The Impact of EPL on Labour Productivity in a General Equilibrium Matching Model

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Abstract

The standard analysis of the impact of EPL on labour market outcomes concentrates mainly on unemployment, disregarding the possible effect on productivity. In this paper we make (a component of) labour productivity endogenous and analyze how the presence of a stringent protection legislation affects labour market in an equilibrium matching model with endogenous job destruction. Indeed, considering labour productivity an endogenous could be important not only in the case of EPL, but also for all kind of personnel policy evaluation. In this framework high labour productivity on one hand is costly in terms of effort, on the other hand is beneficial in terms of lower job destruction. We find that high firing costs partially substitute high labour productivity in reducing job destruction and this, consequently, brings down the optimal level of productivity. Moreover, the impact of EPL on unemployment is ambiguous but numerical exercises show unambiguously how higher firing restrictions reduce different measures of aggregate welfare. To some extent, the clear emergence of these results is full of policy implication and, indeed, rationalizes the recent empirical evidence on the impact of EPL.

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1. Introduction

Recent empirical evidence from European countries and the U.S. shows that the presence of stringent employment protection legislation (EPL) affects significantly the level of productivity. In particular, both cross-country (DeFreitas and Marshall, 1998), Dif-in-Dif (Micco and Pages, 2006; Autor et al., 2006, 2007; Bassanini and Venn, 2007; Bassanini et al., 2009; Lisi, 2009) and other studies (Riphahn, 2004; Ichino and Riphahn, 2005) found that a higher EPL have a negative impact on labour productivity.

Nonetheless, the theoretical analysis of the impact of EPL focused mainly on unemployment and job flows, disregarding the effects on labour productivity. In fact, both standard analysis of labour demand under uncertainty (Bentolila and Bertola, 1990; Bertola, 1990; Bentolila and Saint-Paul, 1992; Bentolila and Dolado, 1994; Boeri and Garibaldi, 2007) and general equilibrium models (Mortensen and Pissarides, 1994, 1999b; Garibaldi, 1998; Pissarides, 2000; Cahuc and Postel-Vinay, 2002) consider the level of productivity an exogenous parameter, not influenced by the presence of firing costs. Indeed, the issue has been already the object of interest of some papers. However, these studies analyze the role of EPL in distorting the adjustment of employment and investment and, in turn, productivity growth (Hopenhayn and Rogerson, 1993; Saint-Paul, 1997, 2002; Bartelsman and Hinloopen, 2005).

In this paper, in the spirit of Ichino and Riphahn (2005), we concentrate more on the behavioral component of productivity, therefore we make (a component of) labour productivity an endogenous object of the model and then study the impact of a stringent protection legislation. Since our concern is to understand the equilibrium impact on productivity, unemployment and welfare, we need to embed the analysis into an equilibrium model of the labour market. To this extent, it is our conviction that the Mortensen and Pissarides matching approach to equilibrium unemployment is the best candidate for this kind of analysis.

In this framework, the matching between any single job vacancy and unemployed worker is a costly and sticky process, governed by a matching function assumed with constant returns. The job productivity has a common component and an idiosyncratic component, due to either demand or technology shocks, which makes the value of product job-specific. The idiosyncratic component follows a jump process characterized by a Poisson arrival frequency and it is drawn by a common price distribution whenever it jumps.

The usual assumption in the literature is that technology is fully flexible at the beginning of creation, but investment is irreversible. Therefore, at the moment of creation firms choose the most profitable job in the market, with the idiosyncratic component at the upper support of the price distribution. Thus, every new match generates a positive surplus, which is divided between wages and profits according to bilateral bargain. However, whenever a shock arrives an existing job cannot be switched to one more profitable and wages are revised in the face of new productivity. Nonetheless, large negative shocks generate a negative surplus, which makes optimal for a firm to destroy the job. In the presence of a stringent protection legislation, modeled as firing costs, job destruction is costly for the firm. Moreover, there exist a zero-profit condition for the opening of new vacancies, which determines the tightness of the market and, along with the destruction rule, the level of unemployment.

In this paper we imagine that an employed worker has to exert effort to produce and this generates disutility. Following this argument, we assume that the common component of productivity is determined by the level of effort exerted by workers. Therefore, high labour productivity on one hand is costly in terms of effort, on the other hand is beneficial in terms of lower job destruction. This is, as far as we are aware, a novelty as the common component of productivity is usually considered an exogenous parameter of the model, not influenced by the level of institutional variables. In the light of the micro-founded nature of the matching approach to equilibrium unemployment, this extension could be a good suggestion to capture in the framework the recent evidence on the impact of EPL on productivity. Moreover, the approach to put labour market outcomes and personnel economics together when we address policy questions has already turned out to be successful (see e.g. Shapiro and Stiglitz, 1984).

An equilibrium is a job destruction and job creation rule, a labour productivity and a level of unemployment implied by the rational expectations behavior of individual firms and workers and by the matching technology. We study how the presence of a stringent protection legislation affects productivity, unemployment and welfare in the aggregate steady-state. We find that high firing costs partially substitute high labour productivity in reducing job destruction and this, consequently, brings down the equilibrium labour productivity. Moreover, the impact of EPL on unemployment is ambiguous but numerical exercises show unambiguously how higher firing restrictions reduce different measures of aggregate welfare. To some extent, the clear emergence of these results is full of policy implication and, indeed, rationalizes the recent empirical evidence on the impact of EPL.

The paper proceeds as follows: in Section 2 we describe the basic theoretical framework and in Section 3 characterize its steady-state. Section 4 studies qualitatively the impact of a stringent protection legislation on the equilibrium level. In Section 5 we conduct some numerical exercise to study the effect on productivity, but also on different measure of aggregate welfare. Section 6 concludes.

2. The theoretical framework

The basic theoretical framework is the matching approach to equilibrium unemployment with endogenous job destruction, in the version of Pissarides (2000). In this economy there is an endogenously sized continuum of jobs, characterized by a common component of productivity p and an idiosyncratic component x. Each product commands in the market a price of px, which evidently differ to each other for the presence of the idiosyncratic component. In the standard versions of the model p is considered an exogenous parameter, capturing the macro events that affect productivity in all jobs by the same amount and in the same direction. Differently, in our interpretation p is the endogenous labour productivity and x is the idiosyncratic condition in the market, due to demand or technology. Therefore, in our model we do not consider p a parameter capturing the macro shocks, because our aim is exactly to study how firing costs affect the level of the behavioural component of productivity. Nonetheless, it is evident that there is no difficulty in introducing such a parameter in our model.

The stochastic process governing the idiosyncratic component x is Poisson with arrival rate λ. Whenever a jump arrives, the new level of x is drawn from the distribution $G(x)$ with finite upper support $\bar{x}$ and no mass point. The Poisson process implies that shocks are persistent, but conditional on change the new draws are independent by the initial level of x.

