Uncertainty over the Immigration Quota System

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Abstract

The European recent legislation on immigration reveal a peculiar paradox on migration policies: from one side as a result of increased labour market competition and concerns about terrorism, the trend of the recent legislation over immigration points to an increasing frontier closure (OECD 1999, 2001). From another side, there is an increase of regolarizations, that is the european policies become less tightened. Our aim is to study why we find counterbalancing and opposite policies in the european immigration legislation. To do this, we have used a recent approach to migration choice that assume that the decision to migrate can be described as a investment decision. Our results show that uncertainty over the immigration quota system can delay the mass entry of immigrants. Therefore, if the government's aim is to delay and/or control entry migration waves, it could control the uncertainty on the information related to the immigration quota.

JEL Classification Numbers: F22, J61, O15, R23.

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1 Introduction

While the barriers to international trade and capital mobility have been largely removed, labour markets are the most tightly regulated area of economic activity (Faini et al., 1999). Migration policies of the EU member states *vis-à-vis* non-EU countries are no exception in this respect, even if the Rome Treaty acknowledged the free mobility of labour as one of the four fundamental freedoms of the Common Market. Boeri and Brücker (2005), studying European Migration, show that rules for legal immigration into the EU from third countries are getting tighter and tighter: "since 1990 there have been 92 reforms of national migration policies in the EU-15, that is, more than five reforms per year. Most of these reforms are marginal in that they adjust specific provisions rather than revising the overall regulatory framework. Furthermore, seven reforms out of ten tighten regulations, for example, by increasing procedural obstacles faced by those applying for visas, reducing the duration of work permits or making family reunification more difficult", or by introducing an immigration quota system[1]. Nevertheless, despite this evidence, another aspect related to migration policy reveals a peculiar paradox on migration policies: since 1990 there have been 26 (39 since 1973) one-shot regularization programs in 10 EU countries (Jachimowicz et al., 2004; Sunderhaus, 2007)[2]. Therefore, from one side as a result of increased labour market competition and concerns about terrorism, the trend of the recent legislation over immigration points to an increasing frontier closure (OECD 1999, 2001). From another side, there is an increase of regolarizations, that is the european policies become less tightened. Then, which kind of policy

[1] Source: Fondazione Rodolfo Debenedetti. See www.frdb.org for details.

[2] In their broadest sense, regularization programs offer those migrants who are in a country without authorization the opportunity to legalize their status.

Irregular migrants, also referred to as "undocumented," "unauthorized," or "illegal," are defined by most states as those migrants who have either entered a country legally and then fallen out of legal status — such as students, temporary workers, rejected asylum seekers, or tourists — or those who have entered illegally, either by crossing a border undetected or with false documents. In either case, irregular migrants do not have a legal right to residence in the state to which they have migrated.

is better for controlling immigration? Is it better to tighten or reduce the rules for legal immigration? Our aim is to answer to these questions, investigating the counterbalancing and opposite policies in the european immigration legislation. To do this, we use a recent approach to migration choice that assume that the decision to migrate can be described as a investment decision (Sjaastad, 1962). In this respect, Burda (1995), following a real option approach, showed that individuals prefer to wait before migrating, even if the present value of the wage differential is positive, because of the uncertainty and the sunk costs associated with migration [3]. Subsequently Khwaja (2002) and Anam *et al.*, (2004) developed Burda's approach by describing the role of uncertainty in the migration decision. Another work that uses real option in migration is Feist (1998), in which the author analyses the option value of the low-skilled workers to escape to the unofficial sector if welfare benefits come too close to the net wage in the official sector. Three recent papers (Moretto and Vergalli, 2005; Vergalli, 2006; Vergalli, 2007) have applied the real option framework to the analysis of the migration dynamics, focussing on the role of communities and network to explain mass migration.

The existence of the quotas seems to be *idiosyncratic* with respect to various aspects of the economic approach. Particularly we can find of it not only for that it pertains to the migration phenomenon, by also concerning foreign investment or also the adoption of licenses regulating the market[4].

[3] Investment is defined as the act of incurring an immediate cost in the expectation of future payoff. However, when the immediate cost is sunk (at least partially) and there is uncertainty over future rewards, the timing of the investment decision becomes crucial (Dixit and Pindyck, 1994, p. 3).

[4] We can find many examples in which quotas assume an important role in the market: capital controls are often imposed to prevent a country's net credit position from exceeding some acceptable level; central banks face limits on the amount of foreign reserves that can be used to enforce an exchange rate target; firms in a fast-growing industry or in a developing economy may be competing for extended periods for a small number of qualified managers or hightly skilled workers; entry of firms is restricted in many industries by regulations aimed at containing market size or by technological constraints on the use of a scarce resource. Similar approaches arise for taxi and liquor licences, fishing and costal trade rights, the number of

The role of quotas has been introduced in the real option theory of investment choice by two pioneering works of Bartolini (1993; 1995). In his first paper (1993), the author develops a general model that considers the investment decision of decentralized profit-maximizing agents, who face investment adjustment costs in a market with stochastic returns and a limit on aggregate investment. The model is consistent with equilibrium models of asset princing under uncertainty but differs from the mainstream assumption of constant investment cost, assuming that, for technological or insitutional reason, the investment cost is constant only until an investment ceiling becomes binding: at that point, in fact, Bartolini shows that cost becomes infinite. His paper shows that a competitive market reacts to this type of externality by generating recurrent runs as aggregate investment fluctuates around its limit: the existence of limits on aggregate investment may induce endogenous and recurrent asset runs. In the second paper (1994), the author examines entry decisions in a market witha quota on foreign entry and ongoing uncertainty about future returns on investment, with a focus on the dynamics of foreign investment before the quota becomes binding. His analysis shows that the way in which entry rights are allocated among foreign firms has important implications for the efficiency of investment. When foreign firms are rationed, the first-best policy holds until the quota becomes binding. When foreign firms are not rationed, they abandon the optimal policy when their investment in the host country reaches a critical threshold. That threshold triggers a "rent run": the quota is immediately filled, foreign firms begin to sell at below marginal cost, and additional welfare losses are incurred.

