The effects of the minimum wage in an economy with tax evasion

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A model of the labor market is built where imperfect detection in case of auditing induces underreporting of earnings. The introduction of the minimum wage makes some workers increase compliance, boosting fiscal revenues. A spike at the minimum wage level appears in the distribution of earnings. The model predicts a positive correlation between the size of the spike at the minimum wage level and the size of the informal economy. Empirical evidence supporting this prediction is presented.

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1 Introduction

What are the fiscal implications of introducing or increasing the minimum wage? How can we explain the very high spike at the minimum wage level appearing in the wage distribution of some countries? This paper contributes to answering these questions by studying the effects of the interaction between tax evasion and minimum wage legislation.

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A simple model of the labour market is built where underreporting of earnings is made possible by imperfect detection of tax evasion. When auditing a firm the tax authority may discover only a part of its true tax liability. Even when fines are imposed on risk neutral firms and the probability of auditing is given, imperfect detection alone is able to generate an internal solution to the tax evasion decision. This is due to the fact that fines are imposed on the difference (if positive) between discovered income and declared income.

The introduction of the minimum wage induces some worker-firm pairs to increase compliance, while pushing others out of the formal labour market into the black economy or into inactivity. The increase in compliance is due to the fact that the minimum wage poses a constraint to reporting behavior, as agents have to choose whether to report nothing or to report at least the minimum wage. When faced with such a restriction agents may prefer to increase their reporting to the minimum wage level than to decrease it to zero. The overall effect when enforcement is not too effective is to unambigously increase fiscal revenues. The distribution of the fiscal burden is also altered. Moreover, an otherwise smooth distribution of declared earnings is transformed by the introduction of the minimum wage into a distribution presenting a spike at the minimum wage level.

The model also predicts a positive correlation between the size of the spike at the minimum wage level and the size of the informal economy. Empirical evidence supporting this prediction is presented.

The minimum wage is the subject of a rich literature and policy debate[1]. Large efforts have also been devoted to the theoretical and empirical study of tax evasion and the shadow economy[2]. However, the aspects of the interaction between minimum wage legislation and tax evasion investigated in this paper have, to the best of my knowledge, never been analysed before.

A World Bank study on labour markets in Eastern Europe and the Former Soviet Union (World Bank, 2005) notices how in several countries in the region "disproportionately high shares of workers cluster on declared wages at or just above the minimum wage (with evidence of additional undeclared incomes above the minimum), creating incentives to sustain a high minimum wage to sustain tax revenue" and calls for further research on this aspect of

[1] See Brown (1999) for a review.

[2] See Andreoni et al. (1998) or Slemrod and Yitzhaki (2002) for surveys on tax evasion and Schneider and Enste (2000) for a survey on the shadow economy.

minimum wage policy. This is indeed the aim of the paper.

The importance for policymakers of contrasting tax evasion and increasing budget revenues by establishing lower bounds to the amount of taxes and social security contributions a person in formal employment has to pay is being stressed by two recent reforms in Bulgaria and Croatia. In Bulgaria minimum wage is fixed by national level negotiations, but since the beginning of 2003 employers have been required to pay social security contributions on the basis of minimum social insurance thresholds, varying along economic activity and occupational groups (Hristoskov, 2004, and Tomev, 2004.) In Croatia, where the minimum wage is not prescribed by law, a minimum basis for calculation of obligatory social security contributions has been established in 2003 (Crnković-Pozaić, 2004.) These reforms have introduced instruments specifically aimed at tackling the issue of underreporting, at least with regards to social security contributions. In countries where such a reform has not been conducted, it is the minimum wage that is playing the role.

The literature on tax evasion has been mainly focused on personal income tax and on the compliance decision by an individual filling the tax declaration form. However, due to the tax withholding and information reporting systems present in many countries, this is not an accurate description for the case of employed labour. Indeed, the rate of non-compliance for wages and salaries at the stage of filling the tax declaration form is often negligible. For instance, Klepper and Nagin (1989) report a mere 0.1% non-compliance for wages and salaries in the US, lower than for any other income category. The study of tax evasion by employed labour is however of particular interest as fiscal imposition on labour in the form of social security contributions (SSC) and personal income tax (PIT) represents the bulk of fiscal revenues in many countries[3]. A contribution of the paper is to present a tractable model of tax evasion by employed labour.

The literature on minimum wage has been particularly concerned about its impact on the distribution of earnings. A spike at the minimum wage level has been observed in several instances (see for instance DiNardo et al., 1996, Rama, 2001, Neumark et al. 2004, Dickens and Manning, 2004). Such a spike has been defined as a "puzzle" for several standard types of labour market models (Brown, 1999) and as an "anomalous finding from the standpoint

[3]In EU15 as a whole labour taxes contributed in 2002 around 50% of total tax receipts (Eurostat, 2004), while in several Central and Eastern European countries social security contributions alone represent more than a third of total tax revenues (UNECE, 2004.)

of the standard model of the low wage labor market" (Card and Krueger, 1995, p. 152.) Proposed rationalizations include reductions in non-wage compensation or increases in required effort to offset a binding minimum wage, flatter earnings profiles, adjustments in the amounts of hours worked. The model presented here proposes an alternative rationale for the observed spike in the framework of a competitive model. The positive correlation between the size of the spike at the minimum wage level and the estimated size of the informal economy in the data presented in section 5 suggests that the mechanism analyzed in this paper indeed contributes to shape the observed distribution of earnings in some countries.

The following section provide some evidence about the relevance of underreporting of earnings. In section 3 some of the related previous literature is discussed. The model is introduced in the fourth section. First it is solved in absence of a minimum wage, then the various effects of introducing the minimum wage are explored. Section 5 investigates empirically the relationship between the size of the spike at minimum wage level and the informal economy. A section exploring the quantitative implications of the model through a simple numerical exercise follows. Section 7 presents some extentions to the basic model. The last section concludes.

2 The relevance of underreporting

Undeclared work is a serious issue in many countries. Reliable data on its extension are difficult to come by, but raw estimates indicate that the phenomenon is relevant, particularly in transition and developing countries, but also in some OECD countries (see Schneider and Enste, 2003, pp. 43-53.)

In a report for the European Commission about "undeclared work in an enlarged union" (Renooy et al., 2004) the authors stress how the practice of paying "envelope wages" above the officially declared minimum "exists in practically all of the Central and Eastern European countries" and in particular in the less developed. For instance, "in Latvia, underreporting (not non-reporting!) of income dominates all other forms of undeclared work", while "in Bulgaria and Romania, it is also a well-known practice used in all sectors of the economy."

According to the Lithuanian statistical office (Statistics Lithuania, 2003) "economic entities in 2002 did not declare at least 23 per cent of wages and salaries formally and informally paid to employees" while "the number of nonregistered employees is considered to reach 104 thousand" (by comparison total employment was 1.4 millions in 2002).

An OECD study of the Baltic countries (OECD, 2003) also reports as a common practice the payment of supplements above the officially declared wage and estimates that in Latvia and Lithuania 20% of private-sector employees earn more than what is officially reported[4]. An employers' survey puts the percentage of wage on which social insurance contributions are actually paid at 86% in Lithuania and at 91% in Latvia. In Bulgaria as of March 2003, 22.6% of people working under an employment contract were estimated to receive actual wages higher than those declared for tax and social security purposes (Tomev, 2004.)

In a poll about wages "under the table" in Russia (Petrova, 2005) 11% of employees reported to receive all of their income under the table, while 8% got part of it under the table. These practices are particularly serious in large cities, where they involve 16% and 17% of employees respectively.

The phenomenon is not limited to CEE economies. An OECD study finds that, among OECD members, beside Hungary, also Mexico and South Korea had "actual social security contribution receipts [...] about 30% short of what could be expected on the basis of scheduled contribution rates and ceilings, compared with total wages and salaries in national accounts which include estimates of undeclared incomes", while Italy, Poland, Spain and Turkey[5] had an estimated shortfall above 20% (OECD, 2004.)

3 Previous literature

The implications for tax evasion of the tax withholding system applied to employees have been studied in a series of papers by Yaniv (1988, 1992, 1995, 1996, 1998, 1999.) Tax evasion stems from separate decisions by the employer and the employee (Yaniv, 1988) or, more in line with the approach proposed here, from a collaboration among the two (Yaniv, 1992.) In the latter article evasion arises when a risk-neutral employer agrees to reduce the tax liability

[4]The Latvian Central Statistical Office publishes data on earnings under the heading "Gross wage of employed *excluding all kinds of irregular payments* by kind of activity" (italics added.)

[5]In Turkey firms that belong to the formal sector are estimated to underreport 28% of their wage bill and for around 50% of employees enrolled in SSK (Social Security Organization) wages reported by employers are at the minimum insurable level (World Bank, 2006.)

of a risk-averse employee by underreporting her wage, in exchange for paying a gross wage below the market rate. In case of detection, a fine is imposed on the employer and the employee has to pay due taxes. Under efficient bargaining and non-decreasing relative risk-aversion by the employee, the contract curve is shown to have a slope such that a higher declared wage corresponds to a higher paid wage. Moreover, with logarithmic utility, the tax withholding system is shown to entail greater evasion than a self-declaration system with the same enforcement parameters. Advanced tax payments are also analysed using prospect theory (Yaniv, 1999). However, the papers by Yaniv are mainly concerned with analyzing the effects of the tax witholding system, while the effect of the minimum wage for evasion behaviour has not been considered.

Fugazza and Jacques (2003) study the effect of the minimum wage in a labour market where tax evasion takes the form of participation in the informal sector. They develop a matching model where workers can search for a job and firms can post vacancies either in the formal or informal sector. Workers differ in the subjective cost of operating in the informal sector and the total number of jobs is given. The authors determine the tightness of the two labour markets and their relative size in terms of active workforce and then study the effect of variations in labour market and fiscal institutions. The model have two different interior equilibria, depending on whether the instantaneous net return of a filled job is higher in the regular or irregular sector. In that framework the introduction of a minimum wage is interpreted as an increase in the exogenously given wage in the formal sector and is found to have an ambiguous effect on the relative size of the two labour markets in both equilibria. On one hand, formal employment is more attractive for workers. On the other hand, to induce firms to post vacancies in the formal sector the relative labour market tightness in the formal vis-à-vis the informal sector has to decrease, thus making search in the formal sector less attractive for workers. The effect on the government budget depends on which equilibrium the economy is in. While Fugazza and Jacques address some of the same issues dealt with in the present paper, the framework of analysis is rather different. In their paper, workers can either completely comply with regulation or be outside of the legal labour market, and wages in the two sectors are exogenously given.

The view that a worker can be involved in both informal and formal activities is held by Cowell (1985). He studies a model where time can be allocated among leisure, work in the formal sector, and work in the informal sector. The effects of fiscal and enforcement parameters on the dimension of the informal sector is investigated. The paper contrasts a self-declaration tax system, where the exogenously given wage is identical in the two sectors of activity, to a Pay As You Earn (PAYE) system, where wages may instead differ. The potential relevance of a discontinuity arising when a worker passes from doing some legal work to being completely in the black economy is stressed. As in the previous paper, wages in the two sectors are exogenously given. Moreover, the implication for the wage distribution of the interaction of tax evasion and minimum wage legislation is explored in none of the above mentioned papers and, to the best of my knowledge, has not been previously addressed in a formal model.

