Skills, Financial Markets and Grow

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Abstract

We study the connections between the development of financial markets and the availability of skilled labour in affecting the rate of growth of the economy. The model at the core of the paper features search-and-matching in financial markets as well as an "increasing varieties" mechanism as the engine of growth. To carry out $R\&D$ projects skilled workers (innovators) need to be combined with financial resources. Thus, in financial markets, innovators search for a suitable financier while financiers search for a suitable innovator.

We find that both more innovators and more efficient financial markets boost the rate of growth. Furthermore, the growth effect from more innovators tend to be higher if overcoming search frictions becomes costless and financial markets are "walrasian". Finally, increasing the efficiency of financial markets affects the distribution of income by improving the rent position of skilled individuals.

1 Introduction

Well functioning financial markets and a well educated population are commonly regarded as necessary prerequisites for a satisfactory performance on growth. On the one hand, since advancements in the knowledge base are the engine of economic progress, sustained growth requires a skilled population. On the other hand, abundant educated persons are not sufficient per se if they are not employed in innovative projects and endowed with sufficient resources to carry out these projects. Indeed, combining skills and resources in the making of innovation represents the ultimate role of financial activities.

In the existing literature, the linkages between finance and skills in the growth process do not appear to have been sufficiently considered. On the one hand, classical endogenous growth theory while emphasizing the role of skills has often dismissed financial arrangements as irrelevant. Lucas (1988), for instance, regards finance as an "over-stressed" determinant of growth. On the other hand, contributions emphasizing the role of finance tend to disregard the relevance of skills as their main focus is on the growth effects from a shift towards riskier investment projects allowed by more efficient financial arrangements [see, for instance, Bencivegna and Smith (1991), Obstfeld (1994) or Acemoglu and Zilibotti (1997)].

In this paper, we attempt to elucidate the roles of skill and finance in the growth process by studying a model where (endogenous) growth spurs from investments in applied industrial research which require both skills and resources to be carried out. We assume that skilled individuals can not borrow these resources nor can they insure the risk inherently attached to research and development ($R\&D$) activities. As a consequence, they need the intervention of "venture capitalists" willing to fund these activities. We also assume that transactions between skilled individuals and capitalists are not undertaken in a "walrasian" environment. Rather, these are subject to information, enforcement and transaction costs which need to be mitigated by resorting to specialised financial intermediaries. It is the bite of these costs, then, which explains the existence of financial markets.

On technical grounds the model borrows from two strands of theoretical literature, the so-called neo-shumpeterian growth literature (Grossman and Helpman, 1991; Aghion and Howitt, 1998) and the search and matching approach to market equilibrium (Pissarides, 2000). In particular, the model exhibits the following two basic features. First, growth originates through an "increasing variety" mechanism. New varieties represent the output of industrial innovation activity. Second, transaction, information and enforcement costs are modeled as search frictions. This means that skilled individuals willing to carry out an innovative project need to search for a suitable venture capitalist. Analogously, venture capitalists willing to fund an innovative project need to search for suitable skilled individuals. The search for the partner takes place in financial markets where all participants buy "matching" services from specialised intermediaries. As a consequence, financial development is modeled as an increase in the efficiency of these intermediaries.

Our main results are the following. First, we find that both more abundant skills and more developed financial markets lead to an higher rate of growth. Second, an increase in the number of skilled individuals has a stronger impact on the rate of growth if financial markets are "walrasian" instead of being plagued with frictions. Third, policies aimed at increasing the efficiency of financial markets lead both to an increase in the rate of growth and to an increase in the income flow accruing to skilled individuals. If these policies are accompanied by measures leading to more skill accumulation, then the effect on growth is magnified while the one on the income of the skilled is reduced. Overall, these results point to a complementarity nexus between skill and financial development in determining the rate of growth.

The plan of the paper is as follows. In section 2 we present the economic environment and derive the relevant behavioural relationships. In sections 3 we study the equilibrium rate of growth under the assumption that the number of skilled individuals is abundant with respect to the supply of venture capital. In section 4, instead, we deal with the more interesting case of scarce skill supply.

In section 5 we summarise the main conclusions and provide a short report on some empirical evidence consistent with our conclusions.

2 The Model

2.1 Assumptions

The economy is composed of three sectors - manufacturing, financial services and a research and development sector ($R\&D$) - and is populated by a measure L of unskilled workers and a measure S of skilled workers. Unskilled workers are employed in all sectors while the skilled are required only to perform $R\&D$ activities. All workers supply inelastically and costlessly a unit amount of labour services. Due to their specialisation, skilled individuals will also be referred as innovators throughout.

The net output of the economy is represented by a unique non-storable final (consumption) good. We assume that individuals live forever, discount the future at rate δ and have logaritmic preferences. Thus, the welfare function for the $j-th$ individual is

$$W_{j,t} = \int_t^{\infty} e^{-\delta(s-t)} \log(C_{j,s}) ds$$

Further, individuals have free access to a market of risk-free consumption loans whose interest rate is indicated by r.

