Endogenous Effort When Unemployment is a Worker Discipline Device*

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Abstract

This paper provides a simple equilibrium dynamic model in which employment evolves according to the rules of the Shapiro-Stiglitz's (1984) shirking model. The proposed framework allows to endogenize the effort decision undertaken by the individual worker and it might resolve the indeterminacy from which is affected the model with exogenous (and constant) effort. Furthermore, exploiting an externality argument, we allow the model to capture different local dynamic patterns in which are enclosed persitent cycles and convergent fluctuations.

Keywords: Efficiency Wages, Effort Decision and General Equilibrium Dynamic Model
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1 Introduction

It is well known that the Shapiro-Stiglitz's (1984) shirking model does not analyse the effort decision undertaken by the individual worker. *Given* a positive level of effort, the

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principles of dynamic programming are used to derive the conditions under which the individual worker finds profitable to behave correctly instead of shirking. The results of this theoretical exercise are widely recognised in the labour economics literature: since the monitoring possibilities of the firms are not perfect, a positive involuntary unemployment rate must persist as an equilibrium phenomenon in order to generate a threat that avoid the possible opportunistic behaviour of the employees. The persistence of the equilibrium unemployment rate is ensured in the (neo)classical way, that is, offering to employed workers a real wage higher than the opportunity cost of labour[1].

More recently, this simple partial equilibrium model has been widely exploited also in the real business cycle (RBC) literature aiming to explore the dynamic implications of non-Walrasian approaches to labour market. In fact, the belief in genuinely involuntary unemployment inspired a lot of contributions dealing with the efficiency wage argument suggested by the shirking motivation. In this literature, the employers adjust real wages in order to meet the incentive compatibility constraint according to which workers provide a *certain* level of effort. However, these models do not analyse in a satisfactory manner the way in which this level of effort is determined and the way in which the economy reacts to a variable effort. In particular, there are models in which the effort provided by the employees results to be constant in equilibrium and during the adjustment process. See for example Danthine and Donaldson (1990) and Nakajima (2005). In other models, when the effort is allowed to vary, the proposed framework distances from the set-up originally introduced by Shapiro and Stiglitz (1984). See, for example, Uhlig and Xu (1996).

In this paper we adopt a different perspective. First of all, following a contribution by Miles Kimball (1994), we derive the evolution law of employment arising from the original shirking model. Thereafter, we complete the picture adding the preferences of a union to whom is devolved the effort decision. In the dynamic model developed in this paper the effort is the only determinant of the real wage[2]. Therefore, the union is called to continuously determine the "dividend" of the asset equation that defines the expected discounted value of utility of the non-shirking employed worker. This task is carried out balancing at the margin the union utility arising from higher employment level with the cost of providing additional effort.

An interesting feature of our model is that the union can be "myopic" when it decides

[1]In this perspective, the shirking motivation for the payment of an efficiency wage provides also a rationalisation for real wages stickiness in presence of involuntary unemployment. See Mankiw and Romer (1991).

[2]As we shall see below, this means to assume a constant marginal productivity of labour per effort units.

effort. Specifically, the hypothesis of myopic behaviour is met enclosing two distinct externality factors in the evolution law of employment. The first is on the unemployment rate, the second on the wage differential in effort terms. As it will be showed below, assuming that the union might neglect the effects of its choices on the average rate of unemployment and on the average level of the wage differential, allows the model to capture different local dynamic patterns in which are enclosed persistent cycles (orbits) and convergent fluctuations[3]. Furthermore, under particular conditions, controlling for the effort might resolve the indeterminacy from which is affected the standard model with exogenous (and constant) effort.

The paper is arranged as follow. Section 2 provides a short overview on the literature dedicated to dynamic models dealing with the efficiency wages hypothesis. Section 3 derives the employment dynamics underlying the Shapiro-Stiglitz's (1984) shirking model. Section 4 builds a dynamic equilibrium model with efficiency wages. Section 5 analyses the equilibrium cyclical patterns of effort and employment. Finally, Section 6 concludes.

2 Related Literature

The efficiency wage hypothesis has been widely exploited in building general equilibrium dynamic models. The incorporation of non-Walrasian features in standard dynamic models developed along the lines of the RBC tradition[4] has been encouraged by the inability of providing theoretical schemes accounting for the observed variability of wages and employment and to explain the persistence of involuntary unemployment.

The seminal paper by Jean-Pierre Danthine and John Donaldson (1990) provides the first RBC model with involuntary unemployment and a suboptimal equilibrium path for employment. This work analyses two different motivations for the payment of efficiency wages, that is, the "gift-exchange" motivation (Akerlof, 1982) and the shirking motivation (Shapiro and Stiglitz, 1984), and each of them are characterised by a different model specification. An interesting result achieved in this contribution is that the efficiency wage hypothesis is not sufficient to generate a real wage resistance consistent with empirical evidence. In particular, this is true in the gift-exchange model where the equilibrium efficiency wage depends on a reference wage which, in turn, is a decreasing function of the

[3]Cycles are generated by the occurrence of a Hopf bifurcation. On the other hand, convergent fluctuations reflect a situation of local indeterminacy.

[4]In what follows, the RBC tradition is interpreted in a broad sense. All the contributions reviewed in this section enclose models with intertemporal optimisation over an infinite horizon.

current unemployment rate. In this case, employment and real wage display the same high correlation displayed by the traditional RBC model[5]. Results that are closer to empirical evidence are achieved by exploiting a very simplified version of the shirking framework in which the equilibrium efficiency wage varies only in response to changes in the stock of capital and to variations in the technology parameter[6].

The cyclical consequences of the shirking motivation for the payment of efficiency wages when the workers' effort is an adjustable variable are examined in a paper by Harald Uhlig and Yexiao Xu (1996). This work develops the idea that if unemployment acts like a threat, that threat should be more pronounced when unemployment is high. Therefore, an increase in the unemployment rate should lead workers with jobs to work harder, making them more efficient. The inclusion of a countercyclical effort adjustment in a otherwise standard RBC model helps in explaining the rather large cyclical employment movements as well as the rather low cyclical movements in real wage rates. However, this results is not very promising because it is achieved within framework that requires implausibly large movements in the technology parameter in order to explain the observed business cycle dynamics[7].

Another interesting contribution is that proposed by Rui Coimbra (1999). This paper, being closer to the Solow's (1979) idea of efficiency wages, is not directly related to the Shapiro-Stiglitz's (1984) framework. However, this is one of the first attempts to inquiry on the possibility of macroeconomic cyclical behaviours in presence of real wage stickiness. In particular, Coimbra proposes a general equilibrium dynamic model aimed to explore the joint implications of efficiency wages, indivisible labour and increasing returns to scale in the production technology. Specifically, the efficiency wages hypothesis is introduced by assuming that firms choose the real wage and the effort constrained by a participation constraint arising from an enforceable contract specifying that the agreed wage will be

[5] See, for example, Kidland and Prescott (1982).

[6] "This exercise is to be viewed as an attempt to ascertain whether a postulated wage inertia, compatible with the spirit of the shirking paradigm, although imposed without fully articulating the underlying model, generates dynamic behavior for the other aggregates of our economy then can be viewed as a fair replication of business cycle regularities." Danthine and Donaldson (1990).

[7] In this model, the Solow's residual is equal to the sum between the log-deviations of effort and the technology parameter from their respective expected values. If the former reacts countercyclically to variations of the latter, the observed pattern of Solow's residuals can be explained assuming that fluctuations in the technology parameter are unplausibly large if compared to those required in a standard RBC model with constant effort. Given that, the authors themselves observe that within the proposed specification an adjustable effort due to efficiency wage consideration is unlikely to play an important role in understanding business cycles.

paid only if a specified effort level will be achieved. On the other hand, the indivisible labour hypothesis is introduced by assuming that workers cannot decide the amount of hours they wish to work[8]. There are two remarkable results achieved in this paper. The first is that with a *constant* effort, the real wage rigidity implied by the efficiency wage hypothesis can indeed explain the persistence of involuntary unemployment but it cannot affect the dynamic properties of the standard RBC model[9]. The second is that the inclusion of the Hansen's (1985) indivisible labour hypothesis can generate endogenous and persistent fluctuations with both an elasticity of inputs substitution and an amount of increasing returns to scale in consonance with the empirical evidence[10].

