How to Reduce Unemployment: 
Notes on Stability and Dynamics*

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Abstract

In this paper I explore the out-of-equilibrium dynamic behaviour of the Farmer’s (2010d) ME-NA model. Specifically, assuming that the micro-economic adjustments are instantaneous, I build a dynamic model in continuous time that describes the macro-economic adjustments of the value of output and the interest rate. Within this framework, I show that the model economy has a unique stationary solution whose dynamics is locally stable. Moreover, simulating the model economy under the baseline calibration, I show that the adjustments towards the steady-state equilibrium occur through convergent oscillations while the most promising way out from a financial crisis combines a fiscal expansion with interventions aimed at easing the trade-off between holding risk and risk-free assets.

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1 Introduction

In a recent work, Farmer (2010d) develops an overlapping generation (OLG) model with a perpetual-youth demographic framework along the lines put forward by Yaari (1965) and Blanchard (1985) aiming at exploring the effectiveness of fiscal policies as possible ways out from the 2008 financial crisis. There he proposes a non-linear apparatus similar to an IS-LM system exploited to perform comparative statics on different policy scenarios. The major results achieved in this contribution is that fiscal policy can be an effective way to increase the value of output and reduce unemployment but there are also better alternatives that do not crowd out private consumption such as central bank interventions that directly influence asset markets by preventing sudden booms and consequent disastrous crashes.\(^1\)

The non-linear apparatus put forward by Farmer (2010d) has some intriguing and non-standard features. First, on the side of the market for goods, the IS schedule is derived under the assumption that the only autonomous component of aggregate demand is the public expenditure while private consumption (saving) depends positively (positively) on the value of output and negatively (positively) on the real interest rate.\(^2\) As a consequence, a demand-constrained equilibrium in the market for goods implies a (non-linear) decreasing relationship between the value of output and the real interest rate denominated as ME schedule.\(^3\) Second, the monetary sector of the model economy, i.e., the LM schedule, is replaced by a no-arbitrage condition such that all interest bearing assets must pay the same return. Under the competitive assumption that the marginal productivity of capital is proportional to the value of output, this condition implies an upward equilibrium relationship between the value of output and the real interest rate denominated as NA schedule. Finally, consistently with the choice of units made by Keynes (1936) in the *General Theory*, the nominal wage is used as the numeraire so that all the nominal variables are measured in money wage units.

In this paper, following the theoretical contributions by Chang and Smyth (1972) and Varian (1977), I explore the out-of-equilibrium dynamic behaviour of the Farmer’s (2010d) ME-NA model. Specifically, assuming that the micro-economic saddle-path adjustments

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\(^1\)Those works are parts of a broader project aimed at providing a new micro-foundation of the *General Theory* grounded on search and business cycle theories. See Farmer (2008a-b, 2010a), Guerrazzi (2010) and Gelain and Guerrazzi (2010).

\(^2\)The model abstracts from investment by assuming that there is a fixed amount of productive capital. See Farmer 2008b.

\(^3\)Non-linearity comes from the hyperbolic relationship between saving (or consumption) and the real interest rate.
implied by the OLG perpetual-youth framework are instantaneous, I build a dynamic model in continuous time that describes the macro-economic adjustments of the value of output and the real interest rate.\(^4\) Within this framework, I show that the model economy has a unique meaningful stationary solution whose dynamics is locally stable.

Moreover, I use the self-made macro-dynamic model to perform some numerical simulations. Specifically, I show that exploiting the baseline calibration suggested by Farmer (2010d), the adjustments to the short-run macro-economic equilibrium occur through convergent oscillations. Finally, I explore the effectiveness of a set of policy interventions that might be helpful in exiting from a financial crisis. My computational experiments suggest that fiscal policies can be an effective way to increase the value of output and reduce unemployment but there might be also smoothing companion interventions that provide for a lower crowding out of private consumption. To be precise, I show that those interventions rely on policies aimed at easing the trade-off between holding risky and risk-free financial assets.