The final good is manufactured by a competitive industry which uses a number of intermediate goods as the sole inputs, these latter are imperfect substitutes according to the following CES technology:

$$Y_t = \left[\int_0^{n_t} x_{i,t}^{\frac{\alpha-1}{\alpha}} di \right]^{\frac{\alpha}{\alpha-1}} \qquad \alpha > 1$$

Y_t represents the amount of final output produced at time t while $x_{i,t}$ represents the amount of the $i-th$ intermediate good which is used as an input. n_t indicates the number of intermediate goods that are marketed and used at t . Parameter α gives the elasticity of substitution between any pair of intermediate goods as well as the price elasticity of their conditional demand. As α increases, intermediate goods become more substitutable and their demand more elastic. Since the CES function is characterised by constant return of scale, the number of firms operating in the final sector is irrelevant.

Each intermediate good is produced by a monopolist which is endowed with a simple linear technology linking $x_{i,t}$ to the amount $l_{i,t}$ of employed unskilled labour:

$$x_{i,t} = l_{i,t}$$

Monopoly arises thanks to the perpetual validity of the patent filed by the innovator who first conceived and designed the intermediate good. Thus, intermediate goods are invented by successful innovators who, in turn, sell their patents to producers willing to become monopolists.

The invention of a new good by an innovator requires bright ideas as well as the investment of economic resources. We assume that any innovator is endowed at all times with a valuable idea which, once transformed into an industrial blueprint, gives birth to a *range* of new intermediate goods (i.e. a *range* of patents). To transform ideas into blueprints, however, innovators need to fuel resources into a *R&D* project. Furthermore, before the project begins, they also need to search for a financier willing to provide these resources. Below, we describe the details of the *R&D* process as well as the functioning of the market for *R&D* funds.

In addition to the engagement of the innovator, *R&D* projects require the employment of c unskilled workers at all times. Paying a wage to these workers along the whole duration of the project represents the obligation of the financier. The duration of the project is stochastic since (successful) termination arrives at a Poisson rate h. Once a blueprint is realised, the innovator files a range of patents and sell them to monopolists. We assume that the number of patents brought about by any single *R&D* project is given by λn_t, i.e. the number of new goods is proportional to the number of existing goods. More existing goods imply a wider range of applications for the new blueprint. This assumption conveys the effect of "building on the shoulder" of previous innovators, it represents a well known externality leading to endogenous growth in models with expanding varieties.

R&D funds are exchanged in an environment plagued with a host of informational and contractual imperfections which originate from the heterogeneity of agents on both sides of the market. Borrowing from Wasmer and Weil (2004) - and several other authors cited by them - we model the outcome of those imperfections as a search-and-matching context. We assume that each innovator needs to search for a suitable financier and that each financier needs to search for a suitable innovator. Accessing the market for *R&D* funds is free both for innovators and for financiers. Once an innovator has successfully terminated a project, she can stay at home or return to market activity and search for a financial partner willing to fund the next project. On the other hand, any financiers can access to market activity by borrowing funds in the market for risk-free consumption loans and searching for a partner in the market for *R&D* funds.

Despite the access to the market for *R&D* funds is free, operating in this market is costly for both kinds of agents. In particular, we assume that the latter must buy services from specialised intermediaries whose role is that of creating contacts between searching agents and favour their matching. We assume that entry in the "intermediation sector" is blockaded and that intermediaries do not require resources for their operations. As a consequence, they earn pure rents in the form of prices charged on searching agents.

The output of intermediaries consists of a flow of matches per unit time. This output is subject to the well known search externality in the sense that the flow of matches depends on the number of searching agents. Thus, let $\mathcal{I}_t$ and $\mathcal{F}_t$ represent the number of searching innovators and financiers at time t, the number of matches is determined by the following constant return to scale matching function:

$$M_t = M(\mathcal{I}_t, \mathcal{F}_t) \tag{1}$$

Let $q(\theta_t) \equiv M_t/\mathcal{I}_t$, $\theta_t = \mathcal{I}_t/\mathcal{F}_t$, represent the instantaneous conditional probability for an unmatched innovator to find a suitable financier. Constant returns imply that q is decreasing with respect to market tightness θ_t with an elasticity which is lower than unit. Analogously, $q(\theta_t)\theta_t$ represents the matching hazard for searching financiers.

Once an innovator and a financier are matched they bargain on the terms of a contract whereby the financier commits to pay a wage to those unskilled workers employed in the $R\&D$ project while the innovator commits to pay back to the financier a portion of the revenues accruing at the patent stage. This portion is bargained over by the two agents as soon as they match and the project begins.

Observe that in this economy innovators are both risk averse and exposed to two kinds of risks. The first risk is related to the uncertain duration of the search phase while unmatched and, as a consequence, to the uncertain amount of search costs. The second risk arises from the uncertain duration of the $R\&D$ project which, in turn, makes uncertain both the timing and the amount of patent revenue. Yet, due to the law of large numbers aggregate uncertainty is absent and we can safely postulate the existence of perfectly competitive insurance markets covering both kinds of risks. The relevance of such an assumption is twofold. First, together with the free access of financiers to the risk free market for consumption loans, it confines imperfections only to the $R\&D$ funds segment of financial markets (the "venture capital" market). Second, from a technical perspective, we may treat innovators as risk-neutral maximisers of their human wealth.[1]

2.2 Manufacturing

Let w_t represent the wage paid to unskilled workers. The price charged by the producer of the $i-th$ intermediate good is given by the imposition of a constant mark-up over the marginal cost:

