THE SUPPLY OF LABOR AND HOUSEHOLD PRODUCTION

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Abstract
Labor supply is seen as an output from household production. Given by the physical effort of a person, working in the market also requires specific inputs. This process may be described with the help of a general technology that comprises joint production. At least one of the outputs is labor supply. With the help of a simplified version of the model, initially the choice among different types of market work is discussed. Within this discussion, it is shown how different estimates of the opportunity cost of time naturally appear, all in standard microeconomic results. Then, the definition of net result of the worker is related to economic rent due to the fact that the consumer-producer can not alter the time endowment.

JEL codes: J2, D13.
Key words: labor supply, household production.

1. Introduction

Labor supply, in standard models, is given by the residual time of the economic agent after the quantity of leisure is chosen. This is generalized in the household production models by defining consumption activities that use, as inputs, purchased goods and own time. Even then, labor supply is modeled in the same way: from total available time in a given period, time in household activities is subtracted so as to get the time sold in the market. An alternative modeling is to have leisure time as a residual.

Another alternative is to have both labor and leisure in the preference function, as in Johnson (1966), Georgescu-Roegen (1968, p.248-249), and Sanson (1987 and 1991). Johnson (1966) introduced a model with leisure and labor in the preference function and at least one input for the production of labor. The paper was highly influential in the literature on urban transportation demand, although gradually superseded by Beckerian models. This was true especially of DeSerpa (1971). He included all forms of time use together with the Beckerian commodities in the preference function. Gronau (1986 and 1987) also includes the labor activity in the preference function in a Beckerian model.

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When labor supply is defined as a time residual, a production function for labor is thus explicitly left out. In such a situation, labor time does not appear in the preference function, at least in Walrasian models, one of which is the household production model of Becker (1965). Besides this, Beckerian models define production functions only for the activities that are listed in the preference function. It is true that the household production model can be interpreted as allowing for a linear production function for the labor activity in which time is the only input. As usual, the level of this activity is measured by the amount of time used in it. However, many types of labor may be considered in which this equivalence is not valid. One example would be labor performed and sold by tasks instead of by the hour. Also, there are specific inputs in the productive process of human work. Therefore, a production function that has only time as input does not cover the general case. It amounts to ignoring the productive process for labor.

Many insights on the production of human work exist in the literature, although in scattered form. Leontief's closed model, which is inspired in the *Tableau Économique* of the physiocrats, treats consumers as a sector that specializes in supplying labor. Due to the unrealistic assumption of fixed coefficients for the consumption of final goods, the more popular model in which consumption and labor are exogenous has superseded it.[1]

As in any household productive activity, labor production requires several inputs, some of which have been studied. Singh, Squire, and Strauss (1986), Suen and Mo (1994), and Caillavet, Guyomard, and Lifran (1994) present models of productive consumption that include, for instance, sleep, nutrition and health expenditures. An earlier suggestion for treating labor as a household output, as an equal to commodities, appeared in DeSerpa (1971, p.829n.1), although the analysis in the paper was made with fixed supply of labor. Gronau (1986 and 1987) treats labor as an activity with its own inputs. However, the analysis is very brief and the results are somewhat different from this paper. In human capital theory, expenditures that increase future capacity of getting income are considered as input or, better, as investment. However, such an analysis is done intertemporally and seems not to include an explicit production function for labor. The literature on collective household production seems to follow Becker's treatment as far as it refers to the production of labor.[2] Therefore, it appears that there is some space for a

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[1] See Dorfman, Samuelson, and Solow (1958, ch. 10) and Pasinetti (1977, ch. 4).

[2] For a recent survey of collective household models, with emphasis on their econometric implementation, see Vermeulen (2002).

rediscussion of the model in which labor supply is treated as any output of the household productive process. In this paper, we summarize and extend research presented in Sanson (1997 and 2002).

The present article is restricted to a static situation. Besides, sleep and leisure activities are treated as inputs that are not specific to labor production since they affect the whole productive process of the household. They are among the activities directly included in the preference function.

The paper attempts the design of a model of labor supply that should fit in the standard consumer and firm theories. Ideally, the model should also fit in the general equilibrium framework. For this, the endowment treatment is fundamental. We will see that the endowment of work capacity leads naturally for an agent that looks for a maximum rent for the human capital. Currently, the household production theory, despite its growing sophistication, still stays only in the dark alleys of the more popular microeconomics texts, intermediary and advanced. That is not consistent with the importance of the theme in empirical applications of economics.