Each firm has only one job that can be either filled and producing some good (state $J(x)$), according to the idiosyncratic level and the behavioural productivity, or vacant and searching for a worker (state *V*), which costs *pc* per unit of time. Firms have full information on technology and market condition, therefore they create always the most profitable job, that is, with the idiosyncratic level at the upper support of the price distribution. Furthermore, the Nash bargaining rule implies that new jobs offer the highest wage as well. However, investment are irreversible and when a shock arrives firms have no choice over their productivity. Filled jobs not always resist to negative productivity shocks and, in particular, they are destroyed whenever the new draw of x falls below a certain level of *reservation productivity* R. This implies that each job has a probability of being destroyed equal to $\lambda G(R)$. Job destruction is not costless, rather whenever a job is destroyed firm has to pay the firing costs pF.

Respectively, each worker can be in one of two states, employed and producing some good (state $W(x)$) or unemployed and searching for a job (state U). Employed worker receives the wage $w(x)$ and has to choose how much effort e to exert in the job, which determines the common component of productivity $p = f(e)$. Even if not necessary, we assume a linear relation $p = e$ between effort and productivity[1]. On the contrary, unemployed worker does not exert effort and benefits only from z, which can be interpreted either as unemployment compensation or as leisure. Wages are the outcome of the Nash bargaining, according to which workers receive a fraction $0 < \beta < 1$of the match surplus, where β can be interpreted as the workers' bargaining power. Since the match surplus is conditional on idiosyncratic productivity, wages are revised whenever a productivity shock occurs. In particular, it is intuitive that both match surplus and wage are increasing function of x. Following the previous literature, we assume that workers are risk neutral and impatient, which implies zero saving and full consumption. Furthermore, exerting effort generates an increasing disutility. Therefore, an employed worker enjoys conditional on x the instantaneous utility

$$u(x) = w(x) - \frac{1}{2}\gamma e^2,$$

where γ is the parameter governing marginal disutility of effort (see e.g. Garibaldi, 2006), whereas the instantaneous utility of the unemployed worker is simply

$$u = z.$$

The number of matches between vacant jobs and unemployed workers is governed by the matching function $m(v, u)$, where v and u are respectively the number of vacant jobs and

[1] Notice that this specification is without loss of generality, given that for the utility function below an additional parameter on the relation $p = \varphi e$ would not be identified, but only $\frac{\gamma}{\varphi^2}$ would be identified.

unemployed workers. Labour force is normalized to 1, so that in this economy the number of unemployed workers u is the unemployment rate. As standard in the literature, we assume that the matching function is twice continuously differentiable, increasing and concave in both its arguments and homogeneous of degree one, with elasticity strictly between $0 < \xi < 1$. By linear homogeneity, the transition rate from vacant to filled job is $m(v, u)/v = m(1, u/v) = q(\theta)$, with $q'(\theta) < 0$, where $\theta = v/u$ identifies the *labour market tightness*. Moreover, the elasticity of $q(\theta)$ is strictly between $-1 < \eta < 0$ and it is related with the elasticity of the matching function (respect to v) by $\eta = \xi - 1$. Similarly, the transition probability from unemployed to employed is $m(v, u)/u = m(v/u, 1) = \theta q(\theta)$, an increasing function of θ[2].

The endogenous variables of the model are the level of market tightness θ, the level of reservation productivity R, the level of effort e and, in turn, labour productivity p and the level of unemployment u. In the next section we derive their steady-state values.

3. Steady-state equilibrium

In steady-state the choices of opening a vacancy and destroying a job for a firm and the level of effort for a worker are based on the asset values of the various conditions. Indeed, these asset values are close to Pissarides (2000), therefore not much time will be spent on their derivation. As said before, the crucial difference in this paper is the introduction of effort in the worker utility function, which formally does not change heavily the asset values, but it does change significantly the subsequent steady-state analysis.

From the assumptions on vacancy cost, idiosyncratic component and firing costs, we have that the asset values of a vacancy and a filled job satisfy the Bellman equations

$$rV = -pc + q(\theta)[J(\bar{x}) - V] \tag{1}$$

$$rJ(x) = px - w(x) + \lambda \int_R^{\bar{x}} J(s)dG(s) - \lambda G(R)pF - \lambda J(x) \tag{2}$$

In (1) a firm has to pay the vacancy cost per unit of time $-pc$ and with probability $q(\theta)$ matches with an unemployed worker, gives up the value of a vacancy V and gets the value of a filled job at the upper support of the price distribution $J(\bar{x})$. In steady-state vacancies are opened until all rents are exhausted. Therefore, the equilibrium zero-profit condition is

[2] In the Appendix we show that all properties of $q(\theta)$ derive exclusively by the standard assumptions of the matching function.

$$rV = 0 \qquad \Longrightarrow \qquad J(\bar{x}) = \frac{pc}{q(\theta)} \tag{3}$$

In (2), conditional on the idiosyncratic component, a firm enjoys the value of product *px* and pay the wage *w(x)*, then with probability λ a shock arrives and a new level of *x* is drawn from the price distribution *G(x)*. In this case the firm has to give up the value *J(x)* and gets the new value *J(s)* if *s* is over the reservation productivity *R*, or destroys the job and pay *pF* otherwise.

Similarly, from the assumptions on unemployment compensation (or leisure) and instantaneous utility function, the asset values of unemployed and employed worker solve

$$rU = z + \theta q(\theta)[W(\bar{x}) - U] \tag{4}$$

$$rW(x) = w(x) - \frac{1}{2}\gamma e^2 + \lambda \int_R^{\bar{x}} W(s)dG(s) + \lambda G(R)U - \lambda W(x) \tag{5}$$

In (4) an unemployed worker enjoys the unemployment compensation *z* and with probability $\theta q(\theta)$ matches with a vacant job, gives up the value *U* and gets the value of employed at the upper support of the price distribution $W(\bar{x})$. In (5), conditional on the idiosyncratic component, an employed worker enjoys the wage *w(x)* but suffers the effort exerted $-\frac{1}{2}\gamma e^2$, then with probability λ a shock arrives and a new level of *x* is drawn from the price distribution *G(x)*. In this case the worker has to give up the value *W(x)* and gets the new value *W(s)* if *s* is over the reservation productivity *R*, or the value of unemployed *U* otherwise. Furthermore, the choice of the effort level is one of rationale expectations, that is, *e* is the effort that maximizes the asset value of being employed.

