NAIRU, capital formation and monetary policy *

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Abstract.. The strategic interaction between central bank and trade unions is integrated into standard medium run general equilibrium models with imperfect competition in good and labour markets. The main results are: (i) not fully indexed social expenditure, plausible in the short and medium run, produce money nonneutrality, but the opposite holds in the long run, when this nominal rigidity is removed; (ii) Rogoff's (1985) conservative central banker may decrease inflation and unemployment; (iii) insufficient capital accumulation (Rowthorn, 1999a; 1999b) is not a generally plausible explanation of European unemployment, but it is subject to milder conditions than that based on a capital-using shift in technology (Blanchard, 1997); (iv) with balanced government budgets, consistent monetary and social policies may achieve a zero "Desired Inflation Rate of Unemployment" (DIRU); (v) in the present European context the minimisation of DIRU would require to enhance social policies.

Keywords: Trade unions, unemployment, capital accumulation, monetary policy, policy games.

JEL classification numbers: J51, E24, E31, E58, J58

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1. Introduction

In the last decade, the theoretical literature on unemployment has been dominated by two main approaches. One is represented by the debate on the (fixed or varying) NAIRU; the other one by the development of policy games between a central bank and one or several (monopoly) trade unions.

The former approach to unemployment has taken place in general equilibrium models with imperfect competition in good and labour markets mainly based on the celebrated work by Layard, Nickell and Jackman (1991), LNJ henceforth, and thus built on the analytical foundations laid down by Blanchard and Kiyotaki (1987). In these models, unemployment comes to depend on the mutual compatibility between individual price decisions by firms and wage setting behaviour by (firms and) trade unions, with monetary policy fixing the general price level (money is neutral).

The extension of this approach to the "medium run" suggested to interpret the European labour market performance (rising unemployment) from the mid-1970s to the end of the 1990s in terms of either a shift in the demand curve for labour produced by a "capital-using" shift in technology (Blanchard, 1997) or an inadequate growth in capital stock in relation to labour force growth and technical progress (Rowthorn, 1999a; 1999b). Much of this controversy had to do with the elasticity of substitution between capital and labour being greater than (or equal to) one, or lower than one.

The second strand of the literature, which developed out of the Barro-Gordon (1983) problem, brought together the debates on the optimal degree of conservativeness in central banking and on the tendency highlighted by Calmfors and Driffill (1988) towards wage moderation and higher employment rates in economies characterised by extreme values (very low and very high) in the degree of centralisation of wage bargaining[1]. In the models belonging to this approach, which all share a short-period perspective, Central Bank independence and the degree of centralisation of wage bargaining explain the existing differences in the macroeconomic performance of the industrialised economies, with firms playing the background role of mechanically fixing

[1] The first aspect of this phenomenon (at low degrees of centralisation) was explained through a cooling effect on wages produced by competition among labourers, whereas the second aspect (at high degrees of centralisation) was seen as due to unions' ability to internalise the costs of high wage claims. Both effects were reckoned as weaker at intermediate (industry) degrees of centralisation.

employment by moving along the demand for labour function (e.g., Hall and Franzese, 1996; Cukierman and Lippi, 1999 and 2001; Guzzo and Velasco, 1999[2]). In this class of models, the nonneutrality of money comes to depend, in particular, on the (direct or indirect) presence of inflation, or other objectives different from real wages and employment, in the utility functions of trade unions (Acocella and Ciccarone, 1997; Acocella and Di Bartolomeo, 2002). The economic set-up underlying the non-cooperative games between a single central bank and possibly several unions has progressively moved away from reduced forms of the AS-AD type to microfounded general equilibrium economies with imperfections in the labour and, possibly, in the good markets (e.g., Coricelli, Cukierman and Dalmazzo, 2000, 2001[3]).[4]

The aim of this paper is to bring together these two lines of research. Our intuition is simple: if a central bank strategically interacts with unions setting the *nominal* wage, the price level which it can bring about influences the real wage, and so the employment level chosen by profit maximising firms. In a general equilibrium analysis with imperfect competition in good and labour markets which stays as close as possible to that by Rowthorn (1999a; 1999b) and thus to Layard, Nickell and Jackman (1991), we wish to understand how monetary policy interacts with the already explored factors which affect prices and unemployment in the "medium run".

To this aim, monetary policy is modelled assuming that there exists several trade unions but only one Central Bank. For the years before EMU this should be interpreted, in line with the standard interpretation of representative agents in labour markets, as encapsulating the average preferences and behaviour of the national monetary policy-makers. Under this assumption, equilibrium outcomes depend on the (average) degree of conservativeness of such (representative) Central Bank. For the EMU period, this average collapses into a European Central Bank characterised by a degree of conservativeness higher than the pre-EMU average one.

The paper is structured as follows. In the next section, we describe the model and, differently from what is usually done in the policy game literature, we solve it by adopting a Nash equilibrium concept. Although we acknowledge that the distinction between leader and follower may allow us to highlight interesting phenomena (e.g., dynamic inconsistency), we believe that the choice of Stackelberg games is not sufficiently justified in a medium run setting. In this case, wage contracts continue to span a time period which is longer than that of monetary policy, so that wage setters can be conceived as taking into account the central bank's reaction to their choices in order to offset its negative effects. At the same time, the longer time dimension of the analysis allows for

[2] As corrected by Lippi (2002) and Guzzo and Velasco (2002).

[3] In these papers, the interaction structure is as follows: first, unions set nominal wages; in a second stage, the central bank chooses the nominal stock of money and affects inflation via money balances; in a final stage each firm takes the general price level as given and sets its own price in monopolistic competition so as to maximise profits.

[4] For a recent survey of this literature, see Berger, de Haan and Eijffinger, S. (2001).

changes in wage contracts which make it reasonable to assume that also the public sector (the central bank) forms expectations on the future behaviour of the private one (the unions). This observation induces us to espouse the point of view of Roemer (1999) and of others in favour of simultaneous players' moves.

In section 3, we use the players' "reaction functions" to identify the condition under which monetary policy is not neutral, i.e, that benefits are not perfectly indexed to prices. In section 4, we focus on the reactions of prices and of the unemployment rate to variations in the main factors shaping the economic environment in the "medium rum", i.e., in the degree of conservativeness of the central bank, in social policy (level of benefits), in capital accumulation and in capital-using shifts in technology. This analysis leads to two main results. First, Rogoff's (1985) conclusion that a conservative central banker may decrease both inflation and unemployment applies. Second, when monetary policy is considered, Rowthorn's explanation of European employment appears more plausible than Blanchard's, even if neither of them holds in general but only in special cases.

In sections 5, we assume balanced government budgets, and we construct a graphical apparatus which allows us to discuss some present European policy options. In section 6, we introduce the concept of "Desired Inflation Rate of Unemployment" (DIRU, the rate of unemployment corresponding to a price level equal to that desired by the Central Bank) and we show that centralised (i.e., mutually compatible) fiscal and monetary policies may always bring about a DIRU equal to zero. Finally, we highlight that in the present European context the minimisation of the DIRU would require to enhance, not to tighten, social policies. Section 7 offers concluding remarks.

2. The model

The economy is populated by: J imperfectly competitive firms producing differentiated goods; L workers who supply labour, hold money balances, receive firms' dividends and consume; J trade unions fixing the nominal wage; a central bank regulating the money supply.

The meaning of the notation is as follows:

$j = 1, \cdots J$	number of firms and goods (each firm produces one good)
$i = 1, \cdots, L$	number of workers/consumers
L_j	labour force in sector j
$L = \sum_j L_j$	total labour force
$L_j = L / J \ \forall j$	the labour force is equally distributed among sectors
N_j	employment in firm j

$N = \sum_j N_j$ total employment

$u = \dfrac{N - L}{L}$ overall rate of unemployment

K_j capital employed by firm *j*,

2.1 The workers/consumers

The representative consumer maximises utility subject to the budget constraint:

$$\max_{C_{ij}, n_i, M_i} U_i = C_i^g \cdot \left(\frac{M_i}{P} \right)^{1-g} - \mu N_i^{\beta}$$

$$\text{s.t.} \qquad \sum_j P_j C_{ji} + M_i = W n_i + \overline{M}_i + \sum_j \pi_{ij}$$

where M_i is the money endowment and n_i is the individual labour input, W is the nominal wage rate, C_i is her consumption of goods, specified as a CES function of the various goods with elasticity parameter θ: $C_i = J^{\frac{1}{1-\theta}} \left(\sum_j C_{ij}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}$; the price index is homogeneous of degree one in individual prices P_j: $P = \left(\frac{1}{m} \sum_j P_j^{1-\theta} \right)^{\frac{1}{1-\theta}}$; π_{ij} is the dividend paid by firm *j* to consumer *i*.

Defining[5] $R_i = W n_i + \overline{M}_i + \sum_j \pi_{ij}$ and solving the problem (see Appendix A), we obtain the demand for commodity *j* and the aggregate demand:

$$Y_j = \left(\frac{P_j}{P} \right)^{-\theta} \frac{1}{J} \cdot \frac{g}{1-g} \cdot \frac{\overline{M}}{P}$$

$$Y = \frac{g}{1-g} \cdot \frac{\overline{M}}{P}$$

2.2 The firms

In order to consider the possibility stressed by Rowthorn (1999a; 1999b) that the elasticity of substitution between labour and capital is less than one, we assume that each firm has a CES production function:

[5] When describing the trade unions, it will be specified that the unemployed workers are assumed to earn benefits, so that their R_i will include such benefits instead of labour income $W_j n_i$.

$$Y_j = \left[\alpha(\Lambda_N N_j)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(\Lambda_k K_j)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

where Λ_N and Λ_K are indices of productive efficiency. It follows that:

$$Y = \left[\alpha(\Lambda_N N)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(\Lambda_k K)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

In the short run, firm *j* takes capital as given, chooses employment in the *j*-th labour marker and monopolistically sets production by taking into account its demand. The firm's problem can thus be written as:

$$\max_{N_j} \frac{\Pi_j}{P} = \frac{P_j}{P} Y_j - \frac{W_j}{P} N_j$$

$$\text{s.t.} \qquad \left(\frac{JY_i}{Y} \right)^{-\frac{1}{\theta}} = \left(\frac{P_j}{P} \right)$$

From the FOC we obtain the labour demand function of firm *j*:

$$\frac{W_j}{P} = \left(\frac{JY_j}{Y} \right)^{-\frac{1}{\theta}} \alpha \left(\frac{\theta - 1}{\theta} \right) \left(\frac{Y_j}{\Lambda_N N_j} \right)^{\frac{1}{\sigma}} \Lambda_N$$

2.3 The unions

There exist one monopoly union in each market *j*. In line with the literature we are taking as our benchmark, their utility functions are derived from those of the workers they represent (in each market *j)* by taking into account only the income deriving from labour (wage bill) or from benefits (yet to be discussed), and disregarding the disutility of labour.

