Income tax, subsidies to education, and investments in human capital in a two-sector economy

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The paper studies a two-sector economy with investments in human and physical capital and imperfect labor markets. Workers and firms endogenously select the sector they are active in, and choose the amount of their investments. To enter the high-skill sector, workers must pay a fixed cost that we interpret as direct cost of education. The economy is characterized by two different pecuniary externalities. Given the distribution of the agents across sectors, at equilibrium, in each sector there is underinvestment in both human and physical capital, due to non-contractibility of investments. A second pecuniary externality is induced by the self-selection of the agents in the two sectors.

When total factor productivities are sufficiently diverse, subsidies to labor income in the low skill sector and fixed taxes on the direct costs of education increase total surplus, while subsidies to labor income in the high skill sector can actually reduce it.

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1. INTRODUCTION

Causes and consequences of investments in human capital have been a central field of research in the last few decades. Several distinct motivations have concurred to focus the analysis on this issue. Among them, the relevance of human capital externalities in growth theory (see, Lucas (1988), Romer (1990), and the subsequent literature), and the questions posed by the dynamics of the wage premium and, more generally, by the evolution of income distribution (see, for instance, Heckman and Krueger (2003), Goldin and Katz (2007), Gordon and Dew-Becker (2008) and their extensive references). The analysis of human capital externalities is still far from settled from both the empirical and the theoretical viewpoints. Empirically, it is not obvious that (at the level of subsidies prevailing in most Western countries) there are significant, positive differences between social and private returns.[2] From a theoretical viewpoint, the precise microeconomic mechanism generating the externality is not fully understood (see, however, Acemoglu (1996)). A better understanding of the nature of the externality has policy relevance. This is true even if one is willing to take for granted that, at least in many countries, there are no significant, unexploited, positive externalities, because this is typically obtained with very high subsidies to education.[3]

In this paper, we extend the microeconomic analysis of the externalities related to investments in human capital. We also derive some results on the welfare effects of different policies: fixed tax/subsidies to education, which affect its direct cost, and tax/subsidies on labor income.[4]

We consider economies with two key features. First, we adopt the notion of human capital put forth in Roy (1951): there are distinct markets for (in our set-up, perfectly non substitutable) skilled and unskilled labor. However, contrary to what is often assumed in Roy models, once a worker has selected the type of human capital she wants to acquire, she still has to decide how much effort to invest. The human capital so acquired translates one-to-one into efficiency units of high skill (low skill, respectively) labor.[5] When agents, through schooling, self-select into different labor markets, the effect of public policies on total surplus works via two different channels. The first is the standard one: their impact on the optimal effort of the agents acquiring a specific skill (we will refer to it as *incentive effect*). The second is their impact on the agents' distribution across markets (following Charlot and Decreuse (2005), we call it *composition effect*). The properties of the economy crucially depend on the interaction of the two effects. In "pure" Roy models (with self-selection, but no choice of the investment effort) only the composition effect is at play. In "pure" efficiency-units models (without self-selection) only the incentive effect is at work. Our model allows us to study the interaction between the two phenomena. The second essential feature of our economy is that investments are not contractible ex-ante, so that agents must base their investment decisions on the (conditional) distribution of their payoffs. Lack of contractibility generates

[2] For the U.S.A., a negative conclusion if reached, for instance, by Heckman, Layne-Farrar, and Todd (1996) and by Acemoglu and Angrist (2001). For E.U. countries, the results in De la Fuente (2003) are also negative. See also Krueger and Lindhal (2000).

[3] In 2005, in the OECD average, 85.5% of the direct cost of education (all levels included) is financed by public sources (see OECD (2008, Table B3.1, p.251)). The EU19 average is 90.5%. At the tertiary level, the percentages are, respectively, 73.1% and 82.5% (Table B3.2b, p. 253).

[4] In both cases, we introduce uniform lump-sum taxes/subsidies on workers, so that the public budget is balanced.

[5] As usual, we can also interpret effort as elastic supply of labor of a given skill.

constrained inefficiency of the market equilibrium, i.e., the equilibrium total surplus is lower than its constrained efficient level.

There are (at least) two classes of economies with these properties. The first is based on a two-sector version of the economy considered in Acemoglu (1996). In his model, firms and workers choose the amount of their investments. Then, they are matched randomly, production takes place, and income distribution is determined by a bargaining process. In our version, there are two separate sectors. In the first one, firms use capital and skilled labor. In the second, capital and unskilled labor. Firms and workers first choose the sector they will be active in. Afterward, each sector is just an Acemoglu's economy (so that agents choose their optimal amount of investments). A second class of economies is characterized by a continuum of separate islands. On each island there is a continuum of workers and firms. They choose simultaneously the sector they are going to be active in, and the amount of their investments. While all the firms are identical ex-ante, workers are identical in each island, but heterogeneous across them. The source of heterogeneity is a parameter affecting individual investments in human capital. Its realization on each island is private information of the workers. After investments take place, competitive labor markets open, wages are determined at their competitive market clearing values, and production takes place. In this set-up, investments are not contractible ex-ante and, when they are made, firms and workers base their actions on the conditional distribution of wages and producers' surpluses. In Appendix 2, we show that the closed form of the rational expectation equilibrium of this economy is, essentially, identical to the one obtained for the previous model.[6] In the paper, we consider the first class of economies to allow for a direct comparison with the results on the one-sector model, reported in Acemoglu (1996).

We adopt the Roy model of human capital. Most of the recent literature takes a different viewpoint, following the efficiency units approach (stemming from Griliches (1970)). This rules out, by assumption, all the consequences of self-selection, which appear, instead, to be relevant from both the theoretical and the empirical viewpoints.[7]

Acemoglu (1996) studies the microeconomic foundations of externalities in human capital accumulation in an efficiency units model. Some of his results are robust to our extension to a two-sector economy. For instance: in both cases, the externality is related to the average level of (sector-specific) human capital, not to its aggregate level (as postulated in Lucas (1988)). With respect to policy prescriptions, however, the differences are sharp: in the one-sector model, subsidies to labor income (or to investments in human capital) are unambiguously beneficial. Only the incentive effect is at play: a subsidy to the investments in human capital of *any subset* of agents increases, as a first order effect, their investments and, therefore,

[6] There are differences with respect of the exact nature of constrained inefficiency. They change the details of the analysis, but not the central message. In fact, the equivalent of Proposition 4 always holds, without any rectriction on the set of parameters.

[7] A survey supporting this claim is in Sattinger (1993). For more recent discussions of the different empirical implications of efficiency units vs. Roy models see, for instance, Carneiro, Heckman, and Vytlacil (2001). They conclude (p.32) that "The data suggest that comparative advantage is an empirically important phenomenon governing schooling choices and that naive efficiency units models of the labor market are empirically inappropriate". Investments in human capital in a two-sector economy with frictions due to random matching have been studied in Sattinger (2003), Charlot and Decreuse (2005), and Mendolicchio, Paolini, and Pietra (2008). The composition effect plays an essential role in their results. However, they consider economies with perfectly inelastic supply of human and physical capital, so that the incentive effects of public policies are absent.

the expected human capital. This has a positive impact on the firms' investment decisions. In turn, this increases the optimal investment of all the workers, so that these subsidies are always Pareto improving. To reformulate the point differently: in one-sector economies, there is a unique (pecuniary) positive externality related to the level of the investments. Any policy increasing the investments of any subset of agents is Pareto improving. In a two-sector economy, the incentive effect of a policy can be strengthened, or weakened, by its composition effect. Specifically, in the final section of the paper, we consider (balanced budget) policies based on tax/subsidies to the direct costs of education, and on skill-contingent subsidies to labor income. If total factor productivities are sufficiently diverse across sectors, subsidies to low skilled labor income always increase total surplus, because their positive effect on individual effort in this sector is strengthened by the composition effect, i.e., by the "improvement of the expected quality" of the pool of workers in both markets. An increase in taxes on the direct costs of education, also increases total surplus, because of its composition effect. On the other hand, subsidies to high skill labor incomes have a positive incentive effect for these workers, but a negative composition effect. To reformulate the point, in two-sector economies there are two distinct pecuniary externalities at play. The one related to the investments in the low-skill sector is always positive for all the agents in the economy. To the contrary, the one related to the investments in the high-skill sector is always negative for low-skilled workers (and firms active in that sector), it may be positive or negative for agents active in the high-skill sector. We provide a robust example where its effect on total surplus is negative.

We consider a simple parametric class of economies, with Cobb-Douglas production functions and quasi-linear utility functions. This allows us to compute explicitly the equilibrium, and the welfare effect of the policies. Because of quasi-linearity of preferences, our analysis abstracts from any substantive consideration of distributional issues, and we focus only on efficiency issues. Quasi-linearity also implies that the lump-sum taxes used to balance the government budget have no incentive effect. Moreover, acquisition of human capital is deterministic and instantaneous, so that there are no opportunity costs of education. An extension of the analysis to a richer environment is possible, but at an high cost in terms of analytical tractability. What matters most, the basic intuition behind the welfare results is strong, and they should be robust (maybe, in a less sharp form) to many possible extensions of the basic set-up.

We take as benchmark an economy with no subsidies to the direct costs of education and no income taxes. We would obtain exactly the same results taking as a benchmark a (more descriptively realistic) economy where there are subsidies on the direct costs of education and a flat tax on labor income. The obvious reinterpretation of our result would be in term of lower direct subsidies and some (small) degree of progressiveness in the tax schedule. We focus the analysis on income subsidies for three reasons: they deliver analytically tractable, closed form, values of the equilibrium variables. With the (obvious) reinterpretation provided above, they are a feature of real world economies. Moreover, the effect of labor income taxes on investment in human capital has been extensively analyzed in the literature, and, therefore, our results can be easily compared with previous work in this field. Finally, analytically identical results can be obtained considering (properly specified) direct subsidies to the effort in education.[8]

[8] Alas, to obtain exactly the same closed form of the equilibrium variables, subsidies must take

There is a very large literature on the effects of subsidies to education and of labor income taxes on accumulation of human capital. The usual arguments favoring subsidies hinge either on their positive externality effects or on the existence of liquidity constraints. While these second phenomenon is in principle important, we (deliberately) abstract from it. The classical analysis of the effects of labor income tax on investments in human capital started with the seminal papers by Ben-Porath (1970), Boskin (1975) and Heckman (1976).[9] A flat labor income tax has a negative impact on human capital accumulation just because of non-deductibility of the direct costs of education. On the other hand, by depressing the net interest rate, in fully specified life-cycle models of consumer behavior, a tax on total income may actually have a positive effect. Eaton and Rosen (1980) extend the analysis to (uninsurable) multiplicative wage uncertainty, pointing out that a flat earning tax affects investments in human capital through its effects on their riskiness and (via an income effect) on the attitude toward risk. The canonical conclusion regarding progressive income taxes (compared with a revenue neutral flat rate) is that they discourage investments at the high skill level, while they may encourage it by the less skilled. More recent theoretical contributions include Anderberg and Andersson (2003), Bovenberg and Jacobs (2005), Jacobs (2003, 2007), Jacobs and Bovenberg (2008), Jacobs, Schindler and Yang (2009) (see, also, Heckman, Lochner and Taber (1998, 1999)). However, in all these papers, there is no self-selection into different skills, so that the key mechanism at work in our economy is absent. Also, notice that, in our set-up, at the equilibrium, workers face no uncertainty, so that the mechanism pointed out in Eaton and Rosen (1980) is absent.

The structure of the paper is the following. Section 2 discusses the general features of the model. Section 3 and 4 discuss equilibria in the benchmark, Walrasian, economy, and in the economy with imperfect labor markets. Section 5 studies the properties in terms of welfare of the equilibria of the economy with frictions. Most of the details are in Appendix 1. In Appendix 2, we sketch the analysis of the "island" model outlined above.

2. THE MODEL

The economy is composed by two separate production sectors, denoted by $s \in \{ne, e\}$. Workers (denoted by a subscript i when we refer to individuals, I when we refer to the set) and firms (denoted by j and J, respectively) can choose to enter one of the two sectors, paying a fixed cost. Workers' costs, (c_I^{ne}, c_I^e), are exogenous, and can be interpreted as private, fixed costs of education (tuitions and the like). We denote firms' costs (d_J^{ne}, d_J^e). They are endogenously determined, and will be discussed later on.

There are two intervals of equal length of workers, $\Omega_I = (0, 1)$, and firms, Ω_J, both endowed with the Lebesgue measure. Let $\nu(\Omega_I^s)$ $(\nu(\Omega_J^s))$ denote the measure of the set Ω_I^s (Ω_J^s, respectively). At equilibrium, each interval is partitioned into two sets, $\{\Omega_I^{ne}, \Omega_I^e\}$ and $\{\Omega_J^{ne}, \Omega_J^e\}$, determined endogenously. In sector s, production requires a firm j (with physical capital k_j^s) and a worker i (with stock of human cap-

on a specific (and non particularly appealing) functional form (see note 13 below). Moreover, incentives of this kind beg the problem of direct observability of individual effort.

[9] As mentioned above, in our set-up, one obtains substantially identical results considering direct (non-linear) subsidies to effort and subsidies to the direct costs of education. Previous, related work includes Blankenau (2005), Blankenau and Camera (2006, 2009), Caucutt and Kumar (2003), Lloyd-Ellis (2000), Sahin (2004), and Su (2004).

ital h_i^s). Once the partitions $\Omega_I^P \equiv \{\Omega_I^{ne}, \Omega_I^e\}$ and $\Omega_J^P \equiv \{\Omega_J^{ne}, \Omega_J^e\}$ are given, each sector of the economy reduces to the set-up studied in Acemoglu (1996). Firms are identical, and choose their investments in physical capital to maximize their expected profits. Workers choose their investments in human capital to maximize their expected utilities.

The economy lasts one period, divided into several subperiods. We consider two versions of the basic model. In the frictionless (or Walrasian) version, in subperiod zero, firms and workers enter (paying a fixed cost) one of the two sectors. In subperiod 1, each firm active in sector s is matched with a worker active in the same sector (we will be more precise on the matching issue later on). Firms and workers can sign binding contracts on the amount of human and physical capital that they will supply. In subperiod 2, investments are carried out. In the final subperiod, exchanges and production take place, and agents are paid on the basis of their marginal product.

In the second version of the model, the one with frictions, the total output of each match is split according to the Nash bargaining solution with (exogenous) weights β and $(1-\beta)$ (for a rationalization of this allocation rule in this context, see the Appendix in Acemoglu (1996)). Moreover, and most important, agents cannot contract with their partner a given level of investment, because they are carried out before matches take place. To summarize: in subperiod 0, agents choose to enter one of the two sectors, paying a fixed entry cost. In subperiod 1, they choose their levels of investment. In subperiod 2, they are randomly matched and then, finally, production and exchanges take place.

When workers are heterogeneous, the friction in the labor market, due to the determination of labor income through bargaining, has a very limited role in determining the efficiency properties of equilibria. The crucial feature is that agents choose their investments, considering the conditional distribution of the investments of a (non trivial) set of potential future partners. Indeed, as long as investments are non-contractible, the same qualitative results hold, even if the spot labor markets are perfectly competitive[10] (see Appendix 2).

Technologies are described by a pair of Cobb-Douglas production functions with constant returns to scale. Therefore, in the Walrasian set-up, equilibrium profits are zero, entry costs d_j are zero for each s, and each firm is indifferent among sectors. Thus, the equilibrium partition is essentially determined by the labor supply side of the model. On the contrary, in the economy with frictions, expected producers' surpluses are positive in both sectors and, as we will show later on, larger in sector e. To avoid additional complications (not really germane to our main issue), we want to consider an economy with full employment at equilibrium. This requires that, at equilibrium, each agent is actually matched with a partner. We assume, as in Acemoglu (1996), that the matching function guarantees with probability one a match to each agent, provided that $\nu(\Omega_I^s) = \nu(\Omega_J^s)$. A commonly used function which delivers this property is $\pi_j^s = \frac{\min\{\nu(\Omega_I^s), \nu(\Omega_J^s)\}}{\nu(\Omega_J^s)}$ (and $\pi_i^s = \pi_j^s \frac{\nu(\Omega_J^s)}{\nu(\Omega_I^s)}$), where π_j^s is the probability of a match for a firm active in sector s. The partition Ω_I^P is determined endogenously. To guarantee full employment, we need that, at each equilibrium, $\nu(\Omega_I^s) = \nu(\Omega_J^s)$. The easiest way to obtain this property is to introduce a feature of the economy such that equilibrium expected profits are always equal in

[10] In Acemoglu (1996), the benchmark is an economy with identical workers and firms. Evidently, in this case, if spot labor markets were competitive, we would end up with the Pareto efficient, complete markets allocation.

the two sectors.[11] One way to obtain this is to assume that the technology exploited in sector ne is free, while the one adopted in sector e is protected by a patent, owned by some outside agent (clearly, nothing would change if each technology were subject to a distinct patent).[12] Rights to use the patent are auctioned off to firms before the match firm-worker obtains. Given that, at an equilibrium, expected profits in both sectors must be identical, the equilibrium royalties must be equal to the (positive) difference between the expected producer's surpluses in the two sectors. Then, at each equilibrium, each firm is indifferent among sectors, so that we can choose Ω_J^P with $\nu(\Omega_I^s) = \nu(\Omega_J^s)$. The property we are looking for.

