Capital-Skill Complementarity, Embodied Technological Progress and Unemployment Inequality

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Abstract

We build an intertemporal general equilibrium framework with a double heterogeneity: simple *vs.* complex jobs and low *vs.* high skilled workers. The key feature of our model is *Capital-Skill Complementarity* in the final sector aggregate production function. A four-factor production function with different substitution elasticities between the two types of capital and the two types of labour is employed. In such a way, biased technological change is introduced *via* CSC and embodied technical progress. We quantitatively assess the contributions of different explanations to the Belgian labour market dynamics over the period $1976 - 1996$ decomposing relative labour demand shifts into a skill-biased effect and a capital-skill complementarity effect. We conclude that the former turns out to be the main force driving the rise in the relative demand for skills, whereas CSC through the growth in the equipment stock accounts for a smaller part of that.

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1 Introduction

Over the last decades striking changes in labour market conditions across skill groups have emerged in all OECD economies. Still remain however important country-specific peculiarities. In the US the skill premium, the ratio of the wage rate of skilled workers to that of unskilled workers, has reached its highest level over postwar period (it has risen by 30% since 1980), whereas the aggregate labour share of income has slightly declined and the unemployment rate has remained remarkably stable. On the contrary, in front of small variations in the wage structure the European economies (except for UK) have witnessed a drop in the labour share (by roughly 5%) and the emergence of significant differences in unemployment rate (on average increased by 7%) across skills[1].

Different factors, institutional and not, may affect the distribution of wages and the behavior of unemployment rates. Nonetheless, in a standard demand-supply framework the discrepancy between the proportion of qualities *available* in the labour force and that *required* by the demand side, constitutes a key element in determining groups' well-being.

Using an education-based skill classification, all industrialized economies have been characterized by a fairly steady increase in the average skill level, thanks to social factors, demographic changes and school laws. Despite their relative scarcity, however, less-educated workers have been suffering deteriorating labour market conditions, in terms of either wage or unemployment rate differentials[2]. On theoretical grounds, these stylized facts appear to be counterintuitive: with a downward-sloping demand for relative labour input, when the relative supply of skilled workers rises, their relative price should decline. No story relying entirely on supply shifts is consistent with the data. As a result, a steep decline in the relative demand for less-skilled workers is needed to justify what we observe. In search of possible sources of such relative

[1] For a comprehensive analysis of the European labour market performance, usual references are Manacorda and Petrongolo (1999), Blanchard and Wolfers (2000) and Gautier (2002). Relative to the US economy, among many others, see Autor *et al.* (1998).

[2] According to the "two sides of the same coin" Krugman theory (1994) the modern technical progress affects skill groups' labour market conditions in different ways depending on the institutional setting. Where skill wages are bargained independently of each other (as in the US and the UK), technological change would result in a rise in the relative skill wage. On the contrary, this would cause an increase in the relative unemployment rate where unskilled wage is indexed on that of skilled workers (as in continental European countries). The differences between labour market institutions in the US and in Europe and their role in the dynamics of European unemployment are discussed in Nickell (1997) and Blanchard and Wolfers (2000).

demand shifts, economists have come up with different but related hypotheses. All are based on the assumption that the structure of the industrialized economies has changed in favour of high-skilled workers. Two explanations widely used in the literature are based on international trade (IT hereafter) and skill-biased technological change ($SBTC$ hereafter). The former attributes the structural transformation to changing international patterns of specialization due to trade liberalization[3] (Leamer, 1996; Wood, 1998; Slaughter, 1998). The latter relies on the occurrence of an *exogenous* technological shock which, enhancing the productivity of skilled workers and reducing that of unskilled ones, causes a shift of the relative labour demand. This would outweighs the growing relative supply, so that the relative skill price would rise rather than drop (Bound and Johnson, 1992; Berman *et al.*, 1994; Autor *et al.*, 1998; Acemoglu, 1998, 2002; Pierrard and Sneessens, 2003). Despite the broad consensus received, the SBTC theory has a "residual" nature[4]. Accordingly, the Capital-Skill Complementarity hypothesis (CSC henceforth) tries to explains labour market variations in terms of changes in observed factor quantities. The CSC story reads as follows. Capital is more complementary to skilled than unskilled workers, hence the growth of capital stock tends to increase the marginal product of skilled labour. Under the CSC assumption, a decrease in the price of investment relative to consumption goods stimulates capital accumulation and finally results in an upward shift of the relative labour demand (Krusell *et al.*, 2000; Lindquist 2003). In such a way, the role of capital accumulation becomes crucial. In the case of SBTC, technological progress affects *new* and *old* capital alike and it is a combination of new capital and new technologies that determines the behavior of the relative demand for skills. Investment in new capital is *not* necessary to implement technological advance. On the contrary, when CSC matters new capital embeds superior technologies and capital accumulation requires more skills, given the different ways in

[3]The effects of trade on inequality across skill groups are usually analyzed in a Heckscher-Ohlin model with skilled and unskilled labour as the two factors and developed and developing economies as the two countries. Trade with developing countries causes the developed economies to specialize in the production of skill-intensive goods, in which they have a comparative advantage because of their relative large supply of skilled workers, and to reduce production of labour-intensive manufactures. As a result, in the developed countries there are a rise in the relative demand for skilled labour and a widening in the market conditions between skilled and unskilled workers.

[4]Bound and Johnoson (1992) interpret SBTC as the residual trend component which causes much of the variation in the skill premium and Krusell *et al.* (2000) emphasize that "[...]*there is no generally accepted economic framework for interpreting skill-biased technological change, measuring its rate of growth, or directly assessing its quantitative importance.*" (p.1029)

which capital interacts with skilled and unskilled labour in the production technology[5]. As a result, CSC reduces the importance of unmeasured trend changes in accounting for the stylized facts: observed wage and unemployment patterns would no more be entirely justified by means of exogenous technological progress, but also by the growth of capital stock.

The objective of this paper is twofold. First, it aims at setting up an intertemporal general equilibrium where the final sector CES production function is characterized by capital-skill complementarity. Instead of the standard capital-labour production function, we develop a four-factor production function with different substitution elasticities between the two types of capital and the two types of labour. In such a way, biased technological change is introduced *via* CSC and embodied technical progress. Second, we decompose the shifts in the relative demand for skills into the capital-skill complementarity effect and a residual skill-biased effect and provide a quantitative assessment of the respective contributions to the Belgian labour market dynamics over the period $1976 - 1996$. We conclude that over the covered period demand shifts are mainly due to skill-biased changes (which results in a constant decrease in the share of unskilled workers), whereas the capital-skill complementarity explanation (due to an acceleration in the growth rate of the stock of capital) accounts for a smaller part of that.

The rest is organized as follows. Section 2 introduces a four-factor nested CD-CES production function. We will show the main differences with respect to the usual Cobb-Douglas framework and important role played by the substitution elasticity parameters in determining model's ability to fit the data. A brief section (3) explains how data on relative equipment price are computed for the Belgian economy. In section 4 the model is presented. Then follows the

[5]Actually, there are two alternative theoretical explanations for the capital-skill relationship. On the one hand, the *skill in adoption* story (Nelson and Phelps, 1966, and Greenwood and Yorukoglu, 1997) predicts that all technological change increases the relative demand for skilled labour, since skills facilitate the implementation and adoption of new technologies. Assuming that new investment goods embed superior technologies, their adoption involves significant learning costs (people must learn how to use them); skilled workers have a competitive (relative to the unskilled) advantage in this implementing process (educated workers would be better at assimilating new ideas), then technological progress causes the relative demand for skills to increase. The relative wage, consequently, should be rising as long as investment-specific technological change is boosting and decreasing as it matures. As soon as unskilled workers become accustomed to using new technologies, firms will replace expensive skilled labour with cheaper unskilled workers. On the other hand, the CSC concept elaborated by Griliches (1969) (recently restated in Krusell *et al.*, 2000) emphasizes the role of skills in the *use* of capital goods that embody technology. Therefore, this would predict that skill groups' inequalities should rise as long as the stock of capital increases. Accordingly, these two different concepts are sometimes regarded as the capital-skill and the equipment-skill complementarity hypothesis, respectively.

calibration (section 5), with particular attention focused on the estimations of the substitution elasticities provided by the literature (5.1). Our results are shown in section 6. Finally, some concluding remarks are drawn in section 7.