Wages are split so that workers receive a fraction β of the total match surplus and are revised whenever a productivity shock occurs. However, with the presence of firing costs the match surplus of a new job is different from that of an existing job, because only in the second case firms save the firing costs for the continuation of the match. Thus, we have to distinguish between the outside w_0 and the inside wage *w(x)*. In the case of a new job the match surplus is

$$S_0(\bar{x}) = J(\bar{x}) - V + W(\bar{x}) - U$$

and the sharing rule implies

$$W(\bar{x}) - U = \beta[J(\bar{x}) - V + W(\bar{x}) - U] \tag{6}$$

Using the relation $p = e$, the zero-profit condition (3), the asset equations for a filled job (2), unemployed (4) and employed worker (5) and the sharing rule (6), gives the outside wage equation (see the Appendix for the derivation)

$$w_0 = (1 - \beta)\left(z + \frac{1}{2}\gamma p^2\right) + \beta p(\bar{x} + c\theta - \lambda F) \tag{7}$$

Differently, in the case of an existing job a firm saves the firing costs for the continuation of the match and thus the match surplus is different

$$S(x) = J(x) - V + pF + W(x) - U$$

and the sharing rule implies

$$W(x) - U = \beta[J(x) - V + pF + W(x) - U]$$

Similar calculation gives the inside wage equation

$$w(x) = (1 - \beta)\left(z + \frac{1}{2}\gamma p^2\right) + \beta p(x + c\theta + rF) \tag{8}$$

Equations (7) and (8) differ only for the impact of firing costs F and this difference indeed emphasizes the conflict between insiders and outsiders. On one hand, inside a match the prospect of paying F leads firms to concede marginally a higher wage to avoid the destruction of job. On the other hand, outside the match the expectation of paying F sooner or later once a job is created leads firms to start the match with a lower wage to partially recoup the future payment. As (7) shows, the impact of F on the outside wage is higher when λ is higher, because the probability of job destruction per unit of time is greater.

The choice of destroying a job is taken inside a match, therefore we have to use the inside wage equation to derive the job destruction condition. Substituting (8) in (2), we get a more explicit expression of the asset value of a filled job as a function of the idiosyncratic component

$$(r + \lambda)J(x) = (1 - \beta)\left(px - z - \frac{1}{2}\gamma p^2\right) - \beta p(c\theta + rF) + \lambda \int_R^{\bar{x}} J(s)dG(s) - \lambda G(R)pF \tag{9}$$

From (9) we can see that the asset value $J(x)$ is a monotonically increasing function of x, which means that there exists a unique value x^* such that $J(x^*) = 0$ and for any x greater (smaller) than x^*, then $J(x) > 0$ ($J(x) < 0$). In the model without firing costs, this implies that the reservation productivity R under which a firm destroys the job satisfies the reservation property $J(R) = 0$. In the model with firing costs, for a firm is optimal to continue even a negative match, as soon as the negative surplus is smaller than the cost of destroying a job pF. That is, with firing costs the reservation property is $J(R) = -pF$ (or $W(R) = U$), which allows us to characterize the reservation productivity R. Subtracting the generic asset equation (9) from the equation evaluated at $x = R$ and using $J(R) = -pF$, we get

$$(r+\lambda)J(R) = (1-\beta)\left(pR - z - \tfrac{1}{2}\gamma p^2\right) - \beta p(c\theta + rF) + \lambda\int_R^{\bar{x}} J(s)dG(s) - \lambda G(R)pF \qquad (10)$$

$$(r+\lambda)[J(x) - J(R)] = (1-\beta)p(x-R)$$

$$J(x) = \frac{(1-\beta)p(x-R)}{(r+\lambda)} - pF \qquad (11)$$

Now, substituting (11) in the integral expression of (10) and dividing by $(1 - \beta)p$, we get an implicit expression for R as a function of market tightness θ, labour productivity p and the parameters of the model

$$R - \frac{z}{p} - \frac{1}{2}\gamma p - \frac{\beta}{1-\beta}c\theta + \frac{\lambda}{r+\lambda}\int_R^{\bar{x}}(x-R)dG(s) + rF = 0 \qquad (12)$$

Equation (12) is the first steady-state condition of the model and in what follow we will refer to this as the job destruction rule (JD), when we emphasize the relation between R and θ, or as the reservation equation (RE), when we emphasize the relation between R and p. The value of pR is the lowest acceptable price to continue a job. From (12), we can see that pR is less than the reservation wage $\left(rU = z + \frac{\beta}{1-\beta}pc\theta\right)$, which is the lowest acceptable wage for a worker. One reason standard in this literature is the presence of some labour hoarding, represented by the integral expression. Given the probability that x might change in the future, for a firm is optimal to continue some currently negative match and wait for a higher price, in order to avoid the hiring cost. As intuitive, labour hoarding is increasing in the probability of a change λ. The second one is the presence of firing costs, which are paid by firms but not enjoyed by workers.

The choice of creating a job is taken outside the match, therefore we have to use the outside wage equation and evaluate the value of a filled job at the upper support of the price distribution. Substituting (7) in (2), subtracting (10) and using $J(R) = -pF$, we get

$$(r+\lambda)[J(\bar{x}) - J(R)] = (1-\beta)p(\bar{x}-R) + \beta pF(r+\lambda)$$

$$J(\bar{x}) = \frac{(1-\beta)p(\bar{x}-R)}{(r+\lambda)} - (1-\beta)pF \qquad (13)$$

Now, inserting the zero-profit condition (3) in (13), we get an implicit expression for θ as a function of the reservation productivity R and the parameters of the model

$$\frac{c}{q(\theta)} = (1-\beta)\left(\frac{\bar{x}-R}{r+\lambda} - F\right) \tag{14}$$

Equation (14) is the second equilibrium condition and we will refer to this as the job creation condition (JC). The left hand side of (14) is the cost of a vacancy for the expected duration of a vacancy. The right hand side is the discounted additional surplus a firm gets from a new job. Therefore, this condition says that in equilibrium the expected hiring cost has to be equal to the expected gain from a new job.

Equations (12) and (14) jointly determine R and θ, as illustrated in Figure 1. Let define (12) as $B(R,\theta,p,\omega) = 0$ and (14) as $D(\theta,R,\omega) = 0$, where ω is the set of parameters. Then we have

$$\frac{\partial R}{\partial \theta} = -\frac{\partial B/\partial \theta}{\partial B/\partial R} = \frac{c\beta/(1-\beta)}{\left[\frac{r+\lambda G(R)}{r+\lambda}\right]} > 0 \tag{15}$$

$$\frac{\partial \theta}{\partial R} = -\frac{\partial D/\partial R}{\partial D/\partial \theta} = \frac{(1-\beta)/(r+\lambda)}{\frac{c}{q(\theta)^2}q'(\theta)} < 0 \tag{16}$$

As (15) shows, the curve JD slopes up because a higher θ increases the probability of finding a job and, thus, the opportunity cost for a worker $\left(\frac{\beta}{1-\beta}c\theta\right)$, who now pretends a higher wage to accept a job and so more jobs are marginally destroyed. As (16) shows, the curve JC slopes down because a higher R increases the probability that a job is destroyed $\lambda G(R)$ and, in turn, reduces the expected gain from a new job $\left((1-\beta)\frac{\bar{x}-R}{r+\lambda}\right)$, so less vacancies are opened.