Finally, inspired to the suggestive literature on self-fulfilling prophecies, we find a recent paper by Tomoyuki Nakajima (2005). This work exploits the efficiency wage apparatus proposed by Alexopoulos (2004) aiming to derive the conditions under which the equilibrium path of the economy is indeterminate. In this contribution, the effort provided by the employees depends on the ratio between the consumption of employed non-shirking workers and employed shirking workers. The firms, aiming to satisfy the incentive compatibility constraint according to which workers behave correctly, are assumed to set the wage rate in order to keep constant such a ratio. Therefore, even effort results to be constant in this model[11]. The indeterminacy of the equilibrium path is achieved by assuming a partial unemployment insurance covering the unemployment risk, that is, postulating a positive difference between the income of employed and unemployed workers[12]. A notable feature of this model is to collect two typical Keynesian topics, that is,

[8] In the seminal paper by Gary Hansen (1985), individuals can either work some given positive number of hours or not at all so that they are unable to work an intermediate number of hours.

[9] This is a just replication of the result achieved by Danthine and Donaldson (1990) by exploiting the gift-exchange paradigm.

[10] See, for example, Caballero and Lyons (1992).

[11] In the efficiency wages apparatus proposed by Alexopoulos (2004), workers that are caught shirking are not fired. Instead, they receive a fixed wage cut. However, as in the Shapiro-Stiglitz's (1984) model, firms offer a wage contract such that - in equilibrium - employed workers do not shirk on the job.

[12] This contribution belongs to the honoured tradition that tries to establish a link between the conditions for indeterminacy and the equilibrium condition for labour market. In the standard neoclassical growth model, the conditions for indeterminacy are met whenever a lower marginal utility from consumption is associated to a higher equilibrium employment. In the seminal work by Benhabib and Farmer (1994) this is achieved exploiting a labour demand with a positive slope steeper than labour supply. In the efficiency wage model developed by Nakajima (2005), the indeterminacy of the equilibrium path is achieved exploiting a NSC with a negative slope implied by the incomplete unemployment insurance. Given that in an efficiency wage model the NSC replaces the labour supply, it is straightforward that with a negative slope a lower marginal utility from consumption is associated to a higher equilibrium

the possibility of business cycles driven by agents' expectations and the occurrence of an equilibrium in which unemployment is involuntary.

3 The Dynamics of the Shirking Model

In this section, exploiting the results achieved in a contribution by Kimball (1994), we analyse the out-of-steady-state employment dynamics arising from the Shapiro-Stiglitz's (1984) shirking model.

Let us begin with the supply side of labour market. The instantaneous utility of the individual worker is given by

$$U(t) = w(t) - e(t)$$

where $w(t)$ is the real wage and $e(t)$ is the disutility of effort[13].

Let b be the instantaneous separation rate and q the detection rate per unit of time worked. Given these specifications of the instantaneous utility function and the instantaneous probabilities, the asset equation for a shirking worker is the following

$$\rho^W V_E^S(t) = w(t) + (b+q)\left[V_u(t) - V_E^S(t)\right] + \dot{V}_E^S(t) \qquad (1)$$

where, $V_E^S(t)$ is the expected discounted value of utility for an employed shirker, $V_u(t)$ is the expected discounted value of utility for an unemployed worker and ρ^W is the real interest rate used by the worker to discount utility flows[14].

On the other hand, the asset equation for a non-shirking employed worker is given by

$$\rho^W V_E^N(t) = w(t) - e(t) + b\left[V_u(t) - V_E^N(t)\right] + \dot{V}_E^N(t) \qquad (2)$$

employment even with a labour demand displaying the traditional (negative) slope. A similar result is showed by Benhabib and Farmer (2000).

[13] In our dynamic effort-elicitation model, the level of effort provided by an employees is assumed to be a sort of commitment. Specifically, when a worker find profitable to behave correctly, he already knows how much effort he has to provide.

[14] The expression "asset equation" is used because the expected discounted value of the utility that we are trying to evaluate is considered as a particular financial asset that is continuously negotiated. Taking the example of the shirking employed worker described by equation (1), the dividend of the financial asset is given by the real wage $w(t)$ while the capital gain (in this case, a loss!) is given by the difference between the expected discounted value of being unemployed and the expected discounted value of being employed not exerting effort, that is, $V_u(t) - V_E^S(t)$.

where $V_E^N(t)$ is the expected discounted value of utility of a non-shirking employed worker.

The no-shirking condition (NSC) may be derived imposing $V_E^N(t) \geqslant V_E^S(t)$. Moreover, since firms do not have to pay a real wage higher than the payment for which workers behave correctly, the NSC will hold as equality all the time, that is

$$V_E^N(t) = V_E^S(t) \qquad \text{for all } t \tag{3}$$

Differentiating equation (3) yields

$$\dot{V}_E^N(t) = \dot{V}_E^S(t) \qquad \text{for all } t \tag{4}$$

Subtracting equation (2) from (1), and using equations (3) and (4) to simplify, yields

$$V_E^N(t) - V_u(t) = \frac{e(t)}{q} \tag{5}$$

Equation (5) states a well known results in the efficiency wage literature: as long as $q < +\infty$ (imperfect monitoring) workers strictly prefer employment to unemployment[15]. Therefore, employed workers enjoy rents, hence the resulting unemployment is involuntary.

Assuming that unemployment benefits are zero, the asset equation for an unemployed worker is the following:

$$\rho^W V_u(t) = m(t)\left[V_E^N(t) - V_u(t)\right] + \dot{V}_u(t) \tag{6}$$

where $m(t)$ is job acquisition rate[16].

Notice that when we assume that the unemployment benefits are zero, the level of effort provided by the individual non-shirking employed worker coincides with his reservation wage. Therefore, the difference between $w(t)$ and $e(t)$ might be thought as a wage differential.

Substituting equation (5) into equation (6) and solving for $w(t)$ yields

$$w(t) = \frac{e(t)}{q}\left[b + \rho^W + m(t)\right] + e(t) \tag{7}$$

Normalizing the size of labor force to unity, the job acquisition rate $a(t)$ can be determined from the employment flow identity, that is

[15]As stressed by Orphanides (1993), equation (5) also suggests that the expected benefit to a worker of providing effort – represented by the value of not losing his job multiplied by the probability of being discovered and fired if he shirks – has to be equal to cost of providing effort.

[16]When the asset equation for a worker who is not employed is written in this form, it is implicitly assumed that in case of hiring the worker will behave correcly.

$$\dot{L}(t) \equiv m(t)[1 - L(t)] - bL(t) \tag{8}$$

Rearranging,

$$m(t) = \frac{\dot{L}(t) + bL(t)}{1 - L(t)} \tag{9}$$

Substituting equation (9) in (7) yields:

$$w(t) = \frac{e(t)}{q}\left[\rho^W + b + \frac{\dot{L}(t) + bL(t)}{1 - L(t)}\right] + e(t) \tag{10}$$

This is the dynamic version of Shapiro-Stiglitz's no-shirking condition (Kimball, 1994). Specifically, equation (10) states that the efficiency wage is given by the reservation wage augmented by a mark-up which depends on the degree of aggregate labour market tightness. Moreover, notice that in an effort-elicitation model, the NSC takes the place of the labour supply curve in a standard model of labour market.

Equation (10) can be used to derive the rate of increase of employment, that is

$$\dot{L}(t) = [1 - L(t)]\left[q\frac{w(t) - e(t)}{e(t)} - \rho^W + b\right] - bL(t) \tag{11}$$

The second term in brackets provides an explicit form for the fraction of unemployed workers finding a job in each instant. The solution of the general equilibrium dynamic model will be derived assuming that

$$\frac{w(t) - e(t)}{e(t)} \equiv e(t)^a \tag{12}$$

where $a \in (0,1)$. In other words, we will assume that the wage differential in effort term is given by a constant-elasticity function enclosing effort as unique argument. This is equivalent to assume a constant marginal productivity of labour per effort units. Therefore, the real wage will be univocally determined by the effort level decision.