The main objective of the article is to formulate a model of household production in which labor is one of the productive activities whose output may be sold in the market. This will be done in the context of Walrasian models, in which the time allocated to work in the market is given as a residual. Section 2 covers the notion of time endowment, with emphasis on its interpretation as a period in which the human organism is available for working. This highlights the fact that, in contrast to other types of Walrasian endowments, the time endowment can not be altered. Section 3 proposes a Beckerian household production model that has, as a distinguishing characteristic, production functions for different types of labor. Section 4 illustrates the general model with the special case of two commodities, where one is a good that does not require time and the other only requires time, being a Beckerian version of the income-leisure model of textbooks. However, the production functions for labor are kept. This model allows for a new discussion of the shadow price of time. Finally, section 5 adapts the notion of profit or net result, used in models of household production from agricultural economics, to the production of labor.

2. The endowment of labor

Before the presentation of the model of labor supply, it will be necessary to define labor as a result of a productive process. A related question is the definition of the endowment of time for each economic agent. Its interpretation is dependent on the definition of labor, and it implies a modification in the concept of opportunity cost of time.

The concept of human work presupposes an activity by a person during a time interval.[3] A person, in fact, is a kind of physical capital that renders services during a given period.[4] Thus this person has an occupation that may include different kinds of work, being infinite the number of possible tasks. What counts is the combination of personal abilities to be used in each task. The human capital is complemented by different inputs to render different goods and services, not necessarily tied to a paid job. As it is taught by the theory of household production, work is then one of the many possible outputs from a productive process centered on a person or a family. The organism itself is the basic capital of the person, and being a worker is to specialize in putting for rent the own work capacity. The market structure in which this work capacity is rented varies with the type of work. It goes from quasi-monopoly markets in which the labor activity is dependent on scarce abilities, to the competitive markets of nonqualified labor.

The second question related to labor production is the nature of the endowment of labor force for each person. This endowment is given by Nature, as far as the time interval considered. Thus, for a period of a month the person has a maximum of man-hours that the organism can be at work, not given by the organism itself but by the time interval. The intensity of work, however, is variable and is dependent on personal limits and the environment in which the organism operates.

Similarly, there are many types of activities that a given person may exercise during the period. Because of this, it is possible to think of a production function for labor in which one of

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[3] See Debreu (1959, p.30-31) and Arrow e Hahn (1971, ch.4). Whinston (1992, p.15-16, 163-164) emphasizes that many textbooks consider time itself as the input. In fact, time only delimits the duration and the direction of the labor process. A favorable interpretation to the textbooks is that an implicit assumption is made. Time is only a short for a number of man-hours of homogeneous human work per period. If every worker delivers the same number of man-hours, then it is enough to count the number of workers. In models of individual supply, the reference to time endowment must have a number of man-hours. Another semantic question is the use of the words labor and work. In order to avoid a discussion that would easily get into physics, these words are synonymous here.

[4] Léon Walras refers to consumable services that are obtainable from personal abilities. See Walras (1983, lessons 15 to 18).

the inputs, the essential one, is the man-hours the person allocates to a given task.[5] A simple example is a gardener who charges for the service by the area covered and then allocates hours of work to the contracted task. From the current viewpoint, the gardener uses, besides the services of gardening physical equipment, the services of the organism itself, treated as capital.

The human organism may be able to carry on more than one activity at the same time or at least during the same period. One example is to work on a laptop during an air flight, where both activities are related to an occupation.[6] Another example is the leisure of the theory class during academic meetings. This possibility is not considered in the standard models, due to the determination of labor supply as a residual. Therefore, selling labor capacity and using it in unpaid activities are mutually exclusive. Separating the endowment of work capacity from the different types of work that are produced allows for a treatment of this capacity as an input that can be jointly used in the many possible activities of a household.

In terms of the Walrasian treatment of endowments, only the availability of the human capital, measured in man-hours, as it is done for any capital good, would be treated as any other good.[7] The labor output, of course, would depend on effort and on the technology implicit in its production function.

A point that the Walrasian endowment approach brings in the discussion is its fixity. Endowments of goods may be altered, and this is the essence of models of excess demand and reservation prices in general equilibrium analysis. In the case of labor capacity, this is not possible. A person may hire external labor in order to substitute for own labor in a given activity, but this does not alter own labor capacity. It just means an alternative use for the saved labor capacity.

[5] This availability of the worker for the execution of alternative tasks may be seen, despite the differences in theoretical paradigms, as the equivalent to the Marxian concept of "abstract human labor". The conversion of abstract labor into different types of labor could be made within the present production function approach. The usual conversion with fixed coefficients would be a special case. An irresistible question is would such a solution avoid the aggregation problems that created insurmountable difficulties for the labor theory of value?