[1]Of course, we do no deal with problems of moral hazard arising from full insurance. Observe, however, that both the search and matching activity and the $R\&D$ activity do not involve unobservable costly efforts. Thus, there is no incentive for the innovator to disattend provisions from insurance contracts.

$$p_{i,t} = \frac{\alpha}{\alpha - 1} w_t$$

Efficient expenditure on intermediate goods by firms operating in the final sector coupled with the absence of profits lead to the following pricing equation for the final good:

$$p_t = \left[\int_0^{n_t} p_{i,t}^{1-\alpha} di \right]^{\frac{1}{1-\alpha}}$$

Let the final output represent the numeraire of the economy - i.e. $p_t = 1$ - the last two equations imply that the wage increases with the number of intermediate goods:

$$w_t = \frac{\alpha - 1}{\alpha} n_t^{\frac{1}{\alpha - 1}} \tag{2}$$

Next, notice that intermediate goods enter symmetrically in the production of the final output. In turn, symmetry and uniform pricing imply that intermediate goods are all employed in the same amount: $x_{i,t} = x_t \quad i = 1....n_t$. Thus, if one indicates with L_Y the measure of unskilled workers constantly devoted to the production of intermediate goods, it must be true that $x_{i,t} = L_Y/n_t$ $i = 1....n_t$.

Substitute the last result in the production function for the final output and find that, thanks to the benefits of specialisation, Y_t increases with the measure of intermediate goods even if the amount of physical resources devoted to manufacturing is constant:

$$Y_t = L_Y n_t^{\frac{1}{\alpha - 1}}$$

Finally, let π_t represents the profit flow accruing to the representative intermediate monopolist:

$$\pi_t = x_{i,t}(p_{i,t} - w_t) = \frac{1}{\alpha} L_Y n_t^{\frac{2-\alpha}{\alpha - 1}} \tag{3}$$

On the one hand, π_t increases with n_t through the increase in the per-unit profit $(p_{i,t} - w_t)$. In fact, due to the benefits from specialisation, wages and prices both increase at the same rate while keeping a constant mark-up. On the other hand, π_t decreases with n_t as physical production x_t shrinks due to "business stealing" from new entrants. The combined effect depends on the size of α, if $\alpha > 2$ business stealing prevails.

2.3 Consumers

Full insurance coupled with free access to the risk-free consumption loans allow the j-th individual to spread over her planning horizon her current (human and non-human) wealth so as to satisfy at all times a standard Euler first order condition:

$$\dot{C}_{j,t}/C_{j,t} = r - \delta$$

Let g_n represent the rate of growth of n_t and, as a consequence, $\frac{1}{\alpha-1}g_n$ the rate of growth of Y_t. Since the Euler condition holds also for aggregate consumption and since aggregate consumption equals aggregate output, the equation above is equivalent to the one below:

$$\frac{1}{\alpha - 1} g_n = r - \delta \tag{4}$$

2.4 R&D

R&D projects present a per-period cost given by cw_t. If success arrives at time t, the innovator patents λn_t new products and immediately sells these patents to manufacturing firms. The value of each patent equals the present value of the ensuing flow of profits, straightforward calculations show that this value is given by $\pi_t/(\delta + g_n)$. In turn, the value of patents allows the computation of the value of the project V_t defined as the amount of expected revenues from the sale of patents less the amount of expected labour costs. We report the result of such a computation below:

$$V_t = \frac{1}{\delta + h}\left[-cw_t + h\ \lambda n_t\ \frac{\pi_t}{\delta + g_n}\right] \tag{5}$$

The amount within brackets represents the per-period expected net return from the project which jointly accrues to the two partners, the innovator and the financier, as long as their partnership continues. This flow is discounted at rate $\delta + h$ as δ accounts for time preference while h for the limited duration of the project.

2.5 Search-and-Matching

Let the couple (I_t^m, F_t^m) represent the value of an innovator and of a financier when they are matched and running a project and the couple (I_t^u, F_t^u) the corresponding values while unmatched.[2]

As soon as two partners match, the financier agrees to pay the flow of labour costs cw_t while the innovator commits to hand a portion $1 - \sigma$ ($\sigma < 1$) of future patent revenues. This portion represents the object over which the two partners bargain.

Since our focus below is on a stationary equilibrium, it is safe to assume that both agents start a new search immediately after a project has been successfully terminated. The value of a matched innovator I_t^m is then equal to the value of an asset which pays a lump sum revenue in case the project terminates plus a continuation value due to the fact that the innovator starts a

[2] There is a slight abuse of language here. In fact, we should refer to I_t^m and I_t^u as the human wealth of the innovator in the corresponding states.

new search upon termination. The lump-sum is given by patent revenues multiplied through σ, the continuation value is given by I_t^u. Analogously, the value of a matched financier is equivalent to that of an asset that presents a per-period holding costs cw_t and that provides a lump sum payment and a continuation value when the match terminates. The lump-sum is given by patent revenues multiplied through $1-\sigma$. Upon termination, the financier becomes unmatched and starts to search for a new innovator. Thus, F_t^u represents the continuation value.

