Educational Mismatch in the Labor Market. A Theory and an Investigation using Unemployment Spells' Duration

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Abstract

This work proposes an investigation of educational mismatch focusing on the study of individuals' unemployment spells. We present evidence for Italy, showing that overeducation is basically an occurrence that follows long periods of unemployment. Using duration models we show that hazard rates of graduates are higher than those of undergraduates only for transitions towards occupations that require the competencies provided by the universities. This process is strictly related to individuals' innate ability. We build up a matching model coupled with endogenous educational and technological choices and we consider the role of university *selectivity* and individual innate ability in determining unemployment duration. We show that a policy that gives more relevance to innate ability in the schooling attainment may boost the creation of graduate-complementary job positions reducing educational mismatch.

Jel classification: J24, J64, I23.

Key Words: Overeducation, Unemployment Spells, Hazard Functions, Matching Models, University Selectivity.

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1 Introduction

This work proposes an investigation of educational mismatch focusing on the study of individuals' unemployment spells. In the economic literature, educational mismatch, in the form of overeducation, describes the extent to which individuals possess a level of education in excess of that required in their specific job. The phenomenon significantly affects graduates workers. In particular, in the UK and in the United States educational mismatch seems to involve a number of workers that ranges between the 17% and the 42% of the whole employed graduate labor force, while in Italy the share of overeducated workers is around 39% (McGuinness, 2006). The understanding of the possible mechanisms at the root of this phenomenon is important, especially for economies characterized by public-funded higher education systems, since resources might be wasted on non productive investments. In this view, the goal of our paper is twofold. Firstly, we present some evidence showing that overeducation is basically an occurrence that follows long unemployment spells. By using data of Italian workers collected in 2006, we show that the hazard rate into employment is higher for graduates than for undergraduates only for transitions towards occupations that require the competencies provided by university attendance. By using competing risk models, we show that the observed individuals' matching differences are due to heterogeneity in terms of their innate ability, family background, and geographical location. Secondly, we build up a general equilibrium model able to fit these empirical findings. Using a two-sectors urn-ball matching model coupled with firms' and individuals' sectorial choice, we show that the occurrence of overeducation may be associated either to matching-frictional problem or inefficient self-selection into education. However, only in the latter case we should observe a positive correlation between individual's innate ability and the probability of exiting towards a right match.

The interest on the overeducation phenomenon has been, and it is, very high among labor economists. From an empirical perspective, almost all the existing works concentrate their efforts in assessing the relevance of the phenomenon in determining wages broadening the human capital framework (among others, Bauer, 2000; Chevalier, 2003; McGuinness and Bennett, 2007; Woodcook, 2008). In terms of macro-analysis, Manacorda and Petrongolo (1999) and Nickell and Bell (1995) investigate the effect of educational mismatch in shaping the dynamics of skilled and unskilled unemployment series within developed countries. To the best of our knowledge, there are no works trying to analyze labor market educational mismatch in terms of unemployment spells. In this context, by pointing out the peculiar unemployment history of workers that end up in positions that do not require their skills, our empirical analysis brings important new insights into the debate concerning the most common explanations for this phenomenon i.e., i) a natural end of long periods of unemployment *versus* ii) a temporary outcome due to frictional unemployment in the graduates' labor market (Sicherman, 1991). From a theoretical point of view, a few papers investigate the occurrence of educational mismatch leading to various explanations of the existing empirical evidence. Andolfatto and Smith (2001) present a model where educational mismatch arises during steady-state transitions due to technological change. The authors argue that "deadwoods" educated workers re-allocate within the unskilled sector while young educated workers enter the skilled sector directly. While their results explain the occurrence of overeducation, they consider overeducated individuals as old workers with a well-matched occupational history, which appears to be at odds with the results of many empirical studies.[1] Moreover the authors assume firms technological decisions are not affected by individuals' educational choices. However, it should be recognized that in order to determine the extent of the phenomenon, the strategic complementarity between educational and technological choices might be relevant. Charlot and Decreuse (2005), Charlot *et al.* (2005), and Moen (1999) go in this direction by presenting models where overeducation arises since self-selection into education is inefficient. Although these works differ one another in many respects, they share some common features. Firstly, they refer to overeducation as a phenomenon that arises when the share of graduates overruns its optimal level. In particular, they discuss the presence of overeducation and the optimal policy to reduce the phenomenon by assuming that once the educational choice has been made, graduates' and undergraduates' labor markets are perfectly segmented (which excludes *ex-ante* the possibility of mismatch) and this appears to be strongly counterfactual. Secondly, in these works the cost of education is assumed to be identical across individuals even when these are heterogeneous in terms of their innate talent. This assumption not only appears in contrast with the well established relation between innate ability and schooling outcomes (Cameron and Heckman, 1998) but, most importantly, it crucially calls for a rise in tuition fees in order to re-establish the social optimum. We enter this literature by presenting a theoretical framework where: i) Education gives rise, besides monetary costs, to costs related to individual innate talent; ii) there is strategic complementarity between individuals' educational choice and firms' technological decision; iii) graduates may be employed in the undergraduate sector. In this setting, we show that an increase in the share of graduates may induce a rise in the technological endowment of firms via a *tightness effect*: The larger the pool of graduates the greater the probability that a firm fills graduate-complementary vacancies. However, this relation is concave since the presence of a too large share of graduates in the labor force implies a *composition effect*: the larger the share of graduates the lower their expected ability.

In terms of policy, our model offers a different perspective with respect to existing studies relying on inefficient self-selection into education as the main cause at the root of educational mismatch/overeducation. Many papers enter the debate concerning the relevance of considering students heterogeneity in terms of ability when evaluating the effect of policies that promote college attendance, suggesting that overeducation may be reduced *only* by raising tuition fees (Charlot and Decreuse, 2005; Hendel *et al.*, 2006). However, this result is based on the assumption that individuals do not have liquidity constraints and, as a consequence, a rise in tuition fees would not affect individuals located at the top of the ability distribution because of their higher returns to education. Indeed, we believe that public education should be evaluated in the light of the fact that liquidity constraints do exist. As far as there is no *ex-ante* correlation between households' wealth and individuals' innate talent, a rise in tuition fees may hit individuals with potentially higher returns to education too. We believe that a useful policy instrument may reside in setting the appropriate level of

[1] Among others, Sicherman (1991) finds that in the US overeducated workers are younger with an up-ward occupational mobility.

selectivity of the higher education system since it shapes the correlation between educational choices and individual ability and it might eventually boost the creation of graduate-complementary job positions reducing educational mismatch and rising the overall expected output.

The outline of the article is as follows. Section 2 contains the descriptive statistics and our empirical investigation. In Section 3 we set up the theoretical model, while in Section 4 we discuss our policy implications. Some concluding remarks are presented in Section 5.

2 The Empirical Investigation

2.1 Data and Duration Patterns

As part of the motivation of our study, and in order to provide some facts describing the pattern of unemployment duration of individuals with different characteristics, we present some empirical evidence using data from a survey carried out by the Italian Institute for Vocational Training of Workers (ISFOL) containing information on the labor market outcomes of a sample of 8156 workers recorded in 2006.[2] This survey provides information on workers' status (employed/unemployed) and on the length of their unemployment spells. The data set records the time needed to obtain the present job (in months) or the censored time for those still unemployed at the time of the interview. Only uninterrupted spells of unemployment are considered. Moreover, our sample consists only of individuals that start working between 2005 and 2006. This data set provides indications to determine if workers are in job positions where the competencies acquired at school are effectively needed and allows us to evaluate the extent of educational mismatch and the unemployment spell duration associated to the characteristics of the job match. Table 1 and Table 2 contain some representative statistics of our sample, while in Table 3 we define our variables. In particular, in Table 2 we describe the characteristics of individuals with different job match qualities. It is important to make clear that our measure of educational mismatch is a subjective one since we consider in a wrong match individuals who affirm that their degree is not a necessary requirement for their job. There exists a substantive literature comparing the outcomes deriving from subjective and objective measures of overeducation (obtained by technical evaluation by professional job analysts of job positions). However, there is no consistent evidence that these different approaches give rise to systematic and significant bias of the incidence or wage effects of overeducation (McGuinness, 2006).

Some preliminary aspects of the duration pattern of unemployed spells may be gathered by inspecting the Kaplan-Meier estimated hazard functions. These functions reflect the percentage of spells ending into employment during time. The general pattern of the hazard functions is non-linear with an increasing exit rate at the beginning of the spell which declines with the elapsed time into unemployment. Figure 1 depicts the empirical hazard functions for individuals with different education levels. The hazard function of graduates lies above that of their less educated counterpart. In general, this reflects a faster transition out of unemployment for more educated people. The same exercise has been

[2] For a detailed description of the survey see Giammatteo (2009).

Table 1: Frequency and Average of variables in the sample. Employed and Unemployed, 2006.

	Employed		Unemployed	
	Frequency	Average	Frequency	Average
Observations	2493	50.3	2471	49.7
Unemployment Spells Duration	2493	9.8	2471	16.2
Female	915	36.7	845	34.2
Age	2493	30.4	2471	32.7
Married	774	31.0	944	38.2
South	658	26.4	970	39.25
Unemployment Benefits	-	-	174	7.0
Father education	203	8.1	117	4.7
Education: High-school	1271	51.0	1249	50.5
Education: Graduate	671	26.9	480	19.4
Secondary school leaving grade	1892	73.6	1760	71.2
High school leaving grade	197	15.5	140	11.2
University leaving grade	211	31.4	144	30.0
Degree on time	249	37.1	142	29.6
Overeducation	1403	56.2	-	-

Note: Unemployment duration is in months. The averages are sample averages. For final marks (secondary, high school, and university) averages are with respect to the number of individuals in the group. Variables' description presented in Table 3.

Table 2: Frequency and Average of variables in the sample. Employed, 2006.

	Employed Right-Match		Employed Wrong-Match	
	Frequency	Average	Frequency	Average
Observations	1090	43.7	1403	56.2
Unemployment Spells Duration	1090	9.29	1403	10.25
Female	413	37.9	502	35.8
Age	1090	29.3	1403	31.2
Married	269	24.7	505	36.0
South	302	27.7	356	25.3
Father education	120	11.1	83	5.9
Education: High-school	540	49.5	731	52.1
Education: Graduate	468	43.0	203	14.5
Secondary school leaving grade	1090	9.3	1403	10.25
High school leaving grade	102	18.9	95	13.0
University leaving grade	149	31.8	62	30.5
Degree on time	176	37.6	73	36.0

Note: Unemployment duration is in months. The averages are sample averages. For final marks (secondary, high school, and university) averages are with respect to the number of individuals in the group. Variables' description presented in Table 3.