[6] In this example, the service of the organism enters simultaneously in the labor activity, which is an output, and in the transportation activity, which can be a case of an input activity to the work activity being sold.

[7] See Arrow and Hahn (1971, p.75-76, 165-166), in which an integration of Becker's model of time allocation is proposed for Walrasian endowments. In that solution, there are several types of possible endowments of leisure time, as proposed in Arrow and Debreu (1954). There is corresponding set of activities or tasks that require time with fixed coefficients. The sum of the excess demands for different types of time use can not be higher than the total endowment of time of the person. There is no restriction on the sign of each excess demand. In the present solution, in contrast, there is only one endowment, labor capacity, which may be used in any activity, including the several types of leisure and labor. Both solutions have in common the limit on time use given by Nature.

In the context of household production theory, the endowment of labor capacity may be interpreted as a fixed factor for a firm. Depending on the technology of a firm, a fixed factor has possible uses for the production of different types of goods, and may be used jointly. In household production theory, the decisions about consumption and production are not separable, as it is supposed in the theory of the firm, in which there is no interaction between the consumption decisions of the owners of firm and their, or of their employees, decisions on production. Thus, the capacity of labor, as a fixed factor in the household production process, affects simultaneously the consumption and production decisions, including labor. The main difference is that this fixed factor is not alterable in the long run. Investments in education, formal or from experience, are equivalent to technological progress. This Walrasian endowment approach in the household production theory results in modifications in the interpretation of the opportunity cost of the labor capacity. In the basic income-leisure model, if a person gives up selling part of the labor capacity in the market to consume it, the best wage given up is the opportunity cost. In these models, it is also supposed that there is joint use of the human capital in several activities. This is so because joint use destroys the one-to-one tradeoff between labor and leisure. With the production function approach for labor supply, a man-hour can have many alternative uses, which include consumption activities or many types of marketable labor. In terms of sacrificed income, now the opportunity cost is given by the best occupation available to the person. However, given the labor supply, there is also an opportunity cost involved in choosing consumption activities. In both situations, it will be seen that the marginal rate of transformation between a pair of activities, associated to a man-hour, as a measure of labor capacity, is now part of the definition of opportunity cost.

3. A general household production model

Take a Beckerian model with a given list of commodities with no perfect substitutes in the market.[8] Suppose that any good bought in the market requires some positive amount of man-hours to be consumed, which turns it into a commodity. The person maximizes the preference function

[8] Had the commodities perfect substitutes, it would be necessary to distinguish, for each of them, between what is consumed domestically and what is sold, as it is done in the international trade theory. See Strauss (1986) and Sanson (1997).

$$u = u(z), \tag{1}$$

where z is a vector that represents the quantities of commodities.

Technology is shown by the transformation function[9]

$$F(z, y, t) = 0, \tag{2}$$

where y is a vector of tradable goods. Inputs are represented by nonpositive elements of the vector. One example of output is labor, in fact the tasks performed in a given occupation. Vector t represents the types of labor that the person performs domestically and in the market through different occupations. If the number of commodities and tradable goods is greater than the number of types of work, then there are work activities that are jointly used in several activities of the household. This possibility is covered by (2), as a general technology of production.

The budget restriction is given by

$$p^T y = 0, \tag{3}$$

where p is a price vector with the same number of components as y. The only source of income is the sale of outputs from the productive process.[10] As the inputs are negative variables, their costs are being covered by (3). In contrast with usual presentations of household production models, this version treats the goods bought in the market as inputs, which, similarly to the advanced theory of the firm, are treated as negative variables.

The labor restriction[11] is simply the sum of all the possible uses of the total available labor:

$$J^T t = 1, \tag{4}$$

where J is the summation vector and the endowment of labor is normalized to unity.

From the Lagrangean function

$$L(z,y,t,\lambda) = u(z) + \lambda_1 F(z, y, t) + \lambda_2 (1 - J^T t) + \lambda_3 p^T y$$

the following first order conditions for a maximum are obtained:[12]

[9] A variant of this generalized production function is used by Pollak and Wachter (1975, p.261).

[10] Income from other sources is supposed to be equal to zero, i.e., $m = 0$.

[11] Calling the usual time restriction as labor restriction is consistent with the argument that a person really has an endowment of labor capacity that the organism may perform during the period under consideration. Walras himself

$$\frac{\partial u}{\partial z} + \lambda_1 \frac{\partial F}{\partial z} = 0^T , \tag{5}$$

$$\lambda_1 \frac{\partial F}{\partial y} + \lambda_3 p^T = 0^T , \tag{6}$$

$$\lambda_1 \frac{\partial F}{\partial t} - \lambda_2 J^T = 0^T . \tag{7}$$

The restrictions (2), (3), and (4) should also be listed as part of this matrix equation system. λ_i represent the three Lagrangean multipliers.