The indirect utility function of consumer *i* is: $U_i = \mu \frac{R_i}{P} - \mu n_i^{\beta}$. The union's objective function is: $V_j = V_j(W/P, A/P, N_j, L_j)$, where L_j is the total union's membership and *A* is the alternative income. We assume that each household can deliver the same labour quantity[6] n_i and that households are equally distributed across the labour markets. In a symmetric equilibrium, if the worker is unemployed it will be $n_i = 0$, whereas it will be $n_i = 1$ if the worker is employed. Since the consumer indirect utility function is linear in income (i.e. no risk aversion), the union maximises its median voter's expected (labour) income and solves the problem:

[6] It must however be remembered that in this model households do not develop a "supply of labour" in the proper sense. It is the union that, once wages are fixed, delivers all the labour input demanded by firms.

$$\max_{W_j} \quad V_j = S_j\left(\frac{W_j}{P}\right) + \left(1 - S_j\right)\frac{A}{P}$$

$$\text{s.t.} \qquad \frac{W_j}{P} = \left(\frac{JY_j}{Y}\right)^{-\frac{1}{\theta}} \alpha\left(\frac{\theta - 1}{\theta}\right)\left(\frac{Y_j}{\Lambda_N N_j}\right)^{\frac{1}{\sigma}} \Lambda_N$$

where S_j is the probability of being employed (which is of course affected by the levels of total labour force and total employment).

If the alternative income A is constant, from the FOC we obtain (see Appendix B):

$$W = \varepsilon_{NS} A \left\{ \varepsilon_{SN} + \left[\alpha\left(\frac{\theta - \sigma}{\sigma\theta}\right)\left(\frac{Y}{\Lambda_N N}\right)^{\frac{1-\sigma}{\sigma}} - \frac{1}{\sigma} \right] \right\}^{-1}$$

Equilibrium in the labour market is obtained by equating labour demand and "supply":

$$\frac{W}{P} = \alpha\left(\frac{\theta - 1}{\theta}\right)\left(\frac{Y}{\Lambda_N N}\right)^{\frac{1}{\sigma}} \Lambda_N = \varepsilon_{NS} \frac{A}{P} \left\{ \varepsilon_{SN} + \left[\alpha\left(\frac{\theta - \sigma}{\sigma\theta}\right)\left(\frac{Y}{\Lambda_N N}\right)^{\frac{1-\sigma}{\sigma}} - \frac{1}{\sigma} \right] \right\}^{-1}$$

where $\varepsilon_{NS} = \dfrac{\partial S_j}{\partial N_j}\dfrac{N_j}{S_j}$.

From this we get the function:

$$LM(u,P): \left(\varepsilon_{SN} - \frac{1}{\sigma}\right)\left(\frac{Y}{\Lambda_N N}\right)^{\frac{1}{\sigma}} + \alpha\left(\frac{\theta - \sigma}{\sigma\theta}\right)\left(\frac{Y}{\Lambda_N N}\right)^{\frac{2-\sigma}{\sigma}} - \frac{\theta\varepsilon_{NS}}{\alpha(\theta - 1)}\frac{A}{P\Lambda_N} = 0$$

It is now possible to use $\left(\dfrac{Y}{\Lambda_N N}\right) = \left[\alpha + (1-\alpha)\left(\dfrac{k}{1-u}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$, where $k = \dfrac{\Lambda_K K}{\Lambda_N L}$ and u is the unemployment rate (equal to 1 – N/L), in order to obtain the equilibrium rate of unemployment u^*.

2.4 The Central Bank

The Central Bank (CB) maximises total consumers' welfare, but also exhibits, in line with all the policy game literature,[7] a specific aversion to inflation. Its objective function is thus:

$$\Omega = \sum_{ij} U_i = \sum_{ij} \left\{ \mu\left[\frac{W_j}{P} n_i + \frac{M_i}{P} + \frac{\sum_j \pi_{ij}}{P}\right] - \mu n_i^{\beta} \right\} - \xi_B (P - P^*)^2$$

[7] See, e.g., Cukierman and Lippi (1999) and Guzzo and Velasco (1999).

where ξ_B is the "degree of conservativeness" of the central bank.

The central bank knows that part of the consumers will be employed and earn a wage, whereas others will be unemployed and earn the alternative income. In a symmetric equilibrium (wages and labour inputs are the same across sectors), the first term in the function is equal to: $\frac{W}{P}N+\frac{M}{P}+\frac{\Pi}{P}=\frac{W}{P}N+\frac{M}{P}+Y-\frac{W}{P}N=\frac{M}{P}+Y$.

Recalling that in such a symmetric equilibrium, if the worker is unemployed it will be $n_i=0$, whereas it will be $n_i=1$ if the worker is employed, and since the N_j coincide with the use of labour inputs by firms, the sum will be equal to the aggregate demand for labour: $\sum_{ij}\mu n_i^{\beta}=\sum_j\left[N_j\mu n_i^{\beta}\right]=\mu\sum_j N_j=\mu N$.

Hence, by taking into account the alternative real incomes *A/P*, the central bank maximises its objective function under the constraints imposed by the economy:

$$\max_M \Omega=\mu\frac{M}{P}+\mu(L-N)\frac{A}{P}+\mu Y-\mu N-\frac{\xi_B}{2}(P-P^*)^2$$

$$\text{s.t.}\qquad Y=\frac{g}{1-g}\cdot\frac{\overline{M}}{P}$$

$$\frac{W_j}{P}=\left(\frac{JY_j}{Y}\right)^{-\frac{1}{\theta}}\alpha\left(\frac{\theta-1}{\theta}\right)\left(\frac{Y_j}{\Lambda_N N_j}\right)^{\frac{1}{\sigma}}\Lambda_N$$

Under symmetry the last constraint becomes:

$$\frac{W}{P}=\alpha\left(\frac{\theta-1}{\theta}\right)\left(\frac{Y}{\Lambda_N N}\right)^{\frac{1}{\sigma}}\Lambda_N$$

The latter equation is a function of *W/P* and *Y*: $N=N(W/P,\quad Y)$. We substitute for $Y=\frac{g}{1-g}\cdot\frac{\overline{M}}{P}$ and from the FOC we obtain the function (see Appendix C):

$$CB(u,P):\mu\left(\frac{Y}{g}-N\right)-\mu\frac{A}{P}L-\xi_B P(P-P^*)=0$$

The second order condition is negative for $g>2/3$ and *P** sufficiently small (Appendix D). By inserting the aggregate demand for labour in the first order condition (so as to make it a function from nominal wage to money), it can be shown that $dM/dW\,|_{FOC}<0$: the Central Bank reacts to an increase in nominal wages by decreasing the money supply.

The economic mechanism set into motion by unions' wage push is analogous to that discussed by Coricelli, Cukierman and Dalmazzo (2000, 2001). The wage increase, for a given price level, lowers the demand for labour, the production in each and every market falls and, given the individual demands for goods, firms increase their relative prices. The central bank reacts to the price increase by contracting the money supply[8], thus further reducing real money balances and the demands for goods, and so employment. It will be shown below that, in the present setting, this reaction by the central bank acts as a fundamental deterring device on unions' wage claims: the fear of the extra unemployment induced by monetary policy increases with the central bank's degree of conservativeness.

3. The policy game

As the game between unions and CB is of the Nash type, we focus on the players' reaction functions or, more precisely, on Cubitt's (1992) quasi-reaction loci:

$$LM(u,P):\left(\varepsilon_{SN}-\frac{1}{\sigma}\right)\left(\frac{Y}{\Lambda_N N}\right)^{\frac{1}{\sigma}}+\alpha\left(\frac{\theta-\sigma}{\sigma\theta}\right)\left(\frac{Y}{\Lambda_N N}\right)^{\frac{2-\sigma}{\sigma}}=\frac{\theta\varepsilon_{NS}}{\alpha(\theta-1)}\frac{A}{P\Lambda_N}$$

$$CB(u,P):\mu\left(\frac{Y}{g}-N\right)-\mu\frac{A}{P}L-\xi_B P(P-P^*)=0$$

From these, the following proposition straightforwardly derives.

Proposition 1. If the level of benefits, or of the alternative income, is set in real terms, then monetary policy is neutral: unions and firms bring about equilibrium in the labour market (unemployment and income), and the CB fixes only the price level, as the LM is vertical in the (P, u) space. If benefits are not perfectly indexed to prices, it is necessary to take into account also the central bank's reaction function, which means that its decisions have real effects.

From this proposition we derive that in this model, as it is usual in the policy games literature, the consideration of inflation as an argument of the unions' objective functions (which

[8] In Coricelli, Cukierman and Dalmazzo (2000) the central bank increases or decreases the money supply (i.e., uses monetary policy to counteract either the effects of the wage increase on unemployment or those on inflation) according to its degree of conservativeness; however, the increase in prices set by firms when wages go up is so high that the overall effect on real money balances is always negative.

here occurs through *A*/*P*) produces nonneutrality (see, e.g., Acocella e Di Bartolomeo, 2002). More precisely, it is the nominal rigidity generated by *A* which allows inflation to enter (indirectly) the union objective function. However, differently from that literature, the degree of price indexation of the alternative income may act as a link between the short and the long run. In the short or medium run, imperfect indexation (together with other nominal rigidities) allows for nonneutrality; whereas over longer time spans, when benefits can be conceived as fully indexed to average inflation, neutrality results.

An early and detailed discussion on the system of benefits is Atkinson and Micklewright (1991), who highlight that such system is a highly composite one and that it includes many different and country-specific items. For this reason, it is difficult to assess empirically the behaviour of an aggregate variable such as that considered in our model[9]. In any case, the European Commission's (1998; 2000) data on expenditures for unemployment in the European countries show a variability in the rates of change which seems to support the assumption that *A* is not perfectly indexed to prices, at least in the short and medium run.

Furthermore, the calculation of dP/du shows that the slope of the BC's reaction function crucially depends upon the relative values of the parameters ξ_B and *A* (Appendix E), that is, the relative behaviour of monetary policy and social policy. In order to understand what happens to prices on the basis of the BC reaction function when ξ_B varies, we implicitly differentiate it and get $\frac{dP}{d\xi_B} = \frac{P(P-P^*)}{\left\{\frac{\mu AL}{P^2} - \xi_B(2P-P^*)\right\}}$. This shows that the price level decreases when ξ_B is sufficiently high relative to *A*, that is when social policy is relatively modest.

The sign of this derivative is also relevant to determine the effects of variations of *A* and ξ_B on the Nash equilibrium value of *u*. The latter depends in the same way on the relative values of ξ_B and *A*, because the equilibrium price level is affected by ξ_B and *A* and, in its turn, the variation in *P* influences the equilibrium level of *u* according to the equation for dP/du derived in Appendix E, which also depends upon the relative behaviour of monetary policy and social policy. Since it is difficult to separately analyse the direct and indirect effects of parameter changes on the equilibrium values of the relevant variables, in the next section we directly calculate the final effects of such changes.

[9] See also OECD (1995).