Both Bartolini's works, adding determined quotas in a real option framework, introduce new insights on the difference between multiple-agent equilibria and centralized surplus-maximizing planner. Without ceilings, when analyzing multiple-agent equilibria, the traditional studies have tipically relied on invisible-hand-like results from the theory of investment under uncertainty: under the assumption of constant investment cost, the investment policy chosen by a centralized surplus-maximizing planner would coincide with the policy chosen

polluting trade permits or ecolabelling permits (Dosi and Moretto, 2001).

by decentralized competitive agents. This equality is not satisfied in Bartolini's model, because of the existence of the ceiling.

In the first part of our paper we use the Bartolini model to describe what could happen in the migration dynamic if the authority imposed a determined quota on the immigration entries. In the second part of the paper, we will ask what could happend if the quota were unknown. The results show that an unknown quota system delays mass entry. Therefore if the government were able to cause noise on policy information (and if it were able to control mean and variance of the uncertainty), it could control also the mass entry. In this sense, if the government aim is to delay entry it is convenient to generate uncertainty. This fact could also explain why the recent legislation on immigration shows the two counterbalancing effects explained above.

This paper is organised as follows. Section 2 summarises the evolution of national immigration policies. Section 3 presents the model and the basic assumptions. Section 4 develops the theoretical framework with known quota and the main results. Section 5 develops the theoretical framework with unknown quota and the main results. Finally, section 6 summarises the conclusions.

2 Evolution in National Immigration Policies

Boeri and Brücker (2005) have developed an aggregate policy index that describes "*the trend in migration policies". The index is obtained by taking the average of the following seven indicators: 1) admission requirements; 2) number of administrations involved; 3) lenghth of first stay; 4) quotas; 5) residence requirement; 6) years to obtain a permanent permit; 7) asylum policy*[5]. By their anaylsis it tourns out that the national immigration policies are becoming more and more tighten[6].

[5] The indexes from 1 to 6 were defined by Fondazione Rodolfo Debenedetti (see www.frdb.org for details) and the index 7 was defined by Hatton (2004).

[6] "*All countries except Greece, [...] denote a tightening in regulations*", see Boeri and Brücker, 2005, page 634.

Nevertheless, let see to the regularization programs adopted in Europe in since 1973 in table 1[7]: since 1990 there have been 26 (39 since 1973) one-shot regularization programs in 10 EU countries.

Country	*Regularizations*	*Years*
Belgium	3	1974-1975, 1995-1999, 2000
France	5	1973, 1979, 1981-1982, 1991, 1997-1998
Germany	2	1996, 1999
Greece	2	1997-1998, 2001
Italy	6	1982, 1987-1988, 1990, 1996, 1998-1999, 2002
Luxembourg	1	2001
Netherlands	5	1964, 1975, 1978-1979, 1991-1994, 1996
Portugal	3	1992-1993, 1996, 2001
Spain	7	1985-1986, 1991, 1994, 1996, 2000, 2001, 2005
United Kingdom	5	1974-1978, 1977, 1987, 1998-1999, 2004
Total	39	

Table 1: Regularization Programs in 10 EU countries.

Threfore, from one side immigration policies are becoming more and more tighten; from another side, there is an increase of regolarizations, that is the european policies become less tightened. Then, which kind of policy is better for controlling immigration? Is it better to tighten or reduce the rules for legal immigration? Or there exists a third policy to conrtol immigration waves? In the following part we introduce a model that try to answer to these questions.

3 Ceilings on entries

3.1 The Model

For ease of exposition, the model is developed by using the familiar terminology of agents' entry decisions under uncertainty[8]. Consider the immigration decision of individuals in a host country subject to uncertain wage gap. Let us summarize the main assumptions:

[7] Table 1 is our elaboration on Jachimowicz et al. (2004, pages 36-40) and Sunderhaus, (2007).

[8] See Dixit-Pindyck (1994), page 253 and Moretto and Vergalli (2005).

1. At any time t, a potential immigrant may decide to migrate ("entry"). Individuals are risk-neutral and discount the future incomes at the riskless interest rate ρ.

2. Each individual can enter by committing forever to a flow cost w or undertaking a single irreversible investment (migration) which requires an initial sunk cost $K = w/\rho$.

3. Indicating by n_t the number of individuals that are in the host country at time t, each of them yields a flow of incomes:

$$\pi\left(\theta_t, n_t\right) \equiv u\left(n_t\right)\theta_t \tag{1}$$

where θ is a multiplicative labour market-specific shock.

4. The function $u(n)$ is twice continuously differentiable in n with the usual properties:

$$u(n) > 0,\ u'(n) < 0$$

$$\lim_{n \to 0} u(n) = +\infty$$

$$\lim_{n \to \infty} u(n) = \underline{\mathrm{u}}>0$$

5. All individuals are identical and their size dn_t is infinitesimally small with respect to the labour market.

6. The labour market-specific shock follows a geometric diffusion process:

$$d\theta_t = \alpha\theta_t dt + \sigma\theta_t dW_t \qquad with\ \theta_0 = \theta\ and\ \alpha, \sigma \succ 0 \tag{2}$$

where dW_t is the increment to a Wiener process, satisfying $E(dW_t) = 0$ and $Var(dW_t) = dt$.

In the first part of the paper we assume that the quota is known: the existence of a limit on the aggregate level of investment induces an externality among the benefit functions of different firms, which causes a possible divergence between the socially-optimal and profit-maximizing policies. In the second part we will relax this limit, by assuming that the quota might be undetermined.