Table 3: Description of Variables

	Description
Employed	Variable indicating if the respondent is employed at the time of the interview.
Unemployed	Variable indicating if the respondent is unemployed at the time of the interview.
Unemployment Spells	Variable indicating the length of unemployment spell to find the present job if the respondent is employed or the length of unemployment spell since starting the job search process if the respondent is unemployed at the time of the interview. Duration is measured in months.
Female	Dummy variable indicating the respondent's sex, Female=1, 0 otherwise.
Age	Respondent's age at the interview.
Married	Dummy variable indicating if the respondent is married, Married=1, 0 otherwise.
South	Dummy variable indicating if the respondent is resident in the South of Italy, South=1, 0 otherwise.
Unemployment Benefits	Variable indicating if the respondent received unemployment benefits during its unemployment spell. Information includes the number of months of benefits duration.
Father education	Dummy variable indicating if the respondent's father is a graduate. Father education=1 if the respondent's father is a graduate, 0 otherwise.
Education: High-school	Dummy variable indicating if the respondent owns a high-school degree. High-School=1, 0 otherwise.
Education: Graduate	Dummy variable indicating if the respondent is a graduate. Graduate=1, 0 otherwise.
Degree subject	A vector of 4 0-1 dummy variables indicating degree subjects: 1) Science=1 if mathematics, science, chemistry, geo-biology, engineering agrarian; 2) Medicine=1 if medicine; 3) Econ.&Law=1 if political science, economics, statistics, law; 4) Humanities=1 if humanities, linguistic, teaching, psychology.
Secondary school leaving grade	Dummy variable for final score at secondary school SS Score=1 if secondary school final score is medium-high; SS Score=0 otherwise.
High school leaving grade	Dummy variable for final score at high school HS Score=1 if high school final score>55/60 or high school final score>90/100; HS Score=0 otherwise.
University leaving grade	Dummy variable for final score at university. University Score=1 if university score$\geq$110/110; University Score=0 otherwise.
Degree on time	Dummy variable indicating if the degree is completed on time (adjusted for course duration), Degree on time=1, 0 otherwise.
Overeducation	Dummy variable for the answer to the question: "Is your degree a required qualification for your job?", Overeducation=1 if the answer is not, 0 otherwise.

done for individuals living in areas with a different level of economic development i.e., the less developed South of Italy in comparison with the rest of the country (Figure 2). In this case, we observe that the hazard function is lower for people located in the South area. Interestingly, if we combine these two bits of information we realize that a graduate from the South of Italy has a higher probability of finding a job than an undergraduate from the North only after about two years of unemployment spells duration (Figure 3). Further, we compare people with different education levels in terms of their unemployment spell duration to obtain a job where their competencies are effectively used. In Figure 4 we show the hazard function of individuals that report transitions toward occupations congruent with their education level. In this case, the hazard rate of graduates lies far above that of undergraduates. In contrast, Figure 5 refers to individuals who terminate their unemployment spells in a wrong match. These figures highlight that although unemployment duration is higher for individuals that exit toward bad occupations, differences between individuals with different education levels are not too pronounced and surprisingly the curve for graduates lies below that of undergraduates. This would imply that when graduates are overeducated they have a spell length higher than that of their undergraduate counterpart. This aspect is even more evident in the South of Italy, as reported in Figure 6 and Figure 7. Although informative, this preliminary analysis based on the empirical estimates of the hazard functions does not control for possible factors at work in shaping duration dependence. In particular, the observed differences between individuals with right and wrong matches might be due to differences in their characteristics in terms of skills, innate ability or family background. These variables may affect the patterns of hazard probabilities and the duration dependence that we see in the data. In order to take these differences into account we evaluate the transitions out of unemployment by estimating proportional hazard competing risk models as discussed in the next paragraph.

2.2 Empirical Strategy

We undertake the empirical analysis using a competing risks model (CRM) where we assume that each subject has an underlying failure time that may be of m different types given by the set $j = \{1, .., m\}$. We assume a competing risks formulation in which independent competing risks determine the duration of unemployment. In our case we suppose that unemployment may terminate by exiting toward employment with a right or a wrong educational match. So in our case $m = 2$. In a CRM with m types of failure there are $m+1$ states $\{0, 1, .., m\}$, where 0 represents the initial state and $\{1, .., m\}$ are possible destination states. We may assume that there exist latent variables $(t_1, t_2, .., t_m)$ which correspond to the spell duration for each possible failure. Destination-specific covariates are denoted by x_j (with $j = 1, 2, .., m$). Since we observe only the shortest duration and the others are censored, the joint survivor function S may be expressed as:

$$\begin{aligned} S_\tau &= \Pr[\tau > t] = \Pr[t_1 > t, ..., t_j > t] \qquad (1) \\ \tau &= \min_j (t_j), \quad t_j > 0 \end{aligned}$$

where τ denotes the spell duration.

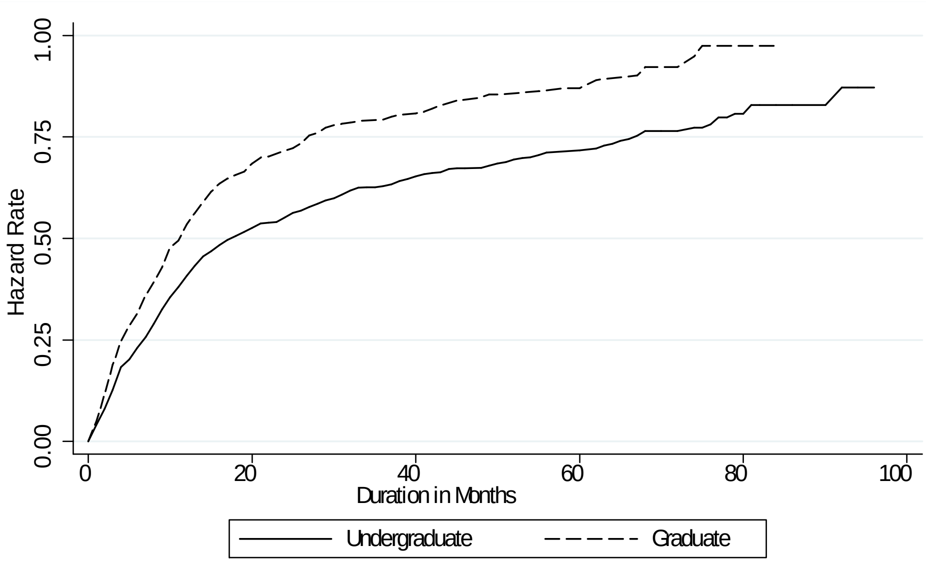

Figure 1: Kaplan-Meier failure estimates by Education.

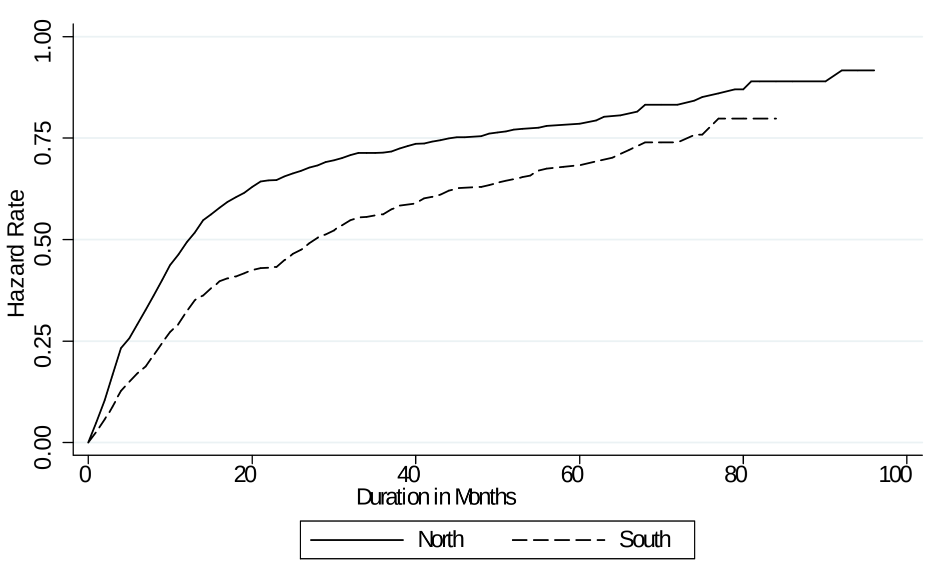

Figure 2: Kaplan-Meier failure estimates by Region.

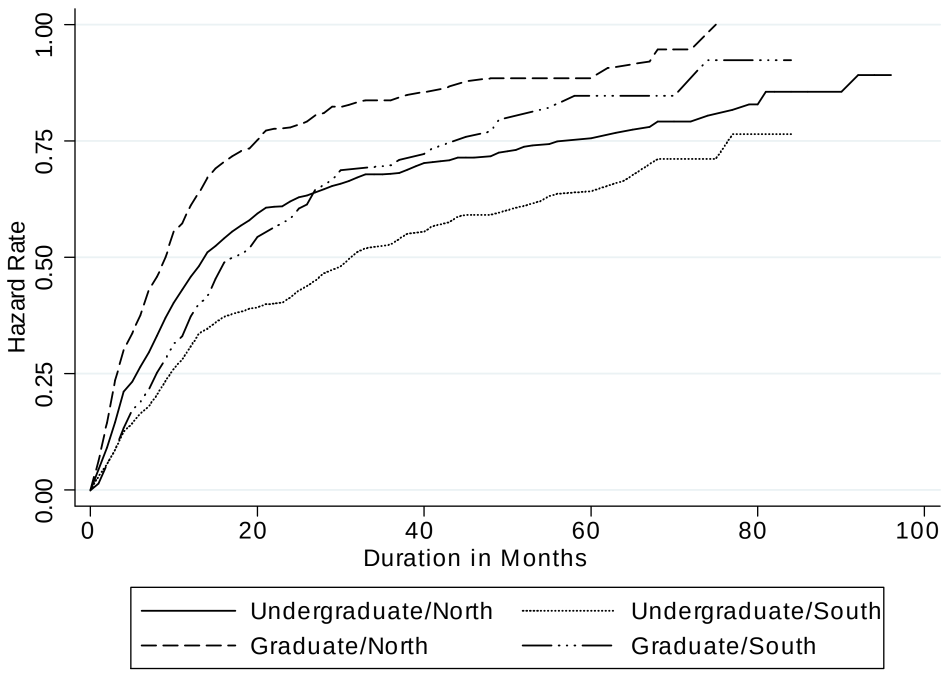

Figure 3: Kaplan-Meier failure estimates by Education and Region.