The paper is arranged as follows. Section 2 develops the theoretical model. Section 3 illustrates the results of some numerical simulations. Finally, section 4 concludes.

## 2 The Model

In the Farmer’s (2010d) OLG perpetual-youth model there is a unit mass of identical firms that produce an homogenous-perishable good whose value in wage units is indicated by \(Z_t\). The production process of this good is described by a constant-returns-to-scale production Cobb-Douglas function in which the capital share is denoted with \(\alpha \in (0, 1)\). Each firm employs 1 unit of productive capital and a variable amount of labour hired on a competitive search market aiming at maximizing its profits.

Symmetrically with firms, in the Farmer’s (2010d) theoretical proposal there is a unit mass of identical households that discount the future at the constant rate \(\beta \in (0, 1)\) and have a constant probability \(\pi \in [0, 1]\) to survive in the subsequent period. The problem of each household is to choose an optimal sequence of consumption expenditure \(\{C_t\}_{t=0}^{+\infty}\) that maximizes an instantaneous logarithmic utility function under the dynamic constraint of a wealth accumulation path. In each period, the financial wealth of the economy is given by sum between the value of the public debt \((B_t)\), the price of a risky asset \((p_k,t)\) and the value of the marginal return on the unit of employed capital \((\alpha Z_t)\). Finally, labour

\(^4\)Interactions between micro- and macro-economic adjustments are a topic for future researches.
income is taxed at the proportional rate $\tau_t$ while $R_t$ is the interest factor.$^5$

Taking into account that each household consume a fixed fraction of its wealth and that wealth is independent of the household age allows to derive the following modified Euler equation:

$$\frac{C_{t+1}}{C_t} = \tilde{\beta} R_t \left( \frac{C_t - \tilde{\alpha}(Z_t (1 - \tau_t (1 - \alpha)) + p_{k,t} + B_t)}{C_t} \right)$$

(1)

where $\tilde{\alpha} \equiv \frac{(1-\beta\pi)(1-\pi)}{1-\pi(1-\beta\pi)}$ and $\tilde{\beta} \equiv \frac{1-\pi(1-\beta\pi)}{\pi}$.

It is worth noting that when the representative household is assumed to live forever, i.e., $\pi = 1$, so that $\tilde{\alpha} = 0$ and $\tilde{\beta} = \beta$, the expression in (1) collapses to a standard Euler equation in which the growth rate of consumption is given by the interest factor corrected by the household subjective discount factor.

The dynamics of the value of public debt is described by the following first-order difference equation:

$$B_{t+1} = R_t (B_t + G_t - \tau_t (1 - \alpha) Z_t)$$

(2)

where $G_t$ is the value of public expenditure.

Substituting (2) in (1) and assuming that that the micro-economic saddle-path adjustments of the household problem have achieved a stable solution allow to derive the following aggregate consumption function:

$$C = \frac{\tilde{\alpha}\tilde{\beta}R}{R\tilde{\beta} - 1} \left( p_k + \frac{B}{R} + Z - G \right)$$

(3)

Taking into account that public expenditure is the only autonomous component of aggregate demand and assuming that the value of income changes at a rate proportional to the excess demand in the goods market, the national account identity suggest that for any given level of $R$ the aggregate dynamics of $Z$ can be described by the following differential equation:

$$\dot{Z} = \gamma \left( G - \left( Z - \frac{\tilde{\alpha}\tilde{\beta}R}{\beta R - 1} \left( p_k + \frac{B}{R} + Z - G \right) \right) \right)$$

(4)

where $\gamma$ is a positive constant that conveys the speed of adjustment of the market for goods.