3.2 Solution with defined quota

For the first result we add the following assumption:

7. There exists an exogenous determined quota N on n.

To obtain our result, let us consider the value of a firm when no entry is taking place: the no-bubble value of the "entry", $V(\theta, n)$, must satisfy the no-arbitrage requirement where time is suppressed if not necessary:

$$\pi(\theta, n) + E[dV(\theta, n)/dt] = \rho V(\theta, n) \tag{3}$$

Assuming $V(\theta, n)$ to be twice-differentiable function of its arguments, It$\widehat{o}$'s Lemma yields E[d$V(\theta, n)$]. Substituting this into (3), the differential equation defining $V(\theta, n)$ when there are n individuals entered in the host country and no entry is taking place, is given by:

$$\frac{1}{2}\sigma^2 V_{\theta\theta}(\theta, n) + \alpha V_\theta(\theta, n) - \rho V(\theta, n) + \pi(\theta, n) = 0 \tag{4}$$

The general solution of (4):

$$V(\theta, n, N) = B(n, N)\,\theta^\beta + \frac{\theta u(n)}{\rho - \alpha} \tag{5}$$

Where the last term $\left(\frac{\theta u(n)}{\rho-\alpha}\right)$ represents the value of migration in the absence of new entry[9], $B(n, N)\,\theta^\beta$ is the correction of the immigrant's value due to the new entry and $B(n, N)$ must therefore be negative. The coefficient $B(n, N)$ can be determined by using the following suitable set of boundary conditions:

[9] that is, the discounted present value of the profit flows over an infinite horizon starting from θ (Harrison 1985, p. 44). See equation (14).

1. First let us define $\theta^*(n)$ as the value at which by the competitive pressure, the value-matching condition require the value of being entered to equal the entry cost K at $\theta = \theta^*(n)$:

$$V(\theta^*(n), n, N) = K \tag{6}$$

2. The second one is the smooth-pasting condition[10]:

$$V_n(\theta^*(n), n, N) = 0 \tag{7}$$

3. The last boundary condition to be considered applies to the value of the N^{th} entry. The value of the N^{th} entry should converge to the value of a migration computed by keeping the number of immigrants fixed at N. From (5) this requires:

$$B(n, N) = 0^{11} \tag{8}$$

Therefore we can obtain this following proposition:

Proposition 1 *The benefit-maximizing entry policy in a market with a quota N is identical to the efficient entry policy until a number $n^*<N$ of individuals have entered the market. At that point a rent run takes place, and the residual quota is instantly filled. The entry policy is given by:*

$$\begin{aligned} \theta^*(n) &= \frac{\beta}{\beta-1}(\rho-\alpha)\frac{K}{u(n)} \qquad for\ n = (0, n^*] && (9) \\ \theta^*(n) &= \frac{\beta}{\beta-1}(\rho-\alpha)\frac{K}{u(n^*)} \equiv (\rho-\alpha)\frac{K}{u(N)} \qquad for\ n = [n^*, N] && (10) \end{aligned}$$

Proof. *See Bartolini (1995) and the Appendix.* ■

[10] By proposition 1 of Bartolini's (1993).

[11] The same assumption can be obtained by comparing (20) with (5). See Bartolini (93) page 928 or Bartolini (95) page 47.

The insights coming from Proposition 1 are showed in figure 1, below.

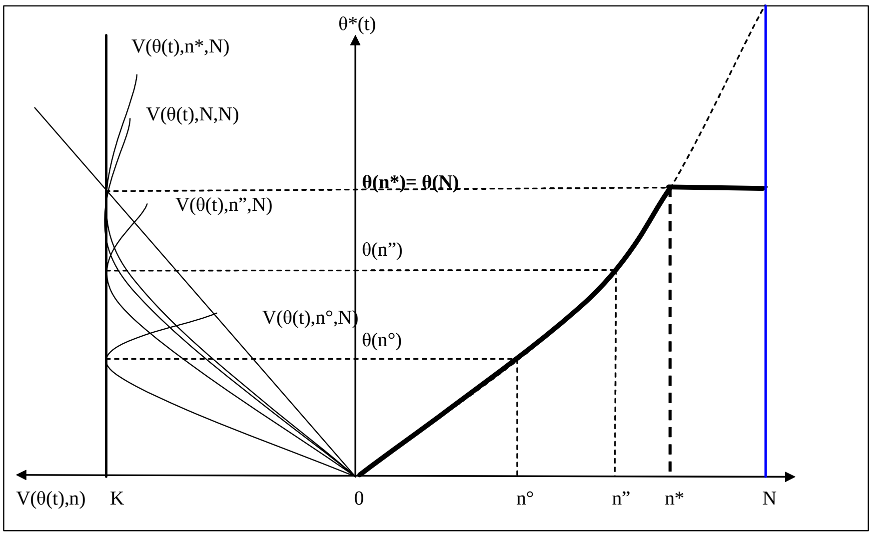

Figure 1

Figure 1 shows:

- in the first quadrant on the left entry value for different θ and n levels;

- on the right, the threshold levels for different numbers of immigrants.

With free entry, labour market competition generates a run that fills the quota when a fraction n^*/N has been filled. Until then, the entry policy is identical to the case without a quota: immigrants initially enter at the optimal pace, knowing that all the potential benefits will dissipated by the early entry of the last $(N - n^*)$ individuals.

4 Solution with undetermined quota

So far we have studied the otpimal policy with a determined quota on the number of individuals in a host country, but what it happens if the limit is uncertain?

To introduce uncertainty over the quota, we replace assumption (7) with the following assumption (7'):

7'. Let us define the Bayesian process on N. If the function of conditioned density is $g(N;n)$[12], then $F(N)$ is the a-priori distribution on N defined on $[0,\infty)$ with $f(0)\geq 0$. Then, we have that:

$$g(N;n)=\frac{f(N)}{1-F(n)}$$

while the probability distribution is:

$$G(N;n)=\frac{F(N)-F(n)}{1-F(n)}$$

Therefore, each individual does not know the exactly level of the limit over the stock . The bound is decided by the Government that has perfectly information about it. So each individual gives a probability on the level of N, distributed according to g$(N;n)$.

