A Search Model in a Segmented Labour Market: the Odd Role of Unions

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Abstract

Assuming random matching productivity, we present a search equilibrium model where each match ends in a vacancy, in a temporary job or in a permanent job. Centralized bargaining set the wage rate of permanent workers whereas firms decide unilaterally the wage rate of temporary workers. In this *segmented* labour market: a) the wage setting function can be downward sloping; b) higher union's bargaining power lead to higher wage and higher unemployment; c) average workers productivity depends positively on union bargaining power.

Keywords: Temporary contract, unemployment, productivity, search model, equilibrium, unions

1 Introduction

Aim of the paper is to analyze the implications of the presence of labour unions for macroeconomic variables in a segmented labour market where permanent workers, covered by contractual arrangements, and temporary workers, paid at their reservation wage, coexist.

Most part of the literature has focused on the effects of various measures of labor market flexibility (in particular, relative to regulation of temporary contracts).

Early literature has analyzed the impact of temporary contracts on unemployment and job turnover focusing on the behavior of employment over the business cycle with a traditional partial equilibrium framework of labor demand under uncertainty. In these models, firms have a stable permanent workforce, and adjust temporary workers to fluctuations in economic activity, so that temporary contracts serve as *buffer stocks* (see among others Bentolila e Saint-Paul (1992), Garibaldi (1998), Boeri (1999), Pissarides e Mortensen (1994)). These studies show that the introduction of temporary contracts has an ambiguous impact on the overall employment but increase employment volatility over the business cycle.

Using Pissarides e Mortensen (1994) matching model with endogenous job destruction, Cahuc e Postel-Vinay (2002) widen its scope by analyzing the consequences of the specific combination of temporary and permanent jobs on unemployment. Assuming that long term contracts, which can be terminated in any period with a fixed *firing cost*, coexist with temporary contracts that can be either terminated at *no cost* or converted into a long-term contract, the paper shows that looser restrictions on the use of temporary contracts have a beneficial impact on employment that can be offset by the increase in job turnover when there are positive firing costs.

In their matching model where firms create *entry level* jobs which can be converted or destroyed after a given period of time Blanchard e Landier (2001) conclude that lowering firing costs for entry-level jobs while keeping them high for regular jobs can have two effects: firms are more willing to hire new workers and see how they perform, but they are also more reluctant to keep them in regular jobs. As a result, transition rates to permanent jobs are low and the excess turnover induced by the coexistence of temporary and permanent contracts can be high enough to offset the efficiency gains of higher flexibility. In other words, the effects of partial reform may be perverse, leading to higher unemployment and lower workers' welfare.

Dolado *et al.* (2007) build on the same "search and matching" literature that assess the effects of EPL reforms in dual labor markets, but focus on the spillover effects of targeted EPL. The Authors develop a model in which two groups of workers with different productivity levels interact through the matching process in a labor market subject to search frictions. In this way, they are able to evaluate the effects of a reduction of firing costs concerning only one group of workers on the targeted and non-targeted workers with the effects of a more general reform affecting all workers. Calibrating the model on Spanish data, the Authors find that targeting firing cost reductions on low productivity workers and in jobs subject to frequent productivity shocks is the most effective way of reducing aggregate unemployment. However, as the authors themselves acknowledge, the model is flawed by some limitations, given that it ignores other possible effects of firing cost reductions on the labor market. In particular, changes in firing costs could affect labor productivity since higher turnover improves the reallocation of production factors and the adoption of new technologies.

In a similar framework, Casquel e Cunyat (2008) design a model with heterogeneous workers according to which temporary contracts can serve different functions depending on workers' skills. Firms hire workers on temporary jobs and later can convert the temporary job in a permanent one or fire the worker. Hence, three equilibria emerge in which (1) the temporary job could be a stapping stone, so that all temporary jobs are converted, (2) all temporary jobs are dead-end jobs or (3) only skilled workers can access permanent jobs (a segmentation equilibrium). The kind of equilibrium prevailing depends crucially on the institutional labor market framework, with lower firing costs or unemployment benefits making it easier for skilled and unskilled workers to gain access to permanent jobs.

In our paper, we focus on the relation between unions' bargaining power and the average workers' productivity.

Labor unions' effects on labor productivity are generally heterogeneous and change substantially across workplaces Freeman e Mendoff (1984). On the one hand, the traditional arguments about the effects of unions refers to the so-called monopoly face of unionism and seems to support a negative effect of unionization on labor productivity, because of regulation, restrictions and rent seeking activities which hamper investment and innovation. On the other hand, according to the so-called collective voice and institutional response face, unions could boost firms' productivity through different channels. In fact, unions could provide an alternative option for expressing discontent other than exiting: this new communication channel between the management and the workforce could foster the adoption of more efficient methods of production Freeman e Mendoff (1984). Besides, an increase in productivity could be the result of either a natural selection process or a reaction to lower expected profits (Addison e Hirsh (1989)).

