

# Structural Estimation of Search Models with Equilibrium Wage-Tenure Contracts<sup>⌘</sup>

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## Abstract

This paper studies the empirical implications of an equilibrium search model in which firms offer optimal wage-tenure contracts and workers are allowed to change their job for a more profitable career profile. We provide numerical simulations and propose a methodology for structural estimation of the Burdett and Coles (2003) equilibrium model. Using numerical simulations and reasonable parameter values, we show the concave shape of the baseline salary scale. Moreover, the functional form of the wage offer and earnings distribution functions and densities are established. We analyse first order predictions of the model using standard OLS estimations finding stronger tenure effects for workers at the bottom of the wage distribution. We also propose an estimation method in two steps. Our non-parametric estimate of transition parameters are equal to 0.08 and 0.02 for transitions to employment and to non-employment respectively.

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# 1 Introduction

This paper studies the empirical implications of an equilibrium search model in which firms offer optimal wage-tenure contracts and workers are allowed to change their job for a more profitable career profile. We provide numerical simulations of the Burdett and Coles (2003) model and look at its first order predictions using standard wage regressions. We also provide non-parametric estimates of transition parameters and propose a methodology for structural estimation of the model.

The contribution by Burdett and Coles (2003) adds considerably important elements to the theoretical analysis of equilibrium wage differentials with increasing wage-tenure profiles. Contrary to what is assumed in previous equilibrium search models (e.g., Burdett and Mortensen, 1998), the wage contracts offered in this environment specify wages as a function of tenure. The wage dispersion result comes from the trade-off between reducing quits and offering smooth consumption streams to risk averse workers. This is basically the main insight of the equilibrium search model. Higher quit probability reduces the employers' expected surplus, so the above trade-off implies a smooth wage-tenure profile.

This framework is of interest for two main reasons. First, estimating the structural parameters of the search model we can exploit equilibrium conditions to analyse the wage distribution. Secondly, it allows to have a further look at the empirical debate regarding the estimate the returns to firm tenure and provide a measure of the relative importance of firm-specific capital and selection effects.<sup>1</sup> In particular, the theoretical framework we adopt allows to clearly distinguish between wage growth due to job-to-job transitions and wage growth due to pure incentive effects. As a result we exclude human capital effects, as productivity in our model is fixed and constant across workers and firms. The dynamic environment considered allows also to shed some light on the evolution of wage inequality over time, and to disentangle the effect of returns to observables versus unobservables characteristics.

Finally, our paper is a first attempt to provide a structural estimation of an equilibrium search model with increasing wage-tenure profiles. Although the empirical literature dealing with structural estimation of search models is now well established (see Eckstein and Van den Berg, 2003 for a survey); to the best of our knowledge, ours represents the first paper that explicitly exploits the closed-form solution proposed for the wage-contracts offer distribution and the equilibrium conditions of the model to estimate returns to

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<sup>1</sup>See Altonji and Shakokto (1987), Topel (1991) and Dustmann and Meghir (2004).

tenure. In a very relevant paper, Postel-Vinay and Robin (2002) do assume in the wage setting mechanism is different from the standard Burdett and Mortensen (1998) model and generate wage-tenure effects, however their empirical analysis is mainly based on cross-section evidence and decomposition of wage variance into individual-, firm- and search components.

We use data from Italian administrative archives (INPS).<sup>2</sup> We first look at first order predictions of the Burdett and Coles (2003) in terms of tenure effects for workers in different quartiles of the wage distribution; then we provide a non-parametric estimation of transition parameters. Finally, we propose the following estimation method for the Burdett and Coles (2003) model. The minimum and maximum observed wage in the sample are consistent estimates of the lower and upper bounds of the distribution of wages posted. Using an utility function of the CRRA type, we provide a structural estimate of the relative risk aversion parameter. The estimation procedure is in five steps. First we use the non-parametric estimate of structural transition parameters from durations terminating into unemployment and those terminating into another job. Then, we construct the likelihood function and maximise with respect to the risk aversion parameter using previous non-parametric estimates and the theoretical wage offer distribution obtained in Burdett and Coles (2003). Having obtained such an estimate, we then predict the wage offer distribution. We also estimate non-parametrically the wage offer distribution from those observations of workers that accepted the job after unemployment. The latter represents the empirical wage offer distribution. Finally, we evaluate the pure tenure effects by comparing the two distributions and performing statistical tests on moments of the two.

The rest of the paper is organised as follows. In the next Section we briefly review the literature and provide an exposition of the Burdett and Coles (2003) equilibrium search model. We also highlight the important novelties introduced in the theoretical field and novel implications for the empirical analysis. In Section 3, after briefly describing the sample, we describe our estimation methodology and our preliminary results. Finally we discuss some further steps for this research.