2 An Analytical Framework to Understand Capital-Skill Complementarity

To explain how CSC works and its implications on the interactions among economic variables, we use a four-input production function with two parts, a Cobb-Douglas over capital structures and a CES function of the three remaining inputs[6]. It is as follows:

$$Y(k_s, k_e, h, l) = Ak_s^{\alpha}\left\{\mu l^{\sigma} + (1-\mu)\left[\lambda h^{\rho} + (1-\lambda)k_e^{\rho}\right]^{\frac{\sigma}{\rho}}\right\}^{\frac{1-\alpha}{\sigma}}. \tag{1}$$

Both factors, capital and labour, are composed of two elements. Capital input is divided in structures, k_s, and equipment, k_e. At the same way, labour input is split in high-skilled, h, and low-skilled, l, workers. Moreover, A is Hicks-Neutral technological progress, μ and λ are the share parameters (and also factor-biased technology parameters) and, finally σ and ρ represent the two elasticity parameters (governing the elasticities of substitution between inputs). Precisely, $\theta_{k_e h, l} = \frac{1}{1-\sigma}$ is the elasticity of substitution between skilled workers (actually, the composite skilled labour/equipment) and unskilled workers, whereas $\theta_{k_e, h} = \frac{1}{1-\rho}$ is the elasticity of substitution between equipment and skilled workers[7].

Assuming competitive labour markets, the skill premium is given by the ratio of marginal products. Computing the first derivatives and rearranging, one obtains the following expression:

$$w = \frac{w^h}{w^l} = \frac{(1-\mu)(1-\lambda)}{\mu}\left(\frac{h}{l}\right)^{\sigma-1}\left[\lambda\left(\frac{k_e}{h}\right)^{\rho} + (1-\lambda)\right]^{\frac{\sigma-\rho}{\rho}}. \tag{2}$$

Equation (2) decomposes the skill premium dynamics into three components. Taking logs of

[6]For the sake of completeness, it must be noted that an alternative to the nested CES is represented by the translog function. As specified in Krusell *et al.* (2000, p.1034), however, the translog approach has two drawbacks: first, it has more parameters that have to be estimated and second, it lacks the property of having constant elasticities of substitution over time.

[7]Note that the specific functional form (1) imposes a symmetry restriction on substitution elasticities. In fact, it forces equipment and skilled labour to have the same elasticity of substitution with respect to unskilled labour.

(2) we have:

$$\ln w = \underbrace{\ln\left[\frac{(1-\mu)(1-\lambda)}{\mu}\right]}_{SkillBiased\ Effect} + \underbrace{(\sigma-1)\ln\left(\frac{h}{l}\right)}_{Relative\ Quantity} + \underbrace{\frac{\sigma-\rho}{\rho}\ln\left[\lambda\left(\frac{k_e}{h}\right)^{\rho}+(1-\lambda)\right]}_{Capital\ Skill\ Comlementarity}. \tag{3}$$

This shows that the elasticity parameters, σ and ρ, are important to understand the behaviour of the skill premium when factor quantities change and enables us to analytically distinguish the CSC effect from the broader Skill-Biased Effect.

As usual, the substitution effect (the *relative quantity* term) makes the relative price change when the relative scarcity varies. In our framework, this relation implies that, everything else being unchanged, the relative demand curve for skills is downward sloping with the (negative) inverse of skilled-unskilled labour input elasticity of substitution as the slope coefficient

$$\frac{\partial ln(w)}{\partial ln(h/l)} = \sigma - 1 = -\frac{1}{\theta_{k_e h,l}}.$$

Such a relationship shows that as long as the two types of labour are not perfect substitutes (that is $\sigma \neq 1$), it is correct to assume a downward-sloping relative demand with respect to their relative price[8]. Therefore, in a demand-supply scheme, an exogenous rightward shift of the relative supply moves the equilibrium point *along* the downward-sloping relative demand curve, leading to a decrease in the skill premium. The extent of the drop depends on how much substitute the two types of labour are in the production process. Considering the extreme situation of perfect substitution between labour inputs, that is $\theta_{k_e h,l} = \infty$, rises in the relative supplies of skilled workers do not affect wage inequality.

The term labeled as *skill-biased effect* shows how the share parameters affect the wage ratio. Decreasing μ amounts to decreasing the share of unskilled workers and at the same time raising that of the aggregate equipment-skilled labour. In our framework, taking apart the effect on relative wage due to capital accumulation and embodied technological progress (expressed by the CSC term), this skill-biased effect may be interpreted as a residual term which includes other sources of skilled-worker favourable changes in the economy structure such as international trade and skill-biased technological change.

The third factor, the proper *capital skill complementarity effect*, is crucial to our story. This shows that as long as $\sigma > \rho$, which corresponds to capital being more skill- than unskill-complement, we have $\partial w/\partial k_e > 0$. So provided that the CSC hypothesis holds, a faster growth in capital equipment tends to increase the skill premium as it raises the relative demand for

[8]A similar conclusion can be also found in Johnson (1997, p.42) and Acemoglu (2001, pp.13-20).

skilled labour. This holds without assuming any exogenous shocks in parameters as it is instead necessary in the SBCT story.

Finally, equation (3) may be also used to understand the continental European side of Krugman's (1994) theory. First we linearize equation (3) and then take derivatives of the resulting expression with respect to time. This results in[9]

$$g_w \approx g_{sbe} + (\sigma - 1) \cdot g_{(u^l - u^h)} - (\sigma - 1) \cdot (g_L - g_H) + \lambda(\sigma - \rho) \left(\frac{k_e}{h}\right)^{\rho} (g_{k_e} - g_h) \tag{4}$$

where g_x is the growth rate of variable x. To reproduce the fixity of the European skill premium, we impose that $g_w = 0$. Consequently, equation (4) becomes

$$g_{(u^l - u^h)} \approx (g_L - g_H) + \theta_{k_e h,l} \cdot g_{sbe} + \lambda(\sigma - \rho)\theta_{k_e h,l} \left(\frac{k_e}{h}\right)^{\rho} (g_{k_e} - g_h). \tag{5}$$

Keeping for a while $g_{sbe} = 0$, given that $g_L - g_H$ is negative, the relative unemployment rate $u^l - u^h$ rises if and only if:

$$\lambda(\sigma - \rho)\theta_{k_e h,l} \left(\frac{k_e}{h}\right)^{\rho} (g_{k_e} - g_h) > g_L - g_H \tag{6}$$

in absolute value. In other words, to have the observed widening relative unskilled unemployment gap, condition (6) requires the rightward relative demand shift due to capital equipment accumulation *via* CSC to be bigger than the rightward movement of the relative skill supply. Whenever this condition is not fulfilled, for our exercise to work the following expression must hold in absolute value

$$\lambda(\sigma - \rho)\theta_{k_e h,l} \left(\frac{k_e}{h}\right)^{\rho} (g_{k_e} - g_h) + \theta_{k_e h,l} \cdot g_{sbe} > g_L - g_H. \tag{7}$$

In this case, the relative labour demand is moved in a skill-favourable direction by the sum of the CSC effect and the Skill-Biased effect[10].

3 Relative Price of Equipment in the Belgian Economy

The previous section clarifies the role of the ratio of capital equipment to skilled labour input in our analysis. This ratio affects labour market inequalities through capital-skill complementarity. The distinction of two types of capital, structures and equipment, reflects the different growth rates they have shown over the last 30 years. The stock of capital equipment has risen

[9] Analytical details are shown in Appendix A.