By using the usual boudary counditions, we get:

Proposition 2 *The benefit-maximizing entry policy in a market with a* ***unknown*** *quota N is identical to the efficient entry policy until a number n^{**} of immigrants has entered the market, such that $n^{*}<n^{**}<N$. At that point a rent run takes place, and the residual quota is instantly filled. The entry policy is given by:*

$$\theta^{*}(n) = \frac{\beta}{\beta-1}(\rho-\alpha)\frac{K}{u(n)} \qquad for\ n=(0,n^{**}] \tag{11}$$

$$\theta^{*}(n) = \frac{\beta}{\beta-1}(\rho-\alpha)\frac{K}{u(n\prime)}\equiv(\rho-\alpha)\frac{K}{u(N)} \qquad for\ n=[n^{**},N] \tag{12}$$

Proof. see the Appendix. ■

Let us observe figure 2: in the quadrant on the left we can observe the value of the immigrants on the horizontal axis and the optimal threshold level on the vertical axis; in the quadrant on the right we have the threshold level and the number of immigrants in the host country. The red line represents the trigger

[12] Each firm knows that the distribution of probability changes each new entry in the market. We then specify the firms' beliefs at any instant of time by a state-dependent distribution and density function of the quota N (Tsur and Zemel, 1994, 1996; Bosello and Moretto, 1999).

depending on the number of immigrants. It is possible to stress that without uncertainty the threshold level becomes flat at n*, as in figure 1. Instead, with the presence of uncertainty, the competitive run happens according to the red line, that is, at $n\prime > n^*$ the run starts. In this sense uncertainty delays mass entry that fills the residual quota. Therefore if the government were able to cause noise on policy information (and if it were able to control mean and variance of the uncertainty), it could control also the mass entry. In this sense, if the government aim is to delay entry it is convenient to generate uncertainty. This fact could also explain why the recent legislation on immigration shows the two counterbalancing effects explained above.

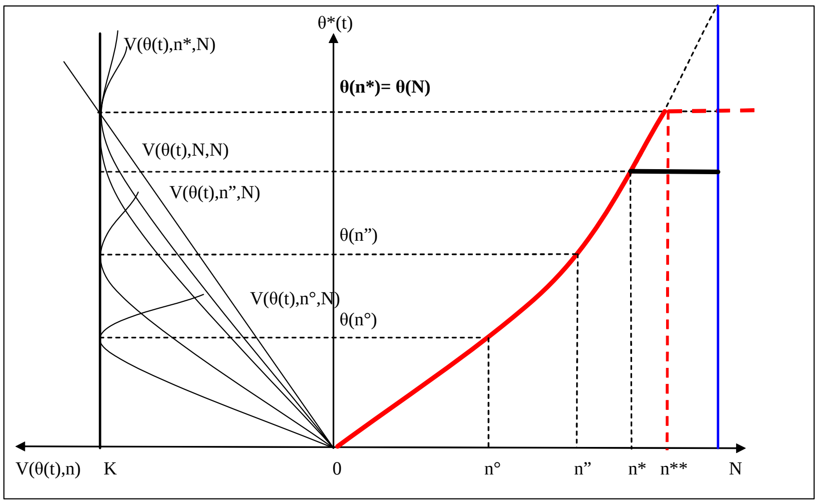

Figure 2

5 Conclusion

The European recent legislation on immigration reveal a peculiar paradox on migration policies: from one side as a result of increased labour market competition and concerns about terrorism, the trend of the recent legislation over immigration points to an increasing frontier closure (OECD 1999, 2001). From another side, there is an increase of regolarizations, that is the european policies become less tightened. Which is the better policies? Our aim is to study why we find counterbalancing and opposite policies in the european immigration legislation. To do this, we have used a recent approach to migration choice that assume that the decision to migrate can be described as a investment decision (Bartolini, 1993; Vergalli, 2007). Our results show that uncertainty over the immigration quota system can delay the mass entry of immigrants. Therefore, if the government's aim is to delay and/or control entry migration waves, it could control the uncertainty on the information related to the immigration quota. Indeed, if governmenet were able to control uncertainty, it could smooth the entry of immigrants. In conclusion, between the two policies adopted (tighten or reduce the rules for legal immigration) there exists a third policy that is to alternate tightening and reduction in order to create uncertainty over the quota system and control entry.

A Appendix

A family of solutions of (4) is given by:

$$V(\theta, n, N) = A(n, N)\,\theta^{\gamma} + B(n, N)\,\theta^{\beta} + F(\theta, n) \tag{13}$$

where β and γ are the positive and negative roots of the quadratic equation in $\lambda:\ (\sigma^2/2)\,\lambda(\lambda - 1) + \alpha\lambda - \rho = 0$ with $1<\beta<\frac{\rho}{\alpha}$ and $A(n, N)$ and $B(n, N)$ are the two families of integration constants; $F(\theta, n)$ is chosen as the discounted expectation of flow payoff computed by keeping the number of immigrants fixed at n:

$$F(\theta, n) = E_0\left[\int_0^{\infty} \pi(n, \theta)\, e^{-\rho t} dt \mid \theta(0) = \theta\right] = \frac{\theta u(n)}{\rho - \alpha} \tag{14}$$

Because the probability of entry goes to zero as θ goes to zero, one boundary condition is that $\underset{\theta\to 0}{Lim} V(\theta, n, N) = 0$, this implies that $A(n, N) = 0$, and then equation (5).

B Proof of Proposition 1.