Equations below give the Bellmans associated to these assets:

$$I_t^m = \frac{1}{\delta + h} h \left[\sigma \; \lambda n_t \; \frac{\pi_t}{\delta + g_n} + I_t^u \right] \tag{6}$$

$$F_t^m = -\frac{cw_t}{\delta + h} + \frac{h}{\delta + h} \left[(1-\sigma) \; \lambda n_t \; \frac{\pi_t}{\delta + g_n} + F_t^u \right] \tag{7}$$

Expressions in brackets represent the lump sum plus continuation value accruing at the time the project ends. Once these expressions are multiplied through h, the result can be interpreted as the expected flow-equivalent of these lump sums. In turn, flows are discounted at rate $\delta + h$ to account for time preference and limited duration.

By analogy, the value of searching agents must be equal to the expected return from search less the expected search costs. This boils down to the following couple of asset equations for I_t^u and F_t^u:

$$I_t^u = \frac{1}{\delta + q(\theta_t)} \left[-c_I w_t + q(\theta_t) I_t^m \right] \tag{8}$$

$$F_t^u = \frac{1}{\delta + \theta_t q(\theta_t)} \left[-c_F w_t + \theta_t q(\theta_t) F_t^m \right] \tag{9}$$

Expressions in brackets give the expected per-period net return from search for both agents. We assume that innovators are charged by intermediaries a per-period search fee of $c_I w_t$ while financiers are charged a fee of $c_F w_t$. Flows accruing to the two searchers are discounted at different rates since search ends at rate $q(\theta_t)$ for innovators and at rate $\theta_t q(\theta_t)$ for financiers.

Let NS_t represents the *net surplus* from the match:

$$NS_t \equiv (I_t^m - I_t^u) + (F_t^m - F_t^u) = V_t - \frac{\delta}{\delta + h}(I_t^u + F_t^u) \tag{10}$$

The first equality holds as a definition, the second comes from substituting equations 5, 6 and 7. We assume that the two parties determine the sharing parameter σ according to a generalised Nash bargaining over the net surplus from the match:

$$\sigma = \arg\max (I_t^m - I_t^u)^{\beta} (F_t^m - F_t^u)^{1-\beta} \quad 0 < \beta < 1 \tag{11}$$

Straightforward differentiation shows that the fraction of surplus accruing to each party is proportional to the party's relative bargaining power:

$$I_t^m - I_t^u = \beta\, NS_t \tag{12}$$

$$F_t^m - F_t^u = (1 - \beta)\, NS_t \tag{13}$$

2.6 Resource Constraints

Constraints arise as a consequence of the limited availability of skilled and unskilled workers. The unskilled worker constraint (U-constraint) is always binding since the marginal productivity of intermediate goods is always positive. By contrast, the skilled worker constraint (S-constraint) needs not to be binding. The search for a financier may be so costly and the expected duration of an $R\&D$ project so long that an innovator may decide not to participate to market activities.

Turning first to the U-constraint, observe that growth at constant rate g_n requires that a measure of g_n/λ projects terminates at all times and, as a consequence, that a measure of $g_n/h\lambda$ projects are running at all times. Thus, the measure of unskilled workers that operate in manufacturing L_Y is given by L less those workers that are constantly employed in the $R\&D$ sectors:

$$L_Y = L - c\frac{g_n}{h\lambda} \tag{14}$$

The S-constraint can be expressed in terms of g_n^{Max}, the maximum rate of growth which can be attained when all innovators participate to market activity. Thus, let $\mathcal{I}^m$ and $\mathcal{I}^u$, with $\mathcal{I}^m + \mathcal{I}^u = S$, represent the constant number of matched and unmatched innovators when the S-constraint is binding and the rate of growth is constant. Since matched innovators become unmatched at rate h while unmatched become matched at rate $q(\theta)$, equilibrium flows between the two states imply the following relationship between $\mathcal{I}^m$ and S:

$$\mathcal{I}^m = \frac{q(\theta)}{q(\theta) + h} S$$

Thus, since $h\mathcal{I}^m$ innovators reach the patent stage at all times, the number of new goods at time t (i.e. $\dot{n}_t$) is given by $h\mathcal{I}^m\ \lambda n_t$:

$$g_n^{Max}(\theta) = h\lambda \frac{q(\theta)}{q(\theta) + h} S \tag{15}$$

2.7 Equilibrium

In the next two sections we first study the equilibrium when the S-constraint is not binding and then pass to characterise the equilibrium when the constraint is binding.

In both equilibria, free entry in the market for $R\&D$ funds implies that the equilibrium value of an unmatched financier is nil:

$$F_t^u = 0 \tag{16}$$

By contrast, the value of unmatched innovators I_t^u is nil in the equilibrium with abundant skilled labour (S-constraint not binding) and non-negative in the equilibrium with scarce skilled labour (S-constraint binding).

Thus, when the S-constraint is not binding, the model is closed by equation 16 and the analogous of a free entry condition for unmatched innovators:

$$I_t^u = 0 \tag{17}$$

Also, in this equilibrium the rate of growth is lower than g_n^{Max}.

By contrast, when the S-constraint is binding, innovators are in short supply so that the rate of growth is equal to g_n^{Max}. In this case the model is closed by the free entry condition 16 and the S-constraint 15.