Under this specification, the production function is given by

$$Y[e(t), L(t)] = \int_0^{+\infty} w(t)\, dL(t) \equiv L(t) f[e(t)]$$

where $f[e(t)] \equiv e(t)[1 + e(t)^a] = w(t)$.

The specification used for the real wage is quite *ad hoc*[17]. However, it allows to capture three interesting elements. First, both effort and labour are essential for production[18]. Second, being an exponential, the wage differential in effort terms is always positive[19]. Third, the wage-effort elasticity is higher than one and increasing in the effort level[20]. Being the inverse, this means that the effort-wage elasticity is lower than the level satisfying the Solow's (1979) condition and decreasing in the effort level[21]. This possibility is showed in a dynamic partial equilibrium model proposed by Faria (2000).

Developing equation (11) using definition (12) leads to

$$\dot{L}(t) = [1 - L(t)]\left[qe(t)^{a} - \rho^{W}\right] - b \tag{13}$$

Equation (13) states that employment increases until the number of unemployed workers finding a job equates the number of full employment layoffs. This happen whenever the job acquisition rate - the second term in brackets - is equal to the wedge between the expected wage differential in effort terms and the worker's interest rate.

As suggested by Kimball (1994), equation (13) indicates a lagged employment response to labour market conditions. Whenever workers accept to work harder, the firms willingness to hire increases. In such a situation, the individual employer will be reluctant to hire new employees, since the motivation of workers who realize that the labour market is a period of boom will be weakened unless they get extra-high wages. Therefore, each individual firm delays new hirings and this moderates the overall hiring rate in the economy.

Consider the case in which effort is constant, that is, $e(t) = e^{*}$ for all t. Denote by L^{*} the stationary solution of equation (13) and consider its local behaviour in the neighbourhood of L^{*}. The eigenvalue of the linearised equation is given by[22]

$$\xi \equiv -\frac{b}{1 - L^{*}} < 0$$

[17]The expression for the marginal productivity of labour revails that firms maximize profits with respect to the employment level, taking the effort level as given.

[18]In fact, $F(0, L) = F(e, 0) = 0$.

[19]Under imperfect monitoring, this means that unemployment is always involuntary.

[20]See the Appendix.

[21]Moreover, the equilibrium efficiency wage will be higher than the level such that the effort-wage elasticity is equal to unity. In fact, the Solow's (1979) model does not contemplate shirking. Therefore, if additional costs related to low-effort are taken into consideration, this will result in an equilibrium effort-wage elasticity lower than unity.

[22]See the Appendix.

Given the sign of the root ξ, the stationary solution L^* is locally asymptotically stable. Notice that in the version of the model with constant effort, employment is not a predetermined variable[23]. Therefore, at the time at which the model is started, a continuum of equilibrium trajectories exist, each of them corresponding to a continuum of possible starting values for employment in the neighbourhood of L^*. In other words, the stationary solution is locally indeterminate.

Suppose to start the model at $t = 0$ and consider the particular equilibrium under which employment moves continuously from whatever $L(0)$ given by history. Let $\phi(t)$ be the solution for employment. Its linear approximation is the following

$$\phi(t) \simeq L^* + \exp(\xi t)\left[L(0) - L^*\right]$$

Given the starting value $L(0) < (>)\, L^*$, employment grows (shrinks) toward L^* just fast enough for the real wage to equal the marginal product of labour per effort units all the times[24]. Some employment trajectories for different values of $L(0)$ are tracked in figure 1.

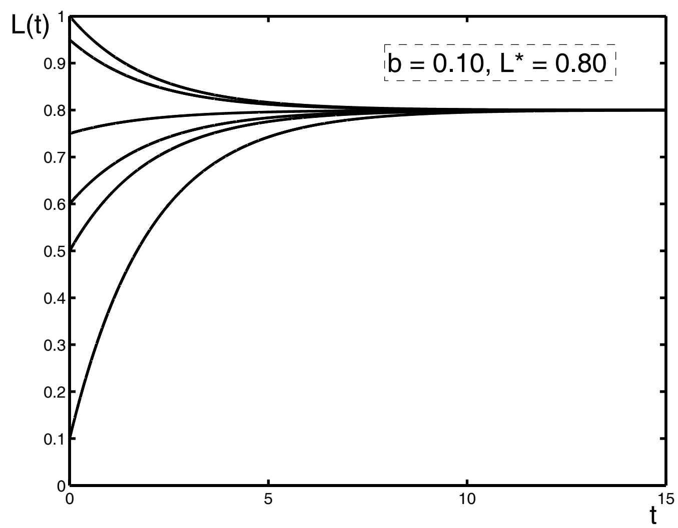

Figure 1: The employment trajectories of the model with constant effort.

[23] "Since $\dot{L}(t)$ enters the structural model only as an expectation, L is nonhistorical". Georges (1995).

[24] The higher is ξ, the higher is the speed of convergence to the stationary solution. Therefore, a higher separation rate and a lower equilibrium unemployment rate make for a more rapid convergence to the stationary solution.

4 An Equilibrium Dynamic Model with Efficiency Wages

In this section, we build a quite standard dynamic model developed in continuous time in which employment evolves according to equation (13). In order to simplify notation, from now on we omit the functional dependence of the variables on time.

The effort level is chosen by a union whose behaviour might be "myopic"[25]. Its preferences are described by the following instantaneous utility function

$$U\left(L,e\right)=\ln L-e \tag{14}$$

Equation (14) indicates that employment increases the union welfare in a logarithmic way, while the cost of exerting effort is linear and normalised to unity for each unit. In other words, the union chooses the dividend of the asset equation that is relevant for employment dynamics assuming that exerting effort is costly for workers but they does not dislike to be employed[26].

The hypothesis that a union is called to choose the work intensity of labour force is not totally new in the labour economics literature, especially if we consider contributions dealing with bargaining problems. See for example Moene (1988) and Cramton and Tracy (1992).

Assuming that the union chooses effort in order to maximise its welfare respecting the employment dynamics arising from the Shapiro-Stiglitz's (1984) shirking model, the "implicit" union problem[27] is given by

$$\max_{e}\int_{0}^{+\infty}\exp\left(-\rho^{P}t\right)\left[\ln L-e\right]dt \tag{15}$$

$$s.to$$

$$\dot{L}=B\left(1-L\right)\left(qAe^{a}-\rho^{W}\right)-b$$

where $B=\left(1-\overline{L}\right)^{\beta-1}$ and $A=\overline{e}^{\alpha-a}$.

[25] Choosing the effort, the union chooses also the real wage rate. Therefore, as anticipated above, the union decision determines univocally the "dividend" of the asset equation that is relevant for the non-shirking employed worker.

[26] In this model, as in the original contribution by Shapiro and Stiglitz (1984), higher employment levels correspond to higher real wage rates.

[27] In the implicit programming approach, the optimization problem to be solved depends on parameters that, in turn, depend on solutions to the problem itself. See Kehoe, Levine, and Romer (1992).

The variables $\overline{L}$ and $\overline{e}$ are, respectively, the average level of employment and the average level of effort. When it decides the effort, they are taken as given by the union. On the other hand, β and α are parameters and they can be higher or lower then unity[28].

Clearly, the factors A and B represent a sort of externality in the union problem. In particular, they help to clarify the sense in which the union behaviour might be myopic: when it chooses effort observing the evolution of employment, the union does not consider the effect of its choice on the average levels of the involved variables[29]. Thereafter, the solution of the union problem is not, in general, Pareto efficient[30].

4.1 The Solution of the "Implicit" Union Problem

The present value Hamiltonian of the implicit union problem is given by the following expression:

$$H = \ln L - e + \Lambda \left[B\left(1-L\right)\left(qAe^{a} - \rho^{W}\right) - b \right] \tag{16}$$

where Λ is the shadow value of employment (co-state variable).

The first order condition for the control variable is the following

$$\Lambda B\left(1-L\right) qAae^{a-1} = 1 \tag{17}$$

Quite intuitively, the union chooses effort equalizing its marginal cost to the marginal net revenue in utility terms deriving from the increase in employment.