Proof. First, imposing the following zero-income condition[13]:

$$V(\theta^*, n, N) = 0 \tag{15}$$

on (5) for $n = N$, yields (9) and $B(N, N) = 0$. Next, differentiating (15) totally with respect to n and using (7):

$$\begin{aligned} 0 &= \frac{\partial V(\theta^*, n, N)}{\partial n} = V_{\theta}(\theta^*, n, N)\,\frac{\partial \theta^*}{\partial n} && (16)\\ &= \left[\frac{u(n)}{\rho - \alpha} + B(n, N)\,\beta\,(\theta^*)^{\beta - 1}\right]\frac{\partial \theta^*}{\partial n} && (17)\end{aligned}$$

[13]Individuals are unable to trade off their decision to enter immediately against the same decision delayed. The entry problem is now subject to the constraint that immigrants must enter just *fast enough* to make expected incomes at entry equal to zero. Thus the smooth-pasting condition (7) is replaced by (15).

(16) splits $[0, N]$ into intervals where one of the following two conditions must hold:

$$H(n) \equiv \left[\frac{u(n)}{\rho - \alpha} + B(n, N) \beta (\theta^*)^{\beta - 1} \right] = 0 \tag{18}$$

or

$$\frac{\partial \theta^*}{\partial n} = 0 \tag{19}$$

where (18) is the smooth-pasting condition. Since $B(N, N) = 0$ and $\frac{u(N)}{\rho - \alpha} \neq 0$, then (18) cannot hold at $n = N$. Therefore, it must be (19) that hold at $n = N$.

Now, define n^* as the largest $n \leq N$ that satisfies (18). For all $n^* \leq n \leq N$, we have $\frac{\partial \theta^*}{\partial n} = 0$, so that all immigrants in the range $[n^*, N]$ must enter at $\theta(t) = \theta^*$. For the range $n < n^*$, (18) holds. Applying this to the general solution (5) gives (9). Finally, the unique solution $n^* < N$ is obtained by combining (9) and (10). Let us demonstrate the uniqueness of n*: V$(\theta(t), N, N)$ equals the discounted income stream with benefit fixed at u(N) :

$$V(\theta, N, N) = \int_0^\infty e^{-\rho t} [\theta u(N)] dt = \frac{\theta(t) u(N)}{\rho - \alpha} \tag{20}$$

comparing (20) with (5) gives $B(N, N) = 0$. To obtain $B(n, N)$, substitute (5) into (7): $B_n(n, N) = -(\theta^*)^{1-\beta} u'(n) / (\rho - \alpha)$.

Integrating this between n and N, gives

$$\int_n^N B_q(q, N) \, dq = -\int_n^N (\theta^*)^{1-\beta} \frac{u'(q)}{\rho - \alpha} dq \tag{21}$$

Using (1), $B(N, N) = 0$, and changing the integration variable on the right-hand of (21), from q to $u(q)$, gives

$$\int_n^N B_q(q, N) \, dq = -\int_n^N \left[\frac{\pi}{u(q)} \right]^{1-\beta} \frac{u'(q)}{\rho - \alpha} du(q) \tag{22}$$

$$B(n, N) = \frac{\pi^{1-\beta}}{\beta(\rho - \alpha)} \left[u^\beta(N) - u^\beta(n) \right] < 0^{14} \tag{24}$$

[14] It is possible to notice that since $B(n, N) < 0$, then the limit for $n \to N$ is:

■

Proof. The smooth-pasting condition becomes:

$$\begin{aligned} H(n) &= \frac{u(n)}{\rho-\alpha} + \beta\theta^{*\beta-1} B(n,N) \\ &= \frac{u(n)}{\rho-\alpha} + \theta^{*\beta-1} \frac{\pi^{*1-\beta}}{(\rho-\alpha)} \left[u^{\beta}(N) - u^{\beta}(n)\right] \\ &= \frac{u(n)}{\rho-\alpha} + (\frac{\pi^*}{\theta^*})^{1-\beta} \frac{1}{(\rho-\alpha)} \left[u^{\beta}(N) - u^{\beta}(n)\right] \end{aligned}$$

When $n = N$, we have $B(N,N) = 0$, therefore $H(N) > 0$. This fact implies that we need $\frac{d\theta^*}{dn} = 0$ in order that the smooth-pasting is verified. The consequence is that the optimal trigger for $n = N$ cannot be $\theta^*(n) = \frac{\beta}{\beta-1}(\rho-\alpha)\frac{K}{u(n)}$[15]. In fact for $n = N$ the trigger is $\theta^*(N) = (\rho-\alpha)\frac{K}{u(N)}$[16].

Let us assume that function H is positive for a $N - y$ (where y may be infinitesimally small), the trigger does not change and we can write the smooth-pasting condition in the following manner:

$$H(N-y) = \frac{u(N-y)}{\rho-\alpha} + (\frac{\pi^*}{\theta^*})^{1-\beta} \frac{1}{(\rho-\alpha)} \left[u^{\beta}(N) - u^{\beta}(N-y)\right] \qquad (25)$$

In this case $\frac{u(N-y)}{\rho-\alpha}$ increases but $\left[u^{\beta}(N) - u^{\beta}(N-y)\right]$ is negative. If $H(N-y)$ is still greater than zero, with $\frac{\pi^*}{\theta^*} = u(N)$ we ought to obtain $\frac{d\theta^*}{dn} = 0$. This procedure continues until we obtain a y (defined by $n^* = N - y$) such that $H(n^*) = 0$.

Let us take the first derivative with respect to y

$$\lim_{n\to N} B(n,N) = 0^- \qquad (24)$$

[15] or $\pi^* = \theta^*(n)u(n) = \frac{\beta}{\beta-1}(\rho-\alpha)K$.

[16] or $\pi^* = \theta^*(N)u(N) = (\rho-\alpha)K$.

$$\begin{aligned}
\frac{dH(N-y)}{dy} &= -\frac{u'(N-y)}{\rho-\alpha} + (\frac{\pi^*}{\theta^*})^{1-\beta}\frac{1}{(\rho-\alpha)}\beta u^{\beta-1}(N-y)\,u'(N-y) \qquad (26)\\
&= \frac{u'(N-y)}{\rho-\alpha}\left[(\frac{\pi^*}{\theta^*})^{1-\beta}\beta u^{\beta-1}(N-y)-1\right]\\
&= \frac{u'(N-y)}{\rho-\alpha}\left[(u(N))^{1-\beta}\beta u^{\beta-1}(N-y)-1\right]\\
&= \frac{u'(N-y)}{\rho-\alpha}\left[\beta(\frac{u(N-y)}{u(N)})^{\beta-1}-1\right]<0
\end{aligned}$$

Quod erat demonstrandum if y increases (moving from N to 0) there exists a value of n^* (i.e. y^* given that N is given) such that $H(n^*)=0$. ■

C Proof of Proposition 2.