More recently, Hirsch (2004) argues that the average effect of unions on labor productivity is negligible, while Freeman (2005) concludes that the union effect on productivity is certainly not negative, even if it is difficult to establish a positive one. Finally, a recent meta-analysis on the relationship between unions and productivity shows that there is a negative association in the United Kingdom and a positive one in the United States (Doucoliagos e Laroche (2003)).

The same Authors (Doucouliagos e Laroche (2003)) apply the meta-analysis and meta-regression analysis to the large literature on unions and productivity growth (29 studies on different countries over a long time period), finding evidence of a negative relationship. Specifically, higher initial levels of unionization are associated with lower productivity growth and also changes in the degree of unionization are negatively related to changes in productivity.

In order to understand the role played by trade unions in a segmented labour market, where firms can offer to workers both a temporary and a permanent contract, we present a search model that take explicitly into account segmentation. In particular, we assume that temporary workers, being paid at their reservation wage, continue to search for a better job whereas permanent workers do not. We concentrate the analysis on union bargaining power and we show that stronger unions raise permanent workers wages, unemployment and the *shakiness* index (defined by the ratio between short-term and total contract) but, by inducing firms to keep on permanent basis only the workers whose match show a very high productivity, they increase the average workers' productivity.

The paper is organized as follows. In next Section we present the model, Section 3 presents a simplified version of the model that allows us to obtain analytical solutions and Section 4 discusses the main results.

2 The model

We define by v the number of vacancies, by u the number of unemployed and by $\theta = \frac{v}{u}$ the tightness in the labour market. We assume that the number of matching, m, is given by a Leontief matching function[1]:

$$m = \eta \quad min(u, v)$$

We also assume that in the labor market there are more workers than jobs, so that $v < u$ holds. With these hypothesis, we obtain that the probability of finding a job for an unemployed $\frac{m}{u} = \eta\theta$ and the probability of filling a vacancy, $\frac{m}{v} = \eta$. Therefore, labour market tightness does not influences firm's probability of finding workers and influences positively workers' probability of finding jobs.

[1] As in Lagos (2000) and Shimer (2007), which assume workers and jobs in fixed proportions.

A match between a worker and a job gives a random productivity x. $F(x)$ is the cumulate distribution function and $\mu = E(x)$ is the average of matching productivity. Given the match arrives, the firm immediately evaluates its productivity and chooses if hiring the worker or not [2].

When a worker is hired, the firm can choose to offer him either a *temporary* or a *permanent* contract.

Assumption 1. *The wage rate of temporary contracts is set by firms at a level such that the utility of temporary workers is equal to the one of the unemployed (participation constraint). The wage rate of permanent workers is bargained at a centralized level and depends on the average permanent workers productivity.*

Assumption 2. *Temporary workers continue to search and they find another job with the same probability of unemployed, permanent workers do not search anymore.*

The contractual arrangement is decided by the firm and it depends on the productivity x_i of the match and on the characteristics of the two available contracts. Let us call with $\underline{x}$ the endogenous bottom level of productivity which makes the match rentable for the firm and define it *hiring productivity* and let us assume that it exists a higher endogenous value of x, $\bar{x}$, the *keeping productivity*, that makes the firm willing to employ the worker on permanent basis. Considering assumptions 1 and 2, a match therefore gives rise to:

- a vacancy with endogenous probability $F(\underline{x})$,
- a temporary contract with endogenous probability $F(\bar{x}) - F(\underline{x})$,
- a permanent contract with endogenous probability $1 - F(\bar{x})$.

Permanent contracts can be hit by a negative shock with exogenous probability λ. The probability that the shock hits temporary workers is assumed to be higher, namely $\lambda + \phi$, because temporary jobs are in the average less productive and because temporary workers are less covered by employment protection legislation. An higher ϕ implies a lower expected duration of temporary jobs.

[2] In a previous version of the model, we assumed that screening required one period of time with similar results.

2.1 Wage setting: temporary workers

Given the above hypothesis and following standard job search models, the asset value of being in the state of unemployment is given by:

$$rU = B + \eta\theta \left[[F(\bar{x}) - F(\underline{x})](W^T - U) + [1 - F(\bar{x})](W^P - U) \right] \quad (1)$$

where B is the per period utility of being unemployed (that we call unemployment benefits thereafter), $\eta\theta$ the probability of finding a job, U, W^T and W^P the average asset values of being in the states of unemployment, temporary job and permanent job, respectively.

