Workers and Firm Sorting into Temporary Jobs^{*}

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Abstract

The liberalization of fixed term contracts in Europe has led to a two tier regime, with a growing share of jobs covered by temporary contracts. The paper proposes a matching model with direct search in which temporary and permanent jobs coexist in a long run equilibrium. When temporary contracts are allowed, firms are willing to open permanent jobs in as much as their job filling rate is faster than that of temporary jobs. From the labour demand standpoint, a simple *trade-off* emerges between an ex-ante job filling rate and ex-post flexible dismissal rate. From the labor supply standpoint, a *trade-off* emerges between an ex-ante lower job finding rate and ex-post larger retention rate. The model features a natural sorting of firms and workers into permanent and temporary jobs. It is also consistent with the observation that workers hired on a permanent contract receive more training. The paper shows that the transition from a rigid to a two a tier regime system is always associated with a transitory fall in unemployment.

• Key Words: Matching Models, Temporary Jobs

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1 Introduction

The liberalization of temporary contracts, or fixed term contracts as are often defined in the policy debate, is the main labour market reform in continental Europe. The liberalization applies only to new hires, so that only new jobs and new vacancies can *potentially* be advertised and filled with temporary contracts. Existing jobs, covered by open ended contracts, are not directly affected by the reform. As a result, a two tier regime has emerged in many continental European markets, with a growing share of temporary contract, which reached 13.6% in 2004 (European Commission [2005]). As the stock of open ended jobs dies out by natural turnover, many observers and policy analysts wonder whether the share of temporary contracts will eventually absorb the entire labor market. This paper argues that the long run implications of a labor market with both temporary and permanent contracts are not fully explored.

In a pure labor demand setting with risk neutral workers and a frictionless labor market, temporary jobs should indeed take over the entire labour market. Boeri and Garibaldi [2006] study theoretically and empirically the transition from a rigid system with only permanent contracts to a dual system with temporary and permanent contracts. In the aftermath of the liberalization, no vacancies covered by permanent contracts are posted, and the stock of temporary contracts absorbs the entire workforce. Similar implications are held by various papers (Cahuc and Postel Vinay [2002] and Blanchard and Landier [2002])¹ and ad hoc assumptions ensure that temporary and permanent contract coexist in equilibrium.

This paper studies firms and workers' sorting into permanent and temporary contracts in an imperfect labor market. Specifically, it studies vacancy posting in permanent and temporary jobs in a world with matching frictions and direct search. From the labor demand standpoint, a filled job with a temporary and flexible contract is more profitable to a firm, since it allows the firm to easily adjust labour in the face of adverse productivity shocks. Free entry in each submarket implies that in equilibrium jobs advertised with permanent contracts display a larger job filling rate. From a labour demand standpoint, a simple *trade-off* emerges between an ex-ante slower job filling rate and ex-post more flexible dismissal rate. In other words, firms that post jobs with temporary contracts face longer job filling rate. This mechanism is akin to wage posting and to the competitive search equilibrium initially proposed by Moen [1997].

From the labour supply stand point, a similar mechanism emerges. For a given wage within the bargaining set, in the spirit of Hall [2005], risk neutral workers with heterogeneous and unobservable reservation utility, prefer to search in the permanent submarket. Yet, in as much as job search in the submarket for temporary workers leads to larger job filling rate, a simple *labor supply trade-off* emerges between an ex-ante lower job finding rate and an ex-post larger retention rate². As a result the model features a natural sorting of firms

¹In Cahuc and Postel Vinay [2002] temporary and permanent contracts coexist in light of a random and exogenous state permission to fill jobs with temporary contracts. In Blanchard and Landier [2001] all jobs start with a temporary contract, and only a fraction is endognouesly converted into a permanent job. Garibaldi and Violante [2005] have similar implications

 $^{^{2}}$ A similar implication, at least from the labor supply standpoint, emerges in the quantitative general equilirbium model proposed by Alonso-Borrego et al. [2005]. The free entry condition in both markets, a key feature of the mechanism of this paper, is not modeled by Alonso-Borrego et al.

and workers into permanent and temporary jobs.

The simple theory has several implications. First, the steady state of the model displays both temporary and permanent jobs, with an equilibrium share of temporary jobs that crucially depends on the average duration of temporary contracts and the structure of productivity shocks. Second, and most important, the liberalization of temporary contracts does not crowd out permanent contracts, and the labour market moves smoothly toward a long run dual system. Third, in the early stage of the transition, as temporary contracts are opened up and their share grows slowly, equilibrium unemployment always fall, and the liberalization of temporary jobs is associated to a "honeymoon effect". Fourth, long run equilibrium unemployment with a dual system can be above or below the equilibrium unemployment with only permanent contracts.

Two important extensions are developed. In the first one, firms have the option to undertake costly training in the face of adverse productivity shocks. The theory clearly shows that workers covered with permanent contracts are more likely to be trained. The second extension studies the transition from temporary to permanent contracts, and shows that the basic trade off survives also when workers can graduate to the permanent market via a temporary job.

The paper proceeds as follows. Section 2 highlights the structure of the model and the basic equations. Section 3 defines and solves the equilibrium of the model. Section 4 studies analytically the transition toward a dual regime and presents a simple set of simulations. Section 5 introduces the possibility of workers' training as a mean to face the productivity shocks. Section 6 studies the model with on the job search. Section 7 discusses the implications of our theory vis-à-vis the existing empirical evidence. Section 8 concludes.

2 The matching framework

- The labour market consists of a mass one of risk neutral workers. Each worker can be employed or unemployed. Workers are subject to natural turnover and separate from their existing job with a Poisson process with arrival rate equal to s.
- Workers differ in their idiosyncratic income from non employed. The outside flow utility is indicated with z, and we assume that z is time invariant and not observable to the firms. z is drawn from a continuous cumulative distribution F(z) with upper support z^u . Since z is not observable, workers are identical vis-à-vis the firms.
- Firms produce with a constant returns to scale technology with labor productivity equal to y_h . Each job has an instantaneous probability λ of experiencing a (permanent) adverse shock. Conditional on an adverse shock, the productivity falls to $y_l < y_h$. We further assume that the wage paid is strictly larger than z^u so that the labour market is viable for each worker.
- Two types of contracts exist in the economy. Temporary contracts and permanent contracts. Temporary contracts can be broken by the firm at will. Firm initiated separation is not possible with

permanent contracts³. Firms that hire workers on permanent contracts must rely on workers' natural turnover for downsizing.

- Firms create jobs by posting costly vacancies, and firms can freely decide to open either temporary or permanent jobs. Keeping open a vacancy, either temporary or permanent, involves a flow cost equal to c. For simplicity, we assume that the vacancy cost is identical for both contracts.
- Temporary and permanent contracts are offered in different *submarkets*. In each submarket, the meeting of unemployed workers and vacant firms is described by a well defined matching function m with constant returns to scale. Submarkets are indexed by $i \in [p, t]$ where p stands for *permanent* and t for *temporary*.
- Unemployed workers can freely move across submarkets but can not search simultaneously across submarkets. In this respect, search is directed toward a specific submarket (this hypothesis will be relaxed in section 6). Unemployed workers searching for a permanent job enjoy a fixed exogenous benefit b > 0. b is not enjoyed when the worker searches in the temporary submarket.
- There are matching frictions in each submarket. We let m(u_i, v_i) be the flow of new matches, where u_i denotes the measure of unemployed workers in submarket i searching for the measure v_i of vacancies; following standard assumptions, we assume that m is concave and homogeneous of degree one in (u_i, v_i) with continous derivatives. Now define h_i = m(u_i, v_i)/u_i = m(1, θ_i) = h(θ_i) as the transition rate from unemployment to employment for an unemployed worker in submarket i and q_i = m(u_i, v_i)/v_i = q(θ_i) as the arrival rate of workers for a vacancy in submarket i. θ_i = v_i/u_i is the submarket specific labour market tightness. The matching function m satisfies the following conditions:

$$\lim_{\substack{\theta_i \to 0}} h(\theta_i) = \lim_{\substack{\theta_i \to \infty}} q(\theta_i) = 0 \qquad i = p, t$$
$$\lim_{\substack{\theta_i \to \infty}} h(\theta_i) = \lim_{\substack{\theta_i \to 0}} q(\theta_i) = \infty \qquad i = p, t$$

- Upon the meeting of an unemployed workers and a vacant firm, each match signs a long term contract that fix a wage for the entire employment relationship without ex-post renogatiation. In the spirit of Hall [2005], any wage within the parties bargaining set, at the time of job creation, can be supported as an equilibrium.
- To make the problem interesting, we restrict our attention to wages such that $y_h > w_p > y_l$ and $y_h > w_t > y_l$. This will ensure that, conditional on the realization of the adverse shock λ , permanent contracts involve a loss to the firm. Further, we will focus on a constant wage across submarket, such that $w_p = w_t = w$.

³The interpretation of dismissal at will in the case of temporary workers is twofold: either firms are allowed to fire whenever the shock occurs, or they're able to set contracts whose duration is exactly $1/(s + \lambda)$

• The equilibrium of the model is characterized by free entry of firms in each submarket, and workers' sorting condition across submarkets.