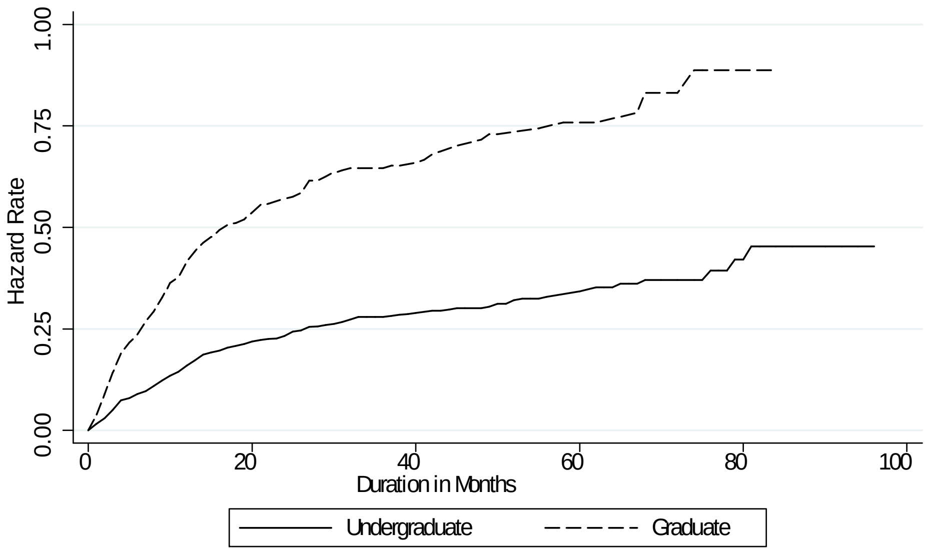

Figure 4: Kaplan-Meier failure estimates. Failure: Right match.

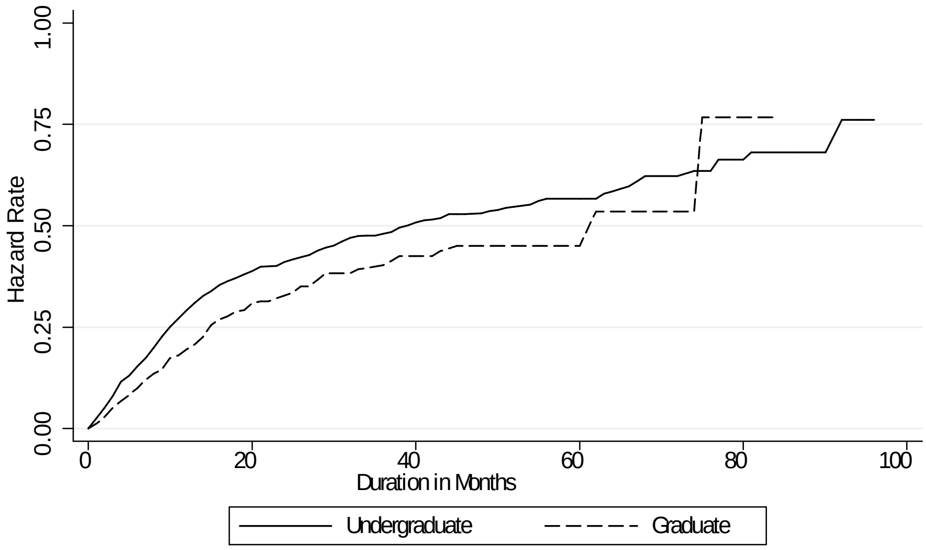

Figure 5: Kaplan-Meier failure estimates. Failure: Wrong match.

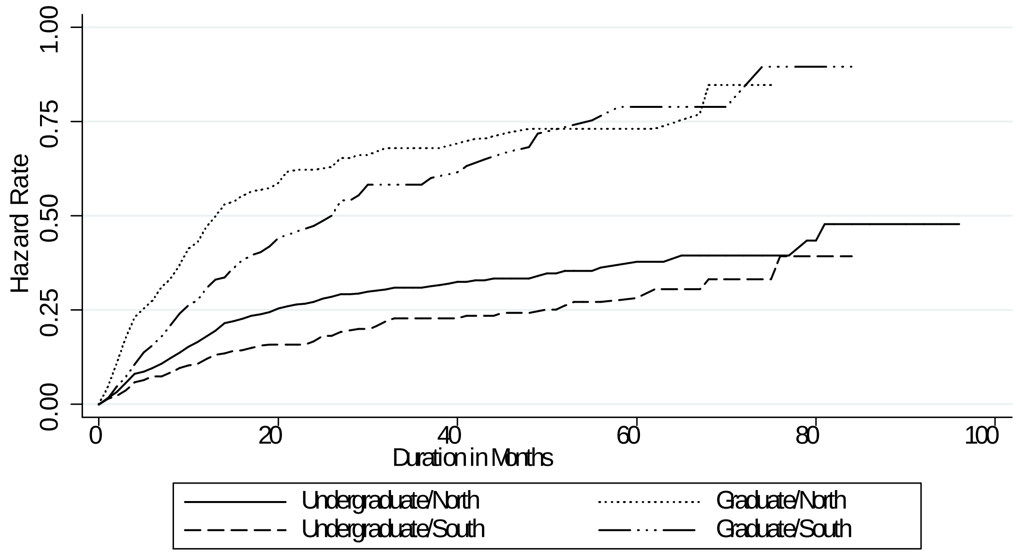

Figure 6: Kaplan-Meier failure estimates by Education and Region. Failure: Right Match.

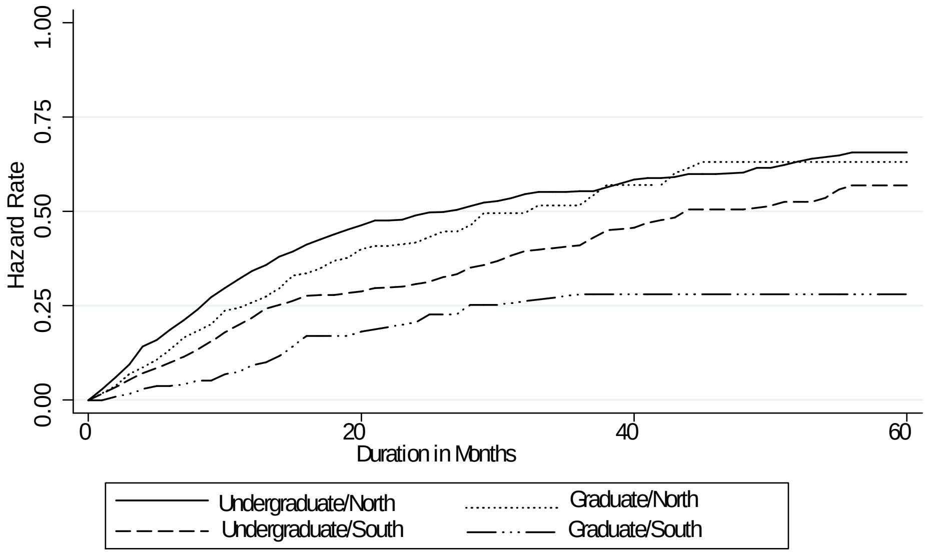

Figure 7: Kaplan-Meier failure estimates by Education and Region. Failure: Wrong match.

Table 4: Hazard Rate. Competing and Independent Risk Estimates of Weibull Model with and without IG frailty

Risk Coefficient Transitions	Cox Model			No Heterogeneity			IG Heterogeneity		
	Risk1	Risk2	Risk3	Risk1	Risk2	Risk3	Risk1	Risk2	Risk3
Age	−0.026***	−0.032***	−0.023***	−0.029***	−0.035***	-0.071***	−0.077***	−0.081***	-0.077***
	(0.002)	(0.004)	(0.003)	(0.003)	(0.004)	(0.002)	(0.03)	(0.009)	(0.004)
Female	-0.098***	-0.122***	-0.085***	-0.105**	-0.127**	-0.332***	-0.333***	-0.301***	-0.331***
	(0.046)	(0.069)	(0.061)	(0.046)	(0.069)	(0.056)	(0.083)	(0.140)	(0.111)
Graduate	0.452***	1.174***	-0.335	0.523***	1.252***	-0.431***	0.810***	2.451***	-0.671***
	(0.075)	(0.098)	(0.129)	(0.075)	(0.098)	(0.128)	(0.145)	(0.198)	(0.238)
Graduate*South	−0.075	0.024	-0.388**	-0.059	0.037	-0.182	−0.026	0.130	-0.512
	(0.012)	(0.142)	(0.199)	(0.108)	(0.142)	(0.198)	(0.201)	(0.283)	(0.355)
University leaving grade	0.034	0.103	-0.165	0.040	0.108	-0.065	0.125	0.232	-0.218
	(0.089)	(0.106)	(0.164)	(0.089)	(0.106)	(0.164)	(0.170)	(0.212)	(0.303)
High School leaving grade	0.251***	0.662***	-0.043	0.280***	0.696***	-0.160	0.390***	1.345***	-0.164
	(0.078)	(0.113)	(0.110)	(0.017)	(0.113)	(0.109)	(0.149)	(0.227)	(0.208)
Secondary school leaving grade	0.089***	0.316***	-0.040	0.082*	0.314***	-0.326***	−0.035	0.536***	-0.289**
	(0.051)	(0.087)	(0.062)	(0.050)	(0.086)	(0.055)	(0.089)	(0.175)	(0.111)
Degree in Humanities	-0.319***	-0.570***	0.185	-0.385***	-0.645***	0.122	-0.666***	-1.279.***	0.227
	(0.096)	(0.121)	(0.162)	(0.096)	(0.121)	(0.161)	(0.182)	(0.243)	(0.300)
Degree in Science	−0.094	−0.147	0.096	−0.081	−0.147	0.108	−0.138	-0.291	0.241
	(0.133)	(0.162)	(0.236)	(0.133)	(0.162)	(0.236)	(0.256)	(0.325)	(0.433)
Degree in Medicine	0.429**	0.653***	-1.728*	0.470**	0.698***	-1.900**	0.744**	1.358**	-2.792**
	(0.177)	(0.183)	(1.006)	(0.177)	(0.183)	(1.005)	(0.347)	(0.367)	(1.401)
South	−0.547***	-0.520***	-0.560***	−0.595***	−0.574***	-0.768***	−1.164***	−1.184***	-1.228***
	(0.055)	(0.093)	(0.068)	(0.055)	(0.093)	(0.066)	(0.099)	(0.184)	(0.125)
Father Education	0.287***	0.256**	0.320**	0.301***	0.281***	-0.242**	0.519***	0.550**	0.519**
	(0.081)	(0.107)	(0.125)	(0.081)	(0.107)	(0.124)	(0.155)	(0.213)	(0.238)
Married	0.024	−0.093	0.108	0.016	−0.101	0.509***	0.305**	0.114	0.514***
	(0.058)	(0.093)	(0.076)	(0.058)	(0.092)	(0.071)	(0.106)	(0.546)	(0.137)
Son	0.081	0.028	0.108	0.102	0.575	-0.685***	−0.350***	−0.066	-0.428**
	(0.067)	(0.101)	(0.092)	(0.068)	(0.101)	(0.060)	(0.108)	(0.214)	(0.143)
α				0.954	0.965	0.839	0.965	1.903	1.698
				(0.015)	(0.023)	(0.017)	(0.023)	(0.049)	(0.054)

Notes: i) Risk1 is failure toward employment; Risk2 is failure toward right match; Risk3 is failure toward wrong match ii) Standard Error in parenthesis; iii)*** 1% significant, ** 5% significant, * 10% significant. iv) α is the coefficient for duration dependence; v) IG indicates the Inverse-Gaussian specification.