Considering a symmetric equilibrium, that is a situation in which $\left(1-\overline{L}\right) = \left(1-L\right)$ and $\overline{e} = e$, equation (17) might be easily solved for Λ. It yields

$$\Lambda = \frac{e^{1-\alpha}}{\left(1-L\right)^{\beta} qa} \tag{17a}$$

Differentiating with respect to time leads to

$$\dot{\Lambda} = \frac{1-\alpha}{\left(1-L\right)^{\beta} qae^{\alpha}}\dot{e} + \frac{\beta}{\left(1-L\right)^{1+\beta} qae^{\alpha-1}}\dot{L} \tag{17b}$$

[28] As it will be clear later, α cannot be equal to unity.

[29] Notice that when β is higher (lower) than unity, the union overestimate (underestimate) the unemployment rate. On the other hand, when α is higher (lower) than unity, the union overestimate (underestimate) the wage differential in effort terms.

[30] A trajectory $\{L\left(t\right), e\left(t\right)\}$ would be Pareto optimal if and only if $\left[1-\overline{L}\left(t\right)\right] = \left[1-L\left(t\right)\right]$ and $\overline{e}\left(t\right) = e\left(t\right)$ for all t.

Dividing (17b) by (17a) yields

$$\frac{\dot{\Lambda}}{\Lambda} = (1-\alpha)\frac{\dot{e}}{e} + \beta\frac{\dot{L}}{1-L} \tag{17c}$$

Therefore, the dynamics of the control variable, that is, the effort, is given by

$$\dot{e} = \frac{e}{1-\alpha}\left(\frac{\dot{\Lambda}}{\Lambda} - \frac{\dot{L}}{1-L}\right) \tag{17d}$$

Notice that in order to avoid an explosive dynamics for e, we have to exclude the case in which α is equal to unity.

Along the optimal path, the shadow value of employment has to satisfy the following rule

$$\dot{\Lambda} = \rho^P\Lambda - \frac{1}{L} + \Lambda B\left(qAe^a - \rho^W\right) \tag{18}$$

Once again, we consider the conditions for a symmetric equilibrium. Dividing each member of equation (18) by Λ and using equation (17) leads to

$$\dot{e} = \frac{e}{1-\alpha}\left[\frac{(1-\beta)\left(qe^\alpha - \rho^W\right)}{(1-L)^{1-\beta}} - \frac{(1-L)^\beta qa}{Le^{1-\alpha}} + \frac{\beta b + \rho^P(1-L)}{1-L}\right] \tag{19}$$

Given definition (12), equation (19) allows to derive the evolution of the real wage[31].

Finally, the transversality condition for this problem is given by

$$\lim_{t\to+\infty}\exp\left(-\rho^P t\right)\Lambda(t)\,L(t) = 0 \tag{20}$$

4.2 Steady-State Relationships

Consider the differential equation for L in a situation of symmetric equilibrium, that is, when $\left(1-\overline{L}\right) = (1-L)$ and $\overline{e} = e$. It yields

$$\dot{L} = (1-L)^\beta\left(qe^\alpha - \rho^W\right) - b \tag{21}$$

In steady-state $\dot{L} = 0$, therefore, solving for e we have

[31] In particular, the dynamics of the real wage is given by the following differential equation

$$\dot{w} = \left[1 + (\alpha+1)\,e^\alpha\right]\dot{e} \tag{19a}$$

$$e=\left[\frac{b+\rho^{W}\left(1-L\right)^{\beta}}{q\left(1-L\right)^{\beta}}\right]^{\frac{1}{\alpha}}\equiv\eta\left(L\right)>0 \tag{22}$$

In our model, the real wage depends only on the effort level[32]. Therefore, it is straightforward that equation (22) provides a particular version of the "canonical" NSC. See figure 2.

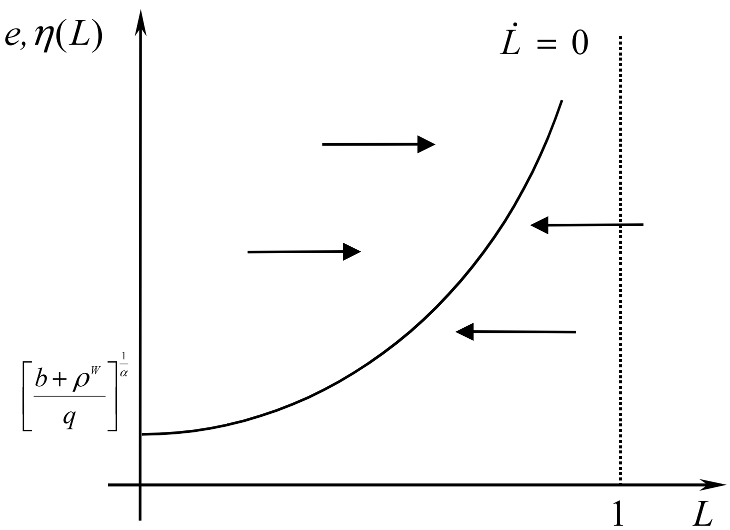

Figure 2: The equilibrium locus for L

The other steady-state condition is derived imposing $\dot{e}=0$. Therefore,

$$\frac{e\rho^{P}}{1-\alpha}-\frac{\left(1-L\right)^{\beta}qae^{\alpha}}{\left(1-\alpha\right)L}+\frac{e\left(1-\beta\right)\left(qe^{\alpha}-\rho^{W}\right)}{\left(1-\alpha\right)\left(1-L\right)^{1-\beta}}+\frac{e\beta b}{\left(1-\alpha\right)\left(1-L\right)}=0 \tag{23}$$

There is not an explicit solution for the equilibrium locus for e. However, exploiting equation (22), it is possible to derive the following expression

$$\frac{\eta\left(L\right)}{1-\alpha}\left(\rho^{P}+\frac{b}{1-L}\right)-\frac{\varphi(L)}{\left(1-\alpha\right)L}\equiv G\left(L\right)=0$$

where $\varphi(L)\equiv a\left[b+\rho^{W}\left(1-L\right)^{\beta}\right]>0$.

[32]See definition (12).

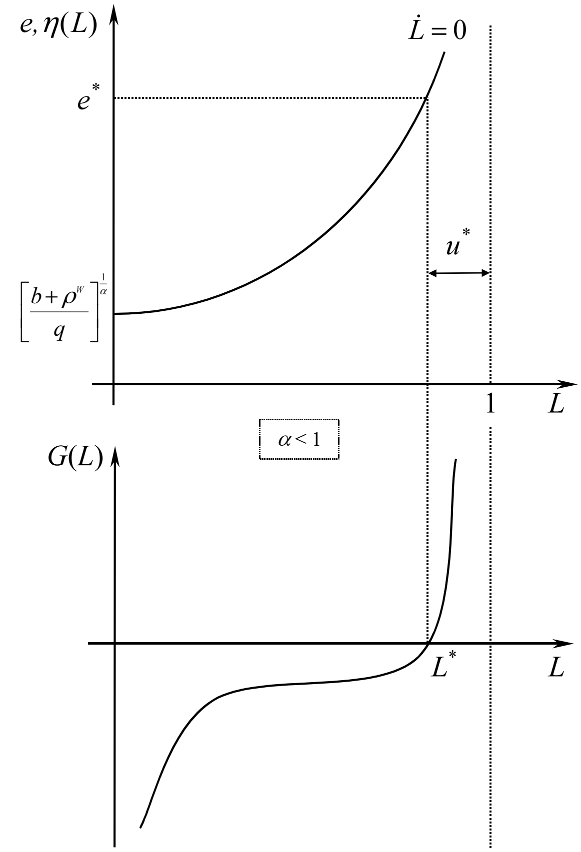

Figure 3: The steady-state with $\alpha < 1$.