2.1 Value Functions and Job Creation in the Permanent Market

Let $U_p(z)$ and $E_p(z)$ denote, respectively, the expected discounted income for an unemployed worker and for an employed one in the permanent market. The Bellman equations are:

$$rU_p(z) = z + b + h(\theta_p)[E_p(z) - U_p(z)]$$
(1)

$$rE_p(z) = w + s[U_p(z) - E_p(z)]$$
 (2)

where r is the pure discount rate, z is the workers' specific outside option and b is the unemployment benefit. Let J_p^h and J_p^l denote, respectively, the present discounted value of a permanent job when productivity is high (y_h) or low (y_l) ; their formal expression read

$$rJ_p^h = y_h - w + \lambda [J_p^l - J_p^h] + s[V_p - J_p^h]$$
$$rJ_p^l = y_l - w + s[V_p - J_p^l]$$

When productivity is high, the firm enjoys an operational profit equal to $y_h - w$. The worker leaves at rate s and the firm gets the expected value of a vacancy formally indicated with V_p . Conditional on a productivity shock λ , the firm has no margin of adjustment and experiences a capital loss equal to the difference between the value of a permanent job in high state and a value in bad state $J_p^l - J_p^h$. In the low state, the firm runs an operational loss $y_l - w$ as long as the worker separates at rate s. The asset equation of a vacancy reads

$$rV_p = -c + q(\theta_p)[J_p^h - V_p]$$

Assuming free entry in the permanent market, $V_p = 0$, we have that

$$c = q(\theta_p) J_p^h \tag{3}$$

The previous condition is one of the key equations of the model. It shows that the flow cost of vacancy posting is equal to expected benefit, where the latter is described as the product of the job filling rate into permanent contract time the value of a filled job.

Finally note that the value of a filled job can be written as

$$J_p^h = \frac{y_h - w}{r + s + \lambda} + \frac{\lambda(y_l - w)}{(r + s)(r + s + \lambda)}$$

$$J_p^l = \frac{y_l - w}{r + s} < 0$$
(4)

The latter expression represents the cost associated to having a permanent contract in case of adverse shock.

2.2 Value Functions and Job Creation in the Temporary Market

Workers employed with a temporary contract are dismissed conditional on the arrival rate λ , so that the value of employment reads

$$rE_t(z) = w + (s+\lambda)[U_t(z) - E_t(z)]$$
(5)

The value of unemployment depends on the specific outside income and faces a transition probability $h(\theta_t)$

$$rU_t(z) = z + h(\theta_t)[E_t(z) - U_t(z)]$$
(6)

Firms in temporary market are free to dismiss workers conditional on the adverse productivity shock; the value of a filled temporary job and of a temporary vacancy read

$$rJ_t^h = y_h - w + (s+\lambda)[V_t - J_t^h]$$
$$rV_t = -c + q(\theta_t)[J_t^h - V_t]$$

Assuming free entry also in the temporary market, $V_t = 0$, we have that

$$c = q(\theta_t) J_t^h \tag{7}$$

Similarly to the condition above, equation (7) says that the flow cost of vacancy in the temporary market is equal to expected benefit, where the latter is described as product of the job filling rate into temporary contract time the value of a filled job.

Before turning to the equilibrium definition, we derive the second key condition of our analysis. Using the free entry condition into the temporary, one can easily show that a filled temporary job has larger value than a permanent job

$$J_t^h = \frac{y_h - w}{r + s + \lambda} > J_{p,h}$$

We are now in a position to establish a key result of our model. The expected value of vacancy depends on the job filling rate and on the value of a filled job. A labour market with both temporary and labour market is such that

$$q(\theta_t)J_t^h = q(\theta_p)J_p^h$$

where we have just proved that $J_t^h > J_p^h$. This result tells that the coexistence of temporary and permanent contract implies that

$$q(\theta_t) < q(\theta_p)$$

Once the job is filled, the firms prefer a flexible contract. They are thus willing to offer both temporary and permanent contract if the job filling rate for permanent contracts is larger than the job filling rate for temporary contracts. Conversely, this result suggests that the job finding rate of a temporary contract is larger, so that

$$h(\theta_t) > h(\theta_p)$$



Figure 1: Workers' endogenous sorting

The previous result is very important for the results of the next section, where we discuss the workers' sorting condition between the two submarkets.

2.3 Workers' Sorting

Workers take as given the job finding $rate^4$ and optimally decide in which submarket to search for a job. Since workers can freely move across submarkets, the optimal allocation will be

$$U(z) = Max[U_p(z), U_t(z)]$$

where the expressions for $U_p(z)$ and $U_t(z)$ are obtained combining (2) with (1) and (5) with (6)

$$rU_p(z) = \frac{(z+b)(r+s) + h(\theta_p)w}{r+s+h(\theta_p)}$$
(8)

$$rU_t(z) = \frac{z(r+s+\lambda) + h(\theta_t)w}{r+s+\lambda + h(\theta_t)}$$
(9)

The values of unemployment, for given job finding rates, are monotonically increasing in z. In what follows, we look for a reservation value of R such that the marginal worker (the one with idiosyncratic outside option z = R) is indifferent between searching for a temporary or a permanent job. If such R exist, workers endogenously sort between the two markets. Note that workers with low z are the workers that place a larger value from labour market participation. Such workers are more willing to take up a job right away, even if such job has shorter duration. The formal value of R is

$$R = w - b \frac{(r+s)[r+s+\lambda+h(\theta_t)]}{(r+s)h(\theta_t) - (r+s+\lambda)h(\theta_p)}$$

⁴Once a functional form for the matching function is chosen, θ_i is completely determined by the behaviour of the firms.

Figure (1), plots the reservation value. As long as the existence condition holds⁵, then R < w and there exists a proportion of workers 1 - F(R) searching in the permanent market. It's easy to see that when b = 0 the reservation outside option is equal to the wage and all workers look for a temporary job.

2.4 Labor Market Flows

Labour supply is the sum of unemployment and employment in each submarket

$$u_t + n_t = F(R)$$
$$u_p + n_p = 1 - F(R)$$

The dynamic evolution of unemployment in the two submarket is given by difference between job creation and job destruction. This implies that

$$\begin{split} \dot{u}_p &= sn_p - h(\theta_p)u_p = s[1 - F(R) - u_p] - h(\theta_p)u_p \\ \dot{u}_t &= (s + \lambda)n_p - h(\theta_t)u_t = (s + \lambda)[F(R) - u_t] - h(\theta_t)u_t \end{split}$$

Unemployment in each submarket is constant when job creation is equal to job destruction; the steady state expressions for the stocks read

$$u_p = \frac{s[1 - F(R)]}{s + h(\theta_p)}$$

$$n_p = \frac{h(\theta_p)[1 - F(R)]}{s + h(\theta_p)}$$

$$u_t = \frac{(s + \lambda)F(R)}{s + \lambda + h(\theta_t)}$$

$$n_t = \frac{F(R)h(\theta_t)}{s + \lambda + h(\theta_t)}$$

3 Equilibrium

The equilibrium is obtained by a triple $\{\theta_t, \theta_p, R\}$, an exogenous wage rate w and a distribution of employment across states that satisfy the set of value functions $\{J_i^h, J_p^l, V_i, E_i(z), U_i(z) \text{ with } i \in [p, t]\}$ and:

• Optimal vacancy posting in each submarket. The value of a vacancy is identical across submarkets and driven down to zero by free entry

$$V_p = V_t = 0$$

This in turn implies:

- Job creation in the permanent market

$$q(\theta_p)J_p^h = c \qquad (JC, permanent)$$

⁵See the appendix.

- Job creation in the temporary market

$$q(\theta_t)J_t^h = c \tag{JC, temporary}$$

which together say that in equilibrium the expected benefit of a permanent job must be equal to the expected benefit of a temporary job.

• Optimal workers' sorting. The marginal worker is indifferent between searching in the market for temporary or permanent jobs

$$U_p(R) = U_t(R) \tag{Sorting}$$

Once a functional form for $m(u_i, v_i)$ is chosen, θ_p and θ_t are determined through job creation conditions; the sorting equation yields R and the last equations in the previous section determine the stocks.

3.1 Comparative Static

Qualitative aspects of the final equilibrium obviously depend on the values taken by the exogenous parameters. In this section we focus our attention upon the effects of changes from a relevant couple of them, namely the unemployment benefit b and the shock occurance rate λ , on the unemployment rate, the labour market tightness, the reservation outside option and the value (for the firm) of a filled job.

