Stochastic Dynamics and Matching in the Old Keynesian Economics: A Rationale for the Shimer's Puzzle*

Marco Guerrazzi‡
Department of Economics
University of Pisa

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Abstract

In this paper, following the micro-foundation of the *General Theory* provided by Farmer (2008a-b, 2009), I build a competitive search model in which output and employment are demand-driven, prices are flexible and the nominal wage is chosen as numeraire. In addition, another "Old Keynesian" feature of my theoretical proposal is that agents are divided into two categories, i.e., wage and profit earners (e.g. Kaldor 1955-1956). The former are assumed to work and consume while the latter arrange production and save by financing an investment expenditure driven by a stochastic process. Calibrating and simulating the model in order to fit the US first-moments data, I show that this framework can provide a rationale for the Shimer's (2005) puzzle, i.e., the relative stability of real wages in spite of the large volatility of labor market tightness.

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‡ Research Fellow, Department of Economics, University of Pisa, via F. Serafini n. 3, 56126 Pisa, Phone +39 050 2212434, Fax +39 050 2212450, E-mail guerrazzi@ec.unipi.it

1 Introduction

Addressing some criticisms raised after the publication of the *General Theory*, Keynes (1937, pp. 211-212) prophetically stated: "I'm more attached to the comparatively simple fundamental ideas which underlie my theory than to the particular forms in which I have embodied them, [...], time and experience and the collaboration of a number of minds will discover the best way of expressing them".

An influential evaluation of the circulation of the Keynesian legacy in the economic profession is given by Leijonhufvud (1966) in *On Keynesian Economics and the Economics of Keynes.* On the one hand, by *Keynesian Economics* Leijonhufvud (1966) means the interpretation of the *General Theory* incorporated into the Hicksian IS-LM apparatus and more recently into the so called "New-Keynesian" paradigm[1]. It is well known that this kind of modeling has a Neo-classical core through which deviations from the "natural" rate of unemployment can be interpreted as optimal reactions to nominal and/or real rigidity. On the other hand, Leijonhufvud (1966) correctly recognizes that those rigidities were not a central argument of the old *Economics of Keynes.*

In this paper I cause to breathe new life into the "Old Keynesian" economics by blending it with modern search and business cycle theories and then testing the resulting amalgam against recent empirical evidence on macroeconomic fluctuations. Specifically, following the micro-foundation of the *General Theory* recently provided by Farmer (2008a-b, 2010), I build a competitive search model in which output and employment are driven by effective demand, the price level is perfectly flexible and the nominal wage rate is chosen as numeraire. As pioneered by Moen (1997), the "competitiveness" of the Farmer's (2008a-b, 2010) demand-driven search framework arises from the fact that agents take prices and other market parameters as given when they decide their optimal behavior. However, in contrast to Moen (1997), the matching technology is assumed to enter the problem of workers and employers as a proper trading externality so that the resulting equilibrium allocation is not - in general - Pareto optimal[2]. As a consequence, the social optimum allocation results from the solution of a well-defined

[1]See, for example, Mankiw and Romer (1991).

[2]The underlying assumption is that that moral hazard factors prevents the creation of competitive markets for search inputs.

planning problem in which all the search externalities are completely internalized.

Another Old Keynesian feature of my theoretical proposal put forward to ease aggregation and simplify dynamics is that economic agents are divided into two broad categories, i.e., wage and profit earners. As originally suggested by Kaldor (1955-1956, pg. 95), the wage-category comprises not only manual labor but salaries as well while the profit-category comprises the income of property owners and not only of entrepreneurs. Thereafter, both categories of agents are modeled with a discrete version of the perpetual youth overlapping-generations (OLG) framework pioneered by Yaari (1965) and Blanchard (1985) and are also assumed to differ in their marginal propensities to consume and their tasks. On the one hand, wage earners dislike savings and consume the whole income raised by supplying a fixed amount of labor services. On the other hand, profit earners are assumed to save the total income raised by arranging the production of goods in order to finance investments and capital accumulation[3]. Moreover, following Farmer (2008a) I formalize the Keynesian "animal spirits" of profit earners by assuming that their investment expenditure is driven by an autonomous stochastic process such as those usually exploited for total factor productivity (TFP) in the real business cycle (RBC) literature.

Given those building blocks, I show that this stochastic framework is consistent with a unique demand constrained equilibrium (DCE) that is not necessarily Pareto-efficient but - on the contrary - can be well characterized by socially-inefficient over or under employment. Furthermore, I emphasize that this theoretical setting might be consistent with a certain degree of endogenous real wage stickiness and that it can be exploited to prove "classical" Keynesian propositions such as the feasibility of effective balanced fiscal policies that successfully restore the social optimum level of (un)employment. Specifically, I show that the former possibility is due to a turning point in the concavity of the equilibrium wage function that might lead to a quite flat trade-off between real wage rates and employment, the latter to a multiplier effect

[3]Following the cash-flow criterion proposed by Abel et al. (1989), these two assumptions place the economy exactly in the border-line region between dynamic efficiency and inefficiency. However, in the analysis that follows nothing would be changed by assuming that profit earners consume a share of their investment expenditure. Finally, the "usual" situation in which investments are financed by the savings of workers is analyzed in the OLG model developed by Farmer (2010, Chapter 4).

that works through changes in the autonomous components of aggregate demand.

Finally, calibrating and simulating the model economy in order to fit the US first-moments data, I show that under the hypothesis that the economy is hit simultaneously by demand and supply shocks the suggested stochastic framework can provide a rationale for the so-called Shimer's (2005) puzzle, i.e., the relative stability of real wage rates in spite of the large volatility of labor market tightness indicators. The added-value of my computational proposal is twofold. First, this striking US business cycle feature can hardly be explained by means of the standard matching model a là Mortensen and Pissarides's (1994) in which real wage rates are the outcome of a generalized Nash (1950) bargaining. Second, in sharp contrast with the contributions by Hall (2005a-b) and Shimer (2005), the puzzle is resolved without assuming any real wage rigidity. In fact, I show that the required real wage stickiness for the amplification of real shocks can endogenously arise from the combination of the Farmer's (2008a-b, 2010) demand-driven competitive search framework with a quite conventional calibration of the model economy.

The paper is arranged as follows. Section 2 describes the theoretical framework. Section 3 provides some numerical simulations. Finally, section 4 concludes.

2 The Model

I develop a demand-driven perpetual youth model with competitive search along the lines traced out by Farmer (2008a-b, 2010). Specifically, exploiting a discrete version of the OLG framework put forward by Yaari (1965) and Blanchard (1985), I consider a model economy in which a unit mass of new agents born in each time period. Thereafter, with a fixed instantaneous probability, each new born agent is randomly assigned to an income earners' category characterized by a specific marginal propensity to consume and a specific task. On the one hand, wage earners are assumed to dislike savings and consume the whole income raised by supplying labor services. On the other hand, profit earners are assumed to save the whole income raised by arranging a stochastic production process by aiming at financing investments and capital accumulation. Moreover, following Farmer (2008a) I assume that the profit earners' investment expenditure measured in wage units evolves according to an autonomous stochastic AR(1) process that - as suggested by Kurz (2008) - might be consistent with a limit posterior of a Bayesian inference for self-fulfilling beliefs.

This simple dynamic setting allows to derive all the properties of a DCE for a model economy with savings and productive investments in a straightforward manner. Furthermore, I exploit this framework to emphasize the possibility of a certain degree of endogenous real wage stickiness and to show how to design balanced fiscal policies that successfully implement the social optimum level of (un)employment.

2.1 Wage Earners

Without loss of generality, I assume the existence of many different generations of identical wage earners indexed by their date of birth. Each generation of wage earners is assumed to survive into the subsequent period with a fixed probability $\pi \in (0,1)$. Moreover, in order to preserve a unit mass population of each income earners' category, every time period a measure $(1-\pi)$ of wage earners is assumed to born and die. As a consequence, the inter-temporal problem of the generation of wage earners born at time 0 is given by

$$\max_{\{C_t\}_{t=0}^{+\infty}} \Gamma_0(\cdot) = E_0\left[\sum_{t=0}^{+\infty} (\pi\theta)^t \log(C_t)\right] \qquad 0<\theta<1 \tag{1}$$

where $E[\cdot]$ is the expectation operator, θ is a discount rate (equal for all the agents) and C_t is the instantaneous level of consumption in real terms.

As stated above, I assume that wage earners dislike saving and consume the whole income raised by providing a fixed amount of search intensity. Therefore, the instantaneous budget constraint for the inter-temporal problem in (1) can be written as

$$p_t C_t = w_t L_t \quad \text{ for all } t \tag{2}$$

where p_t is the current price level, L_t is current employment and w_t is the current nominal wage rate[4].

In each period, wage earners are endowed with a single unit of time H_t that they inelastically allocates to job search activities. Among searching workers, the fraction L_t successfully find a job while the remaining U_t is unemployed. Therefore,

[4]Obviously, I'm implicitly assuming rational expectations.

$$L_t + U_t = H_t \equiv 1 \tag{3}$$

Current employment and labor supply are assumed to be linked by the following expression:

$$L_t = h_t H_t = h_t \tag{4}$$

where h_t is instantaneous hiring effectiveness. Obviously, this market parameter - taken as given by wage earners - can be determined as

$$h_t = \frac{\overline{L}_t}{\overline{H}_t} = \overline{L}_t \tag{5}$$

where $\overline{L}_t$ is aggregate employment while $\overline{H}_t \equiv 1$ is the aggregate labor supply.