Given that temporary worker continue to search (hypotheses 2), the asset value of being employed on temporary basis is[3]:

$$rW^T = w^T + (\lambda + \phi)(U - W^T) + \eta\theta[1 - F(\bar{x})](W^P - W^T) \quad (2)$$

where w^T is the wage rate of temporary workers, λ the exogenous probability of negative shocks and ϕ the probability of exogenous termination. Finally, the asset value of being in a permanent job is:

$$rW^P = w + \lambda(U - W^P) \quad (3)$$

where w is the wage rate of permanent workers. Assume now that $U = W^T$ (hypotheses 1). Considering the symmetric equilibrium it is immediate to see from equations 1 and 2 that $B = w^T$ must hold, i.e. the wage rate of temporary workers equals the per period utility of being unemployed.

2.2 Wage setting: permanent workers

According to Nash bargaining, we assume that the firm and the union maximize their payoffs with respect to w:

$$(W^P - U)^{\alpha}(J^P - V)^{1-\alpha} \quad (4)$$

where J^P is the average asset value of permanent workers and V is the one of vacant jobs. $(W^P - U)$ can be computed using equations 3 and 1 with $W^T = U$. It gives:

$$W^P - U = \frac{w - B}{R + \eta\theta[1 - F(\bar{x})]} \quad (5)$$

[3]Unless necessary, we do not write in the following equation the index i referring to the productivity of the matching. For instance, the wage rate of temporary workers is equal for all of them and therefore not depending on matching productivity.

where $R \equiv r + \lambda$.

The average asset value of permanent workers is given by:

$$rJ^P = \gamma^P(\bar{x}) - w + \lambda(V - J^P) \tag{6}$$

where $\gamma^P(\bar{x})$ is the endogenous average productivity of permanent workers defined as:

$$\gamma^P(\bar{x}) = \frac{\int_{\bar{x}}^{\infty} x f(x) dx}{1 - F(\bar{x})} \tag{7}$$

Equation 6, for $V = 0$ because of firms free entry, gives $J^P = \frac{\gamma^P(\bar{x}) - w}{R}$.

Using the standard technics to maximize equation 4 with respect to w we obtain:

$$w(\bar{x}) = \alpha\gamma^P(\bar{x}) + (1 - \alpha)B \tag{8}$$

As expected, the wage rate of permanent workers is a weighed sum of the average endogenous productivity of permanent workers and unemployment benefits.

2.3 Hiring and keeping thresholds

When a match of productivity x_i happens, the firm chooses wheter hiring or not the worker and, in the former case, which kind of contract offer to him. Hiring that worker on temporary basis gives the following asset value:

$$rJ_i^T = x_i - B + (\lambda + \phi + \eta\theta)(V - J_i^T) \tag{9}$$

because temporary workers leave the firm with probability $\eta\theta$ (where they also find a permanent position with probability $\eta\theta(1 - \bar{x})$). Given $V = 0$ in equilibrium, the firm will hire the worker if $J_i^T \geq 0$ which gives $x_i > B$ as the *hiring* condition. This leads to the definition of the *hiring productivity*:

$$\underline{x} = B$$

Consider now the asset value of the same matching when the firm offers a permanent contract:

$$rJ_i^P = x_i - w + \lambda(V - J_i^P) \tag{10}$$

The *keeping productivity* threshold $\bar{x}$ is defined by comparing the asset value of a permanent contract with the one of a temporary contract. Solving in J_i^T and J_i^P equations 9 and 10 respectively, using $\underline{x} = B$ and $V = 0$, solving

$J_i^T = J_i^P$ in x_i, we obtain the bottom level of productivity which yields the permanent job more profitable than the transitory one:

$$\bar{x}(w, \theta) = w\kappa(\theta) + [1 - \kappa(\theta)]B \tag{11}$$

where $\kappa(\theta) \equiv 1 + \frac{R}{\phi + \eta\theta}$, so that $\frac{d\kappa}{d\theta} < 0$.

Remark 1. *The* keeping productivity *is increasing in the wage rate of permanent workers and it is decreasing in labour market tightness.*

Proof. The results emerge immediately considering that $\kappa(\theta)$ decreases with θ and that $w > B$ because the wage rate of permanent workers must be higher than unemployment benefits. □

A higher labour market tightness gives more opportunities to temporary workers of moving away from the temporary positions, reduces the expected length of temporary contracts and lowers the asset value of temporary jobs. Therefore, it pushes firms to keep more workers on permanent basis.