• From the point of view of the firms, the level of the unemployment benefit does not have any effect on the value of a filled job, nor it does, using the job creation conditions in the two submarkets, with the labor market tightnesses. In symbols

$$q(\theta_i)J_i^h = c \Rightarrow \frac{\partial \theta_i}{\partial b} = \frac{-c[\partial J_i^h/\partial b]}{[J_i^h]^2[\partial q(\theta_i)/\partial \theta_i]} = 0 \text{ since } \partial J_i^h/\partial b = 0$$

On the contrary, an increase in b is expected to make the permanent submarket more attractive for the workers: a fraction of them leaves the temporary market and begins to search for a permanent job, but since the behaviour of the firms (namely: the market tightnesses) has not changed, permanent unemployment increases and temporary unemployment decreases. In fact, using the formal value of Rit is immediate to see that, as long as R < w, $\partial R/\partial b < 0$. This result allows to evaluate the effect on the unemployment rates

$$\frac{\partial u_p}{\partial b} = -\frac{s}{s+h(\theta_p)}\frac{\partial F(z)}{\partial z|z=R}\frac{\partial R}{\partial b} > 0$$

and

$$\frac{\partial u_t}{\partial b} = \frac{s+\lambda}{s+\lambda+h(\theta_t)} \frac{\partial F(z)}{\partial z|z=R} \frac{\partial R}{\partial b} < 0$$

as expected. The effect on total unemployment is consequently ambiguous⁶.

⁶With some algebra it can be shown that an increase in the unemployment benefit increases total unemployment as long as $\lambda < [h(\theta_t) - h(\theta_p)]/h(\theta_p)$.

• Comparative statics about λ is not as clear. If a shock to the productivity of a match becomes more likely, all firms enjoy the operational profit for a shorter period; the value of a filled job, either temporary or permanent, diminishes and firms are less prone to post new vacancies. In formal terms

$$\frac{\partial J_p^h}{\partial \lambda} = \frac{y_l - y_h}{(r + s + \lambda)^2} < 0 \text{ and } \frac{\partial J_t^h}{\partial \lambda} = -\frac{y_h - w}{(r + s + \lambda)^2} < 0$$

Using the result above with job creation conditions in both markets yields the negative reaction of the labor market tightnesses

$$\frac{\partial \theta_i}{\partial \lambda} = \frac{-c[\partial J_i^h/\partial \lambda]}{[J_i^h]^2[\partial q(\theta_i)/\partial \theta_i]} < 0$$

From the point of view of the workers a higher λ makes the duration of a temporary job shorter; a fraction of them would therefore move from the temporary to the permanent tier but, differently from the case of the unemployment benefit, the productivity shock negatively affects the tightness in both submarkets too. In other words a trade off emerges between a higher risk of being fired on the temporary market (which has a negative direct effect upon the reservation outside option) and a possibly too high unemployment duration on the permanent. The net effect of a change in the shock rate upon R is therefore a priori ambiguous and no prediction can be made upon the unemployment rates.

4 Liberalization of Temporary Contracts

While the steady state solution clearly implies a long run coexistence of the two type of contracts, the question linked to the liberalisation of temporary contracts has not yet been discussed. In this section we consider the full transition from a *rigid regime*, a situation where only permanent contracts are allowed, to a *dual regime* where temporary and permanent contracts coexist in equilibrium.

The rigid regime is formally described as a labour market in which only the permanent submarket exists. We define the introduction of temporary jobs as a permanent unexpected shock to the steady state of the rigid market. The functioning of the liberalisation is as follows. At time $\tau = 0$ when the shock occurs the stock of unemployed workers of the old regime is immediately split in two: workers with $z \leq R$ start searching in the temporary submarket, while workers with z above the reservation productivity R stay in the permanent one. Firms immediately post vacancies in order to fully absorbe any rent. Thereafter, the stock of workers smoothly move toward a new steady state with two submarkets. Note that both the reservation utility R as well as market tightnesss in the two submarkets are time invariant, and the dynamics of the model can be described analytically.

To keep track of the dynamics of the model after the introduction of temporary contracts, we will consider separately the behaviour of workers whose outside option is below or above the reservation threshold.

• $z \leq R$. At $\tau = 0$ all unemployed workers with an outside utility below the reservation utility start

searching for a temporary job at the finding rate $h(\theta_t)$. On the demand side, firms post a number of temporary vacancies such that the tightness jumps to its equilibrium level and fill them at rate $q(\theta_t)$. In addition those workers employed with a permanent contract and idiosyncratic utility below R are gradually dismissed at rate s and become unemployed in the temporary submarket. The steady state is reached when all workers with outside utility below the reservation R move to the temporary submarket. Let's define with $n_p(z,\tau)$ and $n_t(z,\tau)$ the share of permanent and temporary contract with outside utility less or equal to z at transition time τ . $u_t(z,\tau)$ is similarly defined for the unemployment stock. This implies that at each point in time the distribution of workers with a low outside option reads

$$F(z) = n_p(z,\tau) + n_t(z,\tau) + u_t(z,\tau), \qquad z \le K$$

and the dynamics of the three functions is described by

$$\begin{split} \dot{n}_p(z,\tau) &= -sn_p(z,\tau), \qquad z \leq R \\ \dot{n}_t(z,\tau) &= h(\theta_t)u_t(z,\tau) - (s+\lambda)n_t(z,\tau), \qquad z \leq R \\ \dot{u}_t(z,\tau) &= sn_p(z,\tau) + (s+\lambda)n_t(z,\tau) - h(\theta_t)u_t(z,\tau), \qquad z \leq R \end{split}$$

where it is clear that there is no inflow into $n_p(z, \tau)$ for $z \leq R$, but simply an outflow that dies out as all permanent jobs with outside utility below R are slowly destroyed at rate s. The flows of temporary contracts is governed by flows that are identical to those of the steady state. During the transition, the unemployment rate into the temporary submarket increases also because of the inflow of old permanent jobs.

• z > R. People with outside utility above the reservation R are either employed with a permanent contract or unemployed and searching for a permanent job. This is true both in the rigid and in the liberalized regime. Accordingly, the distribution of such workers reads

$$F(z) = u_p(z,\tau) + n_p(z,\tau), \qquad z > R$$

where $u_p(z,\tau)$ is the stock of unemployed at time τ and $n_p(z,\tau)$ is the stock of employed workers. The dynamics of these two components is given by

$$\begin{split} \dot{u}_p(z,\tau) &= sn_p(z,\tau) - h(\theta_p)u_p(z,\tau), \qquad z > R \\ \dot{n}_p(z,\tau) &= h(\theta_p)u_p(z,\tau) - sn_p(z,\tau), \qquad z > R \end{split}$$

The system of differential equations can be solved analytically. The details are in the appendix. The readers can find the final results below

$$n_t(z,\tau) = \left\{ \frac{h(\theta_t)h(\theta_p)F(z)}{[s+h(\theta_p)][h(\theta_t)+\lambda]} - \frac{h(\theta_t)F(z)}{h(\theta_t)+s+\lambda} \right\} e^{-[h(\theta_t)+s+\lambda]\tau} + \frac{h(\theta_t)F(z)}{h(\theta_t)+s+\lambda} - \frac{h(\theta_t)h(\theta_p)F(z)}{[s+h(\theta_p)][h(\theta_t)+\lambda]} e^{-s\tau}$$

$$\begin{split} u_t(z,\tau) &= F(z) \left\{ \frac{s}{s+h(\theta_p)} + \frac{\lambda h(\theta_p)}{[s+h(\theta_p)] \left[h(\theta_t) + \lambda\right]} - \frac{s+\lambda}{h(\theta_t) + s+\lambda} \right\} e^{-[h(\theta_t) + s+\lambda]\tau} + \\ &+ \frac{(s+\lambda)F(z)}{h(\theta_t) + s+\lambda} - \frac{\lambda h(\theta_p)F(z)}{[s+h(\theta_p)] \left[h(\theta_t) + \lambda\right]} e^{-s\tau} \\ u_p(z,\tau) &= \frac{s[1-F(z)]}{s+h(\theta_p)} \\ n_p(z,\tau) &= \frac{h(\theta_p)F(z)}{s+h(\theta_p)} e^{-s\tau} + \frac{h(\theta_p)[1-F(z)]}{h(\theta_p) + s} \end{split}$$

Taking the limit as τ goes to infinity and using z = R, one gets easily the expressions for the two tiers steady state (see section 2.4).

4.1 Just a "honeymoon effect"?

Having derived the analytical solution to the transition, we now look into the effects of the liberalisation of temporary contracts, with particular attention to the unemployment rate. Our solution distinguishes between a short run and a long run effect.

In the aftermath of the liberalisation, immediately after the shock, the unemployment rate necessarily falls. The reasoning is as follows. At $\tau = 0$ the stock of unemployed workers is as large as in the rigid regime, but a fraction F(R) of workers starts searching into the temporary submarket where the job finding rate $h(\theta_t)$ is larger. Indeed, market tightness and vacancy posting are a forward looking variable, and immediately jump to exhaust all the rents. While it is true that in the temporary submarket also the separation rate is larger through the destruction rate λ , it takes time for such effect to emerge. Further, market tightness is constant during the transition. As a result unemployment, initially, necessarily falls⁷.

Figures (2) and (3) plot the dynamics of the unemployment and the employment rates for a given set of parameters values⁸. The downward jump represents this "honeymoon effect": on impact, the liberalization of temporary contracts has a positive effect on total employment.