It is worth noting that the expression in (5) conveys a typical "thick market" externality, *i.e.*, the higher (lower) the aggregate employment rate, the easier (harder) for a wage earner to find a job. See Diamond (1982).

The maximization of (1) subject to (2) leads to the following straightforward solution:

$$C_t = \left(\frac{w_t}{p_t}\right) L_t \qquad \text{for all } t \tag{6}$$

With equilibrium search unemployment *ex-ante* homogeneous wage earners might become heterogeneous in the absence of perfect income insurance because in this case the individual wealth would depend on the individual employment history. See Merz (1995) and Andolfatto (1996). However, the assumption that in each period wage earners pool their income in order to subsidize who remain unemployed does not alter the results achieved in the paper.

2.2 Profit Earners

Symmetrically with wage earners, I assume the existence of many different generations of identical profit earners indexed by their date of birth. Thereafter, in each period the representative profit earner is assumed to arrange the production of a homogeneous-perishable good by means of the following constant-returns-to-scale Cobb-Douglas production function:

$$Y_t = A_t K_t^{\alpha} X_t^{1-\alpha} \qquad\qquad 0 < \alpha < 1 \tag{7}$$

where Y_t is the output level, A_t is a productivity shock, K_t is the current stock of capital, α is the capital share and X_t is the fraction of employed wage earners allocated to production activities.

As in the standard RBC theory, the log of the productivity shock is assumed to follow a stochastic AR(1) process. Hence,

$$\ln A_t = \ln \mu + \xi \ln A_{t-1} + \eta_t \tag{8}$$

where μ is a positive drift, ξ measures the persistence of productivity shocks and $\eta_t \sim N\left(0, \sigma_\eta^2\right)$ is a stochastic productivity disturbance.

Given (7) and (8), the inter-temporal problem of profit earners born at time 0 is given by

$$\max_{\{L_t\}_{t=0}^{+\infty}} \Pi_0 (\cdot) = E_0 \left[\sum_{t=0}^{+\infty} \left((1-\pi)\,\theta \right)^t (p_t Y_t - w_t L_t) \right] \qquad 0 < \theta < 1 \tag{9}$$

The expression in (9) suggests that profit earners are assumed to survive into the subsequent period with a fixed probability which is complementary to the surviving probability of wage earners. As a consequence, this means that, in each period, the unit mass of now born agents have a probability $(1-\pi)$ of being wage earners and a probability π of being profit earners.

Following Farmer (2008a-b, 2010), the instantaneous resource constraint for the inter-temporal problem in (9) is assumed to be the following:

$$L_t = X_t + V_t \qquad \text{for all } t \tag{10}$$

where V_t is the fraction of employed wage earners allocated to recruiting activities by the representative profit earner.

In each period, current employment and the fraction of wage earners allocated to recruiting activities are assumed to be linked by

$$L_t = v_t V_t \tag{11}$$

where v_t is the recruiting efficiency parameter.

The expression in (11) is aimed at providing a micro-foundation for applications processing and suggests that profit earners know that V_t recruiters can instantaneously hire $v_t V_t$ wage earners X_t of whom will be employed in production activities. Obviously, this market parameter can be determined as

$$v_t = \frac{\overline{L}_t}{\overline{V}_t} \tag{12}$$

where $\overline{V}_t$ is the fraction of wage earners allocated to recruiting activities by all profit earners in the whole economy.

Given (10) - (12), the inter-temporal problem of profit earners reduces to

$$\max_{\{L_t\}_{t=0}^{+\infty}} E_0 \left[\sum_{t=0}^{+\infty} \left((1-\pi)\,\theta \right)^t \left(p_t A_t K_t^{\alpha} L_t^{1-\alpha} \left(1 - \frac{1}{v_t} \right)^{1-\alpha} - w_t L_t \right) \right] \qquad 0 < \theta < 1 \tag{13}$$

It is worth noting that v_t acts as an externality factor in the profit earners' problem. In other words, profit earners set employment by taking the recruitment efficiency as a given market parameter.

The first-order condition (FOC) for the problem in (13) is given by

$$(1-\alpha)\,\frac{Y_t}{L_t} = \left(\frac{w_t}{p_t} \right) \qquad \text{for all } t \tag{14}$$

Finally, the capital stock is assumed to evolve according to the usual dynamic accumulation law. As a consequence,

$$K_{t+1} = I_t + (1-\delta)\,K_t \qquad 0 < \delta < 1 \tag{15}$$

where $I_t = Y_t - \left(\frac{w_t}{p_t} \right) L_t$ are real investments while δ is the depreciation rate of capital.

In the remainder of the paper, I will assume that the employment rate is *not* a state-variable for the model economy[5]. In other words, I will suppose that at the end of each period all the employed wage earners are fired and (possibly) re-hired at the beginning of the next[6].

[5] As it will become clear later on, the state-variables of the economy are the productivity shock, the capital stock and investment expenditure.

[6] Obviously, this means a 100% labor market turnover rate in each time period.

2.3 Matching

Now I describe how searching wage earners find jobs in the economy as a whole. Specifically, following Farmer (2008a-b, 2010) the aggregate searching technology is assumed to be given by the following instantaneous constant returns-to-scale matching function:

$$\overline{L}_t = \overline{H}_t^{\gamma} \overline{V}_t^{1-\gamma} \qquad\qquad 0 < \gamma < 1 \tag{16}$$

where γ is the matching elasticity.

The expression in (16) suggests that in each period aggregate employment is the result of the matching between all the searching unemployed wage earners and the recruiters employed by all the profit earners. Therefore, in contrast to the standard matching approach popularized by Pissarides (2000), I'm assuming that vacancies are posted by using labor instead of output.

Given the fixed instantaneous search allocation in (3), (16) simplifies to

$$\overline{L}_t = \overline{V}_t^{1-\gamma} \tag{17}$$

The expression in (17) describes how aggregate employment is related to the aggregate recruiting effort arranged by all the profit earners. Given the result in (3), a simple manipulation of (17) allows also to derive a stable trade-off between the aggregate level of recruiters and the total amount of unemployed wage earners which provides a version of the well-known Beveridge curve that, in turn, summarizes the operation of search externalities in the whole economy[7]. Specifically,

$$\overline{V}_t = \left(1 - \overline{U}_t\right)^{\frac{1}{1-\gamma}} \tag{18}$$

where $\overline{U}_t$ is the aggregate unemployment rate. A graphical outlook is given in figure 1.

It is worth noting that this version of the Beveridge curve crosses the vertical axis when all the wage earners are allocated to recruiting activities and there is no unemployment while

[7]Pissarides (2000) derives a Beveridge curve by considering the steady-state of the unemployment evolution law arising from a constant-returns to scale matching function coupled with an exogenous hazard rate for employment contracts. Hansen (1970) derived a negative equilibrium relationship between job vacancies (or recruiters) and unemployment from more primitive assumptions.

it crosses the horizontal axis when all the wage earners are unemployed and no profit earner carries out recruiting activities[8].

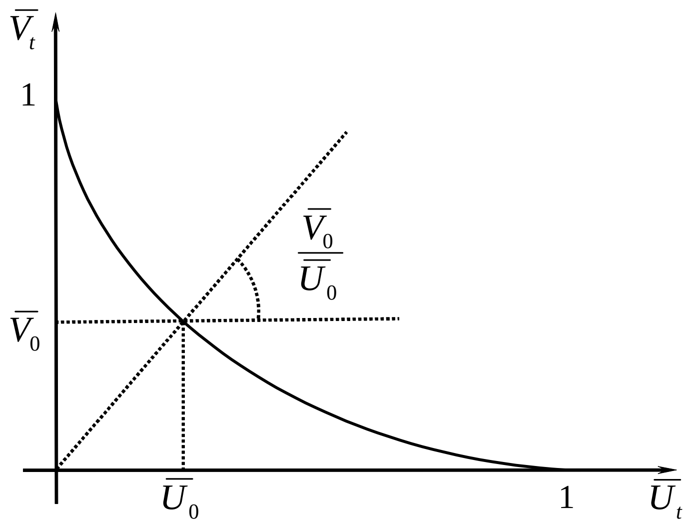


Figure 1: Beveridge curve

Considering a situation of instantaneous "symmetric" equilibrium, i.e., a situation in which in a given time period $\overline{L}_t = L_t$ and $\overline{V}_t = V_t$, then it becomes possible to express the recruiting efficiency parameter as a function of aggregate employment. In fact, exploiting the results in (10), (11) and (17) it is possible to derive

$$v_t = L_t^{-\frac{\gamma}{1-\gamma}} \tag{19}$$

In contrast to (5), the expression in (19) conveys a typical "thin market" externality, i.e., the recruiting efficiency relevant for the profit earners' problem is higher (lower), the lower (higher) is aggregate employment. Again, see Diamond (1982).

2.4 Social Optimum

In order to have an efficient benchmark for the evaluation of realized allocations, I analyze the problem of an omniscient-benevolent social planner who can operate simultaneously the

[8]The stability of the Beveridge curve is argued by Abraham and Katz (1986) while its distance from the vertical and the horizontal axis is discussed by Solow (1998).

production and the matching technologies in (7) and (16) by internalizing the externality factor in (19). In a subsequent part of the paper, this exercise will provide also some insights on the properties of the equilibrium wage function.