Without any loss of generality, we can take the prices of both kinds of output to be equal to 1 and, therefore, omit them.

Finally, notice that there are always three additional equilibria: the ones where all workers and firms are in one of the two sectors, and the one where none is active in any sector. As usual, we will ignore these trivial equilibria.

3. THE FRICTIONLESS ECONOMY

When active in sector s, and matched with worker i with human capital h_i^s, firm j has production function

$$y_{ij}^s = A^s h_i^{s\alpha} k_j^{s(1-\alpha)},$$

with $A^e > A^{ne}$. Let μ be the unit price of physical capital, that we assume to be equal in the two sectors. This implies some loss of generality, but allows for more straightforward computations. Similar results could be obtained for $\mu^e \neq \mu^{ne}$.

If active in sector s, and given a match with worker i, firm j solves optimization problem

$$\text{choose } k_j^s \in \arg\max A^s h_i^{s\alpha} k_j^{s(1-\alpha)} - \mu k_j^s - w_{ij}^s h_i^s, \tag{Π^{Ws}}$$

where we omit the royalties, because, at equilibrium, they must be zero.

For each worker active in sector s, the utility function is

$$U_i^s(C_i^s, h_i^s) = C_i^s - \frac{1}{\delta_i}\frac{h_i^{s(1+\Gamma)}}{1+\Gamma},$$

where C_i^s denotes consumption, h_i^s is the amount of human capital (or the labor supply). Let c_I^s be the (fixed) cost of the investment in sector s human capital. Then, in the absence of taxes and subsidies, if worker i is active in sector s and matched with firm j, $C_i^s = \left(w_{ij}^s h_i^s - c_I^s\right)$. Workers are heterogeneous because of the parameter δ_i. Without any essential loss of generality, we assume that $\delta_i = i$, and that δ_i is uniformly distributed on $(0,1)$. More general assumptions on the distribution of δ_i, or its support, would introduce additional computational complexities without changing any essential result. Given that, in the sequel, we will introduce uniform lump sum taxes, we must (alternatively, and equivalently) either

[11] An alternative solution is to assume that firms cannot move across sectors. A non-null measure of firms is exogenously assigned to each sector. We then pick a matching function which always guarantees that each firm is matched with a worker (and conversely) for each non-trivial partition of the workers. As long as there is a continuum of agents in each sector, this can be done. Of course, this approach would break down if we had a finite number of agents and, anyhow, is based on a very *ad hoc* trick.

[12] Any input used only in sector e and with perfectly inelastic supply would do. We consider the case of a patent to simplify as much as possible the model.

allow for negative consumption, or assume that workers have a strictly positive (and sufficiently large) initial endowment of consumption good. Given the properties of the utility functions, the most convenient solution (purely notation-wise) is the first one.

By a straightforward computation, the equilibrium amount of agent i's investment in human capital in sector s is given by

$$H^{Ws}(\delta_i) \equiv \left[\delta_i \alpha A^{s\frac{1}{\alpha}} \left(\frac{1-\alpha}{\mu} \right)^{\frac{1-\alpha}{\alpha}} \right]^{\frac{1}{\Gamma}},$$

where the superscript W denotes the frictionless, Walrasian economy. Given that, at the equilibrium, profits are always zero and firms are identical, there is no loss of generality in assuming that firm j is always matched with worker $i = j$. With this convention, the (equilibrium) demand for physical capital of firm $j = i$ is

$$K^{Ws}(\delta_i) \equiv \left[\delta_i \alpha A^{s\frac{1+\Gamma}{\alpha}} \left(\frac{1-\alpha}{\mu} \right)^{\frac{1-\alpha+\Gamma}{\alpha}} \right]^{\frac{1}{\Gamma}}.$$

Let's now consider the equilibrium partition Ω_I^P. For convenience (and without any loss of generality), set $c_I^{ne} = 0$ and $c_I^e > 0$. Let

$$V_i^{Ws}(\delta_i, c_I^s) \equiv U_i^s \left(H^{Ws}(\delta_i), K^{Ws}(\delta_i), c_I^s \right),$$

be the level of utility of agent i active in sector s, evaluated at the equilibrium

Worker i chooses to enter sector e if and only if

$$V_i^{We}(\delta_i, c_I^e) - V_i^{Wne}(\delta_i) \geq 0,$$

i.e., by direct computation, if and only if

$$\delta_i \geq \delta^W \equiv \left[\frac{\frac{1+\Gamma}{\Gamma} c_I^e}{\left[\alpha \left(\frac{1-\alpha}{\mu} \right)^{\frac{(1-\alpha)}{\alpha}} \right]^{\frac{1+\Gamma}{\Gamma}} \left[A^{e\frac{1+\Gamma}{\alpha\Gamma}} - A^{ne\frac{1+\Gamma}{\alpha\Gamma}} \right]} \right]^{\Gamma}. \tag{1}$$

Hence, for c_I^e positive and sufficiently small, there is a unique threshold value δ^W, strictly increasing in c_I^e.

Clearly, the physical-human capital ratio is δ_i-invariant, with $\frac{K^{Ws}(\delta_i)}{H^{Ws}(\delta_i)} = \left(\frac{(1-\alpha)A^s}{\mu} \right)^{\frac{1}{\alpha}}$ and $\frac{K^{We}(\delta_i)}{H^{We}(\delta_i)} > \frac{K^{Wne}(\delta_i)}{H^{Wne}(\delta_i)}$, each i.

4. THE ECONOMY WITH FRICTIONS

Given any random variable x^s, with $x^s : \Omega_I^s \to \mathbb{R}$, (or y^s, with $y^s : \Omega_J^s \to \mathbb{R}$), let

$$E_{\Omega_I^s}(x_i^s) \equiv \frac{\int_{\Omega_I^s} x_i^s di}{\nu(\Omega_I^s)}$$

$\left(E_{\Omega_J^s}(y_j^s) \right)$ be the conditional expectation of x_i^s over the set Ω_I^s (of y_j^s over Ω_J^s).

Later on, we will show that, at the equilibrium, it is always $\Omega_I^e = [\widehat{\delta}, 1)$. Therefore, in the sequel, the partitions Ω_I^P and Ω_J^P will be defined by the threshold level $\widehat{\delta}$. To emphasize this, we will use the notation $\Omega_J^s\left(\widehat{\delta}\right)$ and $\Omega_I^s\left(\widehat{\delta}\right)$. We denote δ^F the equilibrium threshold value. The superscript F indicates equilibrium values in the economy with frictions.

For future reference, let's determine the optimal amount of investments assuming that there is a public intervention defined by a pair of vectors $\xi^s \equiv (\tau^s, \zeta^s, \Delta c_I^s, T)$, $\xi \equiv (\xi^e, \xi^{ne})$, describing (possibly) sector specific subsidies and taxes. We assume that there are linear subsidies on labor income (with rates τ^s, $s = ne, e$), and on the cost of the investments in physical capital (with rates ζ^s, $s = ne, e$) and fixed taxes on the direct costs of education, Δc_I^s (we will always set $\Delta c_I^{ne} = 0$). T denotes a (uniform) lump-sum tax, such that the public budget is balanced. We write the subsidy rates as sector specific just to simplify the notation. As we will show, at the equilibrium, the investment in physical capital is always larger in sector e, and the labor income of each worker in sector e is always strictly greater than the one of any worker active in sector ne. Therefore, this subsidy system is (at equilibrium) isomorphic to a system of step-linear subsidies to labor income (i.e., similar to the usual system of progressive income taxes) and to investments in physical capital.[13]

Pick an arbitrary threshold value $\widehat{\delta}$. If active in sector s, firm j selects the value of k_j^s solving optimization problem

$$\begin{aligned} & \max_{k_j^s} E_{\Omega_I^s(\widehat{\delta})}\left(\left(1-\beta\right) A^s h_i^{s\alpha} k_j^{s(1-\alpha)} - \mu\left(1-\zeta^s\right) k_j^s\right) - d_J^s \\ = \; & \left(1-\beta\right) A^s E_{\Omega_I^s(\widehat{\delta})}\left(h_i^{s\alpha}\right) k_j^{s(1-\alpha)} - \mu\left(1-\zeta^s\right) k_j^s - d_J^s. \end{aligned} \tag{Π^{Fs}}$$

Let $E_{\Omega_I^s(\widehat{\delta})}\left(\Pi^{Fs}\left(.\right)\right)$ be the expected surplus (inclusive of d_J^s) in sector s. As mentioned before, we interpret d_J^s as royalties paid to access the technologies used in the two sectors. We set $d_J^{ne} = 0$ and, at equilibrium, d_J^{Fe} is equal to the (positive, as we will show) difference between the conditional expected producer's surpluses in the two sectors. Therefore, each firm is indifferent between the two sectors and has non-negative (conditional) expected profits.

The optimization problem of worker i (if s) is

$$\begin{aligned} & \max_{h_i^s} E_{\Omega_J^s(\widehat{\delta})}\left(U_i^s\left(.\right)\right) \\ = \; & \left(1+\tau^s\right)\beta A^s h_i^{s\alpha} E_{\Omega_J^s(\widehat{\delta})}\left(k_j^{s(1-\alpha)}\right) - \frac{1}{\delta_i}\frac{h_i^{s(1+\Gamma)}}{1+\Gamma} - \left(c_I^s + \Delta c_I^s + T\right). \end{aligned} \tag{U^{Fs}}$$

For completeness, let's make precise our notion of equilibrium.

Definition 1. Given ξ, an equilibrium of the economy with frictions is a threshold value $\delta^F \in (0,1)$, a royalty $d_J^{eF} > 0$, and two pairs of maps $\left\{H^{Fs}\left(\delta_i, \delta^F, \xi\right), K^{Fs}(\delta^F, \xi)\right\}$, $s = ne, e$, such that:

i. $U_i^e\left(H^{Fe}\left(\delta_i, \delta^F, \xi\right), K^{Fe}(\delta^F, \xi)\right) - U_i^{ne}\left(H^{Fne}\left(\delta_i, \delta^F, \xi\right), K^{Fne}(\delta^F, \xi)\right) \geq 0$ if and only if $\delta_i \geq \delta^F$;

ii. $\left[E_{\Omega_I^e(\delta^F)}\left(\Pi^{Fe}\left(\delta^F, \xi\right)\right) - E_{\Omega_I^{ne}(\delta^F)}\left(\Pi^{Fne}\left(\delta^F, \xi\right)\right)\right] = d_J^{eF} > 0$;

[13] Exactly the same closed form of the equilibrim is obtained considering a direct subsidy to effort in education of the form $\tau^s h_i^{s\alpha}$, which would require direct observability of effort.

iii. $\quad K^{Fs}(\delta^F,\xi)$ solves $\left(\Pi^{Fs}\right)$, $s = ne$ for each $j = i$ such that $\delta_i < \delta^F$, $s = e$ for each $j = i$ such that $\delta_i \geq \delta^F$;

iv. $\quad H^{Fs}\left(\delta_i,\delta^F,\xi\right)$ solves $\left(U^{Fs}\right)$, $s = ne$ for $\delta_i < \delta^F$, and $s = e$ for $\delta_i \geq \delta^F$.

In Appendix 1, in eqs. $(A3)$ and $(A4)$, we compute the equilibrium values of human and physical capital in each sector s, for arbitrarily given threshold $\widehat{\delta}$, $\left(H^{Fs}\left(\delta_i,\widehat{\delta},\xi\right), K^{Fs}(\widehat{\delta},\xi)\right)$. Let $V_i^{Fs}\left(\delta_i,\widehat{\delta},\xi\right)$ be the associated level of utility of agent i, if active in sector s. Worker i enters sector e if and only if

$$F\left(\widehat{\delta},\xi,c_I^e\right) \equiv V_i^{Fe}\left(\delta_i,\widehat{\delta},\xi\right) - V_i^{Fne}\left(\delta_i,\widehat{\delta},\xi\right) \geq 0.$$

The equilibrium threshold value δ^F is then obtained solving

$$F\left(\widehat{\delta},\xi,c_I^e\right) \equiv f\left(\widehat{\delta},\xi\right) - a\left(c_I^e + \Delta c_I^e\right) = 0,$$

where

$$\begin{aligned} f\left(\widehat{\delta},\xi\right) \quad &\equiv \quad \widehat{\delta}^{\frac{\alpha}{1-\alpha+\Gamma}}\left(A^e E_{\Omega_I^e(\widehat{\delta})}\left(\delta_i^{\frac{\alpha}{1-\alpha+\Gamma}}\right)^{(1-\alpha)}\right)^{\frac{1+\Gamma}{\alpha\Gamma}}\chi^e(\xi) - \\ &\quad\;\; \widehat{\delta}^{\frac{\alpha}{1-\alpha+\Gamma}}\left(A^{ne} E_{\Omega_I^{ne}(\widehat{\delta})}\left(\delta_i^{\frac{\alpha}{1-\alpha+\Gamma}}\right)^{(1-\alpha)}\right)^{\frac{1+\Gamma}{\alpha\Gamma}}\chi^{ne}(\xi), \end{aligned} \tag{2}$$

with $\chi^s(\xi) \equiv \frac{(1+\tau^s)^{\frac{1}{\Gamma}}(1+\Gamma-(1+\tau^s)\alpha)}{(1-\zeta^s)^{\frac{(1+\Gamma)(1-\alpha)}{\alpha\Gamma}}}$, and $a \equiv \frac{1+\Gamma}{\alpha^{\frac{1}{\Gamma}}\beta^{\frac{1+\Gamma}{\Gamma}}}\left(\frac{\mu}{(1-\alpha)(1-\beta)}\right)^{\frac{(1+\Gamma)(1-\alpha)}{\alpha\Gamma}}$.

Remark 1. Using $(A7)$ in Appendix 1, and given that $E_{\Omega_I^e(\widehat{\delta})}\left(\delta_i^{\frac{\alpha}{1-\alpha+\Gamma}}\right)$ $> E_{\Omega_I^{ne}(\widehat{\delta})}\left(\delta_i^{\frac{\alpha}{1-\alpha+\Gamma}}\right)$, and $A^e > A^{ne}$, at each equilibrium,

$$d_J^{eF} = \left[E_{\Omega_I^e(\delta^F)}\left(\Pi^{Fe}\left(\delta^F,\xi\right)\right) - E_{\Omega_I^{ne}(\delta^F)}\left(\Pi^{Fne}\left(\delta^F,\xi\right)\right)\right] > 0,$$

at $\xi = 0$, as claimed above.

The following Proposition summarizes the fundamental properties of equilibria.

PROPOSITION 1. *Fix (Γ,α,β). Given (A^e,A^{ne},ξ), there is $\widetilde{C} > 0$ such that, for each c_I^e such that $ac_I^e \in \left(0,\widetilde{C}\right)$, there is an equilibrium with threshold value $\delta^F \in (0,1)$. Moreover, given A^{ne}, there is $\underline{A}^e$ such that, for each $A^e > \underline{A}^e$, at $\xi = 0$, the equilibrium is unique and $\frac{\partial f(.)}{\partial \widehat{\delta}}|_{\widehat{\delta}=\delta^F} > 0$, $\frac{\partial \delta^F(.)}{\partial \tau^e} < 0$, $\frac{\partial \delta^F(.)}{\partial \tau^{ne}} > 0$, $\frac{\partial \delta^F(.)}{\partial \Delta c_I^e} > 0$, $\frac{\partial \delta^F(.)}{\partial A^e} < 0$ and $\frac{\partial \delta^F(.)}{\partial A^{ne}} > 0$.*

Proof. In Appendix 1. ∎

In the sequel, we will mostly consider the leading case where $\frac{\partial f(.)}{\partial \widehat{\delta}} > 0$ at each equilibrium threshold.