The results on the long run effects are more ambiguous. Whether total unemployment is permanently lower than in the rigid regime depends on the relative strength of the job finding and job destruction rates in the two submarkets. The unemployment is permanently reduced if⁹

$$\begin{aligned}
 u_{p,old} &> u_p(\tau \to \infty) + u_t(\tau \to \infty) \Rightarrow \\
 \frac{s}{s+h(\theta_p)} &> \frac{s[1-F(R)]}{s+h(\theta_p)} + \frac{(s+\lambda)F(R)}{s+\lambda+h(\theta_t)} \Rightarrow \\
 \lambda &< \frac{s[h(\theta_t) - h(\theta_p)]}{h(\theta_p)}$$
(10)

⁷Analitically this result is obtained by taking the time derivative of u_t and evaluating it at $\tau = 0$; this yields $\partial u_t(\tau)/\partial \tau | \tau = 0 < 0$. Details are in the appendix.

⁸We assumed that the matching function is a Cobb-Douglas one with unemployment elasticity α : $m_i = k_i u_i^{\alpha} v_i^{1-\alpha}$ where $\alpha = 0.5$ and $k_i = 1$. Time is expressed in years. The pure discount rate r is 0.02, worker turnover s is 0.1 and the average waiting time for a productivity shock is about six years ($\lambda = 0.15$). Productivity is either 1 or, conditional on the adverse shock, 0.6. The wage is 0.8 and the exogenous benefit b for the unemployed on permanent market is 30% of the wage. The cost of keeping open a vacancy is 0.3.

⁹The stock of unemployed workers in the old regime is discussed in the appendix



Figure 2: Dynamics of the unemployment rate



Figure 3: Dynamics of the employment rate

i.e. if workers' turnover is not too high. This statement, however, needs to be furtherly discussed. Using the condition for the existence of R one gets

$$\lambda < \frac{(r+s)\left\{h(\theta_t)w - h(\theta_p)w - b[r+s + h(\theta_p)w]\right\}}{b(r+s) + h(\theta_p)w}$$

When b = 0, from the point of view of (10) this condition is not relevant, becoming monotonically binding for increasing values of the unemployment benefit; this means that for small values of the unemployment benefit the coexistence of permanent and temporary contracts does not prevent the labour market from a higher equilibrium unemployment rate.

5 Training

In this section we consider the possibility that firms, in the aftermath of the adverse productivity shock, may be able to jump back to the high productivity by undergoing costly training. Specifically, we assume that when the negative shock occurs firms can jump back to the high level of productivity y_h by paying a lump sum cost T in the form of training. As the wage paid to workers is held fixed, we can abstract from the issue of financing. We will show that there exist two bounds $[T_l, T_u]$ such that if $T_l < T < T_u$ only firms in the permanent submarket decide to train workers. The asset equations in the permanent market read

$$rJ_p^h = y_h - w + s[V_p - J_p^h] + \lambda[\max(J_p^l, J_p^h - T) - J_p^h]$$

$$rJ_p^l = y_l - w + s[V_p - J_p^l]$$

$$rV_p = -c + q(\theta_p)[J_p^h - V_p]$$

where the max operator conditional on the λ shocks highlights the training option. On the temporary market the asset equations read

$$rJ_t^h = y_h - w + s[V_t - J_t^h] + \lambda[\max(V_t, J_t^h - T)]$$
$$rV_p = -c + q(\theta_t)[J_t^h - V_t]$$

We now formally establish under what conditions workers with a permanent job receive training. Since undergoing training trasforms a low productivity job into a high productivity job, a firm with a permanent contract will undergo training if

$$J_p^h - T > J_p^l$$

Simultaneously, a firm with a temporary contract will not undergo training if

$$V > J_t^h - T$$

The first condition implies

$$\frac{y_h - w}{r + s + \lambda} + \frac{\lambda(y_l - w)}{(r + s)(r + s + \lambda)} - T > \frac{y_l - w}{r + s} \Rightarrow T < \frac{y_h - y_l}{r + s + \lambda}$$

while the condition on the temporary workers reads

$$T > \frac{y_h - w}{r + s + \lambda}$$

If the cost of training T is large enough so that the exit strategy turns out to be preferable in the temporary market, but not too large, then only firms in the permanent market are induced to train the workers

$$\frac{y_h - w}{r + s + \lambda} < T < \frac{y_h - y_l}{r + s + \lambda} \tag{11}$$

More generally, it is never the case that workers receive training only in the temporary market. Training may be viable on both markets, only in the permanent, or in none of them, depending on the level of T. When T is bounded as in condition (11) the following interesting results follow:

- The temporary market is not affected by training costs. As a consequence, the value of a filled job is the same as in the model without training.
- The value of filled jobs in the permanent market now reads

$$J_p^h = \frac{y_h - w - \lambda T}{r + s}$$

which is larger than in the model without training, but still lower than J_t^h .

• Free entry makes the equilibrium conditions in the temporary submarket independent on T

$$c = q(\theta_p)J_p^h$$
$$c = q(\theta_t)J_t^h$$

This means that in equilibrium the temporary market tightness is the same as without training, while the permanent tightness has now to be higher. As a consequence, on average, in the model with training the job finding rate is higher, the arrival rate of workers for a vacancy is lower, and the steady state overall unemployment is lower.

6 On the Job Search

This section proposes a further extension of the basic model, as it allows workers (either employed or unemployed) in the temporary tier to search for a permanent job. As we keep the wage constant across submarkets, we do not need to explicitly consider wage determination, one of the (many) difficult issues to be faced when one deals with on the job search (Shimer [2003] and Nagypal [2006]). Nevertheless, the matching function and the definition of market tightness need to be modified and adjusted. In what follows, the number of matches in the permanent submarket reads

$$m_p(u_p + n_t + u_t, v_p) = m_p(u_p + F(R), v_p)$$

where the pool of workers that search for a job is the sum of workers searching only in the permanent market (u_p) and the pool of workers searching in the temporary submarket $(n_t + u_t)$. Since the pool of workers in the temporary submarket is the fraction of them with outside utility below R, the second expression immediately follows. As a result, market tightness in the permanent submarket is given by

$$\theta_p = \frac{v_p}{u_p + n_t + u_t} \tag{12}$$

The matching function in the temporary submarket is unchanged and is simply given by $m_t(u_t, v_t)$, with market tightness $\theta_t = v_t/u_t$.

The value functions in the permanent submarket are defined similarly to those of the baseline model (see section 2.1). The only difference is the expression for θ_p , that is defined as in (12) as a way to take into account the composition of the pool of workers searching for a permanent job. Free entry in the permanent submarket implies that

$$q(\theta_p)J_h^p = c$$

where J_h^p is given by (4).

The value functions for the temporary submarket are different, since workers leave temporary jobs at rate $s + h(\theta_p)$. When business conditions are good, the value function reads

$$rJ_t^h = y_h - w + [s + \lambda + h(\theta_p)][V_t - J_t^h]$$

while the value of a vacancy is simply given by

$$rV_t = -c + q(\theta_t)[J_t^h - V_t]$$

so that free entry implies that

$$q(\theta_t)J_t^h = c$$

where J_t^h is now given by

$$J_t^h = \frac{y_h - w}{r + s + \lambda + h(\theta_p)} \tag{13}$$

The job creation conditions are still the two key equations, but since now J_t^h depends also on θ_p they form a non linear system of two equations in two unknowns that can be solved in cascade¹⁰. The last variable to be determined is the reservation utility R. The value of unemployment in the temporary submarket reads

$$rU_t(z) = z + h(\theta_t)[E_t(z) - U_t(z)] + h(\theta_p)[E_p(z) - U_t(z)]$$

where it is clear that an unemployed worker with low outisde utility searches both in the temporary and in the permanent submarket, and can leave the unemployment pool for both types of jobs. Unemployed

¹⁰Starting from job creation in the permanent submarket one gets θ_p ; using this result with job creation in the temporary submarket also θ_t is obtained.

workers in the permanent submarket behave as in the baseline model, and their asset value equation for the unemployment status is provided by (1). Given the expressions for $E_t(z)$ and $E_p(z)$ and after some steps of algebra (see the appendix for details), the reservation utility R reads

$$R = w - b \frac{r + s + \lambda + h(\theta_t) + h(\theta_p)}{h(\theta_t)}$$

which implies that R < w. Ensuring also that b is small enough¹¹, we can easily estabilish that 0 < R < w. With respect to the base model, the value of a filled temporary job given in (13) is now lower and not necessarely higher than the value of a permanent one; however, assuming that $J_p^h < J_t^h$, the structure and functioning of this model is identical to the model without on the job search. In particular, the basic mechanism that ensure that temporary and permanent jobs coexist in equilibrium survives to this admiteddely more realistic scenario. The fact that the value of a permanent job is unchanged while a temporary one is worth less than before means that firms take into account the possibility that temporary workers leave their job moving toward the permanent tier and are consequently less prone to post temporary vacancies; in equilibrium, this leads to a lower tightness in the temporary submarket where a relatively higher congestion from the point of view of the workers emerges.