The analytical solution of this model is clearly difficult. But some insights can be gained by opening up some of the equations. From the elements of (5), after eliminating the Lagrangean multiplier, it follows that:

$$\frac{\partial u / \partial z_i}{\partial u / \partial z_j} = \frac{\partial F / \partial z_i}{\partial F / \partial z_j} . \tag{8}$$

This means that the marginal rate of substitution between two commodities is equal to the corresponding marginal rate of transformation. This corresponds to an efficiency condition in the household production process.[13]

From (6) and (7), by also eliminating the multipliers, it follows that

$$p_j \frac{\partial F / \partial t_i}{\partial F / \partial y_j} = p_k \frac{\partial F / \partial t_i}{\partial F / \partial y_k} \tag{9}$$

Consider, for instance, y_j and y_k as types of traded labor. Then it is possible to interpret (9) as saying that alternative uses of the labor capacity result in the equality of their values of marginal products.

An optimal condition for a pair of goods bought from the market follows from (6):

made the suggestion that the labor capacity were called leisure to differentiate it from market labor, despite the fact that only a fraction of this activity is truly leisure.

[12] The null vectors may have different dimensions in each equation.

[13] Paretian efficiency conditions have been explicitly incorporated in modeling collective decisions on consumption and the supply of labor in collective models of household production. See Apps and Rees (1997) and Chiappori (1997).

$$\frac{\partial F/\partial y_k}{\partial F/\partial y_\ell} = \frac{p_k}{p_\ell} \tag{10}$$

Now the marginal rate of technical substitution equals the corresponding input price ratio. This condition is associated to cost minimization.

However, let y_ℓ be interpreted as a given type of traded labor and k as an input for this kind of labor. If (10) were then rewritten as

$$p_\ell \frac{\partial F/\partial y_k}{\partial F/\partial y_\ell} = p_k \tag{11}$$

the result would be the equality between the value of marginal product and the respective price of input y_k that is used in the production of labor y_ℓ.

Conditions of type (10) or (11) are familiar from the theory of the firm. However, they can not, at least for a general model such as this, be solved independently from the other equations. The production decisions and the consumption decisions are simultaneously taken.[14]

4. A simple case with occupational choice

A special case of the above model can be cast after the textbook case of income and leisure. In the model, as a simplification, there is only one commodity. The preference function is described by $u(x_3, t_3)$, where x_3 represents a commodity and t_3 represents leisure.[15] For simplification, each unit of this commodity requires one unit of a specific good and no man-hours for its consumption. In this case, the convention of treating all variables as nonnegative is followed. Therefore, the vector of goods is given by $y = (-x_1, -x_2, -x_3, h_1, h_2)$. The variable x_3 only appears in vector $z = (x_3, t_3)$. The production functions of the commodity and of leisure are

[14] For an early discussion of the cases in which recursiveness of these problems is possible, see Singh, Squire, and Strauss (1986).

[15] In $u(x_3, t_3)$, the arguments of the function include two commodities that have specialized inputs. Also, the consumer directly values only this use of time, while the other uses of time, in the production of work, will be determined as a residual from the endowment of labor power. As far as the opportunity cost of time is concerned, the individual will only consider the opportunity cost of sacrificing leisure time. DeSerpa (1971) and Pollak and Wachter (1975, p.271) introduce the time inputs for commodities in the utility function as a way to consider preferences on time use itself. This might be double counting, since a Beckerian commodity is a package that includes the use of own time in its production and consumption. A much older tradition in the literature is to consider the labor supply itself in the preference function, a procedure more related to the present model. On the treatment of labor supply in the preference function, see footnote n.1, above.

simply $z_1 = x_3$ and $z_2 = t_3$. This simplified production function allows for the model to focus on the production and supply of labor. The person has a type of human capital that makes it viable to supply two kinds of market labor. The production function for each labor type is given by $h_j = f_j(x_j, t_j)$, where x_j and t_j represent specific inputs for the labor activity h_j.[16] The variable t_j shows the duration of the availability of the person's own organism for the labor activity h_j. This production function reflects the convention that people carry out tasks that require physical effort. With variable coefficients of production, the substitution between bought inputs and the use of this human capital is possible. The income restriction of this worker is given by

$$p_3 x_3 = w_1 h_1 - p_1 x_1 + w_2 h_2 - p_2 x_2$$

where p_i is the price of goods bought in the market and w_j is the unit wage. This budget restriction may be interpreted as the equality between the expenditure on one commodity and the net income from each type of labor.[17] The labor capacity restriction, usually referred to as time restriction, is given by

$$t_1 + t_2 + t_3 = 1,$$

where t_1 and t_2 refer to the use of labor capacity in two types of occupations or tasks.