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<sup>2</sup>Our aim is also to estimate the model using the English Household Panel (BHPS). Comparing two very different European countries as Italy and the UK, the paper provides also interesting insights about the role of institutional environments in shaping career profiles and labour market outcomes in general. A similar exercise has been fruitfully performed in a recent paper by Flinn (2002) who estimates the relevant parameters of a search model with on-the-job search for Italy and the US, finding that while cross-sectional wage differentials are remarkably higher in the US, lifetime welfare outcomes are surprisingly similar for the two countries. Higher labour market mobility, as reflected in structural parameters of the search model, are important in explaining this finding.

## 2 Equilibrium Search with Wage-Tenure Contracts

The basic Burdett and Mortensen (1998) equilibrium search model provides a rationale for wage differentials across identical workers and firms in an environment in which search frictions matter. If workers are allowed to search on-the-job, and firms engage in dynamic competition posting fixed-wage contracts, the resulting endogenous wage distribution is necessarily dispersed. The main result of the model still holds even if workers and/or firms are heterogeneous. In fact, heterogeneity is needed to get a satisfactory fit of the wage distribution (Bontemps et al., 2000; Bowlus et al., 2001). However, the assumption of a fixed-wage contract is clearly at odds with the increasing and concave wage-tenure profile that we observe in the data and the fact that most jobs do allow for promotions and wage growth over the duration of the match.<sup>3</sup>

In this environment, separations are due to quits or exogenous job destruction. As Mortensen (2003) discusses, quits are not efficient from the employer point of view, as rents are lost upon separation. Postel-Vinay and Robin (2002) explicitly address the issue modelling the wage formation as a sequential auction mechanism in which heterogeneous firms compete for workers matching outside offers. Their model provides a theory of intra-firm wage dispersion and wage dynamics over workers' career with wage-tenure effects. However, the main criticism for their modelling strategy is related to the difficulty of observing outside offers and then matching those offers. In particular, the matching strategy seems to be a difficult assumption in labour markets in which workers are homogeneous.

Stevens (2004) provides a different mechanism for the increasing wage-tenure profile. Assuming firms offer take-it-or-leave-it wage contracts and workers are risk neutral, she demonstrates that the optimal contract is in two steps. First workers are paid a wage strictly lower than their marginal product, then after a probation period, they are paid their marginal product. In this environment, no turnover results, and the distribution of contracts degenerates to a single common contract, i.e., no cross employer wage dispersion survives.<sup>4</sup> On the other hand, wage dispersion inside the firm is due to different employment durations.

Burdett and Coles (2003) re-establish the equilibrium wage dispersion outcome assuming workers are risk averse and are liquidity constrained. Em-

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<sup>3</sup>In this Section we leave aside other theories that are able to explain the concave wage-tenure profile and we just concentrate on the search approach.

<sup>4</sup>In this respect, this result is close in spirit to Diamond (1971).

employers offer wage-tenure contracts that trade-off a steeper profile (backloading of wages) to reduce employees' quit probability with a flatter one that smooths consumption flows. In this model, both intra and inter firms wage dispersion exist. The former is related to different tenure across workers at the same firm, while the latter emerges from a mixed strategy equilibrium in which firms offer different initial wage offers and make the same profits. A salient feature of the model is the existence of a common baseline salary scale that provides different starting points for wage-tenure contracts.

In what follows we introduce the Burdett and Coles (2003) equilibrium search framework and in the next Section we start with our numerical simulations. The model is set in continuous time and the focus is on steady states. Workers and firms are homogeneous and each match produces flow revenue  $p$ . Workers are allowed to search off- and on-the-job; however, the Poisson arrival rate of offers is assumed to be equal across states and equal to  $\lambda$ . Firms offer wage contracts specifying the future wage at each level of tenure at that firm. The rate of time preference is assumed equal to zero for both workers and firms. Firms maximise steady state flow profits. Workers are risk averse, liquidity constrained and infinitely lived. Jobs are exogenously destroyed at rate  $\delta$ .<sup>5</sup> The latter is also the inflow of new workers into the market. Finally  $b$  is the value of leisure and  $p > b > 0$ .

### 3 Empirical Analysis

#### 3.1 Simulation Results

In this Subsection, we provide some results from numerical simulations of the Burdett and Coles (2003) model. In particular we look at equilibrium as defined in Theorem 2 of the original paper without reporting any proof. We use a standard utility function of the CRRA type

$$u(w) = \frac{w^{1-\gamma}}{1-\gamma}$$

In the empirical analysis we provide a method for structural estimation of the relative risk aversion parameter  $\gamma$ . [Note that if  $u(w) = \ln w$ , then  $\gamma = 1$ :] The equilibrium defines a baseline salary scale of the form

$$\frac{dw}{dt} = \frac{\delta}{p_i} \frac{p_i w}{u(w)} - \frac{p_i w}{p_i} \int_0^w \frac{u'(x)}{x} dx; \quad (1)$$

<sup>5</sup>In the paper, Burdett and Coles (2003) assume workers are infinitely lived and have a probability  $\delta$  of death every period. We slightly change the model to take into account of empirical transitions towards a more plausible non-employment state.