$^5$Obviously, the interest factor is 1 plus the real interest rate.
The differential equation in (4) suggests that the value of output increases (decreases) whenever the value of saving is lower (higher) than the value of public expenditure.\(^6\)

In the Farmer (2010d) theoretical proposal the monetary sector of the model economy is replaced by a no-arbitrage condition such that all interest bearing assets must pay the same return. Specifically, there are two financial assets in the economy. The first is a risk-free asset issued on government debt that in each period yields with certainty the interest factor \(R_t\). By contrast, the second is a risky asset whose yield is given by the sum between its market price \(p_{k,t}\) augmented with the marginal return on the unit of employed capital.

The assumption of no risk-less arbitrage opportunities implies that

\[
\frac{p_{k,t+1} + \alpha Z_{t+1}}{p_{k,t}} = R_t
\]  

(5)

Under the assumption that at a micro-economic level \(p_k, Z\) and \(R\) have all achieved a stable solution, (5) implies that for any given level of \(Z\) the aggregate dynamics of \(R\) can be described by the following differential equation:

\[
\dot{R} = \delta (\alpha Z - p_k (R - 1))
\]  

(6)

where \(\delta\) is a positive constant that conveys the speed of effectiveness of the no-arbitrage condition.

The differential equation in (6) implies that the interest factor increases (decreases) whenever the marginal return on the unit of employed capital is higher (lower) than the risk-free yield on the amount of resources invested in the risky-asset.\(^7\)

### 2.1 Steady-State

Now I look for a meaningful pair \((Z^*, R^*)\) such that \(\dot{Z} = \dot{R} = 0\). By ‘meaningful’ I mean a pair \((Z^*, R^*) \in \mathbb{R}^2_{++}\), i.e., a pair characterized by a positive value of output in wage units and a positive real interest rate.\(^8\)

On the one hand, taking the result in (4) into account, \(\dot{Z} = 0\) implies that

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\(^6\)When households live forever public expenditure completely crowds out private expenditure. As a consequence, the value of output adjusts towards the corresponding level of public expenditure.

\(^7\)Assuming a constant equity premium would not change the qualitative dynamics of \(R\). The implications of counter-cyclical equity premia are discussed in Appendix.

\(^8\)The model does not contemplate the possibility of a negative equilibrium real interest rate.
\[ \hat{Z} = G + H \left( \hat{R} \right) \left( p_k + \frac{B}{\hat{R}} \right) \]  

(7)

where \( H \left( \hat{R} \right) \equiv \frac{\hat{\alpha}\hat{\beta}\hat{R}}{R\hat{\beta}(1-\hat{\alpha})-1} \).

The expression in (7) is the downward-sloped ME curve and provides all the pairs \( \left( \hat{Z}, \hat{R} \right) \) such that the market for goods is in equilibrium.

On the other hand, taking the result in (6) into account, \( \hat{R} = 0 \) implies that

\[ \overline{Z} = \frac{p_k (\overline{R} - 1)}{\alpha} \]  

(8)

The expression in (8) is the upward-sloped NA curve and provides all the pairs \( \left( \overline{Z}, \overline{R} \right) \) such that there are no risk-less arbitrage opportunities.

To facilitate the analysis of the stationary solution, I define the following constants:

\[ a \equiv \hat{\beta} \left( 1 - \hat{\alpha} \right) p_k \]
\[ b \equiv \hat{\beta} \left( 1 - \hat{\alpha} \right) \left( p_k + \alpha G \right) + p_k \left( 1 + \alpha \hat{\alpha} \hat{\beta} \right) \]
\[ c \equiv p_k + \alpha \left( G - \hat{\alpha} \hat{\beta} B \right) \]  

(9)

Taking into account of the definitions in (9), a positive stationary interest factor is given by

\[ R^* = \frac{b + \sqrt{b^2 - 4ac}}{2a} \]  

(10)

Obviously, once \( R^* \) is determined, the corresponding value of \( Z^* \) can be found by substituting (10) alternatively in (7) or in (8). It is worth noting that in addition to all the model parameters, the steady-state solution \( (Z^*, R^*) \) also depends on the price of the risky asset, on the value of public debt and on the value of public expenditure.\(^9\) Such a steady-state solution is illustrated in Figure 1.