3 Growth and finance when skills are abundant

3.1 The equilibrium rate of growth

Substitute the two free entry conditions 16 and 17 in equations 10, 12 and 13 and find that the value of matched parties is proportional to V_t:

$$I_t^m = \beta V_t \tag{18}$$

$$F_t^m = (1-\beta) V_t \tag{19}$$

Free entry conditions 16 and 17 and asset equations 8 and 9 also imply:

$$I_t^m = \frac{1}{q(\theta_t)} c_I w_t \tag{20}$$

$$F_t^m = \frac{1}{\theta_t q(\theta_t)} c_F w_t \tag{21}$$

Putting together these results, it is straightforward to show that market tightness θ is independent from V_t and, as a consequence, from the rate of growth of the economy:

$$\theta^* = \frac{\beta}{1-\beta}\frac{c_F}{c_I} \tag{22}$$

The equilibrium rate of growth can be obtained either by equating the RHSs of equations 18 and 20 or the RHSs of equations 19 and 21. We choose the first alternative and, after a bit of manipulation, report below the resulting expression:

$$\frac{c_I w_t}{q(\theta^*)} + \frac{\beta c w_t}{\delta + h} = \frac{1}{\delta + h}\beta\ h\ \lambda n_t\ \frac{\pi_t}{\delta + g_n} \tag{23}$$

This expression can be interpreted as an arbitrage equilibrium condition since it equates the costs and the benefits accruing to the innovator from market participation. The first term on the LHS gives the expected total search cost paid by the innovator, $c_I w_t$ is the search flow cost while $1/q(\theta^*)$ represents the expected duration of search. The second term represents the fraction of wages paid by the innovator to the c unskilled workers that are employed in the $R\&D$ project. Although these wages are charged on the financier, bargaining over the surplus implies that the fraction β of the total wage bill is actually paid by the innovator. This bill is discounted at the rate $\delta + h$ to account for time preference and for the limited duration of the $R\&D$ project. The RHS of the expression represents the benefits from market participation. These benefits amount to the expected return from the sale of patents multiplied through the innovator's share β. Although patent revenues are a lump sum, multiplication through h converts the lump sum into an expected flow. In turn, this flow is discounted at the same rate $\delta + h$ applied on the wage bill.

The cost of market participation is independent from the rate of growth, g_n does not enter the LHS of the equation directly nor it affects the wage rate. By contrast, the rate of growth negatively affects the benefit on the RHS. First, g_n reduces the value of any single patent for any amount of resources L_Y devoted to manufacturing.[3] Second, the rate of growth reduces profits π_t by reducing the amount of unskilled labour L_Y employed in manufacturing (equation 3).

Substitute the U-constraint 14 as well as equations 2 and 3 in the arbitrage condition 23 and express the equilibrium rate of growth g_n^* only in terms of parameters and exogenous variables:

[3] The effect of the rate of growth on the value of a patent is the results of three different mechanisms. First, an higher g_n means that the per-unit profit $(p_{i,t} - w_t)$ increases at a faster rate thanks to the benefits from specialisation. Second, an higher g_n means faster "business stealing". Third, an higher g_n implies an higher interest rae r (equation 4) and, as a consequence, higher discounting ("capitalisation effect" in the growth literature).

In the present context, the first and the third mechanism are in balance so the value of patents may be thought of as shrinking with the rate of growth due to "business stealing".

$$g_n^* = \frac{\lambda h L - \delta(\alpha - 1)\left[c + \frac{c_I}{\beta q(\theta^*)}(h+\delta)\right]}{\alpha c + (\alpha - 1)\frac{c_I}{\beta q(\theta^*)}(h+\delta)} \tag{24}$$

3.2 Comparative statics

It is straightforward to show that the rate of growth increases with respect to the endowment of unskilled labour (L), the intensity of the $R\&D$ spill-over (λ) and, albeit after some computations, the arrival rate of patents (h). The rate of growth also increases if the matching hazard $q(.)$ increases for any given market tightness θ. By contrast, growth decreases with respect to $R\&D$ costs (c), the rate of time preference (δ), the share of total revenues accruing to wage earners (α) and with respect to financiers search costs (c_H) through the effect of the latter on equilibrium market tightness θ^*. The rate of growth also decreases with respect to innovators search costs (c_I). The elasticity of $q(\theta)$ is lower than unit. This, in turn, implies that the amount $c_I/q(\frac{\beta}{1-\beta}\frac{c_F}{c_I})$ increases with respect to c_I.

The endowment of skilled labour S does not affect growth. This is far from unexpected, however, in an equilibrium where skilled workers are in excess supply and the only bottleneck for growth is represented by the imperfections in financial markets.

The effect on the rate of growth from an increase in β is in principle ambiguous. On the one hand, for a given market tightness θ^*, if β increases innovators grab a larger share of revenue from the sale of patents. This stimulates their market participation and, henceforth, the $R\&D$ activity. On the other hand, a lower share of patent revenue for financiers discourages their market participation and increases equilibrium tightness θ^*. In turn, this implies a reduction in the matching rate for innovators $q(\theta^*)$ which has both a negative direct effect on the flow of matches for a given number of participating innovators and a negative induced effect on the incentive to participate.