Table 1: Parameters for the Baseline Salary Scale

$\frac{3}{4}$	$\pm$	$\delta$	$p$	$b$
0.1	0.01	0.1	5	4.6

subject to  $w = \underline{w}$  at  $t = 0$ . The parameters used for the simulation are reported in Table 1. In this specification, while the risk aversion parameter is arbitrarily chosen, transition rates are reported to match empirical results from the equilibrium search literature; in particular, the order of magnitude is the one found in most studies. In Figure 1 we report the evolution of the baseline salary scale. Its concave profile is immediately clear, and different parameters' settings do not change the qualitative results. On the worker's side, the model predicts that the expected value of employment, reported in Figure 2, increases over time and is given by the following expression

$$\frac{dV_s}{dt} = \frac{1}{2} \rho \frac{1}{p_i} \frac{1}{w} \int_{\underline{w}}^{\bar{w}} \frac{u^0(x)}{p_i x} dx; \quad (2)$$

Expected profits per worker are reported in Figure 3 and are given by the expression

$$\pi_w = \frac{1}{\pm} \frac{\rho}{(p_i w)(p_i \bar{w})}; \quad (3)$$

for  $w > \underline{w}$ : The equilibrium wage offer and earnings distribution functions are reported in Figure 4. Their analytical expressions are as follows

$$K(w) = \frac{\pm}{\delta} \frac{\rho}{p_i w} \frac{1}{p_i \bar{w}}; \quad (4)$$

$$F(w) = 1 - \frac{\pm}{\delta} \frac{\rho}{p_i \bar{w}} \frac{1}{p_i w} \frac{1}{2u^0(w)} \int_{\underline{w}}^{\bar{w}} \frac{u^0(x)}{(p_i \bar{w})(p_i x)} dx; \quad (5)$$

Where the latter has a mass point at  $\underline{w}$  where

$$F(\underline{w}) = \frac{\delta + \pm}{\delta} \frac{u(b) - u(\underline{w})}{u^0(\underline{w})};$$

The support of wages paid  $[\underline{w}; \bar{w}]$  is non-degenerate and implies  $\underline{w} < b; \bar{w} < p$  satisfying

$$\frac{\delta + \pm}{\delta} = \frac{p_i \bar{w}}{p_i \underline{w}}; \quad (6)$$

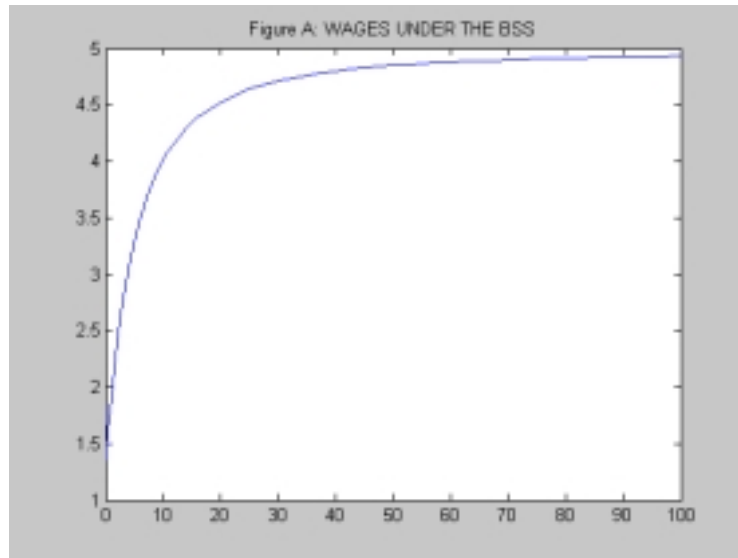


Figure 1: Wages under the Baseline Salary Scale

$$u(w) = u(b) + \frac{p_i w}{2} \int_w^{\bar{w}} \frac{u^0(x) dx}{p_i x} \quad (7)$$

The equilibrium wage  $w^*$  and earnings densities are reported in Figures 5 and 6 respectively using the expressions below

$$k(w) = \frac{p_i w}{2} \frac{(p_i w)^{1-\alpha}}{(p_i w)^{3-\alpha}} \quad (8)$$

$$f(w) = \frac{p_i u^0(w)}{u^0(w)^2} \int_w^{\bar{w}} \frac{u^0(x)}{(p_i w)(p_i x)} dx \quad (9)$$

As in the original Burdett and Mortensen (1998) model, the shape of the earnings density is at odds with the one observed in the data and further research is required to provide a satisfactory fit of the Burdett and Coles (2003) model to the data.