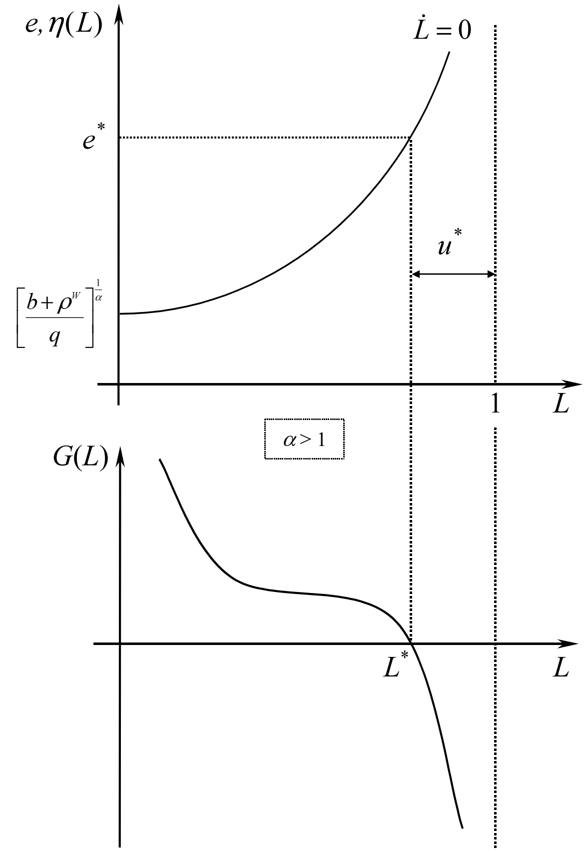

Figure 4: The steady-state with $\alpha > 1$.

Consider the case in which $\alpha < 1$, so that $(1-\alpha) > 0$. Obviously,

$$\lim_{L \to 0^+} G(L) = -\infty \quad \text{and} \quad \lim_{L \to 1^-} G(L) = +\infty$$

Whenever $\alpha > 1$, so that $(1-\alpha) < 0$, the situation is reversed. Therefore,

$$\lim_{L \to 0^+} G(L) = +\infty \quad \text{and} \quad \lim_{L \to 1^-} G(L) = -\infty$$

Since the function $G(L)$ is continue and monotone in the interval $(0,1)$, the steady-state exists and it is unique. See the two panels of figures 3 and 4. In each figure, the distance u^* denotes the equilibrium unemployment rate.

4.3 Local Dynamics

Consider the system of autonomous differential equations given by (21) and (19), that is,

$$\dot{L} = (1 - L)^{\beta} \left(qe^{\alpha} - \rho^{W} \right) - b$$

$$\dot{e} = \frac{e}{1-\alpha} \left[\frac{(1-\beta)\left(qe^{\alpha} - \rho^{W} \right)}{(1-L)^{1-\beta}} - \frac{(1-L)^{\beta} qa}{Le^{1-\alpha}} + \frac{\beta b + \rho^{P}(1-L)}{1-L} \right]$$

In order to analyse the local dynamics of this non-linear system of differential equations, we derive its Taylor first-order approximation around the unique steady-state equilibrium (L^{*}, e^{*}), that is

$$\begin{pmatrix} \dot{L} \\ \dot{e} \end{pmatrix} \simeq \begin{bmatrix} j_{1,1} & j_{1,2} \\ j_{2,1} & j_{2,2} \end{bmatrix} \begin{pmatrix} L - L^{*} \\ e - e^{*} \end{pmatrix}$$

where $j_{i,k}$, $i,k = 1,2$, are the elements of the Jacobian matrix $J \in \Re^{2\times 2}$.

As shown in the appendix, these elements are given by

- $j_{1,1} = -\frac{\beta b}{1-L^{*}} < 0$
- $j_{1,2} = \frac{\alpha q (1-L^{*})^{\beta}}{[\eta(L^{*})]^{1-\alpha}} > 0$
- $j_{2,1} = \frac{\varphi(L^{*})[1-L^{*}(1-\beta)]}{(1-L^{*})(1-\alpha)(L^{*})^{2}} + \frac{b\eta(L^{*})\left[(1-\beta)^{2}+\beta\right]}{(1-\alpha)(1-L^{*})^{2}} > 0 \Leftrightarrow \alpha < 1$
- $j_{2,2} = \frac{(1-\beta)[b+\varphi(L^{*})]+\beta b+\rho^{P}(1-L^{*})}{(1-\alpha)(1-L^{*})} - \frac{qa\alpha(1-L^{*})^{\beta}[\eta(L^{*})]^{\alpha-1}}{(1-\alpha)L^{*}}$

Clearly, $e^{*} \equiv \eta(L^{*})$ and $u^{*} \equiv (1 - L^{*})$ are, respectively, the steady-state level of effort and the steady-state unemployment rate. Moreover, $\varphi(L^{*}) \equiv a \left[b + \rho^{W} (1 - L^{*})^{\beta} \right]$.

It is well-known that the trace of the Jacobian matrix measures the sum of the eigenvalues and the determinant measures their product. For brevity,

$$\xi_{1} + \xi_{2} = j_{11} + j_{22} = \mathrm{TR}(J)$$

$$\xi_{1}\xi_{2} = j_{11}j_{22} - j_{12}j_{21} = \mathrm{DET}(J)$$

The trace and the determinant of our Jacobian matrix are given by the following expressions

$$\text{TR}(J) = \frac{(1-\beta)\left[b+\varphi(L^*)\right]+\alpha\beta b+(1-L^*)\,\rho^P}{(1-\alpha)\,(1-L^*)} - \frac{qa\alpha\,(1-L^*)^\beta\left[\eta\,(L^*)\right]^{\alpha-1}}{(1-\alpha)\,L^*}$$

$$\text{DET}(J) = -j_{1,1}\left[\text{TR}(J)+\frac{\alpha\beta b}{1-L^*}\right]-j_{1,2}j_{2,1}$$

An exact analysis of the sign of TR(J) and DET(J) without specifying the way in which each parameter exactly affect the equilibrium rate of unemployment and the equilibrium level of the effort risks to be ineffective. However, it should be clear that in such an analysis particular attention should be devoted to the effects generated by different values of α and β, each of them chosen in the close neighborhood of unity.

Suppose to keep β fixed and to allow the parameter α to vary. If variations of α lead TR(J) to change sign neutralising systematically the additive effect generated the second term in brackets in the expression for DET(J), the elasticity of the wage differential with respect to effort will distinguish the case in which TR(J) is positive and DET(J) is negative from the case in which TR(J) is negative and DET(J) is positive. In the former case, the steady-state is represented by a saddle point, in the latter by a sink[33]. In order to verify this and other possibilities, we performed some numerical simulations with different values of α and β.

4.4 Numerical Simulations

The expressions for the $j_{i,k}$ elements reveals that the local stability properties of the system and its global dynamics depend only on the model parameters and on the unique root of function $G(\cdot)$[34]. Therefore, using a computation package[35], it is possible to obtain the value of the latent roots (eigenvalues) of the Jacobian matrix J.

Specifically, we performed different simulations of the model aiming to explore the dynamic behaviour of our differential equations system for different values of α and β chosen in the neighborhood of unity. The other parameters were calibrated following similar contributions[36]. In particular, we set $q = 1$, $b = 0.10$, $a = 0.60$, and $\rho^W = \rho^P = 0.03$.

[33] Moreover, as it will be shown below, if it happens to be the case that for a given value of β, the parameter α distinguishes between oscillatory (complex) and non-oscillatory dynamics, the union degree of myopicity in observing the unemployment rate will play the role of bifurcation parameter.

[34] The unique root of the function $G(\cdot)$, in turn, in univocally determined by the parameters of the model.

[35] In this work, we used MATLAB 6.5.

[36] See Georges (2002) and Giammarioli (2003).

First of all, we considered the effects of different values of α, that is, the symmetric equilibrium elasticity of the wage differential with respect to effort. Setting $\beta = 0.8$, we obtained the results resumed in table 1.

α	ξ_1	ξ_2	*Dynamics*
0.8	1.4343	-1.0104	*Saddle*
0.9	2.0941	-1.3209	*Saddle*
0.99	9.8858	-2.7213	*Saddle*
1.01	$-3.5178 + 3.7945i$	$-3.5178 - 3.7945i$	*Sink*
1.05	$-0.68129 + 2.202i$	$-0.68129 - 2.202i$	*Sink*
1.1	$-0.32635 + 1.5905i$	$-0.32635 - 1.5905i$	*Sink*
1.2	$-0.15031 + 1.1336i$	$-0.15031 - 1.1336i$	*Sink*

Table 1: Simulation results for different values of α ($\beta = 0.8$).