Given the formal structure of the model economy, in each time period the Pareto-optimal allocation is defined by the level of employment that maximizes the sum between wage income and realized profits in real terms, i.e., the level of L_t that maximizes the real amount of output for any given level of the productivity shock and the stock of employed capital[9]. As a consequence, the inter-temporal social planner problem is the following:

$$\max_{\{L_t\}_{t=0}^{+\infty}} \Lambda_0(\cdot) = E_0 \left[\sum_{t=0}^{+\infty} \theta^t A_t K_t^{\alpha} X_t^{1-\alpha} \right] \qquad 0 < \theta < 1 \tag{20}$$

The instantaneous resource constraint for the inter-temporal problem in (20) is given by

$$X_t = L_t \left(1 - L_t^{\frac{\gamma}{1-\gamma}} \right) \qquad \text{for all } t \tag{21}$$

As a consequence, the social-optimal level of employment is simply

$$L_S = (1-\gamma)^{\frac{1-\gamma}{\gamma}} \qquad \text{for all } t \tag{22}$$

Obviously, $U_S = 1 - L_S$ provides the social-optimal unemployment rate. Furthermore, it is worth noting that L_S and U_S depend only on the unique parameter of the matching technology[10]. A graphical outlook of the social planner problem is given in figure 2.

The diagram in figure 2 allows to clarify some important features of the production and matching technologies. Obviously, whenever $L_t = 0$ there is no production for the good reason that no wage earner is employed. However, there is no production even when $L_t = 1$. In fact, in this case, taking into account of the result in (19), the aggregate recruiting efficiency would be equal to one. Therefore, it would be optimal for all the profit earners to allocate all the employees to recruiting while no wage earner would be allocated to production activities[11].

[9] In a simple competitive (auction) model of the labor market, the Pareto-optimal allocation would be given by $L_S = 1$, for all t.

[10] Farmer (2008b, 2010) sets $\gamma = \frac{1}{2}$, so that he derives $L_S = U_S = \frac{1}{2}$. A calibration that fits the average long-run US unemployment rate is given in section 3.

[11] In fact, whenever $\overline{U}_t = 0$ the corresponding point on the Beveridge curve in (18) is given by $\overline{V}_t = 1$.

As a consequence, total output is at its maximum level Y_S whenever $L_t = L_S$. Indeed, any additional employed wage earner would not produce additional output but he would be simply employed in recruiting additional recruiters without improving the resulting allocation[12].

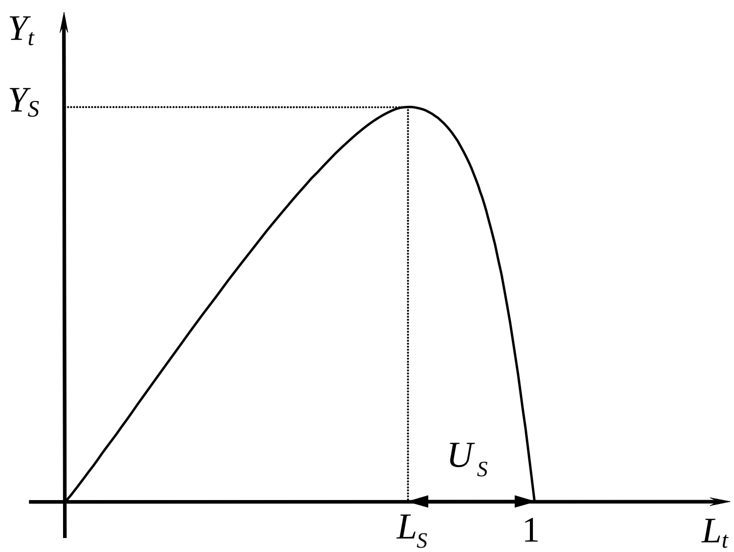


Figure 2: Social optimum level of (un)employment

2.5 Aggregate Demand and Supply

At any given time t, taking into consideration the national account identity, the value of aggregate demand (AD_t) can be obtained by adding the aggregate nominal consumption (c_t) and the nominal investment expenditure ($p_t I_t$)[13]. Hence,

$$AD_t = c_t + p_t I_t \tag{23}$$

Given the result in (6), aggregate nominal consumption is given by

$$c_t = w_t L_t \tag{24}$$

[12] As a consequence, there will be also a social optimal level of recruiters equal to $(1-\gamma)^{\frac{1}{\gamma}}$, for all t.

[13] It is worth noting that the hypothesis of product homogeneity allows to value I_t with the same price index used for Y_t.

Following the choice of units made by Keynes (1936) in the *General Theory* (Chapter 4, pg. 41) and resumed by Farmer (2008a-b, 2010), I use the nominal wage rate w_t as numeraire[14]. Therefore, taking into account the results in (23) and (24) the value of aggregate demand in wage units can be written as

$$\frac{AD_t}{w_t} = L_t + \widehat{I}_t \tag{25}$$

where $\widehat{I}_t \equiv \left(\frac{w_t}{p_t}\right)^{-1} I_t$.

In the model economy under examination, investments are not derived from (rational) optimization. Following Farmer (2008a, pg. 38), I formalize the Keynesian animal spirits of profit earners by assuming that nominal investment expenditure measured in wage units is driven by an autonomous stochastic AR(1) process such the one in (8). Hence,

$$\widehat{I}_t = \kappa + \rho \widehat{I}_{t-1} + \epsilon_t \tag{26}$$

where κ is a positive drift, ρ measures the persistence of the exogenous investment sequence and $\epsilon_t \sim N\left(0, \sigma_\epsilon^2\right)$ is a stochastic demand disturbance[15].

The expression in (26) formalizes in a very simple way a central issue of the *General Theory*, i.e., the idea that investment expenditure evolves exogenously with no regard for expected profits. In fact, in the *Economics of Keynes* the main driving force of investments is given by the state of long-term expectations[16]. Along these lines, modeling a state variable for the beliefs' evolution process of traders committed in forecasting the true liquidation value of a risky asset, Kurz (2008, pp. 778-779) deduces a first-order autoregressive process like the one in (26) as a limit posterior of a Bayesian learning inference in a non-stationary environment.

Let me turn to aggregate supply. In the *General Theory* (Chapter 3, pp. 24-25), Keynes (1936) defined the value of aggregate supply by having in mind the idea of entrepreneurs that compete one another for the factors of production by means of price adjustments. As suggested by Farmer (2008a-b, 2010), in a one-good economy the equation that triggers competition for

[14]The additional macroeconomic consequences of assuming a constant nominal wage rate are explored in Appendix.

[15]Different specifications with some heterogeneous microfoundation flavors are tested in Appendix.

[16]See Keynes (1936), Chapter 12, pg. 149.

the labor factor is the FOC for L_t in (14). Therefore, if the nominal wage rate w_t is chosen as numeraire, then the value of aggregate supply in wage units is simply given by

$$\frac{AS_t}{w_t} = \frac{1}{1-\alpha} L_t \tag{27}$$

The instantaneous equilibrium condition for the goods market, i.e., $AD_t = AS_t$, provides the following solutions for the value of output in wage units and the level of employment:

$$\left(\frac{w_t^*}{p_t^*}\right)^{-1} Y_t^* = \frac{1}{\alpha}\widehat{I}_t \quad \text{and} \quad L_t^* = \frac{1-\alpha}{\alpha}\widehat{I}_t \tag{28}$$

Obviously, $U_t^* = 1 - L_t^*$ provides the corresponding rate of (actual) unemployment. Furthermore, it is worth noting the multiplier effect played by the inverse of the capital share[17]. A graphical outlook of aggregate demand and supply in wage units is given in figure 3.

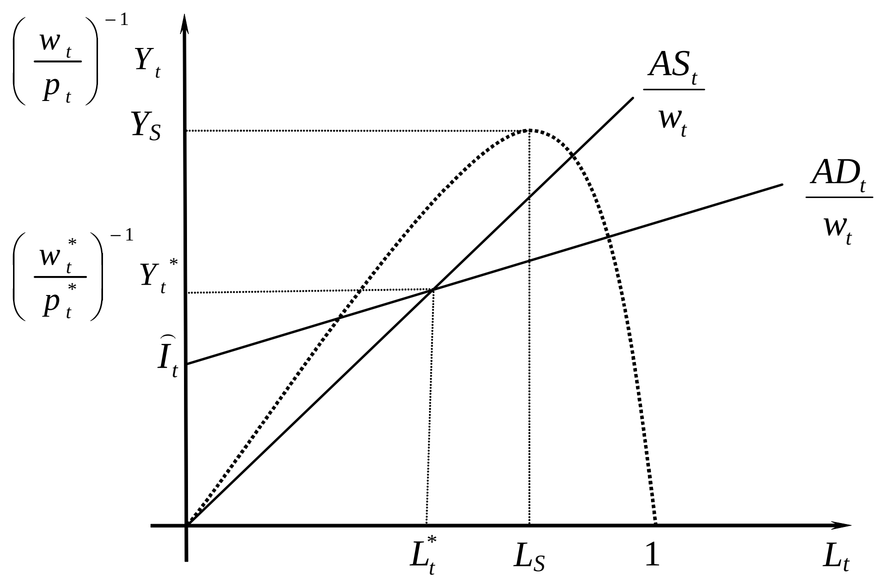


Figure 3: Aggregate demand and supply in wage units

Given the erratic nature of investment expenditure, there is no certainty that actual employment coincides with the social optimum in (22). For example, the diagram in figure 3 shows a situation in which actual employment is lower than the social-optimal level so that

[17]In conventional macroeconomics textbooks, the same effect is played by the inverse of the marginal propensity to save.

the model economy is experiencing inefficient (and involuntary) unemployment[18]. In addition, it is worth noting that (3) and (28) impose precise lower and upper bounds for the stochastic process followed by the investment expenditure in wage units. Specifically,

$$0 \leqslant \widehat{I}_t < \frac{\alpha}{1-\alpha} \quad \text{for all } t \tag{29}$$

The expression in (29) is straightforward and provides useful insights for the calibration of σ_ϵ. In fact, positive solutions for $\left(\frac{w_t^*}{p_t^*}\right)^{-1} Y_t^*$ and L_t^* are not consistent with a negative realization of $\widehat{I}_t$. Moreover, a value of $\widehat{I}_t$ higher than $\frac{\alpha}{1-\alpha}$ will result in an equilibrium employment higher than the available labor supply in the whole economy. Therefore, assuming that expectations are the driving force of the profit earners' investment expenditure, (28) and (29) suggest that animal spirits should be modeled as a bounded sequence of self-fulfilling beliefs[19].