7 Discussion

There is a vast empirical literature on temporary contracts. A large bulk of the evidence looks at the transition rate of temporary contracts into permanent contracts. The transition from fixed-term to permanent contracts has been analyzed by Booth et al. [2002] for the U.K., Güell and Petrongolo [2000] for Spain, and Holmlund and Storrie [2002] for Sweden. Another bulk of the literature focuses on the stepping stone channel played by temporary contracts. Temporary contracts are used by firms to screen applicants and serve as a gateway toward a permanent job; papers with these focus are Booth et al. [2002] and Zijil et al. [2004]. These dimensions of temporary contract, albeit important, are not the key focus of the paper.

The coexistence of temporary and permanent jobs emphasized in this paper relies on a simple trade off between arrival rates and rent values. The empirical implication of this mechanism is the fact, from the worker standpoint, that the job finding rate into the temporary submarket is larger than the job finding rate into permanent jobs. Direct empirical evidence for this mechanism is not straightforward, since the effect depends crucially on unoboservable components. Yet, indirect evidence appears consistent with this basic implications. Güell [2000] estimates hazard rate from unemployment for people that were previously employed with a temporary and a permanent job. She finds that the hazard rate for workers that had a temporary job is consistently larger. This is certainly coherent with the mechanism of the paper. Blanchard

$$b < \frac{h(\theta_t)}{r + s + \lambda + h(\theta_t) + h(\theta_p)}$$

 $^{^{11}\}mathrm{Technically}$ the equilibrium of the model must be such that

and Landier [2002] estimate transition rate of youth French unemployed into permanent and temporary jobs. The transition rate in the late nineties is more than 20 percent for temporary contracts and around 15 percent (or even less) from unemployment into regular jobs.

The short run effects of the liberalisation of temporary contracts have been studied by Boeri and Garibaldi [2006]. They show that most countries that experienced a gradual liberalization of temporary contracts experienced also employment gains. Such honeymoon effect is clearly present in the mechanism analysed in this paper.

Arulampalam and Booth [1998] investigate the relationship between employment flexibility and training using UK data, and find that workers on temporary contracts are less likely to receive work – related training. The Oecd [2002] reports that the diffusion of temporary contracts has been associated to a reduction in training incidence. Brunello et al. [2006] estimate the probability both of taking any training and of receiving employer - sponsored training as a function of educational attainment, gender, tenure, marital status, age (divided in four classes), public/private sector employment, part time/full time status, type of contract (fixed term, casual job and other, with permanent job as the reference), country, industry, firm size and occupation. Controlling for all these effects, as well as country fixed effect, they find temporary workers have a 4 percentage penalty rate in the probability of receiving training. All these results are coherent with the relationship between training and temporary contracts implied by this paper.

The equilibrium of the model depends also on a specific unemployed income paid only in the permanent submarket. In real life labor markets, unemployed income often requires a specific on the job tenure, and our assumption is fully consistent with this fact. Indeed, one of the concern in the policy debate is the low unemployment insurance faced by workers that transit across the temporary tier. Our equilibrium specification exploits this feature to derive a well defined long run equilibrium.

8 Conclusion

The liberalization of fixed term contracts in Europe has led to a two tiers regime, with a growing share of jobs covered by temporary contracts. The paper proposes a matching model with direct search in which temporary and permanent jobs coexist in a long run equilibrium. When temporary contracts are allowed, firms are willing to open permanent jobs in as much as their job filling rate is faster than that of temporary jobs. The model features a natural sorting of firms and workers into permanent and temporary jobs. It is also consistent with the observation that workers hired on a permanent contract receive more training. The transition from a rigid to a two a tier regime system is always associated with a transitory fall in unemployment. The simple theory has several implications. First, the steady state of the model displays both temporary and permanent jobs, with an equilibrium share of temporary jobs that crucially depends on the average duration of temporary contracts and the structure of productivity shocks. Second, and most important, the liberalization of temporary contracts does not crowd out permanent jobs, and the labour market

moves smoothly toward a long run dual system. Third, in the early stage of the transition, as temporary contracts slowly emerge, equilibrium unemployment always fall, and the liberalization of temporary market is associated to a "honeymoon effect". Fourth, long run equilibrium unemployment with a dual system can be above or below the equilibrium unemployment with only permanent contracts.

The model is also consistent with the fact that workers covered with permanent contracts are more likely to be trained, while the mechanism analysed survives also when workers can graduate to the permanent market via a temporary job.

9 Appendix

9.1 Existence

The coexistence of the two submarkets in equilibrium depends on the existence of a positive reservation outside utility strictly lower than the wage.

• Existence of R. Since both $U_p(z)$ and $U_t(z)$ are linear and monotonically increasing in z, R does exist (and moreover is unique) if and only if $U_t(z=0) < U_p(z=0)$ and $\frac{\partial U_p(z)}{\partial z} > \frac{\partial U_t(z)}{\partial z}$. Using (??) and (??), the condition on the slopes says that

$$\frac{r+s}{r+s+h(\theta_p)} > \frac{r+s+\lambda}{r+s+\lambda+h(\theta_t)}$$

and the one on the intercepts reads

$$\frac{h(\theta_t)w}{r+s+\lambda+h(\theta_t)} > \frac{h(\theta_p)w+(r+s)b}{r+s+h(\theta_p)}$$

If b = 0 the two conditions are equivalent and boil down to

$$\frac{r+s}{r+s+\lambda} > \frac{h(\theta_p)}{h(\theta_t)}$$

However, in our hypothesis b > 0; this means that inequality (??) requires a higher intercept for $U_t(z)$, i.e. that this is the strictest condition.

• The existence of a reservation outside option is a necessary but not sufficient condition for the coexistence of temporary and permanent contracts in equilibrium. We already know, in fact, that if $R \ge w$ all workers search for a temporary job. We need then that

$$\begin{aligned} R &< w \Rightarrow w - b \frac{(r+s)[r+s+\lambda+h(\theta_t)]}{(r+s)h(\theta_t) - (r+s+\lambda)h(\theta_p)} < w \Rightarrow \\ (r+s)h(\theta_t) &> (r+s+\lambda)h(\theta_p) \Rightarrow \\ \frac{r+s}{r+s+\lambda} &> \frac{h(\theta_p)}{h(\theta_t)} \end{aligned}$$

and we can conclude that if (??) holds then R exists and is lower than the wage.

9.2 Dynamics

In the rigid market all the workforce is either employed with a permanent contract or unemployed

$$u_p + n_p = 1$$

The differential equations describing the dynamics of these two components therefore does not depend on the outside utility and read

$$\dot{u}_p(\tau) = sn_p(\tau) - h(\theta_p)u_p(\tau)$$

$$\dot{n}_p(\tau) = h(\theta_p)u_p(\tau) - sn_p(\tau)$$

It's easy to see that when the old regime reaches its steady state the stocks amount to

$$u_p = \frac{s}{s + h(\theta_p)}$$

and

$$n_p = \frac{h(\theta_p)}{s + h(\theta_p)}$$

As we pointed out above, in order to fully describe the dynamic behaviour of employed and unemployed workers in both submarkets we need to separately consider people with outside option below and above the reservation value R. In every moment in time the distribution of the formers reads

$$F(z) = n_p(z,\tau) + n_t(z,\tau) + u_t(z,\tau), \quad z \le R$$
(14)

When $\tau = 0$ the stock of workers who start searching in the new submarket is given by the fraction of unemployed workers of the previous regime whose outside option is lower than R

$$u_t(z,\tau=0) = \frac{sF(z)}{s+h(\theta_p)}, \quad z \le R$$
(15)

Since right after the introduction of the new regime nobody works with a temporary contract $(n_t(z, \tau = 0) = 0)$, the initial condition for permanently employed workers with $z \leq R$ can be obtained through (14)

$$n_p(z,\tau = 0) = F(z) - u_t(z,\tau = 0) - n_t(z,\tau = 0) \Rightarrow$$

$$n_p(z,\tau = 0) = F(z) - \frac{sF(z)}{s+h(\theta_p)} - 0 =$$

$$= \frac{h(\theta_p)F(z)}{s+h(\theta_p)}, \quad z \le R$$
(16)

We are now in a position to describe the dynamic behaviour of n_p, n_t and u_t provided $z \leq R$.