Let $g_j(t)dt$ denote the probability of failure to risk j in the interval $(t, t + dt)$ then the total hazard rate $\lambda_\tau(.)$ applicable to all causes is:

$$\lambda_\tau(t) = \sum_{j=1}^{m} g_j(t) \tag{2}$$

If risks are independent, then the hazard rate for a specific cause j is $\lambda_j(t) = g_j(t)$. This means that the probability of failure from cause j in $(t, t + dt)$ conditional on survival to t is the same whether j is one of the risks or it is the only risk. Given independent risks, the hazard rate for failure of j-th type is defined by:

$$\lambda_j(t_j|x_j) = \lim_{dt_j \to 0} \frac{\Pr[t_j \leq T \leq t_j + dt, |T \geq t_j, x_j]}{dt_j} \tag{3}$$

where T indicates duration in the origin state.

We estimate a proportional hazard model of the form:

$$\lambda_j(t|x) = \lambda_{0j}(t) \exp\left(x(t)\beta_j\right) \tag{4}$$

where both the baseline hazard λ_{0j} and β_j are specific to type j hazard.

2.3 The Estimates

Table 4 reports the results of the estimated models. We implement a competing risk analysis where we distinguish two separate destination states: right match job and wrong match job. For comparison, we also report the estimates referred to transition toward employment without separating the destination states. In Table 4 *Risk1* refers to exit towards employment, while *Risk2* and *Risk3* refer to exits towards a right and a wrong match respectively.

For each transition we estimate the Cox model specification and, in order to check the robustness of our results, we also present estimates deriving from Weibull models where we introduce a control for unobserved heterogeneity (Inverse-Gaussian) as suggested in the literature.[3] Apart from isolated cases commented in due course, coefficients appearing in the different specifications are very similar, although those arising from parametric models are slightly bigger than the others, irrespective of the control for unobserved heterogeneity. We introduce explanatory variables which are likely to influence the educational choices as well as the availability and the characteristics of jobs. Hence, our empirical analysis puts special emphasis on the role played by two order of pushing factors in unemployment outflows: personal ability, and local economic characteristics. Although we also have some information on unemployment benefits, we observe that just a minority of unemployed in our sample receive some form insurance (in Table 1, just 7% of unemployed workers). We know that in Italy, unemployment benefits cover only a small fraction of the workforce, mainly workers in open-ended contracts who loose their jobs. Moreover, there is weak evidence on the role of this variable in the Italian labor market (Brugiavini, 2009). This may explain its limited extent among unemployed in our

[3] We focus our comments on the Cox specification since we are aware of the misspecification problems arising when using the Weibull hazard function specification (Cameron and Trivedi, 2005).

sample and its reduced significance in our preliminary estimated regressions, so we decide to discard it.

The estimated coefficients of the unemployment hazard equations which refer to exit toward employment, without distinguishing by exit type, are comparable to those presented in other empirical studies on the topic.[4] In our econometric exercise we notice that, when separating by exit type transitions, graduates have an extremely significant advantage in finding a job congruent with their competencies with respect to their undergraduate peers. Conversely, when transitions towards mismatched occupations are considered, having a university degree does not provide a preferential track. In this case, if we consider our Weibull specifications, we observe that the coefficients associated to the variable which controls for the possession of a degree qualification have negative signs. This would imply that when graduates search in the undergraduates labor market, they may be even penalized in terms of unemployment spell duration. This is in line with the findings observed in Figure 5. Further, graduates located in less developed areas of the country are significantly penalized in exiting the unemployment pool if they compete with undergraduate job searchers. In our model, individual ability is represented, in line with the existing literature, by pre-college and pre-high-school leaving grades.[5] Interestingly, high and secondary school leaving grades are significant in determining transitions towards well matched occupations but they have no role in speeding up unemployment outflows towards wrong matched positions. In this case the parametric specifications detect a significant negative effect of the secondary school leaving grades, implying that more able individuals tend to slow down their transition into bad quality occupations. In line with some previous results, the university leaving grade is not significant in determining labor market transitions.[6] We know that university grades may be biased by the so called "grade inflation" phenomenon, and in Italy many universities tend to inflate their final marks. Family background may impact the individuals' choices in many ways. On the one hand, families may provide the necessary help at early stages of children' growth in setting their schooling ability. On the other hand, they may facilitate the access to high quality universities. Cultural background is extremely relevant in this process even because better educated parents may value their children's education more than the others (Checchi, 2003). Finally, parents may increase individual employment opportunities through informal networks (Sylos Labini, 2008; Guiso *et al.*, 2004). It is reasonable to think that coming from a family that may act as a "network provider" increases employment opportunities. Our results point out that the father education is an important variable in determining employment transitions. Even for mismatched workers, employment opportunities are significantly influenced by family background and this may be in support of the "family network" hypothesis. We are also aware of the existing sharp South-North labor market differences in Italy (Brunello *et al.*, 2001; Bertola and Garibaldi, 2006). The situation is even more severe when considering youth unemployment. Our findings reveal that the process of job finding is more difficult and long lasting in the South of Italy. In particular, the labor market for southern graduates appears as dual. Individuals with high

[4] Among others see Biggeri *et al.* (2001) and Barbieri and Schrerer (2008).

[5] Among others, see Cappellari and Lucifora (2010), McGuinness (2003), and Ordine and Rose (2009).

[6] On this argument see Ordine and Rose (2009).

innate or schooling ability, coming from families with a good background, have job opportunities in well matched jobs without any remarkable difference with respect to their northern counterpart. In contrast, mismatched graduates are more penalized in terms of unemployment spells' duration when compared with northern graduates. Our empirical analysis also controls for gender, showing that being female always imposes a penalty in terms of job opportunities and unemployment spells' length. As expected, fields of education influence unemployment to job transitions in case of right matches. An implication of our results is that mismatched individuals wait for a job position consistently more that their well matched peers and this labor market segmentation is relevant especially for graduate individuals. As far as we want to enter the debate on overeducation as a "stepping stone" towards well matched occupation, and as a temporary occurrence in the individual's working life, we must really be very cautious in clasping this hypothesis.

3 The Theoretical Model

In this section we figure out a possible explanation of the empirical evidence presented so far. In particular we aim at analyzing theoretical underpinnings linking the duration of unemployment to individuals' ability conditional to job match characteristics. As a result, we single out a possible role for the *selectivity* of the university system in shaping this relationship and we address some important policy implications.

Consider an economy characterized by a *continuum* of risk-neutral individuals and firms, who match in the labor market following the lines set out by Mortensen (1986) and Pissarides (1985). Without loss of generality we normalize to 1 the *continuum* of both individuals and firms and we assume the mass of agents remains constant over time. Differently from standard matching models, we assume that i) the matching technology follows an urn-ball model as in Butters (1977) and Hall (1979);[7] ii) before entering the job-market, firms and individuals have to make a technological and educational choice: In particular, individuals/firms decide whether they want to enter the graduate/high-tech or the under-graduate/low-tech market respectively.[8] We assume that individuals are heterogeneous with respect to their innate talent (ability), which determines their productivity on the job and we consider the case in which innate ability is inversely related to the cost (effort) of acquiring education. On the demand side, each firm can post a limited number of vacancies, normalized to 1, and it sets production on the basis of a technological choice T. In particular, a firm can choose the sector where posting a vacancy i.e., it can choose to operate either within the high- or the low-technological sector. In order to simplify notation, from now on we refer to graduate *versus* undergraduate choice for both firms and individuals. However the reader should keep in mind that individuals make an educational choice while firms make a technological choice. Once the educational/technological choices have been made, the pure matching-process starts. We assume that undergraduate individuals can only be matched with

[7] The urn-ball matching function has become extremely popular among labor economists. For interesting applications see Moen (1999) and Gavrel (2009).

[8] Ordine and Rose (2009) allow for a similar choice but they model a static framework of pure-signaling. Moen (1999) allows only for educational choices.

low-tech firms, while graduates can search in both high-tech and low-tech markets. Using this setting, we demonstrate how overeducation is a phenomenon that might characterize standard matching models i.e., it might be attributable to a simple problem of frictional search in the labor market, arising even when self-selection into education is efficient. However, in this case we should not observe any relationship between innate ability and labor market outcomes. In contrast, such a relation may arise when educational choices are socially inefficient. In this case, we highlight the importance of considering, among other policy instruments, the *selectivity* intensity of the university system in order to re-establish social efficiency and to reduce mismatch.

3.1 Individuals

Consider a *continuum* of individuals of mass 1. We assume individuals characterized by heterogeneous innate ability θ. Innate ability is distributed according to a continuous and strictly increasing cumulative distribution $\Gamma(.)$, whose density function is $\gamma(\theta)$, over a support $[\underline{\theta}, \overline{\theta}]$ where $1 \leq \underline{\theta} < \overline{\theta}$ (so $\Gamma(\underline{\theta}) = 0$ and $\Gamma(\overline{\theta}) = 1$). We indicate with $e = \{g, ug\}$ the educational choice made by individuals in order to maximize their expected discounted utility (g stands for graduate while ug stands for undergraduate). For simplicity we assume that an individual has no income if unemployed (no unemployment benefits). As a consequence, once the educational choice has been made, in each period the individual's utility function $W(e)$ is given by:

$$W(e) = \begin{cases} 0 \quad \textit{if unemployed} \\ w_{ug} \ \textit{if employed in a ug position} \\ w_g \ \textit{if employed in a g position} \end{cases} \tag{5}$$

where w_g and w_{ug} indicate wages for graduate and undergraduate positions respectively. The cost of acquiring education ug is normalized to zero while, when individuals decide to acquire education g, on top of monetary costs, they have to sustain a cost $c(\theta) > 0$, with $\frac{\partial c}{\partial \theta} < 0$, related to their innate ability. We assume that monetary costs are the same for all the individuals, while the effort required to achieve a degree qualification is determined by personal ability. From now on, we refer to $|\frac{\partial c}{\partial \theta}|$ as a measure of the *selectivity* of the higher education sector. In words, the more the cost of education rises when ability decreases the more selective may be considered the higher education sector.[9]

3.2 Firms

Consider a *continuum* of firms of mass 1. We indicate with $T = \{g, ug\}$ the firm's investment in graduate (high-tech) and undergraduate (low-tech) vacancy

[9] A complimentary way of interpreting our definition of *selectivity* is the following. Consider the case in which, besides innate ability, the cost of education is related to households' characteristics (i.e. wealth, parents' culture, etc.) and to environmental and socioeconomic features (i.e. local labor market features, opportunity costs, etc.). These elements may determine heterogeneity of schooling costs and imply that the cost of education may change across individuals with similar innate abilities. In this scenario, the dimension of the "ability relevance" in reducing the cost of education ($|\frac{\partial c}{\partial \theta}|$) can be thought of as a proxy of how much ability matters, among other variables, in acquiring education: The smaller the "ability relevance" the lower is the correlation between ability and education. See Ordine and Rose (2009) for a signaling model covering this issue.

respectively. The cost of entering the g sector is given by $\delta > 0$. The cost of entering the ug sector is normalized to zero.[10] We crucially assume that firms are heterogeneous with respect to the cost they have to sustain in order to enter the g sector. In fact, in the growth theory literature, the cost of advanced technology has been considered typically related to the actual firm's technological endowment. The closer is a firm to the technological frontier the lower is the cost it needs to sustain in order to update its technology. The concept of technological frontier has been introduced by Nelson and Phelps (1966). Acemoglu *et al.* (2006) study empirically the relation between R&D expenditure and the distance from the technological frontier and build up a model where firms differ in terms of costs to adopt new technologies. Moreover, Ordine and Rose (2009) assume heterogeneous firms in terms of costs for ability-complementary technology. In our case, we assume that firms are distributed with a continuous and strictly increasing cumulative distribution $\Phi(.)$ whose density function is $\phi(\theta)$, over a support $[\underline{\delta}, \overline{\delta}]$ where $\underline{\delta} < 1 < \overline{\delta}$ (so $\Phi(\underline{\delta}) = 0$ and $\Phi(\overline{\delta}) = 1$).