4 The model

The size of the population is exogenously given and normalized to 1. Every individual has an exogenously given productivity y_i , distributed in the population according to pdf $g(y)$ and cdf $G(y)$ on the support $[\underline{y}, \bar{y}]$, where $\underline{y} \geq 0$. We assume that the labour market is competitive, each firm employs one worker, there is no capital, and that production is equal to the labour input. Moreover, there is free entry of firms, firms can observe workers' productivity, and workers can move from one firm to another at no cost.

Firms are risk-neutral and maximize profits π. In an environment without tax evasion profits for a firm employing a worker with productivity y_i are given by:

$$\pi_i = y_i - w_i$$

where w_i is the gross wage[6]. Firms have an obligation to withhold taxes and social security contributions and transfer them to the authorities. Taxation is at the proportional rate $t \in (0, 1)$. Workers' (indirect) utility depends on their net income, so that they maximize the net wage:

$$I_i = w_i(1 - t)$$

The wedge between the gross wage payed by the firm and the net wage received by the worker, tw_i, is paid to fiscal authorities.

[6]No distinction is made between labour cost and gross wage and the two concepts should be considered equivalent in the paper.

Free entry of firms implies that in equilibrium expected profit are zero and this in turn implies, in the full compliance case, that a worker with productivity y_i would receive a gross wage y_i, from which firm would deduct taxes ty_i leaving the worker a net wage $(1-t)y_i$.

In this economy, however, it is possible to evade taxes and social security contributions by not reporting to the authorities part or all of the workers' earnings. A firm employing a worker with productivity y_i has therefore to decide how much of the worker's product to declare to tax authorities, x_i , and how much to conceal, $y_i - x_i$. If $x_i = y_i$, then the firm is fully compliant with regulations. If $x_i = 0$, then the full product is hidden from the authorities and the firm-worker pair operates completely in the black economy. If $x_i \in (0, y_i)$, then there is underreporting.

A worker-firm pair can thus operate in the formal economy, by declaring a strictly positive income, or be completely in the black market, by declaring nothing. A worker can also decide to be inactive. In this case income is normalized to 0.

Tax authorities may inspect firms to find out whether they comply with fiscal regulation. Notice that fines are imposed on firm in case tax evasion is detected. We assume that there is an exogenously given probability of an audit being performed $\gamma \in [0, 1]$.

However, the fact that an audit is performed does not imply that the authority discovers for sure the true tax liability, but it may find evidence to impute an income $\hat{y}_i \in [0, y_i]$, where y_i is the true product. For instance, Feinstein (1991) estimates that IRS examiners managed on average to detect only half of the tax evasion in the forms they audited, while Erard (1997) rejects the null hypothesis of perfect detection in his empirical investigation of a model where detection can be either complete or null. For analytical convenience, we assume that the detection technology is such that the probability of finding an imputable product $\hat{y}_i$ is uniform over the interval $[0, y_i]$, i.e. $\hat{y}_i \sim U_{[0,y_i]}$.

Given a declaration of x_i and collected evidence of a true tax liability of $\hat{y}_i$ the tax authority imposes to the firm, in case $\hat{y}_i > x_i$, the payment of $\theta t\,(\hat{y}_i - x_i)$, consisting of taxes plus an additional fine proportional to assessed tax evasion, thus $\theta > 1$. In case $\hat{y}_i \leq x_i$ the tax authority cannot prove any tax evasion, so no fine is imposed.

Given a true product y_i and a reported one $x_i \in [0, y_i]$ the expected fine in case of auditing, f_i, is:

$$f_i = t\theta \int_{x_i}^{y_i} (\hat{y}_i - x_i) g(\hat{y}_i) d\hat{y}_i \qquad where \qquad g(\hat{y}_i) = \frac{1}{y_i}$$

then:

$$f_i = t\theta \frac{1}{y_i} \int_{x_i}^{y_i} (\hat{y}_i - x_i) d\hat{y}_i = t\theta \frac{1}{y_i} \frac{1}{2} (y_i - x_i)^2 \tag{1}$$

Notice that a decrease in reported income, x_i, does not increase the probability that any particular level of income is detected, as this probability is given. However, it increases the probability of paying a fine, as a fine is imposed on the difference between detected and reported income, if positive. In Appendix A we present a different detection technology resulting in expected fines equivalent to (1). Given the detection technology, the expected fraction of evaded income, $y_i - x_i$, that is discovered in case of auditing is:

$$\frac{1}{y_i - x_i} \left[\frac{1}{y_i} \int_{x_i}^{y_i} (\hat{y}_i - x_i) d\hat{y}_i \right] = \frac{1}{2} \frac{y_i - x_i}{y_i} \tag{2}$$

i.e. a fraction corresponding to half the ratio of evaded income over true product. The assumption is thus that it is relatively easy to get away with tax-evasion.

In the economy there is a minimum monthly wage $\bar{w}$, with universal coverage. Workers cannot be legally employed at below the minimum wage, in the meaning that their reported gross wage cannot be below the minimum.

The assumption in the model is that the minimum wage is fixed on a monthly basis for full-time work and that no alternative working-time arrangements are available. This simplifies the analysis and captures some features characterizing Central and Eastern European labour markets. However, in section 7.1 the model is extended to the case where the minimum wage is fixed on an hourly basis, labour supply can vary across workers and underreporting can involve both hours of work and hourly wage.

Below we first determine the equilibrium wage and evasion in case of no minimum wage, then with the minimum wage. Several implications of the interaction between minimum wage and underreporting are also explored. For convenience subscripts are suppressed where not necessary.

4.1 Equilibrium without minimum wage

For a firm employing a worker with productivity y , declaring x , and paying a gross wage w the possible realizations of profits are given by[7]:

$$\pi = \begin{cases} y - w & \text{with probability } 1 - \gamma \\ y - w - f & \text{with probability } \gamma \end{cases}$$

where f , the expected fine in case an audit is conducted, is given by (1).

Therefore, expected profits for the firm are:

$$E\pi = y - w - \gamma f \tag{3}$$

substituting (1) into (3) we get:

$$E\pi = y - w - \gamma\theta t \frac{1}{y}\frac{1}{2}(y - x)^2 \tag{4}$$

The firm chooses both w and x to maximize its profits.

The income I for a worker employed in a firm paying a gross wage w and declaring to fiscal authorities x is given by:

$$I = w - xt \tag{5}$$

this expression captures the fact that taxes and social security contributions are deducted from the worker's declared gross wage x, not from his true gross wage w.

Free entry implies that firms will compete by offering to workers the package (w, x) that maximizes (5) until expected profits go to zero. Thus, the wage equation is obtained by equalling expected profits (4) to zero:

$$w = w(x) = y - \gamma\theta t \frac{1}{y}\frac{1}{2}(y - x)^2 \tag{6}$$

the corresponding income for the worker is given by substituting (6) into (5):

$$I = y - \gamma\theta t \frac{1}{y}\frac{1}{2}(y - x)^2 - xt \tag{7}$$

[7] Actually, when an audit is performed possible realizations of profits are a continuum, due to the stochastic nature of the fine. For expositional convenience the expected value of the fine is considered.

and the solution to the reporting decision problem is obtained by maximizing income given by (7), i.e.:

$$\max_{x\in[0,y]} \qquad y-\gamma\theta t\frac{1}{y}\frac{1}{2}(y-x)^2-xt \tag{8}$$

The first order condition is:

$$t-\gamma\theta t\frac{1}{y}(y-x)=0\Leftrightarrow x=(1-\frac{1}{\gamma\theta})y$$

The second-order condition is always satisfied. The boundary condition $x\leq y$ is always satisfied. Notice that full compliance (i.e. $x=y$) does not take place unless $\gamma\theta\rightarrow+\infty$. In Appendix A it is shown that evasion remains at all levels of income even with the probability of auditing depending on reported income, i.e. $\gamma=\gamma(x)$. The condition $x\geq 0$ is satisfied if and only if $\gamma\theta\geq 1$. When enforcement is very weak, so that $\gamma\theta<1$, full evasion will take place, i.e. $x=0$.

To simplify notation the two enforcement parameters are summarized in the following way:

$$\alpha\equiv\frac{1}{\gamma\theta}$$

To summarize, the solution to the reporting problem without minimum wage is given by:

$$x^*=\begin{cases}(1-\alpha)y & if \quad \alpha\leq 1 \quad \text{case 1 - partial evasion}\\ 0 & if \quad \alpha>1 \quad \text{case 2 - full evasion}\end{cases} \tag{9}$$

Thus, the model implies that, irrespective of the specific level of productivity, firms reveal a constant fraction of their production to fiscal authorities.

As $\frac{\partial\alpha}{\partial\gamma}<0$ and $\frac{\partial\alpha}{\partial\theta}<0$, in an interior solution the fraction of product that is evaded decreases as enforcement improves.

By sustituting (9) into (7) we get the worker's income:

$$I^*=\begin{cases}y(1-t)+\frac{1}{2}\alpha yt & \text{case 1 - partial evasion}\\ y(1-\frac{1}{2}\frac{1}{\alpha}t) & \text{case 2 - full evasion}\end{cases} \tag{10}$$

The expected fraction of concealed production that is discovered in case of auditing is, by substituting $x=(1-\alpha)y$ into (2), $\frac{1}{2}\alpha$. Thus, for example, in an economy where 30% of income is concealed, only 15% of evasion is detected on average in case of auditing.

4.2 Effects of introducing the minimum wage

In this section we study what are the effects of introducing a minimum wage $\bar{w}$ in the economy. We focus on the case where there is partial evasion, i.e. $\alpha \in (0,1)$ [8].

4.2.1 Effects on the distribution

With the introduction of a minimum wage (8) becomes:

$$\max_{x \in \{0\} \cup [\bar{w}, y]} \qquad y - \gamma\theta t \frac{1}{y}\frac{1}{2}(y-x)^2 - xt$$

The only difference is in the choice set, that shrinks from $[0,y]$ to $\{0\} \cup [\bar{w}, y]$.

The introduction of the minimum wage divides worker-firms pairs into three categories:

1. High productivity: $y_i > \frac{\bar{w}}{1-\alpha}$
2. Intermediate productivity: $\bar{w} \leq y_i \leq \frac{\bar{w}}{1-\alpha}$
3. Low productivity: $y_i < \bar{w}$

Worker-firm pairs characterized by high productivity would have declared more than the minimum wage anyway, so they are unaffected by it. The minimum wage is instead a binding constraint for worker-firm pairs that would have declared less in its absence. We first analyze the case of low-productivity workers.

[8] For this to be the case, we need $\gamma\theta > 1$. By assumption $\theta > 1$, but γ, the probability of being subject to an audit, may be low, so this condition may seem restrictive. Notice, however, that in this model an audit is extremely ineffective. As already mentioned if, for instance, 30% of income is evaded, during an audit on average only 15% of the evaded income is discovered. Thus, more than a full-fledged investigation, an audit should be rather interpreted in the present set-up as a routine check by fiscal authorities, thus happening much more frequently than a thorough inquiry. Also, notice that in the proposed alternative setting for the auditing technology γ may be any positive number.