[10] Note that a drop in μ (the share of unskilled workers) gives $g_{sbe} > 0$.

much more than that of structures capital. For the U.S. economy, Krusell *et al.* (2000) report that the standard measure of capital structure grew on average at just a 3% rate over the period considered. They also construct a measure of the capital equipment stock using Gordon's capital equipment price (adjusted for quality changes[11]), and show that equipment grew at a 6% before 1975 and at a 7.5% thereafter. This has been caused by a decline in the Gordon equipment price series at a 4.5% through 1975 and at a 6% after that. The authors interpret the decrease in the relative price of equipment as a proxy for equipment-specific technological change. In our framework, such a situation is reproduced by assuming that consumption goods and structures are produced with a constant returns to scale technology whereas equipment is produced with the same technology scaled by equipment-specific technological progress, q_t. The idea is that additional units of investment goods use decreasing amounts of resources in the economy with respect to how much is necessary to produce new structures and consumption goods as a consequence of the faster pace of the technological progress in the equipment industry. In a competitive equilibrium this decrease in the relative marginal costs reflects in a corresponding decrease over time of the relative equipment price of equipment. As a result, the price of capital equipment goods relative to that of consumption goods is equal to $1/q_t$.

Unfortunately, no quality-adjusted price series is available for equipment goods in Belgium. A solution proposed in Pamukçu and Van Zandweghe (2002) consists in using the so-called "harmonised deflator" (Schreyer, 2001) for each ICT capital good in order to obtain quality-adjusted price indices taking the U.S. hedonic indices as a benchmark. It is assumed that the change of the relative price of an equipment asset (expressed as the price level of equipment goods divided by the price level of structures and consumption goods) is the same across countries. Therefore, the rate of change of the harmonised price index of Belgium is as follows:

$$\dot{q}_e^B = \dot{q}_{ne}^B + \dot{q}_e^{US} - \dot{q}_{ne}^{US}$$

[11] Price indices are constructed by comparing prices of sampled products between two periods in time. Two conditions have to be fulfilled for this to yield reliable estimates: the products in the sample have to be representative of a whole product group and they should be comparable between the two periods. In a situation where we should compare two different models, the fundamental question is: how much of an observed price change is due to quality change and how much is a true change in prices? The hedonic method is a statistical tool for developing standardised per unit prices for goods - such as equipment assets - whose quality and characteristics change rapidly over time. This technique redefines heterogenous goods as aggregations of their characteristics, so that changed or new versions of a good can be represented as a new combination of characteristics. In practice, this approach analyses the relationship between price and quality by regressing prices on explanatory variables that represent important product characteristics.

where $\dot{q}_{ne}^{B}$ and $\dot{q}_{ne}^{US}$ are the rates of change of the non-equipment price index of Belgium and the U.S. respectively, whereas $\dot{q}_{e}^{US}$ denotes the growth rate of the U.S. equipment price[12]. Following this approach, we will assume that the pattern of the Belgian relative equipment price has been fairly comparable to that observed in the U.S. economy, so that we can exploit the data in Krusell *et al.* (2000) to implement our analysis.

4 The Model

The analytical framework studied in section (2) as well as most of the related literature rely on the assumption that markets (labour market, in particular) are modeled as perfectly competitive. Differently, our model consists in a intertemporal general equilibrium model with search frictions[13]. Besides Pierrard and Sneessens (2002), which our model is built on, Albrecht and Vroman (2002) and Gautier (2002) analyze the relationship between skill inequality and technological progress using a searching framework. With respect to these three papers, we model the final sector aggregate production function as a four-factor CD-CES whose key feature is capital-skill complementarity. We consider two kinds of capital, structures and equipment, so as to introduce some technological change specific to the production of capital equipment. Moreover, since we are interested in labour market inequality between skill groups, we also distinguish between two types of labour - skilled and unskilled[14].

[12] Pamukçu and Van Zandweghe (2002) admit that the drawback of this method is that a price deflator of business investment that combines price indices of both equipment and non-equipment investment goods should be used as a proxy of $\dot{q}_{ne}$. However, they stress, this is unlikely to cause serious bias to the results, since expenditure on equipment investment is still a relatively small portion of total investment expenditure.

[13] Search frictions make it costly for firms to find suitable workers. In our opinion, this kind of model appears to be appropriate since recruiting costs get even more importance when firms attempt to select skilled workers, especially if one takes into account the possible imperfect correlation between education and skills (on the skill-education mismatching, see Cuadras-Morat and Mateos-Planas, 2003).

[14] It is standard in the literature to define the level of labour skill on the basis of the level of workers' education. Manacorda and Petrongolo (1999) stress that the educational attainment of individuals might be used as a relevant indicator of skill since *"education can be assumed to represent an intrinsic characteristic of the individuals, while other classifications, such as occupation, tend to reflect job's rather than individual's characteristics. Furthermore, education is a time-invariant personal attribute, provided the individuals under analysis have completed their course of study"* (pp.183-185). Apart from few exceptions, most of the models concerning skill biased technological change adopts a dichotomous classification of skills. Despite some drawbacks [*"the disadvantage is one loses much information about the subtleties of wage structure changes from this extreme approach to aggregation."* (Katz and Autor, 1999, p.1516)], this method is easy to implement and allows for a

4.1 Basic Assumptions

We consider an economy with two sectors. In the final sector a representative firm produces a homogeneous good by means of structure and equipment capital and intermediate goods. The final good can be either consumed or accumulated by the representative household. The intermediate sector produces two types goods, simple and complex, with labour as exclusive input. As assumed in Gautier (2002), the intermediate firms are distinguished by their task complexity. To simplify, one can think of the complex-good intermediate firm as using a high-tech production technique, whereas the simple-good firm a low-tech one. Accordingly, there are two kinds of jobs, either complex or simple. Simple jobs can be performed by workers of any level of ability whereas complex job can be carried out only by high-skilled workers.

Firms can open either simple-job or complex-job vacancy, but they have to choose *ex ante* the type of vacancy they want to open[15]. The distribution of this *minimal skill requirement* across vacancies is endogenous and is determined by the free entry conditions[16]. On the contrary, the distribution of worker skills in the labour force is exogenously given. As generally assumed, all firms are single job.

Three differences between our framework and the one of Gautier (2002) and Albrecht and Vroman (2002) can be detected.

Firstly, the modelisation of the bargaining process. Though all share a meeting process with a Nash bargaining approach to wage setting, in Albrecht and Vroman (2002) this process is *undirected*. Specifically, both workers can spend time in searching for both types of jobs, even if whenever a low-skilled worker encountered a high-skill vacancy this match cannot be consummated[17]. Here instead workers' searching efforts are *directed*, in the sense that low-skilled workers never apply for a complex-job vacancy. Since they know that their requests

division of the labour force along the dimension of particular interest.

[15]As stressed in Pierrard and Sneessens (2002, p.2), this amounts to thinking of firms choosing *ex ante* whether she will produce by using a high- or a low-tech technology. This appears plausible since in practice a firm which has opened a service job cannot easily change it to a manufacturing job, nor can a firm easily downgrade its technology.

[16]The free entry conditions state that the steady-state value of keeping a vacancy opened, both for complex and for simple job, is zero. Since opening a vacancy is costless (it only costs keeping it opened), vacancies are opened until the expected income stream is nil.

[17]They use this approach in order to capture the idea that *"given the overall labour market conditions, low-skill workers are better off the greater the fraction of vacancies that require low-skill levels, and vice versa for high-skill workers."* (Albrecht and Vroman (2002, p.285)

would always be rejected, unskilled workers uniquely direct their search effort in the simple-job market. Relevantly, searching effort of skilled workers is determined endogenously. The proportion of time they allocate in each job market is chosen so as to maximize the expected utility. Disadvantageous conditions in the complex-job segment may induce high-skilled workers to devote more effort in searching for simple jobs, even though these are less suited for their talents.

Secondly, on-the-job search is absent in Albrecht and Vroman (2002). Overqualified workers (educated labour performing simple tasks) are not allowed to look for a job more appropriate (and better-paid) to their skills. In our model, on-the-job search intensity is determined by the maximization of the expected utility, achieved by equalizing the opportunity cost of on-the-job search to the marginal utility of leisure. The possibility of keeping on searching for a complex job while working on a simple job deeply affects the simulation results.