Following Acemoglu (1997), the production function is given by:

$$y = y\left(e, T, \theta\right) = \bar{y}[\theta]^{1_{\{T=g \text{ and } e=g\}}}[1]^{1_{\{T=ug \text{ or } e=ug\}}} \tag{6}$$

where $\bar{y}$ is a constant.

Relation (6) indicates that there is homogeneity in the undergraduate sector i.e., when individuals work in the ug sector they produce an output $\bar{y}$ independently on their ability and education. On the other hand, graduate technologies are complementary only to graduate workers and the intensity of such complementarity is given by individual's innate ability.[11] Finally, we indicate with Q the cost of maintaining a vacancy $\forall T$, and we assume that vacancies can be destroyed at no cost.[12] Once the technological decision has been made, each firm realizes a profit $\Pi(T)$ in each period indicated as follows:

$$\Pi(T) = \begin{cases} -Q \quad \textit{if unfilled vacancy } \forall T \\ \bar{y} - w_{ug} - Q \textit{ if filled ug vacancy} \\ \theta\bar{y} \ - w_g - Q \textit{ if filled g vacancy.} \end{cases} \tag{7}$$

3.3 The Interaction Process and the Bellman Equations

The process consists in the following two stages. Firstly, individuals and firms conditional on their own type (ability and distance to the frontier) decide si-

[10] This assumption may easily be justified by thinking that in order to enter the graduate sector, firms are required to have costly technological endowment that should be used by engineers, doctors, investors, etc.; while low-skills complementary machines are typically less costly. See Mokyr (1996) on this argument.

[11] In fact, we are assuming *skill-ability complementary* technologies. This conjecture regarding the centrality of the positive interaction between technologies and ability is largely consistent with empirical evidence. Bartel and Sicherman (1999) find that the education premium in the US over the period 1979-1993 is the result of an increase in demand for innate ability or other unobserved characteristics of more educated workers. Murnane *et al.* (1995) argue that the returns to cognitive skills have risen during the 1980s. Juhn *et al.* (1993) provide evidence regarding observed and unobserved components of skills. The authors show that the premium to unobserved components precedes the increase in the return to education.

[12] We could assume $Q_g \neq Q_{ug}$. However, by assuming $Q_g = Q_{ug} = Q$ we simplify the notation and, since vacancies can be destroyed at no cost, this does not affect our main results.

multaneously the sector they want to enter i.e., they choose between graduate and undergraduate sectors. Once the educational/technological choices have been made, individuals and firms enter the labor market as unemployed and with unfilled vacancies respectively, and then the matching process starts. As a consequence, the relative markets' tightness in the present model is endogenous. We recall that, while undergraduate workers can match only with undergraduate firms, the converse is not true: graduate workers can be matched with undergraduate firms. Once a match is realized, we assume a standard Nash-bargaining (axiomatic) solution for wage determination.

In order to solve the model we proceed backward: Firstly we evaluate the actual expected value for individuals and firms using a standard dynamic programming method; secondly, by using the obtained results we proceed to find the Bayesian Nash Equilibrium (BNE) of the simultaneous first-stage game in which agents decide educational level and technological contents. The intuition behind the BNE solution is straightforward: Consider a firm and its decision of investing in a g position. A firm will do such an investment only if the associated expected payoff is greater than that associated to a ug position. Crucially, this depends on the distribution of θ within individuals that decide to acquire education g and on the relative markets' tightness. At the same time, worker's decision of investing in education g is a function of the number of firms that decide to create g positions. By evaluating the BNE we evaluate the "distributions" of agents that are best response to each other. We will show that the BNE can be efficient in terms of the total expected output of the economy conditional upon the appropriate level of *selectivity* $|\frac{\partial c}{\partial \theta}|$ in the higher education sector. Intuitively, when the mass of graduates increases, firms raise their investment in graduate-complementary positions since the probability of filling a vacancy increases (*tightness effect*). However, when the number of graduates rises, the average innate ability of the graduate labor force decreases. As a consequence, there exists a cutoff level in the mass of graduates above which firms find optimal to reduce their investment in graduate-complementary jobs (*composition effect*). In this respect, is then crucial to evaluate the right policy to implement. Policy designed to promote college attendance may induce an improvement in the overall efficiency only when the *tightness effect* dominates the *composition effect*. However, we prove that under some reasonable assumptions, when the *tightness effect* is the dominant one, we should not find any regularity between individuals' ability and the unemployment spells ending in a right match. In contrast, when the *composition effect* dominates, we should observe unemployment spells' duration of graduates in right positions being inversely related to their innate ability.

3.3.1 The matching functions

We indicate with E_e the employment level per educational groups ($e = \{g, ug\}$). $\dot{E}_e$ indicates the over-time variation of employment levels with:

$$\dot{E}_e = H_e - bE_e \tag{8}$$

where $b > 0$ is the exogenous quitting rate and H_e is the number of hirings per educational level.

Since H_g indicates the overall number of hirings for graduates, and since graduates can be matched in both sectors, we have that:

$$H_g = H_g^R + H_g^O \tag{9}$$

where H_g^R indicates the number of graduates hired in the graduate sector (R stands for "right match"), and H_g^O indicates the number of graduates employed in undergraduate positions (O stands for "overeducated"). By indicating with U_e the number of unemployed workers with education e and with V_T the number of posted vacancies per sector T, we can write the hiring functions as follows:

$$H_{ug} = KV_{ug}^{\eta} U_{ug}^{1-\eta} \tag{10}$$

$$H_g^O = KV_{ug}^{\eta} U_g^{1-\eta} \tag{11}$$

$$H_g^R = \mathbf{a}_g^R(t) U_g. \tag{12}$$

Some clarifications are in order. On one side, since worker's ability is irrelevant in ug jobs there is no room for worker's ranking in this sector and, as a consequence, eqs. (10) and (11) are assumed to be standard (Cobb-Douglas CRTS) matching functions (K is a constant and $0 < \eta < 1$).[13] On the other side, since ability matters in the g sector, here the matching process is described, in line with the existing literature, as an urn-ball model where workers send applications and firms select the best candidates they receive. In the standard urn-ball process, unemployed workers can send only one application each period. Here we keep this assumption by allowing for graduates sending only one application per sector and with a strict preference for a graduate-job. This implies that the graduate sector remains characterized by the standard Poisson distribution that describes the urn-ball process, while the undergraduate sector (where there is no workers' ranking) collapses into a matching model in which only the tightness of the market determines the probability of being employed (see Moen, 1999). The urn-ball process is a convenient tool for describing the labor market when workers are heterogeneous and in our model it makes possible to specify a graduate's exit rate from unemployment toward a right-match as a function of his characteristics. In order to understand where eq. (12) comes from, indicate with $t = U_g/V_g$ the tightness of the graduate sector. Indicate with $a_g^R(t,\theta)$ the probability that an unemployed graduate with ability θ receives a job offer from a g firm. The probability $a_g^R(t,\theta)$ is given by:

$$a_g^R(t,\theta) = \exp\left(-\frac{1-\Gamma(\theta)}{t}\right)\frac{\gamma(\theta)}{t}. \tag{13}$$

with $\frac{\partial a_g^R}{\partial \theta} > 0$. Eq. (13) indicates the probability that a firm does not meet any applicant of ability greater than θ times the probability for a worker with ability θ to meet a firm. By integrating $a_g^R(t,\theta)$ over $[\theta^*, \overline{\theta}]$, whose lower bound θ^* is the threshold-ability determined *ex-ante* in the BNE, we obtain the overall probability of being hired in a g position, called $\mathbf{a}_g^R(t)$ with:

[13] We are aware that there is no micro-foundation for the Cobb-Douglas matching function. However, it is not crucial for our results. All the same, it helps in proving the existence of a steady-state equilibrium.

Table 5: Notation for actual expected values.

Firms	Workers
	$V_{ug}^{E} \Rightarrow$ empl. ug individual;
$V_{ug}^{F} \Rightarrow$ filled ug position;	$V_{ug}^{U} \Rightarrow$ unempl. ug individual;
$V_{ug}^{V} \Rightarrow$ vacant ug position;	$V_{g}^{R} \Rightarrow$ empl. g individual in a right position;
$V_{g}^{F} \Rightarrow$ filled g position;	$V_{g}^{O} \Rightarrow$ empl. g individual in a over position;
$V_{g}^{V} \Rightarrow$ vacant g position;	$V_{g}^{U} \Rightarrow$ unempl. g individual

$$\mathbf{a}_g^R(t) = 1 - \exp\left(-\frac{1}{t}\right) \tag{14}$$

and

$$\frac{\partial \mathbf{a}_g^R(t)}{\partial t} < 0 \tag{15}$$

and this explains eq. (12).

Now, we can set the following formalism:

- $a_{ug} = \frac{H_{ug}}{U_{ug}+U_g} \Rightarrow$ prob. that an undergraduate is employed;
- $a_g^O = \frac{H_g^O}{U_{ug}+U_g} \Rightarrow$ prob. that a graduate is employed in an undergraduate position.
- $a_g^R(t, \theta) \Rightarrow$ prob. that a graduate with ability θ is employed in a right position;
- $\mathbf{a}_g^R(t) = \frac{H_g^R}{U_g} \Rightarrow$ prob. that a graduate is employed in a graduate position.

By indicating with α_T the probability that a T vacancy is filled and using an argument similar to that made when explaining eq. (13), we can write the following expressions:

- $\alpha_g(t, \theta) = \exp - ([1 - \Gamma(\theta)]\, t)\, \gamma(\theta) t \Rightarrow$ prob. that a g vacancy is filled with a type-θ worker;
- $\boldsymbol{\alpha}_g(t) = \int_{\theta^*}^{\overline{\theta}} \alpha_g(t, \theta) d\theta = 1 - \exp(-t) \Rightarrow$ prob. that a g vacancy is filled.
- $\alpha_{ug} = \frac{H_{ug}+H_g^O}{V_{ug}} \Rightarrow$ prob. that an undergraduate vacancy is filled.