After providing basic simulation results of the model we then analyse its quantitative implications in terms of wage growth due to job-to-job transitions and tenure effects. Using 5,000 replications we simulate workers' careers using the baseline model as described above. Our finding indicates that over the working life about 30% of wage growth is due to transition effects, while the remaining 70% is due to tenure effects.

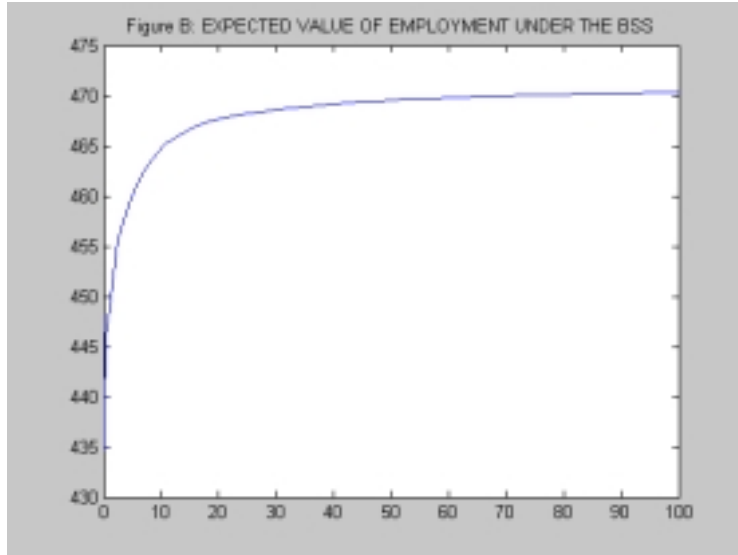


Figure 2: Expected Value of Employment under the Baseline Salary Scale

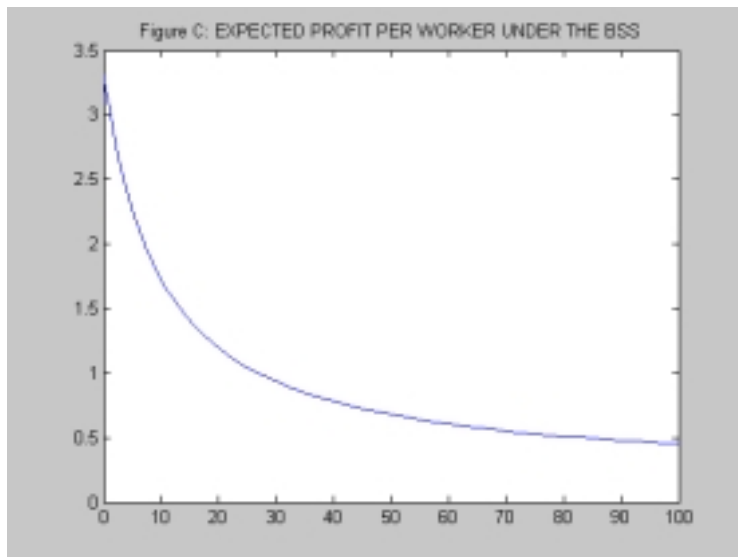


Figure 3: Expected Profit per Worker under the Baseline Salary Scale



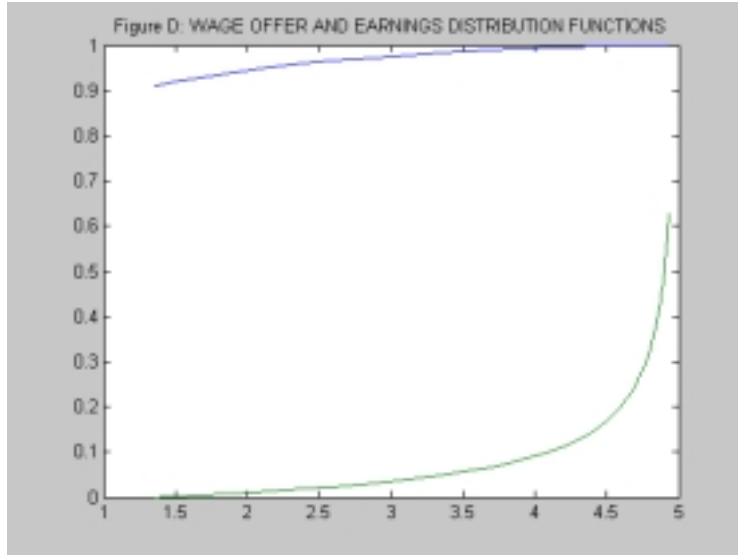


Figure 4: Wage Offer and Earnings Distribution Functions

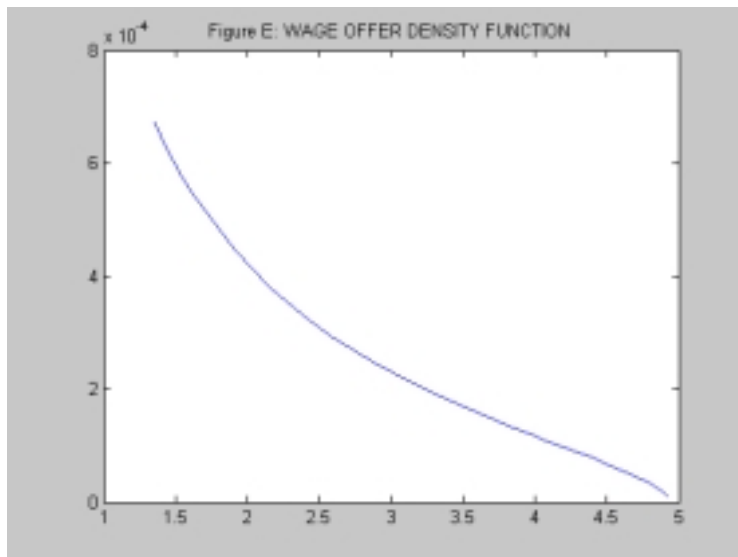


Figure 5: Wage Offer Density

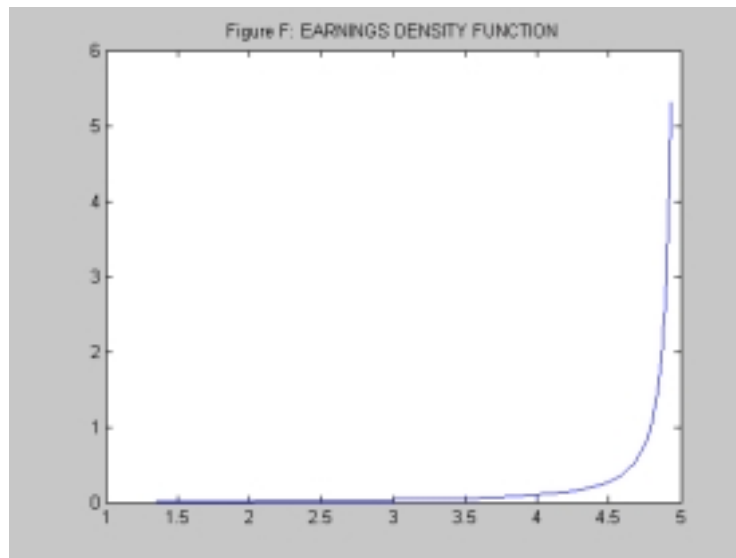


Figure 6: Earnings Density

### 3.2 Data and Sample

The data used in this study is from the Italian Administrative Social Security Archives (INPS).<sup>6</sup> Detailed information about labour market histories of workers employed in the private sector is available for the period 1985-1996. Demographic characteristics of workers are matched with relevant information about the firm they are currently working at, as sector of activity, average number of employees and age of firm. Given the longitudinal structure of the data, it is possible to track the entire career of workers and easily construct the variables object of study. Information about daily and