2.6 Demand Constrained Equilibrium and Wage Function

Now I can provide a formal definition of equilibrium based on the building blocks described above. Following Farmer (2008a-b, 2010), I exploit the term "Demand Constrained Equilibrium" (DCE) in order to describe a demand-driven competitive search model closed by a balance material condition. Hence,

Definition 1 *For each $\widehat{I}_t \in [0, \frac{\alpha}{1-\alpha})$, $A_t > 0$ and $K_t > 0$ a symmetric DCE is given by*

(i) a real wage rate $\left(\frac{w_t^*}{p_t^*}\right)$

(ii) a production plan $\{Y_t^*, V_t^*, X_t^*, L_t^*, U_t^*\}$

(iii) a consumption allocation C_t

(iv) a pair $\{h_t^*, v_t^*\}$

with the following properties:

[18]Obviously, this framework can also describe situations of over-employment in which $L_t > L_S$. In this case, a reduction of employment would be Pareto-improving because it would correspond to a re-allocation of wage earners from recruiting to production activities.

[19]It is worth noting that in this demand-driven competitive search framework the equivalent of the Hosios's (1990) condition works just as a constraint on the actual realization of $\widehat{I}_t$. Specifically, whenever $\widehat{I}_t = \frac{\alpha}{1-\alpha} L_S$ the resulting equilibrium allocation is definitely Pareto-optimal.

- *Feasibility:*

$$Y_t^* = A_t K_t^\alpha \left(X_t^*\right)^{1-\alpha} \tag{30}$$

$$L_t^* = X_t^* + V_t^* \tag{31}$$

$$C_t^* + \widehat{I}_t \left(\frac{w_t^*}{p_t^*}\right) = Y_t^* \tag{32}$$

$$U_t^* = 1 - L_t^* \tag{33}$$

- *Consistency with the optimal choices of wage and profit earners:*

$$C_t^* = \left(\frac{w_t^*}{p_t^*}\right) L_t^* \tag{34}$$

$$(1-\alpha)\frac{Y_t^*}{L_t^*} = \left(\frac{w_t^*}{p_t^*}\right) \tag{35}$$

- *Search market equilibrium:*

$$h_t^* = L_t^* \tag{36}$$

$$v_t^* = \frac{L_t^*}{V_t^*} \tag{37}$$

$$L_t^* = \left(V_t^*\right)^{1-\gamma} \tag{38}$$

It is worth noting that in a DCE all nominal variables are expressed in money wage units and in sharp contrast with the competitive search equilibrium suggested by Moen (1997) no agent has incentives to change its behavior even if L_t^* is different from L_S. Furthermore, the results in (30), (31) and (38) suggest that the equilibrium wage function in (35) can be alternatively written as

$$(1-\alpha)\, A_t K_t^{\alpha} \frac{\left(1-(L_t^*)^{\frac{\gamma}{1-\gamma}}\right)^{1-\alpha}}{(L_t^*)^{\alpha}} = \left(\frac{w_t^*}{p_t^*}\right) \tag{39}$$

where $L_t^* \in [0,1]$.

The expression in (39) shows that for any eligible equilibrium employment rate, a positive (negative) productivity (or supply) shock leads to an increase (decrease) of the corresponding equilibrium real wage rate[20]. Moreover, the results in (28) imply that

$$\lim_{\widehat{I}t\to\left(\frac{\alpha}{1-\alpha}\right)^-} L_t^* = 1 \;\text{ so that } \lim_{\widehat{I}t\to\left(\frac{\alpha}{1-\alpha}\right)^-} \left(\frac{w_t^*}{p_t^*}\right) = 0 \tag{40}$$

$$\lim_{\widehat{I}t\to 0^+} L_t^* = 0 \text{ so that } \lim_{\widehat{I}t\to 0^+} \left(\frac{w_t^*}{p_t^*}\right) = +\infty \tag{41}$$

The expressions in (40) and (41) are straightforward. On the one hand, a full employment DCE, i.e., $L_t^* = H_t = 1$, is characterized by the fact that there is no production while labour is a free-good[21]. In the other hand, a DCE in which nobody is employed is obviously characterized by the absence of production. However, given that labor is so scarce, profit earners would be willing to pay a real wage rate that tends to infinity[22]. A graphical outlook of the equilibrium wage function is given in figure 4.

The diagram in figure 4 allows to clarify some important features of the non-linear expression in (39). First, the equilibrium wage function is strictly decreasing and this suggests a quite conventional trade-off between real wage rates and equilibrium employment[23]. Second, for $L_t^* = L_S$ there is an inflection point in the equilibrium wage function. Specifically, whenever

[20]In addition, capital accumulation boosts real wage rates.

[21]It is worth noting that in this case the trading externalities in the problems of wage and profit earners disappear. In fact, in a full employment DCE $h_t^* = 1$ while $v_t^* \to +\infty$.

[22]Obviously, in this case trading externalities are at their maximum level, i.e., $h_t^* = v_t^* = 0$.

[23]The model economy under examination fulfils what Keynes (1936) in the *General Theory* (Chapter 2, pg. 5) defined as the first postulate of the classical economics. As a consequence, the real wage rate is always equal to the marginal productivity of labor. Moreover, ceteris paribus, an increase (decrease) in employment is always coupled with a decrease (increase) in the real wage rate.

the actual realization of L_t^* is lower (higher) than L_S, the equilibrium wage function is convex (concave).

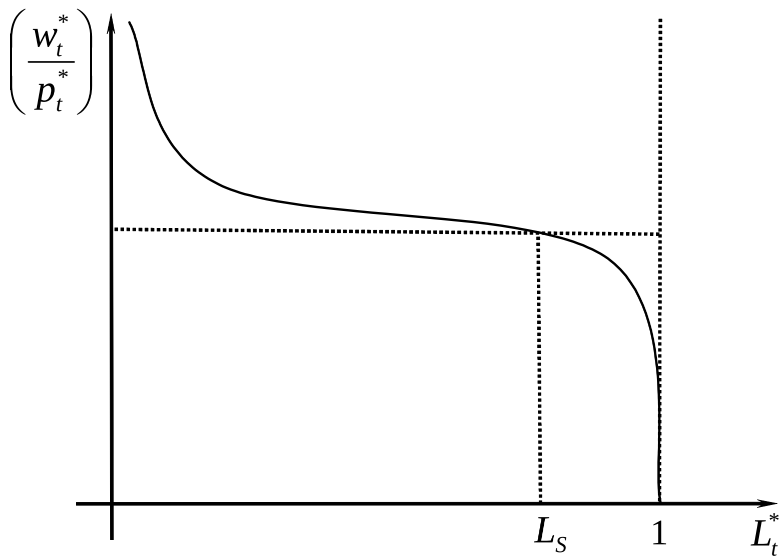


Figure 4: Equilibrium wage function

Having in mind the social planner problem developed above and illustrated in figure 2, the reason why the exact inflection point of the equilibrium wage function has to coincide with the social optimal (un)employment level is straightforward. In fact, equilibrium employment levels lower than L_S are associated with increasing total output because employing additional wage earners improve the resulting allocation. As a consequence, the average and marginal product of labor display the conventional convex decreasing path. However, beyond L_S total total output starts to decrease because employing additional wage earners would not produce additional output but they would be simply employed in recruiting additional recruiters. As a consequence, this leads the average and marginal product of labor to follow a decreasing concave path that quickly converges to zero.

The possibility of a turning point in the concavity of the equilibrium wage function is an intriguing and unexplored feature of the Farmer's (2008a-b, 2010) demand-driven competitive search framework. In fact, this means that equilibrium employment rates in the near proximity of L_S imply a quite flat trade-off between equilibrium real wage rates and (un)employment. As a consequence, even if all agents are price-takers and prices are free to adjust, this delivers an original possible endogenous source of real wage stickiness.

An overall graphical description of a DCE is given in figure 5.