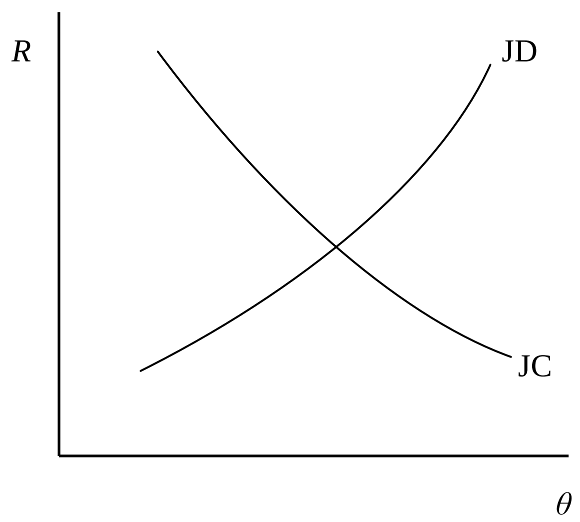

Figure 1

Steady-state reservation productivity and market tightness

So far, the joint determination of R and θ has been done as in the previous literature for a given level of labour productivity p and, indeed, besides the different specification of the worker utility function, no significant novelty are introduced. However, in our model labour productivity is not a parameter but an ulterior unknown. Following our interpretation of p as the behavioural component of productivity, we assumed that its level is determined by the level of effort e exerted by the employed worker and, in particular, that $p = e$.

The choice of effort is rationally taken by worker when he matches with a vacant job, therefore in equilibrium e maximizes the value of being employed at the upper support of the price distribution $W(\bar{x})$. Since our equilibrium is one of rational expectations, when a worker takes this choice he actually knows the job destruction rule R and takes into account the impact on it. Moreover, given the choice of effort is taken individually, the single worker considers the impact on market tightness θ marginally negligible. From this, it can be easily seen that the same effort level maximizes the asset value of unemployment, being U a monotonically increasing function of $W(\bar{x})$

$$\big(r + \theta q(\theta)\big)U = z + \theta q(\theta)W(\bar{x})$$

The maximization of $W(\bar{x})$ in the form of Bellman equation (5) is not a trivial calculus. However, using $p = e$ and the reservation property $W(R) = U$, equations (4) and (5) can be solved for the permanent income form as a function of R, θ, p and the parameters of the model (see the Appendix for the derivation)

$$rW(\bar{x}) = (1-\beta)z + \beta p\left(\bar{x} + c\theta - \tfrac{1}{2}\gamma p\right) + \tfrac{\lambda\beta p}{r+\lambda}\left[G\big(R(p)\big)R(p) + \textstyle\int_{R(p)}^{\bar{x}} s\, dG(s) - \bar{x}\right] \tag{17}$$

As intuitive, since there is a non-zero probability of a productivity shock and, all the more so, of being fired, the permanent income of an employed worker at the upper support of the price distribution is less than the instantaneous utility. This form (17) allows us to take the F.O.C. and characterize the equilibrium condition for labour productivity p

$$\frac{\partial rW(\bar{x})}{\partial p} = 0$$

$$\bar{x} + c\theta - \gamma p + \tfrac{\lambda}{r+\lambda}\left[(G(R)R - \bar{x}) + pG(R)\tfrac{\partial R}{\partial p} + \textstyle\int_{R}^{\bar{x}} s\, dG(s)\right] = 0 \tag{18}$$

Equation (18) represents the equilibrium condition for labour productivity p (or effort e)[3] and from now on will be called the productivity equation (PE). From (18), we can notice that the optimal level of p depends on R and θ, but from (12) and (14) only RE depends on p. Therefore, for any level of market tightness θ, PE and RE jointly determine R and p, as illustrated in Figure 2. The shape of these curves is a bit more complicated then JC and JD, but still intuitive. Let define equation (18) as $M(p, R, \theta, \omega) = 0$. Then we have

$$\frac{\partial R}{\partial p} = -\frac{\partial B/\partial p}{\partial B/\partial R} = -\frac{\frac{z}{p^2} - \frac{\gamma}{2}}{\left[\frac{r + \lambda G(R)}{r + \lambda}\right]} = \begin{cases} > 0, & \forall p > \sqrt{\frac{2z}{\gamma}} \\ = 0, & p = \sqrt{\frac{2z}{\gamma}} \\ < 0, & \forall p < \sqrt{\frac{2z}{\gamma}} \end{cases} \tag{19}$$

$$\frac{\partial p}{\partial R} = -\frac{\partial M/\partial R}{\partial M/\partial p} = -\frac{\frac{\lambda}{r + \lambda}\left[G(R) + pf(R)\frac{\partial R}{\partial p}\left(1 - \frac{\lambda G(R)}{r + \lambda G(R)}\right)\right]}{-\gamma + \frac{\lambda G(R)}{r + \lambda G(R)}\left(\frac{z}{p^2} + \frac{\gamma}{2}\right)} > 0 \tag{20}$$

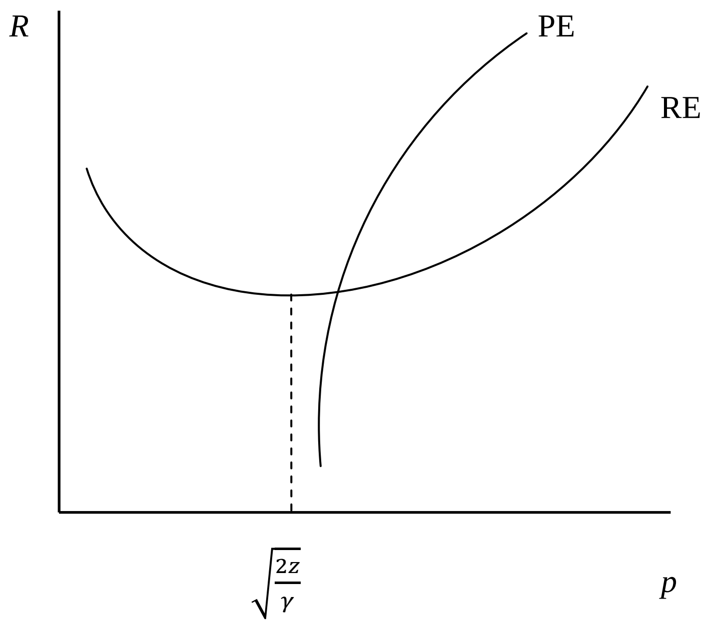

Figure 2

Steady-state reservation productivity and labour productivity

[3] At first sight, the S.O.C. for this maximization problem would depend on the value of parameters

$$\frac{\partial^2 rW(\bar{x})}{\partial p^2} = -\gamma + \frac{\lambda G(R)}{r + \lambda G(R)}\left(\frac{z}{p^2} + \frac{\gamma}{2}\right)$$

However, for a very large set of values, indeed all the plausible ones, numerical computations unequivocally show that the condition $\frac{\partial^2 rW(\bar{x})}{\partial p^2} < 0$ is respected.

From (19), labour productivity p has two opposite effects on optimal reservation productivity R, the *disutility-wage effect* and the *production effect*. On one hand, a higher p increases the disutility of worker and consequently the wage $\left(-\frac{1}{2}\gamma p\right)$, thus more jobs are marginally destroyed. On the other hand, a higher p increases the value of production (pR) and partially compensates a lower x, leading to a fall in R. Nonetheless, because of the increasing marginal disutility of effort, we can establish that when p is low the effect on wage is small and the effect on production dominates, whereas when p is high the disutility increases more than proportionally and the effect on wage dominates. Therefore, RE has a standard u-shape, with a minimum in the point in which *disutility-wage effect* and *production effect* exactly compensate.