yearly wages are available as the total number of days paid during the year, allowing to construct a measure of monthly wage directly comparable across workers. From the dataset, we extract a subsample of workers that entered the labour market between 1985 and 1995. In particular, we focus on blue collars male workers in the metal industry, between 15 and 50 years of age.

We consider a sample of 9,801 male blue-collar workers employed in manufacturing in 1991. We distinguish between movers and stayers. In particular, we look at the sample of workers in 1991 and consider their status at that moment: employed or unemployed. For employed people we first check if their spell is going to end before the observation period or not (censoring). Around 89% of workers in the sample are currently employed. About half

<sup>6</sup>See Casavola et al. (2000) for a complete description of the dataset.

Table 2: Descriptive Statistics

	all		employed			unemployed	
		uncensored		all	censored	all	
		quits	layoffs				
monthly wage	2941 (754)	2923 (754)	2903 (723)	2905 (727)	3024 (639)	2959 (691)	2801 (1126)
tenure	6:58 (3:76)	2:82 (2:10)	5:41 (3:04)	5:07 (3:06)	9:64 (2:25)	7:14 (3:55)	2:22 (2:18)
unempl. duration	{	{	{	{	{	{	1:81 (1:75)
observations	9801	621	4150	4771	3929	8700	1101

Durations are expressed in years. For unemployed, tenure is in previous job. Monetary values are in 000s of Italian Lira. St dev in parenthesis.

of them are censored observations. Descriptive statistics for monthly wages and tenure are in Table 2.