With uncertainty over the stock, equation (23) becomes:

$$E(B(n)) = \frac{\pi^{*1-\beta}}{\beta(\rho-\alpha)}\left[\int_n^\infty u^\beta(N)\,g(N;n)\,dN - u^\beta(n)\right] \qquad (27)$$

and then

$$E(B(n)) = \frac{\pi^{*1-\beta}}{\beta(\rho-\alpha)}\left[\frac{\int_n^\infty u^\beta(N)\,f(N)dN}{1-F(n)} - u^\beta(n)\right]<0 \qquad (28)$$

This constant is negative because it worth $u^\beta(n) > u^\beta(N)$ any $N>n$.

Let us take the limit of $E(B(n))$, in particular:

$$\begin{aligned}
\lim_{n\to\infty} E(B(n)) &= \frac{\pi^{*1-\beta}}{\beta(\rho-\alpha)}\left[\frac{\int_n^\infty u^\beta(N)\,f(N)dN}{1-F(n)} - u^\beta(n)\right]\\
&= \frac{\pi^{*1-\beta}}{\beta(\rho-\alpha)}\left[\frac{-u^\beta(n)\,f(n)}{-f(n)} - u^\beta(n)\right]\\
&= \frac{\pi^{*1-\beta}}{\beta(\rho-\alpha)}\left[+u^\beta(n)-u^\beta(n)\right]=0^-
\end{aligned}$$

This result is in line with (24)[17]. The smooth pasting condition strongly depends on $E(B(n))$. Therefore:

[17] Let us notice that $\lim_{n\to\infty} E(B(n)) = 0$ even if $u(n) \to \underline{u} \geq 0$.

$$E(H(n)) = \frac{u(n)}{\rho-\alpha} + \beta\theta^{*^{\beta-1}} E(B(n)) \tag{29}$$
$$= \frac{u(n)}{\rho-\alpha} + (\frac{\pi^*}{\theta^*})^{1-\beta} \frac{1}{(\rho-\alpha)} \left[\frac{\int_n^\infty u^\beta(N) f(N)dx}{1-F(n)} - u^\beta(n) \right]$$

Let us assume that $u(n) \rightarrow \underline{u} \geq 0$ when $n \rightarrow \infty$: when $n \rightarrow \infty$ we obtain $E(B(n)) = 0$, and in the smooth pasting we have $\frac{u(\infty)}{\rho-\alpha} = \frac{\underline{u}}{\rho-\alpha} \geq 0$. It follows that for an high level of n, $E(H(n)) > 0$, therefore also $\frac{d\theta^*}{dn} = 0$[18].

Let us use the definition of limit: in this case for each real number $\varepsilon > 0$ infinitesimely small, there exists a value $n\prime$ such that for $n > n\prime$ the difference

$$E(H(n')) - E(H(\infty)) < \varepsilon$$

Nevertheless, since now we have $E(H(\infty)) = 0$, we are able to find a value n' such that:

$$E(H(n')) - 0 < \varepsilon$$

If $\varepsilon \rightarrow 0$ it follows that n' is the right value we are searching.

C.1 Case with $\underline{u}$>0

Now, we have to demonstrate that $d\frac{E(H(\infty-y))}{dy} < 0$. We start from $\infty - y$: if also in $\infty - y$ it is worth $E(H(\infty-y)) > 0$ the trigger level remains $\pi^* = \theta^*(\infty)\underline{u} = (\rho-\alpha)K$, then $\frac{\pi^*}{\theta^*} = \underline{u}$. Substituting, we have:

$$E(H(\infty-y)) = \frac{u(\infty-y)}{\rho-\alpha} + (\underline{u})^{1-\beta} \frac{1}{(\rho-\alpha)} \left[\frac{\int_{\infty-y}^\infty u^\beta(x) f(x)dx}{1-F(\infty-y)} - u^\beta(\infty-y) \right]$$

Taking the derivative:

$$d\frac{E(H(\infty-y))}{dy} = -\frac{u'(\infty-y)}{\rho-\alpha} + (\underline{u})^{1-\beta} \frac{1}{(\rho-\alpha)} \times$$
$$\times \left[\frac{-u^\beta(\infty-y) f(\infty-y)(1-F(\infty-y) - \int_{\infty-y}^\infty u^\beta(x) f(x)dx f(\infty-y)}{(1-F(\infty-y))^2} + \beta u^{\beta-1}(\infty-y) u'(\infty-y) \right] =$$

[18] Let us notice that this result is always verified when $\underline{u}$>0 is strictly positive when $n \rightarrow \infty$, but it is verified also for $\underline{u}\geq 0$ by using limit definition.