The last difference is relative to the job competition. We admit that *a priori* a low-skilled worker can be less or more productive than a high-skilled worker in carrying out a simple job, but in order to fucus attention on job competition, we assume that educated labour is more productive on simple jobs. Differently, in Albrecht and Vroman (2002) skilled workers do not have higher productivity than unskilled labour in performing simple jobs. The productivity on simple tasks is the same across the two skill-groups. Nevertheless, the ladder effect is not ruled out. High-skilled workers employed on simple jobs may always get a higher wage rate than less-educated workers because of their better outside option (they can also do complex jobs)[18]. More complicated is the relative productivity assumption made by Gautier (2002). He states that *a priori* there is no reason for high-skilled workers to be relatively better at doing simple jobs. The point is that on-the-job search reduces the match surplus (to be shared between firm and worker) generated when a simple job is occupied by a high-skilled worker. As a result, there exists a critical value (endogenously determined) of the relative productivity above which firms are compensated for the stronger bargaining position owned by high-skilled workers on simple jobs due to their positive quit probability[19]. The variability of the skilled labour productivity on

[18]Depending on the parameter estimates, two types of equilibria may result. The first is called *equilibrium with cross-skill matching* and it is one in which matches between high-skilled workers and low-tech vacancies are mutually beneficial and therefore consummated. The other, which is referred to as *equilibrium with ex-post segmentation*, is characterized by the absence of these "hybrid" matches (skilled workers only perform complex tasks).

[19] *"The higher the probability for a high-skilled worker to find a complex-job, the higher his productivity on simple jobs must be to compensate employers for the shorter expected match duration."* (Gautier, 2002, p.29).

simple jobs turns out to be the key variable. Gautier (2002) shows, quite counter-intuitively, that less-educated workers may benefit from a skilled workers' high productivity on simple jobs since this enhances the expected return for firms from opening a simple-job vacancy; consequently, more vacancies for simple jobs will be opened and the unemployment rate of unskilled workers will go down (there is, therefore, a positive externality).

4.2 Labour Market Flows

The distribution of skills across workers is exogenous. Normalizing the total labour force to one, we indicate with ξ (respectively, $1-\xi$) the fraction of skilled (resp., unskilled) workers. The following account identities hold:

$$\begin{aligned} N_t^c + N_t^{sh} + U_t^h &= \xi \\ N_t^{sl} + U_t^l &= 1-\xi \end{aligned} \tag{8}$$

where N_t^c and N_t^s stands for the total number of complex and simple jobs respectively, whereas N_t^{sh} and N_t^{sl} represent the simple tasks repartition between high-skilled and low-skilled workers, such that $N_t^s = N_t^{sh} + N_t^{sl}$. Finally, U_t^h and U_t^l denote the number of high-skilled and low-skilled unemployed job-seekers.

We assume that the number of complex and simple job matches, M_t^c and M_t^s respectively, is a function of the number of the corresponding job vacancies, V_t^c and V_t^s, and the number of job seekers adjusted by search efficiencies (to have a measure of the *effective* job seekers), by means of the following two matching functions:

$$\begin{aligned} M_t^c &= M^c(V_t^c, sc_t U_t^h + so_t N_t^{sh}) \\ M_t^s &= M^s(V_t^s, ss_t U_t^h + U_t^l). \end{aligned} \tag{9}$$

Search efficiencies, sc_t, ss_t and so_t, deserve some explanation. They are assumed to be increasing functions of the search efforts, eu_t, $1-eu_t$ and eo_t, respectively. The story is the following: each household, both skilled and unskilled, owns one unit of time to spend in searching for a job. There is no labour-leisure choice. When unemployed, members of the household look for a job. Since unskilled workers cannot be hired on complex jobs, they spend all their time-endowment in the simple-job market. On the contrary, given the conditions in the two segments of the labour market (wages, probability to find a job, etc.), skilled workers decide how to split their effort. The fraction of their time spent in searching for a complex and a simple job is indicated with eu_t and $1 - eu_t$ (both between 0 and 1) respectively. Similarly, when employed on simple tasks, skilled workers can spend a fraction ($0 \leq eo_t \leq 1$) of their leisure in searching for jobs more suited for their skills. Labour markets' tightness is denoted by θ_t^c and θ_t^s, where:

$$\begin{aligned}
\theta_t^c &= \frac{V_t^c}{sc_t U_t^h + so_t N_t^{sh}}, \\
\theta_t^s &= \frac{V_t^s}{ss_t U_t^h + U_t^l}.
\end{aligned} \tag{10}$$

The probabilities, p_t^c and p_t^s, of finding a complex or a simple job per unit of search intensity can be written as follows:

$$\begin{aligned}
p_t^c &= \frac{M_t^c}{sc_t U_t^h + so_t N_t^{sh}} = p^c(\theta_t^c), \\
p_t^s &= \frac{M_t^s}{ss_t U_t^h + U_t^l} = p^s(\theta_t^s).
\end{aligned} \tag{11}$$

Similarly, the probabilities , q_t^c and q_t^s, of filling a complex or a simple vacancy are given by:

$$\begin{aligned}
q_t^c &= \frac{M_t^c}{V_t^c} = q^c\left(\frac{1}{\theta_t^c}\right), \\
q_t^s &= \frac{M_t^s}{V_t^s} = q^s\left(\frac{1}{\theta_t^s}\right).
\end{aligned} \tag{12}$$

The probability that a simple job is filled is the sum of the probabilities of hiring a high-skilled worker and a low-skilled worker:

$$\begin{aligned}
q_t^{sh} &= \frac{ss_t U_t^h}{ss_t U_t^h + U_t^l} q_t^s \\
q_t^{sl} &= \frac{U_t^l}{ss_t U_t^h + U_t^l} q_t^s.
\end{aligned} \tag{13}$$

Finally, adding to all this notation the definitions of the two (exogenous) destruction rates, ψ for the complex jobs and χ for the simple jobs, we get the following employment dynamics that can be written in terms of, respectively, vacancies and job-seekers' effort:

$$\begin{aligned}
N_{t+1}^c &= (1-\psi)N_t^c + q_t^c V_t^c, \\
&= (1-\psi)N_t^c + p_t^c[sc_t U_t^h + so_t N_t^{sh}].
\end{aligned} \tag{14}$$

$$\begin{aligned}
N_{t+1}^{sh} &= (1-\chi - so_t p_t^c)N_t^{sh} + q_t^{sh} V_t^s, \\
&= (1-\chi - so_t p_t^c)N_t^{sh} + p_t^s ss_t U_t^h.
\end{aligned} \tag{15}$$

$$\begin{aligned}
N_{t+1}^{sl} &= (1-\chi)N_t^{sl} + q_t^{sl} V_t^s, \\
&= (1-\chi)N_t^{sl} + p_t^s U_t^l.
\end{aligned} \tag{16}$$

4.3 The Intermediate-Good Firms

There are two types of single-job intermediate firms: one uses a high-tech production technique (and therefore she demands for skilled labour) and the other a low-tech production method (and therefore she opens simple vacancies). A complex vacancy can be either filled by an educated worker (FC) or remain unfilled (VC). A simple vacancy can, instead, be filled by a skilled worker (FSH), by an unskilled worker (FSL) or remain unfilled (VS). The standard approach in search models is to attach a continuous time asset value to any possible worker and job state. Let start associating a value function W_t^i to each of the possible five states, $i \in \{VC, VS, FC, FSH, FSL\}$.