>From equation 8, using equation 7 we obtain the implicit definition of the wage setting function:

$$w = \alpha \frac{\int_{\bar{x}(w,\theta)}^{\infty} x f(x) dx}{1 - F(\bar{x}(w, \theta))} + (1 - \alpha)B \tag{12}$$

Remark 2. *The wage setting function can be both decreasing or increasing in labour market tightness. A higher union bargaining power, α, a higher difference between average and marginal productivity of permanent workers, $\gamma(\bar{x}) - \bar{x}$, and a lower probability of being hired on permanent basis, $1 - F(\bar{x})$, imply the positive sign for $\frac{dw}{d\theta}$ to be more likely. Therefore, we can not exclude a negatively sloped wage setting function in the space w, θ.*

Proof. By defining $T = w - [\alpha\gamma(\bar{x}(w, \theta)) + (1 - \alpha)B]$, $\frac{dw}{d\theta} = -\frac{\frac{\partial T}{\partial \theta}}{\frac{\partial T}{\partial w}}$. We obtain $\frac{dw}{d\theta} = -\frac{-\alpha \frac{d\gamma}{d\bar{x}} \frac{d\bar{x}}{d\theta}}{1 - \alpha \frac{d\gamma}{d\bar{x}} \frac{d\bar{x}}{dw}}$. The numerator must be positive because the average productivity of permanent workers increases with the *keeping productivity* $\bar{x}$ and because $\bar{x}$ decreases with θ (see remark 1). Therefore, the sign depends on the opposite of the denominator of the previous equations, so that on $\alpha \frac{d\gamma}{d\bar{x}} \frac{d\bar{x}}{dw} - 1 =$. Considering that $\frac{d\gamma}{d\bar{x}} = \frac{f(X)}{1 - F(X)}[\gamma(X) - X]$ from equation 7, where $f(x)$ is the probability density function of x, we obtain:

$$sign\left(\frac{dw}{d\theta}\right) = sign\left[\alpha \frac{f(\bar{x}(w, \theta))}{1 - F(\bar{x}(w, \theta))} \left[\gamma(\bar{x}(w, \theta)) - \bar{x}(w, \theta)\right] \kappa(\theta) - 1\right]$$

$\square$

The result of indeterminacy of the slope of the wage setting function depends on the existence of the temporary contracts. In fact, a higher labour market tightness induces firms to hire more worker on permanent basis (see equation 11) and, by this way, to reduce the average productivity of permanent workers. Given that the bargaining process take into account the productivity of the average worker, this can also lead to a lower bargained wage rate.

Remark 3. *The wage rate of permanent workers increases in union bargaining power if it decreases in labour market tightness.*

Proof. We can compute $sign\left(\frac{dw}{d\alpha}\right)$ following the same steps seen above in proof 2.3. Hence, we obtain that $\frac{dw}{d\alpha} = -\frac{-[\gamma(\bar{x})-B]}{1-\alpha\frac{d\gamma}{d\bar{x}}\frac{d\bar{x}}{dw}}$. Given that $\gamma(\bar{x}) > B$ and that the denominator is the same as the one of $\frac{dw}{d\theta}$, the sign of $\frac{dw}{d\alpha}$ is opposed to the one of $\frac{dw}{d\theta}$. If the wage rate increases with the union bargaining power, it must also decrease with labour market tightness. $\square$

In conclusion, if an higher union's bargaining power increases the wage rate, as one should expect, the wage rate must depend negatively on labour market tightness.

2.4 Job Creation

Firms enter the market until the asset value of a vacancy, given by:

$$rV = -c + \eta[F(\bar{x}) - F(\underline{x})](J^T - V) + [1 - F(\bar{x})](J^P - V)] \qquad (13)$$

is positive. c is the cost of search and J^T and J^P stand for the expected average asset values of temporary and permanent positions, in turn given by:

$$rJ^T = \gamma^T(\bar{x}, \underline{x}) - B + [\lambda + \phi + \eta\theta](V - J^T) \qquad (14)$$

because $w^T = B$ as shown above. $\gamma^T(\bar{x}, \underline{x})$ is the average productivity of temporary workers, depending on the endogenous thresholds of the *hiring* and *keeping* productivity and it is defined as:

$$\gamma^T(\bar{x}, \underline{x}) = \frac{\int_{\underline{x}}^{\bar{x}} x f(x) dx}{F(\bar{x}) - F(\underline{x})} \qquad (15)$$

The asset value J^T computed from equation 13 and the one computed from equation 14 must be equal for $V = 0$. After substituting J^P computed from equation 6, considering that $w^T = B$, using $\kappa(\theta) = 1 + \frac{R}{\phi+\eta\theta}$, and defining $C \equiv \frac{cR}{\eta}$ we obtain the *job creation condition*:

$$C = \mu - w[1 - F(\bar{x})] - \left(\frac{\gamma^T(\bar{x}, \underline{x}) - B}{\kappa(\theta)} + B \right) F(\bar{x}) \tag{16}$$

where $\bar{x}(w, \theta)$ is defined in equation 11 and μ is the mean of the x distribution.