Finally, by indicating with $r > 0$ the intertemporal interest rate and considering the notation for the actual expected values as indicated in Table 5, we can write down the value functions as follows.

Value functions

- Undergraduate individuals:

$$rV_{ug}^{E} = w_{ug} - b(V_{ug}^{E} - V_{ug}^{U}) \tag{16}$$

$$rV_{ug}^{U} = a_{ug}(V_{ug}^{E} - V_{ug}^{U}). \tag{17}$$

- Firms with undergraduate job-positions:

$$rV_{ug}^{F} = \bar{y} - w_{ug} - Q - b(V_{ug}^{F} - V_{ug}^{V}) \tag{18}$$

$$rV_{ug}^{V} = -Q + \alpha_{ug}(V_{ug}^{F} - V_{ug}^{V}). \tag{19}$$

- Graduate individuals:

$$rV_{g}^{R} = w_{g} - b(V_{g}^{R} - V_{g}^{U}) \tag{20}$$

$$rV_{g}^{O} = w_{ug} - b(V_{g}^{O} - V_{g}^{U}) \tag{21}$$

$$rV_{g}^{U} = a_{g}^{R}(.)(V_{g}^{R} - V_{g}^{U}) + a_{g}^{O}(V_{g}^{O} - V_{g}^{U}). \tag{22}$$

- Firms with graduate job-positions:

$$rV_{g}^{F} = \theta\bar{y} - w_{g} - Q - b(V_{g}^{F} - V_{g}^{V}) \tag{23}$$

$$rV_{g}^{V} = -Q + \alpha_{g}(.)(V_{g}^{F} - V_{g}^{V}). \tag{24}$$

Notice that, apart from eq. (22) these are pretty standard value functions. Eq. (22) indicates that the value of being an unemployed graduates with ability θ includes the probability of being employed in a graduate position ($a_g^R(.)$) and in an undergraduate position (a_g^O).

3.4 Solving the model

The use of Nash-bargaining solution imposes that when a match is realized, the generated surpluses for firm and worker must be equal. Hence by imposing $V_{ug}^E - V_{ug}^U = V_{ug}^F - V_{ug}^V$ we obtain the following expression for the undergraduate workers' wage:

$$w_{ug} = \frac{\bar{y}(r + b + a_{ug})}{a_{ug} + \alpha_{ug} + 2b + 2r}. \tag{25}$$

This expression, that should look familiar to the reader used with matching models, allows us to derive the actual expected value of workers that decide to acquire education ug. Similarly, by imposing $V_g^R - V_g^U = V_g^F - V_g^V$ and combining the relative value functions we obtain the following expression for the graduate workers' wage employed in a right position:

$$w_g = \frac{\theta\bar{y}\left[r+b+a_g^O+a_g^R(.)\right](r+b)+a_g^O\left[r+b+\alpha_g(.)\right]w_{ug}}{(r+b)\left[2r+2b+\alpha_g(.)+a_g^O+a_g^R(.)\right]+a_g^O\left[r+b+\alpha_g(.)\right]}. \tag{26}$$

It is easy to see that if a graduate could not search in the undergraduate sector ($a_g^O = 0$) relation (26) would become a standard wage expression for matching models. By substituting (25) into (26) we get:

$$w_g = \bar{y}\frac{\theta\left[r+b+a_g^O+a_g^R(.)\right](r+b)+a_g^O\left[r+b+\alpha_g(.)\right](r+b+a_{ug})}{(a_{ug}+\alpha_{ug}+2b+2r)\left[(r+b)a_g^R(.)+(2r+2b+\alpha_g(.))(a_g^O+r+b)\right]}. \tag{27}$$

In the Appendix we set the conditions under which eqs. (25) and (27) give rise to a steady-state equilibrium in the matching process. We now proceed (backward) in evaluating the simultaneous decision of individuals and firms concerning educational level and technological sector respectively. We assume that at this stage, agents ground their decisions considering the parameters a_g^O, a_{ug}, $a_g^R(.)$, α_{ug}, and $\alpha_g(.)$ as if they were at their steady-state value. Put differently, we are assuming agents choose their strategy in order to maximize the payoffs they would obtain in the steady-state.

The first-stage game

Individuals and firms have to decide, conditional on their ability and distance to the frontier, the level of education and the technology they want to acquire respectively. Once they make their choice, they enter the labor market as unemployed individuals and as firms with unfilled vacancies and then the matching process starts.

We describe the interaction process using a game in normal form in Figure 8. The game is Bayesian since each agent knows his own type (ability/distance to the frontier) and just the distribution of types of player to whom he may be matched. In Figure 8 we indicate with $E[V_g^V|\theta]$ the payoff of a g firm that matches a g worker because individual's ability is revealed only when a match is realized. Moreover, since g firms and ug workers cannot match each other we set $V_g^V = -\delta$ and $V_{ug}^U = 0$ when $e = ug$ and $T = g$. Notice that, in this interaction process we look for pure strategies of firms and individuals that are best responses to each other, conditional to the type of players. As a consequence, the BNE gives us the shares of individuals and firms that acquire higher education and invest in graduate positions respectively and it provides a measure of the relative tightness of the two sectors.

Proposition 1 *It exists a unique BNE of the game in Figure 8 in which only individuals with ability $\theta \geq \theta^*$ set $e = g$ and only firms with $\delta \leq \delta^*$ set $T = g$.*

Proof. Consider the firm's choice first. Indicate with γ the probability (it is a density) that the individual sets $e = g$. In this case, a firm invests in g position only if:

$$\delta \leq \gamma E[V_g^V|\theta] - V_{ug}^V. \tag{28}$$

		Firm	
		(ϕ) *g*	($1-\phi$) *ug*
(γ)	*g*	$V_g^U - c(\theta)$, $E[V_g^V \mid \theta] - \delta$	$V_{ug}^U - c(\theta)$, V_{ug}^V
Individual			
($1-\gamma$)	*ug*	0, $-\delta$	V_{ug}^U, V_{ug}^V

Figure 8: The individual-firm Bayesian game in normal form.

Given our assumption on the monotonicity of $\Phi(.)$, we can indicate with δ^* the cutoff level of distance to the frontier for which relation (28) is satisfied. Now, indicate with ϕ the probability that a firm set $T = g$ and consider the individual's educational choice. Setting $e = g$ is optimal for an individual only if:

$$c(\theta) \leq \phi V_g^U. \tag{29}$$

Given our assumption on the monotonicity of $\Gamma(.)$ and given that $\frac{\partial c}{\partial \theta} < 0$, we can indicate with $\theta^*$the cutoff ability level for which relation (29) is satisfied. Hence, the following pair characterizes the BNE of the game in Figure 8:

$$\begin{cases} \gamma = 1 - \Gamma(\theta^*) \\ \phi = \Phi(\delta^*). \end{cases} \tag{30}$$

■

Intuitively, a firm invests in a g position only if the associated expected payoff is greater than that associated to a ug position. Crucially, this depends on the distribution of θ within individuals that decide to acquire education g, on the relative markets' tightness, and on firm's distance from the technological frontier. At the same time, the worker's decision of investing in education g is a function of the number of firms that decide to create g positions and of the level of his own ability. Relation (30) contains the shares that are best response to each other and these can be considered as the shares of agents that represent the only steady-state of the interaction process.[14]

In order to evaluate the equilibrium efficiency, we focus on the cutoff level δ^* i.e., the share of firms that satisfies relation (28) as an equality. In fact, δ^* indicates the share of firms that create graduate-complementary positions

[14]See Osborne and Rubinstein (1994) p. 38-39 on the interpretation of BNE as a steady-state equilibria.

and, as it appears from eq. (6) the greater δ^* the greater the expected output produced in the economy, since *ceteris paribus* a g firm realizes a greater output than its ug counterpart. Put differently, the wider the share of g firms the better is the performance of the considered economy in terms of expected output.

In order to make explicit relation (28) consider the following steps. By using eqs. (19) and (25) we have that:

$$rV_{ug}^V = -Q + \alpha_{ug}\frac{\bar{y}}{a_{ug} + \alpha_{ug} + 2b + 2r}. \tag{31}$$

Similarly, by taking expectations of both sides of eq. (24) conditional upon $\theta > \theta^*$ and by substituting into that eq. (26) we obtain the following expression:

$$rE[V_g^V|\theta > \theta^*] = -Q + \boldsymbol{\alpha}_g(t)\frac{\left[r + b + a_g^O\right] E[\theta|\theta \geq \theta^*]\bar{y} - a_g^O w_{ug}}{(r+b)\left[a_g^R(.) + \alpha_g(.) + a_g^O + 2b + 2r\right] + a_g^O(r + b + \alpha_g(.))}. \tag{32}$$

By substituting eq. (25) into eq. (32) and using eq. (31), we can write the cutoff level δ^* in relation (28) as follows:

$$\delta^*(\theta^*) = \Gamma(\theta^*)\frac{Q}{r} + \frac{\bar{y}}{rF}\left[\frac{[1-\Gamma(\theta^*)]\,\boldsymbol{\alpha}_g(t)\left[FE[\theta|\theta > \theta^*]G - (r + b + a_{ug})a_g^O\right]}{P} - \alpha_{ug}\right] \tag{33}$$

where F, G, and P summarize strictly positive constants.[15] Relation (33) represents the *best response function* in terms of share of firms investing in graduate positions. Since we are evaluating the best response δ^* when the share of graduates is $\Gamma(\theta^*)$, eq. (33) describes the BNE of the game. Notice that in eq. (33) we have that:

$$E[\theta|\theta \geq \theta^*] = \frac{\int_{\theta^*}^{\bar{\theta}} \theta\gamma(\theta)d\theta}{1 - \Gamma(\theta^*)} \tag{34}$$

and, as a reminder:

$$\boldsymbol{\alpha}_g(t) = \int_{\theta^*}^{\bar{\theta}} \alpha_g(t,\theta)d\theta. \tag{35}$$

Eq. (34) expresses firm's expectation on graduates' ability which positively depends on θ^*, since the higher the cutoff ability level, the higher is the expected productivity of graduates. Eq. (35) represents the probability of filling a g position which is inversely related to θ^*: In this case, as the cutoff point θ^* rises, the probability of filling a vacancy reduces. We can now evaluate how the share δ^* changes *in equilibrium* as θ^* changes.