Unsurprisingly, our descriptive evidence shows that on average, employed workers have higher wages and longer tenure than unemployed ones. It is also interesting to note that across employed people, those that quit in the future earn lower wages than those that stay, but higher than those that terminate the job for other reasons. On the other hand, the tenure variable indicates that quitters have shorter job durations (2.82) and that those that are stable in their jobs have almost ten years of tenure.

### 3.3 Results

The first step of our analysis is the estimation of raw returns to tenure using a standard OLS estimator.<sup>7</sup> We estimate the following regression

$$\log w_i = \beta_1 + \beta_2 \text{tenure}_i + \beta_3 \text{tenure}_i^2 + \beta_4 \text{quantile}_i + \beta_5 \text{ten}_i \times \text{quant}_i + \epsilon_i \quad (10)$$

where  $\epsilon_i$  is a disturbance error. We start regressing the wage on a quadratic in tenure and then add the other regressors, the position in the wage distribution and an interaction term for the position in the distribution and tenure. In the first three columns of Table 3 we show regression results for the all sample, then in the remaining columns we run the same regressions separately for employed and unemployed workers. For unemployed people, we consider

<sup>7</sup>Needless to say, we are well aware of standard endogeneity problems we had to deal with for unbiased estimates. At this stage, we consider our linear regressions as descriptive evidence.

Table 3: Wage Regressions I

	All			Employed			Unemployed		
tenure	0.018 (7.83)	0.002 (2.80)	0.004 (4.71)	0.014 (5.51)	0.002 (2.43)	0.002 (2.95)	0.033 (2.75)	0.017 (3.48)	0.014 (2.76)
tenure <sup>2</sup>	-0.000 (-0.61)	-0.000 (-4.67)	-0.000 (-1.61)	0.000 (0.78)	-0.000 (-3.32)	-0.000 (-2.38)	-0.001 (-0.72)	-0.001 (-2.72)	-0.002 (-3.01)
quantile		0.075 (251.9)	0.080 (139.5)		0.073 (260.7)	0.075 (123.7)		0.090 (71.13)	0.087 (47.55)
tenure $\times$ quant			-0.000 (-9.29)			-0.000 (-2.73)			0.000 (1.66)
obs	9801	9801	9801	8700	8700	8700	1101	1101	1101
R <sup>2</sup>	0.07	0.87	0.87	0.07	0.89	0.89	0.03	0.82	0.82
F	412.27	23207	17578	349.16	24718	18554	17.72	1752.8	1317.4

Dependent variable: log of monthly wage. t-statistics in parentheses.

tenure they had before unemployment.<sup>8</sup>

Our estimates show that tenure has always a positive statistically significant effect on wages; the expected concave shape of the wage-tenure profile is also found with a negative sign for tenure squared. One additional year of tenure increases wage by around 1% in the all sample. However, some differences emerge among currently employed and unemployed workers. As far as employed workers are concerned, it is no surprise to find that their current wages are higher if tenure is higher. However, the result for unemployed workers is quite striking. Our results show that when they find a job, the effect of previous tenure on their re-employment wages is positive indicating that firm-specific capital is transferable and they don't seem to incur any big loss upon losing their previous jobs. This result is at odds with many studies that show that previous tenure has a negative effect on re-employment wages.

Controlling for the position in the wage distribution (columns 2 and 3) we observe a sharp reduction in the magnitude of the tenure effect (for the whole sample it drops from 0.018 to 0.002) and a clear positive effect on the wage. In other words we (trivially) verify that higher positions in the wage distribution are associated with higher wages. Moreover, comparing workers in the same part of the distribution, we find that higher tenure always increases wage.

<sup>8</sup>Although we run some regressions distinguishing between censored and uncensored tenure spells, at this stage we don't include them in the analysis as results are very similar to those reported in the paper.

Table 4: Wage Regressions II: Employed Workers

	Quits			Layoffs		
tenure	0:034 (1:88)	i 0:003 (i 0:60)	i 0:004 (i 0:65)	0:016 (5:54)	0:001 (1:97)	0:002 (2:32)
tenure <sup>2</sup>	i 0:000 (i 0:24)	0:000 (0:79)	0:000 (0:71)	0:000 (0:64)	i 0:000 (i 2:84)	i 0:000 (i 2:02)
quantile		0:075 (70:40)	0:075 (42:25)		0:073 (250:0)	0:074 (110:1)
ten $\times$ quant			0:000 (0:25)			i 0:000 (i 2:20)
obs	621	621	621	8079	8079	8079
R <sup>2</sup>	0.07	0.89	0.89	0.08	0.89	0.89
F	23.95	1796.2	1345.1	356.49	22911	17193

Dependent variable: log of monthly wage. t-statistics in parentheses.