Remark 4. *The relationship between the wage rate of permanent workers, w, and the labour market tightness, θ, alongside the* Job creation condition *is negative.*

Proof. The relationship can be computed using the same tools seen in proof 2.3. We obtain the following result:

$$\frac{dw}{d\theta} = \frac{F(\bar{x})}{1 - F(\bar{x})} \frac{\frac{d\kappa}{d\theta}}{\kappa(\theta)^2} [\gamma^T(\bar{x}) - B]$$

that, according to the results of traditional search models, is negative. In fact $\frac{d\kappa}{d\theta} < 0$ and $\gamma^T(\bar{x}) > B$ (if not, temporary workers would have been paid more than their average productivity). □

The main conclusion obtained in the previous section is that if the wage setting function is increasing in union bargaining power it must be decreasing in labour market tightness and viceversa whereas alongside the job creation function the relationship between the wage rate and labour market tightness is always negative.

The whole solution of the model is given by the wage setting function of equation 12 and the job creation condition of equation 16.

2.5 Simplified case

In order to obtain meaningful results, let us now propose some specific assumptions to simplify the model. Obviously, the results proposed in this section represent simply a particular case whose results can nevertheless not be ignored.

In particular, we assume that x is uniformly distributed in the range [0,1]. These assumption gives $\gamma(\bar{x}) = \frac{1+\bar{x}}{2} = \frac{1+w\kappa(\theta)+(1-\kappa(\theta)B}{2}$ and allow us to solve in w the wage setting function defined in equation 12:

$$w(\alpha, \theta, B) = B + \frac{\alpha}{2 - \alpha\kappa(\theta)}(1 - B) \tag{17}$$

The derivative of $w(\alpha, \theta, B)$ with respect to B and α is positive, whereas its derivatives with respect to θ is negative[4].

We can conclude that the existence of a dual labor market where temporary workers are not covered by centralized bargaining makes the wage setting function decreasing in labour market tightness.

To further simplify the model assume $B = 0$, so that $\underline{x} = w^T = 0$ and substitute in the equation 16. Consider that, given the above simplifications, $\bar{x} = w\kappa(\theta)$ and substitute it in equation 16. We obtain a simplified version of the *job creation condition*.

$$\kappa(\theta) = \frac{2w - \xi}{w^2} \qquad JCC \tag{18}$$

where $\xi \equiv 1 - 2C$.

The above simplification give also raise to a further simplified version of the wage setting function shown in equation 17:

$$w(\alpha, \theta) = \frac{\alpha}{2 - \alpha\kappa(\theta)} \qquad WSF \tag{19}$$

As show in the appendix A, it is possible to demonstrate that the stable state steady equilibrium gives:

$$w^* = \frac{\alpha}{4}\left[3 + \Gamma(\alpha, \xi)\right]$$

$$\theta^* = \frac{R}{\eta}\left[\frac{2}{\alpha}\frac{1 + \Gamma(\alpha, \xi)}{3 + \Gamma(\alpha, \xi)} - 1\right]^{-1} - \frac{\phi}{\eta} \tag{20}$$

$$\bar{x}^* = \frac{1}{2}\left[1 + \Gamma(\alpha, \xi)\right] \tag{21}$$

where:

$$\Gamma(\alpha, \xi) = \sqrt{9 - 8\frac{\xi}{\alpha}}$$

In order to have real solutions, $\alpha \geq \frac{8}{9}\xi$ must hold. Moreover, in order to have $\bar{x} < 1$ (otherwise all workers where employed on temporary basis), it should be that $\Gamma(\alpha, \xi) < 1$. This implies $\alpha < \xi$. Real and significant solutions therefore exist only if $\alpha < \xi \leq \frac{9}{8}\alpha$. These strong restrictions derive from the simplification of the model presented above.

We can compute the effects of a variation in union bargaining power on the economic system.

[4] Note that $0 < \frac{\alpha}{2 - \alpha\kappa(\theta)} < 1$ must always hold in order to have $B < w < 1$.

Remark 5. *Stronger unions raise the wage rate and the* keeping productivity *and reduce labour market tightness. Higher difference in the probability of termination of the contract between temporary and permanent contract (ϕ) reduces labour market tightness without affecting the wage rate and the* keeping productivity.