Low productivity A worker with productivity below the minimum wage, $y_i < \bar{w}$, can only work in the black market or be inactive. The possibility of a worker paying back part of his wage to the firm is thus excluded. The main results are qualitatively unaffected by this modelling choice.

From (10) we get income in case of work in the black market, i.e. full evasion:

$$I_{bm} \equiv y_i(1 - t\frac{1}{2\alpha}) \tag{11}$$

Income in case of inactivity is assumed to be 0.

The labour market status is chosen by comparing income in the two cases, giving the following condition:

$$I_{bm} > 0 \Leftrightarrow \alpha > \frac{t}{2}$$

Then, if $\alpha > \frac{t}{2}$ workers with productivity below the minimum wage choose to work in the black market, as the expected return is positive, otherwise they withdraw from the labour market. Thus, the prediction is that, for a given tax rate, in economies where enforcement is quite effective the minimum wage pushes workers into inactivity, and therefore has a negative impact on efficiency, as productive labour stays idle. Instead, in economies with not very effective enforcement the minimum wage hasn't a negative impact on efficiency as workers continue to produce in the black market.

Intermediate productivity The possibility to declare the minimum wage and thus participate in the formal labour market is instead available for worker-firm pairs whose optimal declaration in case of no minimum wage regulation is less than $\bar{w}$, but with productivity above $\bar{w}$, i.e.

$$(1 - \alpha)y_i \leq \bar{w} \leq y_i \Leftrightarrow \bar{w} \leq y_i \leq \frac{\bar{w}}{1 - \alpha} \tag{12}$$

Income in case of declaring $\bar{w}$ is given by subsituting $x = \bar{w}$ in (7):

$$I_{mw} \equiv y_i(1 - t) + (y_i - \bar{w})\, t - t\frac{1}{\alpha y_i}\frac{1}{2}\,(y_i - \bar{w})^2 \tag{13}$$

Declaring a wage higher than the minimum is never optimal. Moreover, as $I_{mw} > 0$ for productivities satisfying (12), these workers will never go into inactivity. The choice is thus between declaring the minimum wage or work in the black market and declare 0. The comparison between income in case of declaring the minimum wage and income in the black market as given by (11) gives the following condition:

$$I_{mw} \geq I_{bm} \Leftrightarrow y_i \geq \frac{1}{2(1-\alpha)}\bar{w} \equiv y_{mw} \tag{14}$$

As the choice between employment at the minimum wage and employment in the black market is relevant only for workers satisfying (12) to determine the behaviour once a minimum wage is introduced it is necessary to position y_{mw} in the interval $[\bar{w}, \frac{\bar{w}}{1-\alpha}]$:

- $y_{mw} < \frac{\bar{w}}{1-\alpha} \Leftrightarrow \frac{1}{2(1-\alpha)}\bar{w} < \frac{\bar{w}}{1-\alpha}$, this condition is always satisfied.
- $y_{mw} > \bar{w} \Leftrightarrow \frac{1}{2(1-\alpha)}\bar{w} > \bar{w} \Leftrightarrow \alpha > \frac{1}{2}$

We then have two possible states:

1. $\alpha > \frac{1}{2}$: then $y_{mw} \in (\bar{w}, \frac{\bar{w}}{1-\alpha})$ i.e. some of the workers affected by the minimum wage and with productivity higher than the minimum wage prefer to decrease evasion and declare the minimum, while others prefer to go into the black market. For this to be the case the degree of underreporting has to be very high. While this may be the case for some economies, we consider the other case to be more relevant.

2. $\alpha \leq \frac{1}{2}$: then $y_{mw} \leq \bar{w}$ i.e. all workers affected by the minimum wage and with productivity higher than the minimum wage prefer to increase compliance and declare the minimum.

Thus, the behaviour of workers with intermediate productivity is characterized in the following way:

$$\begin{cases} \alpha \leq \frac{1}{2} & \Rightarrow \text{declare } \bar{w} \\ \alpha > \frac{1}{2} & \text{if } y_i \text{ s.t} \begin{cases} \bar{w} \leq y_i < \frac{1}{2(1-\alpha)}\bar{w} & \Rightarrow \text{declare } 0 \\ \frac{1}{2(1-\alpha)}\bar{w} \leq y_i < \frac{\bar{w}}{1-\alpha} & \Rightarrow \text{declare } \bar{w} \end{cases} \end{cases}$$

The introduction of the minimum wage restricts the choice set for declared earnings. Some, possibly all, worker-firm pairs prefer to increase compliance by declaring the minimum wage and paying the corresponding taxes rather than decrease it to zero by declaring nothing.

The results are summarized in proposition below.

Proposition 1 *The introduction of the minimum wage in an economy with underreporting of earnings induces some, possibly all, workers whose productivity is above the minimum wage, but who would have declared less if there was no minimum wage, to increase compliance by increasing their declared earnings to the minimum wage level. Workers with higher productivity are unaffected. Workers with productivity below the minimum wage work in the black market if enforcement is not too effective, otherwise they withdraw from the labour force.*

The distribution of declared earnings x before the minimum wage is given by:

$$g_x(x) = \begin{cases} g(\frac{x}{(1-\alpha)}) & \underline{y}(1-\alpha) < x < \bar{y}(1-\alpha) \\ 0 & otherwise \end{cases}$$

after the introduction of the minimum wage it is given by:

$$g_{mw}(x) = \begin{cases} \int_{(1-a)\underline{y}}^{\bar{w}\max\left\{\frac{1}{2},1-\alpha\right\}} g_x(x)dx & if \quad x = 0 \\ \int_{\bar{w}\max\left\{\frac{1}{2},1-\alpha\right\}}^{\bar{w}} g_x(x)dx & if \quad x = \bar{w} \\ g_x(x) & if \quad \bar{w} < x \leq \bar{y}(1-\alpha) \\ 0 & otherwise \end{cases}$$

or equivalently:

$$g_{mw}(x) = \begin{cases} \int_{\underline{y}}^{\bar{w}\max\left\{\frac{1}{2(1-\alpha)},1\right\}} g(y)dy & if \quad x = 0 \\ \int_{\bar{w}\max\left\{\frac{1}{2(1-\alpha)},1\right\}}^{\frac{\bar{w}}{1-a}} g(y)dy & if \quad x = \bar{w} \\ g(\frac{x}{(1-\alpha)}) & if \quad \bar{w} < x \leq \bar{y}(1-\alpha) \\ 0 & otherwise \end{cases}$$

Thus, a "smooth" distribution of productivity is associated with a "smooth" distribution of declared earnings without a minimum wage. However, with the introduction of the minimum wage, two spikes appear at the minimum wage level and at zero. We can thus state the following:

Proposition 2 *In a perfectly competitive labour market with underreporting of earnings a spike at the minimum wage level appears in the distribution of declared earnings.*

4.2.2 Effects on fiscal quantities

The minimum wage divides worker-firm pairs into three categories: those declaring nothing, those declaring the minimum wage, and the unaffected. Here we first determine payments to fiscal authorities for each category. Then, we use the analysis of the distribution of declared earnings done above to find out the effects of the minimum wage on fiscal revenues.

Payments to fiscal authorities Total payments, P, to the fiscal authorities include taxes, T , and expected fines, F. For worker-firm pairs not affected by the minimum wage these quantities are:

$$P_1 = (1 - \frac{\alpha}{2})ty \begin{array}{l} \nearrow \; T_1 = (1-\alpha)ty \\ \searrow \; F_1 = \frac{\alpha}{2}ty \end{array}$$

Underreporting gives to worker-firm pairs with relatively high productivity the opportunity to reduce the "effective"[9] tax rate by a factor $\frac{\alpha}{2}$.

For worker-firm pair declaring the minimum wage fiscal payments are given by:

$$P_2 = t\bar{w} + t\frac{1}{2\alpha}\frac{(y-\bar{w})^2}{y} \begin{array}{l} \nearrow \; T_2 = t\bar{w} \\ \searrow \; F_2 = \frac{1}{2\alpha}\frac{(y-\bar{w})^2}{y}t \end{array}$$

The remaining category is represented by worker-firm pairs that are either in the black economy (when $\alpha \geq \frac{t}{2}$) or do not participate to the labour market (when $\alpha < \frac{t}{2}$).

For workers in the black market fines are the only type of payment, so:

$$P_3 = F_3 = t\frac{1}{2\alpha}y$$

Workers who withdraw from the labour market do not contribute to public finances, so:

$$P_4 = F_4 = 0$$

Notice that $\frac{P_3}{y} \geq \frac{P_2}{y} \geq \frac{P_1}{y}$ in the relevant intervals[10]. Expected payments as a portion of income are the highest for worker-firm pairs in the

[9] In the meaning of total expected payments to fiscal authorities, including fines, over total income.

[10] In particular, $\frac{P_2}{y} \geq \frac{P_1}{y} \forall y$, $\frac{P_3}{y} \geq \frac{P_1}{y} \forall y$, $\frac{P_3}{y} \geq \frac{P_2}{y} \Leftrightarrow y \geq \frac{w}{2(1-\alpha)}$. As only workers with productivity $y_i \geq \max(\bar{w}, \frac{\bar{w}}{2(1-a)})$ will declare the minimum wage, then $\frac{P_3}{y} \geq \frac{P_2}{y}$ for the relevant interval.

black economy, the lowest for worker-firm pairs not affected by the minimum wage. Thus, considering expected total payments, it is possible to state the following:

Proposition 3 *The interaction of minimum wage and underreporting transforms a nominally neutral tax system into a regressive one.*

Effects of the minimum wage on revenues The way the working population splits into the three categories analyzed above has been established in section 4.2.1. Here, to establish the effect of the minimum wage on revenues, we analyze the "high underreporting" case, $\alpha \geq \frac{t}{2}$, and the "low underreporting" case, $\alpha < \frac{t}{2}$, separately.

High underreporting When $\alpha \geq \frac{t}{2}$ total revenues R are given by:

$$R = \int_0^{\max(\bar{w},\frac{\bar{w}}{2(1-a)})} t\frac{1}{2\alpha}yg(y)dy + \int_{\max(\bar{w},\frac{\bar{w}}{2(1-a)})}^{\frac{\bar{w}}{1-a}} [t\bar{w} + t\frac{1}{2\alpha}\frac{(y-\bar{w})^2}{y}]g(y)dy + \int_{\frac{\bar{w}}{1-a}}^{\bar{y}} (1-\frac{\alpha}{2})tyg(y)dy$$

The effects of increasing the minimum wage on total revenues depends on whether $\alpha \lesseqgtr \frac{1}{2}$. In case $\alpha > \frac{1}{2}$ a worker with productivity equal to the minimum wage prefers being employed in the black market than declaring the minimum wage. Then,

$$\frac{\partial R}{\partial \bar{w}} = \int_{\frac{\bar{w}}{2(1-a)}}^{\frac{\bar{w}}{1-a}} [1 - \frac{1}{\alpha}\frac{(y-\bar{w})}{y}]tg(y)dy$$

As $[1 - \frac{1}{\alpha}\frac{(y-\bar{w})}{y}] > 0 \Leftrightarrow y < \frac{\bar{w}}{1-a}$, then $\frac{\partial R}{\partial \bar{w}} > 0$. The variation is due to higher payments by workers affected by minimum wage.