$$\left[-\frac{u^{\beta}(\infty-y)\,f(\infty-y)}{(1-F(\infty-y)}-\frac{\int_{\infty-y}^{\infty}u^{\beta}(x)\,f(x)dxf(\infty-y)}{(1-F(\infty-y))^{2}}+\beta u^{\beta-1}(\infty-y)\,u'(\infty-y)\right]$$

$$d\frac{E(H(\infty-y))}{dy}=\frac{u'(\infty-y)}{\rho-\alpha}\left[-1+(\underline{\mathrm{u}})^{1-\beta}\beta u^{\beta-1}(\infty-y)\right]+$$

$$+(\underline{\mathrm{u}})^{1-\beta}\frac{1}{(\rho-\alpha)}\left[-\frac{u^{\beta}(\infty-y)\,f(\infty-y)}{(1-F(\infty-y))}-\frac{\int_{\infty-y}^{\infty}u^{\beta}(x)\,f(x)dxf(\infty-y)}{(1-F(\infty-y))^{2}}\right]=$$

$$d\frac{E(H(\infty-y))}{dy}=\frac{u'(\infty-y)}{\rho-\alpha}\left[\beta\frac{u^{\beta-1}(\infty-y)}{(\underline{\mathrm{u}})^{\beta-1}}-1\right]+ \tag{30}$$

$$+(\underline{\mathrm{u}})^{1-\beta}\frac{1}{(\rho-\alpha)}\left[-\frac{u^{\beta}(\infty-y)\,f(\infty-y)}{(1-F(\infty-y))}-\frac{\int_{\infty-y}^{\infty}u^{\beta}(x)\,f(x)dxf(\infty-y)}{(1-F(\infty-y))^{2}}\right]<0$$

Also in this case, there exists a value $n\prime=N-y\prime$ such that $E(H(n\prime))=0$ by using:

$$\frac{u(n\prime)}{\underline{\mathrm{u}}}=\frac{\beta}{\beta-1} \tag{31}$$

D Comparison with respect to Bartolini:

To demonstrate that the point n^{*} with certain quota, is lower than $n\prime$ with uncertainty over the stock, we must to show two disequalities:

1. The derivative (26) with certain limit must be *greater* than the derivative with uncertainty (30): this fact shows that the function (29) increases more rapidely than (25);

2. The value of $H(N-y)$ must be greater than the expected value $E[H(\infty-y)]$ for any $y>y^{*}$: This fact, combined with point 1, implies that the two functions do not intersect and that there exists a $y\prime$ such that $E[H(\infty-y)]=0$.

   given that $E[H(\infty-y)]$ is lower than $H(N-y)$ and there exists a value y^{*} in order that $H(N-y)=0$.

To demonstrate the first disequality, let us re-write the equation of the derivative of function H, with certain quota N:

$$\frac{dH(N-y)}{dy} = \frac{u'(N-y)}{\rho-\alpha}\left[\beta(\frac{u(N-y)}{u(N)})^{\beta-1} - 1\right] < 0 \tag{32}$$

and compare (32) with the same equation with uncertainty over the stock (33):

$$d\frac{E(H(\infty-y))}{dy} = \frac{u'(\infty-y)}{\rho-\alpha}\left[\beta\frac{u^{\beta-1}(\infty-y)}{(\underline{\mathrm{u}})^{\beta-1}} - 1\right]$$
$$+(\underline{\mathrm{u}})^{1-\beta}\frac{1}{(\rho-\alpha)}\left[-\frac{u^{\beta}(\infty-y)f(\infty-y)}{(1-F(\infty-y))} - \frac{\int_{\infty-y}^{\infty}u^{\beta}(x)f(x)dxf(\infty-y)}{(1-F(\infty-y))^2}\right] < 0 \tag{33}$$

that is:

$$\frac{dH(N-y)}{dy} > d\frac{E(H(\infty-y))}{dy} \tag{34}$$

For the second disequality, let us stress the analysis with respect to any point y greater than y^*.

$$H(N-y) = \frac{u(N-y)}{\rho-\alpha} + (u(N))^{1-\beta}\frac{1}{(\rho-\alpha)}\left[u^{\beta}(N) - u^{\beta}(N-y)\right] \tag{35}$$

$$E(H(\infty-y)) = \frac{u(\infty-y)}{\rho-\alpha} + (\underline{\mathrm{u}})^{1-\beta}\frac{1}{(\rho-\alpha)}\left[\frac{\int_{\infty-y}^{\infty}u^{\beta}(x)f(x)dx}{1-F(\infty-y)} - u^{\beta}(\infty-y)\right] \tag{36}$$

(36) is lower than (35) if and only if:

$$\frac{\int_{\infty-y}^{\infty}u^{\beta}(x)f(x)dx}{1-F(\infty-y)} < 0$$

that is trivial to demonstrate by using the neoclassical properties. This result can be specularly showed with respect to n, knowing that $n = N - y$ in the following Figure 3.

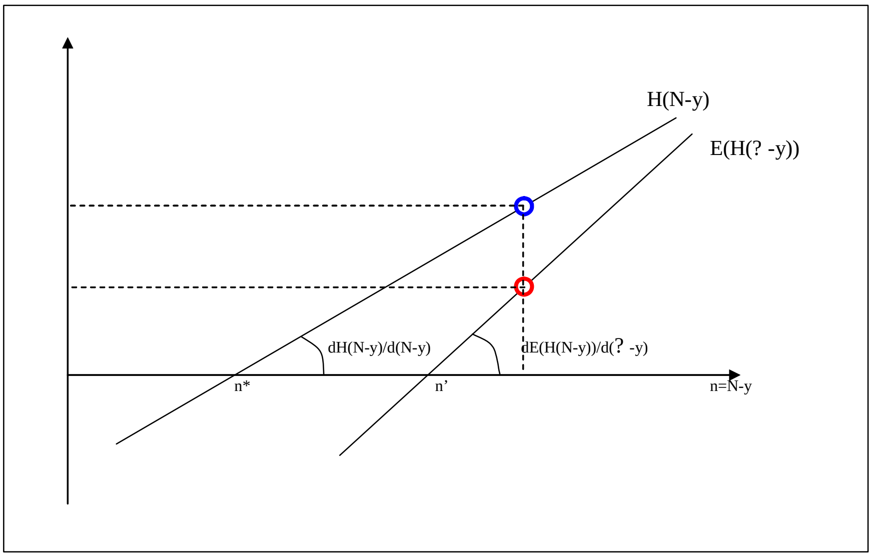

Figure 3

References

[1] Alvarez-Plata, P., H. Brücker and B. Siliverstovs, (2003), "Potential Migration from Central and Eastern Europe into the EU-15 – An Update", Report for the European Commission, DG Employment and Social Affairs, Brussels.