Proof. Differentiating equations 20 we can write:

$$\frac{d\theta}{d\alpha} = \frac{1}{\alpha}\frac{(3\alpha-2\xi)\Gamma(\alpha,\xi)+(9\alpha-10\xi)}{[(3\alpha-2)-(2-\alpha)\Gamma(\alpha,\xi)]^2} \quad \frac{8R}{\eta\Gamma(\alpha,\xi)}$$

whose sign is positive if $(3\alpha-2\xi)\Gamma+(9\alpha-10\xi) > 0$. Solving for α the inequality, we obtain $\alpha > \xi$. However, as we have seen above, significant solutions of the model exist only if $\alpha < \xi$. Therefore, we conclude that $\frac{d\theta}{d\alpha} < 0$ must always hold.

Differentiating equation 21, we obtain:

$$\frac{d\bar{x}}{d\alpha} = \frac{2\xi}{\alpha\Gamma_1} > 0$$

the same sign holds for $\frac{dw}{d\alpha}$. □

3 Flow, unemployment and productivity in the simplified case

In steady state, the stocks of unemployed, u, temporary workers, L^T, and permanent workers, L^P, depend on the flow conditions.

The number of unemployed people remains constant if:

$$\eta\theta(1-\underline{x})u = \lambda L^P + (\lambda+\phi)L^T \tag{22}$$

where the left hand side represents the number of unemployed that find a job, given by the probability of finding a job for an unemployed ($\eta\theta(1-\underline{x})$) times the number of unemployed (u) whereas the right hand side represents the number of permanent and temporary workers that loose a job.

The number of workers employed on temporary basis is constant if:

$$[\lambda+\phi+\eta\theta(1-\bar{x})]L^T = \eta\theta(\bar{x}-\underline{x})u \tag{23}$$

where the right hand side represents the number of workers who leave the temporary position, given by the temporary workers who loose the job because of negative shocks ($(\lambda+\phi)L^T$) plus the ones who leave the temporary position finding a permanent job ($\eta\theta(1-\bar{x})L^T$), and the right hand side represents the number of workers entering in the temporary position, given by the number of matchings whose productivity is between $\bar{x}$ and $\underline{x}$.

The number of permanent workers is constant if:

$$\lambda L^P = \eta\theta(1-\bar{x})(L^T + u) \tag{24}$$

Given that we are mainly interested in the effects of α, we set, as before, $\underline{x} = 0$ and we further simplify the model by assuming $\phi = 0$. Solving equation 23 on L^T, substituting it in equation 22 and considering $L^P = 1 - u - L^T$, we obtain the equilibrium unemployment rate:

$$u(\theta) = \frac{\lambda}{\lambda + \eta\theta} \tag{25}$$

Remark 6. *The unemployment rate is decreasing in the probability of finding a job so that it is increasing in the union bargaining power.*

Proof. The negative relationship between θ and α has been shown in remark 5. □

Given the unemployment rate, we can obviously solve for L^T using equation 23 and for L^P using equation 24. The *shakiness index*, defined $\frac{L^T}{L^P+L^T}$ is given by:

$$s = \frac{\lambda\bar{x}}{\lambda + \eta\theta(1-\bar{x})}$$

Remark 7. Shakiness *in the labour market depends positively on $\bar{x}$ and negatively on θ so that in depends positively on the unions bargaining power, α*

Proof. s depends positively on $\bar{x}$ and negatively on θ. By considering remark 5, the proof cames immediately. □

Let us now consider the average worker's productivity, defined as

$$y = \frac{\frac{1+\bar{x}}{2}L^P + \frac{\bar{x}}{2}L^T}{L^P + L^T}$$

substituting 23 and 24 and rearranging, we obtain:

$$y = \frac{1}{2}\frac{\lambda + \eta\theta(1-\bar{x}^2)}{\lambda + \eta\theta(1-\bar{x})} \tag{26}$$

Remark 8. *The average per worker productivity is increasing in θ and increasing in $\bar{x}$ is $y > \bar{x}$ holds.*

Proof. Defining $D = \lambda + \eta\theta(1-\bar{x})$, and differentiating equation 26 with respect to θ, we obtain $\frac{dy}{d\theta} = \frac{\eta\lambda\bar{x}(1-\bar{x})}{2D^2} > 0$. The derivative of equation 26 with respect to $\bar{x}$, can be written as follows: $\frac{dy}{dw} = \frac{\eta\theta(y-\bar{x})}{D}$.