In case $\alpha \leq \frac{1}{2}$ a worker with productivity equal to the minimum wage prefers declaring the minimum wage than being employed in the black market. Then,

$$\frac{\partial R}{\partial \bar{w}} = [\frac{1}{2\alpha} - 1]t\bar{w}g(\bar{w}) + \int_{\bar{w}}^{\frac{\bar{w}}{1-a}} [1 - \frac{1}{\alpha}\frac{(y-\bar{w})}{y}]tg(y)dy$$

As $[1 - \frac{1}{\alpha}\frac{(y-\bar{w})}{y}] > 0 \Leftrightarrow y < \frac{\bar{w}}{(1-\alpha)}$ and $\frac{1}{2\alpha} - 1 > 0 \Leftrightarrow \alpha < \frac{1}{2}$, then $\frac{\partial R}{\partial \bar{w}} > 0$. In this case there is an additional term, representing the effect of pushing into the black market worker-firm pairs previously in the official economy.

Low underreporting When $\alpha < \frac{t}{2}$ total revenues are given by:

$$R = \int_{\bar{w}}^{\frac{\bar{w}}{1-a}} [t\bar{w} + t\frac{1}{2\alpha}\frac{(y-\bar{w})^2}{y}]g(y)dy + \int_{\frac{\bar{w}}{1-a}}^{\bar{y}} (1-\frac{\alpha}{2})tyg(y)dy$$

Then,

$$\frac{\partial R}{\partial \bar{w}} = -t\bar{w}g(\bar{w}) + \int_{\bar{w}}^{\frac{\bar{w}}{1-a}} [1 - \frac{1}{\alpha}\frac{(y-\bar{w})}{y}]tg(y)dy$$

The first term represents the loss due to the withdrawal of workers from the labour market. In this case the net effect of an increase in the minimum wage depends on the shape of the distribution.

We can then state the following proposition:

Proposition 4 *When underreporting is high, revenues increase with the minimum wage, i.e.* $\frac{\partial R}{\partial \bar{w}} \geq 0$. *When underreporting is low the effect of increasing the minimum wage on revenues depends on the productivity distribution.*

The intuition is straightforward: maximization of workers' net income is equivalent to minimization of transfers to the government. Choice is limited to the possible declaration space $\{0\} \cup [\bar{w}, +\infty)$. Increasing the minimum wage shrinks the possible declaration space, so that the newly chosen compliance after the increase in the minimum wage cannot make workers better off. When the increase in the minimum wage does not have a negative impact on production, i.e. it does not "shrink the pie", this implies that the government cannot be made worse off, i.e. revenues cannot decrease. This can be counterbalanced by a decrease in revenues due to reduced total production when an increase in the minimum wage pushes low productivity workers out of the labour market.

This implies that countries where underreporting is serious because of limited enforcement capacity can use the minimum wage to boost fiscal revenues, without having to worry too much about the impact on efficiency[11]. As enforcement improves the minimum wage becomes a less effective fiscal instrument and efficiency issues become more prominent. However, equity issues are also at stake, as the minimum wage increases revenues by extracting more payments from low productivity workers.

In Bulgaria, for instance, in 2003 social security contributions payments increased by almost 20% in 2003 "[a]s a result from the registration of the labor contracts and the introduction of the minimum insurance income upon principal economic activities and qualification groups of professions, as well as from the improved economic situation" (NSSI).

4.3 The link between the size of the underground economy and the spike at the minimum wage

Both the size of the spike at the minimum wage level and the size of the underground economy relative to the economy as a whole are determined by the interplay of the productivity distribution, the fiscal enforcement parameters as summarized by α, and the minimum wage, $\bar{w}$. In this section we study the link between the size of the underground economy and the size of the spike.

The spike at the minimum wage The size of the spike at the minimum wage level is given by:

$$S = \int_{\max\left(\frac{1}{2(1-\alpha)}\bar{w},\bar{w}\right)}^{\frac{\bar{w}}{1-a}} g(y)dy$$

[11] The assumption is that productivity is the same in the formal and informal sectors. See section 7.3.1 for a discussion on this issue.

A decrease in enforcement parameters initially increases unambigously the size of the spike, but as the process goes on the effect depends on the shape of the distribution:

$$\frac{\partial S}{\partial \alpha}=\begin{cases} \frac{\bar{w}}{(1-a)^2}g(\frac{\bar{w}}{1-a})>0 & if \quad 0<\alpha\leq\frac{1}{2} \\ \frac{\bar{w}}{(1-a)^2}g(\frac{\bar{w}}{1-a})-\frac{\bar{w}}{2(1-\alpha)^2}g(\frac{\bar{w}}{2(1-a)}) & if \quad \frac{1}{2}<\alpha<1 \end{cases}$$

A decrease in enforcement parameters induces to declare the minimum wage workers previously declaring more, increasing the size of the spike. If enforcement is weak enough, however, some workers previously declaring the minimum wage prefer to go into the black economy, reducing the size of the spike. The condition for the size of the spike to increase as enforcement parameters decrease in this case is:

$$\tfrac{1}{2}<\alpha<1 \quad \Rightarrow \quad \tfrac{\partial S}{\partial \alpha}>0 \Leftrightarrow g(\tfrac{\bar{w}}{1-a})>\tfrac{1}{2}g(\tfrac{\bar{w}}{2(1-a)})$$

Assuming that the distribution of productivity is single peaked, if the minimum wage is binding for workers with productivity lower than the mode, then the condition is satisfied. As S is continuous the eventual switch of the economy from $\alpha\leq\frac{1}{2}$ to $\alpha>\frac{1}{2}$ as α increases is not an issue.

The effect on the size of the spike of an increase of the minimum wage depends in general on the shape of the distribution:

$$\frac{\partial S}{\partial \bar{w}}=\begin{cases} \frac{1}{1-a}g(\frac{\bar{w}}{1-a})-g(\bar{w}) & if \quad 0<\alpha\leq\frac{1}{2} \\ \frac{1}{1-a}g(\frac{\bar{w}}{1-a})-\frac{1}{2(1-\alpha)}g(\bar{w}) & if \quad \frac{1}{2}<\alpha<1 \end{cases}$$

As $\bar{w}$ increases some workers previously declaring the minimum wage are pushed out of the formal labour market, thus decreasing the size of the spike, while some previously declaring more declare the minimum wage, increasing the size of the spike.

The conditions for the size of the spike to increase as the minimum wage increases are in this case:

$$\begin{cases} 0<\alpha\leq\frac{1}{2} \quad \Rightarrow \quad \frac{\partial S}{\partial \bar{w}}>0 \Leftrightarrow g(\frac{\bar{w}}{1-a})>(1-a)\,g(\bar{w}) \\ \frac{1}{2}<\alpha<1 \quad \Rightarrow \quad \frac{\partial S}{\partial \bar{w}}>0 \Leftrightarrow g(\frac{\bar{w}}{1-a})>\frac{1}{2}g(\bar{w}) \end{cases}$$

Also in this case the conditions are satisfied if the minimum wage is binding for workers with productivity lower than the mode and the distribution of productivity is single peaked.[12]

[12] The analysis can also be conducted in terms of the size of the spike relative to the size

The informal economy To investigate the impact of α and $\bar{w}$ on the size of the informal economy it is necessary to distinguish between the high and low underreporting cases.

High underreporting The size of the underground economy is given by:

$$U = \underbrace{\int_{\underline{y}}^{\max(\frac{1}{2(1-\alpha)}\bar{w},\bar{w})} yg(y)dy}_{\text{black economy}} + \underbrace{\int_{\max(\frac{1}{2(1-\alpha)}\bar{w},\bar{w})}^{\frac{\bar{w}}{1-a}} (y-\bar{w})g(y)dy + \alpha\int_{\frac{\bar{w}}{1-a}}^{\bar{y}} yg(y)dy}_{\text{underreporting}} \tag{15}$$

As the size of the economy for a given distribution of productivity is fixed at $Y = \int_{\underline{y}}^{\bar{y}} yg(y)dy$ the derivatives of U, $\frac{U}{Y}$, $\frac{U}{Y-U}$ (size of informal economy relative to formal economy) all have the same sign, so we focus only on the effects of α and $\bar{w}$ on U.

A decrease in enforcement, i.e. an increase in α, increases the size of the informal economy:

$$\frac{\partial U}{\partial \alpha} = \begin{cases} \int_{\frac{\bar{w}}{1-a}}^{\bar{y}} yg(y)dy > 0 & if \quad \frac{t}{2} \leq \alpha \leq \frac{1}{2} \\ \frac{\bar{w}}{2(1-\alpha)^2}\bar{w}g(\frac{\bar{w}}{2(1-a)}) + \int_{\frac{\bar{w}}{1-a}}^{\bar{y}} yg(y)dy > 0 & if \quad \frac{1}{2} < \alpha < 1 \end{cases}$$

This is due to the fact that workers unaffected by the minimum wage evade more. Moreover, when enforcement is already low, i.e. $\frac{1}{2} < \alpha < 1$, some

of the officially employed workforce, where the latter is given by:

$$L = \int_{\max(\frac{1}{2(1-\alpha)}\bar{w},\bar{w})}^{\bar{y}} g(y)dy$$

for $\alpha > \frac{1}{2}$ a marginal increase in α decreases the size of the officially employed workforce, so the condition for $\frac{S}{L}$ to increase with α is looser than the one for S.

$$\begin{cases} 0 < \alpha \leq \frac{1}{2} & \Rightarrow & \frac{\partial \frac{S}{L}}{\partial \alpha} > 0 \\ \frac{1}{2} < \alpha < 1 & \Rightarrow & \frac{\partial \frac{S}{L}}{\partial \alpha} > 0 \Leftrightarrow g(\frac{\bar{w}}{1-a}) > \frac{1}{2}g(\frac{\bar{w}}{2(1-a)})(1-\frac{S}{L}) \end{cases}$$

the same is true w.r.t. $\bar{w}$:

$$\begin{cases} 0 < \alpha \leq \frac{1}{2} & \Rightarrow & \frac{\partial \frac{S}{L}}{\partial \bar{w}} > 0 \Leftrightarrow g(\frac{\bar{w}}{1-a}) > (1-a)\,g(\bar{w})(1-\frac{S}{L}) \\ \frac{1}{2} < \alpha < 1 & \Rightarrow & \frac{\partial \frac{S}{L}}{\partial \bar{w}} > 0 \Leftrightarrow g(\frac{\bar{w}}{1-a}) > \frac{1}{2}g(\bar{w})(1-\frac{S}{L}) \end{cases}$$

workers previously declaring the minimum wage go into the black economy. As U is continous the eventual switch of the economy from $\alpha \leq \frac{1}{2}$ to $\alpha > \frac{1}{2}$ as α increases is not an issue.