• In the rigid market a fraction of workers was employed with a permanent contract even if endowed with a low outside option. From $\tau = 0$ onwards, once they are fired (what happens at rate s), they

start searching in the temporary submarket with no possibility to come back to the permanent tier

$$\begin{split} \dot{n}_p(z,\tau) &= -sn_p(z,\tau) \Rightarrow \\ \dot{n}_p(z,\tau) + sn_p(z,\tau) &= 0 \Rightarrow \\ \int e^{s\tau} [\dot{n}_p(z,\tau) + sn_p(z,\tau)] d\tau &= b_1 \Rightarrow \\ n_p(z,\tau) e^{s\tau} + b_0 &= b_1 \Rightarrow n_p(z,\tau) = B e^{-s\tau}, \quad z \leq R \end{split}$$

where b_0 and b_1 are constants of integration. Using (16) and solving for B

$$n_p(z,0) = B = \frac{h(\theta_p)F(z)}{s + h(\theta_p)}, \quad z \le R$$

therefore

$$n_p(z,\tau) = \frac{h(\theta_p)F(z)}{s+h(\theta_p)}e^{-s\tau}, \quad z \le R$$
(17)

i.e. the initial stock of permanent workers with a low outside option decreases at rate s down to zero; in fact

$$\lim_{\tau \to \infty} n_p (z \le R, \tau) = 0$$

• Since in the rigid market only permanent contracts were allowed, right after the shock the stock of temporary workers is an empty set, but immediately firms post temporary vacancies and fill them at rate $h(\theta_t)$ by hiring from the stock of "temporary" unemployed. Temporary matches are then destroyed at rate $s + \lambda$

$$\dot{n}_t(z,\tau) = h(\theta_t)u_t(z,\tau) - (s+\lambda)n_t(z,\tau)$$

using (14) and (17) one gets

$$\begin{split} \dot{n}_t(z,\tau) &= h(\theta_t)[F(z) - n_p(z,\tau) - n_t(z,\tau)] - (s+\lambda)n_t(z,\tau) \Rightarrow \\ \dot{n}_t(z,\tau) + [h(\theta_t) + s+\lambda]n_t(z,\tau) &= h(\theta_t) \left[F(z) - \frac{h(\theta_p)F(z)}{s+h(\theta_p)} e^{-s\tau} \right] \Rightarrow \\ e^{[h(\theta_t) + s+\lambda]\tau} \left\{ \dot{n}_t(z,\tau) + [h(\theta_t) + s+\lambda]n_t(z,\tau) \right\} = e^{[h(\theta_t) + s+\lambda]\tau} h(\theta_t) \left[F(z) - \frac{h(\theta_p)F(z)}{s+h(\theta_p)} e^{-s\tau} \right] \Rightarrow \\ n_t(z,\tau)e^{[h(\theta_t) + s+\lambda]\tau} + b_0 &= \int h(\theta_t)F(z)e^{[h(\theta_t) + s+\lambda]\tau} d\tau - \int \frac{h(\theta_t)h(\theta_p)F(z)}{s+h(\theta_p)} e^{[h(\theta_t) + \lambda]\tau} d\tau \Rightarrow \\ n_t(z,\tau)e^{[h(\theta_t) + s+\lambda]\tau} &= b_0 + \frac{h(\theta_t)F(z)}{h(\theta_t) + s+\lambda} e^{[h(\theta_t) + s+\lambda]\tau} + b_1 - \frac{h(\theta_t)h(\theta_p)F(z)}{[s+h(\theta_p)][h(\theta_t) + \lambda]} e^{[h(\theta_t) + \lambda]\tau} + b_2 \Rightarrow \\ n_t(z,\tau) &= Be^{-[h(\theta_t) + s+\lambda]\tau} + \frac{h(\theta_t)F(z)}{h(\theta_t) + s+\lambda} - \frac{h(\theta_t)h(\theta_p)F(z)}{[s+h(\theta_p)][h(\theta_t) + \lambda]} e^{-s\tau} \quad z \leq R \end{split}$$

A unique solution for the dynamics of $n_t(z, \tau)$ is obtained imposing the initial condition $n_t(z, 0) = 0$, solving for B and substituting the expression below into the previous equation

$$B = \frac{h(\theta_t)h(\theta_p)F(z)}{[s+h(\theta_p)][h(\theta_t)+\lambda]} - \frac{h(\theta_t)F(z)}{h(\theta_t)+s+\lambda}, \quad z \le R$$

The stock of temporary workers therefore grows from zero to

$$\lim_{\tau \to \infty} n_t(z,\tau) = \frac{h(\theta_t)F(z)}{h(\theta_t) + s + \lambda}, \quad z \le R$$

• The dynamic behaviour of u_t is not simply the reverse of n_t . The stock of workers looking for a temporary job grows also because people with $z \leq R$ eventually loose their permanent job at rate s and move to the temporary tier

$$\dot{u}_t(z,\tau) = sn_p(z,\tau) + (s+\lambda)n_t(z,\tau) - h(\theta_t)u_t(z,\tau), \quad z \le R$$

Again, using (14) with (17)

$$\begin{split} \dot{u}_t(z,\tau) &= sn_p(z,\tau) + (s+\lambda)[F(z) - u_t(z,\tau) - n_p(z,\tau)] - h(\theta_t)u_t(z,\tau) \Rightarrow \\ \dot{u}_t(z,\tau) + [h(\theta_t) + s+\lambda]u_t(z,\tau) &= (s+\lambda)F(z) - \frac{\lambda h(\theta_p)F(z)}{s+h(\theta_p)}e^{-s\tau} \Rightarrow \\ e^{[h(\theta_t) + s+\lambda]\tau} \left\{ \dot{u}_t(z,\tau) + [h(\theta_t) + s+\lambda]u_t(z,\tau) \right\} = e^{[h(\theta_t) + s+\lambda]\tau} \left[(s+\lambda)F(z) - \frac{\lambda h(\theta_p)F(z)}{s+h(\theta_p)}e^{-s\tau} \right] \Rightarrow \\ u_t(z,\tau)e^{[h(\theta_t) + s+\lambda]\tau} + b_0 &= \int (s+\lambda)F(z)e^{[h(\theta_t) + s+\lambda]\tau} d\tau - \int \frac{\lambda h(\theta_p)F(z)}{s+h(\theta_p)}e^{[h(\theta_t) + \lambda]\tau} d\tau \\ u_t(z,\tau) &= Be^{-[h(\theta_t) + s+\lambda]\tau} + \frac{(s+\lambda)F(z)}{h(\theta_t) + s+\lambda} - \frac{\lambda h(\theta_p)F(z)}{[s+h(\theta_p)][h(\theta_t) + \lambda]}e^{-s\tau}, \quad z \leq R \end{split}$$

Imposing the initial condition (15) and solving for B one gets a unique solution for the dynamics of $u_t(z,\tau)$

$$B = F(z) \left\{ \frac{s}{s + h(\theta_p)} + \frac{\lambda h(\theta_p)}{[s + h(\theta_p)] [h(\theta_t) + \lambda]} - \frac{s + \lambda}{h(\theta_t) + s + \lambda} \right\}, \quad z \le R$$

The stock of unemployed workers on the temporary market therefore goes from the initial level

$$u_t(z,\tau=0) = \frac{sF(z)}{s+h(\theta_p)}, \quad z \le R$$

to its steady state value

$$\lim_{\tau \to \infty} u_t(z,\tau) = \frac{(s+\lambda)F(z)}{s+\lambda+h(\theta_t)}, \quad z \le R$$

Let us now turn to the stock of workers with z > R. People with large outside utility never move from the permanent tier; in every moment in time they are either employed or unemployed with a permanent contract

$$1 - F(z) = u_p(z,\tau) + n_p(z,\tau), \quad z > R$$
(18)

The initial stock of unemployed workers searching for a permanent job is given by the proportion of unemployed workers in the old regime with z > R

$$u_p(z,\tau=0) = \frac{s[1-F(z)]}{s+h(\theta_p)}, \quad z > R$$
(19)

Using (18) one gets the initial condition for $n_p(z > R, \tau)$

$$n_p(z,\tau=0) = \frac{h(\theta_p)[1-F(z)]}{s+h(\theta_p)}, \quad z>R$$

• The stock of unemployed with z > R increases when permanently employed workers with large outside option leave their jobs and decreases when they find a new one

$$\dot{u}_p(z,\tau) = sn_p(z,\tau) - h(\theta_p)u_p(z,\tau), \quad z > R$$

using (18)

$$\begin{split} \dot{u}_{p}(z,\tau) + [s+h(\theta_{p})]u_{p}(z,\tau) &= s[1-F(z)] \Rightarrow \\ e^{[s+h(\theta_{p})]\tau} \left\{ \dot{u}_{p}(z,\tau) + [s+h(\theta_{p})]u_{p}(z,\tau) \right\} &= s[1-F(z)]e^{[s+h(\theta_{p})]\tau} \Rightarrow \\ u_{p}(z,\tau)e^{[s+h(\theta_{p})]\tau} + b_{o} &= s[1-F(z)]\int e^{[s+h(\theta_{p})]\tau}d\tau \Rightarrow \\ u_{p}(z,\tau) &= Be^{-[s+h(\theta_{p})]\tau} + \frac{s[1-F(z)]}{s+h(\theta_{p})}, \quad z > R \end{split}$$

As usual, a unique solution is obtained through the imposition of the initial condition in (19); solving by B one gets

$$B = \frac{s[1 - F(z)]}{s + h(\theta_p)} - \frac{s[1 - F(z)]}{s + h(\theta_p)} = 0 \Rightarrow$$
$$u_p(z, \tau) = \frac{s[1 - F(z)]}{s + h(\theta_p)}, \quad z > R$$

The stock of unemployed workers in the permanent market does not depend on time: its level is constant during the transition to the new steady state.