By differentiating eq. (33) with respect to θ^* using the Leibniz' rule for differentiation of definite integrals we get:

[15] $F = a_{ug} + \alpha_{ug} + 2b + 2r$; $G = (r + b + a_g^O)$; $P = (r+b)\left[a_g^R(.) + \alpha_g(.) + a_g^O + 2b + 2r\right] + a_g^O(r + b + \alpha_g(.))$.

$$\frac{\partial \delta^*}{\partial \theta^*} = \frac{1}{r}\left(\underbrace{\gamma(\theta^*)\left[Q + \frac{\boldsymbol{\alpha}(t)\bar{y}G}{P\left[1-\Gamma(\theta^*)\right]}\right]\left[\int_{\theta^*}^{\bar{\theta}} \theta\gamma(\theta)d\theta - \theta^*\right]}_{>0 \; composition \; effect} \right. \tag{36}$$
$$\left. \underbrace{-\frac{\overline{y}}{FP}\left[\left(FE[\theta|\theta>\theta^*]G - (r+b+a_{ug})a_g^O\right)\left(\gamma(\theta^*)\boldsymbol{\alpha}(t) + \alpha(t,\theta^*)\left[1-\Gamma(\theta^*)\right]\right)\right]}_{<0 \; tightness \; effect} \right).$$

Relation (36) indicates how a variation in the best response in terms of share of graduates (θ^*) affects *in equilibrium* the share of firms investing in graduate positions. Assuming satisfied second order conditions, we can indicate with θ^*_{wm} the share of graduates that *ceteris paribus* maximizes firms' investments in graduate positions i.e.:

$$\frac{\partial \delta^*}{\partial \theta^*}|_{\theta^*=\theta^*_{wm}} = 0. \tag{37}$$

It is important to note that only the appropriate *selectivity* level $|\frac{\partial c}{\partial \theta}|$ can ensure that θ^*_{wm} is actually achieved in equilibrium. On the contrary, if this is not the case, we can have equilibria characterized by $\theta^* \neq \theta^*_{wm}$.

In Figure 9 and Figure 10 we represent graphically the best response $\delta^*(\theta^*)$ in the two possible scenarios that may arise:

- Case a) - **Tightness dominance**: $\theta^* > \theta^*_{wm}$ (Figure 9). In this case a reduction in the selectivity level of the higher education sector ($|\frac{\partial c}{\partial \theta}| \downarrow$) induces a rise in the share of graduates ($\theta^* \downarrow$) that in turn induces an increase in the share of firms investing in graduate positions. The overall expected output of the economy increases but the effect on overeducation cannot be uniquely defined since, in this scenario both demand and supply of graduates increase.

- Case b) - **Composition dominance**: $\theta^* < \theta^*_{wm}$ (Figure 10). In this case an increase in the selectivity level of the higher education sector ($|\frac{\partial c}{\partial \theta}| \uparrow$) induces a reduction in the share of graduates ($\theta^* \uparrow$) and this generates an increase in the share of firms investing in graduate positions. Differently from before, here the effect of such a policy induces an improvement in the overall expected output and, simultaneously, a reduction in overeducation since less graduates have more firms looking for them in the labor market.

4 Policy Implications

According to our model, in order to suggest the right policy to maximize output and reduce overeducation, it is crucial to figure out the specific scenario that characterizes the presence of overeducation.

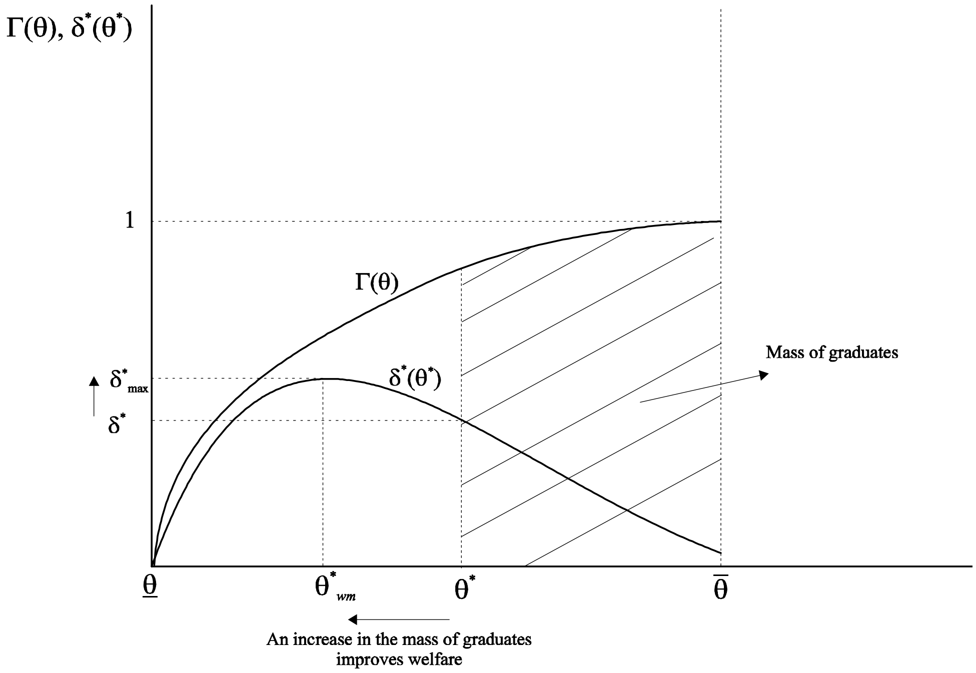

Figure 9: An inefficient Bayesian equilibrium: The case of tightness dominance

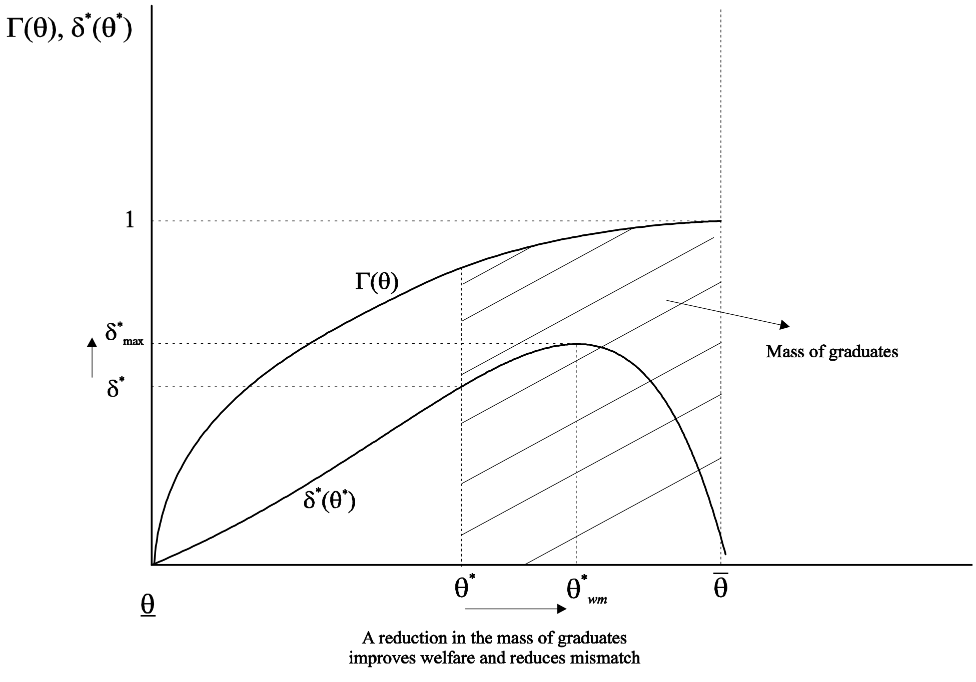

Figure 10: An inefficient Bayesian equilibrium: The case of composition dominance

To this end, assume that individuals' innate ability raises production up to a given level over which the output remains constant. This implies that from firms' perspective the distribution of individuals $\Gamma(.)$ has a mass point over which a rise in innate ability does not affect production. Now, assume that this mass point is in θ^*_{wm}, namely all individuals with ability $\theta \geq \theta^*_{wm}$ have the same ability θ^*_{wm}. These assumptions imply that the economy is characterized by a mass of equally "high-productive" individuals and this mass is large enough so that below θ^*_{wm} the *tightness effect* is always dominated by the *composition effect*.[16] In this scenario, we can write the probability for a graduate with ability θ of being employed in a right position as follows:

$$a_g^R(t,\theta) = \begin{cases} \exp\left(-\frac{1-\Gamma(\theta)}{t}\right)\frac{\gamma(\theta)}{t} \text{ if } \theta^* \leq \theta < \theta^*_{wm} \\ \\ \frac{\gamma(\theta^*_{wm})}{t} \text{ if } \theta \geq \theta^*_{wm} \end{cases} \tag{38}$$

where the bottom line of eq. (38) indicates that individuals with ability $\theta \geq \theta^*_{wm}$ have a probability of being employed in a right position that is constant with respect to θ since they are identical in terms of productivity. In contrast, the first line relates the exit rate of graduates towards right matched positions following the urn-ball process as in eq. (13) since when $\theta < \theta^*_{wm}$ workers' ranking applies.

Now, first consider the case of "tightness dominance" illustrated in Figure 9. In this scenario, from firms' perspective all graduates have an ability level equal to θ^*_{wm} i.e., they have the same productivity. Hence, by approximating the average individual's unemployment spell terminated in a right match with $S_g^R = 1/a_g^R(.)$ we have that:

$$S_g^R = \frac{t}{\gamma(\theta^*_{wm})}. \tag{39}$$

In words, in the presence of a tightness problem, namely too few graduates in the labor market, we should observe that the average unemployment spell of well matched individuals does not depend on their innate ability θ. In this case, the higher education sector is too selective, and the share of individuals that in equilibrium decide to acquire higher education is constrained with respect to the social optimum.

On the other side, consider the case of "composition dominance" illustrated in Figure 10. Here, *in equilibrium* graduates are heterogeneous in terms of their productivity since $\theta^* < \theta^*_{wm}$. In this case we have that the average individuals' unemployment spell terminated in a right match is given by:

$$S_g^R = \begin{cases} \exp\left(\frac{1-\Gamma(\theta)}{t}\right)\frac{t}{\gamma(\theta^*_{wm})} \text{ if } \theta^* \leq \theta < \theta^*_{wm} \\ \\ \frac{t}{\gamma(\theta^*_{wm})} \text{ if } \theta > \theta^*_{wm} \end{cases} \tag{40}$$

with $\frac{\partial S_g^R}{\partial \theta} < 0$ and an absolute minimum in $\theta = \theta^*_{wm}$. In words, only in the presence of "composition dominance" we actually find differences in graduates'

[16] We are only assuming that the economy is characterized by a large share of equally high-productive individuals. See Moen (1999) for a similar assumption concerning the presence of a mass point in the distribution of individuals' ability.

unemployment spells due to individuals' innate ability. When the *selectivity* level of the university system is too low, the unemployment spells' length terminating in a right match is inversely related to individuals' ability.