Remark 2. Given $\xi = 0$ and $(A^{ne}, \Gamma, \alpha, \beta)$, for A^e sufficiently close to A^{ne} the economy can exhibit multiple equilibria. In Example A1 (in Appendix 1), we construct an economy with $\frac{\partial f(.)}{\partial \widehat{\delta}} > 0$ for $\widehat{\delta}$ sufficiently close to 0, and $\frac{\partial f(.)}{\partial \widehat{\delta}} < 0$ for $\widehat{\delta}$ sufficiently close to 1. Given that $\frac{\partial f(.)}{\partial \widehat{\delta}}$ is a continuous function on $(0,1)$, $f(.)$ has at least one local maximum, $\overline{\delta}$. Evidently, each economy with c_I^e such that $ac_I^e < f(\overline{\delta})$, and close enough to $f(\overline{\delta})$, has at least two equilibria.

Remark 3. Let $\xi = 0$. Consider a sequence of equilibrium thresholds $\left\{\delta^F\left(A^{ev}\right)\right\}_{v=1}^{v=\infty}$ associated with any sequence $\{A^{ev}\}_{v=1}^{v=\infty}$ with $A^{ev} \to A^{ne}$. If $\left(1 - \left(\frac{1}{\gamma}\right)^{\frac{\alpha}{1-\alpha+\Gamma}}\right) > \frac{ac_I^e}{(1-\alpha+\Gamma)A^{ne}}$, $\delta^F\left(A^{ev}\right) \to \widetilde{\delta} \in (0,1)$. Hence, investments in human capital of type e at equilibrium may be positive even when this skill is completely useless, from the technological view point. This result is somehow similar to what happens in signalling models. However, in this economy there is no asymmetry of information, and therefore the mechanism behind investments in technically useless skills is different, and it is crucially related to lack of contractibility. It is an open issue how general is this asymptotic property of the generalized Roy model in non-Walrasian economies.[14]

Remark 4. Fix $\xi = 0$. Modulo a redistribution of output, the Walrasian allocation is the unique Pareto efficient allocation of this economy (i.e., δ^W coincides with its Pareto optimal level). With elastic supply of human and physical capital, no allocation rule (i.e., no value of β) can guarantee Pareto efficiency of the equilibrium allocation, because $K^{Fs}(\widehat{\delta})$ is δ_i-invariant, while, in the Walrasian economy, $K^{Ws}(\delta_i)$ is correlated with δ_i.

Remark 5. At $\xi = 0$, using (A3) and (A4) in Appendix 1, the physical/human capital ratio is given by

$$\frac{K^{Fs}(\delta^F)}{H^{Fs}\left(\delta_i, \delta^F\right)} = \frac{K^{Ws}\left(\delta_i\right)}{H^{Ws}\left(\delta_i\right)} \left[\frac{(1-\beta)^{\frac{1}{\alpha}} E_{\Omega_I^s(\delta^F)}\left(\delta_i^{\frac{a}{1-\alpha+\Gamma}}\right)^{\frac{1}{\alpha}}}{\delta_i^{\frac{1}{1-\alpha+\Gamma}}}\right].$$

In sector ne, and for sufficiently small δ_i, the term in square brackets is always greater than one, so that $\frac{K^{Fne}(\delta^F)}{H^{Fne}\left(\delta_i,\delta^F\right)} > \frac{K^{Wne}(\delta_i)}{H^{Wne}(\delta_i)}$, for δ_i small enough. This immediately implies that agents with a sufficiently low δ_i are always better off at the equilibrium of the frictional economy. Hence, the Walrasian equilibrium allocation is not Pareto superior to the one of the economy with frictions. Of course, it still dominates it in terms of total surplus.[15]

Remark 6. The threshold value δ^F can be either lower or higher than its value in the Walrasian economy, as we establish with the following example.

Example 1. Let $\xi = 0$. Consider the economy with $A^e = 2, A^{ne} = 1$, $\alpha = \beta = 1/2$, and $\Gamma = 1$. By direct computation, using (1) and (2), δ^F and δ^W are obtained

[14] Clearly, this result could partly depend upon the specific features of our model, i.e., existence of two separate sectors, and perfect lack of substitutability between human capitals of different skills.

[15] Given the structure of preferences, if total surplus in the economy with frictions were larger than the one of the Walrasian economy, we would contraddict the first fundamental theorem of welfare economics.

solving

$$0 = \frac{27}{8192}\left(\sqrt[3]{\widehat{\delta}}\left(4\frac{1-\widehat{\delta}}{1-\widehat{\delta}}\right)^2 - \widehat{\delta}\right) - c_I^e \equiv f^F\left(\widehat{\delta}\right) - c_I^e$$

in the economy with frictions, and

$$0 = \frac{15}{32}\widehat{\delta} - c_I^e \equiv f^W\left(\widehat{\delta}\right) - c_I^e$$

in the Walrasian economy. They are shown in Figure 1 ($f^W\left(\widehat{\delta}\right)$ is described by the solid line). One can verify that, for $c_I^e < 0.019$, $\delta^F < \delta^W$, while, for $c_I^e > 0.019$, the opposite occurs.

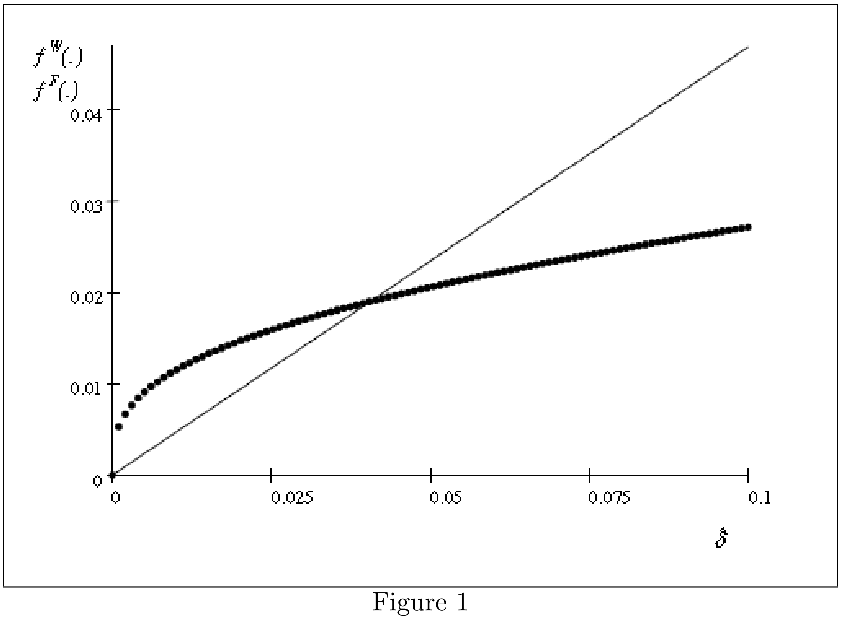

Figure 1

Our main purpose is to analyze the policy implications of self-selection into distinct labor markets. Anyhow, it is worthwhile to briefly consider the comparative statics of equilibria. Let $\phi \equiv (\xi, A^e, A^{ne})$. Let $w^s\left(\delta_i, \delta^F, \phi\right)$ be worker i's wage in sector s. The standard deviation, $\sigma_{\Omega_I^s(\delta^F)}\left(\delta^F, \phi\right)$, measures the variability of wages within sector s. $WP_{\Omega_I^e(\delta^F)}\left(\delta^F, \phi\right)$ is the wage premium. In general, there are three different notions of wage premium: the marginal ratio $\frac{w^e\left(\delta^F, \delta^F, \phi\right)}{w^{ne}\left(\delta^F, \delta^F, \phi\right)}$, $E_{\Omega_I^e(\delta^F)}\left(\frac{w^e\left(\delta_i, \delta^F, \phi\right)}{w^{ne}\left(\delta_i, \delta^F, \phi\right)}\right)$ (the average over the agents who actually invested) and $E_{\Omega_I^{ne}(\delta^F)}\left(\frac{w^e\left(\delta_i, \delta^F, \phi\right)}{w^{ne}\left(\delta_i, \delta^F, \phi\right)}\right)$ (the average over the ones who actually did not invest). The wage function is multiplicative in δ_i. Hence, the three values coincide, so that we can unambiguously talk of "wage premium".

PROPOSITION 2. *Fix* $(\Gamma, c_I^e, \alpha, \beta)$. *Assume that* $\frac{\partial f(.)}{\partial \widehat{\delta}}|_{\widehat{\delta}=\delta^F} > 0$. *At* $\xi = 0$, *the following sign restrictions are satisfied:*[16]

$$\left[\begin{array}{cccccc} & d\tau^e & d\tau^{ne} & \Delta c_I^e & dA^e & dA^{ne} \\ E_{\Omega_I^e(\delta^F)}\left(H^{Fe}(.)\right) & ? & + & + & ? & + \\ E_{\Omega_I^{ne}(\delta^F)}\left(H^{Fne}(.)\right) & - & + & + & - & + \\ E_{\Omega_I^e(\delta^F)}\left(w^e(.)\right) & ? & + & + & ? & + \\ E_{\Omega_I^{ne}(\delta^F)}\left(w^{ne}(.)\right) & - & + & + & - & + \\ \sigma_{\Omega_I^{ne}(\delta^F)}(.) & - & + & + & - & + \\ WP_{\Omega_I^e(\delta^F)}(.) & + & - & - & + & - \end{array}\right].$$

Proof. In Appendix 1. ∎

The mechanism explaining these results is based on the interaction of incentive and composition effect. For instance, consider the effect of a change in the parameter A^{ne}. Its increase stimulates effort in education of low-skilled workers, and pushes up the threshold δ^F. Via the composition effect, it improves the (conditional) expected human capital of both low and high skilled workers. This, in turn, stimulates investments in physical capital. The positive feed-backs strengthen these initial impacts. Hence, the effect on expected human capital and wages in both sectors are positive. For the wage premium, both direct and composition effects are negative. The standard deviation of wages of unskilled workers increases because both effects are positive.

In the table, we omit the standard deviation of the wages of skilled workers. For this variable, the sign of the composition effect varies over the parameter space, so that it is impossible to reach any well-defined result.[17]

5. EFFICIENCY PROPERTIES OF THE ECONOMY WITH FRICTIONS

In Remark 4, we have seen that the equilibria of the economy with frictions are Pareto inefficient. We will now show that they do not satisfy either a weaker criterion of *constrained optimality* (CO in the sequel) which takes into account the imperfections which characterize the economy. Most interesting is the analysis of their inefficiency in terms of amount, and type, of investments. In the sequel, we will mainly refer to the investments in human capital. Similar considerations hold for the ones in physical capital.

In our set-up, inefficiencies can be of two different types. First, an individual can choose an *amount* of investment different from the CO one, *given* the partition Ω_I^P associated with the CO allocation. We will refer to this possible source of inefficiency as *underinvestment* (or *overinvestment*) *in educational effort.* Secondly, an agent can choose to invest in a *type* of education different from the one assigned to her at the CO allocation. We will say that there is *underinvestment in educational level* when agent i invests in education ne, while, at the CO allocation, she should invest in education level e.

[16] Each cell reports the sign of the derivative of the function on the row with respect to the variable on the column.

[17] For reasonable values of the parameters, $\alpha = \frac{2}{3}$ and $\Gamma > \frac{1}{2}$, some numerical simulations show that the composition effect has the sign opposite to the one of $\frac{\partial \delta^F}{\partial \phi}$. Therefore, $\frac{\partial \sigma^e(.)}{\partial \phi}$ is positive for $\phi' \in \{\tau^e, A^e\}$, negative for $\phi' \in \left\{\tau^{ne}, \Delta c_I^e, A^e\right\}$.

In the one-sector model, equilibria are unambiguously characterized by under-investment. In our set up, the same effect is at work: in each sector, given δ^F (or any arbitrary $\widehat{\delta}$), an increase in the investments of firms and workers leads to a Pareto improvement. The argument is identical to the one exploited by Acemoglu (1996): fix δ^F and consider a small change in h_i^s and k_j^s, each i and j. The changes in utilities and producers' surplus evaluated at the equilibrium pair (h_i^{Fs}, k_j^{Fs}) (and taking into account that $k_j^{Fs} = k^{Fs}$, each j and s) are given by

$$0 < \left(\alpha\beta A^s \left[\frac{k^{Fs}}{h_i^{Fs}}\right]^{1-\alpha} - \frac{1}{\delta_i} h_i^{Fs\Gamma}\right) dh + \left((1-\alpha)\beta A^s \left(\frac{h_i^{Fs}}{k^{Fs}}\right)^{\alpha}\right) dk, \quad (3)$$

and

$$\begin{aligned} 0 \quad &< \quad \left((1-\alpha)(1-\beta) A^s \frac{E_{\Omega_I^s(\delta^F)}\left(h_i^{Fs\alpha}\right)}{k^{Fs\alpha}} - \mu\right) dk \\ &\quad + \left(\alpha(1-\beta) A^s \frac{k^{Fs(1-\alpha)}}{E_{\Omega_I^s(\delta^F)}\left(h_i^{Fs(1-\alpha)}\right)}\right) dh, \end{aligned} \quad (4)$$

respectively. The inequalities hold because the first terms in parenthesis in (3) and (4) are zero (at the optimal solutions of $\left(\Pi^{Fs}\right)$ and (U^{Fs})), while the second terms are positive. Hence, given any $\widehat{\delta}$, there is underinvestment in educational effort and physical capital, in each sector. This establishes, in a more direct way, the Pareto inefficiency of equilibria in the economy with frictions.

In the two-sector case, there is a second potential source of inefficiency, because changes in the value of $\widehat{\delta}$ may also entail Pareto improvements. An increase in the threshold value $\widehat{\delta}$ increases the conditional expected amount of human capital in both sectors at the same time and, consequently, induces an increase in the amount of physical investments of firms in both sectors. Indeed, given that $\delta_i^{\frac{\alpha}{1-\alpha+\Gamma}}$ is strictly monotonically increasing,

$$\frac{\partial E_{\Omega_I^s(\widehat{\delta})}\left(\delta_i^{\frac{\alpha}{1-\alpha+\Gamma}}\right)}{\partial \widehat{\delta}} > 0, \text{ for each } s \text{ and } \widehat{\delta}, \quad (5)$$

and, consequently, using $(A3)$ and $(A4)$,

$$\frac{\partial H^{Fs}\left(\delta_i, \widehat{\delta}, \xi\right)}{\partial \widehat{\delta}}|_{\xi=0} > 0 \text{ and } \frac{\partial K^{Fs}\left(\widehat{\delta}, \xi\right)}{\partial \widehat{\delta}}|_{\xi=0} > 0, \text{ for each } s \text{ and } \widehat{\delta}. \quad (6)$$

More relevant, from $(A5)$, $(A6)$ and (5), for each i and $\widehat{\delta}$,

$$\frac{\partial V_i^{Fs}\left(\delta_i, \widehat{\delta}, \xi\right)}{\partial \widehat{\delta}}|_{\xi=0} > 0 \text{ and } \frac{\partial E_{\Omega_I^s(\widehat{\delta})}\left(\Pi^{Fs}\left(\delta_i, \widehat{\delta}, \xi\right)\right)}{\partial \widehat{\delta}}|_{\xi=0} > 0, \quad (7)$$

where $E_{\Omega_I^s(\widehat{\delta})}\left(\Pi^{Fs}\left(\delta_i, \widehat{\delta}, \xi\right)\right)$ is the (ex-post) surplus of the firm matched with worker i in sector s.

These properties do not suffice to establish our claim, because a change in the threshold induces a jump in the producer's surplus for the firms shifting from one sector to the other. However, as we will formally establish below (in Proposition 4), under suitable restrictions on equilibria, sufficiently small increases of the threshold value increase aggregate surplus.