$$
\begin{aligned}
W_t^{VC} &= -a + E_t\left[q_t^c \frac{W_{t+1}^{FC}}{1+r_{t+1}} + (1-q_t^c)\frac{W_{t+1}^{VC}}{1+r_{t+1}}\right], && (17)\\
W_t^{VS} &= -b + E_t\left[q_t^{sh} \frac{W_{t+1}^{FSH}}{1+r_{t+1}} + q_t^{sl}\frac{W_{t+1}^{FSL}}{1+r_{t+1}} + (1-q_t^{sh}-q_t^{sl})\frac{W_{t+1}^{VS}}{1+r_{t+1}}\right], && (18)\\
W_t^{FC} &= y^c c_t^c - w_t^c + E_t\left[(1-\psi)\frac{W_{t+1}^{FC}}{1+r_{t+1}} + \psi\frac{W_{t+1}^{VC}}{1+r_{t+1}}\right], && (19)\\
W_t^{FSH} &= y^s c_t^s - w_t^{sh} + E_t\left[(1-\chi-p_t^c so_t)\frac{W_{t+1}^{FSH}}{1+r_{t+1}} + (\chi+p_t^c so_t)\frac{W_{t+1}^{VS}}{1+r_{t+1}}\right], && (20)\\
W_t^{FSL} &= \nu y^s c_t^s - w_t^{sl} + E_t\left[(1-\chi)\frac{W_{t+1}^{FSL}}{1+r_{t+1}} + \chi\frac{W_{t+1}^{VS}}{1+r_{t+1}}\right], && (21)
\end{aligned}
$$

where a and b represent the cost of keeping a complex and a simple vacancy opened. These costs are indexed on the corresponding wage rate:

$$
\begin{aligned}
a_t &= a_0 \cdot w_t^c && (22)\\
b_t &= b_0 \cdot w_t^s. && (23)
\end{aligned}
$$

a_0 and b_0 are strictly positive real constants and w_t^c and w_t^s the wage rates paid for a complex and a simple job respectively. The marginal productivity of high-skilled workers performing complex jobs is y^c, the wage is w_t^c and a unit of the high-tech intermediate good is sold to the representative final firm at the price c_t^c. Similarly, the marginal productivity of more-educated workers performing simple jobs is y^s, the wage w_t^{sh} and the market price of a unit of low-tech intermediate goods is c_t^s. Finally, the marginal productivity of less-educated workers is νy^s and their wage is w_t^{sl}. ν denotes the relative efficiency of skilled workers on simple jobs, whose meaning and importance have been explained in section 4.1.

At equilibrium, the free-entry conditions ($W_t^{VS} = W_t^{VC} = 0$) determine the number of simple-job rather than complex-job vacancies that intermediate firms choose to open.

4.4 The Representative Final-Good Firm

The representative final firm produces a homogeneous final good, that can be consumed or accumulated by the representative household. Four inputs are used in the production process: capital structure k_s, capital equipment k_e, complex intermediate goods Q^c, and simple intermediate goods Q^s. Input demands are obtained from the usual profit maximization. For the two intermediate goods, FOCs equalize input marginal products to their respective market-clearing prices:

$$\frac{\partial Y}{\partial Q_t^c} = c_t^c; \qquad \frac{\partial Y}{\partial Q_t^s} = c_t^s. \tag{24}$$

Since there are two types of capital, we must equalize the expected net rate of return on investment in structures with that on investment in equipment. This represents the arbitrage condition on capital investment:

$$(1-\delta_s) + \frac{\partial Y_t}{\partial k_{st}} = E_t\left(\frac{q_t}{q_{t+1}}\right)(1-\delta_e) + q_t\frac{\partial Y_t}{\partial k_{et}} \tag{25}$$

where δ_s and δ_e are the depreciation rates for capital structures and capital equipment respectively. In equation (25) the left hand side represents the rate of return on structures investment which is the sum of the undepreciated capital structures and the marginal product of structures. The right hand side instead is the return rate on equipment investment. This is also composed of two elements: the marginal product of equipment investment multiplied by the equipment-specific technological progress (q_t), and the undepreciated capital equipment corrected by expected rate of change in the relative price of of equipment. $E_t\left(\frac{q_t}{q_{t+1}}\right)$ represents the expected capital loss on undepreciated equipment.

The market-clearing property of the intermediate good prices and the normalization to 1 of the marginal productivity of labour input in the intermediate good sector ($y^c = y^s = 1$) allow us to write the following relationship:

$$Q_t^c = N_t^c; \qquad Q_t^s = N_t^{sh} + \nu N_t^{sl} \tag{26}$$

and therefore to substitute the notation of intermediate complex, Q_t^c, and simple, Q_t^s, goods with the quantity of labour employed in order to get that amount of output (N_t^c and $N_t^{sh}+\nu N_t^{sl}$, respectively).

4.5 The Representative Household

The representative household offers labour services to the intermediate firms and capital services to the representative final firm. His value function can be expressed in terms of four state variables, as follows[20]:

$$W_t^H = W^H(k_{st}, k_{et}, N_t^c, N_t^{sh}, N_t^{sl}). \tag{27}$$

The representative household optimization program consists in choosing the optimal values of the decision variables, namely the consumption level, C_t, and the search efforts on the labour market, eu_t and eo_t. As said above, low-skilled unemployed workers spend all their searching-time (normalized to 1) on the simple-job market. The value function satisfies the following Bellman equation:

$$W_t^H = \max_{C_t, eu_t, eo_t} \{U(C_t) - D(eo_t)N_t^{sh} + \beta E_t[W_{t+1}^H]\}, \tag{28}$$

where $U(\cdot)$ is an increasing and concave utility function, $D(\cdot)$ is an increasing and convex disutility function and β is the subjective discount factor. The optimization is subject to the following constraints:

$$\begin{aligned}
N_t^c + N_t^{sh} + U_t^h &= \xi, \\
N_t^{sl} + U_t^l &= 1 - \xi, \\
N_{t+1}^c &= (1-\psi)N_t^c + p_t^c[sc_t U_t^h + so_t N_t^{sh}], \\
N_{t+1}^{sh} &= (1 - \chi - so_t p_t^c)N_t^{sh} + p_t^s ss_t U_t^h, \\
N_{t+1}^{sl} &= (1-\chi)N_t^{sl} + p_t^s U_t^l
\end{aligned}$$

and to the flow budget constraint (income = expenditure) in terms of consumption units

$$w_t^c N_t^c + w_t^{sh} N_t^{sh} + w_t^{sl} N_t^{sl} + w_t^u (U_t^h + U_t^l) + \left(r_t + \frac{\delta_e}{q_t}\right) k_{e,t} + (r_t + \delta_s) k_{s,t} + \Pi_t =$$

$$= \frac{I_{e,t}}{q_t} + I_{s,t} + C_t + T_t, \tag{29}$$

where w_t^u stands for the benefit received by an unemployed worker, Π_t for the profits (value added net of labour and vacancy costs) redistributed by intermediate firms and T_t for the (lump sum) taxes levied to finance the unemployment benefits. The laws of motion for aggregate

[20] As specified in Pierrard and Sneessens (2002, p.10, footnote n.5), assuming a representative household implies assuming perfect unemployment insurance, an usual assumption in this type of models.

structures and equipment capital are respectively

$$k_{s,t+1} = (1-\delta_s)k_{s,t} + I_{s,t} \tag{30}$$

$$k_{e,t+1} = (1-\delta_e)k_{e,t} + I_{e,t} \tag{31}$$

where $I_{s,t}$ is aggregate investment in new capital structures and $I_{e,t}$ the corresponding variable for capital equipment. The FOCs of the optimization problem can be written as follows:

$$U'_{C_t} = \beta(1+r_{t+1})U'_{C_{t+1}} \tag{32}$$

$$0 = E_t\left[p_t^c sc_{eu_t} W^H_{N^c_{t+1}} - p_t^s ss_{1-eu_t} W^H_{N^{sh}_{t+1}}\right] \tag{33}$$

$$D_{eo_t} = \beta p_t^c so_{eo_t} E_t\left[W^H_{N^c_{t+1}} - W^H_{N^{sh}_{t+1}}\right] \tag{34}$$

From the envelope theorem, the following additional dynamic relationships arise:

$$W^H_{N^c_t} = U_{C_t}(w_t^c - w_t^u) + \beta(1-\psi - sc_t p_t^c)E_t\left[W^H_{N^c_{t+1}}\right] - \beta ss_t p_t^s E_t\left[W^H_{N^{sh}_{t+1}}\right] \tag{35}$$

$$\begin{aligned} W^H_{N^{sh}_t} &= U_{C_t}(w_t^s - w_t^u) - D(eo_t) + \beta(so_t - sc_t)E_t\left[W^H_{N^c_{t+1}}\right] + \\ &\quad + \beta(1-\chi - so_t p_t^c - ss_t p_t^s)E_t\left[W^H_{N^{sh}_{t+1}}\right] \end{aligned} \tag{36}$$

$$W^H_{N^{sl}_t} = U_{C_t}(w_t^s - w_t^u) + \beta(1-\chi - p_t^s)E_t\left[W^H_{N^{sl}_{t+1}}\right] \tag{37}$$