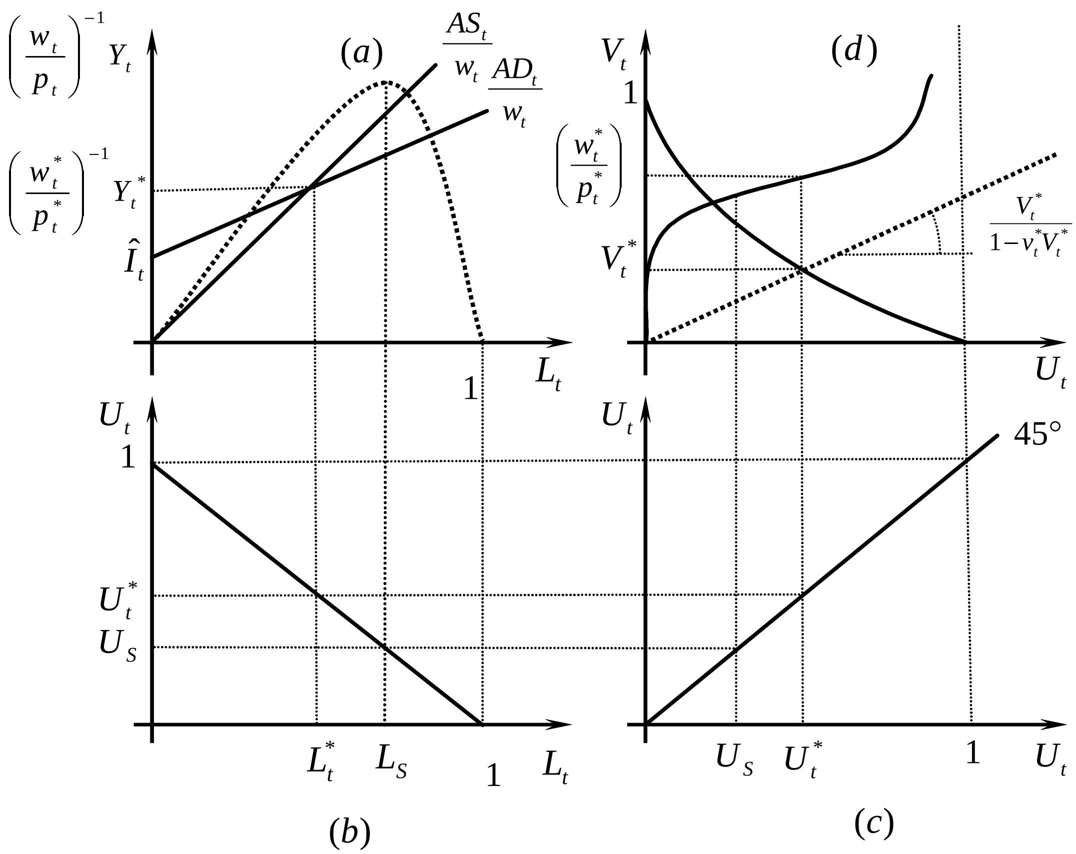


Figure 5: Demand constrained equilibrium

In panel (a) of figure 5 there is the equilibrium in the goods market. In panel (b) there is the one-to-one relationship between employment and unemployment. In panel (c) there is a 45-degree line. Finally, in panel (d) there is the Beveridge curve and the (mirrored) equilibrium wage function.

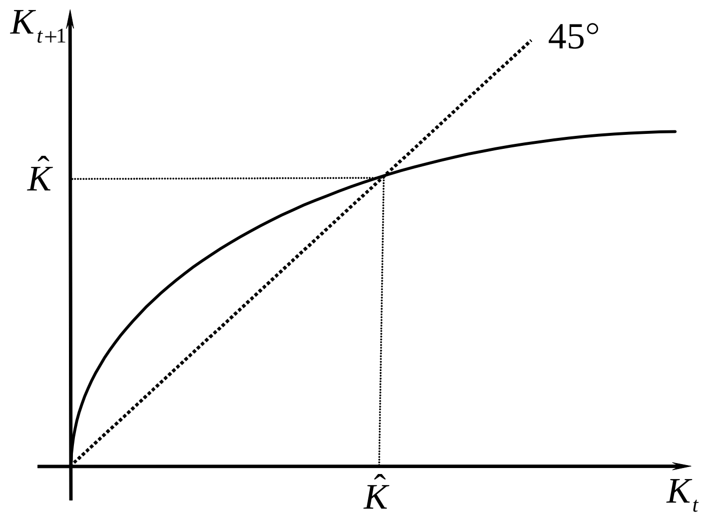
Observing the diagrams in figure 5, it is quite clear that - ceteris paribus - an increase (decrease) in the nominal expenditure in wage units, i.e., a positive (negative) demand shocks, leads to an increase (decrease) in employment and in the recruiters-unemployment ratio and to a decrease (increase) of the equilibrium real wage rate.

2.7 Stochastic Dynamics

Given (15), (28), (31), (36) and (38), the global dynamics of the model economy is described by the following stochastic first-order difference equation for the capital stock:

$$K_{t+1} = \alpha A_t K_t^{\alpha} \left(\frac{1-\alpha}{\alpha} \widehat{I}_t \right)^{1-\alpha} \left(1 - \left(\frac{1-\alpha}{\alpha} \widehat{I}_t \right)^{\frac{\gamma}{1-\gamma}} \right)^{1-\alpha} + (1-\delta) K_t \tag{42}$$

The phase line of the capital stock motion is illustrated in figure 6.



The stochastic steady-state solution of (42) is given by

$$\widehat{K} = \left(\frac{\alpha}{\delta} A_t \right)^{\frac{1}{1-\alpha}} \left(\frac{1-\alpha}{\alpha} \widehat{I}_t \right) \left(1 - \left(\frac{1-\alpha}{\alpha} \widehat{I}_t \right)^{\frac{\gamma}{1-\gamma}} \right) \tag{43}$$

It is worth noting that in the model economy under examination the realizations of the stochastic processes that summarize, respectively, productivity shocks and the Keynesian animal spirits hypothesis contribute to determine the instantaneous steady-state equilibrium[24].

[24]This is in sharp contrast with the previous literature on self-fulfilling prophecies in which the hypothesis of animal spirits (or sun-spots) leads to a multiplicity of equilibrium paths that converge to a unique stationary solution. See Farmer (1993).

2.8 Optimal Fiscal Policies

Given the results in (28) and (29), a DCE can be characterized by any employment level L_t^* in the closed interval $[0, 1]$. However, I shown that a omniscient-benevolent social planner who could simultaneously operate the production and the matching technologies would always choose the employment level L_S. Therefore, unless the realization of the stochastic process that describes the profit earners' animal spirits is consistent with the social planner solution, a DCE will be alternatively characterized by inefficient over or under employment.

In order to provide a remedy for those sub-optimal outcomes I follow Farmer (2010, Chapter 4) and I augment the model economy with a public sector responsive for fiscal policies. Specifically, exploiting the multiplier effect of changes in the autonomous components of aggregate demand, I assume that wage earners' nominal income is taxed at the proportional tax rate τ_t and subsidized with a nominal lump-sum transfer T_t measured in wage units. In this particular case, the value of aggregate demand in wage units becomes equal to the following expression:

$$\frac{AD_t}{w_t} = (1 - \tau_t)\, L_t + T_t + \widehat{I}_t \tag{44}$$

The existence of a public sector can raise some problematic issues. On the one hand, government spending leads to the creation of public goods. In the other hand, deficit spending raises the issue of discharging the public debt. In order to bypass those problems, I focus only on balanced budged fiscal policies. Therefore, I will be concerned only on fiscal policies in which the tax rate and the lump-sum transfer in wage units are linked in the following way:

$$T_t = \tau_t L_t \tag{45}$$

where the expression on the RHS is simply the nominal revenue in wage units of the fiscal policy under examination.

Assuming that the public authority that designs the fiscal policy knows the social optimal level of (un)employment in (22) and that it is also able to observe the actual realization of $\widehat{I}_t$, then it becomes possible to derive a pair $\left\{\tau_t^S, T_t^S\right\}$ that successfully implements the social-optimal level of (un)employment with a balanced budget fiscal policy. Specifically, the instantaneous equilibrium in the goods market at the social-optimum level of employment L_s

implies that the social-optimal level of the proportional tax rate is given by

$$\tau_t^S = \frac{\widehat{I}_t}{L_S} - \frac{\alpha}{1-\alpha} \tag{46}$$

while $T_t^S = \tau_t^S L_S$ is the social-optimal lump-sum transfer measured in wage units.

The social optimal fiscal policy works as follows. Whenever the actual realization of $\widehat{I}_t$ is too low (high) to support the social-optimum level of employment, the social-optimum balanced budget fiscal policy is implemented by a positive (negative) income tax and a positive (negative) lump-sum transfer in wage units.

3 Some Computational Experiments: A Rationale for the Shimer's Puzzle

In recent years, a lot of computational efforts have been devoted to the explanation of the cyclical behavior of equilibrium unemployment and vacancies. Among the others, there is an influential paper by Shimer (2005) that questions the predictive power of the standard matching model *a* là Mortensen and Pissarides's (1994) by arguing that its theoretical framework cannot generate the observed business-cycle-frequency fluctuations in unemployment and job vacancies in response to shocks of plausible magnitude[25].

Using quarterly data from different sources, Shimer (2005) measures, inter-alia, the autocorrelation and the volatility of unemployment, job vacancies and real wage rates for the US economy in the period from 1951 to 2003. Some of his empirical findings are summarized in table 1.

One of the most striking finding emphasized by Shimer (2005) is that the standard deviation of the vacancy-unemployment ratio is almost 20 times as large as the standard deviation of real wage rates over the period under examination. The so-called Shimer's (2005) puzzle arises from the fact that the matching model a là Mortensen and Pissarides (1994) in which real wage rates are the outcome of a generalized Nash (1950) bargaining predicts that this two variables should

[25]In other words, the Mortensen and Pissarides's (1994) model would lack for an amplification mechanism for real shocks.

have nearly the same volatility[26].