To complete the analysis of the welfare properties of equilibria, it is convenient to introduce an explicit notion of (constrained) efficiency. As usual in economies with frictions, we consider the metaphor of a benevolent planner choosing an allocation while facing constraints aiming to capture the ones the agents face in the decentralized economy. We provide two results. First, we show that there are constrained optimal allocations (Proposition 3), and that they can be attained with an appropriate system of taxes and subsidies. The amount of subsidies and taxes is entirely dictated by the features of the CO allocation, and they can be (in fact, are) quite large. That's why, in Proposition 4, we study the effects of *small* taxes and subsidies on total surplus evaluated at the market equilibrium, taking as given the actual demand and supply functions of the agents.

Bear in mind that, in the sequel, we always consider changes in total surplus. We are not concerned with actual Pareto improvements. However, given that utility functions are quasi-linear, an increase of total surplus immediately translates (modulo an appropriate - and $i-$contingent - system of lump-sum taxes and transfers) into a Pareto improvement. Also, given the structure of the economy, the systems of taxes and transfers can be designed so to guarantee a balanced budget.

5.1. Constrained optimal allocations

The objective function of the planner is given by the sum of expected utilities and producers' surpluses of the agents, i.e.,

$$\begin{aligned} P\left(h_i^s, k_j^s, \Omega_I^s, \Omega_J^s\right) \equiv & \sum_s \int_{\Omega_I^s(\widehat{\delta})} \left[\beta E_{\Omega_J^s(\widehat{\delta})}\left(A^s h_i^{s\alpha} k_j^{s(1-\alpha)}\right) - \frac{1}{\delta_i}\frac{h_i^{s(1+\Gamma)}}{1+\Gamma} - c_I^s\right] di \\ & + \sum_s \int_{\Omega_J^s(\widehat{\delta})} \left[(1-\beta) E_{\Omega_I^s(\widehat{\delta})}\left(A^s h_i^{s\alpha} k_j^{s(1-\alpha)}\right) - \mu k_j^s\right] dj. \end{aligned}$$

The policy instruments are the partitions Ω_I^P and Ω_J^P and a pair of maps $(H^{COs}\left(\delta_i, \widehat{\delta}\right), K^{COs}\left(\widehat{\delta}\right))$. We restrict the partitions to have the structure $\Omega_I^e\left(\widehat{\delta}\right) = \left\{i \in \Omega_I | \delta_i \geq \widehat{\delta}\right\}$, and $\Omega_J^e\left(\widehat{\delta}\right) = \left\{j \in \Omega_J | j = i,\ i \in \Omega_I^e\left(\widehat{\delta}\right)\right\}$. Given that firms are (ex-ante) identical, the informational constraints embedded into the definition of $P(.)$, and the properties of the (implicit) matching function, to impose this structure on Ω_J^P does not entail any loss of generality. Also, observe that, given that firms are identical, expected total surplus and realized total surplus coincide.

PROPOSITION 3. *Under the maintained assumptions, each economy with frictions has a CO allocation. Equilibrium allocations are never CO, and they are characterized by underinvestment in the amount of physical capital and in educational effort. Both under and overinvestment in educational level are possible.*

Proof. In Appendix 1. ∎

The source of inefficiency considered by Acemoglu (1996) reappears in our setup, because, given any threshold level $\widehat{\delta}$, $H^{COs}\left(\delta_i, \widehat{\delta}\right) > H^{Fs}\left(\delta_i, \widehat{\delta}\right)$, for each δ_i, and $K^{COs}\left(\widehat{\delta}\right) > K^{Fs}\left(\widehat{\delta}\right)$. On the other hand, the relation between the CO value of the threshold, δ^{CO}, and its equilibrium level, δ^F, is not univocal. Example A2, in Appendix 1, shows an economy such that, for c_I^e sufficiently small $\delta^F < \delta^{CO}$, while the opposite occurs for c_I^e sufficiently large.[18]

Remark 7. In our set-up (as well as in Acemoglu (1996)), equilibria of the economy with frictions are constrained inefficient for each value of β, because, at $\xi = 0$, even if $\delta^{CO} = \delta^F$,

$$\frac{H^{Fs}\left(\delta_i, \delta^{CO}\right)}{H^{COs}\left(\delta_i, \delta^{CO}\right)} = (1-\beta)^{\frac{1-\alpha}{\alpha\Gamma}} \beta^{\frac{1}{\Gamma}}, \text{ for each } s \text{ and } i,$$

and

$$\frac{K^{Fs}\left(\delta^{CO}\right)}{K^{COs}\left(\delta^{CO}\right)} = (1-\beta)^{\frac{1-\alpha+\Gamma}{\alpha\Gamma}} \beta^{\frac{1}{\Gamma}}, \text{ for each } s.$$

Evidently, both ratios are different from 1, for each $\beta \in (0,1)$. In the usual random matching model, efficiency obtains when the Hosios' condition is satisfied, i.e., when β is equal to the absolute value of the elasticity of the matching function. In our economy there is always full employment of all the resources, so that no congestion externality is at work. Therefore, the Hosios' condition has no connection with Pareto efficiency.[19]

It is easy to see that the CO distribution of investments in human and physical capital can be attained with an appropriate system of subsidies to investments in physical capital and labor income, and of fixed taxes or subsidies on the educational choice. Moreover, given that preferences are quasi-linear, the system of tax and subsidies can be balanced using uniform lump-sum taxes (T) on workers (notice that, in the absence of positive endowments of consumption goods, this could entail negative consumption for some subset of agents).

Corollary 1. *There is a (balanced budget) system of taxes and subsidies ξ such that the associated equilibrium allocation is CO.*

Proof. See Appendix 1. ∎

5.2. The effect of income taxes and subsidies to education on total surplus

We conclude considering the welfare effects of alternative, balanced budget, tax schemes. In particular, we study the effect on total surplus of local changes in the vector ξ, in a neighborhood of $\xi = 0$. We just consider the effects of $(\tau, \Delta c_I^e)$. Assume that $\frac{\partial F}{\partial \widehat{\delta}}|_{\widehat{\delta}=\delta^F} > 0$, and that δ^F is not "too high". Then, an increase in the cost of education (redistributing the revenues as lump-sum transfer), or an increase of the subsidies to labor income in the "low skill" sector ne (financed with lump-sum taxes) always has a positive effect on total surplus. On the contrary, an increase in the subsidy to labor income in the high skill sector (again, financed with lump-sum

[18] In the example, the surplus associated with the market equilibrium is always increasing in the threshold value, even when $1 > \delta^F > \delta^{CO}$. This is because, in computing the values of δ^F and δ^{CO}, we use $\left(H^{Fs}(.), K^{Fs}(.)\right)$ in one case, $\left(H^{COs}(.), K^{COs}(.)\right)$ in the other.

[19] As observed in Acemoglu (1996, p. 789), given any threshold $\widehat{\delta}$, the externalities are related to "the value of the future matches and are always positive".

taxes) may decrease it. The intuition behind the result is fairly simple. A subsidy $\tau^{ne} > 0$ has a direct, positive effect on effort in this sector, and a positive, indirect, effect on effort in both sectors, because it induces an increase in the equilibrium value of $\delta^F(\xi)$. For the same reason, a tax on higher education $\Delta c_I^e > 0$ has an indirect, positive effect on effort in both sectors. Therefore, they always lead to an increase in total surplus. The third policy ($\tau^e > 0$) makes sector e more attractive to workers. Therefore, it induces some workers with $\delta_i < \delta^F(0)$ to switch to sector e. This has an unambiguous, negative effect on the welfare of the workers remaining in sector ne (and on the expected profits in this sector). The negative effect on the welfare of the workers in sector e, due to the composition effect, may actually overcome the positive effect of the incentives in this sector, too. More generally, the net effect on total surplus is ambiguous, and there are economies where subsidies in the high skill sector induce a lower total surplus. This is established in Proposition 4 and by a final example.

In showing these results, the main difficulty is that a change in the threshold induces a discontinuous jump in the expected producer's surplus for the firms changing sectors. We provide *one* sufficient condition which guarantees that, at the equilibrium, the total surplus is increasing in the value of the threshold. This condition is far from necessary for our results. Given (α, β, Γ), the threshold value δ^F must be below some upper limit $\overline{\delta}$. Hence, this is essentially a restriction on the ratio $\frac{A^e}{A^{ne}}$. The implicit restriction on the equilibrium threshold is not unreasonable. For instance, for $\alpha = \frac{2}{3}$, we certainly have $\frac{\partial P(\widehat{\delta},\xi)}{\partial \widehat{\delta}}|_{\widehat{\delta}=\delta^F} > 0$ if $\delta^F < 0.6$ and $\Gamma = 0.2$, if $\delta^F < 0.35$ and $\Gamma = 0.5$ and so on. The critical value $\overline{\delta}$ is decreasing in α and Γ.[20]

Up to now we have consider a sector-contingent vector of subsidy rates (τ^e, τ^{ne}). This is certainly an unusual feature. However, the same results can be obtained with a standard system of step-linear subsidies, of the type

$$T(w) = \begin{cases} \tau^{ne} w & \text{if } w \leq w^{ne}\left(\delta^F, \delta^F\right) \\ \tau^{ne} w^{ne}\left(\delta^F, \delta^F\right) + \tau^e\left(w - w^{ne}\left(\delta^F, \delta^F\right)\right) & \text{if } w > w^{ne}\left(\delta^F, \delta^F\right) \end{cases},$$

where w denotes a generic labor income, while $w^s\left(\delta^F, \delta^F\right)$ is the labor income, in sector s, of the worker with $\delta_i = \delta^F$ at the equilibrium associated with the threshold level δ^F. All that is required to establish this result is to observe that, at $\xi = 0$, at the equilibrium associated with any δ^F the actual labor income of each worker in sector e is larger than the labor income of any worker in sector ne. Indeed, by direct computation, the labor income of worker i active in sector s is given by

$$\begin{aligned} \beta Y^s(\delta_i, \delta^F) &= \left[\beta\left(\frac{(1-\alpha)(1-\beta)}{\mu}\right)^{\frac{(1+\Gamma)(1-\alpha)}{\alpha\Gamma}} (\alpha\beta)^{\frac{1}{\Gamma}}\right] \\ &\quad \times A^{s\,\frac{1+\Gamma}{\alpha\Gamma}} E_{\Omega_I^s(\delta^F)}\left(\delta_i^{\frac{\alpha}{1-\alpha+\Gamma}}\right)^{\frac{(1+\Gamma)(1-\alpha)}{\alpha\Gamma}} \delta_i^{\frac{\alpha}{1-\alpha+\Gamma}}, \end{aligned} \tag{8}$$

and, evidently, $\beta Y^e(\delta_i', \delta^F) > \beta Y^{ne}(\delta_i'', \delta^F)$ for each $\delta_i' \geq \delta^F \geq \delta_i''$.

[20] An alternative sufficient condition is that β is "large enough". Notice that, for the class of economies considered in Appendix 2, it is always $\frac{\partial P(\delta^*,\xi)}{\partial \delta^*}|_{\delta^*=\delta^F} > 0$. Here, we need additional restrictions because of discontinuity in expected producer's surplus.

Remark 8. We are taking as a reference point an economy where $\xi = 0$. Evidently, if we introduce a flat tax rate t on labor incomes, we would obtain exactly the same results changing the marginal tax rates.

To conclude, let's make formal the heuristic argument above. Given ξ, workers and firms choose their individually optimal behavior. Let $S(\xi)$ be the expected total surplus corresponding to the equilibrium associated with the vector ξ of policy instruments. Let $\delta^F(\xi)$ be the equilibrium threshold associated with the vector ξ. Then

$$\begin{aligned} S(\xi) \quad \equiv \quad & \sum_s \int_{\Omega_J^s\left(\delta^F(\xi)\right)} E_{\Omega_I^s\left(\delta^F(\xi)\right)}\left(\Pi_j^{Fs}\left(\delta_i, \delta^F(\xi), \xi\right)\right) dj \\ & + \sum_s \int_{\Omega_I^s\left(\delta^F(\xi)\right)} V_i^{Fs}\left(\delta_i, \delta^F(\xi), \xi\right) di, \end{aligned} \tag{9}$$

with total lump-sum taxes given by

$$T(\xi) = \left[\sum_s \tau^s \int_{\Omega_I^s\left(\delta^F(\xi)\right)} w^s(\delta_i, \delta^F(\xi), \xi) di - \Delta c_I^e \nu\left(\Omega_I^e\left(\delta^F(\xi)\right)\right)\right],$$

so that the budget is balanced.

PROPOSITION 4. *Consider an equilibrium associated with* $\xi = 0$ *and such that* $\frac{\partial f(.)}{\partial \widehat{\delta}}|_{\widehat{\delta}=\delta^F(0)} > 0$ *and*

$$\frac{1 - \delta^F(0)^{\frac{1+\Gamma}{1-\alpha+\Gamma}}}{\left(1 - \delta^F(0)\right)\delta^F(0)^{\frac{\alpha}{1-\alpha+\Gamma}}} \geq \frac{1}{1-\alpha}.$$

Then,
i. $\Delta c_I^e > 0$*, and sufficiently small, increases total surplus,*
ii. $\tau^{ne} > 0$*, and sufficiently small, increases total surplus,*
iii. $\tau^e > 0$*, and sufficiently small, may decrease total surplus.*

The proofs of (i, ii) are in Appendix 1, where we also establish that the welfare effect of a subsidy τ^e is, in general, indeterminate. We now provide a strategy to construct economies where an increase in τ^e *decreases* total surplus. The third statement, therefore, is established by the following example.

EXAMPLE 3. *Welfare-reducing subsidies to investments in human capital in the high skill sector.*

From eq. (2), the sign of $\frac{\partial f}{\partial \widehat{\delta}}|_{\widehat{\delta}=\delta^F(.)}$ depends upon the parameters $\left(\frac{A^e}{A^{ne}}, \alpha, \Gamma\right)$, and the equilibrium level $\delta^F(.)$, while it doesn't depend directly on β. Moreover, given $\left(\frac{A^e}{A^{ne}}, \alpha, \Gamma\right)$, the effect of changes of β on the value of $\delta^F(.)$ can always be neutralized by appropriate changes in the parameter c_I^e.

Clearly,

$$\begin{aligned} \frac{\partial S(.)}{\partial \tau^e} \quad = \quad & \left(\frac{\partial S(.)}{\partial \delta^F}\right)\frac{\partial \delta^F(\xi)}{\partial \tau^e} + \sum_s \int_{\Omega_I^s\left(\delta^F(.)\right)} \frac{\partial V^{Fs}(\delta_i, \delta^F, \xi)}{\partial \tau^s} di \\ & + \sum_s \int_{\Omega_J^s\left(\delta^F(\xi)\right)} \frac{\partial E_{\Omega_I^s\left(\delta^F(\xi)\right)}\left(\Pi^{Fs}\left(\delta^F, \xi\right)\right)}{\partial \tau^s} dj \end{aligned}$$

We want to construct an economy such that, at the equilibrium, $\frac{\partial S(.)}{\partial \tau^e} < 0$. The last four terms of $\frac{\partial S(.)}{\partial \tau^e}$ are positive, and they are easily seen to be bounded above (looking at their parametric structure in $(A5)$ and $(A7)$) . Hence, the required result is established if we can construct an equilibrium with $\left(\frac{\partial S(.)}{\partial \delta^F}\right) \frac{\partial \delta^F(\xi)}{\partial \tau^e} < 0$ and arbitrarily large in absolute value. As established in Example A1 in Appendix 1, for $\frac{A^e}{A^{n^e}}$ sufficiently small, there are economies such that $\frac{\partial f(.)}{\partial \widehat{\delta}} > 0$, for $\widehat{\delta}$ sufficiently small and $\frac{\partial f(.)}{\partial \widehat{\delta}} < 0$ for $\widehat{\delta}$ large enough. Given that the function $\frac{\partial f(.)}{\partial \widehat{\delta}}$ is continuous, this implies that, for economies in this set, there is $\overline{\delta}$ such that $\frac{\partial f(.)}{\partial \widehat{\delta}}|_{\widehat{\delta}=\overline{\delta}} = 0$ and $\frac{\partial f(.)}{\partial \widehat{\delta}} > 0$ at each $\widehat{\delta} < \overline{\delta}$. Given the values of all the parameters, but the actual direct cost of education, c_I^e, pick a sequence $\{c_I^{ev}\}_{v=1}^{\infty}$ such that $c_I^{ev} < \overline{c}_I^e = f(\overline{\delta})$, for each v, and $c_I^{ev} \to \overline{c}_I^e$. Along this sequence, by construction, $\delta^F(c_I^{ev}) < \overline{\delta}$, and $\delta^F(c_I^{ev}) \to \overline{\delta}$. Therefore, $\frac{\partial f(.)}{\partial \widehat{\delta}}|_{\widehat{\delta}=\delta^F(c_I^{ev})} > 0$, for each v, and $\lim_{v \to \infty} \frac{\partial f(.)}{\partial \widehat{\delta}}|_{\widehat{\delta}=\delta^F(c_I^{ev})} = 0$. By the implicit function theorem, $\frac{\partial \delta^F(.)}{\partial \tau^e} = -\frac{\frac{\partial f(.)}{\partial \tau^e}}{\frac{\partial f(.)}{\partial \widehat{\delta}}|_{\widehat{\delta}=\delta^F}}$. Given that $\frac{\partial f(.)}{\partial \tau^e}$ is positive and bounded away from zero (see Proposition 1), the sequence $\frac{\partial \delta^F(.)}{\partial \tau^e}$ associated with $\left\{\delta^F(c_I^{ev})\right\}_{v=1}^{\infty}$ is negative and divergent.