In order to optimize, it is convenient to substitute the production functions in the budget restriction and write the following Lagrangean function:

$$L = u(x_3, t_3) + \lambda_1 \left[w_1 f_1(x_1, t_1) + w_2 f_2(x_2, t_2) - p_1 x_1 - p_2 x_2 - p_3 x_3 \right] + \lambda_2 (1 - t_1 - t_2 - t_3)$$

The first order conditions in relation to x_j's and t_j's are:

$$\frac{\partial u}{\partial x_3} - \lambda_1 p_3 = 0 \qquad (12)$$

$$\frac{\partial u}{\partial t_3} - \lambda_2 = 0 \qquad (13)$$

[16] Notice that $f_j(.)$ is being used as a generic symbol for a production function. It should not be taken as a partial derivative.

[17] Johnson (1966, p.142-143) has the trip to work as a time use that should be added to the work time. Also, the cost of the trip should be subtracted from the wage obtained in each trip, although this result is dependent on the transport input being given by a fixed coefficient. The present model could describe this particular case by using a Leontief production function for the labor activity. This production function, in fact, is the one used for each commodity in the first Beckerian models.

$$\lambda_1\left(w_j \frac{\partial h_j}{\partial x_j} - p_j\right) = 0\,,\ \ j \in \{1,2\}\,, \tag{14}$$

$$\lambda_1 w_j \frac{\partial h_j}{\partial t_j} - \lambda_2 = 0\,,\quad j \in \{1,2\}\,. \tag{15}$$

Eliminating the Lagrangean multipliers and omitting the restrictions, the first order conditions may be rewritten as:

$$\frac{\partial u/\partial t_3}{\partial u/\partial x_3} = \frac{w_1 \partial h_1/\partial t_1}{p_3}\,, \tag{16}$$

$$w_1 \frac{\partial h_1}{\partial t_1} = w_2 \frac{\partial h_2}{\partial t_2}\,, \tag{17}$$

$$w_j \frac{\partial h_j}{\partial x_j} = p_j\,,\quad\ j = 1,\,2\,. \tag{18}$$

At least theoretically, these four equations plus the budget restriction, the work restriction, and the two production functions allow for the determination of the value of the following variables: x_1, x_2, x_3, t_1, t_2, t_3, h_1, and h_2.

Equation (16) is analogous to (8) in the general model. However, the production function of the commodity is extremely simplified, as already noted, since $z_3 = x_3$. Thus, instead of a marginal rate of transformation between a pair of commodities, (16) shows the ratio between the opportunity cost of labor capacity used in a leisure activity, $w_1\, \partial h_1/\partial t_1$, and the price of the good used as input for commodity z_3. Equation (16) may be rewritten as

$$w_1 \frac{\partial h_1}{\partial t_1} = p_3 \frac{\partial u/\partial t_3}{\partial u/\partial x_3}$$

and it may be interpreted as the equality between the value of the marginal product of the labor capacity used in the first occupation and the opportunity cost of leisure time in terms of the commodity. It is no surprise that the equilibrium conditions of household production recall the conditions for the firm, since consumption and production decisions are interrelated. It also provides, at least in the one commodity model, an alternative estimate for the opportunity cost of leisure.

Equation (17) is equivalent to (9) and shows that the value of marginal productivity of labor capacity for each occupation is equal in equilibrium. They are also equal to the opportunity cost of leisure as just seen above. This benefit of each unit of work refers only to market revenue. It is a consequence of not considering the preferences of the consumer-producer for the different types of work. Introducing h_j in $u(.)$ could do this. It would extend the labor-leisure models that were referred to in the Introduction.

Finally, equations in (18), similar to (11), say that the value of the marginal product of each input bought in the market is equal to its marginal cost. If the unit wage for each occupation were isolated in each equation, it would be equal to the corresponding marginal cost. This result is here independent from preferences, as it did in the general model.

It is also possible to examine the marginal rates of technical substitution in the production of each type of work. They are given by

$$\frac{\partial h_j / \partial t_j}{\partial h_j / \partial x_j} = \frac{w_j \partial h_j / \partial t_j}{p_j}, \quad j = 1, 2 \qquad (19)$$

As expected, the marginal rates of technical substitution between the two inputs for each occupation are equal to a price ratio of inputs. Yet, the labor capacity price is an opportunity cost, given by the revenue that could be obtained by selling that work unit in the market. Equations in (19) are also variants of (9), now defined for pairs of labor inputs.