		$\ln U$	$\ln(\text{vacancies})$	$\ln\left(\frac{\text{vacancies}}{U}\right)$	$\ln\left(\frac{w}{p}\right)$
Standard deviation		0.190	0.202	0.382	0.020
Quarterly autocorrelation		0.936	0.940	0.941	0.878
Correlation matrix	$\ln U$	1	-0.894	-0.971	-0.408
	$\ln(\text{vacancies})$	–	1	0.975	0.364
	$\ln\left(\frac{\text{vacancies}}{U}\right)$	–	–	1	0.396
	$\ln\left(\frac{w}{p}\right)$	–	–	–	1

Table 1: The Shimer's puzzle

A first important stream of contributions, such as Hall (2005a-b) and the very Shimer (2005), tried to reconcile the matching model to data by introducing some real wage rigidity in order to generate a stronger amplification of real shocks, i.e., in order to amplify the effects of productivity shocks on labor market tightness indicators. Specifically, the former gives up the Nash bargaining rule by postulating that retributions follow a wage norm constrained inside a well-determined bargaining set. By contrast, the latter preserves the Nash rule but it supposes that the workers' bargaining power – usually assumed to be fixed – moves counter-cyclically.

In the remainder of this section, I explore the implications of a radically different approach. Specifically, I provide some computational experiments aimed at checking whether the demand-driven competitive search economy described in section 2 is able to provide a rationale for the Shimer's (2005) puzzle, i.e., the relative stability of real wage rates in spite of the large volatility of labor market tightness indicators. Since in the Farmer's (2008a-b, 2010) framework labor instead of output is used to post job vacancies, my indicator of unfilled jobs will be given by the rate of recruiters employed by profit earners (V), while the indicator of labor market tightness will be given by the ratio between recruiters and unemployment $\left(\frac{V}{U}\right)$. Moreover, my

[26] The intuition for this result is that a real wage rate bargained between the worker and his employer according to the Nash rule absorbs a great deal of productivity shocks. As a consequence, vacancies and unemployment are only partially affected by the stochastic disturbances that affect the value of produced output.

numerical experiments will be carried out by assuming that in each period private consumption and investments include government, net transfers and net exports[27].

Finally, for the sake of comparability, the volatility of artificial series obtained with the suggested model economy will be computed with the same numerical procedure proposed by Shimer (2005). A short summary of this procedure is given in Appendix.

3.1 Model Calibration

The model economy is calibrated in order to fit the first-moments of US data. Specifically, I use the set of parameter values collected in table 2.

Parameter	Description	Value
α	*capital share*	0.3000
δ	*capital depreciation*	0.0250
γ	*matching elasticity*	0.9862
μ	*productivity drift*	1.0000
ξ	*productivity persistence*	0.9500
σ_η	*productivity variance*	0.0076
κ	*investment drift*	0.2018
ρ	*investment persistence*	0.5000
σ_ϵ	*investment variance*	0.0030

Table 2: Calibration

The parameter values illustrated in table 2 comes from different sources. First, the values of α and δ are calibrated as in Kydland and Prescott (1982). Second, the value of ρ is the quarterly figure provided by Farmer (2008a) for the US economy. Assuming that expectations are the main driver of profit earners' investment expenditure, this autocorrelation value fulfills the Kurz's (2008) Bayesian criterion for beliefs' inference. In fact, according to this criterion, what is actually learnable in a non-stationary economy can be described by a stable process

[27] The same simplifying assumption can be also found in Farmer and Guo (1995).

and this explicitly suggests a value of ρ inside the unit circle[28]. Finally, the values of μ, ξ and σ_η are calibrated as in Chang (2000).

The values of the other parameters have been chosen in order to fulfill the following requirements. First, the value of γ delivers a social-optimum unemployment rate around 5.81% a figure consistent with the historical average US unemployment rate. In fact, it is computed as a simple historical mean of the seasonally-adjusted US unemployment rate for the people of 16 years old and over provided by the Bureau of Labor Statistics[29]. Second, the value of κ combined with the Farmer's (2008a) value of ρ delivers an expected value of investment expenditure in wage units consistent with the social-optimum unemployment rate derived as described above. Therefore, the implicit hypothesis that drives my simulations is that profit earners' investment expenditure fluctuates around the level that provides the social-optimum employment rate implied by the suggested matching elasticity. Finally, the value of σ_ϵ is consistent with the restrictions on the stochastic process for $\widehat{I}_t$ in (29).

3.2 Simulation Results

I begin my computational experiments by allowing only for shocks to the value of aggregate demand in wage units. In other words, using the set of parameters in table 2, I implement the Shimer's (2005) procedure to the artificial series generated by the model economy under the simplifying assumption that it is hit only by demand shocks so that $A_t = A_{t-1} = 1$, for all t and $\sigma_\eta = 0$. The results are collected in table 3 (standard errors in parentheses).

The numerical results for a model economy driven only by the profit earners' animal spirits suggest some interesting and preliminary conclusions. First, the simple theoretical setting in section 2 is able to get the ranking of the observed standard deviations of all the involved variables: labor market tightness (real wage) is the most (less) volatile series. Second, the model matches the observed sign of the correlation coefficients of all the variables. Specifically,

[28]In *A Treatise on Probability*, Keynes (1921, Chapter 32, pg. 391) had a similar intuition. In fact, he does explicitly stated: "No one supposes that a good induction can be arrived at merely by counting cases. The business of strengthening the argument chiefly consists in determining whether the alleged association is *stable*, when the accompanying conditions are varied." (italics from the author).

[29]Series identification: LNS14000000. See http://www.bls.gov.

it is worth noting the negative correlation between unemployment and recruiters (Beveridge curve). Third, the model with only demand shocks overstate real wage stickiness: the standard deviation of labor market tightness is about 35 times the standard deviation of real wage rates. Finally, the model tends to understate the volatility of unemployment and to overstate the autocorrelation of all the variables.

		$\ln U$	$\ln V$	$\ln\left(\frac{V}{U}\right)$	$\ln\left(\frac{w}{p}\right)$
Standard deviation		0.0476 (0.0031)	0.1370 (0.0087)	0.5018 (0.0477)	0.0143 (0.0017)
Quarterly autocorrelation		0.9987 (0.0000)	0.9899 (0.0014)	0.8951 (0.0150)	0.9998 (0.0000)
Correlation matrix	$\ln U$	1	-0.9949 (0.0012)	-0.9927 (0.0049)	-0.9748 (0.0091)
	$\ln V$	–	1	0.9957 (0.0044)	0.9503 (0.0159)
	$\ln\left(\frac{V}{U}\right)$	–	–	1	0.9600 (0.0130)
	$\ln\left(\frac{w}{p}\right)$	–	–	–	1
Initial conditions: $K_0 = 32.3313$; $\widehat{I}_0 = 0.4036$					

Table 3: Demand shocks

I close my computational experiments by augmenting the algorithm in order to allow for demand *and* productivity shocks generated in order to have orthogonal disturbances, i.e., $E_t[\eta_t \epsilon_t] = 0$, for all t. Using the configuration of parameters in table 2 and implementing the Shimer's (2005) procedure, I derive the numerical results in table 4 (standard errors in parentheses).

The simulations results for a model economy hit simultaneously by demand and supply shocks are straightforward. In the suggested theoretical framework - taking into account of the result in (39) - productivity shocks affect only the volatility of real wage rates. Therefore, whenever compared to the figure in table 3, the value of the standard deviation of $\left(\frac{w}{p}\right)$ becomes larger and closer to the Shimer's (2005) empirical findings[30]. In fact, taking a significance level of 95%, the hypothesis that my simulated value is equal to its observed counterpart cannot be

[30]The slight differences for the volatility and autocorrelation of the other variables with respect to the figures in table 3 are due only to the fact that the simulation were ran with a different seed.

rejected. Moreover, the ratio between the volatility of the labor market tightness indicator and real wage rates becomes quite close to the puzzling value indicated in table 1.

		$\ln U$	$\ln V$	$\ln\left(\frac{V}{U}\right)$	$\ln\left(\frac{w}{p}\right)$
Standard deviation		0.0476 (0.0031)	0.1369 (0.0087)	0.5012 (0.0479)	0.0259 (0.0030)
Quarterly autocorrelation		0.9987 (0.0000)	0.9899 (0.0014)	0.8952 (0.0149)	0.9998 (0.0000)
Correlation matrix	$\ln U$	1	-0.9950 (0.0012)	-0.9928 (0.0044)	-0.5386 (0.0896)
	$\ln V$	–	1	0.9958 (0.0040)	0.5248 (0.0881)
	$\ln\left(\frac{V}{U}\right)$	–	–	1	0.5303 (0.0888)
	$\ln\left(\frac{w}{p}\right)$	–	–	–	1
Initial conditions: $K_0 = 32.3313$; $\widehat{I}_0 = 0.4036$; $A_0 = 1.0000$					

Table 4: Demand and supply shocks

3.3 Discussion

An intuitive rationale for my computational results can be given as follows. On the one hand, the inflection point in the equilibrium wage function in (39) happens to be exactly in L_S. Obviously, this means that in the near proximity of the social optimum level of (un)employment - which implicitly provides the barycentre of my simulations - the trade-off between real wage rates and equilibrium employment is quite flat. On the other hand, the value of the matching elasticity in table 2 implies a steeper trade-off between equilibrium recruiters (vacancies) and unemployment. As a consequence, the difference between the slopes of these two fundamental macroeconomic relationships arising from my model calibration suggests a straightforward explanation for the mechanism of amplification of real shocks needed to reconcile the Farmer's (2008a-b, 2010) micro-founded theoretical framework with the Shimer's (2005) empirical findings.