To prove our result we still need to show that there are economies such that, in a neighborhood of $\overline{\delta}$, $\frac{\partial S(.)}{\partial \widehat{\delta}}|_{\widehat{\delta}=\delta^F} \geq \varepsilon$, for some $\varepsilon > 0$. In the proof of Proposition 4, we have defined the expression $\Delta S^e(\delta^F)$, governing the sign of $\frac{\partial S(.)}{\partial \widehat{\delta}}|_{\widehat{\delta}=\delta^F}$. It is easy to check that, given δ^F, $\Delta S^e(\delta^F)$ is strictly positive for β close enough to 1. Pick a value of β, $\overline{\beta}$, such that, at $\overline{\delta}$, $\frac{\partial S(.)}{\partial \widehat{\delta}} > \varepsilon$, for some $\varepsilon > 0$. By continuity, $\frac{\partial S(.)}{\partial \widehat{\delta}} > \varepsilon$ at each δ close enough to $\overline{\delta}$. Change the sequence $\{c_I^{ev}\}_{v=1}^{\infty}$ to neutralize the change in $\delta^F(c_I^{ev})$ due to the new value of β. For the economy so constructed, for some c_I^e, it must be $\frac{\partial S(.)}{\partial \tau^e} < 0$ and arbitrarily large in absolute value, as claimed.

6. APPENDIX 1

A1: Equilibrium in the economy with frictions

We start with an arbitrary threshold $\widehat{\delta}$. The first order conditions (FOCs in the sequel) of problem (Π^{Fs}) imply that

$$k_j^s(E_{\Omega_I^s(\widehat{\delta})}(h_i^{s\alpha}), \xi) = \left[\frac{(1-\beta)(1-\alpha) A^s E_{\Omega_I^s(\widehat{\delta})}(h_i^{s\alpha})}{\mu(1-\zeta^s)}\right]^{\frac{1}{\alpha}} \tag{A1}$$

The FOCs of optimization problem (U^{Fs}) imply that

$$h_i^s\left(E_{\Omega_J^s(\widehat{\delta})}(k_j^{s1-\alpha}), \xi\right) = \left[\delta_i \alpha\beta(1+\tau^s) A^s E_{\Omega_J^s(\widehat{\delta})}(k_j^{s1-\alpha})\right]^{\frac{1}{1-\alpha+\Gamma}}. \tag{A2}$$

Given that firms in sector s are, ex-ante, identical, $k_j^s(.) = k^s(.)$, and $E_{\Omega_J^s(\widehat{\delta})}(k_j^s(.)^{1-\alpha}) = k^s(.)^{1-\alpha}$. Then, solving $(A1)$ and $(A2)$, we obtain

$$\begin{aligned} K^{Fs}(\widehat{\delta}, \xi) &= \left[\frac{(1-\alpha)(1-\beta)}{\mu(1-\zeta^s)} E_{\Omega_I^s(\widehat{\delta})}\left(\delta_i^{\frac{\alpha}{1-\alpha+\Gamma}}\right)\right]^{\frac{1-\alpha+\Gamma}{\alpha\Gamma}} \\ &\quad \times ((1+\tau^s)\alpha\beta)^{\frac{1}{\Gamma}} A^{s\frac{1+\Gamma}{\alpha\Gamma}}, \end{aligned} \tag{A3}$$

and

$$H^{Fs}\left(\delta_i,\widehat{\delta},\xi\right) = \left[\frac{(1-\alpha)(1-\beta)}{\mu(1-\zeta^s)}E_{\Omega_I^s(\widehat{\delta})}\left(\delta_i^{\frac{\alpha}{1-\alpha+\Gamma}}\right)\right]^{\frac{1-\alpha}{\alpha\Gamma}} \times\delta_i^{\frac{1}{1-\alpha+\Gamma}}\left((1+\tau^s)\alpha\beta\right)^{\frac{1}{\Gamma}}A^{s\frac{1}{\alpha\Gamma}}. \tag{A4}$$

Using these function, agent i's utility, if active in sector s, is

$$\begin{aligned} V_i^{Fs}\left(\delta_i,\widehat{\delta},\xi\right) &\equiv U_i^s(H^{Fs}(\delta_i,\widehat{\delta},\xi),K^{Fs}(\widehat{\delta},\xi)) = -\left(c_I^s+\Delta c_I^s+T\right) \\ &+\left[\frac{(1-\alpha)(1-\beta)}{\mu(1-\zeta^s)}E_{\Omega_I^s(\widehat{\delta})}\left(\delta_i^{\frac{\alpha}{1-\alpha+\Gamma}}\right)\right]^{\frac{(1+\Gamma)(1-\alpha)}{\alpha\Gamma}} \\ &\times\delta_i^{\frac{\alpha}{1-\alpha+\Gamma}}\beta^{\frac{1+\Gamma}{\Gamma}}A^{s\frac{1+\Gamma}{\alpha\Gamma}}\left[(1+\tau^s)\alpha\right]^{\frac{1}{\Gamma}}\frac{1+\Gamma-(1+\tau^s)\alpha}{1+\Gamma}. \end{aligned} \tag{A5}$$

Similarly, given an arbitrary $\widehat{\delta}$, firm j (ex-post) surplus, if active in sector s and matched with worker i, is

$$\begin{aligned} \Pi_j^{Fs}\left(\delta_i,\widehat{\delta},\xi\right) &= (1-\beta)A^{s\frac{1+\Gamma}{\alpha\Gamma}}\left((1+\tau^s)\alpha\beta\right)^{\frac{1}{\Gamma}}\left(\delta_i^{\frac{\alpha}{1-\alpha+\Gamma}}-(1-\alpha)E_{\Omega_I^s(\widehat{\delta})}\left(\delta^{\frac{\alpha}{1-\alpha+\Gamma}}\right)\right) \\ &\times\left(\frac{(1-\alpha)(1-\beta)}{\mu(1-\zeta^s)}E_{\Omega_I^s(\widehat{\delta})}\left(\delta_i^{\frac{\alpha}{1-\alpha+\Gamma}}\right)\right)^{\frac{(1+\Gamma)(1-\alpha)}{\alpha\Gamma}}. \end{aligned} \tag{A6}$$

Its expected value is

$$E_{\Omega_I^s(\widehat{\delta})}\left(\Pi_j^{Fs}\left(\delta_i,\widehat{\delta},\xi\right)\right) = \left[\frac{(1-\alpha)(1-\beta)}{\mu(1-\zeta^s)}E_{\Omega_I^s(\widehat{\delta})}\left(\delta_i^{\frac{\alpha}{1-\alpha+\Gamma}}\right)\right]^{\frac{1-\alpha+\Gamma}{\alpha\Gamma}} \times\frac{\mu(1-\zeta^s)a\left((1+\tau^s)\alpha\beta\right)^{\frac{1}{\Gamma}}A^{s\frac{1+\Gamma}{\alpha\Gamma}}}{(1-\alpha)}. \tag{A7}$$

Proof of Proposition 1. Pick the partition $\Omega_I^P\left(\widehat{\delta}\right)$ induced by any arbitrary $\widehat{\delta}$. Assume that there is an agent i' such that $\delta_{i'}=\widehat{\delta}$ at $\widehat{\delta}$ solving $\left(f\left(\widehat{\delta},\xi\right)-ac_I^e\right)=0$. It is easy to check that $V_i^{Fe}\left(\delta_i,\widehat{\delta},\xi=0\right)-V_i^{Fne}\left(\delta_i,\widehat{\delta},\xi=0\right)\geq 0$ if and only if $\delta_i\geq\widehat{\delta}$. Hence, each equilibrium partition $\Omega_I^P\left(\delta^F\right)$ such that $\Omega_I^s\left(\delta^F\right)\neq\emptyset$, each s, satisfies $\Omega_I^e\left(\delta^F\right)=\left\{i\in\Omega_I|\delta_i\geq\delta^F\right\}$, where δ^F is the (unique) threshold value defining the partition.

Let $\gamma\equiv\frac{1+\Gamma}{1-\alpha+\Gamma}$. By direct computation, for each threshold $\widehat{\delta}$,

$$E_{\Omega_I^e(\widehat{\delta})}\left(\delta_i^{\gamma-1}\right)=\frac{1}{\gamma}\frac{1-\widehat{\delta}^{\gamma}}{1-\widehat{\delta}} \text{ and } E_{\Omega_I^{ne}(\widehat{\delta})}\left(\delta_i^{\gamma-1}\right)=\frac{\widehat{\delta}^{\gamma-1}}{\gamma}.$$

Evidently, both functions are continuous at each $\widehat{\delta}\in(0,1)$. Given that they are conditional expectations of a strictly increasing function, both are strictly increasing in

$\widehat{\delta}$. Clearly, $f\left(\widehat{\delta},\xi=0\right)$ is continuous at each $\widehat{\delta}\in(0,1)$. Given that $E_{\Omega_I^s(\widehat{\delta})}\left(\delta_i^{\gamma-1}\right)$, each s, is bounded, $\lim_{\widehat{\delta}\to 0} f\left(\widehat{\delta},\xi=0\right)=0$. Given that $\lim_{\widehat{\delta}\to 1}\frac{1-\widehat{\delta}^{\gamma}}{1-\widehat{\delta}}=\frac{\partial\left(\widehat{\delta}^{\gamma}\right)}{\partial\widehat{\delta}}|_{\widehat{\delta}=1}=\gamma$,

$$\lim_{\widehat{\delta}\to 1} f\left(\widehat{\delta},\xi=0\right)=\left(\gamma A^{e\frac{1+\Gamma}{\alpha\Gamma}}-A^{ne\frac{1+\Gamma}{\alpha\Gamma}}\right)(1-\alpha+\Gamma)\equiv\bar{C}>0.$$

Hence, by the intermediate value theorem, for each c_I^e such that $ac_I^e\in\left(0,\bar{C}\right)$, there is an interior equilibrium, with threshold δ^F given by the solution to $F(\delta^F,\xi=0,c_I^e)=0$.

Evidently, $\frac{\partial F(.)}{\partial\Delta c_I^e}=-a<0$, and, by direct computation,

$$\frac{\partial f(.)}{\partial\tau^s}|_{\xi=0}=\delta^{F\gamma-1}\left(A^s E_{\Omega_I^s(\delta^F)}\left(\delta_i^{\gamma-1}\right)^{(1-\alpha)}\right)^{\frac{1+\Gamma}{\alpha\Gamma}}\frac{(1+\Gamma)(1-\alpha)}{\Gamma}(-1)^{\varphi(s)}>0,$$

with $\varphi(e)=2$ and $\varphi(ne)=1$, so that $\frac{\partial f(.)}{\partial\tau^e}|_{\xi=0}>0$ and $\frac{\partial f(.)}{\partial\tau^{ne}}|_{\xi=0}<0$. Unfortunately, the sign of $\frac{\partial f(.)}{\partial\widehat{\delta}}|_{\xi=0}$ depends upon the specific parameters of the economy. As established in Example A1 below, there are economies where $\frac{\partial f(.)}{\partial\widehat{\delta}}|_{\xi=0,\widehat{\delta}=\overline{\delta}}$ is positive at some $\overline{\delta}$, negative at some other $\underline{\delta}$. By choosing appropriately the parameter c_I^e, we can construct economies with $\delta^F=\overline{\delta}$, for each $\overline{\delta}\in(0,1)$. This shows that there are economies with $\frac{\partial f(.)}{\partial\widehat{\delta}}|_{\xi=0,\widehat{\delta}=\delta^F}>0$, and others with $\frac{\partial f(.)}{\partial\widehat{\delta}}|_{\xi=0,\widehat{\delta}=\delta^F}<0$.

By direct computation, at $\xi=0$,

$$\begin{aligned}\frac{\partial f(.)}{\partial\widehat{\delta}} &= (\gamma-1)\frac{1}{\widehat{\delta}}f\left(\widehat{\delta}\right)+(1-\alpha+\Gamma)\frac{(1-\alpha)(1+\Gamma)}{\alpha\Gamma}\frac{\widehat{\delta}^{\gamma-1}}{\widehat{\delta}}\\ &\times[A^{e\frac{1+\Gamma}{\alpha\Gamma}}E_{\Omega_I^e(\widehat{\delta})}\left(\delta_i^{\gamma-1}\right)^{\frac{(1-\alpha)(1+\Gamma)}{\alpha\Gamma}}\eta_\alpha^e\left(\widehat{\delta}\right)\\ &-A^{ne\frac{1+\Gamma}{\alpha\Gamma}}E_{\Omega_I^{ne}(\widehat{\delta})}\left(\delta_i^{\gamma-1}\right)^{\frac{(1-\alpha)(1+\Gamma)}{\alpha\Gamma}}\eta_\alpha^{ne}\left(\widehat{\delta}\right)],\end{aligned}$$

where $\eta_\alpha^s\left(\widehat{\delta}\right)$ is the elasticity of $E_{\Omega_I^s(\widehat{\delta})}\left(\delta_i^{\gamma-1}\right)$ with respect to $\widehat{\delta}$. By direct computation, $\eta_\alpha^{ne}\left(\widehat{\delta}\right)=(\gamma-1)$, while $\eta_\alpha^e\left(\widehat{\delta}\right)=\frac{-\gamma\widehat{\delta}^{\gamma}\left(1-\widehat{\delta}\right)+\widehat{\delta}\left(1-\widehat{\delta}^{\gamma}\right)}{\left(1-\widehat{\delta}\right)\left(1-\widehat{\delta}^{\gamma}\right)}$. With a straightforward manipulation, we obtain that

$$\begin{aligned}&\frac{\Gamma\widehat{\delta}^{\frac{\Gamma-1}{\Gamma}}}{1-\alpha+\Gamma}\left[\frac{\gamma^{(1-\alpha)}}{A^{ne}}\right]^{\frac{1+\Gamma}{\alpha\Gamma}}\frac{\partial f(.)}{\partial\widehat{\delta}}\\ =&\left(\frac{A^e}{A^{ne}}\left(\frac{1-\widehat{\delta}^{\gamma}}{\left(1-\widehat{\delta}\right)\widehat{\delta}^{\gamma-1}}\right)^{(1-\alpha)}\right)^{\frac{1+\Gamma}{\alpha\Gamma}}\left(\frac{(1-\alpha)(1+\Gamma)}{\alpha}\eta_\alpha^e\left(\widehat{\delta}\right)+\Gamma(\gamma-1)\right)-1,\end{aligned}$$

If $\eta_\alpha^e\left(\widehat{\delta}\right)\geq 0$ at each $\widehat{\delta}\in(0,1)$, and $\frac{1-\widehat{\delta}^{\gamma}}{(1-\widehat{\delta})\widehat{\delta}^{\gamma-1}}$ is bounded away from zero, the right hand side of the eq. above is always positive, for $\frac{A^e}{A^{ne}}$ sufficiently large. Therefore, for A^e large enough, $\frac{\partial f(.)}{\partial\widehat{\delta}}>0$ at each $\widehat{\delta}$ and, in particular, at each equilibrium threshold. Evidently, if $\frac{\partial F(.)}{\partial\widehat{\delta}}\left(=\frac{\partial f(.)}{\partial\widehat{\delta}}\right)>0$ at each solution to $F(\widehat{\delta},\xi,c_I^e)=0$, the solution must be unique. Moreover, by the implicit function theorem, $\frac{\partial f(.)}{\partial \widehat{\delta}}|_{\widehat{\delta}=\delta^F} > 0$ at each equilibrium implies that $\delta^F(.)$ satisfies $\frac{\partial \delta^F(.)}{\partial \tau^e}|_{\xi=0} < 0$, $\frac{\partial \delta^F(.)}{\partial \tau^{ne}}|_{\xi=0} > 0$, $\frac{\partial \delta^{F(.)}}{\partial \Delta c_I^e}|_{\xi=0} > 0$, $\frac{\partial \delta^{F(.)}}{\partial A^e}|_{\xi=0} < 0$ and $\frac{\partial \delta^{F(.)}}{\partial A^{ne}}|_{\xi=0} > 0$, as claimed.