Similarly, reservation productivity R affects labour productivity p for the *continuation value effect*. In fact, a marginal increase in R does not change the instantaneous utility of worker, but obviously it does change his continuation value. In particular, a higher R not only increases the probability of being fired $(\lambda G(R))$, shortening the expected period of employment, but also decreases the probability of finding a job $(\theta q(\theta))$, increasing the expected period of unemployment. Both these impacts affect negatively the continuation value and, therefore, the worker chooses p so as to address optimally its level, knowing that R is chosen optimally by firms through (12). This continuation value effect is included in the numerator of (20) and it is greater for R and p high, which implies that PE has a shape as in Fig. 2.

The last equation of the model is the steady-state condition for unemployment, usually called the Beveridge curve. There are different ways to derive this condition, here we state it in terms of flows in and flows out unemployment. In equilibrium the number of workers who enter unemployment $(1-u)\lambda G(R)$ equals the number of workers who leave unemployment $u\theta q(\theta)$, so the steady-state condition is

$$u = \frac{\lambda G(R)}{\lambda G(R) + \theta q(\theta)} \tag{21}$$

Equation (21) is the final condition of the model and implies that in equilibrium for any R and θ there is a unique unemployment rate u and, in turn, a unique number of vacant jobs v.

The Beveridge curve is often drawn in vacancy-unemployment space by a downward sloping and convex curve. Indeed, as highlighted by Mortensen and Pissarides (1994), in the matching model with endogenous job destruction the precise shape of the Beveridge curve is ambiguous. In particular, differentiation of (21) shows that there are two opposite effects

$$\frac{\partial v}{\partial u} = -\frac{\partial T/\partial u}{\partial T/\partial v} = -\frac{-\lambda G(R) + (1-u)\lambda f(R)\left(\frac{\partial R}{\partial \theta}\frac{\partial \theta}{\partial u}\right) + \theta q(\theta)\eta}{(1-u)\lambda f(R)\left(\frac{\partial R}{\partial \theta}\frac{\partial \theta}{\partial v}\right) - q(\theta)(1+\eta)} = -\frac{<0}{\gtreqless 0} \tag{22}$$

On one hand, more vacancies increase the number of job matches, implying a lower unemployment rate, captured by the second term of the denominator of (22). On the other hand, more vacancies increase the number of jobs destroyed, implying a higher unemployment rate, the first term of the denominator. Since the empirical evidence seems to identify this form, it is common to assume that the matching effect is stronger than the destruction one and to draw the Beveridge curve as a downward sloping and convex curve. Moreover, numerical simulations of the model with the equilibrium values fairly always confirm the conventional shape. As usual, in Figure 3 we draw the Beveridge curve with a straight line through the origin, representing all the possible values for *v* and *u* compatible with the equilibrium market tightness θ.

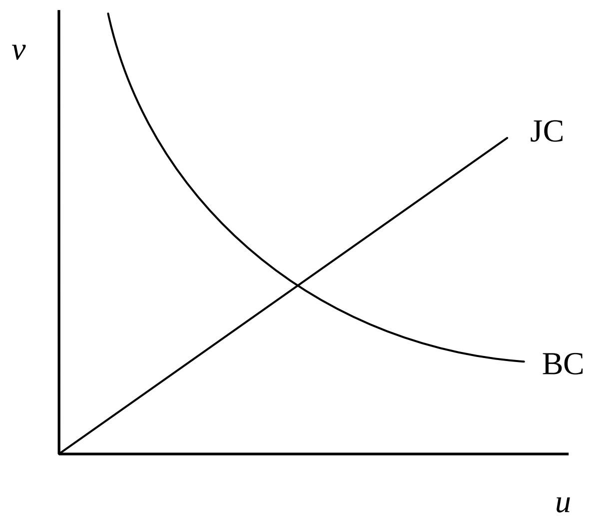

Figure 3

Steady-state unemployment and vacancies

In conclusion, we are ready to define the rational expectations equilibrium of the model:

Steady-state equilibrium – The rational expectation equilibrium is a quadruple (R^*, θ^*, p^*, u^*) that satisfies the job destruction condition (12), the job creation condition (14), the productivity equation (18) and the Beveridge curve (21) implied by the rational expectations behavior of individual firms and workers and by the matching technology.

For any value of labour productivity p, equations (12) and (14) determine reservation productivity R and market tightness θ. Then, from all these equilibrium triple, equation (18) identifies the unique value of equilibrium productivity p, compatible with job creation and job destruction conditions. Finally, with knowledge of R and θ, from (21) there is a unique value of equilibrium unemployment u and, in turn, a unique value of v.

To avoid to weigh down the content of the paper, here we do not address rigorously the analysis of the dynamics of the model out-of-steady-state, however some remarks are proper. The usual assumptions in this kind of analysis are that firms are able to open up or close vacancies instantaneously and that wage can be renegotiated at any time; that is, vacancies and wage are jump variables. These assumptions ensure that the zero-profit condition from a new vacancy (3) and the sharing rule (8) hold out of equilibrium as well. Similarly, the natural assumptions to make for the other two unknowns of the model are that firms can shut down unprofitable jobs instantaneously and that workers exert the optimal level of effort at any time; that is, reservation productivity and labour productivity are jump variables as well. These assumptions imply that the reservation property (12) and the optimal productivity (18) hold both in and out of steady state. Differently, the dynamic behaviour of unemployment, governed by the job flows in and out, is anyhow constrained by the matching technology, which does not allow jumps in job creation. Therefore, unemployment is the unique sticky variable of the model, because of the friction in the job creation process due to the matching technology.

Finally, from (12), (14) and (18) it can be easily seen that neither the job destruction condition, nor the job creation condition, nor the productivity equation, depends on sticky variables and so all these endogenous (R, θ, p) indeed do not exhibit transitional dynamics but must be on their steady state values even during the adjustments, being all the dynamics discharged on vacancies and unemployment. Notice that market tightness is still a jump variable even if unemployment is sticky, but this only because firms can adjust instantaneously the optimal vacancies during the transitional dynamics of unemployment. Therefore, with these premises it is natural to imagine the out-of-steady-state dynamics as a saddle path, with one stable root for unemployment and three unstable ones for the other endogenous[4].

[4] A much more rigorous analysis of the transitional dynamics in this kind of models has been pursued in Pissarides (1985 or 1990) and can be found also in Pissarides (1990). Nonetheless, here we follow the same line and arguments of Pissarides (2000).

4. Qualitative analysis

In this section we address the main question of the impact of EPL on steady-state and, in particular, on endogenous labour productivity. However, to highlight the relevance of the extension pursued in the paper, we start pre-emptively the analysis of the impact of F considering p a parameter and only subsequently we allow p to change.