Before addressing the issue of robustness, it is worth noting that the insights arising from my simulation results are not completely new. In fact, augmenting the search model à la Mortensen and Pissarides (1994) with a monopolistically competitive demand side and assuming that the model economy is hit simultaneously by monetary and productivity shocks, Barnichon (2007, 2009) finds that a great deal of the Shimer's (2005) puzzle might be due to a misidentification of the disturbances that drive (un)employment when firms are demand-constrained. In other words, taking into account of demand shocks vis-à-vis supply shocks can help in explaining the relative stability of real wage rates in spite of the large volatility of labor market tightness indicators even without invoking any exogenous real wage rigidity[31].

3.4 Robustness

In order to have an intuition about the model calibration robustness displayed in table 2, I simulate two different versions of model economy characterized by different parameter values but preserving the assumption of simultaneous demand and supply shocks. The first takes into consideration the matching elasticity value suggested for expositional purposes by Farmer (2008b, 2010) (i.e., $\gamma = \frac{1}{2}$) by leaving all the other parameters unaltered. In this way equilibrium unemployment fluctuates around its long-run historical mean while the social optimum level of (un)employment is one-half of the labor force. The second explores the consequences of assuming that the investment expenditure drift is such that the expected equilibrium unemployment is equal to the expected equilibrium rate of recruiters (i.e., $\kappa = 0.204\,7$) leaving all the other parameters unaltered[32]. In this way the social optimum level of unemployment is consistent with its long-run historical mean while equilibrium unemployment fluctuates around a value of about 4.46%.

The results of those simulations can be briefly summarized as follows. In the first case, the labor market tightness indicator volatility is slightly higher than the volatility of real wage rates and in addition the model economy displays a counterfactual positive correlation between unemployment and real wage rates. In the second case, the model economy displays correlation

[31] In fact, Barnichon (2007) obtains artificial series closer to the Shimer's (2005) empirical findings when he does consider the flexible Nash-bargained real wage rate.

[32] It is worth noting that this is a straightforward way to consider the Beveridge's (1944) definition of full employment inside the Farmer's (2008a-b, 2010) demand-driven competitive search framework. In fact, in *Full Employment in a Free Society*, Beveridge (1944) defined full employment as a state of affairs in which the number of vacant jobs is equal to the number of unemployed workers.

coefficients with a sign consistent with their observed counterpart but the volatility of the labor market tightness indicator becomes explosive and not statistically significant[33].

In conclusion, the simulation results of these two different versions of the model economy suggest that the good quality of the figures in table 4 comes from an investment expenditure that fluctuates around a social optimum level of (un)employment that is fairly distant from the explosive full employment allocation.

4 Concluding Remarks

This paper aims at providing a stochastic-dynamic matching model with an Old Keynesian flavor and testing it against recent empirical evidence on macroeconomic fluctuations. Specifically, following the micro-foundation of the *General Theory* recently provided by Farmer (2008a-b, 2010), I build a competitive search model in which output and employment are driven by effective demand, the price level is perfectly flexible and the nominal wage rate is chosen as numeraire. Moreover, another Old Keynesian feature of my theoretical proposal originally stressed by Kaldor (1955-1956) is the assumption that agents were divided into two categories with different propensity to consume and different tasks, i.e., wage and profit earners. The former are assumed to consume the whole wage income raised by supplying a fixed amount of labor services. By contrast, the latter are assumed to save the whole income raised by arranging the production of goods in order to finance an investment expenditure that is driven by an autonomous stochastic process such as those exploited for TFP in the RBC theory.

This simple setting with stochastic dynamics allow to derive in a straightforward manner all the properties of a DCE for a model economy with savings and productive investments. Furthermore, I emphasize that this framework might be consistent with a certain degree of endogenous real wage stickiness and I show how to design effective balanced fiscal policies that successfully restore the social optimum level of (un)employment.

Finally, I show that under the assumption that the economy is hit simultaneously by demand and supply shocks, this simple demand-driven competitive search model can provide a rationale for the so-called Shimer's (2005) puzzle. Specifically, calibrating and simulating the

[33]The complete results of those simulations are available from the author.

model economy in order to fit the US first-moments data, I demonstrate that the suggested stochastic theoretical framework can well be consistent with a degree of wage stickiness that easily rationalizes the relative stability of real wage rates in spite of the large volatility of labor market tightness indicators. In fact, I show that the real wage stickiness required to resolve the Shimer's (2005) puzzle can endogenously arise from the combination of the Farmer's (2008a-b, 2010) demand-driven competitive search framework with a quite conventional calibration of the model economy.

5 Appendixes

In this section, I provide a short description of the Shimer's (2005) numerical procedure implemented in section 3. Moreover, I explore the macroeconomic consequences for the relationship between unemployment and inflation of assuming a constant nominal wage rate. Finally, I test the implication for the cyclical volatility of real wage rates and labor market tightness of two additional hypotheses for the evolution of the investment expenditure measured in wage units, i.e., the accelerator hypothesis and the chaotic behavior.

5.1 Shimer's Procedure

The numerical procedure proposed by Shimer (2005) to assess the volatility of unemployment, vacancies, labor market tightness and real wages runs as follows. First, generate $1,212$ observations for each of the involved variables, i.e., unemployment, recruiters, labor market tightness and real wage rates. Second, throw away the first $1,000$ observations for each simulated series in order to have 212 data points which correspond to data for the 53 quarters from 1951 to 2003. Third, take the log of each series and compute their autocorrelation. Forth, detrend the log of the model-generated data by using the Hodrick-Prescott (HP) filter with a smoothing parameter set at $100,000$. Fifth, compute the deviations of all the artificial series from their respective HP trend and take the respective standard errors and correlation. Sixth, repeat the previous steps for $10,000$ times. Finally, take the mean and the standard deviation of all the variables obtained in the third and the fifth step[34].

5.2 Phillips Curve

Following the choice of units made by Keynes (1936) in the *General Theory* (Chapter 4, pg. 41) and resumed by Farmer (2008a-b, 2010), the theoretical model in section 2 is developed by using the nominal wage rate w_t as numeraire. An almost "natural" extension is to consider the possibility that the nominal wage is completely fixed and assume that all the adjustments to real wage rates occur through changes in the price level. Hence,

$$w_t = \overline{w} \text{ for all } t \tag{A.1}$$

Given (A.1), the Farmer's (2008a-b, 2010) theoretical framework can be used to test the soundness of another fundamental macroeconomic relationship, i.e., the Phillips curve. As it is well-known, the most common representation of a Phillips curve is given by a relationship between inflation and the unemployment rate such as

$$\frac{p_t - p_{t-1}}{p_{t-1}} = f(U_t) + \chi_t \tag{A.2}$$

where χ_t is an erratic component while $f(\cdot)$ is a given function.

Using the set of parameters in table 2, I derive a non-parametric estimation of $f(\cdot)$ obtained with a normal kernel[35]. Retrieving artificial data from a typical replication a là Shimer (2005) obtained with demand and supply shocks, I derive the results illustrated in figure A.1.

The non-parametric regression in figure A.1 shows a Phillips curve with a quite conventional shape, i.e., the unemployment and the inflation rate are linked by a weak decreasing relationship. Moreover, it is worth noting that given the value of the matching elasticity, the social optimal unemployment rate corresponds to a nearly null inflation rate. As a consequence, under fixed nominal wage rates, U_S is a good approximation of the NAIRU.

[34]The MAT LAB$^{\text{TM}}$ 6.5 code that applies this procedure to the theoretical model developed in section 2 is available from the author.

[35]The estimation is carried out with R 2.2.0. by exploiting the statistical package *sm*. See Bowman and Azzalini (1997). Moreover, the kernel bandwidth is obtained by assuming the normality of the probability density function of the regressor.

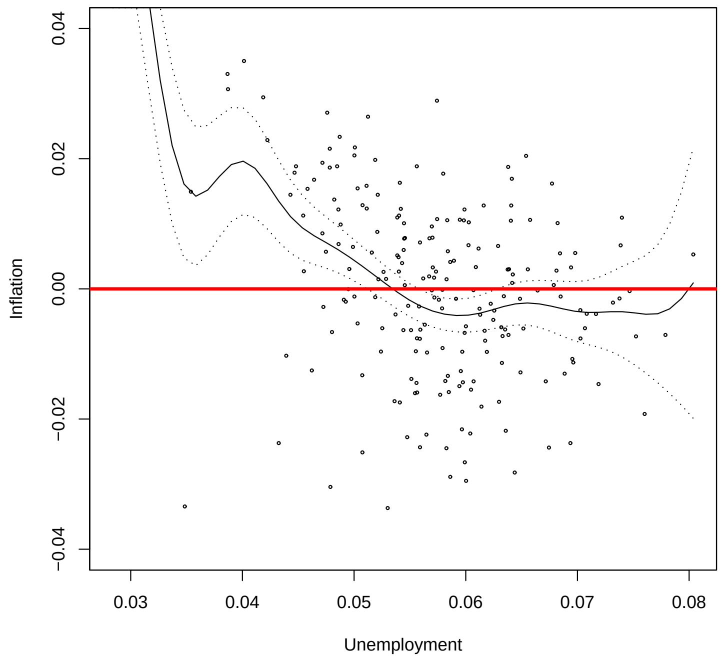


Figure A.1: Phillips curve

5.3 Investment Accelerator

Following Farmer (2008a), the theoretical model in section 2 is developed by assuming that the profit earners' investment expenditure measured in wage units is driven by a stochastic AR(1) process that - as suggested by Kurz (2009) - might be consistent with a limit posterior of a Bayesian inference for self-fulfilling beliefs. An intriguing alternative could be the acceleration business cycle hypothesis pioneered by Clark (1917) and recently revisited by Puu et al. (2005). A convenient specification is the following:

$$\widehat{I}_t = d\,(Y_{t-1} - Y_{t-2}) + \widehat{I}_a \tag{B.1}$$

where d is a positive constant while $\widehat{I}_a$ is the autonomous investment expenditure in wage units.