Hence, to conclude, we need two additional results (we omit the index "I" to simplify notation):

Fact 1. $\eta_\alpha^e(\delta) \geq 0$, at each $\delta \in (0,1)$.

By direct computation, $\eta_\alpha^e(0) = 0$ and $\eta_\alpha^e(1) = \frac{\gamma-1}{2} > 0$. Hence, either there is $\overline{\delta} \in (0,1)$ such that $\eta_\alpha^e\left(\overline{\delta}\right) = 0$ or $\eta_\alpha^e(\delta) > 0$ for each $\delta \in (0,1)$, as claimed. Consider the numerator of $\eta_\alpha^e(\delta)$, call it $g(\delta)$,

$$g(\delta) = -\gamma\delta^\gamma (1-\delta) + \delta (1-\delta^\gamma).$$

Given that the denominator is strictly positive for each $\delta \in (0,1)$, $\eta^e(\delta) \leq 0$ if and only if $g(\delta) \leq 0$. Clearly, $g(0) = g(1) = 0$. Given that

$$\frac{\partial g(.)}{\partial \delta} = \left(1 - \gamma^2\delta^{\gamma-1} + \left(\gamma^2 - 1\right)\delta^\gamma\right),$$

$\frac{\partial g(.)}{\partial \delta}|_{\delta=0} > 0$ and $\frac{\partial g(.)}{\partial \delta}|_{\delta=1} = 0$. Moreover,

$$\frac{\partial^2 g(.)}{\partial \delta^2}|_{\delta=1} = \gamma\left(\gamma^2-1\right)\delta^{\gamma-1} - \gamma^2(\gamma-1)\delta^{\gamma-2} = \gamma(\gamma-1) > 0,$$

so that $\delta = 1$ is a local minimum of $g(\delta)$. Hence, if there is a $\widetilde{\delta} \in (0,1)$ such that $g\left(\widetilde{\delta}\right) = 0$, there must also be a $\overline{\delta} \in (0,1)$ such that $g\left(\overline{\delta}\right) = 0$ and $\frac{\partial g(.)}{\partial \delta}|_{\delta=\overline{\delta}} > 0$. Given that, by assumption, $\overline{\delta} \in (0,1)$, $\overline{\delta} \neq 0$, and, therefore, $\frac{g(\overline{\delta})}{\overline{\delta}} = 0$, and $\left(\frac{\partial g(.)}{\partial \delta}|_{\delta=\overline{\delta}} - \frac{g(\overline{\delta})}{\overline{\delta}}\right) > 0$. However,

$$\begin{aligned} 0 &< \frac{\partial g(.)}{\partial \delta}|_{\delta=\overline{\delta}} - \frac{g\left(\overline{\delta}\right)}{\overline{\delta}} = -\gamma^2\overline{\delta}^{\gamma-1} + \left(\gamma^2-1\right)\overline{\delta}^\gamma + \gamma\overline{\delta}^{\gamma-1}\left(1-\overline{\delta}\right) + \overline{\delta}^\gamma \\ &= \left(\gamma - \gamma^2\right)\left(1-\overline{\delta}\right)\overline{\delta}^{\gamma-1} < 0, \end{aligned}$$

because $\gamma > 1$. A contradiction. Hence, $g(\delta) > 0$ and, therefore, $\eta_\alpha^e(\delta) > 0$, at each $\delta \in (0,1)$.

Fact 2. Let $G(\delta) \equiv \left(\frac{1-\delta^\gamma}{1-\delta}\frac{\delta}{\delta^\gamma}\right)$. Then, $G(\delta) > \gamma > 1$, for each $\delta \in (0,1)$.

The result is quite obvious from the geometrical viewpoint. Alternatively, observe that $\lim_{\delta\to 0} G(\delta) = +\infty$ and $\lim_{\delta\to 1} G(\delta) = \gamma$. Hence, to establish the Fact, it suffices to show that $\frac{\partial G(\delta)}{\partial \delta} < 0$ at each $\delta \in (0,1)$. By direct computation,

$$\frac{\partial G(\delta)}{\partial \delta}|_{\delta=\widehat{\delta}} = \frac{\gamma}{\left(1-\widehat{\delta}\right)\widehat{\delta}^\gamma}\left(\frac{1}{\gamma}\frac{1-\widehat{\delta}^\gamma}{1-\widehat{\delta}} - 1\right) = \frac{\gamma}{\left(1-\widehat{\delta}\right)\widehat{\delta}^\gamma}\left(E_{\Omega_I^e(\widehat{\delta})}\left(\delta^{\gamma-1}\right) - 1\right) < 0.$$

EXAMPLE A1. We show that there are economies such that $\frac{\partial f(.)}{\partial \widehat{\delta}}|_{\widehat{\delta}=\delta^F} < 0$. Fix $\xi = 0$. Let $\alpha = \frac{1}{2}$, $\Gamma = 10$, $A^{ne} = 1$, and $A^e = 11/10$. By direct computation,

$$f\left(\widehat{\delta}\right) = 10.5\left(\frac{105}{110}\right)^{\frac{11}{10}}\left[\widehat{\delta}^{\frac{1}{21}}\left(\frac{11}{10}\left(\frac{1-\widehat{\delta}}{1-\widehat{\delta}}\right)^{\frac{1}{2}}\right)^{\frac{11}{5}} - \widehat{\delta}^{\frac{1}{10}}\right].$$

$\frac{\partial f(\widehat{\delta})}{\partial \widehat{\delta}}$ is strictly positive for $\widehat{\delta}$ sufficiently small, and negative for all $\widehat{\delta}$ larger than some critical value $\overline{\delta}$. For instance, one can check that $\frac{\partial f(\widehat{\delta})}{\partial \widehat{\delta}}|_{\widehat{\delta}=\frac{1}{2}} < 0$, while $\left\{\frac{\partial f(\widehat{\delta})}{\partial \widehat{\delta}}|_{\widehat{\delta}}\right\}_{v=0}^{\infty}$ with $\widehat{\delta}^v \to 0$ is unbounded above. Clearly, choosing appropriately c_I^e, we can construct an economy with $\delta^F = \frac{1}{2}$, i.e., such that $\frac{\partial f(\widehat{\delta})}{\partial \widehat{\delta}}|_{\widehat{\delta}=\delta^F} < 0$.

Proof of Proposition 2. Set $\zeta^s = 0$. Let $B \equiv \left[\frac{(1-\alpha)(1-\beta)}{\mu}\right]^{\frac{1-\alpha}{\alpha\Gamma}} (\alpha\beta)^{\frac{1}{\Gamma}}$ and $\gamma = \frac{1+\Gamma}{1-\alpha+\Gamma}$. Using $(A3)$ and $(A4)$, the equilibrium values of the relevant variables are

$$E_{\Omega_I^s(\delta^F)}\left(H^{Fs}\left(\delta_i, \delta^F(\phi), \phi\right)\right) = (1+\tau^s)^{\frac{1}{\Gamma}} B E_{\Omega_I^s(\delta^F)}\left(\delta_i^{\gamma-1}\right)^{\frac{1-\alpha}{\alpha\Gamma}} E_{\Omega_I^s(\delta^F)}\left(\delta_i^{\frac{1}{1-\alpha+\Gamma}}\right) A^{s\frac{1}{\alpha\Gamma}},$$

$$E_{\Omega_I^s(\delta^F)}\left(w^s(\delta_i, \delta^F(\phi), \phi)\right) = \beta B^{(1+\Gamma)} E_{\Omega_I^s(\delta^F)}\left(\delta_i^{\gamma-1}\right)^{\frac{1}{(\gamma-1)\Gamma}} (1+\tau^s)^{\frac{1}{\Gamma}} A^{s\frac{1+\Gamma}{\alpha\Gamma}},$$

$$\begin{aligned}\sigma_{w^s}\left(\delta_i, \delta^F(\phi), \phi\right) &= \left(\beta B^{(1+\Gamma)} E_{\Omega_I^s(\delta^F)}\left(\delta_i^{\gamma-1}\right)^{\frac{(1+\Gamma)(1-\alpha)}{\alpha\Gamma}} (1+\tau^s)^{\frac{1}{\Gamma}} A^{s\frac{1+\Gamma}{\alpha\Gamma}}\right) \\ &\quad \times\sqrt{E_{\Omega_I^s(\delta^F)}\left(\delta_i^{2\gamma-2}\right) - E_{\Omega_I^s(\delta^F)}\left(\delta_i^{\gamma-1}\right)^2},\end{aligned}$$

and

$$WP\left(\delta_i, \delta^F(\phi), \phi\right) = \left(\frac{1+\tau^e}{1+\tau^{ne}}\right)^{\frac{1}{\Gamma}} \frac{A^{e\frac{1+\Gamma}{\alpha\Gamma}}}{A^{ne\frac{1+\Gamma}{\alpha\Gamma}}}\left(\frac{1-\delta^{F\gamma}}{\left(1-\delta^F\right)\delta^{F\gamma-1}}\right)^{\frac{(1+\Gamma)(1-\alpha)}{\alpha\Gamma}}.$$

Notice that the wage premium is $i-$invariant, as claimed in the text. Bear in mind that, by assumption, $\frac{\partial f(.)}{\partial \widehat{\delta}}|_{\widehat{\delta}=\delta^F} > 0$, so that $\frac{\partial \delta^F(.)}{\partial \tau^e} < 0$, $\frac{\partial \delta^F(.)}{\partial \tau^{ne}} > 0$, $\frac{\partial \delta^F(.)}{\partial \Delta c_I^e} > 0$, $\frac{\partial \delta^F(.)}{\partial A^{ne}} > 0$, and $\frac{\partial \delta^F(.)}{\partial A^e} < 0$.

Let $\eta_1^s\left(\delta^F\right)$ be the elasticity with respect to δ^F of $E_{\Omega_I^s(\delta^F)}\left(\delta_i^{\frac{1}{1-\alpha+\Gamma}}\right)$, and $\eta_{2\alpha}^s\left(\delta^F\right)$ the one of $E_{\Omega_I^s(\delta^F)}\left(\delta_i^{2\gamma-2}\right)$. By direct computation, $\eta_{2\alpha}^{ne}\left(\delta^F\right) = 2(\gamma-1)$, while $\eta_1^{ne}\left(\delta^F\right) = \frac{1}{1-\alpha+\Gamma}$. By Fact 1 above, $\eta_\alpha^e\left(\delta^F\right) > 0$. A similar argument establishes that $\eta_1^e\left(\delta^F\right) > 0$ and $\eta_{2\alpha}^s\left(\delta^F\right) > 0$.

In the sequel we use the generic expression $\frac{\partial E_{\Omega_I^s(\delta^F)}\left(H^{Fs}\left(\delta_i,\delta^F,\phi\right)\right)}{\partial \phi'}$ to denote the derivative of $E_{\Omega_I^s(\delta^F)}\left(H^{Fs}(.)\right)$ with respect to one of the parameters of the vector ϕ, keeping δ^F fixed. Similarly, for the other functions and for $\frac{\partial \delta^F(\phi)}{\partial \phi'}$.

Consider first average human capital. At $\xi = 0$, by direct computation,

$$\begin{aligned}\frac{\partial E_{\Omega_I^s(\delta^F)}\left(H^{Fs}(.)\right)}{\partial \phi'} &= \frac{\partial E_{\Omega_I^s(\delta^F)}\left(H^{Fs}(.)\right)}{\partial \phi'} \\ &\quad + \frac{E_{\Omega_I^s(\delta^F)}\left(H^{Fs}(.)\right)}{\delta^F(.)}\left(\frac{1-\alpha}{\alpha\Gamma}\eta_\alpha^s\left(\delta^F\right)+\eta_1^s\left(\delta^F\right)\right)\frac{\partial \delta^F(\phi)}{\partial \phi'}.\end{aligned}$$

For wages,

$$\frac{\partial E_{\Omega_I^s(\delta^F)}\left(w^s(.)\right)}{\partial \phi'} = \frac{\partial E_{\Omega_I^s(\delta^F)}\left(w^s(.)\right)}{\partial \phi'} + \frac{E_{\Omega_I^s(\delta^F)}\left(w^s(.)\right)}{(\gamma-1)\Gamma}\eta_\alpha^s\left(\delta^F\right)\frac{\partial \delta^F(\phi)}{\partial \phi'}.$$

For the wage premium,

$$\frac{\partial WP(.)}{\partial \phi'} = \frac{\partial WP(.)}{\partial \phi'} + \frac{\partial WP(.)}{\partial \delta^F}\frac{\partial \delta^F(\phi)}{\partial \phi'},$$

where, by Fact 2 above,

$$\frac{\partial WP(.)}{\partial \delta^F} = \frac{WP(.)\,\gamma\left[E_{\Omega_I^e(\delta^F)}\left(\delta_i^{\gamma-1}\right)-1\right]}{\left(1-\delta^F\right)\delta^{F\gamma}} < 0,$$

for each $\delta \in (0,1)$. Finally, consider the standard deviations of wages in sector *ne*. By direct computation,

$$\begin{aligned}\frac{\partial \sigma^{ne}(.)}{\partial \phi'} &= \frac{\partial \sigma^{ne}(\delta^F,\phi)}{\partial \phi'} + \frac{\partial \delta^F(.)}{\partial \phi'}\frac{\frac{1}{2}\frac{\sigma^{ne}(.)}{\delta^F(\phi)}}{\sqrt{E_{\Omega_I^{ne}(\delta^F)}\left(\delta_i^{2\gamma-2}\right)-E_{\Omega_I^{ne}(\delta^F)}\left(\delta_i^{\gamma-1}\right)^2}} \\ &\quad [\left(2\frac{(1+\Gamma)(1-\alpha)}{\alpha\Gamma}\eta_\alpha^{ne}\left(\delta^F\right)+\eta_{2\alpha}^{ne}\left(\delta^F\right)\right)E_{\Omega_I^{ne}(\delta^F)}\left(\delta_i^{2\gamma-2}\right) \\ &\quad -2E_{\Omega_I^{ne}(\delta^F)}\left(\delta_i^{\gamma-1}\right)^2\left(\frac{(1+\Gamma)(1-\alpha)}{\alpha\Gamma}+1\right)\eta_\alpha^{ne}\left(\delta^F\right)]\end{aligned}$$

Taking into account the values of $\eta_{2\alpha}^{ne}\left(\delta^F\right)$ and $\eta_\alpha^{ne}\left(\delta^F\right)$, the term in square brackets is equal to

$$\frac{2}{\Gamma}\left[E_{\Omega_I^{ne}(\delta^F)}\left(\delta_i^{2\gamma-2}\right)-E_{\Omega_I^{ne}(\delta^F)}\left(\delta_i^{\gamma-1}\right)^2\right] > 0.$$

The Proposition follows immediately.