The effect of an increase in the minimum wage on the size of the informal economy depends in general on the shape of the distribution:

$$\frac{\partial U}{\partial \bar{w}} = \begin{cases} \bar{w}g(\bar{w}) - G(\frac{\bar{w}}{1-a}) + G(\bar{w}) & if \quad \frac{t}{2} \leq \alpha \leq \frac{1}{2} \\ \frac{1}{2(1-\alpha)}\bar{w}g(\frac{\bar{w}}{2(1-\alpha)}) - G(\frac{\bar{w}}{1-a}) + G(\frac{\bar{w}}{2(1-\alpha)}) & if \quad \alpha > \frac{1}{2} \end{cases}$$

An increase in the minimum wage pushes some workers previously declaring the minimum wage into the black economy, thus increasing informality, but also forces workers continuing to declare the minimum to declare more of their true income, thus reducing informality. Which effect prevails depends on the shape of the distribution.

Low underreporting When $0 < \alpha < \frac{t}{2}$ the size of the underground economy is given by:

$$U = \int_{\bar{w}}^{\frac{\bar{w}}{1-a}} (y - \bar{w})g(y)dy + \alpha \int_{\frac{\bar{w}}{1-a}}^{\bar{y}} yg(y)dy \qquad (16)$$

The derivative w.r.t. α is the same as in the high underreporting case, but, as it is evident by comparing (15) with (16), there is a discontinuity in the size of the informal economy at $\alpha = \frac{t}{2}$. When enforcement parameters increases (i.e. α decreases) so that there is a switch in the economy from the high underreporting status to the low underreporting status, the size of the informal economy drops discretely as workers previously in the black market withdraw from the labour market. This jump goes in the same direction as the derivative, so that we can state that the size of the informal economy always decreases as enforcement parameters increase.

In the low underreporting case the size of the economy for a given distribution of productivity is not fixed anymore, as it is given by $Y = \int_{\bar{w}}^{\bar{y}} yg(y)dy$, with $\frac{\partial Y}{\partial \bar{w}} = -\bar{w}g(\bar{w}) < 0$. As the minimum wage increases, workers with productivity below the minimum wage withdraw from the labour market into non-activity, lowering total production.

Given the sign of $\frac{\partial U}{\partial \bar{w}}$, the sign of the derivative of $\frac{U}{Y}$ w.r.t. $\bar{w}$ is given by the sign of $\frac{\partial U}{\partial \bar{w}}Y - \frac{\partial Y}{\partial \bar{w}}U = \frac{\partial U}{\partial \bar{w}}Y + \bar{w}g(\bar{w})U$. The derivatives of $\frac{U}{Y}$ and $\frac{U}{Y-U}$ (size of informal economy relative to formal economy) have the same sign.

In case of low underreporting $\frac{\partial U}{\partial \bar{w}}$ is given by:

$$\frac{\partial U}{\partial \bar{w}} = -[G(\frac{\bar{w}}{1-a}) - G(\bar{w})] < 0$$

An increase in the minimum wage decreases the absolute size of the informal economy. When workers with productivity lower than the minimum wage withdraw from the labour market, an increase in the minimum wage has the only effect to increase compliance by active workers, thus shrinking the size of the informal economy. However, in this case the economy as a whole also shrinks. The sign of the derivative of the relative size of the informal economy w.r.t. the minimum wage is given by:

$$sign[\partial(\frac{U}{Y})/\partial\bar{w}] = sign[-[G(\frac{\bar{w}}{1-a}) - G(\bar{w})]Y + \bar{w}g(\bar{w})U]$$

which depends on the shape of the productivity distribution.

Given the analysis above, it is possible to state the following proposition:

Proposition 5

1. *The size of the informal economy increases as enforcement decreases.*
2. *The effect of an increase in the minimum wage on the size of the informal economy relative to the formal economy is ambigous.*
3. *Assuming that the distribution of productivity is single peaked, a minimum wage binding for workers with productivity lower than the mode is a sufficient condition for the size of the spike at the minimum wage level to increase as enforcement decreases and as the minimum wage increases.*
4. *Provided that enforcement is not very weak the size of the spike increases as enforcement decreases.*

Thus, if enforcement is not very weak or given a single peaked distribution of productivity and a not too high minimum wage, an increase in α increases both the spike at the minimum wage level and the size of the informal economy, inducing a positive correlation between the two. Such a positive correlation is documented in the following section.

5 The link between informal economy and spike

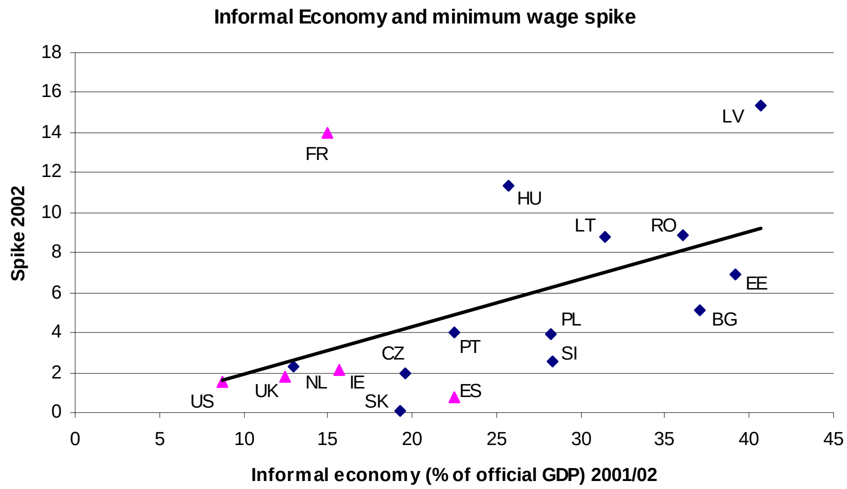
The model predicts that the enforcement parameters (as summarized by α) should induce a positive correlation between the spike at the minimum wage level and the size of the informal economy relative to the formal economy. In this section some supporting evidence is presented.

The two figures below present the relationship of the spike at the minimum wage[13] with the size of the informal economy relative to the formal economy[14] and ratio of the minimum wage to the average wage[15]. The country included are all the countries for which Eurostat reports data on the minimum wage and Schneider reports estimates of the informal economy. The sample includes 16 European countries and the US. Ten of the European countries are Central and Eastern European, where statutory minimum wage arrangements are common.

[13] Proportion of full-time employees with earnings on the monthly minimum wage (source: Eurostat). Notice that the data collected by Eurostat are obtained from administrative sources. For data point indicated with a triangle the definition is different: part-time workers are included (France, Spain), minimum wage is fixed on an hourly base (France, Ireland, UK, USA), earnings below the minimum wage are also included (UK, USA). See Eurostat (2004) for details.

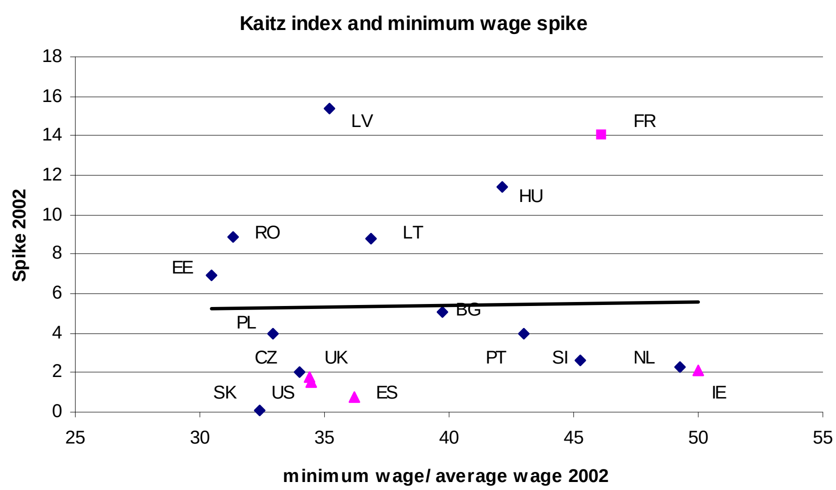
[14] Informal economy as % of official GDP (source: Schneider 2005). A short description of the method used to estimate the size of the informal economy is given in Appendix B (see Schneider, 2005, for details).

[15] Minimum monthly wage as a proportion of average monthly earnings in industry and services (source: Eurostat). In what follows this is indicated as the Kaitz index. For France the figure has been calculated by the author dividing the hourly gross wage by the average gross hourly wage for a full-time employee in industry, trade and services (data source: INSEE.)



Sources: informal economy: Schneider (2005); spike: Eurostat

A positive correlation clearly appears between the size of the spike at the minimum wage level and the estimated size of the informal economy. Instead there appear to be no clear relationship between the size of the spike and a measure of how "biting" the minimum wage is in the wage distribution.



Source: Eurostat, except France - Kaitz Index: own calculations

These findings are confirmed by regressing the size of the spike on the size of the informal economy and the Kaitz index. The former is significant, while the latter is not. Therefore, we can conclude that the positive relationship between the spike and the informal economy is not driven by the minimum wage. The model suggests that this relationship is instead driven by their common dependance on enforcement parameters.

The regression implies that a 1% increase in the size of the informal economy is associated with a 0.28% increase in the share of employees earning the minimum wage.

Table 1: Regression results

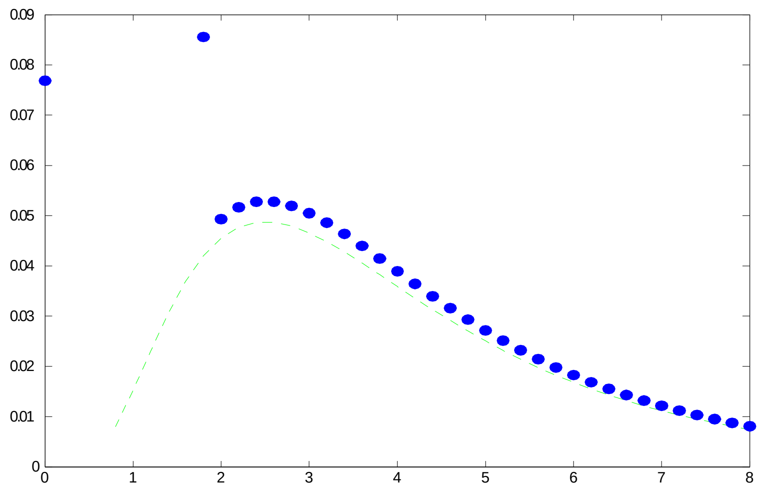
Variable	**Coefficient**	**(Std. Err.)**
Informal	0.279*	(0.113)
Kaitz Index	0.179	(0.179)
Intercept	-8.337	(8.381)
N	17	
R^2	0.302	
$F_{(2,14)}$	3.035	

Significance levels : † : 10% ∗ : 5% ∗∗ : 1%

6 A numerical exercise

In this section the quantitative properties of the model are explored.

Worker's productivity is assumed to be distributed across 37 categories in the range 1-10, with the distance between adjacent productivity categories being 0.25. In the baseline scenario, the distribution of the workforce across the different categories is generated by normalizing the corresponding values of a lognormal with parameters (1.5; 0.6). Tax and social security contributions are assumed to be equivalent to 30% and enforcement parameters are such that without a minimum wage all agents evade 20% of their income, i.e. $\alpha = 0.2$. The minimum wage is assumed to be equal to the income declared by the 6th productivity category, i.e. 1.8.