To investigate the relative strength of these effects on the rate of growth we focus on the function $f(\beta) \equiv \beta q(\frac{\beta}{1-\beta}\frac{c_F}{c_I})$ and - after inspecting equation 24 - observe that the rate of growth g_n^* depends negatively on $f(\beta)$.

Proposition 1

$f(\beta)$ is U-shaped and has its minimum at $\beta^* = 1 - \eta(\theta^*) < 1$ with $\eta(\theta) \equiv q'(\theta)\theta/q(\theta)$.

Proof

By straightforward differentiation.□

Thus, the rate of growth reaches its maximum when the innovators share is β^*. In the labour market search literature the equality $\beta^* = 1 - \eta(\theta^*)$ is traditionally referred as the "Hosios condition" while β^* is interpreted as the sharing rule which leads to efficient unemployment by internalising all search externalities (Pissarides, 2000). In the present context, β^* maintains an "efficiency" interpretation as it represents the sharing rule which minimises equilibrium search costs $c_I/\beta q(\theta^*)$. The resulting rate of growth, however, should by no means be termed as efficient due to presence of a further externality, the one which is conveyed by the "building on the shoulder" effect.

In the remainder of this section, we would like to discuss the growth effect of financial development. In principle, we could model financial development either as an increase in the matching rate q for any given market tightness or as a reduction in search costs. In the first case, the market for $R\&D$ funds becomes more efficient in the sense that imperfections have less bearing. In the second case, the market becomes more efficient because overcoming imperfections becomes less costly. This difference, however, is largely immaterial both on theoretical grounds - as growth effects are qualitatively similar - as well as from an empirical perspective. Is information more detailed or is it available at a lower cost in financially developed countries? In other terms, the two cases appear observationally equivalent as they both lead to an increase in the overall amount of credit flowing to innovative firms.[4]

In the absence of a clear distinction between the two options, we choose to model financial development as a proportional reduction in cost parameters c_I and c_F. The main rationale for this choice is analytical simplicity. A shift in these parameters affects the balance between the return and the overall cost from market participation (equation 23) but it does not affect the skilled resource constraint for a given degree of market tightness (equation 15). By contrast, an exogenous shift in matching hazards affects both these margins and proves more difficult to analyse in the next section when we deal with the equilibrium under skill shortage.

Thus, under skill abundance, financial development - i.e. a uniform proportional reduction of c_I and c_F - leads to higher growth. Accessing financial markets becomes cheaper and this motivates the entry of more financiers as well as more innovators. Market tightness remains constant so that increased participation leads to more matches and more $R\&D$ projects. An higher rate of growth follows.

4 Growth and finance under skill shortage

4.1 The equilibrium rate of growth

When skills are in short supply, the value of an unmatched innovator is positive so that the net surplus from the match is:

$$NS_t = V_t - \frac{\delta}{\delta + h} I_t^u \tag{25}$$

[4]The amount of credit to private firms represents a popular metric used to measure financial development in the applied literature (Levine, 2005).

This equation needs to be considered together with the surplus sharing rule 12 and the asset equation 8 to form a system that may be solved with respect to the triple (NS_t, I_t^u, I_t^m). Below we report the solution for NS_t:

$$NS_t = \frac{V_t + c_I w_t / (\delta + h)}{1 + q(\theta)\beta / (\delta + h)} \tag{26}$$

Thus, the net surplus from the match does not coincide with the value from the *R&D* project due to the positive continuation value of innovators once the match terminates. Notice that, in contrast with the previous equilibrium , the surplus from the match depends on market tightness θ. In particular, the surplus increases with tightness as the latter reduces the matching hazard for unmatched innovators and, as a consequence, reduces their value. The surplus increases since the value of the unmatched innovator I_t^u affects NS_t negatively (equation 25). On the other hand, as in the previous equilibrium, the net surplus decreases with respect to g_n since the latter reduces the value of *R&D* projects V_t.

Use NS_t from equation 26 to determine the value of a matched financier F_t^m (equation 13), then substitute F_t^m in the asset equation for F_t^u (equation 9) and apply the free entry condition 16:

$$(1-\beta)\left[h\frac{\lambda n_t\ \pi_t}{\delta + g_n} - c_I\left(\theta^*/\theta - 1\right)w_t\right] = \frac{c_F w_t}{q(\theta)\theta}(\delta + h) + (1-\beta)\,c w_t \tag{27}$$

This expression equates the benefit and the cost accruing to a financier from market participation. It is equivalent to the free entry condition $F_t^u = 0$ and will also be referred below as the equilibrium "arbitrage" condition. The RHS represents the cost, the first term gives the search cost while the second the fraction of *R&D* costs paid by the financier. The LHS represents the benefit, the first term in the square brackets is the revenue from the sale of patents, the second term corrects for the positive value of unmatched innovators since it holds that $I_t^u = c_I\left(\theta^*/\theta - 1\right)w_t$. In an equilibrium with skill shortage market tightness θ is lower than the equilibrium value under no-shortage θ^*. Thus, the amount $(\theta^*/\theta - 1)$ is positive and becomes nil when shortage disappears, in this case equation 27 becomes similar to equation 23.