These results provide different estimates for the opportunity cost of time, as summarized in the following set of equalities:

$$w_j \frac{\partial h_j}{\partial t_j} = p_j \frac{\partial h_j / \partial t_j}{\partial h_j / \partial x_j} = p_3 \frac{\partial u / \partial t_3}{\partial u / \partial x_3}, \quad j = 1, 2$$

The first set of equalities says that the values of the marginal product of working in different occupations are the same. They are also equal to the product between the price of an input x_j and the marginal rate of technical substitution of t_j for x_j. This expression can be rewritten in terms of the marginal cost of producing h_j and the marginal productivity of working in occupation j. The marginal cost is given by the ratio of the price of an input and its marginal productivity:

$$p_j \frac{\partial h_j / \partial t_j}{\partial h_j / \partial x_j} \equiv \frac{p_j}{\partial h_j / \partial x_j} \frac{\partial h_j}{\partial t_j}$$

Finally, the opportunity cost is equal to the product between the price of commodity x_3 and the marginal rate of substitution of commodity x_3 for leisure, t_3. There are thus three different ways for estimating the opportunity cost of time: two of them from the productive process of tradable work and one from consumption.[18] In the maximizing position of the consumer-producer, they are all equal. Of course, out of equilibrium they can be different and serve as a guide in decisions on where to better allocate time. In more general models, with production functions also for the Beckerian goods, there will be a greater number of alternative estimates of the opportunity cost of time.

The relationship of this model with others can be made in several ways. First, the person may specialize in one occupation. This may occur due to preferences, technology, relative prices or social rules. Then the model could be specified in nonlinear programming terms, with the presence of nonnegativity conditions for each variable and inequalities for some of the restrictions.

Second, to obtain the usual income-leisure model, it would be sufficient to suppose a linear production function for labor, with labor supply given by the number of man-hours sold in the market, t_j. The use of other specific inputs in the production of labor would also be ignored.

In short, the income-leisure model just presented is general enough to also include production functions for labor. It seems to be useful for illustrating the mix of preference and production decisions in the supply of labor. It also seems to give some alternative insights in the discussion of the opportunity cost of time.

As a final characterization of the labor supply, it is useful to discuss a few results in the comparative statics of the specialized model. The study of the compensated demand faces two difficulties. To begin with, the equivalent to the budget restriction is now a nonlinear function, when the production functions of labor are substituted in the income restriction. Then, as opposed

[18] These expressions could serve as a basis for a graphical illustration of the simplified model. With the three alternative uses of the endowment of work capacity, three different individual demand curves could be drawn. The first of them would reflect the demand for leisure and would give the own demand for work capacity. With it, it would be possible to compute a reservation price that could be zero. The other two demand curves would reflect the demand for work capacity as inputs to the production of the two types of market work. The horizontal addition of the

to the traditional income-leisure model, the elimination of the labor capacity restriction is not simple, so it is kept separate here. The definition of the substitution matrix is now dependent on which exogenous variable and restriction is used for defining the compensated functions. As usual, the nonlabor income from the income restriction will be chosen.

In building the Slutsky equations, whether decomposing for commodities, for household outputs or for inputs it makes a difference. Only commodities enter the utility function, therefore it is natural to interpret a compensated demand as a movement along an indifference curve. Compensated demands for inputs imply movements along isoquants, and the decomposition could be made in terms of substitution and output effects. As for the output, it is not usual to decompose the price effect. Instead, the short-run and long-run supply functions are studied. Along these lines, the Slutsky decomposition will only be discussed for the commodities. In the income restriction of the simplified model, the value of nonlabor income is given by

$$m = p_1x_1 + p_2x_2 + p_3x_3 - w_1h - w_2h_2$$

Minimizing this value, subject to a given utility level and the remaining restrictions, results in the compensated, or Hicksian, demand functions: $x_3^H\left(p_1, p_2, p_3, w_1, w_3, u\right)$ and $t_3^H\left(p_1, p_2, p_3, w_1, w_3, u\right)$. Establishing the identity between the Marshallian and Hicksian demand functions and applying the Envelope Theorem yield the following Slutsky equations for the commodity x_3:

$$\frac{\partial x_3}{\partial p_i} = \frac{\partial x_3^H}{\partial p_i} - x_i \frac{\partial x_3}{\partial m}, \; i = 1, 2, 3$$

$$\frac{\partial x_3}{\partial w_j} = \frac{\partial x_3^H}{\partial w_j} + h_j \frac{\partial x_3}{\partial m}, \; j = 1, 2$$

The only price that directly acts upon the quantity demanded of this commodity is p_3, even then only if the budget restriction is linearized around the equilibrium position.[19] All the other prices, including wages, act through the nonlinear budget, as usual in Beckerian models in which the

three individual demand curves would cross the vertical line at the endowment point that would represent the supply of work capacity, and, there, it would determine the opportunity cost of time for the consumer-producer.