Distribution of declared earnings

The figure shows the distribution of declared earnings among the official workforce before and after the introduction of the minimum wage. Without the minimum wage (dashed line), declared earnings are in the range 0.8-8, as 20% of product is evaded.

With the introduction of the minimum wage the distribution of declared earnings (dots) changes. The minimum wage creates two spikes, at the minimum wage level and at zero. As the minimum wage reduces the size of the official workforce by truncating it from below, the distribution of declared earnings is shifted upward above the minimum wage. Notice that in the figure the spike at the minimum wage is the percentage of *official workforce* declaring the minimum wage. Instead, the spike at zero is the percentage of the *population* not taking part to the official labour market.

The table reports the size of the two spikes. Other indicators are also calculated. In the model developed in this paper, the minimum wage is assumed to apply to the workforce as a whole, thus the Kaitz index is simply the minimum wage divided by the average declared wage. The percentage increase in total fiscal revenues (taxes and fines) due to the introduction of a minimum wage is also calculated. Finally, the size of the informal economy as a percentage of the formal economy is presented, a measure that is consistent with the estimates of the informal economy reported in Appendix B. In the baseline scenario, without a minimum wage the informal economy would be 25% of the formal economy, as 20% of income would be evaded. With the

Spike mw[1]	Spike zero[2]	Kaitz index[3]	Δ revenues[4]	Informal economy[5]
Scenario 1 - Baseline				
μ=1.5 σ=0.6 α=0.2 mw=1.8				
8.6	7.7	46.5	4.52	28
Scenario 2 - High evasion				
μ=1.5 σ=0.6 α=0.3 mw=1.575				
11.5	4.6	47.3	1.41	44
Scenario 3 - High minimum wage				
μ=1.5 σ=0.6 α=0.2 mw=2.2				
15.3	11.4	55.2	7.58	29.4
Scenario 4 - High evasion, high minimum wage				
μ=1.5 σ=0.6 α=0.3 mw=1.9259				
18.7	7.7	56.2	2.76	44.6
Scenario 5 - Spread-out distribution				
μ=1.5 σ=0.8 α=0.2 mw=1.8				
10.7	14.3	46.2	8.33	30.7

1: as percentage of workforce in formal employment
2: as percentage of total population
3: minimum wage over average declared wage
4: percentage change in total fiscal revenues due to the introduction of the minimum wage
5: size of the informal economy as percentage of official economy

minimum wage, the informal economy is equivalent to 28% of the formal economy.

Four other scenarios are explored. In the "high evasion" scenario, enforcement is assumed to be weaker, so that 30% of income would be evaded without a minimum wage constraint, i.e. $\alpha = 0.3$. The minimum wage remains equivalent to the income declared by the 6th productivity category[16]. The portion of the population affected by the minimum wage is the same as in the baseline scenario, as only a reshuffle between workers declaring zero and workers declaring the minimum wage takes place[17]. As established by Proposition 4, the size of the spike at the minimum wage level increases, together with the size of the informal economy.

In the "high minimum wage" scenario the minimum wage is assumed to be equivalent to the income declared by the 8th productivity category, i.e. 2. In this case the minimum wage bites deeper into the wage distribution.

[16] Due to the increase in evasion, though, the actual level of the minimum wage is lower than in the baseline scenario.

[17] Notice that the size the spike at the minimum wage level and the size of the spike at zero do not add up to the same number in scenarios 1 and 2 and in scenarios 3 and 4 only because the reported spike at the minimum wage level is expressed as percentage of the official workforce, while the spike at zero as percentage of total population.

As established by Proposition 4 the size of the spike at the minimum wage level increases. The significance from a fiscal point of view is also increased compared to the baseline scenario, as established by Proposition 4.

The "high evasion, high minimum wage" scenario combines the previous two variations. In this case, both the spike at the minimum wage level and the size of the informal economy reach very high levels.

In the last scenario the distribution generating the frequencies is changed, in particular the standard deviation parameter is increased to 0.8. The resulting sizeable change in some of the indicators points to the fact that the quantitative implications of the model are sensitive to the assumption about the underlying distribution of productivity. However, these simple numerical simulations show that the model is able to match the very high spike at the minimum wage observed in some countries and that the fiscal implications of imposing a minimum wage can be sizeable, even if only people with the lowest productivity are affected.

7 Extentions and discussion

In this section the robustness of the mechanism giving rise to a spike at the minimum wage level is discussed and some extentions are proposed.

7.1 Working time

A minimum wage fixed on an hourly basis in an environment in which firms could declare the amount of hours worked with full flexibility and no risk of detection in case of underreporting would pose an extremely loose constraint on reporting behavior. However, the minimum wage can still play its role against underreporting of earnings if there are legislative constraints on the amount of hours that can be reported or incentives not to declare a minimal amount of hours[18] or if misreporting hours of work can also be detected and punished.

In this section we consider the latter case.

[18] According to Eurostat data from LFS the share of part-timers in Central and Eastern European countries is generally low, at around 7% of employees. More interestingly, according to the Hungarian UI Exit to Job Survey 64.7% of the low-wage UI recipients who found a job in April 2001 received a fixed salary, 33.8% were paid an hourly wage and only 1.5% concluded a business contract with the employer (Kertesi and Kollo, 2003.)

Suppose a worker with hourly productivity y_i supplies inelastically h_i hours of work per period. However, the worker-firm pair can choose to report product per hour $x_i \in [0, y_i]$ and hours of work $\tau_i \in [0, h_i]$.

The audit and detection technologies are exactly the same in the two dimensions. In case of audit the tax authorities manage to impute $\hat{x}_i \in [0, y_i]$ and $\hat{\tau}_i \in [0, h_i]$. The probabilities of detection are assumed to be distributed uniformly over the relevant intervals, so that $g_{\hat{x}_i}(\hat{x}_i) = \frac{1}{y_i}$ and $g_{\hat{\tau}_i}(\hat{\tau}_i) = \frac{1}{h_i}$, and, for analytical convenience, they are assumed to be independent. The corresponding c.d.f. are indicated as $G_{\hat{x}_i}$ and $G_{\hat{\tau}_i}$.

The imposed fine, f_i, depends on the detected and declared hours of work and product per hour. In particular, it is possible to distinguish four cases:

1. $\hat{x}_i < x_i$ and $\hat{\tau}_i < \tau_i \Rightarrow f_i = 0$
2. $\hat{x}_i < x_i$ and $\hat{\tau}_i > \tau_i \Rightarrow f_i = t\theta \left(\hat{\tau}_i - \tau_i\right) x_i$
3. $\hat{x}_i > x_i$ and $\hat{\tau}_i < \tau_i \Rightarrow f_i = t\theta \left(\hat{x}_i - x_i\right) \tau_i$
4. $\hat{x}_i > x_i$ and $\hat{\tau}_i > \tau_i \Rightarrow f_i = t\theta \left(\hat{x}_i\hat{\tau}_i - x_i\tau_i\right)$

In case 2 and 3 underreporting is discovered in one dimension only and the fine is imposed on the assessed underreporting in that dimension multiplied by the declared value on the other dimension.

Thus, given a declaration (x_i, τ_i) the expected fine is given by (subscript are suppressed where not necessary):

$$f = t\theta \left[\begin{array}{c} \int_x^y \int_\tau^h \left(\hat{x}\hat{\tau} - x\tau\right) g(\hat{x}, \hat{\tau}) d\hat{\tau} d\hat{x} + \tau G_{\hat{\tau}}(\tau) \int_x^y \left(\hat{x} - x\right) g_{\hat{x}}(\hat{x}) d\hat{x} + \\ + x G_{\hat{x}}(x) \int_\tau^h \left(\hat{\tau} - \tau\right) g_{\hat{\tau}}(\hat{\tau}) d\hat{\tau} \end{array} \right]$$

Where $g(\hat{x}, \hat{\tau}) = g_{\hat{x}}(\hat{x}) g_{\hat{\tau}}(\hat{\tau})$. Given the hypothesis on the distributions, the expected fine is equal to:

$$f = t\theta \frac{1}{y} \frac{1}{h} \frac{1}{4} \left[\left(h^2 + \tau^2\right) \left(y^2 + x^2\right) - 4\tau y x h \right] \tag{17}$$

In what follows the equilibria with and without the minimum wage are characterized.

7.1.1 Equilibrium without minimum wage

In the formulation proposed here income is given by:

$$I = yh - x\tau t - \gamma f \tag{18}$$

Where f is given by (17.) Going through the same type of steps as in the main model, the equivalent to (8) is given by:

$$\max_{x\in[0,y],\tau\in[0,h]} yh - x\tau t - \gamma t\theta \frac{1}{y}\frac{1}{h}\frac{1}{4}\left[\left(h^2+\tau^2\right)\left(y^2+x^2\right) - 4\tau yxh\right] \tag{19}$$

The first order conditions $\frac{\partial I(x,\tau)}{\partial x} = \frac{\partial I(x,\tau)}{\partial \tau} = 0$ are simultaneously satisfied iff:

$$\tau^* = h\sqrt[2]{(1-2\alpha)} \qquad x^* = y\sqrt[2]{(1-2\alpha)}$$

Where $\alpha = \frac{1}{\gamma\theta}$. To have an interior solution it is necessary that $\alpha < \frac{1}{2}$, otherwise full evasion in both dimensions takes place. In what follows it is assumed that $\alpha < \frac{1}{2}$, i.e. enforcement is strong enough to avoid full evasion.

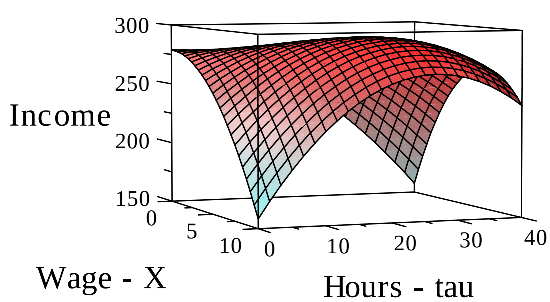
The income corresponding to reporting (x^*, τ^*) is:

$$I^* = yh(1-t) + \alpha yht \tag{20}$$

The second order condition is such that:

$$\frac{\partial^2 I}{\partial x^2} < 0; \ \frac{\partial^2 I}{\partial \tau^2} < 0; \ D\mid_{x^*,\tau^*} = \frac{\partial^2 I(x^*,\tau^*)}{\partial x^2}\frac{\partial^2 I(x^*,\tau^*)}{\partial \tau^2} - \left[\frac{\partial^2 I(x^*,\tau^*)}{\partial x \partial \tau}\right]^2 > 0$$

so that the function is locally concave at (x^*, τ^*). However, the income function is not globally concave. The figure below gives an example of how the function looks like.



$$y = 10; h = 40; \alpha = 0.33; t = 0.4$$

To establish whether (x^*, τ^*) is indeed the global maximum point it is necessary to check the value of the function along the boundaries.

Full evasion in one dimension First we analyze the boundaries within the axes.