The diagram in Figure 1 allows to explore the equilibrium effects of fiscal policies and variations in the value of financial wealth. On the one hand, a fiscal expansion (restriction), i.e., an increase (decrease) of \( G \), leads the ME curve to shift outward (inward). As a consequence, the new steady-state solution will be characterized by a higher (lower) value of \( Z^* \) and a higher (lower) value of \( R^* \). On the other hand, a financial boom (crash), i.e., an increase (decrease) of \( p_k \), leads to a movement in both curves. Specifically, the

\(^9\)For each value of \( G \) and \( B \), the stationary solution of the difference equation in (2) allows to derive the tax rate on labour income that sustains \( (Z^*, R^*) \) as a steady-state equilibrium.
ME curve shifts outward (inward) while the NA curve rotates in a clockwise (counterclockwise) direction. As a consequence, the new steady-state solution will be characterized by a higher (lower) value of $Z^*$ while the effect on $R^*$ is ambiguous.

![Diagram](image)

**Figure 1:** Steady-state

### 2.2 Local Dynamics

The first-order Taylor expansion of the non-linear dynamic system that describes the macro-economic adjustments of the model economy is given by

$$
\begin{pmatrix}
\dot{Z} \\
\dot{R}
\end{pmatrix} =
\begin{bmatrix}
\gamma \left( \frac{R^* \tilde{\beta}(\tilde{\alpha} - 1) + 1}{R^* \tilde{\beta} - 1} \right) & -\gamma \frac{\tilde{\alpha} \tilde{\beta}(p_k + Z^* - G + B \tilde{\beta})}{(R^* \tilde{\beta} - 1)^2} \\
\frac{\alpha}{p_k} & -\delta p_k
\end{bmatrix}
\begin{pmatrix}
Z - Z^* \\
R - R^*
\end{pmatrix}
$$

(11)

On the one hand, the trace of the Jacobian matrix in (11) is the following:

$$
- \left( \gamma \left( \frac{R^* \tilde{\beta}(\tilde{\alpha} - 1) + 1}{1 - R^* \tilde{\beta}} \right) + \delta p_k \right) < 0
$$

(12)

On the other hand, its determinant is given by

$$
\gamma \delta \left( \frac{R^* \tilde{\beta}(\tilde{\alpha} - 1) + 1}{1 - R^* \tilde{\beta}} \right) p_k + \frac{\alpha \tilde{\alpha} \tilde{\beta} (p_k + Z - G + B \tilde{\beta})}{(R^* \tilde{\beta} - 1)^2} > 0
$$

(13)
Since the trace is negative while the determinant is positive, it is possible to conclude that the dynamic system has two negative roots so that the stationary solution is a sink. As a consequence, \((Z^*, R^*)\) is locally stable.

3 Numerical Simulations

In this section I report the results of some numerical simulations of the dynamic model in (11). Specifically, using the baseline calibration suggested by Farmer (2010d) I explore the macro-economic adjustment towards the social optimal allocation that in the model economy fulfils the role of full employment (e.g. Farmer 2010a-b). Thereafter, I consider the effectiveness of a set of policy interventions such as fiscal expansions and financial market regulations aimed at exiting from a financial crisis.

3.1 Calibration

The baseline calibration of the model economy suggested by Farmer (2010d) is illustrated in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>capital share</td>
<td>0.34</td>
</tr>
<tr>
<td>( \beta )</td>
<td>discount rate</td>
<td>0.97</td>
</tr>
<tr>
<td>( \pi )</td>
<td>surviving probability</td>
<td>0.98</td>
</tr>
</tbody>
</table>

Table 1: Baseline calibration

The value of \( \alpha \) implies a labour share of two-third, i.e., a figure consistent with a century of US data and widely exploited in real business cycle literature (e.g. Gordon 1988 and Kydland and Prescott 1982). Moreover, the value of \( \beta \) is chosen to match an average annual real interest rate of 4%. Finally, the value of \( \pi \) implies an expected average life-span for households of 50 years, i.e., a figure that lines up with the average life-span of an American citizen computed with the population distribution in 2008.