[19] Theorem 6 from Blomquist (1989) considers nonlinear budget restrictions in which a term involving a market price and the corresponding good can be additively separated and how some substitution effects even so become predictable. With this theorem, it seems possible to predict that the substitution term $\partial x_3^H / \partial p_3$ is nonpositive. For the other terms in the Slutsky decompositions, it would be necessary to deal with a linearized expression of the budget restriction, based on shadow prices for the commodities and the alternative uses of time. For these prices the standard results in comparative statics are valid. But such prices would be functions of the market prices and these indirect

production functions do not have the property of constant returns to scale. The commodity t_3, being time used outside the market, does not have a price. Here, wage can not be used as usual. The Slutsky decomposition can then be made only with respect to prices of inputs or wages. The expressions are similar to the ones above, except for the variables in the numerator of each derivative. All the effects work through the nonlinear budget function.

5. The net result from labor supply

Provided specific inputs to the productive process of labor are identifiable, it is possible to define gross and net revenue from market work. This recalls farm models in which part of the total income is given by the net result, also called profit, of the goods sold in the market.[20] However, given the discussion of the endowment of work, given by Nature, this net result has the nature of an economic rent.

The concept of economic or pure profit in the theory of the firm is clearly defined by the net results obtained over the opportunity cost of own capital applied in the firm. Capital is defined in terms of the market value of the property rights on physical and financial capital. When this capital is fixed in the long run, the pure profit is called economic rent, a concept that follows from a null opportunity cost. However, this absence of opportunity cost is only valid when the owner has no alternative personal use for the resource; said differently, only when the reservation price is zero.

Making an analogy among wages, profits and rent might seem odd, almost a provocation, given the characteristics of the corresponding factor markets. Even neoclassical economists have kept the analysis of each factor separate, in light of the sociological characteristics of each factor market. So, the purpose of the following observations is to find alternative interpretations for the costs of supplying labor. This is, in fact, a practical question in income taxation, where endless discussions occur on what kinds of deductions should be allowed.

The net result for each occupation is equal to $w_j h_j - p_j x_j$ in the model of the previous section. Notice that this is not the profit concept, as just defined, since it does not consider the

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effects should be treated separately. General results connecting the prices of goods to commodity demand are unlikely.

[20] See Squire, and Strauss (1986, p.18, 71-72). There, the definition is used for a farm output and referred to as profit or net result.

opportunity cost of using this time in nonmarket activities or perhaps in a different occupation with a better remuneration. Instead, the net result is equivalent to the concept of quasi-rent, as used in the theory of the firm, being given by the difference between total revenue and variable costs.

The expressions of the net results for the two occupations of the income-leisure model are obtainable from the budget and time restrictions. Initially, multiply both sides of the time restriction by the shadow price of a man-hour, then add the result to the right-hand side of the budget restriction, and finally write:

$$p_3 x_3 + w_1 \frac{\partial h_1}{\partial t_1} t_3 = w_1 \frac{\partial h_1}{\partial t_1} + \sum_{j=1}^{2} \left(w_j h_j - w_1 \frac{\partial h_1}{\partial t_1} t_j - p_j x_j \right) \qquad (20)$$

The left-hand side of (20) shows the expenditures on inputs for the commodities, including the opportunity cost of time dedicated to one of them, leisure. The right-hand side shows the redefined Beckerian full-income, also in terms of the shadow wage. It is given by the value of the endowment of labor capacity plus the economic profit attained in each occupation.

However, this discussion is made in relation to specific inputs for labor. In the case of a firm, the production of traded goods is supposedly separate from the other activities of the person. It is exactly when consumption and production activities are separable that the idea of firms as a means for the social division of labor can be spoken of. It is as a consequence of this possibility that the concept of circular flows between firms and consumers may be used.