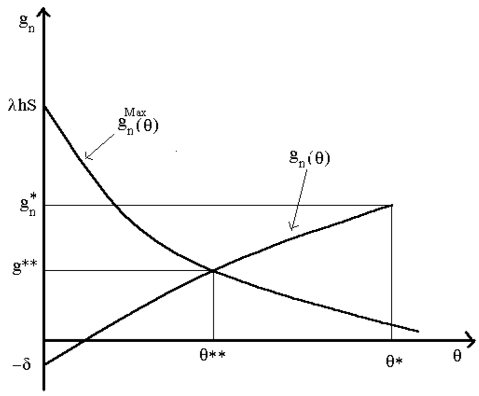
Substitute the expressions for π_t and w_t in the above condition and rearrange:

$$g_n(\theta) = \frac{h\lambda L - \delta(\alpha - 1)\left[c + c_I P(\theta)\right]}{\alpha c + c_I(\alpha - 1)P(\theta)} \tag{28}$$

$$P(\theta) \equiv \left(\frac{\theta^*}{\theta} - 1\right) + \frac{c_F}{c_I}\frac{h + \delta}{\theta q(\theta)}\frac{1}{1-\beta} \tag{29}$$

In contrast with equation 24 in the previous section, equation 28 does not provide the equilibrium rate of growth since market tightness θ is endogenous. This is a consequence of the matching surplus NS_t being dependent on tightness. The relationship between growth and market tightness represented by $g_n(\theta)$ is positive and turns out to be entirely conveyed through $P(\theta)$. What is the reason for $g_n(\theta)$ to be increasing? First, an higher θ increases the search hazard for financiers so that their value while unmatched tends to increase. Second, an higher θ decreases the search hazard for innovators and, as a consequence, decreases their market value. In turn, this implies an increase in the surplus from the match and a second reason for the value of unmatched financiers to increase. In equilibrium, however, the value of unmatched financiers must be zero so the rate of growth must increase to reduce the surplus of the match through a reduction of V_t.

Having established a positive "arbitrage" relationship between g_n and θ, to close the model we need a further link between these two variables. As discussed before, this second relationship is provided by the S-constraint in the equation 15. In figure 1 below we depict the S-constraint along with the function $g_n(\theta)$:

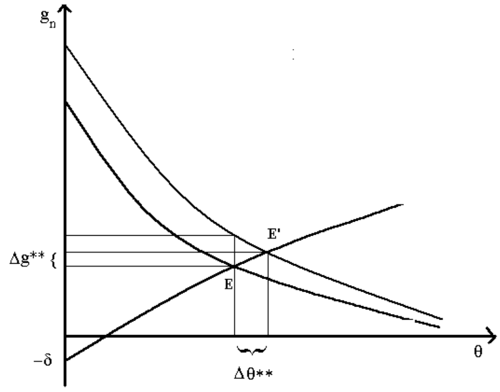


The equilibrium rate of growth

4.2 Comparative statics

In this subsection we characterise the comparative statics of the equilibrium by focusing on those parameters that appear more sensitive to policy intervention. In particular, we offer a qualitative evaluation of the effects coming from an increase in the endowment of skilled labour as well as from a move towards more developed financial markets.

An increase in S moves upwards the $g_n(\theta)$ locus (with a proportional shift) while leaving unaffected the "arbitrage" condition. Figure 2 below illustrates the equilibrium shift from point E to point E' and the ensuing increase both for the rate of growth g_n^{**} and for market tightness θ^{**}:



An increase in the skill endowement.

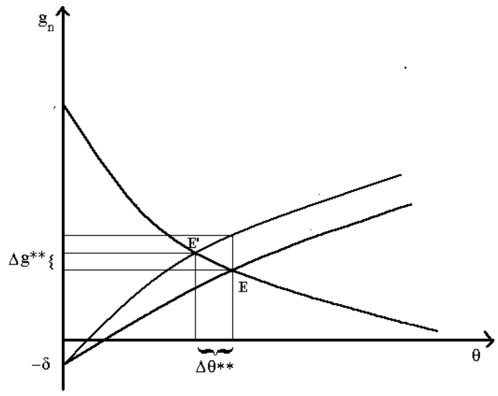
The initial impact of more skilled workers is a reduction in market tightness and a decrease in their market value. Both effects, in turn, lead to an increased participation of financiers and to an higher number of *R&D* projects per unit time. The figure makes it clear, however, that the increased participation of financiers is not sufficient to maintain market tightness at the initial level. Also, the rate of growth does not increase by as much as if tightness had remained constant. These results are explained by the fact that higher growth reduces the return from *R&D* projects and, henceforth, reduces incentives to participate for both kinds of agents. In turn, lower incentives reduce participation for financiers but reveal to be inconsequential for the participation of innovators as long as they remain in short supply and their market value is positive. Summing up, if S increases there are more skilled workers searching for a match but their matching hazard decreases since the entry of new financiers is not sufficient to avoid the increase in market tightness. The net effect on growth is positive.