In addition to the parameter values collected in Table 1, I complete the calibration of the dynamic model by setting \( \gamma = 1 \) and \( \delta = 0.5 \). Those parameter values do not alter the steady-state solution. However, they convey the idea that the equilibrium in the market for goods is achieved faster than the no-arbitrage condition in (5).

10The MATLAB 6.5 code is available from the author.

11Think to the Mundell-Fleming version of the IS-LM model. After a policy shock, the equality between the home and the foreign interest rate (which is a genuine no-arbitrage condition) happens at the end of
3.2 Results

I begin my computational experiments by exploring the macro-economic adjustments towards the social-optimal allocation. Specifically, Farmer (2010d) defines the social-optimal allocation by setting $B = G = 0$ and $p_k = 12.2$. Those figures, combined with the parameter values in Table 1, implies that $Z^* = 1.44$ while $R^* = 1.04$. The macro-economic adjustments towards the social-optimal allocation are illustrated in the two panels of Figure 2.

![Figure 2: Macro-economic adjustments towards the social-optimal allocation](image)

$B = G = 0$, $p_k = 12.2$, $Z(0) = 1.43$, $R(0) = 1.03$, $Z^* = 1.44$, $R^* = 1.04$

The two diagrams in Figure 2 suggest that the adjustments to the short-run macro-economic equilibrium occur through convergent oscillations. Obviously, this is due to the fact that under the baseline calibration in Table 1 the two negative roots of the dynamic system in (11) are complex-conjugate.

Now I explore the effects of a fiscal expansion aimed at restoring the social-optimal level of $Z$ in the afterwards of a financial crash driven by a 20% drop in $p_k$. Taking into account a government debt equal to zero, the drop in $p_k$ leads to a corresponding 20% fall in the value of output but leaves the real interest rate unaltered. According to the baseline calibration in Table 1, to restore the social-optimal level of $Z$ the government has to increase the level of $G$ from zero to 0.88. This permanent increase in government purchases can be financed by raising the tax rate on labour income without affecting the value of public debt. The effects of such a fiscal expansion are illustrated in the two panels of Figure 3.

all the adjustments.

$^{12}$Taking into account that Farmer (2010d) finds a social-optimal (un)employment rate equal to 5%, the social-optimal value of output is equal to $(1 - 0.05)(1 - \alpha)^{-1}$. 

9
The two diagrams in Figure 3 show that the suggested fiscal stimulus will restore the social-optimal value of output in about 4 units of computational time after having followed a transitional path characterized by a strong deflation. In addition, the new stationary solution is characterized by a permanent increase of the real interest rate of 1%. In comparison with the social-optimal allocation described in Figure 2, this higher level of the real interest rate will crowd out a fraction of private consumption.

I close my computational experiments by exploring the consequences of a milder fiscal expansion carried out in combination with the introduction of a positive tax rate $T \in (0, 1)$ on the return of the risky financial asset. Specifically, I consider a fiscal stimulus equal to 50% of the government increase considered above joined with a 18% tax rate on the return of the risky asset. The effects of such a policy package are illustrated in Figure 4.

The two diagrams in Figure 4 show that the suggested policy package is able to restore the social-optimal value of output with a smoother transition path with respect to the pure fiscal expansion described in Figure 3. Moreover, being characterized by the same level of the real interest rate of the social-optimal allocation described in Figure 2, this policy package will not crowd out private consumption. As a consequence, it is quite likely that the combination of those public interventions will be strictly preferred to a pure fiscal expansion.