4. Comparative statics

In order to carry out some exercises in comparative statics by considering changes in the parameters shaping the economy in the "medium rum", we rewrite the two functions:

$$LM(u,P) = \left(\varepsilon_{SN} - \frac{1}{\sigma}\right)\left(\frac{Y}{\Lambda_N N}\right)^{\frac{1}{\sigma}} + \alpha\left(\frac{\theta-\sigma}{\sigma\theta}\right)\left(\frac{Y}{\Lambda_N N}\right)^{\frac{2-\sigma}{\sigma}} - \frac{\theta\varepsilon_{NS}}{\alpha(\theta-1)}\frac{A}{P\Lambda_N} = 0$$

$$CB(u,P) = \mu\left(\frac{Y}{g} - N\right) - \mu\frac{A}{P}L - \xi_B P(P-P^*) = 0$$

To start with, we wish to understand the effects on equilibrium prices and unemployment of shifts in monetary policy "regimes" (i.e., from a less to a more inflation averse central bank), such as that occurred in Europe from the post oil shock period to the creation of the monetary union. To this aim, we calculate the Jacobian relative to the Nash equilibrium and with respect to variations in ξ_B, assuming from now on that it is always $P > P^*$ (as it was in Europe in the 1980s-1990s):

$$\begin{bmatrix} \partial CB/\partial u & \partial CB/\partial P \\ \partial LM/\partial u & \partial Lm/\partial P \end{bmatrix}\begin{bmatrix} du/d\xi_B \\ dP/d\xi_B \end{bmatrix} = -\begin{bmatrix} \partial CB/\partial\xi_B \\ \partial LM/\partial\xi_B \end{bmatrix}$$

which we write as:

$$\mathbf{A}\begin{bmatrix} du/d\xi_B \\ dP/d\xi_B \end{bmatrix} = -\begin{bmatrix} \partial CB/\partial\xi_B \\ \partial LM/\partial\xi_B \end{bmatrix}$$

The signs of the derivatives are (Appendix F): $\partial Lm/\partial u < 0$; $\partial LM/\partial P > 0$; $\partial CB/\partial u < 0$; $\partial CB/\partial P = \left\{\frac{\mu AL}{P^2} - \xi_B(2P-P^*)\right\}$; $\partial CB/\partial\xi_B < 0$; $\partial LM/\partial\xi_B = 0$. The matrix $\mathbf{A}^{-1}$ is:

$$A^{-1} = \left(\frac{1}{\frac{\partial CB}{\partial u}\frac{\partial LM}{\partial P} - \frac{\partial CB}{\partial P}\frac{\partial LM}{\partial u}}\right)\begin{bmatrix} \partial LM/\partial P & -\partial CB/\partial P \\ -\partial LM/\partial u & \partial CB/\partial u \end{bmatrix}$$

so that:

$$\begin{bmatrix} du/d\xi_B \\ dP/d\xi_B \end{bmatrix} = \left(\frac{-1}{\frac{\partial CB}{\partial u}\frac{\partial LM}{\partial P} - \frac{\partial CB}{\partial P}\frac{\partial LM}{\partial u}}\right)\begin{bmatrix} \partial LM/\partial P & -\partial CB/\partial P \\ -\partial LM/\partial u & \partial CB/\partial u \end{bmatrix}\begin{bmatrix} -P(P-P^*) \\ 0 \end{bmatrix}$$

Being $\frac{\partial CB}{\partial u} < 0; \frac{\partial LM}{\partial P} > 0; \frac{\partial LM}{\partial u} < 0$, the sign of the determinant $\Delta = \left(\frac{-1}{\frac{\partial CB}{\partial u}\frac{\partial LM}{\partial P} - \frac{\partial CB}{\partial P}\frac{\partial LM}{\partial u}} \right)$ depends upon the sign of $\partial CB / \partial P$. The following cases are possible:

1. If $\partial CB / \partial P < 0$ then $\Delta > 0$
2. Se $\partial CB / \partial P > 0$ then:

   2a. If $\partial CB / \partial P >> 0$ (positive and high) it can be $\Delta < 0$

   2b. If $\partial CB / \partial P > 0$ (positive but low) it can be $\Delta > 0$

Being:

$$du / d\xi_B = \Delta[-P(P-P^*)]\partial LM / \partial P$$

the sign of $du / d\xi_B$ crucially depends upon the relative values of A and ξ. In particular:

1. $du / d\xi_B > 0$ when $\partial CB / \partial P >> 0$, that is when $\frac{\mu AL}{P^2} >> \xi_B (2P - P^*)$,

2. $du / d\xi_B < 0$ when $\begin{cases} \partial CB / P < 0 \quad \rightarrow \quad \frac{\mu AL}{P^2} < \xi_B (2P - P^*) \\ \partial CB / P > 0 \text{ (but low)} \quad \rightarrow \quad \frac{\mu AL}{P^2} > \xi_B (2P - P^*) \text{ (but low)} \end{cases}$

The same conclusion applies as for the sign of:

$$dP / d\xi_B = \Delta[-P(P-P^*)](-\partial LM / \partial u)$$

which leads to the following:

Proposition 2. Rogoff's (1985) conclusion holds also in this model: a conservative central banker may decrease both inflation and unemployment.

This result is the same as that obtained by Coricelli, Cukierman and Dalmazzo (2000, proposition 6) in a Stackelberg context. Also the economic explanation runs along similar lines, as the fear of a tightening of monetary policy after a wage increase induces unions' to moderate wage claims, so as to bring about lower inflation and unemployment. Of course, the more conservative is the central bank the stronger is the deterring effect of the deflationary threat. Proposition 2 shows that Rogoff's result holds also when LNJ's union's objective function is adopted in a Nash setting. The difference with previous findings is that, in our model, the direction (although not the intensity) of the Central Bank's response to a wage increase is independent of its degree of conservativeness: the money supply is always reduced.

We now calculate the Jacobian with respect to variations in *A*:

$$\begin{bmatrix} du/dA \\ dP/dA \end{bmatrix} = \left(\frac{-1}{\frac{\partial CB}{\partial u}\frac{\partial LM}{\partial P} - \frac{\partial CB}{\partial P}\frac{\partial LM}{\partial u}} \right) \begin{bmatrix} \partial LM/\partial P & -\partial CB/\partial P \\ -\partial LM/\partial u & \partial CB/\partial u \end{bmatrix} \begin{bmatrix} \partial CB/\partial A \\ \partial LM/\partial A \end{bmatrix}$$

where $\frac{\partial CB}{\partial A} = -\frac{\mu L}{P} < 0$ and $\frac{\partial LM}{\partial A} = -\frac{\theta \varepsilon_{SN}}{\alpha(\theta-1)\Lambda_N P} < 0$.

The sign of $du/dA = \Delta\left[\frac{\partial CB}{\partial A}\frac{\partial LM}{\partial P} - \frac{\partial CB}{\partial P}\frac{\partial LM}{\partial A}\right]$ essentially depends upon that of $\frac{\partial CB}{\partial P}$. If $\frac{\partial CB}{\partial P} < 0$ and $\Delta > 0$ it follows $du/dA < 0$. This requires a ξ_B high relative to *A*.

Hence, social policy may be successful in reducing unemployment when the central bank is highly conservative. This occurs because the increase in *A* expands demand without inducing unions to set wages too high (as they would instead do in economies of the LNJ type) in the fear of a strong deflationary response by a very inflation averse central bank. There thus appears to exist a balance between monetary discipline and policies supporting the income of the unemployed. This conclusion is however softened by the realisation that also the sign of $dP/dA = \Delta\left[\frac{\partial CB}{\partial A}\left(-\frac{\partial LM}{\partial u}\right) - \frac{\partial CB}{\partial u}\frac{\partial LM}{\partial A}\right]$ essentially depends upon that of $\frac{\partial CB}{\partial P}$: assuming $\frac{\partial CB}{\partial P} < 0$ and $\Delta > 0$, the sign of *dP/dA* is uncertain[10].

In order to understand the medium run consequences of introducing monetary policy into the Blanchard-Rowthorn economic set up, we now examine the effects of changes in *k* and α. We first calculate the Jacobian with respect to variations in *k*:

$$\begin{bmatrix} du/dk \\ dP/dk \end{bmatrix} = \left(\frac{-1}{\frac{\partial CB}{\partial u}\frac{\partial LM}{\partial P} - \frac{\partial CB}{\partial P}\frac{\partial LM}{\partial u}} \right) \begin{bmatrix} \partial LM/\partial P & -\partial CB/\partial P \\ -\partial LM/\partial u & \partial CB/\partial u \end{bmatrix} \begin{bmatrix} \partial CB/\partial k \\ \partial LM/\partial k \end{bmatrix}$$

The signs of the relevant derivatives are (Appendix F) $\frac{\partial CB}{\partial k} > 0$ and $\frac{\partial LM}{\partial k} > 0$. It follows that: $du/dk = \Delta\left[\frac{\partial CB}{\partial k}\frac{\partial LM}{\partial P} - \frac{\partial CB}{\partial P}\frac{\partial LM}{\partial k}\right]$. If ξ_B is high and *A* is low, $\frac{\partial CB}{\partial P} < 0$, $\Delta > 0$ and

[10] This result is somewhat different from that obtained by Berthold and Fehn (1998), who study the incentives to labor market reforms under various monetary regimes within a highly simplified Barro-Gordon set-up. They show that reforms to liberalise the labor market are stronger under discretionary (nation-wide) monetary policy than in a monetary union; they also show that reductions in employment protection, or in unemployment benefits and related social expenditure, always reduce equilibrium unemployment and inflation. The difference with our results will become even sharper in section 5 below, where balanced budgets are assumed.

$du/dk > 0$. This result, which is the opposite of Rowthorn's, is triggered by the LNJ's increase in wages generated by investment in new physical capital (Rowthorn, 1999a, p. 414), which is here even sharper due to the substitution of the right to manage with the monopoly union assumption. The Central Bank reacts to this increase by decreasing the money supply, demand goes down in all markets, individual prices and the price level decrease, real wages rise and employment on the existing equipment falls. The rise in real wages turns out to be higher than that considered by Rowthorn, where prices are hold fix; it is so high that the loss of jobs on the existing equipment is greater than the extra jobs created on the new equipment, in spite of the possibly low elasticity of substitution.

For low values of ξ_B relative to A (as it was possibly the case in the period considered by Rowthorn), $\frac{\partial CB}{\partial P}$ may be positive and Δ may have either positive or negative sign. This might bring about the same result (in terms of du/dk) as Rowthorn's, but conclusions are not clear-cut, as all depends upon the relative values of the relevant derivatives.