In Table 4 we run the same regression as in equation (10) considering only the sample of currently employed workers. We distinguish between those that quit at the end of the period and those that are laid-off, or end up to unemployment. Again, separating the sample we do observe interesting differences. First, we find that the tenure effect is twice as high for quitters than for layoffs (0.034 against 0.016). This result is compatible with the first order prediction of the Burdett and Coles (2003) model that predicts steeper wage-tenure profiles to retain workers. This is the incentive effect firms have to exploit to avoid workers to leave. Controlling for the position in the wage distribution we observe that tenure doesn't have any significance in explaining wages.

Our results suggest that tenure effects and transition probabilities can play different roles for workers in different part of the wage distribution. To further investigate this, we run a regression separately for workers in each quartile of the wage distribution and check the effect of tenure. Results are in Table 5.

The undoubtedly interesting result is that tenure effects are strong and significant only for workers at the bottom of the wage distribution. Estimations for the all sample indicate that returns to tenure become negative for workers at the top of the distribution. Again interesting differences emerge when comparing employed and unemployed workers. In particular, we observe that the statistical significance of our coefficient is very low for quitters and for all workers but in the first quartile. This probably indicates that tenure profiles have to be steeper in this part of the distribution to avoid workers to quit.

Table 5: Tenure Effects by Quartile

quartile	all	employed		unemployed	
		all	quits	layoffs	
first	0:012 (5:69)	0:006 (2:70)	0:013 (0:82)	0:006 (2:40)	0:030 (3:80)
second	0:000 (0:64)	0:000 (0:45)	0:000 (0:02)	0:000 (i 0:25)	0:002 (i 0:65)
third	0:001 (i 1:93)	0:002 (i 2:02)	0:008 (1:31)	0:001 (i 1:54)	0:004 (i 1:29)
fourth	0:008 (i 2:26)	0:000 (0:18)	0:032 (i 1:31)	0:000 (i 0:16)	0:020 (0:93)

Dependent variable: log of monthly wage. t-statistics in parentheses.

Table 6: Transition Parameters

	Employed		Unemployed
	quits ( $\lambda_1$ )	layoffs ( $\lambda$ )	$\lambda_0$
transition parameter	0.07	0.02	0.16
average wage	2923.76	2961.99	2801.36
observations	621	8079	1101

Before proceeding to structural estimation, in Table 6 below we report non-parametric estimates of our transition parameters with average wages for separate groups. We calculate the inverse of the duration of employment and unemployment. Our results show that the arrival rate of offers when unemployed is higher than the same rate for employed workers indicating that search is more effective in that state (0.16 against 0.07). The job destruction rate is estimated to be around 2% per month. Since the theoretical model has a common arrival rate of offers  $\lambda = \lambda_0 = \lambda_1$ , we proceed calculating  $\lambda$  as a weighted average between  $\lambda_0$  and  $\lambda_1$ ; where weights are proportion of workers in the two states (11% and 89% respectively). Our estimate of  $\lambda$  is 0:08. In Table 6 we also report figures for average wages of workers in each state. As expected, workers that were previously unemployed earn a lower wage indicating that the distribution of wages for employed workers is shifted to the right. Interestingly, the average wage for quitters is lower and the average wage of workers that terminate their job into non-employment, indicating that on average workers with lower wages are more likely to quit.

### 3.4 Estimation Method

In what follows we describe our estimation method.

2 The minimum  $\underline{w}$  and maximum  $\overline{w}$  observed wage in the sample are consistent estimates of the lower and upper bounds of the distribution of wages posted in the model. In the theoretical model, the "foot on the door effect" implies that  $\underline{w} < b$ : Moreover,  $\overline{w} < p$ .

2 The utility function is of the CRRA type

$$u(w) = \frac{w^{1-\gamma} - 1}{1-\gamma}$$

In the empirical analysis we provide a structural estimate of the relative risk aversion parameter  $\gamma$ . [Note that if  $u(w) = \ln w$ , then  $\gamma = 1$ ]