The results enclosed in table 1 suggest that α discriminates essentially between oscillatory (complex) and non-oscillatory dynamics. In fact, for $\alpha < 1$ the Jacobian matrix displays two real roots of opposite sign. This means that the steady-state in represented by a saddle point. As suggested by Georges (1995), this kind of local dynamics might resolve the indeterminacy displayed by the standard model with constant effort[37]. In fact, representing a one-dimensional stable manifold, a saddle point implies that the trajectory converging to the steady-state is unique while all the others diverge. In this case, for every $L(0)$ in the neighborhood of L^* there will be a unique $e(0)$ in the neighborhood of e^* generating a trajectory converging to (L^*, e^*). This value of $e(0)$ should be selected in order to satisfy the transversality condition (20) and it will place the system on the *stable branch* of the saddle point (L^*, e^*). Therefore, when the steady-state is represented by a saddle point, the equilibrium path is locally determinate[38]. See figure 5.

[37]See section 3.

[38]If we think to the wage differential in effort terms as an employment adjustment cost, we observe that controlling for the effort resolves the indeterminacy as long as this cost is concave.

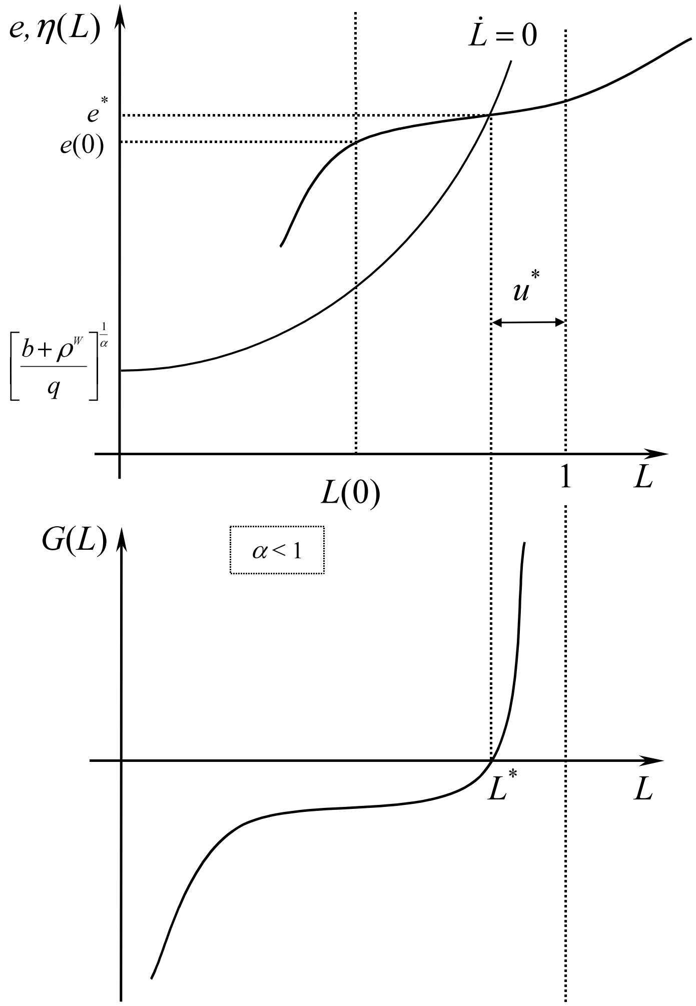

Figure 5: The saddle path

On the other hand, when $\alpha > 1$ the Jacobian matrix displays two complex conjugate roots. Therefore, the adjustment of employment and effort occurs through oscillations. However, the real part of the complex roots is negative. This means that the steady-state is represented by a sink so that oscillations are convergent. In a bidimensional space of variables, the sink represents the case of indeterminacy. In fact, a sink describes the situation in which the *stable manifold* has the same dimension of the space of variables. This means that there will be a continuum of equilibrium paths $\{L(t), e(t)\}$, indexed by $e(0)$, since any path converging to the steady-state (L^*, e^*) necessarily satisfies the transversality condition (20). In other words, in the neighborhood of (L^*, e^*) all the trajectories are optimal. See figures 6 and 7.

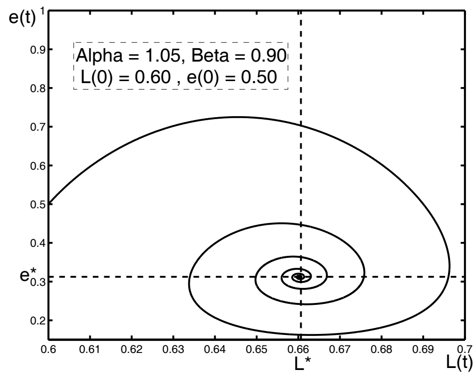

Figure 6: The sink (A).

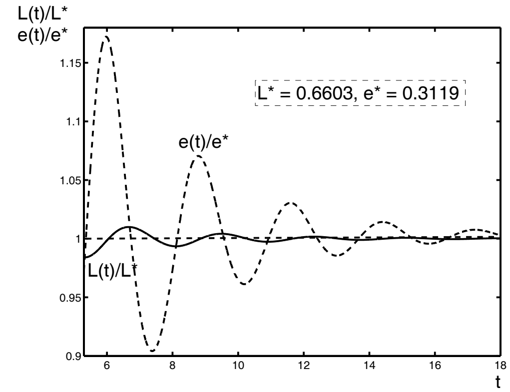

Figure 7: The sink (B).

An indeterminate equilibrium allows for the possibility of business cycles driven by self-fulfilling beliefs[39]. In this case, in fact, the fundamentals of the economy are not able to pin down an unique equilibrium path. Therefore, when the conditions for indeterminacy are met, the description of the environment provided by the union problem above is somehow incomplete. What we could have missed is the forecasting rule used by agents to predict the future. If in the neighborhood of the steady-state all the trajectories are optimal, the path actually followed by the economy could be the one that - consistently with the final equilibrium position (L^*, e^*) - allowed for the continuos validation of the agents' prophecies[40].

Finally, we performed some numerical simulations fixing the value of α but allowing for different values of β. The most interesting results can be obtained considering the case of oscillatory dynamics, that is, the case in which α is higher than unity[41]. In particular, we set $\alpha = 1.05$. The simulation results are resumed in table 2.

[39] See Farmer (1993).

[40] Following this perspective, the forecasting rule (or beliefs function) used by agents to predict the future is the tool that allow to solve the path multiplicity.

[41] I would be possible to show that as long as $\alpha < 1$, the steady-state of the model remains a saddle point no matter which is the value of β.

β	ξ_1	ξ_2	*Dynamics*
0.8	$-0.68129 + 2.202i$	$-0.68129 - 2.202i$	*Sink*
0.9	$-0.31411 + 2.2361i$	$-0.31411 - 2.2361i$	*Sink*
0.99	$-0.014293 + 2.2227i$	$-0.014293 - 2.2227i$	*Sink*
1	$0.015779 + 2.2193i$	$0.015779 - 2.2193i$	*Source*
1.05	$0.16878 + 2.1961i$	$0.16878 - 2.1961i$	*Source*
1.1	$0.31471 + 2.1645i$	$0.31471 - 2.1645i$	*Source*
1.2	$0.59146 + 2.0779i$	$0.59146 - 2.0779i$	*Source*

Table 2: Simulation results for different values of β ($\alpha = 1.05$).

The results enclosed in table 2 suggest that *inside* the oscillatory dynamics, the parameter β discriminates between local stability and instability. In fact, for $\beta < 1$ the real part of the two complex conjugate roots is negative. As stated before, this implies that the steady-state is a sink. On the other hand, when $\beta > 1$ the real part of the complex roots becomes positive. This means that the steady-state is a source, that is, locally unstable. See figure 8 and 9.

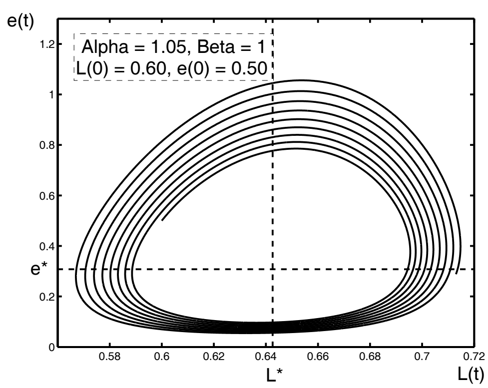

Figure 8: The source (A).