Finally, we wish to evaluate, in our policy game, the effect on unemployment of Blanchard's capital-using shift in technology. The Jacobian with respect to variations in α is:

$$\begin{bmatrix} du/d\alpha \\ dP/d\alpha \end{bmatrix} = \left(\frac{-1}{\frac{\partial CB}{\partial u}\frac{\partial LM}{\partial P} - \frac{\partial CB}{\partial P}\frac{\partial LM}{\partial u}} \right) \begin{bmatrix} \partial LM/\partial P & -\partial CB/\partial P \\ -\partial LM/\partial u & \partial CB/\partial u \end{bmatrix} \begin{bmatrix} \partial CB/\partial \alpha \\ \partial LM/\partial \alpha \end{bmatrix}$$

where (Appendix F): $\frac{\partial CB}{\partial \alpha} > 0$ when $k > 1-u$ and $\frac{\partial CB}{\partial \alpha} < 0$ when $k < 1-u$; and $\frac{\partial LM}{\partial \alpha} > 0$ when $k > 1-u$ or when k < 1-u but $k \cong 1-u$, whereas $\frac{\partial LM}{\partial \alpha} < 0$ when $k < 1-u$ and k not close to (1 – u). It follows that the sign of $du/d\alpha = \Delta\left[\frac{\partial CB}{\partial \alpha}\frac{\partial LM}{\partial P} - \frac{\partial CB}{\partial P}\frac{\partial LM}{\partial \alpha}\right]$ is difficult to establish[11]. This is due also to the need to consider, besides the signs of the derivatives in Δ, also the value of k relative to that of (1 – u). The conclusion to draw is that, when monetary policy is considered, Rowthorn's explanation of European employment is subject to milder conditions to be respected than Blanchard's,[12] as it is not subject to contraints on k relative to (1-u).

[11] Also the sign of $dP/d\alpha$ is difficult to ascertain.

[12] It is worth reminding that in LNJ's and in Rowthorn's models an increase in α has the same effects on wages as an increase in k, so that an economic reasoning analogous to that developed in the case of investment spending may be proposed.

5. Balanced Budgets

The exercises of the previous sections assume that the benefits for the unemployed may be financed in unspecified ways (for instance, budget deficits), and that the monetary authorities do not take into account the tax burden on incomes. These assumptions, only preliminarily acceptable when considering the previous decades, must be changed in order to generalise the analysis, and to make it more suited to study the present medium run European scenario. In particular, we will present a graphical apparatus which allows us to highlight some unconventional policy opportunities.

In this section we hence reconsider the comparative statics effects of changes in the relevant medium run variables and parameters by making the two following assumptions: (i) fiscal budgets are balanced,[13] i.e. total benefits are financed through lump-sum taxes, $T=(L-N)A$, where $T=tN$ and t is the individual fiscal burden on employed workers; (ii) the union cares for workers' income net of taxes and the Central Bank cares for total wealth net of taxes.

When taken together, the two new assumptions lead to a new formulation for the union's objective function:

$$V_j = S_j\left(\frac{W_j}{P}-\frac{t}{P}\right)+\left(1-S_j\right)\frac{A}{P}$$

Moreover, in the central Bank's objective function, the sum of the incomes of individuals (providing positive utility) cancels out with the total lump-sum taxes (providing negative utility) paid out by households. The reaction curves of the players can thus be written as:

$$LM:\left(\varepsilon_{SN}-\frac{1}{\sigma}\right)\left(\frac{Y}{\Lambda_N N}\right)^{\frac{1}{\sigma}}+\alpha\left(\frac{\theta-\sigma}{\sigma\theta}\right)\left(\frac{Y}{\Lambda_N N}\right)^{\frac{2-\sigma}{\sigma}}-\frac{\theta\varepsilon_{NS}}{\alpha(\theta-1)P\Lambda_N}A\frac{1}{(1-u)}=0$$

$$BC:\quad P=\left\{(1-u)L\frac{\mu}{\xi_B}\left(\frac{1}{g}\left(\alpha+(1-\alpha)\left(\frac{k}{1-u}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}\Lambda_N-1\right)\right\}^{1/2}$$

It is worth noting that also in this case it is $dM/dW|_{FOC}<0$. The Central Bank continues not to accommodate increases in nominal wages: the same economic mechanism as that described

[13] This could of course be a way to take explicitly into account the European "stability pact constraint".

at the end of section 2 induces unions to moderate their claims, the more so the higher is the central bank's degree of conservativeness. It is also straightforward to check that Proposition 1 continues to hold when budget considerations are explicitly introduced: if benefits are fully indexed, unemployment is entirely determined by real variables and parameters, and monetary policy is neutral. Hence, balanced budgets do not alter the mechanism through which monetary policy affects the level of activity: the nominal rigidity of *A* appears to be relevant also when fiscal constraints are explicitly introduced.

The Jacobian relative to the Nash equilibrium and with respect to variations in ξ_B is now:

$$\begin{bmatrix} du/d\xi_B \\ dP/d\xi_B \end{bmatrix} = \left(\frac{-1}{\frac{\partial CB}{\partial u}\frac{\partial LM}{\partial P} - \frac{\partial CB}{\partial P}\frac{\partial LM}{\partial u}} \right) \begin{bmatrix} \partial LM/\partial P & -\partial CB/\partial P \\ -\partial LM/\partial u & \partial CB/\partial u \end{bmatrix} \begin{bmatrix} \partial CB/\partial \xi_B \\ \partial LM/\partial \xi_B \end{bmatrix}$$

The signs of the derivatives are (Appendix G): $\frac{\partial CB}{\partial P} = -1$; $\frac{\partial CB}{\partial u} < 0$; $\frac{\partial LM}{\partial P} > 0$; $\frac{\partial LM}{\partial u} < 0$,

so that the sign of the determinant $\Delta = \left(\frac{-1}{\frac{\partial CB}{\partial u}\frac{\partial LM}{\partial P} - \frac{\partial CB}{\partial P}\frac{\partial LM}{\partial u}} \right)$ is certainly positive. Moreover,

being: $\frac{\partial CB}{\partial \xi_B} < 0$ and $\frac{\partial LM}{\partial \xi_B} = 0$, it follows that:

$$du/d\xi_B < 0 \text{ and } dP/d\xi_B < 0.$$

This shows that also Proposition 2 continue to hold in the balanced budget context. This result is graphically summarised in Figure 1: an increase in ξ_B shifts the *CB* curve (drawn linear for simplicity) downward to the left and decreases its slope.

FIGURE 1 HERE

The Jacobian with respect to variations in *A* is:

$$\begin{bmatrix} du/dA \\ dP/dA \end{bmatrix} = \left(\frac{-1}{\frac{\partial CB}{\partial u}\frac{\partial LM}{\partial P} - \frac{\partial CB}{\partial P}\frac{\partial LM}{\partial u}} \right) \begin{bmatrix} \partial LM/\partial P & -\partial CB/\partial P \\ -\partial LM/\partial u & \partial CB/\partial u \end{bmatrix} \begin{bmatrix} 0 \\ \partial LM/\partial A \end{bmatrix}$$

The signs of the derivatives are (Appendix G): $\frac{\partial CB}{\partial A}=0$ and $\frac{\partial LM}{\partial A}<0$. Remembering that the sign of the determinant is positive, this leads to:

$du/dA=-\frac{\partial CB}{\partial P}\frac{\partial LM}{\partial A}\Delta<0$ and $dP/dA=\frac{\partial CB}{\partial u}\frac{\partial LM}{\partial A}\Delta>0$. This result is again graphically summarised in Figure 1: an increase in *A* shifts the *LM* curve upward and decreases its slope.

The effect of *A* on *u* is due to the impact of union's wage policy. Being *A* financed through the taxes paid by active workers, when *A* rises also *t* increases and the real wage net of taxes falls. As discussed in the previous section, in the case excluding tax payments, unions moderate their wage claims only when the central bank is conservative enough (i.e., so much as to decrease "excessively" the money supply, and hence demand and employment), because some extra jobs are in any case made available by the expansion in demand. With balanced budgets unions are instead induced to act always in that way because a higher *A* does not induce, *per se*, an increase in demand and any increase in wages would raise unemployment. Since a greater *A*/*P* implies a smaller increase in *t*/*P* - being the latter equal in equilibrium to $\left(\frac{L}{N}-1\right)\frac{A}{P}$ - the unions' utility increases, leaving room for a decrease in wages and an increase in employment: the increased welfare of the unemployed members is counterbalanced by an increase in the number of employed workers.

The Jacobian with respect to variations in *k* is:

$$\begin{bmatrix} du/dk \\ dP/dk \end{bmatrix}=\left(\frac{-1}{\frac{\partial CB}{\partial u}\frac{\partial LM}{\partial P}-\frac{\partial CB}{\partial P}\frac{\partial LM}{\partial u}}\right)\begin{bmatrix} \partial LM/\partial P & -\partial CB/\partial P \\ -\partial LM/\partial u & \partial CB/\partial u \end{bmatrix}\begin{bmatrix} \partial CB/\partial k \\ \partial LM/\partial k \end{bmatrix}$$

Since it is (Appendix G): $\frac{\partial CB}{\partial k}>0$, $\frac{\partial LM}{\partial k}>0$ and $\Delta>0$, it follows that: $du/dk>0$. This result is again different from Rowthorn's. The explanation is analogous to that provided in the previous section. Investment in new physical capital induces unions to increase nominal wages, with the central bank reacting by decreasing the money supply and so eventually the price level. The consequent rise in real wages is such that the final effect on employment is negative, in spite of the possibly low elasticity of substitution.

The Jacobian with respect to variations in α is:

$$\begin{bmatrix} du/d\alpha \\ dP/d\alpha \end{bmatrix}=\left(\frac{-1}{\frac{\partial CB}{\partial u}\frac{\partial LM}{\partial P}-\frac{\partial CB}{\partial P}\frac{\partial LM}{\partial u}}\right)\begin{bmatrix} \partial LM/\partial P & -\partial CB/\partial P \\ -\partial LM/\partial u & \partial CB/\partial u \end{bmatrix}\begin{bmatrix} \partial CB/\partial \alpha \\ \partial LM/\partial \alpha \end{bmatrix}$$

As in the equivalent case analysed in the previous section, the sign of $du/d\alpha$ depends upon the relative values of k and 1-u (Appendix G); for example, if $k > 1 - u$ then $\frac{\partial CB}{\partial \alpha} > 0$ and $\frac{dLM}{da} > 0$, so that $du/d\alpha > 0$. The explanation for this anti-Blanchard result is analogous to that provided in the case of variations in k, since also the increase in α triggers the mechanism generated by higher wage claims.

6. Desired Inflation Rate of Unemployment

The two curves *LM* and *CB* can be used for policy analysis. The first question we wish to address is whether there exist values for the policy variable *A* and the parameter ξ_B which guarantee a "Desired Inflation Rate of Unemployment" (DIRU: the value of u corresponding to $P = P^*$) equal to zero. The answer to this question is trivial, since the curves *LM* and *CB* shift in the (P, u) space, as shown in the previous section, when *A* and ξ_B change. It is then straightforward that it would always be possible to find appropriate values of *A* and ξ_B which make the two curves crossing on the vertical axis at $P = P^*$, where DIRU is equal to zero. Of course, such combination of fiscal policy and inflation aversion may be difficult to obtain, especially in the European context, where there exists only one Central Bank with given statutes and several Governments (with possibly different objective functions) setting the national social policies which make up the "average" *A* considered here[14].