Indeed, the impact of firing costs on job creation and job destruction, considering p a parameter, retraces basically the analysis of Pissarides (2000). From (12) and (14) we have that

$$\frac{\partial R}{\partial F} = -\frac{\partial B/\partial F}{\partial B/\partial R} = -\frac{r}{\left[\frac{r+\lambda G(R)}{r+\lambda}\right]} < 0 \tag{23}$$

$$\frac{\partial \theta}{\partial F} = -\frac{\partial D/\partial F}{\partial D/\partial \theta} = -\frac{(1-\beta)}{\frac{c}{q(\theta)^2}q'(\theta)} < 0 \tag{24}$$

As (23) and (24) show, firing costs reduce both R and θ. The impact on R is due to the fact that destroying a job is more costly, whereas the impact on θ is because, once a job is created, firm will pay sooner or later the firing costs and this reduces the expected profit from a new job. To get the equilibrium impact we need to consider the overall impact of F, so we differentiate (12) and (14) respectively as $B(R^*, \theta(R^*, F), p, F, \omega) = 0$ and $D(\theta^*, R(\theta^*, F), F, \omega) = 0$ and we get

$$\frac{\partial R^*}{\partial F} = -\frac{\partial B/\partial F}{\partial B/\partial R^*} = -\frac{\frac{\beta q(\theta)^2}{|q'(\theta)|}+r}{\left[\frac{r+\lambda G(R^*)}{r+\lambda}\right]-\left[\frac{\beta q(\theta)^2}{(r+\lambda)\,q'(\theta)}\right]} < 0 \tag{25}$$

$$\frac{\partial \theta^*}{\partial F} = -\frac{\partial D/\partial F}{\partial D/\partial \theta^*} = -\frac{(1-\beta)\left(\frac{r}{r+\lambda G(R)}-1\right)}{\frac{c}{q(\theta^*)^2}q'(\theta^*) - \frac{c\beta}{r+\lambda G(R)}} < 0 \tag{26}$$

Therefore, in equilibrium firing costs reduce both job destruction and job creation. In particular, the equilibrium impact on job destruction (25) is even stronger than the initial impact (23) because higher firing costs reduce market tightness and in turn wage, so less jobs are destroyed marginally (see (15) and (24)). On the other hand, the equilibrium impact on job creation (26) is weaker than the initial impact (24) because firing costs increases the duration of jobs and this partially attenuates the loss of the expected profit due to F (see (16) and (23)). The equilibrium impact is illustrated in Figure 4, where higher F shifts JD down and JC left. As the diagram shows, job destruction decreases unambiguously whereas the effect on job creation would seem ambiguous, but we know from (26) that job creation decreases as well.

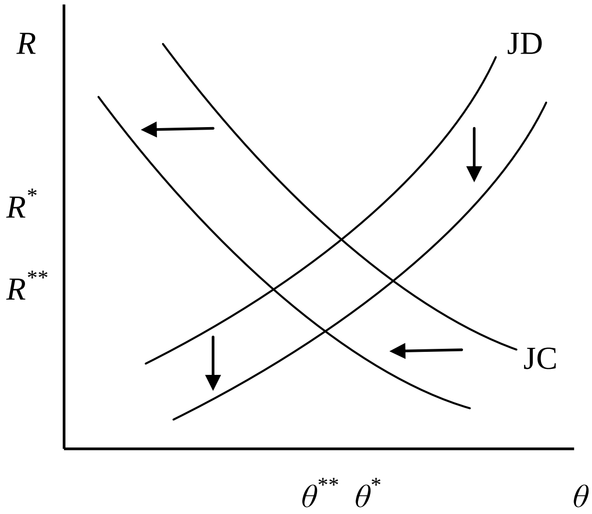

Figure 4

Impact of firing costs on reservation productivity and market tightness (with p fixed)

Because of the symmetric impact on job creation and job destruction, the impact of firing costs on unemployment in these models is usually ambiguous, as differentiation of (21) shows

$$\frac{\partial u}{\partial F} = \frac{\lambda f(R)\theta q(\theta)\frac{\partial R}{\partial F} - \lambda G(R) q(\theta)\xi\frac{\partial \theta}{\partial F}}{[\lambda G(R) + \theta q(\theta)]^2} = \gtreqless 0$$

The equilibrium impact is illustrated in Figure 5. Higher firing costs shift the Beveridge curve in and rotate the job creation line clockwise, therefore the impact on unemployment is ambiguous, but vacancy decreases unambiguously.

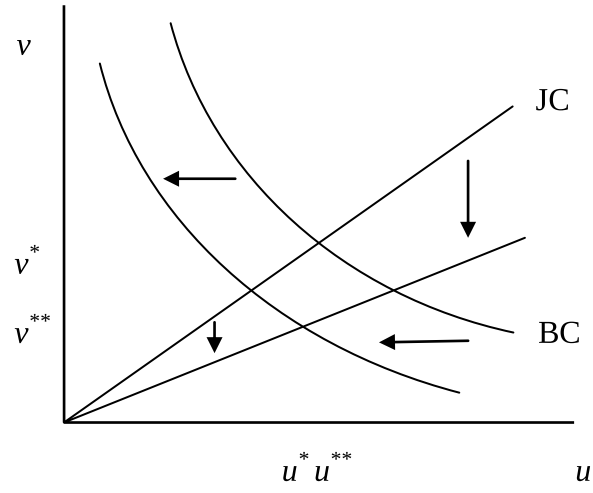

Figure 5

Impact of firing costs on unemployment and vacancies

So far we considered p a parameter unaffected by firing costs and basically we get the same results of the previous literature without significant novelty. Nonetheless, in our model labour productivity is an endogenous object, so now we allow p to respond to a change in F. Intuitively, we expect that firing costs affect in some way labour productivity for different reasons. Firstly, as (7) and (8) show firing costs affect directly the actual and future wage. Moreover, they affect indirectly wage through the probability of finding a job $\big(\theta q(\theta)\big)$. Finally, they influence the probability of being fired $\big(\lambda G(R)\big)$ affecting the continuation value of (17).

From (18) we have that the initial impact of firing costs on optimal productivity is null, that is

$$\frac{\partial p}{\partial F} = -\frac{\partial M/\partial F}{\partial M/\partial p} = -\frac{0}{-\gamma + \frac{\lambda G(R)}{r+\lambda G(R)}\left(\frac{z}{p^2}+\frac{\gamma}{2}\right)} = 0$$

The economic intuition of this result is that firing costs have a negative effect on the outside wage and a positive one on the inside wage, so in expectations these two impacts on the permanent income of a new worker compensate, as showed by (17). This interpretation is made evident by the difference between (17) and the permanent income of a worker inside a match

$$rW(x) = (1-\beta)z + \beta p\left(x + c\theta + rF - \frac{1}{2}\gamma p\right) + \frac{\lambda\beta p}{r+\lambda}\left[G\big(R(p)\big)R(p) + \int_{R(p)}^{\bar{x}} s\, dG(s) - x\right]$$

where firing costs certainly have a positive effect on wage and, in turn, on labour productivity. Thus, all the effect of F on p is induced by the impact on the other endogenous. And in fact, as long as R and θ do not vary there is no change on the continuation value and the permanent income of a new worker, so there is no impact on labour productivity. To get the equilibrium impact we differentiate (18) as $M(p^*, R(\theta, p^*, F), \theta(R, F), \omega) = 0$ and we get the following:

$$\frac{\partial p^*}{\partial F} = -\frac{\partial M/\partial F}{\partial M/\partial p^*} = -\frac{\frac{\partial M}{\partial R}\frac{\partial R}{\partial F}+\frac{\partial M}{\partial \theta}\frac{\partial \theta}{\partial F}}{\frac{\partial M}{\partial p^*}+\frac{\partial M}{\partial R}\frac{\partial R}{\partial p^*}} = -\frac{<0}{<0} \tag{27}$$

Proposition 1 – A higher level of EPL reduces the equilibrium labour productivity through the impact on reservation productivity and market tightness[5].