[2] J. Atsu Amegashie, (2004), "A political economy model of immigration quotas", *Journal Economics of Governance*, Issue Volume 5, pp. 255-267

[3] Anam, M., S. H. Chiang and L. Hua, (2004), "Uncertainty and International Migration: an Option cum Portfolio Model", mimeo

[4] Bartolini, L., (1993), "Competitive runs", *European Economic Review*, n. 37, 921-948

[5] Bartolini, L., (1995), "Foreign Investment Quotas and Rent Extraction Under Uncertainty", *Journal of International Economics*, n. 38, 25-49

[6] Byerlee, D., (1974) "Rural-Urban Migration in Africa: Theory, Policy, and Research Implications", *International Migration Review* 8, 543-566.

[7] Boeri T., and H. Brücker, (2005), "Migration, Co-ordination Failures and EU Enlargement", *Economic Policy, n. ,*

[8] Boeri T., and H. Brücker, (2005), "Migration, Co-ordination Failures and EU Enlargement", DIW Berlin Discussion Paper, 481

[9] Borjas, G., J., (1994), "The Economics of Immigration", *Journal of Economic Literature,* 32, pp. 1667-1717.

[10] Borjas, G., J., (1990), "Friends and Strangers", Basic Books

[11] Borjas, G., J., (1994), "The Economics of Immigration", *The Journal of Economic Literature*, 32(4), pp. 1667-1717

[12] Borjas, G., J., (2003), “The Labor Demand Curve is Downward Sloping: Reexamining the Impact of Immigration on the Labor Market”, *The Quarterly Journal of Economics,* 118 (4), pp. 1335-1374

[13] Bosello F., and Moretto M. (1999), “Dynamic Uncertainty and Global Warming Risk”, *FEEM Working Paper* n. 80.99

[14] Burda, M. C., (1995), “Migration and the Option Value of Waiting”, *The Economic and Social Review*, 27, pp. 1-19

[15] Dixit, A. and R. Pindyck, (1998), "Expandibility, Reversibility, and Optimal Capacity Choice", (????)

[16] Dixit, A. and R. S. Pindyck, (1993), “Investment under Uncertainty”, Princeton (NJ): Princeton University Press

[17] Dosi C. and M. Moretto, (2001), "Is Ecolabbeling a Reliable Environmental Policy Measure?", *Environmental and Resource Economics*, 19, 113-127

Dustmann, C., (2003), "Return Migration, Wage Differentials, and the Optimal Migration Duration", *European Economic Review, 47, 353-369.*

[18] Feist, H., (1998), "Wage-distance Regulation in Social-welfare Programs: an Option-theory Perspective", *Journal of Economics*, 68, pp. 271-293

[19] Friedberg, R. M., (2001), "The Impact of Mass Migration on the Israeli Labor Market", *The Quarterly Journal of Economics*, Vol. 116, No. 4, pp. 1373-1408

[20] Harrison J. M., (1985), "Brownian Motion and Stochastic Flow Systems", New York: John Wiley & Son

[21] Hill, K., J., (987), "Immigrant Decisions Concerning Duration of Stay and Migratory Frequency", *Journal of Development Economics* 25, 221-234

[22] King R., (1993) Mass Migration in Europe. The Legacy and the Future, Belhaven Press, London

[23] Khwaja, Y., (2002), "Should I Stay or Should I Go? Migration Under Uncertainty: A Real Option Approach", mimeo

[24] Jachimowicz, M., G., Demetrios and K. O. Papademetriou, (2004), "Observations on Regularization and the Labor Market Performance of Unauthorized and Regularized Immigrants", Migration Policy Institute, Paper Prepared for the European Commission, DG Employment and Social Affairs

[25] Levinson, A., (2005), "Why Countries Continue to Consider Regularization", Migration Information Source

[26] Moretto, M. and S., Vergalli, (2005), "Migration Dynamics", Feem Working Paper n. 108

[27] OECD, (1999), Trends in International Migration, SOPEMI, Paris.

OECD, (2001), Trends in International Migration, SOPEMI, Paris.

[28] OECD, (2004), Trends in International Migration, SOPEMI, Paris.

[29] Russo, G., (2006), "Barriers to Immigration and Cultural Homogeneity: Is Anything Wrong?", mimeo, presented at European Society of Population Economics, Università di Verona, 2006.

[30] Scheve, K., F., and Slaughter, M., J., (2001), "Labour Market Competition and Individual Preferences Over Immigration Policy", *Review of Economics and Statistics* 83, 133-145.

[31] Sjaastad, L. A., (1962), "The Costs and Returns of Human Migration", *The Journal of Political Economy*, 70(5), Part 2, pp. 80-93

[32] Sunderhaus, S., (2007), "Regularization Programs for Undocumented Migrants", *Migration Letters*, 4(1), pp. 65-76

[33] Thierry, X. and G. I., Rogers, (2004), "Recent Immigration Trends in France and Elements for a Comparison with the United Kingdom", *Population*, Vol. 59, n. 5, pp. 668-672

[34] Todaro, P. M., (1969), “A Model of Labour Migration and Urban Unemployment in Less Developed Countries”, *The American Economic Review*, 59 (1), pp.138-148

[35] Tsur Y., and A., Zemel (1994), “Endangered Species and Natural Resource Exploitation: Extinction vs. Coexistence”, *Natural Resource Modelling*, 8, 389-413

[36] Tsur Y., and A. Zemel (1998), “Pollution Control in an Uncertain Environment”, *Journal of Economic Dynamic and Control*, 22, 967-975

[37] Vergalli, S., (2006), "The Role of Community in Migration Dynamics", Feem Working Paper n. 04.2006

[38] Vergalli, S., (2007), "Entry and Exit Strategies in Migration Dynamics", University of Brescia Discussion Paper n. 0701