The present European situations can however be easily characterised by means of our diagrammatic apparatus. First, notice that a sufficient condition to have $P = P^*$ is $\xi_B = \infty$. This is so because the slope of the CB decreases when ξ_B increases, until it becomes horizontal for $\xi_B = \infty$. When analysed with our model, the independence of the European Central Bank, and the drastic increase in its ξ_B in the 1990s, as compared to the average of the previous degrees of conservativeness held by national central banks, imply the tendency for the *CB* curve to become horizontal. Once P^* is obtained in such a way, the problem arises of how to achieve also a DIRU = 0 (or the lowest possible one). It is obvious, in fact, that in this case the rate of unemployment which obtains depends only on where the *LM* curve is placed in the (P, u) space, that is, at what

[14] It would of course be analytically preferable to have the value of *A* set by governments so as to maximise their objective functions, but such an extension lies beyond the aims of this preliminary attempt.

level the value of A is set. In other terms, when $\xi_B = \infty$, there exist infinite possible values for the DIRU.

FIGURE 2 HERE

Figure 2 shows that any proposal to tighten quantitative European social policy must rest on the belief that the *LM* curve crosses the vertical axis above the point corresponding to $P = P^*$. This belief is however weak: if the *LM* curve crossed the vertical axis above point *E*, the rate of unemployment would be lower than zero, which is, strictly speaking, impossible. If this result could be taken in a loose way, but in any case suggestive of what is actually at stage, we could interpret the point $u = 0$ as that corresponding to a "natural" rate of unemployment, in which case an *LM* curve crossing the vertical axis above point *E* would correspond to a period of tensions in the labour market, increasing real wages and wage shares[15]. This is not however our present experience: the whole debate on European unemployment is based on the widespread view that it is above its "natural" value.

If this is indeed the case, if the Central Bank independence is to be maintained and if its degree of conservativeness is not to be decreased (e.g., through changes in its statutes), our analysis leads to a neat policy conclusion: the minimisation of the DIRU would require to *enhance*, not to tighten, social policies. This conclusion should of course be interpreted as not simply related to unemployment benefits, but to all the alternative incomes, including possible forms of basic guaranteed incomes and pensions.

7. Conclusions

The introduction of strategic interaction between monetary policy-makers and trade unions into medium run macroeconomic models with imperfect competition in good and labour market leads to neat results, some of which confirm and extend previous findings, some others having a more unconventional flavour.

Within the former set, the main conclusion is that our policy game supports Rogoff's (1985)

[15] It is straightforward to check that the expression for the wage share in our model is identical to that in Rowthorn (1999b).

idea that a conservative central banker may decrease both inflation and unemployment. This result extends that obtained by Coricelli, Cukierman and Dalmazzo (2000) in a Stackelberg context; this should not be surprising, as it is well known that the Nash interaction structure favours the emergence of results of the Barro and Gordon (1983) type. It is worth stressing that this proposition holds also when benefits enter the unions' utility function (as in LNJ), irrespectively of how social policy is financed.

Some of the less conventional results provide both positive and normative insights. Among the former ones, the first one to emphasise is that the degree of price indexation of social expenditure addressed to the unemployed represents a channel for monetary policy nonneutrality. In the short and medium run, imperfect indexation (together with other nominal rigidities) allows monetary variables to have real effect, but in the long run, when benefits can be reasonably taken as fully indexed to inflation, neutrality results. It must be stressed that this result applies not only when the way social expenditure is financed is not explained, but also when government budget constraints are taken into account, as it should be in a medium run analysis, when the issue of the sustainability of the public debt cannot be disregarded.

Another contribution to the medium run analysis provided by the endogenisation of monetary policy is represented by the critical elements it adds to the debates on the causes of European unemployment, in particular on the effects of insufficient capital accumulation and of a capital-using shift in technology. Our model raises doubts on the generality of the former (i.e., Rowthorn's) explanation, which appears to be possible (but not certain, as it depends upon the relative strength of several influences) only for low values of ξ_B relative to A and if balanced budgets are ruled out (as it was possibly the case in the period considered by Rowthorn). Doubts are raised also for Blanchard's technology driven explanation, which is more difficult to support than that by Rowthorn. The effect of a capital-using shift in technology on prices and unemployment is in fact harder to establish, since it depends also on the value of the stock of capital (in efficiency units) relative to the unemployment rate, irrespectively of the way benefits are financed.

On the normative side, by assuming balanced budgets and introducing the concept of DIRU, we show that there exist values for per-capita social expenditure and inflation aversion which guarantee a DIRU equal to zero. These values may be however difficult to obtain in the presence of a European Central Bank with given and high inflation aversion and several Governments setting national social policies. When inflation aversion is high, the central bank is however able to achieve the desired price level and the rate of unemployment which results depends only on the value of social expenditure, i.e., there exists a continuum of values for the DIRU. This raises the question of whether in the present European context the minimisation of the DIRU would require to tighten or to expand social policies. With respect to this issue, our analysis leads to the unconventional conclusion that, if the central bank independence is to be maintained and if its degree of conservativeness is not to be decreased, the minimisation of the DIRU would require to strengthen social policies.

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FIGURE 1

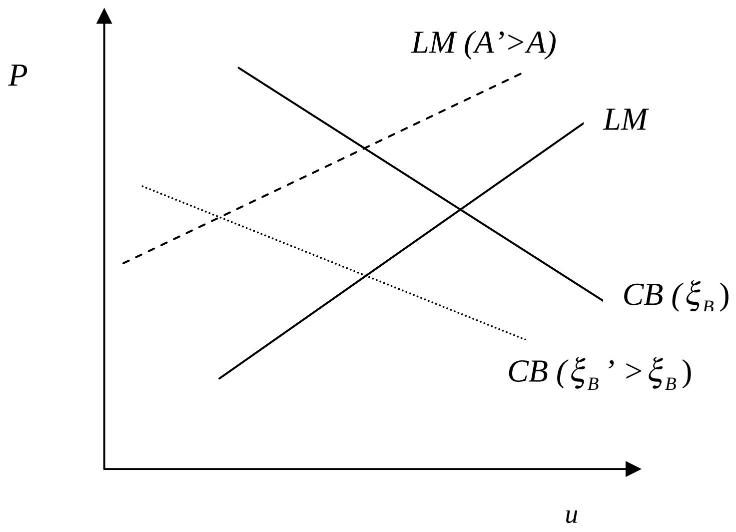
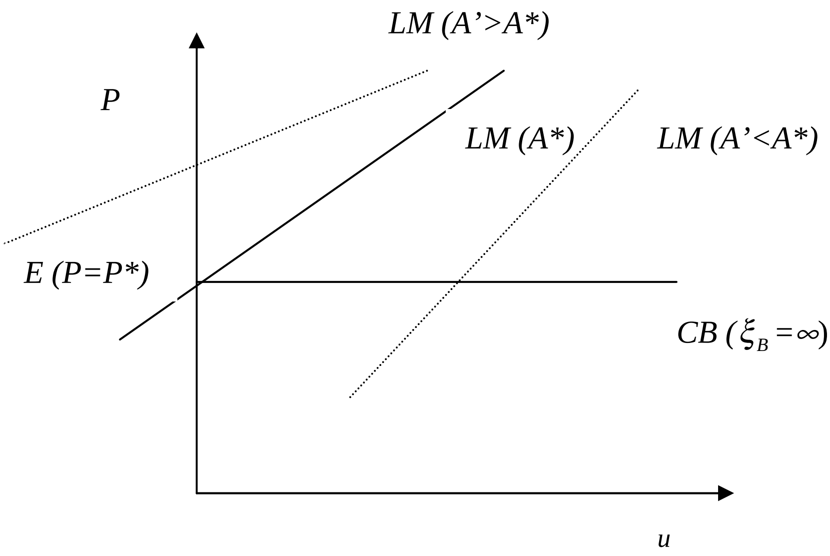


FIGURE 2



Appendix A. The household's problem

U_i is maximised by taking n_i, and thus R_i, as given, so as to obtain the optimum for consumption C_i and money holding M_i. The FOC are:

$$\begin{cases} \dfrac{\partial U_i / \partial C_i}{\partial U_i / \partial M_i} = P \\ PC_i + M_i = R_i \end{cases}$$

from which we get:

$$C_i = \frac{g}{1-g} \cdot \frac{M_i}{P}$$

$$M_i = R_i - \frac{g}{1-g} M_i$$

thus obtaining the household's demand for money holding and the consumption function:

$$M_i = (1-g)R_i \qquad C_i = g\frac{R_i}{P}$$

We now differentiate with respect to C_{ij} the CES part of the utility function and obtain the FOC:

$$\begin{cases} \dfrac{\partial U_i / \partial C_{ik}}{\partial U_i / \partial C_{ih}} = \dfrac{P_k}{P_h} = SMS_{hk} \\ \sum_j P_j C_{ji} = -M_i + R_i \end{cases}$$

being $SMS_{hk} = \left(\frac{C_h}{C_k}\right)^{1/\theta}$ these equations imply:

$$\frac{P_k}{P_k} = \left(\frac{C_h}{C_k}\right)^{1/\theta} \text{ and } \qquad \frac{C_h}{C_k} = \left(\frac{P_k}{P_k}\right)^{\theta}$$

By use of this and the budget constraint, we derive the demand function of household *i* for good *j*:

$$C_{ji} = \left(\frac{P_i}{P}\right)^{-\theta} \left(\frac{g}{J}\right)\left(\frac{R_i}{P}\right)$$

And thus:

$$C_{ij} = \left(\frac{P_j}{P}\right)^{-\theta} \left(\frac{g\overline{R}_i}{JP}\right)$$

$$C_i = g\frac{R_i}{P}$$

$$M_i = (1-g)R_i$$

which leads to:

$$Y_j = \left(\frac{P_j}{P}\right)^{-\theta} \frac{1}{J} \cdot \frac{g}{1-g} \cdot \frac{\overline{M}}{P}$$

$$Y = \frac{g}{1-g} \cdot \frac{\overline{M}}{P}$$

$$\frac{Y_j}{Y} = \frac{1}{J}\left(\frac{P_j}{P}\right)^{-\theta}$$

Now, since $C_i = g\frac{R_i}{P}$, $M_i = (1-g)R_i$ are linearly dependent on R, we can write: $V_i = C_i^g \left(\frac{M_i}{P}\right)^{1-g}$. The ratio $\frac{V_i}{R_i / P}$ is constant, so that we can write $\mu = \frac{V_i}{R_i / P}$ and $U_i = \mu \frac{R_i}{P} - \mu n_i^{\beta}$.