□

Remark 9. *There exist a given level of union bargaining power that maximizes workers productivity.*

Proof. In equation 26 both θ and $\bar{x}$ are function of the union bargaining power α (see equations 20 and 21. Differentiating equation 26 with respect to α, we obtain:

$$sign\left(\frac{dy}{d\alpha}\right) = sign\left(2\theta(y-\bar{x})\frac{d\bar{x}}{d\alpha} + (1-\bar{x})(1+\bar{x}-2y)\frac{d\theta}{d\alpha}\right) \tag{27}$$

where $\frac{d\bar{x}}{d\alpha} > 0$ and $\frac{d\theta}{d\alpha} < 0$, as shown in the remark 5.

Substituting the two derivatives displayed in remark 5 in the right hand side of equation 27 we obtain a cumbersome result whose sign is, in general, not definable.

We also know that, in order to have meaningful solutions, $\alpha < \xi < \frac{9}{8}\alpha$ must hold. We follow the strategy of evaluating equation 27 for the minimum and the maximum value of x_i.

For $\alpha = \xi$, we have $\Gamma = 1$. It is easy to compute that $\frac{d\theta}{d\alpha} = 0$ and that $\frac{d\bar{x}}{d\alpha} = \frac{2}{\alpha}$. Furthermore, from equation 21 we obtain $\bar{x} = 1$ and from equation 26 we obtain $y = \frac{1}{2}$. Therefore, $y - x < 0$ and $\frac{dy}{d\alpha} < 0$. When α takes its maximum value, the average workers' productivity is decreasing in α.

For $\alpha = \frac{8}{9}\xi$, or, more precisely, taking the limit for ξ tending to $\frac{9}{8}\alpha$, Γ tends to zero and $\frac{d\theta}{d\alpha}$ tends to zero, too. Therefore, $\frac{dy}{d\alpha} > 0$ if $y > x$. But, in that case, $\bar{x} = \frac{1}{2}$ (see equation 11). We must show that, conditionally to $\alpha = \frac{8}{9}\xi$, $y > \frac{1}{2}$ holds, so that $\frac{1}{2}\frac{\lambda+\frac{3}{4}\eta\theta}{\lambda+\frac{1}{2}\eta\theta} > \frac{1}{2}$.This condition is always respected.

We can therefore conclude that for α at its minimum acceptable value the per worker productivity increases with union bargaining power and that for α at its maximum value per worker productivity decreases with union bargaining power. □

4 Discussion

The model developed above highlighted the role payed by unions in a segmented labour market divided between permanent workers, covered by contractual arrangements, and temporary workers, paid at their reservation wage.

In this setting, in the hypothesis that a higher union bargaining power leads to a higher wage rate, the wage setting function must be downward sloping with respect to labour market tightness. This result is in contrast with the theoretical literature and can be explained by considering that a higher tightness makes more difficult to replace temporary workers (who are the more likely to leave jobs) and push firms to keep more workers on permanent basis. By this way, the marginal and average productivity of permanent workers decrease and the bargained wage rate, that is a weighted sum of the average productivity and the reservation wage, must decrease too.

In the stable equilibrium, stronger unions raise the permanent workers' wage rate, unemployment and the ratio of temporary on total workers and reduce labour market tightness, as expected. By allocating the most productive matching in the permanent position, unions allows for an increase in workers productivity. In fact, it exists a given level of unions' bargaining power which maximizes per-capita product. Therefore, even dealing with a decreasing wage setting function, we obtain results that are consistent with the economic literature.

The result concerning workers' productivity is probably the more interesting. In the setting presented above, the way in which workers are allocated between temporary and permanent jobs determines workers' productivity. The two extreme cases coherent with the hypotheses of the model are the one where half of the workers are employed on temporary basis and the one where all workers are employed on temporary basis. The first case happens when unions have the minimum bargaining power, the second one coincides with the strongest unions. In the two cases, the average productivity of workers coincides with the average productivity of the average match. Firms evaluation of matching productivity and the choice of offering a permanent or a temporary position to each worker is crucial in determining the average productivity. Stronger unions, pushing firms toward less permanent jobs (because the wage rate of permanent workers becomes higher), allows a "better" allocation of workers and a higher average productivity. Nevertheless, they increase unemployment. The price to pay for having higher productivity are an higher unemployment and shakiness. The reforms aimed to raise productivity may explain the contemporaneous reduction of unemployment rates and productivity and the increase in the ratio of temporary on permanent workers in many European Countries.

Further developments of the model should remove some hypotheses that, even if allowed us to solve analytically the model, are probably too restrictive, namely: a) the absence of firing costs; b) the wage rate of temporary workers equal to the reservation wage.