4.6 Wage Determination: Nash Bargaining

Three types of matching exist in this framework: complex (FC) and simple (FSH) jobs performed by high-skilled workers and simple jobs carried out by unskilled workers (FSL). High-skilled wage rate is determined every period through a renegotiation between the intermediate firm and the representative household. The bargaining process yields a repartition of the surplus obtained from the match between the two agents. The share that each part gets depends on its bargaining power. The wage rate is fixed by a standard Nash product maximization problem:

$$\max_{w_t^c} \left(\frac{W^H_{N^c_t}}{U_{C_t}}\right)^{\eta^c} (W_t^{FC} - W_t^{VC})^{(1-\eta^c)} \tag{38}$$

where η^c represents the high-skilled worker's bargaining power. Exploiting the free-entry conditions, the first order condition of this maximization problem gives

$$W^H_{N^c_t} = \eta^c(W^H_{N^c_t} + U_{C_t}W_t^{FC}) \tag{39}$$

As far as the simple job wage rate is concerned, we eliminate the possibility of individual bargaining on the simple job market and, in contrast, assume a rigid relative wage. The simple-job wage rate is identical for both less- and more-educated workers and indexed on the complex-job wage rate. Specifically:

$$w_t^s = w_t^{sl} = w_t^{sh} = \gamma w_t^c \tag{40}$$

The last relationship implies assuming an exogenously fixed relative wage rate, γ[21].

Finally, the level unemployment benefit, w_t^u is assumed to be the same for all workers and it is determined as a fraction, w^u, of the average wage rate:

$$w_t^u = w^u \frac{N_t^c w_t^c + N_t^{sh} w_t^{sh} + N_t^{sl} w_t^{sl}}{N_t^c + N_t^{sl} + N_t^{sh}} \tag{41}$$

5 Calibration

The following functions are used to implement the simulation exercise:

$M_t^c = m^c (V_t^c)^{\zeta^c} (sc_t U_t^h + so_t N_t^{sh})^{(1-\zeta^c)}$	matching function - complex jobs
$M_t^s = m^s (V_t^s)^{\zeta^s} (ss_t U_t^h + U_t^l)^{(1-\zeta^s)}$	matching function - simple jobs
$a_t = a_0 \cdot w_t^c$	vacancy cost - complex jobs
$b_t = b_0 \cdot w_t^s$	vacancy cost - simple jobs
$Y_t = AK_{s,t}^{\alpha} \left\{ \mu L_t^{\sigma} + (1-\mu) \left[\lambda H_t^{\rho} + (1-\lambda) K_{e,t}^{\rho} \right]^{\frac{\sigma}{\rho}} \right\}^{\frac{1-\alpha}{\sigma}}$	production function
$U_t = ln(C_t)$	istantaneous utility function
$D_t = \tau eo_t$	istantaneous disutility function
$sc_t = \phi_0^{sc} + \phi_1^{sc} \sqrt{eu_t}$	search efficiency in complex jobs
$ss_t = \phi_0^{ss} + \phi_1^{ss} \sqrt{1 - eu_t}$	search efficiency in simple jobs
$so_t = \phi_0^{so} + \phi_1^{so} \sqrt{eo_t}$	on-the-job search efficiency

The model is in quarterly data (1 period = 1 quarter). Most of the parameters are calibrated on Belgian data for the second half of the 90s. Table (1) summarizes the parameters calibration of our model. The psychological discount factor (β) is set to 0.99 as in most RBC literature, implying an annual interest rate of 4%. Given the Cobb-Douglas part over capital structures in our production function (1), parameter α represents the output elasticity to structures capital and coincides with the output share of capital structures. We set α so as to get this

[21]OECD (1996) data on earnings distribution confirm that in continental European countries the $D9/D5$ (ratio of the highest earning decile to the fifth decile) and the $D5/D1$ (ratio of the fifth earning decile to the lowest decile) ratios have remained fairly stable all over the period studied.

Symbol	Value	Symbol	Value
Labour Force Composition			
ξ	0.67		
Job Destruction Rate			
ψ	0.025	χ	0.05
Matching Functions			
m^c	0.2	ζ^c	0.40
m^s	0.3	ζ^s	0.40
Production Functions			
λ	0.365	μ	0.324
ρ	-0.3753	σ	0.4882
A	0.5	α	0.16
y^c and y^s	1.00	ν	0.8
Instantaneous Disutility of on-the-job search		Psychological Discount Factor	
τ	0.37	β	0.99
Search Efficiencies			
ϕ_0^{sc}	0.1	ϕ_1^{sc}	0.65
ϕ_0^{ss}	0.4	ϕ_1^{ss}	0.74
ϕ_0^{so}	0.75	ϕ_1^{so}	0.6
Vacancy Costs			
a_0	0.4	b_0	0.2
Wages Determination			
η^c	0.60		
w^u	0.34	γ	2/3
Capital Depreciation Rates			
δ_s	0.05	δ_e	0.075

Table 1: Parameters Calibration [year of reference: 1996]

share at 17%. In line with Krusell *et al.* (2000), we choose a depreciation rate for equipment ($\delta_e = 0.075$) higher than that for structures ($\delta_s = 0.075$). These values imply a steady-state ratio of aggregate capital to output equal to 10. The labour productivities in the simple and complex intermediate goods firms (y^s and y^c) are normalized to 1. To focus on job competition low-skilled workers' productivity on simple jobs is assumed to be lower than that of high-skilled workers on the same task ($\nu = 0.8$). In such a way, the intermediate firms producing simple goods would always prefer to fill a job vacancy with a skilled worker. Recruiting costs parameters (a_0 and b_0) are chosen to satisfy two conditions: first, selecting a suitable high-skilled worker for complex vacancies is more resource-consuming than filling a simple vacancy for a low-tech firm (that is, $a_0 > b_0$). Second, total vacancy costs represent 2.5% of output and 4.5% of total labour costs.

We define as high-skilled a worker whose education level is equal to a upper-secondary degree or more. As such, the proportion of high-skilled workers in the labor force (ξ) is set to 0.67 as reported in Sneessens and Shadman (2000), that is the value estimated for Belgium in $1995-96$. The elasticities of job matches with respect to vacancies (ζ^c and ζ^s) are taken from Van der Linden and Dor (2001). Using 1997 Belgian data they estimate this elasticity (without making the distinction between high- and low-skilled workers) at 0.4. This value is also used to set high-skilled workers' share of a match surplus. As in Merz (1995) and Andolfatto (1996), the bargaining power of a worker has the same value of the coefficient of unemployment in the matching function (that is, $\eta^c = 1 - \zeta^c = 0.6$). Van der Linden and Dor (2001) also provide the figure of the 1997 Belgian average replacement ratio ($w^u = 0.34$). The value of the job destruction rate estimated by the same authors (0.013 with monthly data), instead, may only be taken as a lower bound since they do not distinguish between simple and complex jobs. Here we assume that the complex job destruction rate ($\psi = 0.025$) is lower than that for simple jobs ($\chi = 0.05$). The ratio of simple to complex wages, γ, is fixed at 2/3 in line with the figure reported in OECD (1996) for Belgian $D9/D5$ and $D5/D1$ ratios.

The remaining parameters (the two matching efficiencies, m^c and m^s, the six parameters of the search efficiencies, ϕ_0^{sc}, ϕ_1^{sc}, ϕ_0^{ss}, ϕ_1^{ss}, ϕ_0^{so} and ϕ_1^{so}, and the disutility parameter, τ) are calibrated so as to satisfy the following six steady-state conditions: a low-skilled unemployment rate of roughly 0.20; a high-skilled unemployment rate of about 0.7; an effective probability to find a complex job equal to 0.40 and that to find a simple job equal to 0.20; almost equal probabilities to fill complex or simple vacancies (around 0.50).