Appendix B: The unions' problem

The FOC is:

$$\frac{W_j}{W_j - A} + \varepsilon_{SN}\left(\frac{\partial N_j}{\partial (W_j / P)}\frac{W_j / P}{N_j}\right) = 0$$

From the FOC we get:

$$\frac{\partial W_j / P}{\partial N_j} = -\frac{1}{\theta}\left(\frac{JY_j}{Y}\right)^{-\frac{1}{\theta}-1}\alpha\left(\frac{\theta-1}{\theta}\right)\left(\frac{Y_j}{\Lambda_N N_j}\right)^{\frac{1}{\sigma}}\Lambda_N\left[\frac{J}{Y}\frac{\partial Y_j}{\partial N_j}\right] +$$

$$+\frac{1}{\sigma}\left(\frac{JY_j}{Y}\right)^{-\frac{1}{\theta}}\alpha\left(\frac{\theta-1}{\theta}\right)\left(\frac{Y_j}{\Lambda_N N_j}\right)^{\frac{1}{\sigma}-1}\Lambda_N\left[\frac{1}{\Lambda_N N_j}\frac{\partial Y_j}{\partial N_j} - \frac{Y_j}{\Lambda_N N_j^2}\right]$$

$$-\frac{1}{\theta}\left(\frac{JY_j}{Y}\right)^{-\frac{1}{\theta}-1}\alpha\left(\frac{\theta-1}{\theta}\right)\left(\frac{Y_j}{\Lambda_N N_j}\right)^{\frac{1}{\sigma}}\Lambda_N\left[\frac{J}{Y}\frac{\partial Y_j}{\partial N_j}\right]$$

It should be noted that the derivative of $\left(\frac{JY_j}{Y}\right)^{-\frac{1}{\theta}}$ with respect to N_j is equal to $-\frac{1}{\theta}\left(\frac{JY_j}{Y}\right)^{-\frac{1}{\theta}-1}\left[\frac{J}{Y}\frac{\partial Y_j}{\partial N_j}\right]$ because it is assumed, in line with Rowthorn (1979) and LNJ, that firms cannot influence the aggregate level of demand

It follows that:

$$\frac{\partial W_j / P}{\partial N_j} = -\frac{1}{\theta}\left(\frac{JY_j}{Y}\right)^{-\frac{1}{\theta}}\alpha\left(\frac{\theta-1}{\theta}\right)\left(\frac{Y_j}{\Lambda_N N_j}\right)^{\frac{1}{\sigma}}\Lambda_N\left[\frac{1}{Y_j}\frac{\partial Y_j}{\partial N_j}\right] +$$

$$+\frac{1}{\sigma}\left(\frac{JY_j}{Y}\right)^{-\frac{1}{\theta}}\alpha\left(\frac{\theta-1}{\theta}\right)\left(\frac{Y_j}{\Lambda_N N_j}\right)^{\frac{1}{\sigma}}\Lambda_N\left[\frac{1}{Y_j}\frac{\partial Y_j}{\partial N_j} - \frac{1}{N_j}\right]$$

that is:

$$\frac{\partial W_j / P}{\partial N_j} = \left(W_j / P\right)\left[\left(\frac{1}{\sigma} - \frac{1}{\theta}\right)\frac{\partial Y_j}{\partial N_j}\frac{1}{Y_j} - \frac{1}{\sigma N_j}\right]$$

and thus:

$$\frac{\partial W_j / P}{\partial N_j}\frac{N_j}{W_j / P} = \left[\left(\frac{1}{\sigma} - \frac{1}{\theta}\right)\frac{\partial Y_j}{\partial N_j}\frac{N_j}{Y_j} - \frac{1}{\sigma}\right]$$

Since it is:

$$\frac{\partial Y_j}{\partial N_j} = \left(\frac{\sigma}{\sigma-1}\right)\left(\alpha(\Lambda_N N_j)^{\frac{\sigma-1}{\sigma}}(1-\alpha)(\Lambda_K K_j)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}-1} \cdot \left(\frac{\sigma-1}{\sigma}\right)\alpha(\Lambda_N N_j)^{\frac{-1}{\sigma}}\Lambda_N =$$

$$= \alpha Y_j^{\frac{1}{\sigma}} \cdot (\Lambda_N N_j)^{\frac{-1}{\sigma}}\Lambda_N = \alpha\left(\frac{Y_j}{\Lambda_N N_j}\right)^{\frac{1}{\sigma}}\Lambda_N$$

that is:

$$\frac{\partial Y_j}{\partial N_j}\frac{N_j}{Y_j} = \alpha\left(\frac{Y_j}{\Lambda_N N_j}\right)^{\frac{1-\sigma}{\sigma}}$$

we obtain:

$$\frac{\partial W_j / P}{\partial N_j}\frac{N_j}{W_j / P} = \alpha\left(\frac{1}{\sigma}-\frac{1}{\theta}\right)\left(\frac{Y_j}{\Lambda_N N_j}\right)^{\frac{1-\sigma}{\sigma}} - \frac{1}{\sigma}$$

But since it is:

$$\left(\frac{Y_j}{\Lambda_N N_j}\right) = \left[\alpha + (1-\alpha)k_j^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \quad \text{where } k_j = \frac{\Lambda_K K_j}{\Lambda_N N_j}$$

substitution into the previous equation gives:

$$\frac{\partial W_j / P}{\partial N_j}\frac{N_j}{W_j / P} = \alpha\left(\frac{1}{\sigma}-\frac{1}{\theta}\right)\left(\alpha + (1-\alpha)k_j^{\frac{\sigma-1}{\sigma}}\right)^{-1} - \frac{1}{\sigma}$$

Simple manipulation leads to:

$$\frac{\partial W_j / P}{\partial N_j}\frac{N_j}{W_j / P} = -\frac{\frac{\alpha}{\theta} + \left(\frac{1-\alpha}{\sigma}\right)k_j^{\frac{\sigma-1}{\sigma}}}{\alpha + (1-\alpha)k_j^{\frac{\sigma-1}{\sigma}}}$$

and so:

$$\frac{\partial N_j}{\partial W_j / P}\frac{W_j / P}{N_j} = -\frac{\alpha + (1-\alpha)k_j^{\frac{\sigma-1}{\sigma}}}{\frac{\alpha}{\theta} + \left(\frac{1-\alpha}{\sigma}\right)k_j^{\frac{\sigma-1}{\sigma}}}$$

that is:

$$\frac{\partial N_j}{\partial W_j / P}\frac{W_j / P}{N_j} = -\left[\sigma + \frac{\alpha\left(1-\frac{\sigma}{\theta}\right)}{\frac{\alpha}{\theta} + \left(\frac{1-\alpha}{\sigma}\right)k_j^{\frac{\sigma-1}{\sigma}}}\right]$$

Hence from the FOC we obtain:

$$\frac{W_j}{W_j - A} = \varepsilon_{NS}\left[\sigma + \frac{\alpha\left(1-\frac{\sigma}{\theta}\right)}{\frac{\alpha}{\theta}+\left(\frac{1-\alpha}{\sigma}\right)k_j^{\frac{\sigma-1}{\sigma}}}\right]$$

We now move to aggregate values:

$$k = \frac{\Lambda_K K}{\Lambda_N L} = \frac{\Lambda_K J\overline{K}}{\Lambda_N J\overline{L}} = \frac{\Lambda_K \overline{K}}{\Lambda_N \overline{L}} = \overline{k}$$

but since $N/L = 1\text{-}u$, it is:

$$\frac{\Lambda_K K}{\Lambda_N N} = \frac{\Lambda_K K}{\Lambda_N L(1-u)} = \frac{k}{1-u}$$

Simmetry guarantees that all the endogenous variables are equal to their average values (denoted with a bar), we get:

$$\frac{\partial \overline{N}}{\partial W/P}\frac{W/P}{\overline{N}} = -\left[\sigma + \frac{\alpha\left(1-\frac{\sigma}{\theta}\right)}{\frac{\alpha}{\theta}+\left(\frac{1-\alpha}{\sigma}\right)\left(\frac{k}{1-u}\right)^{\frac{\sigma-1}{\sigma}}}\right]$$

and so:

$$\frac{W}{W - A} = \varepsilon_{NS}\left[\sigma + \frac{\alpha\left(1-\frac{\sigma}{\theta}\right)}{\frac{\alpha}{\theta}+\left(\frac{1-\alpha}{\sigma}\right)\left(\frac{k}{1-u}\right)^{\frac{\sigma-1}{\sigma}}}\right]$$

If $A = (1-\varphi u)W + \varphi u B$, we obtain Rowthorn's (1999) equation:

$$\frac{1}{\varphi u\left(1-\frac{B}{W}\right)} = \varepsilon_{NS}\left[\sigma + \frac{\alpha\left(1-\frac{\sigma}{\theta}\right)}{\frac{\alpha}{\theta}+\left(\frac{1-\alpha}{\sigma}\right)\left(\frac{k}{1-u}\right)^{\frac{\sigma-1}{\sigma}}}\right]$$

which can be written as:

$$W=\left\{1-\frac{1}{\varepsilon_{NS}\left[\sigma+\dfrac{\alpha\left(1-\dfrac{\sigma}{\theta}\right)}{\dfrac{\alpha}{\theta}+\left(\dfrac{1-\alpha}{\sigma}\right)\left(\dfrac{k}{1-u}\right)^{\frac{\sigma-1}{\sigma}}}\right]\varphi u}\right\}^{-1}B$$

Alternatively, the monopoly union problem could be tackled by starting from:

$$\frac{W_j}{W_j-\overline{A}}=-\varepsilon_{NS}\left[\alpha\left(\frac{\theta-\sigma}{\sigma\theta}\right)\left(\frac{Y_j}{\Lambda_N N_J}\right)^{\frac{1-\sigma}{\sigma}}-\frac{1}{\sigma}\right]^{-1}$$

$\overline{A}$ is the *average* alternative income: $\overline{A}=(1-\varphi u)\overline{W}+\varphi u\overline{B}$, where B is equal in all sectors: $\overline{B}=B/J$ with B equal to the aggregate value. Symmetry provides:

$$\frac{W}{W-A}=-\varepsilon_{NS}\left[\alpha\left(\frac{\theta-\sigma}{\sigma\theta}\right)\left(\frac{Y}{\Lambda_N N}\right)^{\frac{1-\sigma}{\sigma}}-\frac{1}{\sigma}\right]^{-1}$$

and so:

$$\frac{1}{1-(B/W)}=-\frac{\varepsilon_{NS}\varphi u}{\left[\alpha\left(\dfrac{\theta-\sigma}{\sigma\theta}\right)\left(\dfrac{Y}{\Lambda_N N}\right)^{\frac{1-\sigma}{\sigma}}-\dfrac{1}{\sigma}\right]}$$

that is:

$$W=\frac{\varepsilon_{NS}\varphi u}{\varepsilon_{SN}\varphi u+\left[\alpha\left(\dfrac{\theta-\sigma}{\sigma\theta}\right)\left(\dfrac{Y}{\Lambda_N N}\right)^{\frac{1-\sigma}{\sigma}}-\dfrac{1}{\sigma}\right]}B$$

and:

$$\left(\varepsilon_{SN}-\frac{1}{\sigma\varphi u}\right)\left(\frac{Y}{\Lambda_N N}\right)^{\frac{1}{\sigma}}+\frac{\alpha}{\varphi u}\left(\frac{\theta-\sigma}{\sigma\theta}\right)\left(\frac{Y}{\Lambda_N N}\right)^{\frac{2-\sigma}{\sigma}}=\frac{\theta\varepsilon_{NS}}{\alpha(\theta-1)}\frac{B}{P\Lambda_N}$$