• The dynamics of $n_p(z > R, \tau)$ is its exact reverse

$$\dot{n}_p(z,\tau) = h(\theta_p)u_p(z,\tau) - sn_p(z,\tau), \quad z > R$$

Using (18)

$$\begin{split} \dot{n}_p(z,\tau) &= h(\theta_p)[1 - F(z) - n_p(z,\tau)] - sn_p(z,\tau) \Rightarrow \\ \dot{n}_p(z,\tau) + [h(\theta_p) + s]n_p(z,\tau) &= h(\theta_p)[1 - F(z)] \Rightarrow \\ e^{[h(\theta_p) + s]\tau} \left\{ \dot{n}_p(z,\tau) + [h(\theta_p) + s]n_p(z,\tau) \right\} &= h(\theta_p)[1 - F(z)]e^{[h(\theta_p) + s]\tau} \Rightarrow \\ n_p(z,\tau)e^{[h(\theta_p) + s]\tau} + b_0 &= h(\theta_p)[1 - F(z)] \int e^{[h(\theta_p) + s]\tau} d\tau \Rightarrow \\ n_p(z,\tau) &= Be^{-[h(\theta_p) + s]\tau} + \frac{h(\theta_p)[1 - F(z)]}{h(\theta_p) + s}, \quad z > R \end{split}$$

The imposition of the initial condition for $\tau = 0$ yields the unique value of B

$$\frac{h(\theta_p)[1-F(z)]}{s+h(\theta_p)} = B + \frac{h(\theta_p)[1-F(z)]}{h(\theta_p)+s} \Rightarrow B = 0 \Rightarrow n_p(z > R, \tau) = \frac{h(\theta_p)[1-F(z)]}{h(\theta_p)+s}, \quad z > R$$

So also the dynamic equation of $n_p(z > R, \tau)$ does not depend on time; nonetheless we have to keep in mind that the full dynamics for n_p depends also on workers with $z \le R$. We are now in a position to describe the whole dynamics of the system. $n_t(z,\tau)$ and $u_t(z,\tau)$ are fully determined by workers with $z \leq R$, while $u_p(z,\tau)$ by the ones with z > R; $n_p(z,\tau)$ depends on both

$$n_t(z,\tau) = n_t(z \le R,\tau) = \left\{ \frac{h(\theta_t)h(\theta_p)F(z)}{[s+h(\theta_p)][h(\theta_t)+\lambda]} - \frac{h(\theta_t)F(z)}{h(\theta_t)+s+\lambda} \right\} e^{-[h(\theta_t)+s+\lambda]\tau} + \frac{h(\theta_t)F(z)}{h(\theta_t)+s+\lambda} - \frac{h(\theta_t)h(\theta_p)F(z)}{[s+h(\theta_p)][h(\theta_t)+\lambda]} e^{-s\tau}$$

$$\begin{aligned} u_t(z,\tau) &= u_t(z \le R,\tau) = F(z) \left\{ \frac{s}{s+h(\theta_p)} + \frac{\lambda h(\theta_p)}{[s+h(\theta_p)] \left[h(\theta_t) + \lambda\right]} - \frac{s+\lambda}{h(\theta_t) + s+\lambda} \right\} e^{-[h(\theta_t) + s+\lambda]\tau} + \\ &+ \frac{(s+\lambda)F(z)}{h(\theta_t) + s+\lambda} - \frac{\lambda h(\theta_p)F(z)}{[s+h(\theta_p)] \left[h(\theta_t) + \lambda\right]} e^{-s\tau} \end{aligned}$$

$$u_p(z,\tau) = u_p(z > R,\tau) = \frac{s[1 - F(z)]}{s + h(\theta_p)}$$

$$n_p(z,\tau) = n_p(z \le R,\tau) + n_p(z > R,\tau) = \frac{h(\theta_p)F(z)}{s + h(\theta_p)}e^{-s\tau} + \frac{h(\theta_p)[1 - F(z)]}{h(\theta_p) + s}$$

Taking $\lim_{\tau\to\infty}$ and using z=R one gets the expressions for the two tiers steady state.

9.3 The "honeymoon effect"

In order to prove the existence of what we called the "honeymoon effect" of the introduction of temporary jobs we take the time derivative of the equation describing the dynamics of total unemployment and evaluate it at $\tau = 0$; more precisely, since permanent unemployment does not display any dynamics (see the subsection above), we will focus on the behaviour of temporary unemployment. If the liberalisation of temporary contracts leads to an immediate reduction of total unemployment, the time derivative of temporary unemployment evaluated at $\tau = 0$ must be negative. From section 9.2 we know that

$$\begin{split} u_t(\tau) &= F(R) \left\{ \frac{s}{s+h(\theta_p)} + \frac{\lambda h(\theta_p)}{[s+h(\theta_p)] \left[h(\theta_t) + \lambda\right]} - \frac{s+\lambda}{h(\theta_t) + s+\lambda} \right\} e^{-[h(\theta_t) + s+\lambda]\tau} + \\ &+ \frac{(s+\lambda)F(R)}{h(\theta_t) + s+\lambda} - \frac{\lambda h(\theta_p)F(R)}{[s+h(\theta_p)] \left[h(\theta_t) + \lambda\right]} e^{-s\tau} \Rightarrow \end{split}$$

$$\begin{aligned} \frac{\partial u_t(\tau)}{\partial \tau} &= -[s+\lambda+h(\theta_t)]F(z)\left\{\frac{s}{s+h(\theta_p)} + \frac{\lambda h(\theta_p)}{[s+h(\theta_p)][\lambda+h(\theta_t)]} - \frac{s+\lambda}{s+\lambda+h(\theta_t)}\right\} \cdot \\ & \cdot e^{-[s+\lambda+h(\theta_t)]\tau} + \frac{s\lambda h(\theta_p)F(z)}{[s+h(\theta_p)][\lambda+h(\theta_t)]}e^{-s\tau} \end{aligned}$$

imposing $\tau = 0$

$$\begin{aligned} \frac{\partial u_t(\tau)}{\partial \tau | \tau = 0} &= (s + \lambda) F(z) - \frac{s[s + \lambda + h(\theta_t)]F(z)}{s + h(\theta_p)} - \frac{\lambda h(\theta_p)F(z)[s + \lambda + h(\theta_t)]}{[s + h(\theta_p)][\lambda + h(\theta_t)]} + \\ &+ \frac{s\lambda h(\theta_p)F(z)}{[s + h(\theta_p)][\lambda + h(\theta_t)]} = \end{aligned}$$

Omitting the common denominator, which is not relevant for the sign of the expression above, one gets

$$\begin{split} [s+h(\theta_p)][\lambda+h(\theta_t)](s+\lambda)F(z)-s[\lambda+h(\theta_t)][s+\lambda+h(\theta_t)]F(z)+\\ &\quad -\lambda h(\theta_p)F(z)[s+\lambda+h(\theta_t)]+s\lambda h(\theta_p)F(z)= \end{split}$$

$$= h(\theta_p)[\lambda + h(\theta_t)](s + \lambda)F(z) - s[\lambda + h(\theta_t)]h(\theta_t)F(z) + - \lambda h(\theta_p)F(z)[s + \lambda + h(\theta_t)] + s\lambda h(\theta_p)F(z) = = [\lambda + h(\theta_t)]F(z)[h(\theta_p)(s + \lambda) - sh(\theta_t) - \lambda h(\theta_p)] = = [\lambda + h(\theta_t)]F(z) \{s[h(\theta_p) - h(\theta_t)]\} < 0$$

9.4 Search on the job

The proof of the existence of the equilibrium in the model with on the job search follows the lines of section 9.1: we need to find the conditions for the existence of a positive reservation outside utility that is strictly lower than the wage. Once θ_t and θ_p are determined by sequentially solving the job creation conditions system (see section 6), both U_t and U_p are linear functions of z; a positive R therefore exists when the intercept of U_t is larger than the intercept of U_p and its slope is smaller¹². We will then prove that under the same conditions not only R is positive, but is also strictly lower than w.

The value functions for the supply side of the permanent submarket look as in section 2.1

$$rE_p(z) = w + s[U_p(z) - E_p(z)]$$

 $rU_p(z) = z + b + h(\theta_p)[E_p(z) - U_p(z)]$

so that the value of unemployment for a permanent worker reads

$$U_p(z) = \frac{(z+b)(r+s) + h(\theta_p)w}{r[r+s+h(\theta_p)]}$$

In the temporary submarket the asset equations are a bit more complicated, since workers leave their temporary jobs not only because of natural turnover, but also when a permanent vacancy becomes available

$$rE_t(z) = w + h(\theta_p)[E_p(z) - E_t(z)] + (s + \lambda)[U_t(z) - E_t(z)]$$

$$rU_t(z) = z + h(\theta_t)[E_t(z) - U_t(z)] + h(\theta_p)[E_p(z) - U_t(z)]$$

Using $E_t(z)$, $E_p(z)$ and $U_p(z)$ one gets the expression for $U_t(z)$

$$U_t(z) = \frac{[r+s+\lambda+h(\theta_p)]z}{[r+h(\theta_p)][r+s+\lambda+h(\theta_t)+h(\theta_p)]} + \frac{\{(r+s)h(\theta_t)+h(\theta_p)h(\theta_p)+h(\theta_p)[r+s+\lambda+h(\theta_p)]\}w}{(r+s)[r+h(\theta_p)][r+s+\lambda+h(\theta_t)+h(\theta_p)]} + \frac{h(\theta_p)s[(z+b)(r+s)+h(\theta_p)w]}{r(r+s)[r+h(\theta_p)][r+s+h(\theta_p)]}$$

We are now ready to go through the steps of the proof.