1 . Substituting $x = 0$ in (18), we get $I|_{x=0} = yh - \frac{t}{4\alpha yh}(h^2 + \tau^2)\, y^2$, that is maximized for $\tau = 0$;

2. Substituting $\tau = 0$ in (18), we get $I|_{\tau=0} = yh - \frac{t}{4\alpha yh}(y^2 + x^2)h^2$, that is maximized for $x = 0$

Thus, when there is total evasion in one dimension, then it is optimal to have total evasion in the other dimension as well. A positive declaration would only represent a lower bound on the fine to be paid. Therefore, we need to compare I^* given by (20) with the income corresponding to total evasion given by substituting $x = 0, \tau = 0$ in (18):

$$I^*_{bm} = yh - \gamma \frac{t\theta}{4} hy \tag{21}$$

For $\alpha < \frac{1}{2}$ we always have that $I^* > I^*_{bm}$.

Full compliance in one dimension This case is parallel to the case analyzed in the main model, where indeed it is assumed that there is full reporting of the amount of hours worked.

3. In case $x = y$, then I is maximized for $\tau = (1 - \alpha)h$, resulting in an income $I^*|_{x=y} = yh(1 - t) + \frac{1}{2}\alpha tyh$

4. In case $\tau = h$, then I is maximized for $x = (1 - \alpha)y$, resulting in the same income as un the previous case.

Thus, the income when there is total compliance in one dimension is $I^*_{fc} = I^*|_{x=y} = I^*|_{\tau=h}$. It is straightforward to show that $I^* > I^*_{fc}$.

So, the analysis at the boundaries shows that (x^*, τ^*) is indeed the global maximum point.

7.1.2 Equilibrium with a minimum hourly wage

Given an hourly minimum wage $\bar{w}$, problem (19) becomes:

$$\max_{x\in\{0\}\cup[\bar{w},y],\tau\in[0,h]} yh - x\tau t - \gamma t\theta\frac{1}{y}\frac{1}{h}\frac{1}{4}\left[\left(h^2+\tau^2\right)\left(y^2+x^2\right) - 4\tau yxh\right]$$

Parallel to the main model, workers are split in three categories:

1. High productivity: $y_i > \frac{\bar{w}}{\sqrt[2]{(1-2\alpha)}}$ unaffected

2. Intermediate productivity: $\bar{w} \leq y_i \leq \frac{\bar{w}}{\sqrt[2]{(1-2\alpha)}}$ can choose whether to increase or decrease their compliance with wage regulation.

3. Low productivity: $y_i < \bar{w}$

High productivity workers are unaffected by the introduction of the minimum wage as they would have declared higher hourly earnings anyway. Low productivity workers are expelled from the formal labour market and can choose black market activity or inactivity. The choice is made by comparing income in the two cases, given by (21) and 0 respectively. This gives rise to the following condition:

$$I^*_{bm} = yh - \frac{1}{4\alpha}thy > 0 \Leftrightarrow \alpha > \frac{t}{4} \tag{22}$$

As in the main model, if enforcement is very effective (low α) then the minimum wage has an efficiency cost as workers with positive productivity withdraw into idleness. If enforcement is instead not too effective workers with hourly productivity below the minimum wage work completely underground.

In what follows we analyze the behaviour of workers with intermediate productivity.

Declaring the minimum wage When $x = \bar{w}$ the amount of declared hours maximizing income is given by $\tau_{mw} = \frac{2y\bar{w}}{y^2+\bar{w}^2}(1-\alpha)h$, giving an income:

$$I^*_{mw} = yh - \frac{th}{(y^2+\bar{w}^2)}\frac{1}{\alpha}\frac{1}{y}\frac{1}{4}\left[(y^2+\bar{w}^2)^2 - (2y\bar{w})^2(1-\alpha)^2\right]$$

Being underground A worker firm pair can always chose to be completely in the informal economy i.e. $x = \tau = 0$. We have seen that this the best that can be done when there is full evasion in at least one dimension. Income in case of full evasion is given by (21).

The choice between full evasion and declaring the minimum wage is made by comparing income in the two cases. It turns out that:

$$I_{mw}^{*} > I_{bm}^{*} \Leftrightarrow y_i > \frac{1}{\sqrt[2]{[4(1-\alpha)^2 - 1]}}\bar{w} = y_{mw}$$

As the minimum wage constraint is binding only if $y_i < \frac{\bar{w}}{\sqrt[2]{(1-2\alpha)}}$ and $y_{mw} < \frac{\bar{w}}{\sqrt[2]{(1-2\alpha)}} \forall \alpha < \frac{1}{2}$, then there is always a productivity interval where workers prefer to increase their compliance to the minimum wage that to decrease it by declaring zero.

To complete the analysis we should also analyze the remaining boundaries.

Fully reporting In case $x = y$ the maximum income that can be achieved is I_{fc}^{*}, where $I_{mw}^{*} > I_{fc}^{*}$ and $I_{bm}^{*} > I_{fc}^{*}$ for workers whose productivity is such that they are affected by the minimum wage. In case $\tau = h$ the maximum income that can be achieved is for sure less than I_{fc}^{*} and thus less than I_{mw}^{*} and I_{bm}^{*}. Thus, the choice faced by this type of workers is indeed between increasing compliance to the minimum wage level or decreasing it to zero.

In this section the model has been extended by allowing hours of work to be underreported, subject to the same detection technology as earnings. Also in this case the introduction of the minimum wage induces some workers to increase compliance, producing a spike at the minimum wage level. Proposition 2 is thus robust to this extention. As the minimum wage acts as an effective constraint for the low-productivity part of the workforce, Proposition 3 and Proposition 4 extend to this more general setting.

7.2 Alternative structures of the labour market

The model assumes a specific structure of the labour market, where the equilibrium distribution of wages with and without the minimum wage is easily characterized as a worker's earnings are independent of other workers' earnings. The mechanism presented is however more general. As far as workers maximize net income, firms maximize expected profits, and the product generated by a firm-worker pair is independent from the reporting decision, then there is an incentive to minimize the expected total payment (taxes plus expected fines) to tax authorities.

The tax system introduces a wedge between net take-home pay to the worker and labour cost for the firm and the decision on how much to report aims at minimizing such wedge, irrespectively of how savings from tax evasion are then distributed. Thus, the mechanism proposed can be extended to other models of the labour market. In some models of the labour market the problem can be most naturally framed as minimization of expected total labour cost given a net wage; in other models, like in the one developed here, as a maximization of net wage given expected total labour cost; in still other models, like the ones with bargaining, as the maximization of surplus net of payments to the fiscal authorities.

Ignoring general equilibrium effects on the distribution of wages, the introduction of the minimum wage poses a binding constraint for whose firms that would have reported a lower wage in its absence. Given the trade-off beneath the reporting decision, it is likely that a mass point at the minimum wage level will emerge in the distribution of declared earnings due to the interaction between underreporting and minimum wage alone and that the minimum wage, by restricting the choice set of worker-fim pairs, make the government better off, i.e. increases revenues. However, in models of endogenous wage dispersion like Burdett and Mortensen (1998) or Bhaskar and To (2003), where a worker's wage depends on other workers' wages, the general equilibrium effects of the introduction of a minimum wage make the analysis more complex.

7.3 The black economy

The model presents no discontinuity when a firm-agent pair leaves the formal economy and goes completely underground. It may however be argued that being completely in the black economy is substantially different than being part of the official economy. In particular, we analyze the implication of possible discontinuities in two key variables: productivity and expected fines. In the analysis we assume that enforcement parameters are such that there is underreporting.

7.3.1 Productivity discontinuity

While it seems unlikely that the product generated by a firm-worker pair is dependent on the reporting behavior in case of simple underreporting, it is more plausible that entering completely into the black economy may have an effect. More difficult access to the legal protection system to enforce contracts and property rights, inability to tap formal credit, restricted possibility to advertise, no access to support programs (like training schemes, subsidies to R&D) for enterprises are some of the factors that may cause a decrease in the surplus once a firm goes underground. On the other side, the avoidance of official regulation and red tape may boost the product of firms fully in the underground economy (see Loayza, 1996, for a review). The relative relevance of the pros and cons depends on the specific situation of a country. For instance, an ineffective court system and a credit market that is not accessible for some types of enterprises (like SME) even if registered may decrease the disadvantage of being underground.

To extend the model to take into account this potential discontinuity is straightforward. Assume that productivity is:

$$\begin{cases} y_i & if \;\; x_i > 0 \\ y_i + d & if \;\; x_i = 0 \end{cases} \; or \begin{cases} y_i & if \;\; x_i > 0 \\ \eta y_i & if \;\; x_i = 0 \end{cases}$$

In case $d < 0$ or $\eta < 1$ the cons of being in the black market outweight the pros. When there is no minimum wage nothing changes. When there is a minimum wage $\bar{w}$, then worker-firm pair have a greater incentive to increase compliance to the minimum wage level, instead of going into the black market, thus reiforcing the tendency to show a spike at the minimum wage level.

In case $d > 0$ or $\eta > 1$ (and $\alpha > \frac{t}{2}$), being in the black market provides an advantage compared to being in the official economy. In case of an addictive productivity difference, when there is no minimum wage worker-firm pairs characterized by low productivity, i.e. with $y_i < d\frac{(2\alpha-t)}{(1-\alpha)^2}$, will go into the black market, for higher productivity pairs instead nothing changes. When there is a minimum wage, a positive productivity advantage of being in the black market reduces the incentive for firms to declare the minimum wage level instead of going into the black economy, but as far as the minimum wage is high enough compared to the productivity differential, in particular for $\frac{\bar{w}}{d} > \frac{2\alpha - t}{t(1-\alpha)}$, then there is still a spike at the minimum wage level. In case the productivity difference is multiplicative, for the no minimum wage case, a productivity advantage low enough, i.e. $\eta < 1 + \frac{t(1-\alpha)^2}{2\alpha - t}$, is necessary to avoid that all agents go into the black market. In such circumstances, the incentives to declare the minimum wage are reduced, but do not dissapear. In particular, a spike at the minimum wage level will anyway be present.

7.3.2 Discontinuity in expected fines

A discontinuity at zero declaration may also exist with regard to the expected fine. Again, it is not a priori obvious in which direction such a discontinuity may work. On one side, the non-existence of a company in official registers may make more difficult to localize it and perform an audit. On the other side, once an audit is performed, to prove underreporting is much more difficult than proving non-reporting, as in the latter case the operation of a firm without registration constitutes evidence in itself. Discontinuities may also exist in the fine applied in case of detection, with complete underreporting likely to be punished more harshly than partial underreporting. Assume that the expected fine is:

$$\begin{cases} \gamma f & if \;\; x_i > 0 \\ \rho\gamma f & if \;\; x_i = 0 \end{cases} \text{ where } f \text{ is given by (1)}$$

In case $\rho > 1$ being in the black market gives rise to higher expected fines due to higher probability of auditing or higher fines imposed in case of detection. Without minimum wage, nothing changes. With a minimum wage, the incentive to declare the minimum are stronger.