If the alternative income A were constant, we would get:

$$W = \frac{\varepsilon_{NS}}{\varepsilon_{SN} + \left[\alpha\left(\frac{\theta-\sigma}{\sigma\theta}\right)\left(\frac{Y}{\Lambda_N N}\right)^{\frac{1-\sigma}{\sigma}} - \frac{1}{\sigma}\right]} A$$

Appendix C: The CB's reaction function

In order to calculate $\partial N / \partial M$, we implicitly differentiate $\frac{W}{P} = \alpha\left(\frac{\theta-1}{\theta}\right)\left(\frac{Y}{\Lambda_N N}\right)^{\frac{1}{\sigma}} \Lambda_N$ with respect to N and M:

$$\frac{W}{P} - \alpha\left(\frac{\theta-1}{\theta}\right)\left(\frac{\frac{g}{1-g}\frac{M}{P}}{\Lambda_N N}\right)^{\frac{1}{\sigma}} \Lambda_N \quad \rightarrow N\left(\frac{W}{P}, \frac{M}{P}\frac{g}{1-g}\right)$$

and so:

$$dN\left[-\left(-\frac{1}{\sigma}\right)\alpha\left(\frac{\theta-1}{\theta}\right)\left(\frac{Y}{\Lambda_N N}\right)^{\frac{1-\sigma}{\sigma}} \Lambda_N \cdot \frac{Y}{\Lambda_N N^2}\right] + dM\left[-\frac{1}{\sigma}\alpha\left(\frac{\theta-1}{\theta}\right)\left(\frac{Y}{\Lambda_N N}\right)^{\frac{1-\sigma}{\sigma}} \Lambda_N \cdot \frac{\frac{g}{1-g}\frac{1}{P}}{\Lambda_N N}\right] = 0$$

that is:

$$dN\left[-\left(-\frac{1}{\sigma}\right)\alpha\left(\frac{\theta-1}{\theta}\right)\left(\frac{Y}{\Lambda_N N}\right)^{\frac{1}{\sigma}} \Lambda_N \cdot \frac{1}{N}\right] + dM\left[-\frac{1}{\sigma}\alpha\left(\frac{\theta-1}{\theta}\right)\left(\frac{Y}{\Lambda_N N}\right)^{\frac{1}{\sigma}} \Lambda_N \cdot \frac{1}{M}\right] = 0$$

from which we get:

$$dN\left[\frac{1}{N}\right] + dM\left[-\frac{1}{M}\right] = 0$$

that is: $\frac{dN}{dM} = \frac{N}{M}$

The FOC is thus:

$$\mu\frac{1}{P} + \mu\left(\frac{1}{P}\frac{g}{1-g}\right) - \mu A(L-N)\left(\frac{Y}{M^2}\frac{1-g}{g}\right) - \mu\frac{A}{P}\frac{N}{M} - \mu\frac{N}{M} - \xi_B(P-P^*)\frac{g}{(1-g)Y} = 0$$

and, by multiplying both sides by M:

$$\frac{M}{P}\frac{\mu}{1-g}\mu\frac{A}{P}(L-N) - \mu\frac{A}{P}N - \mu N - \xi_B P(P-P^*) = 0$$

that is:

$$\frac{M}{P}\frac{\mu}{1-g} - \mu\frac{A}{P}L - \mu N - \xi_B P(P-P^*) = 0$$

and so:

$$\mu\left(\frac{Y}{g} - N\right) - \mu\frac{A}{P}L - \xi_B P(P-P^*) = 0$$

Appendix D: SOC for the CB

We take the FOC:

$$\mu\frac{1}{P}+\mu\left(\frac{1}{P}\frac{g}{1-g}\right)-\mu A(L-N)\left(\frac{Y}{M^2}\frac{1-g}{g}\right)-\mu\frac{A}{P}\frac{N}{M}-\mu\frac{N}{M}-\xi_B(P-P^*)\frac{g}{(1-g)Y}=0$$

and differentiate keeping Y constrant. Knowing that $dN/dM = N/M$, we get:

$$\frac{\partial^2\Omega}{\partial M^2}=-\mu\left(\frac{Y}{M^2}\frac{g}{1-g}\right)+\mu A(L-N)2\left(\frac{Y}{M^3}\frac{1-g}{g}\right)+\mu A\frac{N}{M}\left(\frac{Y}{M^2}\frac{1-g}{g}\right)+\mu\frac{A}{P}\frac{N}{M^2}+$$

$$-\mu\frac{A}{P}\frac{1}{M}\frac{N}{M}+\mu A\frac{N}{M}\left(\frac{Y}{M^2}\frac{1-g}{g}\right)-\mu\left(\frac{Y}{M^2}\right)-\mu\frac{1}{M}\frac{N}{M}+\mu N\frac{1}{M^2}-\xi_B\left(\frac{g}{(1-g)Y}\right)^2$$

that is:

$$\frac{\partial^2\Omega}{\partial M^2}=-\mu\left(\frac{Y}{M^2}\frac{g}{1-g}\right)+\frac{2}{M^2}\mu\left(\frac{A}{P}\right)(L-N)+\frac{1}{M^2}\mu\frac{A}{P}N+\frac{1}{M^2}\mu\frac{A}{P}N+$$

$$-\frac{1}{M^2}\mu\frac{A}{P}N+\frac{1}{M^2}\mu\frac{A}{P}-\xi_B\left(\frac{g}{(1-g)Y}\right)^2$$

and so:

$$\frac{\partial^2\Omega}{\partial M^2}=-\mu\left(\frac{Y}{M^2}\frac{g}{1-g}\right)+2\frac{\mu}{M^2}\left[\left(\frac{A}{P}\right)(L-N)+\frac{A}{P}N\right]-\xi_B\left(\frac{g}{(1-g)Y}\right)^2$$

which implies:

$$\frac{\partial^2\Omega}{\partial M^2}=\frac{\mu}{M^2}\left[-\left(\frac{Y}{1-g}\right)+2\frac{A}{P}L\right]-\xi_B\left(\frac{g}{(1-g)Y}\right)^2$$

Now, being $\frac{P^2}{M^2}=\left(\frac{g}{(1-g)Y}\right)^2$, we get:

$$\frac{\partial^2\Omega}{\partial M^2}=\frac{\mu}{M^2}\left[-\left(\frac{Y}{1-g}\right)+2\frac{A}{P}L-\xi_B P^2\right]$$

By substituting from the FOC: $\frac{A}{P}L=\left(\frac{Y}{g}-N\right)-\frac{\xi_B}{\mu}P(P-P^*)$

We have:

$$\frac{\partial^2\Omega}{\partial M^2}=\frac{\mu}{M^2}\left[-\left(\frac{Y}{1-g}\right)+2\frac{Y}{g}-2N-2\frac{\xi_B}{\mu}P(P-P^*)-\xi_B P^2\right]$$

which is less than zero for $\left[-\left(\frac{Y}{1-g}\right)+2\frac{Y}{g}-2N-2\frac{\xi_B}{\mu}P(P-P^*)-\xi_B P^2\right]<0$, that is for $Y\frac{2-3g}{g(1-g)}<0$, and so for $g>2/3$ (and P* sufficiently small).

Appendix E: Sign of dP/du

By implicitly differentiating:

$$\left(\frac{Y}{g}-N\right)=\left\{L(1-u)\left(\frac{1}{g}\left(\alpha+(1-\alpha)\left(\frac{k}{1-u}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}\Lambda_N-1\right)\right\}=L(1-u)\left(\frac{1}{g}\left(\frac{Y}{\Lambda_N N}\right)\Lambda_N-1\right)$$

we get:

$$dP\left\{\frac{\mu AL}{P^2}-\xi_B(2P-P^*)\right\}+$$
$$du\left\{(-L)\left(\frac{Y/\Lambda N}{g}\Lambda-L\right)+L(1-u)\frac{1}{g}\left(\frac{\sigma}{\sigma-1}\right)\left(\frac{Y}{N\Lambda}\right)^{\frac{1}{\sigma}}(1-a)\left(\frac{\sigma-1}{\sigma}\right)\left(\frac{k}{1-u}\right)^{-1/\sigma}\left(\frac{k}{(1-u)^2}\right)(-1)\Lambda\right\}=0$$

which we write as:

$$dP\left\{\frac{\mu AL}{P^2}-\xi_B(2P-P^*)\right\}+du\left\{\frac{\partial FOC_{BC}}{\partial u}\right\}=0$$

It is straightforward that $\left\{\frac{\partial FOC_{BC}}{\partial u}\right\}<0$, so that:

$$\frac{dP}{du}=-\frac{\frac{\partial FOC_{BC}}{\partial u}}{\left\{\frac{\mu AL}{P^2}-\xi_B(2P-P^*)\right\}}$$

Appendix F: Signs of the derivatives in the Jacobians

… with respect to ξ_B

$$\partial Lm / \partial u = I + II$$

where:

$$I = \left(\frac{1}{\sigma-1}\right)(1-a)\left(\frac{\sigma-1}{\sigma}\right)\left(\frac{k}{1-u}\right)^{-1/\sigma}\left(\frac{k}{(1-u)^2}\right)(-1)Q^{\frac{2-\sigma}{\sigma-1}} X < 0$$

$$II = Q^{\frac{1}{\sigma-1}}\left\{\alpha\left(\frac{\theta-\sigma}{\theta\sigma}\right)(-1)Q^{-2}(1-a)\left(\frac{\sigma-1}{\sigma}\right)\left(\frac{k}{1-u}\right)^{-1/\sigma}\left(\frac{k}{(1-u)^2}\right)(-1)\right\} < 0$$

$$Q = \alpha + (1-\alpha)\left(\frac{k}{1-u}\right)^{\frac{\sigma-1}{\sigma}}$$

$$X = \left[\varepsilon_{SN} - \frac{1}{\sigma} + \alpha\left(\frac{\theta-\sigma}{\sigma\theta}\right)\left(\alpha + (1-\alpha)\left(\frac{k}{1-u}\right)^{\frac{\sigma-1}{\sigma}}\right)^{-1}\right] = \left[\varepsilon_{SN} - \frac{1}{\sigma} + \alpha\left(\frac{\theta-\sigma}{\sigma\theta}\right)Q^{-1}\right]$$