2 The estimation procedure is in 3 steps:

1. Get a non-parametric estimate of  $\lambda$  and  $\mu$  from durations terminating into unemployment and those terminating into another job. We call them  $\hat{\lambda}$  and  $\hat{\mu}$ :
2. We construct the likelihood function and maximise with respect to  $\gamma$  using non parametric estimates of the transition parameters  $\hat{\lambda}$  and  $\hat{\mu}$  and the theoretical  $F(w)$  obtained by Burdett and Coles (2003). The likelihood for employed workers is the multiplication of the probabilities below

$$\log L(\gamma) = \sum_{i=1}^n \left[ \lambda + \mu (1 - F(w_i)) \right] \exp \left[ -(\lambda + \mu (1 - F(w_i))) \text{dur}_i \right] \prod_{j=1}^n [1 - F(w_j)]^{1 - j} \prod_{i=1}^n \mathbb{1}_{\{w_i > w_{i-1}\}}$$

where

$$e = \frac{\lambda}{\lambda + \mu}$$

is the sampling probability and dur is duration of the spell. The equilibrium wage earnings density is obtained by differentiating  $K(w)$  in the paper and is equal to

$$k(w) = \frac{\lambda}{2\mu} \frac{(p - w)^{1-\gamma}}{(p - w)^{3-\gamma}} \quad (11)$$

Employed durations are assumed to be exponentially distributed, and  $1 - F(w)$  is equation (21) in the paper and reads

$$1 - F(w) = \frac{\lambda}{\mu} \int_w^{\overline{w}} \frac{p - x}{p - w} \frac{1}{2u^0(w)} \frac{u^0(x)}{(p - w)(p - x)} dx \quad (12)$$

Finally the common productivity parameter is a weighted average of the upper and lower bounds of the wage offer distribution, where weights depend on transition parameters.

$$p = \frac{1 - \beta_2}{1 - \beta_1 + \beta_2} w_i + \frac{\beta_2}{1 - \beta_1 + \beta_2} w_{i+1} \quad (13)$$

3. Having  $\beta_1, \beta_2, \beta_3$  then predict  $F(w)$ ; call it  $F^d(w)$ ; [At this stage it is also possible to structurally estimate all the three parameters again, we don't do it now.]
4. Estimate non parametrically  $F(w)$  from those that accepted the job after unemployment. [We do it using a standard empirical cdf.]
5. Compare the two distributions and perform statistical tests on moments of the two.

Before proceeding with estimation, it is important to remind some identification issues we have to deal with. Papers by Dey and Flinn (2003), Flinn (2003), Eckstein and Van den Berg (2003) and Heckman and Flinn (1982) are essential references in this respect.

## 4 Final Comments

In this paper, we study the empirical implications of an equilibrium search model in which firms offer optimal wage-tenure contracts and workers are allowed to change their job for a more profitable career profile. We offer a very preliminary exposition of the structural estimation approach and first numerical simulations of the Burdett and Coles (2003) model. We also provide some preliminary evidence of tenure effects and quitting behaviour of workers using standard OLS methods.

The paper by Burdett and Coles (2003) adds considerably important elements to the theoretical analysis of equilibrium wage differentials with increasing wage-tenure profiles. Contrary to what is assumed in previous equilibrium search models (e.g., Burdett and Mortensen, 1998), the wage contracts offered in this environment specify wages as a function of tenure. The wage dispersion result comes from the trade-off between reducing quits and offering smooth consumption streams to risk averse workers basically following the main insight of the basic equilibrium search model. Higher quit probability reduces the employers expected surplus, so the above trade-off implies a smooth wage-tenure profile.



Using data from Italian Administrative archives (INPS), we estimate the raw effect of tenure of a magnitude of 1% per year. Controlling for the position in the wage distribution, we find that the positive effect of tenure is still significant but much smaller. Surprisingly we find that the effect of tenure is positive also for previously unemployed workers; we interpret this as an indicator of transferability of skills. Using the sample of currently employed workers, we compare quitters and layoffs finding that tenure effects are stronger for the first group. We interpret this as evidence of the incentive effect firms use to retain workers as predicted by the Burdett and Coles (2003) model. What is more, we find that tenure effects are important only for workers that are at the bottom of the wage distribution.

Finally we propose the following estimation method for the Burdett and Coles (2003) model. The minimum and maximum observed wage in the sample are consistent estimates of the lower and upper bounds of the distribution of wages posted. Using an utility function of the CRRA type, we provide a structural estimate of the relative risk aversion parameter. The estimation procedure is in five steps. First we get a non-parametric estimate of structural transition parameters: we find that the arrival rate of offers when unemployed is higher than the same rate for employed workers indicating that search is more effective in that state (0.16 against 0.07). The job destruction rate is estimated to be around 2% per month. Then, we construct the likelihood function and maximise with respect to the risk aversion parameter using previous non-parametric estimates and the theoretical wage offer distribution obtained in Burdett and Coles (2003). Having obtained such an estimate, we then predict the wage offer distribution. We also estimate non-parametrically the wage offer distribution from those observations of workers that accepted the job after unemployment. The latter represents the empirical wage offer distribution. Finally, we evaluate the pure tenure effects by comparing the two distributions and performing statistical tests on moments of the two.

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