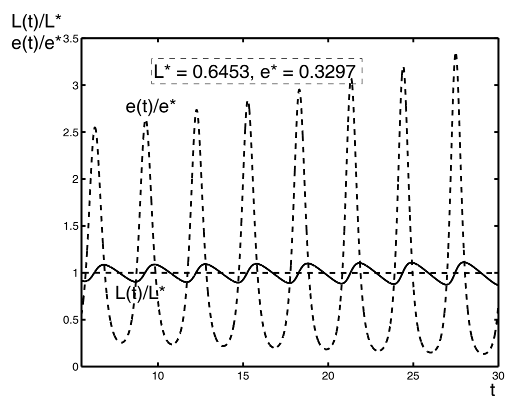

Figure 9: The source (B).

The results in table 2 suggest also that β is a bifurcation parameter for our dynamic system. In fact, the real part of the complex roots changes sign according to the value of β suggesting the occurrence of a Hopf bifurcation[42]. In particular, the complex roots become pure imaginary as β crosses the unity and this happens for values of β very close

[42]See, for example, Gandolfo (1997), Chapter 25.

to unity from the left. Using the calibration specified above and fixing $\alpha = 1.05$, the exact bifurcation value is $\beta_H = 0.9952$[43]. See figure 10.

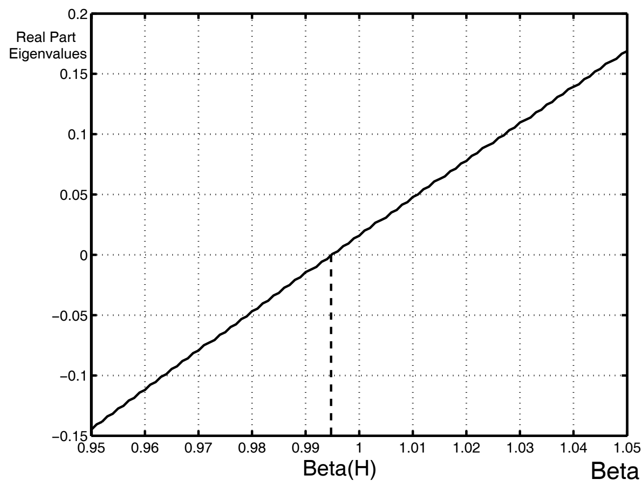

Figure 10: The real part of the complex conjugate eigenvalues

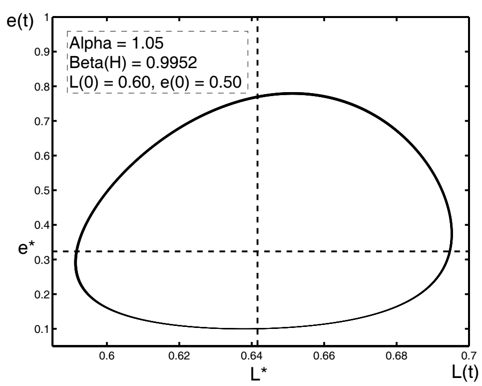

Figure 11: The limit cycle (A).

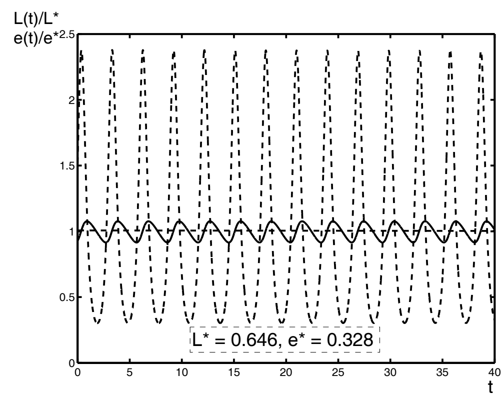

Figure 12: The limit cycle (B).

The occurrence of a Hopf bifurcation for values of β very close to unity from the left suggests that when the union is very slightly myopic in observing the unemployment rate, the optimal trajectory is represented by a closed orbit, that is, a limit cycle[44]. See figure 11 and 12.

[43]It would be possible to show that the bifurcation value of β depends on the value of the two discount rates. The lower are ρ^W and ρ^P, the closer to unity is the value of β such that the complex conjugate eigenvalues become pure imaginary.

[44]Using the discount rate as bifurcation parameter, a similar result was showed by Georges (2002) in

In our model the real wage is a non-linear function of the effort only. Therefore, whenever the optimal trajectory is given by a limit cycle, a similar circular relationship holds also between the employment and the real wage. Since such a circular relationship implies a very low degree of correlation between the employment and the wage, this kind of dynamic macroeconomic behaviour as been suggested as a rationalisation for the Dunlop-Tarshis observation according to which real wages are acyclical. See, for example, Coimbra (1999).

Observing figures 7, 9 and 12, we notice that our model delivers trajectories for the effort - and therefore for the real wage - displaying fluctuations that are wider than the corresponding fluctuations in the employment level. This results is at odds with the empirical evidence[45] and it should be due to the quite *ad hoc* manner in which effort determines the wage differential[46]. In fact, when α is higher than unity, the production technology displays a strong degree of increasing returns. However, the comparison of the different volatility degree displayed by the real wage and the employment is outside the aims of this contribution.

5 Is Equilibrium Unemployment a Worker Discipline Device?

Whenever it is possible to find a path converging to the unique steady-state (L^*, e^*), our model can be exploited to perform comparative statics exercises[47]. In particular, our framework might be useful to analyse the equilibrium effects of an improving in the monitoring technology, that is, the effects of an increase in the parameter q. Consider the case in which α is lower than unity[48]. Higher values of q leads the curve $\eta(L)$ to shift downward while the curve $G(L)$ shifts to the right. Therefore, the new equilibrium will be characterised by a higher employment rate and a lower level of effort. See the two panels of figure 13.

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a model without microfoundations. On the contrary, given particular parameter values, the limit cycle exhibited in our model is the result of an optimising behaviour.

[45]See, for example, Danthine and Donaldson (1990).

[46]See equation (12).

[47]Obviously, this happen when the steady-state (L^*, e^*) is a saddle point $(\alpha < 1)$ or a sink $(\alpha > 1, \beta < \beta_H)$.

[48]It would be possible to show that the equilibrium effects of an improved monitoring technology are the same even if α is higher than unity.

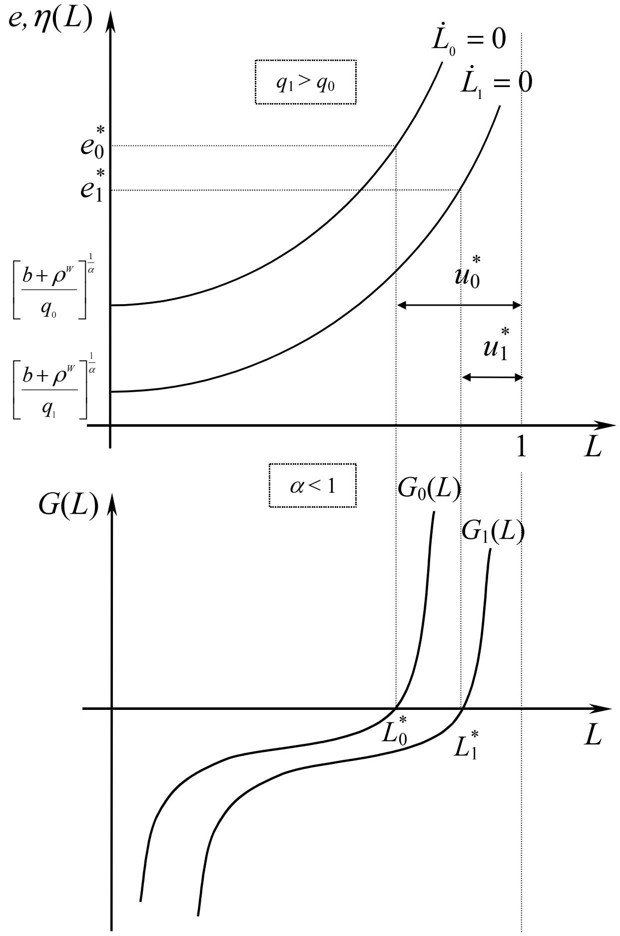

Figure 13: The effects of an improved monitoring technology.