A2: Inefficiency properties of the economy with frictions

The optimal choice k_j^s is clearly $j-$invariant and, by assumption, $\nu\left(\Omega_I^s\left(\widehat{\delta}\right)\right) = \nu\left(\Omega_J^s\left(\widehat{\delta}\right)\right)$. Hence, the planner' s objective function can be rewritten as

$$\begin{aligned} P\left(h_i^s, k^s, \widehat{\delta}\right) \equiv & \sum_s \int_{\Omega_I^s(\widehat{\delta})} \left(\beta A^s h_i^{s\alpha} k^{s(1-\alpha)} - \frac{1}{\delta_i}\frac{h_i^{s(1+\Gamma)}}{1+\Gamma}\right) di - c_I^s \nu\left(\Omega_I^s\left(\widehat{\delta}\right)\right) \\ & + \sum_s \left((1-\beta) A^s \frac{\int_{\Omega_I^s(\widehat{\delta})} h_i^{s\alpha} di}{\nu\left(\Omega_I^s\left(\widehat{\delta}\right)\right)} k^{s(1-\alpha)} - \mu k^s \right) \nu\left(\Omega_J^s\left(\widehat{\delta}\right)\right) \\ = & \sum_s \int_{\Omega_I^s(\widehat{\delta})} \left(A^s h_i^{s\alpha} k^{s(1-\alpha)} - \frac{1}{\delta_i}\frac{h_i^{s(1+\Gamma)}}{1+\Gamma}\right) di \\ & - \left(c_I^s + \mu k^s\right) \nu\left(\Omega_I^s\left(\widehat{\delta}\right)\right). \end{aligned}$$

Its optimization problem is

$$\max_{\left(h_i^s, k^s, \widehat{\delta}\right)} P\left(h_i^s, k^s, \widehat{\delta}\right). \tag{P}$$

It is convenient to decompose (P) into three problems. First, given an arbitrary value $\widehat{\delta}$, we determine the maps $\left(H^{COs}\left(\delta_i, \widehat{\delta}\right), K^{COs}\left(\widehat{\delta}\right)\right)$ solving, for each s, the optimization problem

$$\begin{aligned} \max_{\left(h_i^s, k^s\right)} P_{\widehat{\delta}}^s\left(h_i^s, k^s\right) \equiv & \int_{\Omega_I^s(\widehat{\delta})} \left[A^s h_i^{s\alpha} k^{s(1-\alpha)} - \frac{1}{\delta_i}\frac{h_i^{s(1+\Gamma)}}{1+\Gamma}\right] di \\ & - \left(c_I^s + \mu k^s\right) \nu\left(\Omega_I^s\left(\widehat{\delta}\right)\right). \end{aligned} \tag{$P_{\delta^*}^s$}$$

Next, given the value functions $P^s(\widehat{\delta})$ of the two problems $\left(P_{\widehat{\delta}}^s\right)$, $s = e, ne$, we recast problem (P) as

$$\max_{\widehat{\delta}} \overline{P}(\widehat{\delta}) \equiv P^e(\widehat{\delta}) + P^{ne}(\widehat{\delta}), \tag{$\overline{P}$}$$

finding the optimal value of $\widehat{\delta}$, δ^{CO}.

Proof of Proposition 3. Given that optimization problem $\left(P_{\widehat{\delta}}^s\right)$ is concave, each s, its solution is completely characterized by the FOCs:

i. $\frac{\partial P_{\widehat{\delta}}^s(h_i^s, k^s)}{\partial h_i} = \alpha A^s k^{s(1-\alpha)} h_i^{s(\alpha-1)} - \frac{1}{\delta_i} h_i^{s\Gamma} = 0,$

ii. $\frac{\partial P_{\widehat{\delta}}^s(h_i^s, k^s)}{\partial k} = (1-\alpha) A^s k^{s(-\alpha)} \int_{\Omega_I^s(\widehat{\delta})} h_i^{s\alpha} di - \mu \int_{\Omega_I^s(\widehat{\delta})} di = 0,$

which imply

a. $K^{COs}\left(\widehat{\delta}\right) = A^{s\frac{1+\Gamma}{\Gamma\alpha}} \alpha^{\frac{1}{\Gamma}} \left(\frac{1-\alpha}{\mu} E_{\Omega_I^s}\left(\delta_i^{\frac{\alpha}{1-\alpha+\Gamma}}\right)\right)^{\frac{1-\alpha+\Gamma}{\alpha\Gamma}},$

b. $H^{COs}\left(\delta_i, \widehat{\delta}\right) = \delta_i^{\frac{1}{1-\alpha+\Gamma}} \alpha^{\frac{1}{\Gamma}} A^{s\frac{1}{\alpha\Gamma}} \left(\frac{1-\alpha}{\mu} E_{\Omega_I^s}\left(\delta_i^{\frac{\alpha}{1-\alpha+\Gamma}}\right)\right)^{\frac{1-\alpha}{\alpha\Gamma}}.$

Comparing a–b to $(A3)$–$(A4)$, $K^{COs}\left(\widehat{\delta}\right) > K^{Fs}(\widehat{\delta})$ and $H^{COs}\left(\delta_i, \widehat{\delta}\right) > H^{Fs}\left(\delta_i, \widehat{\delta}\right)$, for each $\widehat{\delta}$, δ_i and s. Therefore, equilibria are always characterized by underinvestment in physical capital and in the effort in education.
Demand and supply functions are clearly well-defined and continuous at each $\widehat{\delta} \in (0,1)$ and $P^s(\widehat{\delta})$ has the same properties. Hence, problem $\left(\overline{P}\right)$ has a solution, either internal or at one of the boundary points, and, therefore, CO allocations exist.

Compare a market allocation and any CO allocation. If they have the same threshold value $\widehat{\delta}$, $K^{COs}\left(\widehat{\delta}\right) \neq K^{Fs}(\widehat{\delta})$ and the market allocation is not CO. Otherwise, the threshold values are different, and constrained inefficiency follows immediately.

EXAMPLE A2. Using $\gamma = \frac{1+\Gamma}{1-\alpha+\Gamma}$, $\frac{1}{b} \equiv \alpha^{\frac{1}{\Gamma}} \left(\frac{1-\alpha}{\mu}\right)^{\frac{(1-\alpha)(1+\Gamma)}{\alpha\Gamma}}$, and $K^{COs}\left(\widehat{\delta}\right)$ $H^{COs}\left(\delta_i, \widehat{\delta}\right)$, we can rewrite the objective function of problem $\left(\overline{P}\right)$ as

$$\overline{P}(\widehat{\delta}) = \sum_s \frac{\alpha\Gamma}{1+\Gamma} v\left(\Omega_I^s(\widehat{\delta})\right) A^{s\frac{1+\Gamma}{\Gamma\alpha}} E_{\Omega_I^s(\widehat{\delta})}\left(\delta_i^{\gamma-1}\right)^{\frac{1-\alpha+\Gamma}{\alpha\Gamma}} - v\left(\Omega_I^e(\widehat{\delta})\right) bc_I^e,$$

and, from the (necessary) FOC,

$$\begin{aligned} -\gamma\frac{\partial \overline{P}(\widehat{\delta})}{\partial\widehat{\delta}} &= \widehat{\delta} A^{e\frac{1+\Gamma}{\Gamma\alpha}} E_{\Omega_I^e(\widehat{\delta})}\left(\delta_i^{\gamma-1}\right)^{\frac{(1+\Gamma)(1-\alpha)}{\alpha\Gamma}} \left(1-(1-\alpha)\frac{E_{\Omega_I^e(\widehat{\delta})}\left(\delta_i^{\gamma-1}\right)}{E_{\Omega_I^{ne}(\widehat{\delta})}\left(\delta_i^{\gamma-1}\right)}\right) \\ & -\alpha\widehat{\delta} A^{ne\frac{1+\Gamma}{\Gamma\alpha}} E_{\Omega_I^{ne}(\widehat{\delta})}\left(\delta_i^{\gamma-1}\right)^{\frac{1-\alpha+\Gamma}{\alpha\Gamma}} - \gamma bc_I^e = 0. \end{aligned}$$

Hence, δ^{CO} is either the solution to $\gamma\frac{\partial\overline{P}(\widehat{\delta})}{\partial\widehat{\delta}} = 0$, or $\delta^{CO} \in \{0,1\}$, while δ^F is the solution to

$$\begin{aligned} &\left(\delta^{F\gamma-1} A^{e\frac{1+\Gamma}{\alpha\Gamma}} E_{\Omega_I^e(\delta^F)}\left(\delta_i^{\gamma-1}\right)^{\frac{(1-\alpha)(1+\Gamma)}{\alpha\Gamma}} - \delta^{F\gamma-1} A^{ne\frac{1+\Gamma}{\alpha\Gamma}} E_{\Omega_I^{ne}(\delta^F)}\left(\delta_i^{\gamma-1}\right)^{\frac{(1-\alpha)(1+\Gamma)}{\alpha\Gamma}}\right) \\ &\times \left(\beta(1-\beta)^{\frac{1-\alpha}{\alpha}}\right)^{\frac{1+\Gamma}{\Gamma}} - \gamma bc_I^e = 0. \end{aligned}$$

Let $\Gamma = \mu = A^{ne} = 1$, while $\alpha = \beta = \frac{1}{2}$, and $A^e = 2$. Let

$$M^F(\widehat{\delta}) \equiv 2^4 \left(\frac{3}{4}\frac{1-\widehat{\delta}}{1-\widehat{\delta}}\right)^2 \left(\widehat{\delta} - \frac{1}{2}\frac{1-\widehat{\delta}}{1-\widehat{\delta}}\right) - \frac{1}{2}\widehat{\delta}\left(\frac{3}{4}\widehat{\delta}\right)^2$$

and

$$M^{CO}(\widehat{\delta}) \equiv \frac{1}{16}\left(\widehat{\delta}2^4\left(\frac{3}{4}\frac{1-\widehat{\delta}}{1-\widehat{\delta}}\right)^2 - \widehat{\delta}\left(\frac{3}{4}\widehat{\delta}\right)^2\right).$$

Then, the equilibrium conditions above can be rewritten as $M^{CO}(\widehat{\delta}) - \frac{32}{3}c_I^e = 0$, for δ^{CO}, and $M^F(\widehat{\delta}) - \frac{32}{3}c_I^e = 0$, for δ^F.

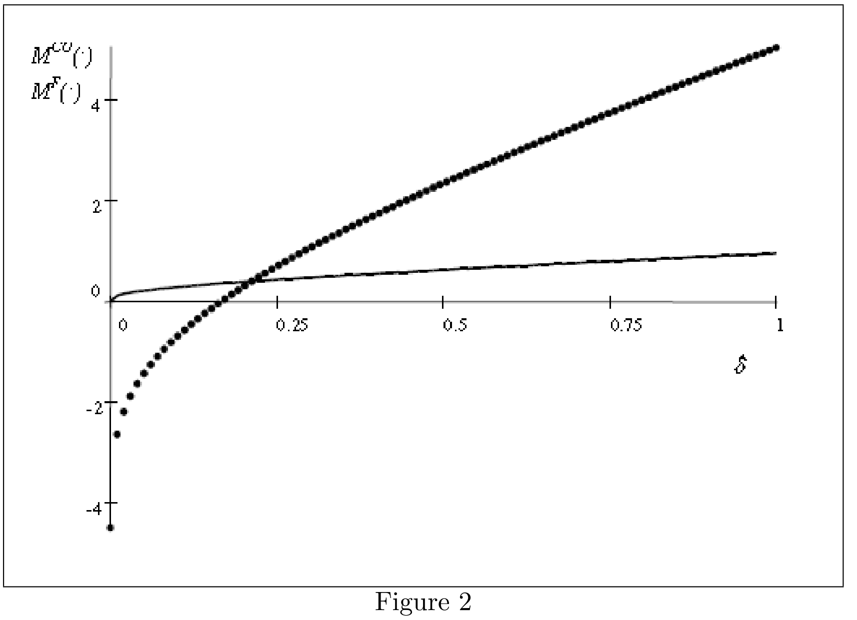

Figure 2

Figure 2 reports the graphs of $M^{CO}(\widehat{\delta})$ (the dotted line) and $M^F(\widehat{\delta})$. Notice that, in the relevant range, $M^{CO}(\widehat{\delta})$ is concave, so that $M^{CO}(\widehat{\delta}) - \frac{32}{3} c_I^e = 0$ is also a sufficient condition for the optimal solution δ^{CO}. For $\widehat{\delta}$ sufficiently small, $M^F(\widehat{\delta}) > M^{CO}(\widehat{\delta})$, while for $\widehat{\delta}$ sufficiently large $M^{CO}(\widehat{\delta}) > M^F(\widehat{\delta})$. Hence, for c_I^e sufficiently small $\delta^F < \delta^{CO}$, while the opposite occurs for c_I^e sufficiently large.

Proof of Corollary 1. Obviously, there are many different tax-subsidy schemes implementing the CO allocation. We will focus the analysis on linear subsidies on labor income and investments in physical capital, and on fixed fees and lump-sum taxes. Fix $\overline{\zeta}^e = \overline{\zeta}^{ne} = \beta$ and $\overline{\tau}^e = \overline{\tau}^{ne} = \frac{1-\beta}{\beta}$. It is easy to check that, given any threshold value $\widehat{\delta}$, the FOCs of the individual optimization problem in the actual economy imply that the FOCs of the (constrained) planner's optimization problem are satisfied. Let $\delta^F(\xi)$ be the market threshold value associated with ξ.

By direct computation, at the CO allocation, expected profits are zero in both sectors. Hence, firms are indifferent among sectors. Therefore, at each optimal solution $\widehat{\delta} \in (0,1)$, the FOCs of optimization problem $(\overline{P})$ are simply given by

$$-\left[U_i^e(H^{COe}\left(\delta_i = \widehat{\delta}, \widehat{\delta}\right), K^{COe}\left(\widehat{\delta}\right)) - U_i^{ne}(H^{COne}\left(\delta_i = \widehat{\delta}, \widehat{\delta}\right), K^{COne}\left(\widehat{\delta}\right)) - c_I^e\right]$$
$$+\sum_s \int_{\Omega_I^s(\widehat{\delta})} \frac{\partial U_i^s(H^{COs}\left(\delta_i = \widehat{\delta}, \widehat{\delta}\right), K^{COs}\left(\widehat{\delta}\right))}{\partial \widehat{\delta}} = 0.$$

Set

$$\overline{\Delta c}_I^e = \sum_s \int_{\Omega_I^s(\widehat{\delta})} \frac{\partial U_i^s(H^{COs}\left(\delta_i = \widehat{\delta}, \widehat{\delta}\right), K^{COs}\left(\widehat{\delta}\right))}{\partial \widehat{\delta}}.$$

Then, given education fees equal to $\left(c_I^e + \overline{\Delta c}_I^e\right)$, $\delta^F(\xi) = \delta^{CO}$ and the equilibrium level of total surplus coincides with its CO level. Finally, redistribute the total net revenues (or costs) of the fee-subsidy scheme across workers using i-invariant lump-sum taxes, so to balance the budget.