[5] At first sight, there might be an ambiguity on the denominator of (27). However, both graphical analysis and numerical computations with a large set of values, indeed the most plausible ones, unequivocally show that the (27) is negative.

The economic intuition of this result is the following. As (12) shows, labour productivity has a negative impact on reservation productivity through the *production effect*, therefore in the choice of the optimal p the *production effect* induces worker to choose marginally a higher p to shut down R. When we analyze the impact of firing costs on reservation productivity we can easily realise that the effect is of the same magnitude of the *production effect*. To see this point let multiply (12) for p and concentrate on the *production effect* and the *firing costs effect*, ignoring for a while the other elements

$$pR + rpF = 0 \tag{28}$$

As (28) shows, the *production effect* is partially substituted by the *firing costs effect* in lowering R and so a higher F, amplifying the relevance of the *disutility effect*, induces worker to choose marginally a lower p. Moreover, a higher F reduces θ and consequently both outside and inside wage, inducing worker to choose a lower p (see (24)). The equilibrium impact is illustrated in Figure 6, when a higher F shifts RE down and PE left.

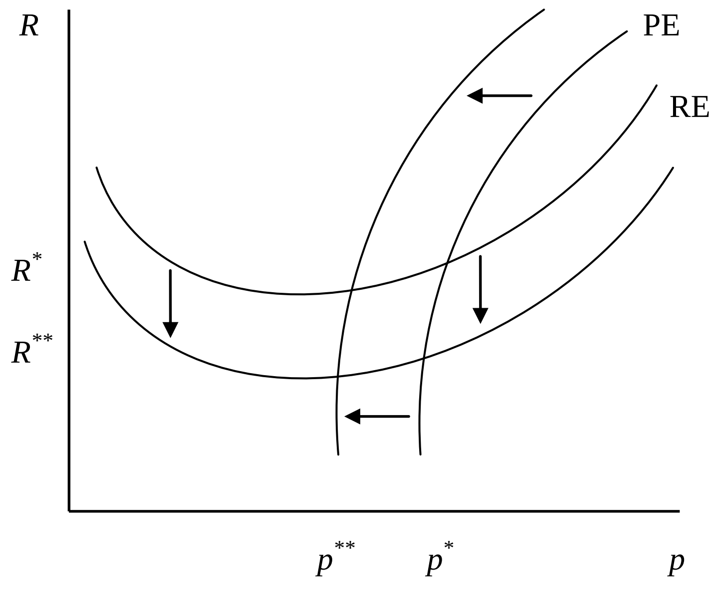

Figure 6

Impact of firing costs on reservation productivity and labour productivity

Considering p an endogenous object leads us to reassess the equilibrium impact of F on job creation and job destruction. In particular, now we differentiate (12) and (14) respectively as $B(R^*, \theta(R^*, F), p(R^*, \theta), F, \omega) = 0$ and $D(\theta^*, R(\theta^*, F, p(R, \theta^*)), F, \omega) = 0$ and we get

$$\frac{\partial R^*}{\partial F} = -\frac{\partial B/\partial F}{\partial B/\partial R^*} = -\frac{\frac{\partial B}{\partial \theta}\frac{\partial \theta}{\partial F} + \frac{\partial B}{\partial F}}{\frac{\partial B}{\partial R^*} + \frac{\partial B}{\partial \theta}\frac{\partial \theta}{\partial R^*} + \frac{\partial B}{\partial p}\frac{\partial p}{\partial R^*}} < 0 \tag{29}$$

$$\frac{\partial \theta^*}{\partial F} = -\frac{\partial D/\partial F}{\partial D/\partial \theta^*} = -\frac{\frac{\partial D}{\partial R}\frac{\partial R}{\partial F} + \frac{\partial D}{\partial F}}{\frac{\partial D}{\partial \theta^*} + \frac{\partial D}{\partial R}\frac{\partial R}{\partial \theta^*} + \frac{\partial D}{\partial R}\frac{\partial R}{\partial p}\frac{\partial p}{\partial \theta^*}} < 0 \tag{30}$$

where we can easily establish the following $|(29)| > |(25)|$ and $|(30)| < |(26)|$, that is:

Proposition 2 – Compared to the standard equilibrium with p as a parameter of the model, in the equilibrium with endogenous labour productivity EPL reduces even more job destruction, but reduces less job creation.

The economic intuition of this result is that, as we have seen before (28), the presence of a more stringent protection legislation reduces the role of the *production effect* and amplifies that of the *disutility-wage effect*, leading to a lower labour productivity which decreases both outside and inside wage and, in turn, the optimal reservation productivity. Consequently, lower job destruction increases the expected duration of job and partially attenuates the loss of the expected profit due to a more severe legislation, leading to a smaller reduction of job creation. The equilibrium impact on R and θ with endogenous labour productivity and the difference with p exogenous is illustrated in Figure 7.

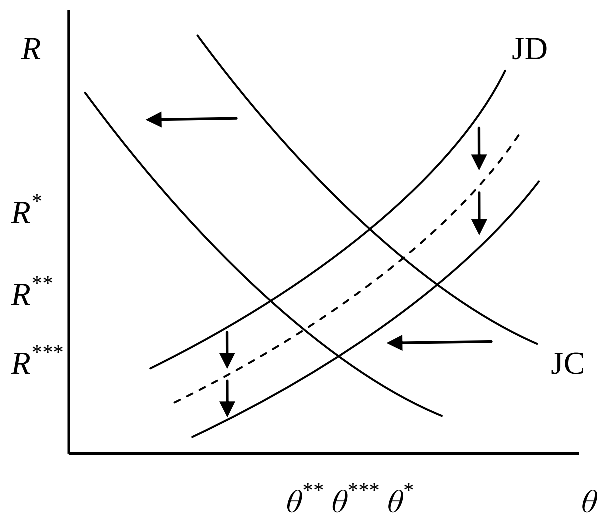

Figure 7

Impact of firing costs on reservation productivity and market tightness (with *p* endogenous)

In conclusion, considering p an endogenous variable changes only quantitatively the equilibrium impact of F on job creation and job destruction, but not the direction. However, firstly the extension of the model with endogenous labour productivity should be important per sé, especially in the light of the recent empirical evidence on the impact of EPL on labour productivity. Moreover, as will be clear in the next section, considering p an endogenous is very much relevant for the quantitative exercise and, in particular, for the welfare analysis and policy implications not only concerning EPL, but also for all kinds of policy evaluation.

