who report their travel time from home to workplace, they calculate average travel time between each pair of regions (more see in Spiekermann et al. (2000) and citations therein).

Fig. A.1: Distribution of commuting flows. 0 – West Germany, 1 – East Germany




a) Left panel – by origin, right panel – by destination.

Tab. A.1: Estimation results of the zero-inflated negative binomial model (East dummy (origin and destination) is interacted with parameters of the wage distribution). Dependent variable – commuting flows.

| **variable** | **coef.** | **robust st. er.** |
|---|---|---|
| East origin | **5.09** | 2.16 |
| East destination | -3.30 | 4.05 |
| Log travel time | **-2.66** | 0.02 |
| Log median wage (origin) | -0.05 | 0.25 |
| Log D5/D2 (origin) | 0.72* | 0.44 |
| Log D8/D5 (origin) | **1.56** | 0.70 |
| Log median wage (destination) | **3.00** | 0.51 |
| Log D5/D2 (destination) | **-2.16** | 0.96 |
| Log D8/D5 (destination) | **12.68** | 1.63 |
| Log POP (origin) | **0.99** | 0.02 |
| Log POP (destination) | **1.00** | 0.05 |
| Spatially weighted POP (x100) | **-0.17** | 0.03 |
| Spatially weighted median wage | **-0.12** | 0.01 |
| const | **-10.21** | 1.45 |
| **inflate** | | |
| East origin | -10.39 | 35.80 |
| East destination | 101.05 | 76.20 |
| Log travel time | **0.95** | 0.25 |
| Log median wage (origin) | -1.08 | 5.16 |
| Log D5/D2 (origin) | -4.37 | 4.78 |
| Log D8/D5 (origin) | -13.78* | 7.42 |
| Log median wage (destination) | 24.05 | 15.09 |
| Log D5/D2 (destination) | **-240.64** | 113.38 |

| | | |
|---|---|---|
| Log D8/D5 (destination) | 185.36 | 121.93 |
| Log POP (origin) | **-1.16** | 0.12 |
| Log POP (destination) | **-1.68** | 0.27 |
| Spatially weighted POP (x100) | **-2.80** | 0.56 |
| Spatially weighted median wage | **0.91** | 0.20 |
| const | -82.53 | 104.95 |
| $\alpha$ | **1.21** | 0.02 |
| Vuong test | 12.43 | |
| *N* | 190,532 | |