In case $\rho \in (0, 1)$ being in the black market gives rise to lower expected fines due to lower probability of auditing. Unless the advantage of being in the black market is not too high, every agent goes underground. In particular for $\rho > (2 - \alpha)\,\alpha$ the equilibrium without minimum wage will not change, while in case of minimum wage, the incentives to declare the minimum wage instead of going into the black economy are reduced, but do not dissapear, with a spike at the minimum wage level remaining.

7.4 Entitlements from social security

Social security contributions usually provides entitlements in the form of pensions, unemployment benefits, health insurance, maternity benefits and so on. If workers value such entitlements, then their existence represents an incentive to contribute and should be taken into account when analyzing the evasion decision. Entitlements are usually partly linked to contributions and partly independent of them. Below, the implications for the model for each case are analyzed.

7.4.1 Proportional transfers

Suppose that workers receive from social security institutions a transfer proportional to their declared wage, ϑx. In theory the value for workers of this could be more than its cost, i.e. $\vartheta > t$. This may be the case when social security funds run a deficit or are subsidized by the general budget (and thus by fiscal imposition on a different tax base) or when workers highly value these transfers (for instance because they provide some insurance, that, due to some market failure, cannot be purchased separately.) In this case, however, there is no reason to evade taxes, so we assume, more realistically, that $\vartheta < t$.

Equation (5) becomes:

$$I = w - tx + \vartheta x$$

In case also equation (1) is modified, so that fines are paid only on the amount of evasion net of foregone benefits, then the model is simply modified by substituting $(t - \vartheta)$ to t. In case fines continue to be paid on evaded taxes, then, the solution to (8) becomes:

$$x = (1 - \alpha + \alpha \frac{\vartheta}{t}) y$$

Not surprisingly, evasion declines, while a positive correlation between the tax rate and the portion of income that is evaded appears. This is consistent with the results reported by Alm et al. (1990) in their study about Jamaican employees tax evasion and avoidance. They find that "the tax base rises with higher benefit for payroll tax contributions and falls with higher marginal tax rates", albeit estimated elasticities are small. As for the effects of the minimum wage, the productivity threshold above which workers prefer to declare the minimum wage is lower in case of transfers proportional to contributions, thus possibly increasing the size of the spike.

7.4.2 Lump-sum transfers

Here the case of a lump-sum transfer δ is analyzed. The transfer is assumed to be conditional on formal working status. In absence of a minimum wage, the only effect of a lump-sum transfer is to displace complete evasion emerging when enforcement is weak with a minimal declaration, as to qualify for the transfer by being formally part of the workforce. More interestingly, in case of minimum wage, a transfer conditional on formal working status represents a further incentive to declare the minimum instead of going into the black market and thus reduces the productivity threshold above which workers prefers to declare the minimum wage. In particular the threshold becomes:

$$y_{mw} = \frac{1}{2(1-\alpha) + 2\alpha\frac{\delta}{t\bar{w}}}\bar{w}$$

The lump-sum transfer δ should be intended as the difference between transfers conditional on being employed and transfers conditional on being not employed (unemployment benefits or other forms of social support.) In case $\delta < 0$ then the threshold would be higher as being formally employed would mean giving up some net transfer, but the effects of the minimum wage will not disapper as far as the monetary loss in case of official employment status is low enough compared to the minimum wage, in particular for $\frac{|\delta|}{\bar{w}} < t\frac{(1-\alpha)}{2\alpha}$.

8 Conclusions

The paper develops a tractable model of underreporting of earnings by employed labour and works out the implications of introducing minimum wage regulation in such an environment.

The interaction between tax evasion and minimum wage gives rise to a spike at the minimum wage level. This is a mechanism that has never been proposed in the literature, that works in a perfectly competitive labour market and that can account for the double digit spike present in some countries.

In addition the model contributes to the policy discussion on minimum wage in countries where underreporting of earnings is a relevant phenomenon. In particular it is shown that the introduction of the minimum wage can indeed boost fiscal revenues by extracting more resources from the lower end of the productivity distribution.

The model makes a new prediction about the correlation between the size of the spike at the minimum wage level and the size of the informal economy that finds support in the data.

The paper also contributes to the literature on tax evasion by showing that imperfect detection alone is able to generate an internal solution to the tax evasion decision, even with fixed probability of an audit and risk neutrality by the agent subject to it.

In Tonin (2006) an empirical investigation is conducted on the effects on disposable income of increasing the minimum wage implied by the model.

The optimal auditing strategy in case of imperfect detection and the optimal minimum wage in a labour market structure where the minimum wage has a role beyond its fiscal impact are the subjects of future research.

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Appendix A

An alternative setting for imperfect detection

Tax authority devotes an exogenously given $\gamma \geq 0$ units of "auditing resources" to every firm-worker pair. The more resources are used, the more income is discovered in expectations. In particular, if γ unit of resources are used, then discovered income $\hat{y}$ is distributed with uniform probability in the interval $[(1-a^{-\gamma})y, y]$ where $a > 1$ measures the effectiveness of auditing.

- if $\gamma = 0$ (no resources) the interval is $[0, y]$ (even with no resources there is the possibility of discovering - may be interpreted as emergence of evidence from other investigations or receiving denunciation or other costless way of getting evidence)
- if $\gamma \to +\infty$ the (degenerated) interval is $[y, y] = \{y\}$ i.e. the full income is discovered for sure

The pdf of the distribution over the interval is $g(\hat{y}) = \frac{1}{y-(1-a^{-\gamma})y} = \frac{a^{\gamma}}{y}$, so:

$$g(\hat{y}) = \begin{cases} \frac{a^{\gamma}}{y} & \hat{y} \in [(1-a^{-\gamma})y, y] \\ 0 & otherwise \end{cases}$$

Provided the tax authority devotes resources γ to a taxpayer characterized by true income y and declared income x then the expected fine is:

$$f = \begin{cases} t\theta \int_{x}^{y} (\hat{y} - x) g(\hat{y}) d\hat{y} & if \quad x \geq (1-a^{-\gamma})y \\ \left[(1-a^{-\gamma})y - x\right] t\theta + t\theta \int_{x}^{y} (\hat{y} - x) g(\hat{y}) d\hat{y} & if \quad x < (1-a^{-\gamma})y \end{cases}$$

as the part of undeclared income below $(1-a^{-\gamma})y$ is discovered with certainty and a fine is imposed on it, then it will never be the case that $x < (1-a^{-\gamma})y$, provided the taxpayer knows the detection technology and γ.

Thus, concentrating on $x \geq (1-a^{-\gamma})y$ we have:

$$f = t\theta \int_x^y (\hat{y} - x) g(\hat{y}) d\hat{y} = t\theta \frac{a^\gamma}{y} \frac{1}{2} (y - x)^2$$

Then,

$$I = y - a^\gamma t\theta \frac{1}{y} \frac{1}{2} (y - x)^2 - xt$$

that is equivalent to (7), where the probability of an auditing being performed $\gamma \in [0, 1]$ is substituted by the coefficient $a^\gamma \geq 0$, where γ is the amount of resources devoted to auditing and a indicated how fast the amount of discovered income increases with auditing effort.

Audit conditional on report x

Probability of performing an audit can be conditioned on declared income x, so $\gamma = \gamma(x)$

Proposition 6 *As far as $\gamma\theta < +\infty$ it is impossible to induce any taxpayer to fully comply.*

Proof. Given an income y and a probability of audit $\gamma(x) \in [0, 1]$ a taxpayer prefers to declare y , i.e. to fully comply, than declaring $x \in [0, y)$ iff

$(1 - t)y \geq y - xt - \gamma(x)t\theta \frac{1}{y} \frac{1}{2} (y - x)^2 \Leftrightarrow \theta\gamma(x) \geq \frac{2}{(1 - \frac{x}{y})} = \gamma^*_{x,y}$

As $\lim_{\frac{x}{y} \to 1^-} \gamma^*_{x,y} = \lim_{x \to y^-} \gamma^*_{x,y} = +\infty$ then as far as $\gamma\theta < +\infty$ there is a neighborhood of y at which the above condition cannot hold and thus taxpayers prefer to declare $x < y$ than y. ■

In the alternative setting proposed in this appendix the equivalent condition not to have full compliance even in case of devoted "auditing resources" conditional on declared income is $\alpha^{\gamma(x)}\theta < +\infty$.

The above proposition implies that whatever auditing policy is implemented, at any income level there will be some evasion. So, for any auditing policy there is room for the minimum wage to exert its influence. However, a fixed cost for the taxpayer of being subject to an audit, together with a higher probability of being audited in case of non-compliance than in case of full compliance, would undo the result.

Appendix B

	Informal economy as % of official GDP			Proportion of full-time employees with earnings on the minimum wage (%)				
	1999/2000	2001/02	2002/03	1999	2000	2001	2002	2003
Bulgaria	36.9	37.1	38.3	:	:	:	5.1	:
Czech Republic	19.1	19.6	20.1	:	1.6	1.7	2	2
Estonia	38.4	39.2	40.1	:	6.5	7.4	6.94	6.4
Hungary	25.1	25.7	26.2	:	3.88	8.43	11.38	8.1
Latvia	39.9	40.7	41.3	:	13.89	16.71	15.36	13.63
Lithuania	30.3	31.4	32.6	:	8.2	7.8	8.8	10.15
Poland	27.6	28.2	28.9	:	:	2.89	3.96	:
Romania	34.4	36.1	37.4	:	6.5	6.1	8.9	12.2
Slovakia	18.9	19.3	20.2	:	:	0.22	0.1	0.41
Slovenia	27.1	28.3	29.4	:	2	2.6	2.6	2.7
Netherlands	13.1	13.0	12.6	2.2	2.1	2.2	2.28	2.24
Portugal	22.7	22.5	21.9	7.5	6.2	4	4	5.69
France[1,2]	15.2	15.0	14.5	12.8	13.6	13.9	14	13.4
Ireland[1]	15.9	15.7	15.3	13.7	:	2.2	2.1	3.1
Spain[2]	22.7	22.5	22.0	2.6	1.4	0.85	0.78	0.86
United Kingdom[1,3]	12.7	12.5	12.2	2.5	1.4	1	1.8	1.2
United States[1,3]	8.7	8.7	8.4	2.4	1.8	1.5	1.5	1.4

1: minimum wage is fixed on an hourly base;
2: part-time workers included in the data on earnings;
3: earnings at or below the minimum wage;

Source: Schneider (2005) *Source: Eurostat (2004)*

The relative size of the shadow economy is estimated by Schneider (2005) using a DYMIMIC (Dynamic Multiple Indicator, Multiple Causes) approach, where the size of the hidden economy is a latent variable. For transition countries cause variables used in the structural model are: share of direct taxation, share of indirect taxation (both in % of GDP); share of public administrative employment in % of total employment as a proxy for burden of state regulation or state interference; unemployment rate and GDP per capita. For highly developed OECD countries additional cause variables used are the burden of social security payments, the tax morale, quality of state institutions and an index of the regulation of the labor market. Employment rate (% of the population between 18 and 64), annual growth rate of GDP, and annual growth rate of local currency per capita are used as indicator variables in the measurement model. The absolute size of the shadow economy is calculated combining the estimates for the relative size obtained through the above mentioned method with available estimates for the size of the informal economy obtained through a currency demand approach and available for some countries. For details on the method see Schneider (2005).