According to the empirical evidence presented in Section 2, we may consider the scenario of "composition dominance" as that describing the possible causes at the root of educational mismatch that characterizes the Italian labor market. Indeed, the duration of unemployment terminated in a right match seems to be related to individuals' innate ability. In this scenario we point out the importance of considering, among other instruments, the *selectivity* level of the university system to implement policies targeted to reduce possible inefficiencies arising from self-selection into education. In this view, many papers (among others see Charlot and Decreuse, 2005; and Hendel *et al.*, 2006) stress the importance of raising tuition fees in order to limit access to higher education and to raise welfare. In our framework, we highlight the possibility of thinking about an alternative policy instrument, whose effectiveness in shaping the correlation between educational choices and individual ability should not depend on the presence of liquidity constraints or heterogeneity in the households' wealth. Furthermore, we stress the importance of considering some possible implications of policies aimed at increasing the share of graduates in the labor force when these go through a reduction in educational contents and students' effort. In this case, the overeducation phenomenon may by exacerbated conditional to the structure of the industrial sector in terms of technological endowment.

5 Conclusions

This paper considers the issue of overeducation, a phenomenon affecting almost all developed countries. We undertake the analysis of overeducation through an investigation of individuals' unemployment spells with the intent of deriving some new insights by exploiting the characteristics of mismatched graduates in terms of their unemployment history. We start presenting some evidence on unemployment spells duration of Italian workers and we highlight that hazard rates of graduates are higher than those of undergraduates only for transitions towards occupations that require the competencies provided by the universities and this process is strictly related to innate ability and geographical location. We build up a matching model coupled with endogenous educational and technological choices and we highlight the role of university *selectivity* in determining unemployment duration of mismatched workers in two different scenarios. We show that, in the case of "tightness dominance" a reduction in the *selectivity* level of the higher education system may induce a rise in the share of graduates, leading to an increase in the share of firms investing in graduate positions. In contrast, in the case of "composition dominance" a reduction in the *selectivity* level of the higher education sector induces an increase in the share of graduates and this might generate a reduction in the share of firms investing in graduate positions because of a too low expected productivity of graduates. We show that overeducation may characterize both scenarios. However, only in the latter case we should observe an inverse relationship between graduates' ability and the duration of unemployment spells terminated in a right match in the labor market. Overall, we figure out the importance of considering the negative effect that may arise in the presence of policies aimed at increasing the share of graduates in the labor force when these go through a reduction in educational contents and students' effort. At the same time, we pose some criticisms on measures aimed at rendering more efficient individual self-selection into education through adjustments of tuition fees. We point out the importance of considering the *selectivity* of the higher education sector as a policy instrument that may affect both the extent of educational mismatch and the overall performance of the economy in terms of output and productivity.

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6 Appendix

Proof of the existence of a steady-state employment level for graduate and undergraduate workers

We divide the proof in two parts. In Part 1 we set the existence of a steady state (s.s.) employment level for undergraduate workers. In Part 2 we set the conditions under which when undergraduates employment is in a s.s. graduates employment level is in a s.s. too.

Part 1. From eqs. (16) and (17) we know that.

$$V_{ug}^E - V_{ug}^U = \frac{w_{ug}}{r + a_{ug} + b}. \tag{41}$$

Since in equilibrium $V_{ug}^E - V_{ug}^U = V_{ug}^F - V_{ug}^V$, using eq. (25) we can write eq. (19) as follows:

$$rV_{ug}^V = -Q + \alpha_{ug}\frac{\bar{y}}{a_{ug} + \alpha_{ug} + 2b + 2r}. \tag{42}$$

Since we assumed that vacancies can be destroyed at no costs, in a (s.s.) where $\dot{E}_{ug} = 0$, we must have $rV_{ug}^V = 0$. Here we prove that eq. (42) is strictly decreasing in E_{ug} with a positive value in $E_{ug} = 0$ and a negative value in $E_{ug} = \Gamma(\theta)$, i.e. it must exist an employment level E_{ug} in which $rV_{ug}^V = 0$. In equation (42) consider $a_{ug} = \frac{H_{ug}}{U_{ug}+U_g}$. In a s.s. we must have that $bE_{ug} = H_{ug}$ hence we can write a_{ug} as follows:

$$a_{ug} = \frac{bE_{ug}}{1 - E_{ug} - E_g} \tag{43}$$

with i) $\lim\limits_{E_{ug}\to 0} a_{ug} \to 0$; ii) $\lim\limits_{E_{ug}\to\Gamma(\theta)} a_{ug} \to \underset{>0}{\frac{b\Gamma(\theta)}{1-\Gamma(\theta)-E_g}}$; iii) $\frac{\partial a_{ug}}{\partial E_{ug}} > 0$.

Now, consider $\alpha_{ug} = \frac{H_{ug}+H_g^O}{V_{ug}}$. Since $bE_{ug} = H_{ug}$ and $H_g^O = KV_{ug}^{\eta}U_g^{1-\eta}$ we can write α_{ug} as follows:

$$\alpha_{ug} = \frac{bE_{ug}}{V_{ug}} + KV_{ug}^{\eta-1}U_g^{1-\eta}. \tag{44}$$

By using the fact that from eq. (10) $V_{ug} = \left(\frac{bE_{ug}}{KU_{ug}^{1-\eta}}\right)^{1-\eta}$ we can write eq. (44) as:

$$\alpha_{ug} = (bE_{ug})^{\frac{\eta-1}{\eta}} K\left[\Gamma(\theta) - E_{ug}\right]^{\frac{1-\eta}{\eta}} \left[1 + \frac{K\left[1-\Gamma(\theta)-E_g\right]^{1-\eta}}{K^{\eta-1}\left[\Gamma(\theta)-E_{ug}\right]^{\eta-1}}\right] \tag{45}$$

with i) $\lim\limits_{E_{ug}\to 0} \alpha_{ug} \to +\infty$; ii) $\lim\limits_{E_{ug}\to\Gamma(\theta)} \alpha_{ug} \to 0$ (as a reminder $\eta < 1$); iii) $\frac{\partial \alpha_{ug}}{\partial E_{ug}} < 0$.

Now, we can evaluate eq. (42) as a function of E_{ug}. Given our results concerning a_{ug} and α_{ug} we have that:

i) $\lim\limits_{E_{ug}\to 0} rV_{ug}^V \to \bar{y} - Q$; ii) $\lim\limits_{E_{ug}\to\Gamma(\theta)} rV_{ug}^V \to -Q$; iii) $\frac{\partial V_{ug}^V}{\partial E_{ug}} < 0$.

As a consequence it exists a level E_{ug} in which eq. (42) is equal to zero and this value is a s.s.. *Q.E.D.* It is important to note a_{ug} and α_{ug} are both functions of E_g as well. As a consequence, the steady state value of E_{ug} is a function the employment level in the graduate sector E_g. In particular, from eqs. (43) and (45) we have that $\frac{\partial \alpha_{ug}}{\partial E_g} < 0$ and $\frac{\partial a_{ug}}{\partial E_g} > 0$. As a consequence, from eq. (42) it is easy to check that $\frac{\partial V_{ug}^V}{\partial E_g} < 0$. Using the implicit function theorem we have that in the s.s. $\frac{\partial E_{ug}}{\partial E_g} < 0$. In words, since graduates can be employed in the undergraduate labor market, the larger the share of employed graduates, the lower the share of employed undergraduates. In Figure 11, we draw the line representing the s.s. ($\dot{E}_{ug} = 0$) in the undergraduate labor market and we indicate with $\underline{E}_{ug}$ the s.s. of employed undergraduates when all graduates are employed.

Part 2. Graduates can be in a s.s. only if $\dot{E}_g = 0$. We know that

$$\dot{E}_g = H_g^R + H_g^O - bE_g \tag{46}$$

with $H_g^R = \mathbf{a}_g^R(t)U_g$ and $H_g^O = a_g^O(U_{ug} + U_g)$. We can write (46) as follows:

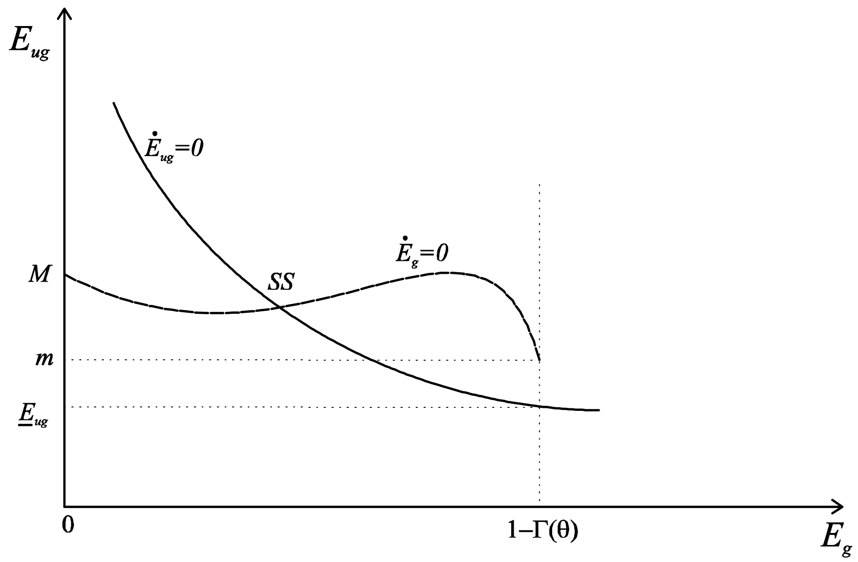

Figure 11: A steady-state equilibrium in graduate and undergraduate labor markets.

$$\dot{E}_g = \mathbf{a}_g^R(t)(1-\Gamma(\theta)-E_g) + a_g^O(1-E_g-E_{ug}) - bE_g \tag{47}$$

which implies that $\dot{E}_g = 0$ only if:

$$E_{ug} = 1 + \frac{1-\Gamma(\theta)}{a_g^O}\mathbf{a}_g^R(t) - E_g\left[\frac{1+b+\mathbf{a}_g^R(t)}{a_g^O}\right] \tag{48}$$

where:

$$\lim_{E_g \to 0} E_{ug} = \underbrace{1 + \frac{1-\Gamma(\theta)}{a_g^O}\mathbf{a}_g^R(\frac{1-\Gamma(\theta)}{V_g})}_{M} \tag{49}$$

$$\lim_{E_g \to 1-\Gamma(\theta)} E_{ug} = \underbrace{1 - \frac{1-\Gamma(\theta)}{a_g^O}(1+b)}_{m} \tag{50}$$

with $M > m\ \forall a_g^O \in [0,\infty)$. Since eq. (48) describes a continuous function, if m is greater than $\underline{E}_{ug}$, there must exist at least one point (in Figure 11 we represent the case of a single point SS) representing a pair (E_g, E_{ug}) that is a s.s. for both markets. Moreover, if

$$a_g^O\frac{\partial \mathbf{a}_g^R}{\partial E_g}(1-E_g) < \frac{\partial a_g^O}{\partial E_g}\left[\mathbf{a}_g^r(t) + (1+b+\mathbf{a}_g^r(t))E_g\right] + a_g^O(1+b+\mathbf{a}_g^R(t)) \tag{51}$$

eq. (48) is monotonically decreasing and the s.s. is unique. *Q.E.D.*