Proof of Proposition 4. Using the properties of the two sets $\Omega_I^s\left(\delta^F\right)$ and $\Omega_J^s\left(\delta^F\right)$, we can rewrite $S(\xi)$ as

$$S(\delta^F(\xi),\xi) \equiv \sum_s \left(\int_{\Omega_I^s\left(\delta^F(\xi)\right)} V^{Fs}(\delta_i,\delta^F,\xi)di + \int_{\Omega_J^s\left(\delta^F(\xi)\right)} E_{\Omega_I^s\left(\delta^F(\xi)\right)}\left(\Pi^{Fs}\left(\delta^F,\xi\right)\right)dj \right).$$

Remember that the net sum of taxes and subsidies is zero. Therefore,

$$\begin{aligned}
\frac{\partial S(.)}{\partial \tau^s} &= \left(\frac{\partial S(.)}{\partial \delta^F}\right)\frac{\partial \delta^F(\xi)}{\partial \tau^s} + \sum_s \int_{\Omega_I^s\left(\delta^F(.)\right)} \frac{\partial V^{Fs}(\delta_i,\delta^F,\xi)}{\partial \tau^s}di \\
&\quad + \sum_s \int_{\Omega_J^s\left(\delta^F(\xi)\right)} \frac{\partial E_{\Omega_I^s\left(\delta^F(\xi)\right)}\left(\Pi^{Fs}\left(\delta^F,\xi\right)\right)}{\partial \tau^s}dj \\
\frac{\partial S(.)}{\partial \Delta c_I^e} &= \frac{\partial S(.)}{\partial \delta^F}\frac{\partial \delta^F(.)}{\partial \Delta c_I^e}.
\end{aligned}$$

From eqs $(A5)$ and $(A7)$, the last two terms of $\frac{\partial S(.)}{\partial \tau^s}$ are positive. By direct computation,

$$\begin{aligned}
\frac{\partial S(.)}{\partial \delta^F} &= -\left[V_i^{Fe}\left(\delta_i=\delta^F,\delta^F,\xi\right) - V_i^{Fne}\left(\delta_i=\delta^F,\delta^F,\xi\right)\right] \\
&\quad -\left[\Pi_j^{Fe}\left(\delta_i=\delta^F,\delta^F,\xi\right) - \Pi_j^{Fne}\left(\delta_i=\delta^F,\delta^F,\xi\right)\right] \\
&\quad + \sum_s \int_{\Omega_I^s\left(\delta^F\right)} \frac{\partial V_i^{Fs}(.)}{\partial \delta^F}di + \sum_s \int_{\Omega_J^s\left(\delta^F\right)} \frac{\partial E_{\Omega_I^s\left(\delta^F\right)}\left(\Pi_j^{Fs}(.)\right)}{\partial \delta^F}dj.
\end{aligned}$$

By definition of δ^F, the first term in square brackets is zero. We have already established (see eqs. (7)) that the last four terms are positive. $\Pi_j^{Fne}\left(\delta_i=\delta^F,\delta^F,\xi\right)$ is also positive, because

$$\Pi_j^{Fne}\left(\delta_i=\delta^F,\delta^F,\xi\right) \geq E_{\Omega_I^{ne}\left(\delta^F\right)}\left(\Pi_j^{Fne}(.)\right) \geq 0.$$

Hence, a *sufficient* condition for $\frac{\partial S(.)}{\partial \delta^F} > 0$ is

$$\begin{aligned}
0 \leq \Delta S^e &\equiv \int_{\Omega_I^e\left(\delta^F\right)} \frac{\partial V_i^{Fe}(.)}{\partial \delta^F}di + \int_{\Omega_J^e\left(\delta^F\right)} \frac{\partial E_{\Omega_I^e\left(\delta^F\right)}\left(\Pi_j^{Fs}(.)\right)}{\partial \delta^F}dj \\
&\quad -\Pi_j^{Fe}\left(\delta_i=\delta^F,\delta^F,\xi\right).
\end{aligned}$$

Define

$$B^e = \left[\frac{1}{\gamma}A^{e\frac{1+\Gamma}{\alpha\Gamma}}(\alpha\beta)^{\frac{1}{\Gamma}}\left(\frac{(1-\alpha)(1-\beta)}{\mu}E_{\Omega_I^e\left(\delta^F\right)}\left(\delta^{\gamma-1}\right)\right)^{\frac{(1+\Gamma)(1-\alpha)}{\alpha\Gamma}}\right].$$

Using $\gamma \equiv \frac{1+\Gamma}{1-\alpha+\Gamma}$, and $(A6)$, the ex-post profits of the firm matched with worker i such that $\delta_i = \delta^F$ are

$$\Pi_j^{Fe}\left(\delta_i = \delta^F, \delta^F, \xi\right) = (1-\beta)\left(\gamma\delta_i^{F\gamma-1} - (1-\alpha)\left(\frac{1-\delta^{F\gamma}}{1-\delta^F}\right)\right) \times B^e.$$

Using $(A5)$ and $(A7)$,

$$\int_{\Omega_I^e(\delta^F)} \frac{\partial V_i^{Fe}(.)}{\partial \delta^F} di = B^e \frac{(1+\Gamma)(1-\alpha)}{\gamma\alpha\Gamma}\beta\left(\frac{1-\delta^{F\gamma}}{1-\delta^F} - \gamma\delta^{F\gamma-1}\right)$$

and

$$\int_{\Omega_J^e(\delta^F)} \frac{\partial E_{\Omega_j^e(\delta^F)}\left(\Pi_j^{Fe}(.)\right)}{\partial \delta^F} dj = \frac{1-\alpha}{\Gamma} B^e (1-\beta)\left(\frac{1-\delta^{F\gamma}}{1-\delta^F} - \gamma\delta^{F\gamma-1}\right).$$

Hence,

$$\begin{aligned}\frac{\Gamma\Delta S^e}{(1+\Gamma)\,\delta^{F\gamma-1}B^e} &= \left[\beta\frac{(1-\alpha)}{\gamma\alpha}\left(\frac{1-\delta^{F\gamma}}{\left(1-\delta^F\right)\delta^{F\gamma-1}} - \gamma\right)\right] \\ &\quad +(1-\beta)\left[(1-\alpha)\frac{1-\delta^{F\gamma}}{\left(1-\delta^F\right)\delta^{F\gamma-1}} - 1\right] > 0.\end{aligned}$$

The first term in square brackets is strictly positive (because, as shown in Fact 2 above, $\frac{1-\delta^{F\gamma}}{\left(1-\delta^F\right)\delta^{F\gamma-1}}$ is bounded below by γ). By assumption, the second term is positive. Notice that the inequality is always satisfied for β large enough.

When $\frac{\partial f(.)}{\partial \widehat{\delta}}|_{\widehat{\delta}=\delta^F} > 0$, $\frac{\partial \delta^F(.)}{\partial \Delta c_I^e} > 0$ and $\frac{\partial \delta^F(.)}{\partial \tau^{ne}} > 0$, so that $\frac{\partial S(\xi)}{\partial \tau^{ne}} > 0$ and $\frac{\partial S(\xi)}{\partial \Delta c_I^e} > 0$. It follows that a subsidy to labor income in sector ne, and/or an increase in the fixed cost of education c_I^e, increases the expected total surplus .

On the other hand, $\frac{\partial \delta^F(.)}{\partial \tau^e} < 0$ and, therefore, under the maintained assumptions, the sign of $\frac{\partial S(.)}{\partial \tau^e}$ is undefined.

7. APPENDIX 2: COMPETITIVE SPOT LABOR MARKET

There is a continuum of separated islands, denoted by $\ell \in (0,1)$. On each island there is an interval $(0,1)$ of identical workers and firms. Firms (denoted by a pair $(j,\ell) \in (0,1)\times(0,1)$) are identical across islands. Workers (denoted by a pair $(i,\ell) \in (0,1)\times(0,1)$) are identical within an island (i.e., with respect to the index ℓ), but heterogeneous across islands, because of the parameter δ_i, whose realization in a given island is private information of the workers[21]. First, firms and workers choose the type and amount of their investments. Next, investments are mutually observable, (island specific) labor markets open and clear at the competitive

[21] In the sequel we implicitly assume that the realization of $E_{\Omega_\ell^s(\widehat{\delta})}\left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}}\right)$ coincides with its theoretical value. Using appropriate assumptions on the random variables δ_i, this can be guaranteed.

wage. Given that, ex-ante, the realization δ_i is not observable, firms choose their investments taking into account the (conditional on $\widehat{\delta}$) distribution of the human capital of the workers. Preferences and production functions are as above. Given that firms are identical, they all have the same optimal level of investments in each sector.

Each worker chooses her behavior solving: given the equilibrium maps $(w_i^s(\delta_i), s_\ell(j))$ and the equilibrium threshold value $\overline{\delta}$,

$$choose \left\{ \overline{s}_{i\ell}, \overline{h}_{i\ell}^s \right\} \in \arg\max_s \left\{ \max_{\overline{h}_{i\ell}^s} E_{\Omega_\ell^s(\overline{\delta})} \left(U_{i\ell}^s \left(C_{i\ell}^s, h_{i\ell}^s \right) \right) \ s.to\ C_{i\ell}^s = w_i^s(\delta_i) \overline{h}_{i\ell}^s - c_I^s \right\}. \tag{U2}$$

where $\overline{s}_{i\ell} \in \{e, ne\}$ denotes her choice of the optimal sector.

Given the equilibrium maps $(w_i(\delta_i), s_\ell(i))$ and the equilibrium threshold value $\overline{\delta}$, each firm solves optimization problem

$$choose \left\{ \overline{s}_{j\ell}, \left(\overline{k}_{j\ell}^s, \overline{h}_{j\ell}^s \right) \right\} \in \arg\max_s \left\{ \max_{\left(\overline{k}_{j\ell}^s, \overline{h}_{j\ell}^s\right)} E_{\Omega_\ell^s(\overline{\delta})} \left(A^s h_{j\ell}^{s\alpha} k_{j\ell}^{s(1-\alpha)} - w_i^s(\delta_i) h_{j\ell}^{s\alpha} - \mu k_{j\ell}^s \right) \right\}, \tag{Π2}$$

with $\overline{s}_{j\ell} \in \{e, ne\}$.

Definition 2. A rational expectations equilibrium is a pair of maps $(w_i^{ne}(\delta_i), w_i^e(\delta_i))$, a threshold value $\overline{\delta}$, and maps $\{s_\ell(i), h_\ell^s(i)\}$ and $\{s_\ell(j), k_\ell^s(j), h_\ell^s(j)\}$ such that

i. $E_{\Omega^e(\overline{\delta})}\left(U_{i\ell}^e\left(\overline{C}_{i\ell}^e, \overline{h}_{i\ell}^e\right)\right) - E_{\Omega^{ne}(\overline{\delta})}\left(U_{i\ell}^{ne}\left(\overline{C}_{i\ell}^{ne}, \overline{h}_{i\ell}^{ne}\right)\right) \geq 0$ if and only if $\delta_i \geq \overline{\delta}$,

ii. for each (i, ℓ), $(s_\ell(i), h_\ell^s(i))$ solves $(U2)$,

iii. for each (j, ℓ), $\{s_\ell(j), (k_\ell^s(j), h_\ell^s(j))\}$ solves $(\Pi 2)$,

iv. for each ℓ, $\int_{(0,1)} h_\ell^s(i) di = \int_{(0,1)} h_\ell^s(j) dj$, for each j.

We start solving for the ex-post competitive equilibrium, contingent on the aggregate investments in physical capital. A straightforward computation shows that the equilibrium wage map is defined by

$$w^s\left(\delta_i, \overline{K}_j^s\right) = \frac{\left(\alpha A^s \overline{K}_j^{s(1-\alpha)}\right)^{\frac{\Gamma}{1-\alpha+\Gamma}}}{\delta_i^{\frac{1-\alpha}{1-\alpha+\Gamma}}}.$$

Moreover, given an arbitrary threshold value $\widehat{\delta}$, ex-ante, expected profits of a firm active in sector s are given by

$$E_{\Omega^s(\widehat{\delta})}(\Pi_{j\ell}) = (\alpha A^s)^{\frac{1}{1-\alpha}} \left(\frac{1-\alpha}{\alpha}\right) E_{\Omega^s(\widehat{\delta})} \left(\frac{1}{w^s\left(\delta_i, \overline{K}_j^s\right)^{\frac{1}{1-\alpha}}} \right) k_{j\ell} - \mu k_{j\ell}.$$

Hence, the zero expected profit condition imposes

$$(\alpha A^s)^{\frac{1}{1-\alpha}} \left(\frac{1-\alpha}{\mu\alpha}\right) E_{\Omega^s(\widehat{\delta})} \left(\frac{1}{w^s\left(\delta_i, \overline{K}_j^s\right)^{\frac{1}{1-\alpha}}} \right) = 1.$$

Replacing into this condition $w^s\left(\delta_i, \overline{K}_j^s\right)$, and taking into account that $\overline{K}_j^s$ is $j-$invariant, we get

$$\overline{K}_j^s = \left(\frac{1-\alpha}{\mu\alpha}\right)^{\frac{1-\alpha+\Gamma}{\alpha\Gamma}} (\alpha A^s)^{\frac{1+\Gamma}{\alpha\Gamma}} E_{\Omega^s(\widehat{\delta})}\left(\delta_i^{\frac{\alpha}{1-\alpha+\Gamma}}\right)^{\frac{1-\alpha+\Gamma}{\alpha\Gamma}}.$$

Replacing into $w^s\left(\delta_i, \overline{K}_j^s\right)$, we obtain the map

$$w^s\left(\delta_i, \widehat{\delta}\right) = \frac{(\alpha A^s)^{\frac{1}{\alpha}}\left(\frac{1-\alpha}{\mu\alpha}\right)^{\frac{1-\alpha}{\alpha}} E_{\Omega^s(\widehat{\delta})}\left(\delta_i^{\frac{\alpha}{1-\alpha+\Gamma}}\right)^{\frac{1-\alpha}{\alpha}}}{\delta_i^{\frac{1-\alpha}{1-\alpha+\Gamma}}}.$$

Finally, given $\widehat{\delta}$, the value of the indirect utility function (if s) is

$$\begin{aligned} V_i^s\left(\delta_i, \widehat{\delta}\right) &= \left(\frac{\Gamma}{1+\Gamma}\left(\frac{1-\alpha}{\mu\alpha}\right)^{\frac{(1-\alpha)(1+\Gamma)}{\alpha\Gamma}} \alpha^{\frac{1+\Gamma}{\alpha\Gamma}}\right)\delta_i^{\frac{\alpha}{1-\alpha+\Gamma}} \\ &\quad \times A^{s\frac{1+\Gamma}{\alpha\Gamma}} E_{\Omega^s(\widehat{\delta})}\left(\delta_i^{\frac{\alpha}{1-\alpha+\Gamma}}\right)^{\frac{(1-\alpha)(1+\Gamma)}{\alpha\Gamma}} - c_I^s. \end{aligned}$$

The map defining the equilibrium threshold is then

$$\begin{aligned} 0 &= \widehat{\delta}^{\frac{\alpha}{1-\alpha+\Gamma}}\left(A^e E_{\Omega_I^e(\widehat{\delta})}\left(\delta_i^{\frac{\alpha}{1-\alpha+\Gamma}}\right)^{(1-\alpha)}\right)^{\frac{(1+\Gamma)}{\alpha\Gamma}} \\ &\quad -\widehat{\delta}^{\frac{\alpha}{1-\alpha+\Gamma}}\left(A^{ne} E_{\Omega_I^{ne}(\widehat{\delta})}\left(\delta_i^{\frac{\alpha}{1-\alpha+\Gamma}}\right)^{(1-\alpha)}\right)^{\frac{(1+\Gamma)}{\alpha\Gamma}} - bc_I^e, \end{aligned} \tag{10}$$

with $b = \left(\frac{1+\Gamma}{\Gamma\left(\frac{1-\alpha}{\mu\alpha}\right)^{\frac{(1-\alpha)(1+\Gamma)}{\alpha\Gamma}}\alpha^{\frac{1+\Gamma}{\alpha\Gamma}}}\right)$.

Modulo a multiplicative term, this expression is identical to eq. (2) in the text. Evidently, the qualitative properties of equilibria are identical in the two classes of economies. There is, however, an important difference with respect to efficiency. According to the definition of CO introduced above, the supply of human and physical capital is CO, contingent on the threshold value, i.e., if $\delta^F = \delta^{CO}$ the market equilibrium would be CO. However, it is easy too see that it is always $\delta^F < \delta^{CO}$, i.e., there is always overinvestment in the education level. Indeed, given that expected profits are always zero, the planner objective function reduces to

$$P(\widehat{\delta}) = \int_0^{\widehat{\delta}} V_i^{ne}\left(\delta_i, \widehat{\delta}\right) d\delta_i + \int_{\widehat{\delta}}^1 V_i^e\left(\delta_i, \widehat{\delta}\right) d\delta_i,$$

with FOC

$$\begin{aligned} \frac{\partial P(\widehat{\delta})}{\partial \widehat{\delta}} &= -\left[V_i^e\left(\delta_i = \widehat{\delta}, \widehat{\delta}\right) - V_i^{ne}\left(\delta_i = \widehat{\delta}, \widehat{\delta}\right)\right] \\ &\quad + \int_0^{\widehat{\delta}} \frac{\partial V_i^{ne}\left(\delta_i, \widehat{\delta}\right)}{\partial \widehat{\delta}} d\delta_i + \int_{\widehat{\delta}}^1 \frac{\partial V_i^e\left(\delta_i, \widehat{\delta}\right)}{\partial \widehat{\delta}} d\delta_i. \end{aligned}$$

The last two terms are always strictly positive. Hence, $\frac{\partial P(\widehat{\delta})}{\partial \widehat{\delta}} = 0$ (a necessary condition for an interior optimum) requires

$$\left[V_i^e\left(\delta_i = \widehat{\delta}, \widehat{\delta}\right) - V_i^{ne}\left(\delta_i = \widehat{\delta}, \widehat{\delta}\right)\right] > 0.$$

Therefore, if $\frac{\partial f(.)}{\partial \widehat{\delta}} > 0$ (i.e., if $\frac{A^e}{A^{ne}}$ is large enough), we always have overinvestment in education level.

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