The accelerationist evolution law for the investment expenditure in (B.1) suggests that in each period $\widehat{I}_t$ is assumed to be proportional to the lagged change in output plus an autonomous component. As a consequence, assuming that expectations are the main driver of the investment expenditure, this means that profit earners' beliefs improve (get worse) whenever they observe an increase (decrease) in output.

In the remainder of this sub-section, I provide a computational experiment aimed at checking the dynamic properties of the model economy described in section 2 under the hypothesis that investment expenditure follows the accelerator principles. Specifically, I implement the Shimer's (2005) procedure to the artificial series generated by the model economy under the assumption that $\widehat{I}_t$ follows the second-order process in (B.1) while TFP is stochastic.

The process in (B.1) is calibrated as follows. First, in order to have an investment expenditure that fluctuates around its social optimum level I set $\widehat{I}_a = \frac{\alpha}{1-\alpha} L_S$. Thereafter, I calibrate the acceleration parameter in order to avoid explosive dynamics. Specifically, I set $d = 0.10$. Using the remainder of the parameters' configuration in table 2 and implementing the Shimer's (2005) procedure, I derive the numerical results in table B.1 (standard errors in parenthesis).

		$\ln U$	$\ln V$	$\ln\left(\frac{V}{U}\right)$	$\ln\left(\frac{w}{p}\right)$
Standard deviation		0.0300 (0.0017)	0.0873 (0.0049)	0.3112 (0.0181)	0.0212 (0.0030)
Quarterly autocorrelation		0.9990 (0.0000)	0.9917 (0.0013)	0.9038 (0.0134)	0.9998 (0.0000)
Correlation matrix	$\ln U$	1	-0.9980 (0.0000)	-0.9984 (0.0000)	-0.1439 (0.05063)
	$\ln V$	–	1	0.9997 (0.0000)	0.1416 (0.0502)
	$\ln\left(\frac{V}{U}\right)$	–	–	1	0.1423 (0.0503)
	$\ln\left(\frac{w}{p}\right)$	–	–	–	1
Initial conditions:	$K_0 = 32.3313$ $K_1 = 32.3283$	;	$\widehat{I}_0 = 0.4036$ $\widehat{I}_1 = 0.4046$	;	$A_0 = 1.000$ $A_1 = 1.001$

Table B.1: Investment accelerator and supply shocks

The numerical results for a model economy with investment accelerator and stochastic productivity shocks suggest the following conclusions. First, the suggested setting gets the ranking of the observed standard deviations of all the variables: labor market tightness (real wage) is the most (less) volatile series. Second, the model matches the actual sign of the correlation coefficients of all the variables. Again, it is worth remarking the negative correlation between unemployment and recruiters (Beveridge curve). Third, the model with investment accelerator understates the volatility of unemployment and recruiters but provides standard deviations for labor market tightness and real wage rates quite close to their observed counterpart. Finally, expect for real wage rates, the model overstates the autocorrelation of all the variables.

5.4 Chaotic Investments

The last option for the evolution of the investment expenditure in wage units that I test is a first-order quadratic deterministic process such as

$$\widehat{I}_t = a\widehat{I}_{t-1}\left(b - \widehat{I}_{t-1}\right) \tag{C.1}$$

where a and b are positive constants.

As suggested in an influential paper by May (1976), the process in (C.1) is suited to describe phenomena with the tendency to growth (shrink) at low (high) levels of $\widehat{I}_t$. Therefore, assuming that expectations are the main driver of the investment expenditure, this would mean that profit earners have the self-fulfilling belief that periods of "bull" markets are necessarily followed by periods of "bear" markets and vice versa. Moreover, it is well possible to find pairs $\{a, b\}$ such that $\widehat{I}_t$ follows a very complicated (chaotic) dynamics[36].

In the remainder of this sub-section, I provide a computational experiment aimed at checking the dynamic properties of the model economy described in section 2 under the hypothesis that investment expenditure follows a chaotic process. Specifically, I implement the Shimer's (2005) procedure to the artificial series generated by the suggested model economy under the assumption that $\widehat{I}_t$ follows the deterministic quadratic process in (C.1) parameterized in order to display a chaotic dynamics while TFP is stochastic.

A pair $\{a, b\}$ such that the process in (C.1) evolves according to a chaotic dynamics can be found as follows. First, exploiting the result in (29), b is set at the level $\frac{\alpha}{1-\alpha}$. In this way, $\widehat{I}_t$ is automatically constrained in its meaningful interval. Second, given the value of α in table 2 and the value of b fixed in such a manner, I find a value of a that delivers deterministic fluctuations

[36] An interesting application of chaos theory to investments and economics is given by Peters (1994).

by drawing the corresponding bifurcation diagram[37]. See figure C.1.

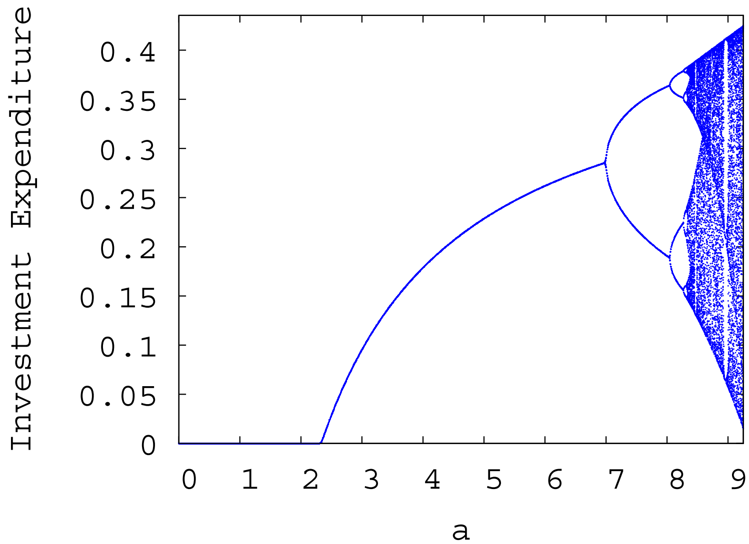


Figure C.1: Bifurcation diagram

The bifurcation diagram in figure C.1 shows that, for example, $a = 8.4$ is a value that delivers a-periodic deterministic oscillations[38]. Using the remainder of the parameters' configuration in table 2 and implementing the Shimer's (2005) procedure, I derive the numerical results in table C.1 (standard errors in parentheses).

The numerical results for a model economy with chaotic investments and stochastic productivity shocks suggest the following conclusions. First, all the uncertainty of the artificial series rely in the real wage rates and their covariance with the other involved variables. Obviously, this is due to the fact that unemployment and recruiters are driven by a deterministic process while real wage rates are affected by the stochastic AR(1) process for TFP. Second, the suggested setting gets the ranking of the observed standard deviations of all the variables: labor market tightness (real wage) is the most (less) volatile series. Third, the model matches the actual sign of the correlation coefficients of all the variables. Again, it is worth noting the negative

[37]This can be easily done by using, for example, the computational package E&F CHAOS. See Diks *et al.* (2008)

[38]According to the proposed calibration, the maximum of the non-linear map for $\widehat{I}_t$ is given by the 90% of $\frac{\alpha}{1-\alpha}$.

correlation between unemployment and recruiters (Beveridge curve). Fourth, the model with chaotic investments tends to overstate the volatility of all the variables but it understates the relative volatility of labor market tightness with respect to real wage rates. Finally, except for real wage rates, the model understates the autocorrelation of all the variables.

		$\ln U$	$\ln V$	$\ln\left(\frac{V}{U}\right)$	$\ln\left(\frac{w}{p}\right)$
Standard deviation		0.5646 (0.0000)	0.7510 (0.0000)	0.7983 (0.0000)	0.1592 (0.0029)
Quarterly autocorrelation		0.5398 (0.0000)	0.3338 (0.0000)	0.2799 (0.0000)	0.9535 (0.0016)
Correlation matrix	$\ln U$	1	-0.9611 (0.0000)	-0.9630 (0.0000)	-0.9524 (0.0025)
	$\ln V$	–	1	0.9999 (0.0000)	0.9910 (0.0024)
	$\ln\left(\frac{V}{U}\right)$	–	–	1	0.9909 (0.0024)
	$\ln\left(\frac{w}{p}\right)$	–	–	–	1
Initial conditions: $K_0 = 30.3454$; $\widehat{I}_0 = 0.3736$; $A_0 = 1.0000$					

Table C.1: Chaotic investments and supply shocks

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