To clarify the interaction between financial market imperfections and the skills-growth relationship, we make a detour and study the effect of an increase in S in a "walrasian" context. Thus, let us assume that search costs tend to nil in a context of skill shortage. It is rather easy to check that the rate of growth from the arbitrage relationship 28 becomes independent from θ:

$$\lim_{c_I, c_F \to 0} g_n(\theta) = \frac{h\lambda L - \delta(\alpha - 1)c}{\alpha c}$$

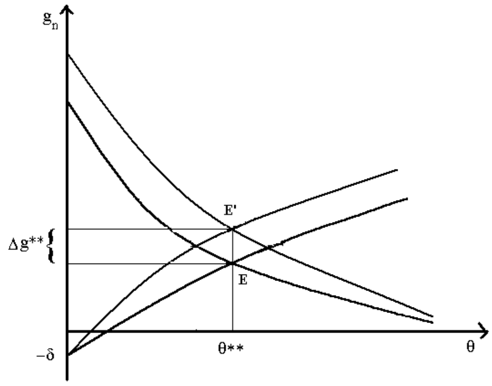
This is the expression for equilibrium growth found by Grossman and Helpman (G-H) (1991) in a context where financial markets are perfect and *R&D* does not require skilled workers. In the present context, instead, skilled workers are both necessary and in short supply. This means that the G-H rate of growth can not be achieved. Furthermore, since the G-H rate equates the costs and the benefits from market participation, the fact that it can not be achieved implies that, in equilibrium, there is a permanent positive incentive to participate both for financiers and for innovators. In turn, since the latter are in short supply, such a permanent incentive leads to a disproportion between innovators and financiers in the form of equilibrium market tightness tending to zero. With zero market tightness, the search hazard for innovators is infinite so that innovators are never unmatched and operate at all times in the $R\&D$ sector. As a consequence, the equilibrium rate of growth is given by $h\lambda S$, which means that growth reacts with a unit elasticity to variations in the skill endowment. This result should be contrasted with the one illustrated in figure 2. The figure, in fact, makes it clear that with imperfect financial markets the elasticity between growth and S is lower than unit. Summing up, skills are more effective in affecting growth if financial markets are "walrasian" in the sense that search costs are absent.

Turning to financial development, we observe that a proportional decrease in search costs shifts upward the "arbitrage" condition while leaving unaffected the S-constraint. This leads to a decrease in market tightness and to an increase in growth. Figure 3 below depicts the shift in equilibrium:



Financial development.

Lower search costs increase on impact incentives to participate for innovators as well as for financiers. However, only the number of financiers can increase since innovators are in short supply. This leads to a reduction in market tightness and to an increase in the value of innovators. Both effects, in turn, counteract the initial boost in incentives experienced by financiers from the reduction in search costs. The figure makes it clear that without the reduction in tightness, financial development would produce a stronger effect on growth. Or, by a reversed perspective, that financial development would be more effective in case it were accompanied by measures aimed at increasing the skill endowment. Figure 4 illustrates the combined effect of more skills and more developed markets:



Combining policies.

We conclude this section by noting that the development of financial market is itself a determinant of whether the equilibrium features skill shortage or skill abundance. With well developed financial markets the S-constraint and the arbitrage line are likely to cross on the left of θ^*. By contrast, if financial markets are sufficiently underdeveloped, such a crossing takes place on the right and θ^* so that the skill abundance equilibrium arises. Again a complementarity arises: an increase in skills is neutral for growth in an economy plagued with frictions that are strong enough to make skills redundant.

4.3 The value of skills

In the model, skills earn positive rents - $I^u > 0$ - as long as they are in short supply or, more formally, as long as equilibrium market tightness is lower than θ^*. Furthermore, as observed, being in short supply does not depend exclusively on S. Rather, it depends on S *and* on how developed are financial markets. As long as the equilibrium remains on the left of θ^*, an increase in S raises market tightness and, henceforth, decreases rents. By contrast, for a given S, more developed financial markets reduce tightness and increase rents.

Thus, development in financial markets benefits the whole economy by increasing the rate of growth but is bound to improve disproportionately the welfare of skilled as opposed to unskilled workers. This represents a further result that hints at the complementarity between skills and well functioning financial markets in the growth process.

5 Concluding remarks and empirical evidence (provisional)

The analysis conducted so far can be summarised as follows. First, the rate of growth increases with the endowment of skills if financial markets are developed enough so that the ensuing equilibrium is characterised by skill shortage. Further, an increase in skills is very effective at increasing the rate of growth if financial markets are walrasian in the sense that overcoming frictions is costless.

By contrast, the effectiveness of skills deteriorates if overcoming frictions is costly. Second, the rate of growth benefits from more developed financial markets although part of this development feeds into higher rents accruing to skilled individuals. Thus, to achieve satisfactory growth and distributive effects from financial development, the latter should be complemented by measures devoted to boost the skill endowment of the economy.

The complementarity between abundant skills and efficient financial markets in promoting economic growth is documented in the study of Evans, Green and Murinde (2002). Empirical work on the link between financial development and growth has usually emphasized the role of liquidity constraints in human capital investments. According to this view, financial development enhances growth by loosening these constraints and enabling more skill accumulation. De Gregorio (1996), however, finds that financial development increases the rate of growth even when the latter is conditioned on human capital variables. This suggests that financial development affects growth (also) through channels that are different from the financing of human capital accumulation. In this paper, we have proposed one of the possible alternatives. In our model, financial development affects growth by promoting the invention of new goods - and the opening of new markets - from a given endowment of skills. This accords with the finding of Rajan and Zingales (1998) whereby financial development promotes economic activity more through the opening of new firms than through the expansion of existing firms.

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