¹²In principle, the existence of a positive R would be shown also under the opposite conditions, i.e. a higher intercept and a smaller slope for U_p ; however, as a few steps of algebra will make clear, the slope of U_p is always larger than the one of U_t .

• Condition on the slopes: $\partial U_p/\partial z > \partial U_t/\partial z$

$$\frac{(r+s)}{r[r+s+h(\theta_p)]} > \frac{r+s+\lambda+h(\theta_p)}{[r+h(\theta_p)][r+s+\lambda+h(\theta_t)+h(\theta_p)]} + \frac{sh(\theta_p)}{r[r+h(\theta_p)][r+s+h(\theta_p)]}$$

Using and omitting the common denominator (which is not relevant for the sign) one gets

$$\begin{split} (r+s)[r+h(\theta_p)][r+s+\lambda+h(\theta_t)+h(\theta_p)] - r[r+s+\lambda+h(\theta_p)][r+s+h(\theta_p)]+ \\ & -sh(\theta_p)[r+s+\lambda+h(\theta_t)+h(\theta_p)] > 0 \Rightarrow \end{split}$$

$$\begin{split} [r^2 + rh(\theta_p) + rs] [\lambda + h(\theta_t)] &- r\lambda [r + s + h(\theta_p)] > 0 \Rightarrow \\ r^2 h(\theta_t) + rh(\theta_t) h(\theta_p) + rsh(\theta_t) > 0 \quad always \end{split}$$

• Condition on the intercepts: $U_p(0) < U_t(0)$

$$\frac{b(r+s)+h(\theta_p)w}{r[r+s+h(\theta_p)]} < \frac{\{(r+s)h(\theta_t)+h(\theta_p)h(\theta_t)+h(\theta_p)[r+s+\lambda+h(\theta_p)]\}w}{(r+s)[r+h(\theta_p)][r+s+\lambda+h(\theta_p)+h(\theta_t)]} + \frac{h(\theta_p)sb(r+s)+h(\theta_p)sh(\theta_p)w}{r(r+s)[r+h(\theta_p)][r+s+h(\theta_p)]}$$

Multiplying both sides by the common denominator the expression reads

$$\begin{split} (r+s)[r+h(\theta_p)][r+s+\lambda+h(\theta_p)+h(\theta_t)][b(r+s)+h(\theta_p)w]+\\ &-r[r+s+h(\theta_p)]\left\{(r+s)h(\theta_t)+h(\theta_p)h(\theta_t)+h(\theta_p)[r+s+\lambda+h(\theta_p)]\right\}w+\\ &-[r+s+\lambda+h(\theta_p)+h(\theta_t)][h(\theta_p)sb(r+s)+h(\theta_p)sh(\theta_p)w]<0; \end{split}$$

$$\begin{split} [r^2 + rh(\theta_p) + rs][r + s + \lambda + h(\theta_p) + h(\theta_t)]b(r + s) + \\ &+ (r + s)[r + h(\theta_p)][r + s + \lambda + h(\theta_p) + h(\theta_t)]h(\theta_p)w + \\ &- rw[r + s + h(\theta_p)](r + s)h(\theta_t) - rw[r + s + h(\theta_p)]h(\theta_t)h(\theta_p) + \\ &- rw[r + s + h(\theta_p)]h(\theta_p)[r + s + \lambda + h(\theta_p)] - [r + s + \lambda + h(\theta_p) + h(\theta_t)][h(\theta_p)swh(\theta_p)] < 0; \end{split}$$

$$\begin{split} [r^2 + rh(\theta_p) + rs][r + s + \lambda + h(\theta_p) + h(\theta_t)]b(r + s) + \\ &+ [r^2 + rh(\theta_p) + rs][r + s + \lambda + h(\theta_p) + h(\theta_t)]h(\theta_p)w - rw[r + s + h(\theta_p)](r + s)h(\theta_t) + \\ &- rw[r + s + h(\theta_p)][h(\theta_t)h(\theta_p)] - rw[r + s + h(\theta_p)]h(\theta_p)[r + s + \lambda + h(\theta_p)] < 0; \end{split}$$

$$\begin{split} [r^2 + rh(\theta_p) + rs][r + s + \lambda + h(\theta_p) + h(\theta_t)]b(r + s) + \\ &- w[r^3h(\theta_t) + 2r^2sh(\theta_t) + rs^2h(\theta_t) + rsh(\theta_p)h(\theta_t) + r^2h(\theta_p)h(\theta_t)] < 0; \end{split}$$

$$[r^{2} + rh(\theta_{p}) + rs][r + s + \lambda + h(\theta_{p}) + h(\theta_{t})]b(r + s) - wrh(\theta_{t})[(r + s)^{2} + h(\theta_{p})(r + s)] < 0;$$

$$[r + s + \lambda + h(\theta_{p}) + h(\theta_{t})]b < wh(\theta_{t}) \Rightarrow$$

$$b < \frac{wh(\theta_{t})}{[r + s + \lambda + h(\theta_{p}) + h(\theta_{t})]}$$

$$(20)$$

that is the condition for the existence of a positive reservation outside option.

By equating $U_p(z)$ to $U_t(z)$ and solving for z = R, we are now in a position to determine its exact value:

$$\begin{aligned} \frac{(R+b)(r+s)+h(\theta_p)w}{r[r+s+h(\theta_p)]} &= \frac{[r+s+\lambda+h(\theta_p)]R}{[r+h(\theta_p)][r+s+\lambda+h(\theta_t)+h(\theta_p)]} + \\ &+ \frac{\{(r+s)h(\theta_t)+h(\theta_t)h(\theta_p)+h(\theta_p)[r+s+\lambda+h(\theta_t)+h(\theta_p)]\}w}{(r+s)[r+h(\theta_p)][r+s+\lambda+h(\theta_t)+h(\theta_p)]} + \frac{h(\theta_p)s[(R+b)(r+s)+h(\theta_p)w]}{r(r+s)[r+h(\theta_p)][r+s+\lambda+h(\theta_t)+h(\theta_p)]}; \end{aligned}$$

Collecting terms with R and multiplying both sides by the common denominator one gets

$$\begin{split} (r+s) \left\{ \begin{array}{c} (r+s)[r+h(\theta_p)][r+s+\lambda+h(\theta_p)+h(\theta_t)]-r[r+s+h(\theta_p)][r+s+\lambda+h(\theta_p)]+\\ -sh(\theta_p)[r+s+\lambda+h(\theta_p)+h(\theta_t)] \\ \end{array} \right\} R = \\ & = wr[r+s+h(\theta_p)] \left\{ (r+s)h(\theta_t)+h(\theta_t)h(\theta_p)+h(\theta_p)[r+s+\lambda+h(\theta_p)] \right\} + \\ & + [r+s+\lambda+h(\theta_p)+h(\theta_t)][h(\theta_p)sb(r+s)+h(\theta_p)sh(\theta_p)w] + \\ & - [b(r+s)+h(\theta_p)w](r+s)[r+h(\theta_p)][r+s+\lambda+h(\theta_p)+h(\theta_t)]; \end{split}$$

For simplicity we separately consider the two sides of the equation; starting from the rhs

$$w\left\{\begin{array}{c}r[r+s+h(\theta_p)](r+s)h(\theta_t)+r[r+s+h(\theta_p)]h(\theta_p)h(\theta_t)+r[r+s+h(\theta_p)]h(\theta_p)[r+s+\lambda+h(\theta_p)]+\\-h(\theta_p)[r+s+\lambda+h(\theta_t)+h(\theta_p)][r^2+rs+rh(\theta_p)]\end{array}\right\}+\\-b(r+s)[r+s+\lambda+h(\theta_p)+h(\theta_t)][r^2+rs+rh(\theta_p)]=$$

$$w[r^{2} + rs + rh(\theta_{p})](r + s)h(\theta_{t}) - b(r + s)[r^{2} + rs + rh(\theta_{p})][r + s + \lambda + h(\theta_{p}) + h(\theta_{t})] = [r^{2} + rs + rh(\theta_{p})](r + s) \{h(\theta_{t})w - b[r + s + \lambda + h(\theta_{p}) + h(\theta_{t})]\}$$

The lhs in turn reads

$$(r+s)R\left\{[r^2+rs+rh(\theta_p)][r+s+\lambda+h(\theta_p)+h(\theta_t)]-[r^2+rs+rh(\theta_p)][r+s+\lambda+h(\theta_p)]\right\} = h(\theta_t)(r+s)[r^2+rs+rh(\theta_p)]$$

so that

$$\begin{array}{lll} Rh(\theta_t) &=& h(\theta_t)w - b[r+s+\lambda + h(\theta_p) + h(\theta_t)] \Rightarrow \\ R &=& w - b \frac{r+s+\lambda + h(\theta_p) + h(\theta_t)}{h(\theta_t)} \end{array}$$

which implies that R < w; moreover, under condition (20), 0 < R < w.

10 Reference

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