It is worthwhile making a last remark. Our calibration gives a proportion of high-skilled workers employed on simple jobs (the ladder or job competition effect) of roughly 12%. This figure may seem rather low (as also pointed out in Pierrard and Sneessens (2002), which gets even a smaller value, 8%) if one looks at other works studying the Belgian economy (Delmotte *et al.*, 2001). They find that a quarter of all hired workers might be considered as overeducated, in other words having an education level higher than that demanded by firms when advertising a vacancy. Anyway, a deeper analysis of the figure reveals that this is overestimated since firms are likely to indicate the minimum skill requirement as a lower bound rather than a strict prerequisite.

5.1 The CES Parameters Estimates

As said in section (2), the nested CES here employed has four key coefficients: the two share parameters, μ and λ, and the two elasticity parameters, ρ and σ.

As far as the share parameters are concerned, μ affects the quantity of output that goes to unskilled labour whereas λ divides the remaining between equipment and skilled labour. Their values are calibrated on the basis of the resulting steady-state input shares of output. We assume an output repartition between inputs common to most continental European countries in the mid 90s, that is some points more than 0.30 to aggregate capital input and less than 0.70 to aggregate labour input. The latter is further divided in a rough 0.20 to unskilled labour and 0.50 to skilled workers.

Relative to the standard Cobb-Douglas framework, CES allows for setting elasticities of substitution different from unity[22]. The related literature provides a lot of estimates for these parameters. Usual assumption is that $\sigma > \rho$ corresponding to $\theta_{k_e h,l} > \theta_{k_e,h}$. As it has been proved in section (2), this is the needed condition to implement CSC. Table (2) summarizes the estimations of σ presented in the literature and suggests that this may range between 0.25 and 0.401, with the corresponding elasticity of substitution between the two kinds of labour, $\theta_{k_e h,l}$, running from 1.35 to 1.66.

[22]Despite this fact, Manacorda and Petrongolo (1999), starting from a linear homogeneous CES aggregate production function with two labour inputs, run a regression of the relative input demand referred to six OECD countries and find the elasticity of substitution to be very close to one with all tests being not significant. As a result, they state that the production function can be legitimately approximated by a Cobb-Douglas specification.

Authors	Preferred estimate of σ	Resulting $\theta_{(HK_e,L)}$
Ciccone and Peri [2003]	0.36	1.5625
Fallon and Layard [1975]	0.33	1.4925
Katz and Murphy [1992]	0.29	1.4084
Murphy *et al.* [1998]	0.27	1.3698
Johnson [1997]	0.34	1.5151
Krusell *et al.* [2000]	0.401	1.6667
Caselli and Coleman [2002]	0.25	1.3333

Table 2: Estimates of the two labour inputs substitution elasticity

Differently, estimates of the elasticity of substitution between capital equipment and skilled labour are less plentiful and more variable. Very often this elasticity is set less than one and, accordingly the related elasticity factor in the CES, ρ, negative. In agreement with this are the figures in Fallon and Layard (1975), $\rho = -0.72$, and in Krusell *et al.* (2000), $\rho = -0.495$. More in general, ρ ranges between -0.8 and 0.4. Accordingly, the admissible values for $\theta_{k_e,h}$ are in the [0.55, 1.66] interval.

Our choices for both these parameters (precisely, $\sigma = 0.4882$ and $\rho = -0.3753$) are those estimated by Lindquist (2003) for the Swedish economy in the period $1967 - 99$. Therefore, the elasticity of substitution between unskilled labour and the composite equipment/skilled labour input, $\theta_{k_eh,l}$, is 1.95. This implies that they are strong substitute for each other in the production process. The elasticity of substitution between capital equipment and skilled labour is lower than unity ($\theta_{k_e,h} = 0.73$) showing complementarity between these two inputs.

6 Findings

Our objective in this section is to check to what extent the rise in low-skilled unemployment observed after 1975 can be explained by changes in relative labour demand due to the growth in equipment stock, and in relative labour supply. The point is to assess whether the rise in the ratio of equipment to skilled worker is sufficient to generate *via* the CSC effect a rightward shift of the relative demand large enough to more than compensate the relative supply shift (that is, whether condition (6) is fulfilled). Only in such a case the entire dynamics of the Belgian relative unemployment rate might be explained in terms of capital-skill complementarity.

Krusell *et al.* (2000) draw the conclusion that, under CSC, input changes in the U.S. labour market can alone account for most of the variations in the skill premium over the last 30 years[23]. To check if this is true in our case, we recalculate the equilibrium values of the unemployment rates and the other endogenous variables by changing only two parameters, the proportion of high-skilled workers α, and the equipment-specific technological progress q. This exercise aims at reproducing the changes observed in the Belgian labour market over the period $1977 - 1996$[24].

Year	α	$1/q$	μ	Equipment per Skilled Worker	U^h/α	$U^l/(1-\alpha)$	Ladder Effect	Labour Share	w^c
Actual Data									
1976	0.27	1	NA	NA	4.7%	6.8%	NA	$\sim$ 70%	NA
1996	0.67	0.25	NA	NA	6.8%	20.1%	NA	$\sim$ 65%	NA
$1976 - 1996$	$+0.45$	-75	NA	$+280$	$+2.1$	$+13.3$	NA	-5.0	NA
Model with *only* Capital-Skill Complementarity									
$1976 - 1996$	0.0	-75	0.0	$+273$	$+6.67$	$+22.67$	-2.6	$+3.8$	$+31.58$
Model with *some* Change in Labour Force Proportion and Capital-Skill Complementarity									
$1976 - 1996$	$+0.14$	-75	0.0	$+222$	0.0	0.0	$+3.3$	$+1.0$	$+25.02$
Model with *full* Change in Labour Force Proportion, Capital-Skill Complementarity *and* Skill-Biased Effect									
$1976 - 1996$	$+0.45$	-75	-0.17	$+290$	$+4.9$	$+12.4$	$+7.8$	-4.0	$+49.68$

NA: Not Available

Table 3: Labour Force Composition, Equipment per Skilled Worker and Unemployment Rates - Comparison between Actual and Simulated Data

As shown in table (3), we divide our exercise in three steps. First, we move q so as to implement the estimated drop in the Belgian relative equipment price. Under embodiment technological progress and CSC, this causes a rise in the equipment per skilled worker (similar in the magnitude to that reported in Krusell *et al.*, 2000) which in turn affects labour market inequality. Given the fixity of the relative supply, the equipment stock growth leads to a remarkable rise in the unemployment rate of unskilled workers. Surprisingly, we also observe

[23]Note that, in any case, the simplifying assumption made in Krusell *et al.* (2000) to abstract from the household sector in order to focus uniquely on the aggregate production function would make their analysis hardly comparable with ours that is developed in a general equilibrium framework.

[24]Main data source: OECD (1996, 1999).

an increase in the quantity of unemployed high-skilled workers. Without the ladder effect, this result would be unreliable. Once job-competition is taken into account, instead, improving conditions in the complex-job segment of the labour market induce high-skilled workers to change the optimal allocation of their searching effort. Both the fraction of time they spend in looking for a complex job, eu, and the fraction of leisure time they devote to search for a more suitable job, eo, double. There is hardly any high-skilled worker employed on a simple job. We observe a further remarkable decrease (-2.6%) in the already low crowding-out value. The complex-job congestion effect makes high-skilled unemployment rate raise though the relative demand has shifted in their favour.

Second we reproduce the observed rise in the labour force proportion of high-skilled workers by increasing α. The result (not reported in the table) shows that the only CSC effect seems to generate a rightward shift of the relative demand which is not so strong as necessary to fully compensate for the opposite forces induced by the change in the composition of skills. Indeed, the rise in the relative employment quantity remains too moderate to more than absorb the increasing number of high-skilled people in the economy (this means that equation (6) does not hold in our exercise). Consequently, the counterfactual outcome is a decrease in the unskilled unemployment rate, due to the sharp drop of their relative quantity in the economy and the insufficient decline in their relative demand.