The result illustrated in figure 13 suggests that in our model the effort is counter-cyclical, that is, equilibria characterised by higher (lower) unemployment rates are also characterised by higher (lower) levels of effort. As in the original model by Shapiro and Stiglitz (1984), this is consistent with the idea according to which unemployment acts like a threat and that threat is more pronounced when the unemployment rate is higher. This possibility is contemplated also in Uhlig and Xu (1996).

6 Concluding Remarks

This paper provided a simple dynamic framework that allow to endogenise the effort decision undertaken by the individual worker in the Shapiro-Stiglitz's (1984) shirking model. In particular, we derived the evolution law of employment arising from that partial equilibrium model assuming that the wage differential in effort terms was represented by a constant-elasticity function having effort as unique argument. Thereafter, we completed the picture adding the preferences of a union to whom is devolved the effort decision. Finally, we allowed this decisional process to be blurred by externalities on the unemployment rate and on the wage differential. These externalities allowed for the possibility of a "myopic" union behaviour.

The solution of the dynamic model that we developed shows that the way in which the wage differential in effort terms reacts to effort variations and the degree of myopicity of the union in observing the unemployment rate play a central role in determining the local dynamic behaviour of the economy.

In particular, when an increase in effort is associated to a less than proportional increase in the wage differential, the steady-state equilibrium of the model is represented by a saddle point. Therefore, in this case, controlling for the effort might resolve the indeterminacy from which is affected the standard model with constant effort. This results provides a sort of extension for the method proposed by Georges (1995) aimed to resolve indeterminacy in perfect foresight models.

On the other hand, when an increase in effort is associated to a more than proportional increase in the wage differential, the dynamics of the model becomes oscillatory: effort (and joint with it the real wage) and employment fluctuate over time. With an oscillatory dynamics, the union degree of myopicity in observing the unemployment rate is essential in order to distinguish between local stability and instability. In particular, when the union underestimate (overestimate) the unemployment rate, the local dynamics of the model is stable (unstable). As suggested by the literature on self-fulfilling prophecies, the case of complete stability (sink) allows to explain business fluctuations driven by the self-fulfilling beliefs of the agents and may justify the "animal spirit hypothesis" of business cycles.

Moreover, we showed that when the union underestimation of the unemployment rate is quite low, the optimal trajectory is represented by a closed orbit (limit cycle) implied by the occurrence of a Hopf bifurcation[49]. Since in our model the effort is the

[49] As long as we assume a positive discount, the occurrence of a Hopf bifurcation without myopicity is impossible using the specification of preferences described by equation (14).

only determinant of the real wage, such a circular relationship holds also between the labour earning and the employment level. This kind of local dynamics has been suggested to explain the low degree of correlation between employment and real wages originally observed by Dunlop (1938) and Tarshis (1939).

Finally, we found that - under our specification - the effort is countercyclical. As in the original contribution by Shapiro and Stiglitz (1984), unemployment acts like a threat, and this threat is more effective when the rate of unemployment is high. Therefore, stable equilibria characterised by higher (lower) unemployment rates are also characterised by higher (lower) levels of effort.

A possible development of this analysis should be the inclusion of capital in the general framework. In particular, the production side of the model should be modified in order to include the stock of productive capital together with the effort and the labour inputs. Such an inclusion necessarily would impose to take into consideration the dynamics of capital accumulation. In this perspective, the effort could be used as a determinant of the Solow's (1957) residual. A model of this kind, accounting for involuntary unemployment, should provide an improved understanding of the business cycle.

Another development could be the endogenization of other parameters, in particular q and b. Specifically, the shirking detention rate could be related to the amplitude of the real wage so that higher wage payments lead firms to improve their monitoring technology. On the other hand, introducing money in the model, the separation rate could be related to monetary shocks.

A Appendix

In our model, the wage-effort elasticity is given by

$$\varepsilon_{w,e} \equiv \frac{\partial w}{\partial e}\frac{e}{w} = 1 + \Phi(e) > 1 \tag{A1}$$

where $\Phi(e) \equiv \frac{ae^a}{1+e^a}$.

Obviously, whenever e is positive, $\Phi'(e) > 0$ ■

B Appendix

When the effort is constant, the equality $\dot{L}(t) = 0$ implies that

$$e^* = \left[\frac{b + \rho^W (1 - L^*)}{q (1 - L^*)}\right]^{\frac{1}{\alpha}} \tag{B1}$$

Thereafter,

$$\left.\frac{\partial \dot{L}(t)}{\partial L}\right|_{L=L^*} = -q\frac{b + \rho^W (1 - L^*)}{q (1 - L^*)} + \rho^W = -\frac{b}{1 - L^*} \equiv \xi \quad \blacksquare \tag{B2}$$

C Appendix

The elements of the Jacobian matrix are given by the derivatives with respect to L and e of the differential equations (21) and (19), each of them evaluated in the unique steady-state (L^*, e^*). The derivatives are the following

$$\frac{\partial \dot{L}}{\partial L} = -\beta (1 - L)^{\beta - 1} \left(qe^\alpha - \rho^W\right) \tag{C1}$$

$$\frac{\partial \dot{L}}{\partial e} = \alpha (1 - L)^\beta qe^{\alpha - 1} \tag{C2}$$

$$\frac{\partial \dot{e}}{\partial L} = \frac{qae^\alpha \left[1 - L (1 - \beta)\right]}{(1 - \alpha) L^2 (1 - L)^{1-\beta}} + \frac{e (1 - \beta)^2 \left(qe^\alpha - \rho^W\right)}{(1 - \alpha)(1 - L)^{2-\beta}} + \frac{e\beta b}{(1 - \alpha)(1 - L)^2} \tag{C3}$$

$$\frac{\partial \dot{e}}{\partial e} = \frac{\rho^P}{1 - \alpha} - \frac{(1 - L)^\beta qa\alpha e^{\alpha - 1}}{(1 - \alpha) L} + \frac{(1 - \beta)\left[qe^\alpha (1 + \alpha) - \rho^W\right]}{(1 - \alpha)(1 - L)^{1-\beta}} + \frac{\beta b}{(1 - \alpha)(1 - L)} \tag{C4}$$

Exploiting equation (22) and the definition of $\varphi(L)$, it is possible to derive the expressions in the text, that is

$$\left.\frac{\partial \dot{L}}{\partial L}\right|_{L=L^*, e=e^*} \equiv j_{1,1} = -\frac{\beta b}{1 - L^*} \tag{C5}$$

$$\left.\frac{\partial \dot{L}}{\partial e}\right|_{L=L^*, e=e^*} \equiv j_{1,2} = \frac{\alpha q (1 - L^*)^\beta}{\left[\eta (L^*)\right]^{1-\alpha}} \tag{C6}$$

$$\left.\frac{\partial \dot{e}}{\partial L}\right|_{L=L^*,e=e^*} \equiv j_{2,1} = \frac{\varphi(L^*)\left[1-L^*\left(1-\beta\right)\right]}{\left(1-L^*\right)\left(1-\alpha\right)\left(L^*\right)^2} + \frac{b\eta\left(L^*\right)\left[\left(1-\beta\right)^2+\beta\right]}{\left(1-\alpha\right)\left(1-L^*\right)^2} \tag{C7}$$

$$\left.\frac{\partial \dot{e}}{\partial e}\right|_{L=L^*,e=e^*} \equiv j_{2,2} = \frac{\left(1-\beta\right)\left[b+\varphi(L^*)\right]+\beta b+\rho^P\left(1-L^*\right)}{\left(1-\alpha\right)\left(1-L^*\right)} - \frac{qa\alpha\left(1-L^*\right)^\beta\left[\eta\left(L^*\right)\right]^{\alpha-1}}{\left(1-\alpha\right)L^*} \tag{C8}$$

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