4.1 Appendix A

The whole solution of the model is therefore represented by the system of two equations (18 and 19) in two endogenous variables ($\kappa(\theta)$, w).

By solving the WSF and the JCC defined above in $\kappa(\theta)$ and by equating the two solutions, we end up with an equation in w that, once solved, gives the following two roots:

$$w_i^* = \frac{3\alpha \pm \sqrt{\alpha(9\alpha - 8\xi)}}{4} \qquad for \quad i = 1,2 \quad (i = 1 : minus \quad i = 2 : plus)$$

Given this result, we can compute the value of $\kappa(\theta)$:

$$\kappa^*(\theta)_i = \left(\frac{2}{\alpha}\right) \frac{\alpha \ \pm \sqrt{\alpha(9\alpha - 8\xi)}}{3\alpha \pm \sqrt{\alpha(9\alpha - 8\xi)}} \qquad for \quad i = 1,2 \quad (i = 1 : minus \quad i = 2 : plus)$$

and, given $\bar{x^*}_i = w_i^* \kappa^*(\theta)_i = 2$ the value of the *keeping productivity*:

$$\bar{x^*}_i = \frac{\alpha \pm \sqrt{\alpha(9\alpha - 8\xi)}}{2\alpha} \qquad for \quad i = 1,2 \quad (i = 1 : minus \quad i = 2 : plus)$$

The two roots give real solutions if $\xi \leq \frac{9}{8}\alpha$ where, if the equal sign holds, the roots coincide. The *keeping productivity*, $\bar{x}^*$, is included in the $0 - 1$ interval if $\alpha < phi$, so that $\alpha < \xi < \frac{9}{8}\alpha$.

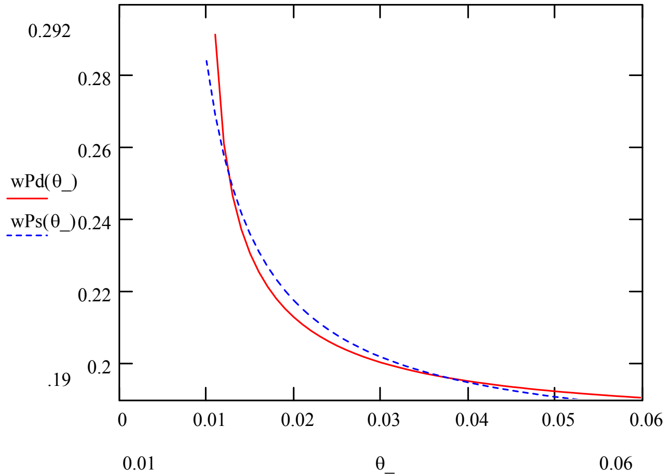
Assume now that two solutions exist. The selection between these two solutions can be obtained considering the dynamic of the model. In particular, assume that the wage rate of equation 19 adjusts to labour market tightness with one period lag, so that $w_t = \frac{\alpha}{2 - \alpha\kappa(\theta_{t-1})}$. Computing $\frac{w_t - wt-1}{wt-1}$, we obtain:

$$\frac{w_t - w_{t-1}}{w_{t-1}} = -\frac{2w_{t-1}\left(w_{t-1} - \frac{3}{2}\alpha\right) + \alpha\xi}{2w_{t-1}\left(w_{t-1} - \alpha\right) + \alpha\xi}$$

where both the numerator and the denominator show a minimum in w_{t-1}. The denominator is always positive because the minimum of w_{t-1} is attained for $w_{t-1} = \frac{\alpha}{2}$ and the value of w_{t-1} calculated at the minimum give $w_{t-1}^{MIN} = \alpha(\xi - \frac{\alpha}{2}) > 0$ because $\xi > \alpha$.

Therefore the sign of the variation rate of wages depends on the opposite of the sign of the numerator, so that $\frac{w_t - wt-1}{w_{t-1}} > 0$ if $2w_{t-1}\left(w_{t-1} - \frac{3}{2}\alpha\right) +$

Figure 1: The wage setting function (wPs) and the job creation condition (wPd)



$\alpha\xi < 0$. Solving this quadratic form for w_{t-1}, we obtain that $\frac{w_t - w_{t-1}}{w_{t-1}} > 0$ for $w_1^* < w_{t-1} < w_2^*$, where w_1^* and w_2^* are the two roots of the system presented at the beginning of this appendix (see also figure 4.1) and that it decreases for external values of the two roots. We can therefore conclude that the wage rate varies until the only stable equilibrium w_2^* is attained and that, for value of $w_{t-1} < w_1^*$ the system does not have solutions.

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