To assess how much of the rise in the labour force proportion of high-skilled workers is actually matched by the relative labour demand shift entirely due to the equipment growth and the CSC effect, we modify this second exercise. The actual change in the number of skilled workers is no more fully reproduced; instead α is increased until the high-skilled unemployment rate remains unchanged. Unexpectedly, this exercise also produces a constant low-skilled unemployment rate. It implies that the CSC effect produces effects favourable to skilled workers just sufficient to compensate a 60% increase in the ratio of high- to low-skilled labour forces (corresponding to a 0.14-point rise in α). If the relative quantity of unskilled workers keeps declining their unemployment rate will start diminishing due the usual substitution effect.

The comparison between these results and those of the first exercise (only rise in q) deserves some attention. A rise in the proportion of high-skilled workers in the total labor force leads to more output, more crowding-out, and also to a decrease in all unemployment rates. The first two results may be expected, and similar to those obtained for instance in Gautier (2002). The last one may be *a priori* more surprising. Our representation of the production process and the endogenous determination of searching efforts provide an explanation for this. Let first ignore job competition effects and consider perfectly segmented labor markets. It is intuitively clear that an increase in the supply of high-skilled labor will lead to an increase in the number of employed workers of that type, although their unemployment rate may increase. An increase in high-skilled employment will ceteris paribus increase the marginal productivity of low-skilled workers (in final goods terms). The result will be a lower low-skilled unemployment rate. Once we introduce the possibility of job competition, the lower low-skilled unemployment rate and the increased tightness on the simple job market gives high-skilled workers a stronger incentive to look for a simple job (this is reflected in a sharp drop in their complex-job searching effort, eu, which halves with respect to the previous case). There is thus more crowding-out effect which may be large enough to change the sign of the effect of the labour force change on the high-skilled unemployment rate. Therefore all unemployment rates decrease.

Finally, we try to improve model's ability to replicate actual Belgian data by introducing an *exogenous* skill-biased effect. As explained through equation (3), this amounts to lowering the unskilled labour share parameter, μ. In such a way, we include in our framework other sources of changes in the economy structure favourable to skilled labour such as international trade and skill-biased technological change. These additional effects strengthen the CSC effect causing a further increase in the relative demand for skills[25].

This final exercise reproduces pretty well the changes observed over the period considered (implying that equation (7) is satisfied). The growth in the capital equipment stock (+290%) is comparable to the one occurred in the U.S. economy. As a result, the low-skilled unemployment rate strongly raises (+12.4% in the simulation, +13.3% in the data), whereas the high-skilled unemployment rate shows only a moderate rise (+5%). The resulting increase in the average unemployment rate (+6%) is close to the level estimated for continental Europe (+7% on average). The proportion of simple jobs held by high-skilled workers (crowding-out effect) increases from 4% to 11.8%. Lastly, we have a decrease (−4%) in the output share of the aggregate labour input, in line with the evidence for the continental European labour markets (−5% on average).

[25] A similar analysis is carried out in Dupuy (2003) for the Dutch economy over the same period. He focuses his attention on the 1969−96 skill premium in the Netherlands and finds that in the first years (1969−1982) demand shifts are mainly due to skill-biased technological change while thereafter, the capital-skill complementarity explanation only accounts for 1/3 of the shifts in the demand for skilled workers, SBTC accounting for 2/3.

7 Concluding Remarks

Over the last three decades, OECD countries have witnessed either widening wage inequality or rising unemployment rate. In both cases, unskilled workers has suffered most of the burden. Focusing on continental European economies, low skilled unemployment rates have more than doubled since the mid 70*s*. Two main reasons have been put forward to explain the observed dynamics: (a) the occurrence of a skill-biased technological change that has led to a recomposition of the labour demand in favor of skilled workers; (b) the rise in the number of "overqualified" workers employed on simple jobs, resulting in a crowding out (the so-called ladder effect) of lower educated labour.

This model highlights the role of the capital-skill complementarity in order to achieve a deeper understanding of the various mechanisms at work. For this purpose we set up a dynamic general equilibrium model with a double heterogeneity (low *vs* high skilled workers and simple *vs* complex jobs) and with endogenous job competition. It has been considered a four-input nested CD-CES production function that allows to take into account the capital-skill complementarity assumption. This amounts to setting a lower elasticity of substitution between capital equipment and skilled labour relative to that between capital equipment and unskilled labour.

To evaluate the importance of capital-skill complementarity we stimulate the growth rate of the capital equipment stock by decreasing the relative price of capital goods. This attempts tries to entirely rationalize the stylized facts by means of endogenous capital accumulation. Firstly, we exclude the widely used *exogenous* skill-biased effect, for which still remains the necessity of providing an explanation on both theoretical and empirical grounds. We showed that the growth in the equipment stock through the capital-skill complementarity hypothesis represents an essential part of the story. Moreover endogenous search behaviour and job competition effects seem to play a crucial role; they may have considerably amplified the biased effects of the embodied technological progress on the low-skilled unemployment rate. Nevertheless our results suggest that capital-skill complementarity might not be exhaustive. In our dynamic general equilibrium model in fact the skill-biased CSC effects seem to remain weak and, consequently, unable to overturn the pressure exerted on labour market inequalities by skill composition changes of the labour force. Other skill-favourable changes in the economy structure such as internationalization and increased trade with low wage countries and more capital intensive production which further hint the labour market conditions of the unskilled workers also need to be taken into account.

Appendix A

First we add and subtract to equation (3) the log ratio of the quantity of unskilled workers to that of skilled workers in the labor force, $\ln L/H$, multiplied by $(\sigma - 1)$. So we get:

$$\ln w = \ln \left[\frac{(1-\mu)(1-\lambda)}{\mu}\right] + (\sigma-1)\ln\left(\frac{h/H}{l/L}\right) - (\sigma-1)\ln\left(\frac{L}{H}\right) + \frac{\sigma-\rho}{\rho}\ln\left[\lambda\left(\frac{k_e}{h}\right)^{\rho} + (1-\lambda)\right] \tag{42}$$

Noting that h/H (resp. l/L) is the complement to unity of the high- (resp. low-) skilled unemployment rate, we write

$$\ln w = \ln \left[\frac{(1-\mu)(1-\lambda)}{\mu}\right] + (\sigma-1)\ln\left(\frac{1-u^h}{1-u^l}\right) - (\sigma-1)\ln\left(\frac{L}{H}\right) + \frac{\sigma-\rho}{\rho}\ln\left[\lambda\left(\frac{k_e}{h}\right)^{\rho} + (1-\lambda)\right] \tag{43}$$

Rearranging this is approximately equal to

$$\ln w \approx \ln \left[\frac{(1-\mu)(1-\lambda)}{\mu}\right] + (\sigma-1)\ln(1-u^h+u^l) - (\sigma-1)(\ln L - \ln H) + \frac{\sigma-\rho}{\rho}\ln\left[\lambda\left(\frac{k_e}{h}\right)^{\rho} + (1-\lambda)\right] \tag{44}$$

Second a first order Taylor's expansion is used to simplify the last term. The resulting expression is

$$\ln w \approx \ln \left[\frac{(1-\mu)(1-\lambda)}{\mu}\right] + (\sigma-1)\ln(1-u^h+u^l) - (\sigma-1)(\ln L - \ln H) + \lambda\frac{\sigma-\rho}{\rho}\left(\frac{k_e}{h}\right)^{\rho} \tag{45}$$

Then we compute the first derivative with the respect to time and indicate by g_x the growth rate of variable x

$$g_w \approx g_{sbe} + (\sigma-1)\cdot g_{(u^l-u^h)} - (\sigma-1)\cdot(g_L - g_H) + \lambda(\sigma-\rho)\left(\frac{k_e}{h}\right)^{\rho}(g_{k_e} - g_h) \tag{46}$$

where g_{sbe} represents the change over time of the ratio $\frac{(1-\mu)(1-\lambda)}{\mu}$. Finally we set $g_w = 0$ so as to obtain equation (5) in the text

$$g_{(u^l-u^h)} \approx (g_L - g_H) + \theta_{k_eh,l}\cdot g_{sbe} + \lambda(\sigma-\rho)\theta_{k_eh,l}\left(\frac{k_e}{h}\right)^{\rho}(g_{k_e} - g_h) \tag{47}$$

where $\theta_{k_eh,l} = \frac{1}{1-\sigma}$.

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