And:

$$\partial Lm / \partial P = \left\{-\frac{\theta\varepsilon_{NS}}{\alpha(\theta-1)P^2}\frac{A}{\Lambda_N}(-1)\right\} > 0$$

$$\partial BC / \partial u = \left\{\frac{\partial FOC_{BC}}{\partial u}\right\} < 0$$

where:

$$\frac{\partial FOC}{\partial u} = \left\{(-L)\left(\frac{Y/\Lambda_N N}{g} - 1\right) + L(1-u)\frac{1}{g}\left(\frac{\sigma}{\sigma-1}\right)\left(\frac{Y}{N\Lambda_N}\right)^{\frac{1}{\sigma}}(1-a)\left(\frac{\sigma-1}{\sigma}\right)\left(\frac{k}{1-u}\right)^{-1/\sigma}\left(\frac{k}{(1-u)^2}\right)(-1)\Lambda_N\right\}$$

$$\partial CB / \partial P = \left\{\frac{\mu AL}{P^2} - \xi_B(2P - P^*)\right\}$$

$$\partial BC / \partial \xi_B = -P(P - P^*) < 0$$

$$\partial Lm / \partial \xi_B = 0$$

… with respect to *k*

$$\frac{\partial BC}{\partial k} = \frac{L(1-u)\mu\Lambda_N}{g}\left(\frac{\sigma}{\sigma-1}\right)Q^{\frac{1}{\sigma-1}}(1-\alpha)\left(\frac{\sigma-1}{\sigma}\right)\left(\frac{k}{1-u}\right)^{-\frac{1}{\sigma}}\left(\frac{1}{1-u}\right) > 0$$

$$\frac{\partial Lm}{\partial k} = III + IV > 0$$

where:

$$III = X\left[\left(\frac{1}{\sigma-1}\right)Q^{\frac{2-\sigma}{\sigma-1}}(1-\alpha)\left(\frac{\sigma-1}{\sigma}\right)\left(\frac{k}{1-u}\right)^{\frac{-1}{\sigma}}\left(\frac{1}{1-u}\right)\right] > 0$$

$$IV = Q^{\frac{1}{\sigma-1}}\left\{\alpha\left(\frac{\theta-\sigma}{\theta\sigma}\right)(-1)Q^{-2}(1-\alpha)\left(\frac{\sigma-1}{\sigma}\right)\left(\frac{k}{1-u}\right)^{\frac{-1}{\sigma}}\left(\frac{1}{1-u}\right)\right\} > 0$$

… with respect to α

$$\frac{\partial BC}{\partial \alpha} = \frac{L(1-u)\mu\Lambda_N}{g}\left(\frac{\sigma}{\sigma-1}\right)Q^{\frac{1}{\sigma-1}}\left[1-\left(\frac{k}{1-u}\right)^{\frac{\sigma-1}{\sigma}}\right]$$

so that:

$\frac{\partial BC}{\partial \alpha} > 0$ when $k > 1-u$ and $\frac{\partial BC}{\partial \alpha} < 0$ when $k < 1-u$.

$$\frac{dLm}{d\alpha} = \left\{\frac{1}{\sigma-1}Q^{\frac{2-\sigma}{\sigma-1}}X\left[1-\left(\frac{k}{1-u}\right)^{\frac{\sigma-1}{\sigma}}\right] + Q^{\frac{1}{\sigma-1}}\left[\left(\frac{\theta-\sigma}{\theta\sigma}\right)Q^{-1} - \alpha\left(\frac{\theta-\sigma}{\theta\sigma}\right)Q^{-2}\left[1-\left(\frac{k}{1-u}\right)^{\frac{\sigma-1}{\sigma}}\right]\right] + \frac{\theta\varepsilon_{NS}}{\alpha^2(\theta-1)}\frac{A}{P\Lambda_N}\right\}$$

that is: $\frac{dLM}{d\alpha} = V + VI + VII$, where:

$$V = \frac{1}{\sigma-1}Q^{\frac{2-\sigma}{\sigma-1}}X\left[1-\left(\frac{k}{1-u}\right)^{\frac{\sigma-1}{\sigma}}\right]$$

$$VI = Q^{\frac{3-2\sigma}{\sigma-1}}\left[\left(\frac{\theta-\sigma}{\theta\sigma}\right)\left(\frac{k}{1-u}\right)^{\frac{\sigma-1}{\sigma}}\right]$$

$$VII = \left\{\frac{\theta\varepsilon_{NS}}{\alpha^2(\theta-1)}\frac{A}{P\Lambda_N}\right\}$$

so that $\frac{\partial Lm}{\partial \alpha} > 0$ when $k > 1-u$ and when I is low, that is for $k \cong 1-u$, whereas $\frac{\partial Lm}{\partial \alpha} < 0$ when $k < 1-u$.

Appendix G. Signs of the derivatives in the Jacobians (Balanced Budgets)

… with respect to ξ_B

$$\frac{\partial BC}{\partial P} = -1$$

$$\frac{\partial BC}{\partial u} = I + II < 0$$

where:

$$I = -\frac{1}{2}(1-u)^{-1/2}\left[L\frac{\mu}{\xi_B}\left[\frac{1}{g}\left(\frac{Y}{N\Lambda_N}\right)\Lambda_N - 1\right]\right]^{1/2} < 0$$

$$II = \frac{1}{2}\left[L(1-u)\frac{\mu}{\xi_B}\left[\frac{1}{g}\left(\frac{Y}{N\Lambda_N}\right)\Lambda_N - 1\right]\right]^{-1/2}\frac{1}{g}\left(\frac{\sigma}{\sigma-1}\right)\left(\frac{Y}{N\Lambda_N}\right)^{\frac{1}{\sigma}}(1-a)\left(\frac{\sigma-1}{\sigma}\right)\left(\frac{k}{1-u}\right)^{-1/\sigma}\left(\frac{k}{(1-u)^2}\right)(-1)\Lambda_N < 0$$

$$\partial Lm/\partial P = \left\{-\frac{\theta\varepsilon_{NS}}{\alpha(\theta-1)P^2}\frac{A}{\Lambda_N}(-1)\frac{1}{1-u}\right\} > 0$$

$$\frac{\partial Lm}{\partial u} = III + IV + V < 0$$

where:

$$III = \left(\frac{1}{\sigma-1}\right)(1-a)\left(\frac{\sigma-1}{\sigma}\right)\left(\frac{k}{1-u}\right)^{-1/\sigma}\left(\frac{k}{(1-u)^2}\right)(-1)Q^{\frac{2-\sigma}{\sigma-1}}X < 0$$

$$IV = Q^{\frac{1}{\sigma-1}}\left\{\alpha\left(\frac{\theta-\sigma}{\theta\sigma}\right)(-1)Q^{-2}(1-a)\left(\frac{\sigma-1}{\sigma}\right)\left(\frac{k}{1-u}\right)^{-1/\sigma}\left(\frac{k}{(1-u)^2}\right)(-1)\right\} < 0$$

$$V = -\frac{\theta\varepsilon_{SN}}{\alpha(\theta-1)P\Lambda_N}A(1-u)^{-2} < 0$$

$$\frac{\partial BC}{\partial \xi_B} = -\frac{\mu}{2\xi_B^2}\left(\frac{\mu}{\xi_B}\right)^{-1/2}\left[L(1-u)\left[\frac{1}{g}\left(\frac{Y}{N\Lambda_N}\right)\Lambda_N - 1\right]\right]^{1/2} < 0$$

$$\frac{\partial Lm}{\partial \xi_B} = 0$$

… with respect to *A*

$$\frac{\partial BC}{\partial A} = 0 \qquad \frac{\partial Lm}{\partial A} = -\frac{\theta\varepsilon_{SN}}{\alpha(\theta-1)\Lambda_N P}\left(\frac{1}{1-u}\right) < 0$$

… with respect to *k*

$$\frac{\partial BC}{\partial k}=\frac{1}{2}\left[L(1-u)\frac{\mu}{\xi_B}\left[\frac{1}{g}\left(\frac{Y}{N\Lambda_N}\right)\Lambda_N-1\right]\right]^{-1/2}\frac{L(1-u)\mu\Lambda_N}{g\xi_B}\left(\frac{\sigma}{\sigma-1}\right)\left(\frac{Y}{N\Lambda}\right)^{\frac{1}{\sigma}}(1-a)\left(\frac{\sigma-1}{\sigma}\right)\left(\frac{k}{1-u}\right)^{-1/\sigma}\left(\frac{1}{1-u}\right)>0$$

$$\frac{\partial LM}{\partial k}=V+VI>0 \qquad \text{where:}$$

$$VI=X\left[\left(\frac{1}{\sigma-1}\right)Q^{\frac{2-\sigma}{\sigma-1}}(1-\alpha)\left(\frac{\sigma-1}{\sigma}\right)\left(\frac{k}{1-u}\right)^{\frac{-1}{\sigma}}\left(\frac{1}{1-u}\right)\right]>0$$

$$VII=Q^{\frac{1}{\sigma-1}}\left\{\alpha\left(\frac{\theta-\sigma}{\theta\sigma}\right)(-1)Q^{-2}(1-\alpha)\left(\frac{\sigma-1}{\sigma}\right)\left(\frac{k}{1-u}\right)^{\frac{-1}{\sigma}}\left(\frac{1}{1-u}\right)\right\}>0$$

… with respect to α

$$\frac{\partial BC}{\partial \alpha}=\frac{1}{2}\left[L(1-u)\frac{\mu}{\xi_B}\left[\frac{1}{g}\left(\frac{Y}{N\Lambda_N}\right)\Lambda-1\right]\right]^{-1/2}\frac{L(1-u)\mu\Lambda_N}{g\xi_B}\left(\frac{\sigma}{\sigma-1}\right)\left(\frac{Y}{N\Lambda_N}\right)^{\frac{1}{\sigma}}\left[1-\left(\frac{k}{1-u}\right)^{\frac{\sigma-1}{\sigma}}\right]$$

$$\frac{dLM}{da}=\left\{\frac{1}{\sigma-1}Q^{\frac{2-\sigma}{\sigma-1}}X\left[1-\left(\frac{k}{1-u}\right)^{\frac{\sigma-1}{\sigma}}\right]+Q^{\frac{1}{\sigma-1}}\left[\left(\frac{\theta-\sigma}{\theta\sigma}\right)Q^{-1}-\alpha\left(\frac{\theta-\sigma}{\theta\sigma}\right)Q^{-2}\left[1-\left(\frac{k}{1-u}\right)^{\frac{\sigma-1}{\sigma}}\right]\right]+\frac{\theta\varepsilon_{NS}}{\alpha^2(\theta-1)}\frac{A}{P\Lambda_N}\right\}$$

so that: $\frac{dLM}{da}=VIII+IX+X$, where:

$$VIII=\frac{1}{\sigma-1}Q^{\frac{2-\sigma}{\sigma-1}}X\left[1-\left(\frac{k}{1-u}\right)^{\frac{\sigma-1}{\sigma}}\right]$$

$$IX=Q^{\frac{3-2\sigma}{\sigma-1}}\left[\left(\frac{\theta-\sigma}{\theta\sigma}\right)\left(\frac{k}{1-u}\right)^{\frac{\sigma-1}{\sigma}}\right]$$

$$X=\left\{\frac{\theta\varepsilon_{NS}}{\alpha^2(\theta-1)}\frac{A}{P\Lambda_N}\right\}$$