But it is an age-old wisdom that the supply of labor is different from the supply of other tradable goods and services, especially in the way the person is involved. Suppose a person owns a capital good, such as a lawn mower. It can be used to sell services of gardening or it can be put to rent.[21] That is not the case with labor. When a person is hired for work, the organism itself is the source of the service. Thus to separate decisions is impossible, since all consumption that is not specific to work is also consumption by the same organism. The procedure of supposing a specific production function for labor can only capture part of the inputs. It leaves out most of the

[21] Renting a capital good involves a social arrangement with recognized property rights. Even then, there is the work of administering these property rights or at least of checking the services of those hired to manage these property rights. But the amount of work and the degree of effort required are certainly smaller than the operation of most capital goods, although the stress involved in the uncertainties of returns on financial capital might be high, especially for risk-averse persons.

inputs that are necessary for keeping alive and in the good shape the human organism. It seems that the most that can be done by the analyst, in computing the profit of an occupation, is to find the contribution of this output to total income.

Note, however, that the contribution of the net revenue of market work to total income is defined after the equilibrium of the consumer-producer is found. It has the nature of an economic rent, and it defines the net product or *le produit net* of the person, as the physiocrats would say. Perhaps, this can be illustrated by rewriting the above special model in terms of cost functions. Take t_j as given and define the restricted or short-run cost functions, defined without the opportunity costs of time.[22] Minimize $p_j x_j$, subject to $f_j(x_j, t_j) = h_j$ for each kind of market work. This will result in the restricted cost functions $c_j = c_j(h_j, t_j, p_j)$. Now, the budget restriction becomes

$$p_3 x_3 = \sum_{j=1}^{2} \left(w_j h_j - c_j(h_j, t_j, p_j) \right)$$

Thus by subtracting variable costs from the total revenue of a given type of market work, the consumer-producer gets the economic rent for a given level of work capacity (time) allocated to that activity. Given (t_1, t_2), the value of the goods used as inputs for commodities has the nature of an economic rent. This is consistent with the Walrasian approach of considering a person as a kind of capital good, as discussed above. A consumer-producer, specialized in selling work and with no income from other property rights, might have a very low level of expenditures on consumption of goods beyond the necessary for working. This would occur with different abilities and for a marginal worker.

However, the real income of a person also involves leisure or, as Becker would put it, commodities that are intensive in the use of time. As seen above, this consumption of leisure is the origin of the opportunity cost of time when this is used as input in order to sell work. This opportunity cost is then the equivalent to a marginal cost of labor. The revenue from work above these variable costs of labor is then the equivalent to an economic net result for the worker and it would be the equivalent to the economic profit for capital goods. People can not be sold in an economy where slavery is banned. Anyway, it there were slavery a person would be the

[22] It would be a straightforward extension to use net revenue functions. Nevertheless, cost functions are better suited for the analysis of the returns to human capital.

equivalent to a capital good. Thus, for a free person, the present value of the net result that might be obtained along the working life is in fact the value of freedom.

7. Conclusion

Time itself is not a productive input in household production. Its role is simply to delimit the interval of time in which production and consumption occur. The true input is the work done during such a time interval or period. However, as a simplification, it is customary to describe labor input by the number of hours during which a person exerts such activity. Besides that the amount of work depends on the level of effort of the person during a given period. Thus, the production of work by a person seems fit for a description by a production function, in which the idea of maximum output for a given set of specific inputs is essential.

This paper initially presented a general Beckerian model with a technology that includes the possibility of joint production and productive consumption. At least one of the outputs of this productive process is labor, meant to be sold in the market. Implicit in the model is fact that the effort level depends on attaining the production frontier. With this specification the household production model, including labor supply, might be more amenable for its integration into general equilibrium theory.

The model is illustrated by the special case of income-leisure, adapted for the inclusion of production functions for two types of labor. With this model, it is possible to obtain objective estimates for the shadow price of the labor capacity from relations that involve marginal productivities of the inputs in labor output.

Finally, the notion of net result is adapted for the production of labor. Because of the unalterable endowment of labor, the net result is related to economic rent. However, the own use of this endowment creates the equivalent to quasi-rent or economic profit, although this expression is inadequate for a kind of capital that has no market in a modern society, except for its services. From the viewpoint of the consumer-producer, a way to increase the net result is by reducing costs of producing labor. Investments in technological progress at the domestic factory may reduce these costs. Another form, perhaps with better gains in the long run, is to invest in differentiating the kind of market labor to be offered. It is as if the person tried to become a different type of person as far as the supply of work is concerned. Instead of economies of scale, which is not possible to explore, given the fixity of the time endowment, the consumer-producer creates new outputs, by differentiating the labor supply along Chamberlinian lines.

There are various extensions of the model that could be made. First, it is the comparative statics, especially with view to econometric work, but also for the purpose of integration of the model in general equilibrium theory. Second, some of the results from the literature on collective household models might be changed.

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