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13 In this case the NA curve becomes $\mathcal{Z} = p_k (R - 1) (\alpha (1 - T))^{-1}$ where $(\mathcal{Z}, R)$ is the set of pairs such that $\dot{R} = 0$.

14 This result is quite robust. Indeed, it also survives by exploiting the book and the working paper parameterization of the model economy. See Farmer (2010a, Chapter 7) and Farmer (2010c).
Figure 4: Effects of a fiscal expansion plus taxation of the risky asset $B = 0$, $G = 0.44$, $T = 0.18$, $p_k = 9.76$, $Z(0) = 1.152$, $R(0) = 1.04$, $Z^* = 1.44$, $R^* = 1.04$

4 Concluding Remarks

In this paper, following the theoretical contributions by Chang and Smyth (1972) and Varian (1977), I explore the out-of-equilibrium dynamic behaviour of the Farmer’s (2010d) ME-NA model. Specifically, assuming that the micro-economic saddle-path adjustments are instantaneous, I build a dynamic model in continuous time that describes the macro-economic adjustments of the value of output and the real interest rate. Within this framework, I show that the model economy has a unique meaningful stationary solution whose dynamics is locally stable.

Moreover, I use the macro-dynamic model to perform some numerical simulations by exploiting the baseline calibration suggested by Farmer (2010d). Those computational experiments suggest that the adjustments to the short-run macro-economic equilibrium occurs through convergent oscillations. Moreover, the scrutiny of different policies to exiting from a financial crisis suggests that fiscal policies can be an effective way to increase the value of output and reduce unemployment but there might be also smoothing companion interventions that provide for a lower crowding out of private consumption. Specifically, I show that those interventions rely on policies aimed at easing the trade-off between holding risky and risk-free financial assets.

5 Appendix: Counter-Cyclical Equity Premia

The theoretical analysis developed in section 2 does not explicitly consider the possibility of an equity premium. Under the assumption of a positive differential between the return
of risky and risk-free assets, the differential equation for the interest factor becomes the following:

\[ \dot{R} = \delta (\alpha Z (1 + \theta) - p_k (1 - R)) \quad \theta > 0 \]  \hspace{1cm} (A.1)

where \( \theta \) is the equity premium.

Theory and circumstantial evidence seem to suggest that risk compensations are counter-cyclical (e.g. Bansal 2008). A linear counter-cyclical equity premium can be conveyed as

\[ \theta (Z) = \rho - Z \frac{\rho}{Z_{\text{max}}} \quad \rho > 0 \]  \hspace{1cm} (A.2)

where \( \rho \) is the upper bound between the yields of risky and risk-free assets while \( Z_{\text{max}} \equiv \frac{1}{1 - \alpha} \).

Substituting (A.2) in (A.1) leads to the following \( \cap \)-shaped NA curve:

\[ \overline{R} = 1 + \frac{\alpha (1 + \rho)}{p_k} Z - \frac{\alpha (1 - \alpha)}{p_k} \rho Z^2 \]  \hspace{1cm} (A.3)

where \((Z, R)\) is the set of pairs such that \( \dot{R} = 0 \).

**Figure A.1:** Multiple equilibria

The expression in (A.2) suggests that in the shut-down allocation the equity premium is at its maximum level. Thereafter, it linearly decreases vanishing in the full employment allocation.
A non-linear NA curve such as the one in (A.3) allows for the possibility of multiple stationary solutions. An example is illustrated in Figure A.1.

The diagram in Figure A.1 shows a situation in which there are two distinct stationary solutions. Specifically, there is a stable (unstable) stationary solution \((Z_1^*, R_1^*)\) \(((Z_2^*, R_2^*))\) characterized by a low (high) value of output and a high (low) level of the real interest rate. This isomorphism with the Diamond (1982) model reminds the need of coordination among public interventions already stressed with the simulation results in section 3.

References