5. Quantitative analysis

In this section we attempt a rough calibration of the model to evaluate quantitatively the impact of firing costs on labour market performance, but also on some measure of aggregate welfare. As usual in this literature, we adopt the following Cobb-Douglas matching function with constant returns to scale, generally the specification most suited to match the data on job creation (see e.g. Layard, Nickell and Jackman, 1991, for the UK; Blanchard and Diamond, 1989, for the US)

$$m(u,v) = Au^{\alpha}v^{1-\alpha}$$

The distribution of the idiosyncratic component of productivity is taken uniform over the support [0, 1][6], i.e. $G(x) = \left(x - \underline{x}\right)/\left(\overline{x} - \underline{x}\right) = x$. Following the literature, the baseline parameters reported in Table 1 are set so as to match some typical features of the empirical data (see e.g. Davis and Haltiwanger, 1992). The parameters of the matching function are set as usual at A = 0.15 and α = 0.5, close to empirical estimates. The workers' bargaining power is set at β = 0.5 equal to the elasticity of the matching function, so as to get constrained efficiency at least in the economy without firing costs. To generate in the simulation reasonable job flows, the arrival rate of the idiosyncratic shock is set λ = 0.081 (see Mortensen and Pissarides, 1994).

[6] Usually in the literature (with labour productivity exogenous) the idiosyncratic component is an additive component of total price and the distribution is taken uniform over $\left[\underline{x},\ \overline{x}\right]$, whit $\underline{x}$ a negative number. However, the level of labour productivity is fixed so that the total price is quite everywhere positive (see e.g. Mortensen and Pissarides, 1994, 1999a,b). In our model we make a preference for the interpretation of the idiosyncratic component as a multiplicative component of total price (see Lilien, 1982; Blanchard and Diamond, 1989; Pissarides, 2000), therefore we adopt a positive support for the distribution, so that the total price is always positive. Nonetheless, both the interpretations maintain the same mechanism underpinning the reservation productivity.

Table 1

Baseline parameters

A	α	β	Λ	r	z	c	γ	$[\underline{x}, \overline{x}]$
0.15	0.5	0.5	0.081	0.03	0.35	0.05	0.5	[0, 1]

Similarly, the preference parameter governing the disutility of effort is set at $\gamma = 0.5$, which induces an increasing disutility of effort but generates very reasonable values of utility and labour productivity. Finally, in our simulation we consider a semester as the unit of time and, accordingly, we set the interest rate at r = 0.03 (see e.g. Cahuc and Postel-Vinay, 2002).

In order to assess the impact of firing costs on labour market performance, we compute[7] different equilibrium of the model with *F* varying from 0 to 4. This should cover a significant range, from the *laissez–faire* case to the *substantial firing restrictions* case, where firing costs are more than three times the semester wage (see e.g. for Italy Garibaldi, 2006). In Table 2 we report the equilibrium values of unemployment rate, job flows, labour productivity, reservation productivity, market tightness and unemployment spell duration for different levels of firing restrictions.

Table 2

Impact of *F* on labour market equilibrium

	U	*JF*	*P*	*R*	θ	*ud*
F = 0	0.212	5.5	2.29	0.87	3.03	3.83
F = 1	0.205	4.9	2.09	0.77	2.57	4.15
F = 2	0.197	4.3	1.92	0.67	2.15	4.55
F = 3	0.188	3.7	1.76	0.56	1.75	5.03
F = 4	0.176	3.1	1.62	0.47	1.39	5.65

First, we can see that more stringent firing restrictions reduce significantly the equilibrium labour productivity. In particular, a level of firing costs equal to two times the wage (*F* = 2) is enough to reduce labour productivity more than 10% respect to the *laissez–faire* case, whereas in the *substantial firing restrictions* case the reduction is even of the 30%. Similarly, firing costs reduce both reservation productivity and market tightness and, in turn, job flows. As we can see, job flows in the *substantial firing restrictions* case are less than 60% of those in the

[7] Fix point algorithm written in Matlab available under request by the author.

laissez–faire case. Nonetheless, as standard in these models (e.g. Mortensen and Pissarides, 1999a), the overall impact on unemployment is positive, because the impact on job destruction overcomes that on job creation. It is worth noting as the difference in the level of job flows between the economies with low firing costs (5.5 – 4.9) and those with high firing costs (3.7 – 3.1), seems to match very reasonably the real data in the U.S., the quintessential frictionless country, and the European countries, where notoriously firing restrictions are consistent. Finally, mirror to the decrease on job creation, higher firing costs increase significantly the unemployment spell duration. In particular, in the *substantial firing restrictions* case the unemployment duration increases more than 50% respect to the *laissez–faire* case.

In Table 3 we show the equilibrium values of reservation productivity and market tightness in the model with p exogenous, along with the values for the complete specification. In the model with exogenous labour productivity, we set p at the equilibrium level get in the *laissez–faire* case (p = 2.29) and we do not allow p to respond to change in our policy tool F. In this way we make clear what happen to job creation and job destruction when we allow labour productivity to adjust optimally to change in firing costs. As we can see, this numerical exercise confirms exactly the result of the qualitative analysis (see (29), (30) and Figure 7). In particular, when we allow p to respond optimally to change in F, this leads to an even stronger reduction of the equilibrium reservation productivity, but to a smaller reduction of the equilibrium market tightness.

Table 3

Job creation and Job destruction with p fixed (p = 2.29)

	R (p = 2.29)	θ (p = 2.29)	R	θ
F = 0	0.87	3.03	0.87	3.03
F = 1	0.78	2.08	0.77	2.57
F = 2	0.69	1.26	0.67	2.15
F = 3	0.61	0.59	0.56	1.75
F = 4	0.53	0.11	0.47	1.39

Finally, to assess the impact of firing costs on well-being of the economy, we compute the value of different measures of aggregate welfare from the *laissez–faire* to the *substantial firing restrictions* case. In particular, we consider two main measures of aggregate welfare, the first concerning the production net of recruiting costs (Y – RC), the second the utility of agents

(*AWF*). Our consistent measures of production and aggregate utility in the economy are (see the Appendix for the derivation)

$$Y = u\,\theta q(\theta)\,p\overline{x}\left[1 + (1-\lambda)\left(1-u-u\,\theta q(\theta)\right)\right] + \left(1-u-u\,\theta q(\theta)\right)p\,E(x \mid x \geq R)$$
$$\left[\lambda + (1-\lambda)\left(1-u\,\theta q(\theta)\right)\right] \tag{31}$$

$$AWF = u\,\theta q(\theta)\,rW(\overline{x})\left[1 + (1-\lambda)\left(1-u-u\,\theta q(\theta)\right)\right] + \left(1-u-u\,\theta q(\theta)\right)rW\left(E(x \mid x \geq R)\right)$$
$$\left[\lambda + (1-\lambda)\left(1-u\,\theta q(\theta)\right)\right] + u\,rU \tag{32}$$

where $E(x \mid x \geq R)$ indicates the conditional expectation of x over the truncated distribution $[R, \overline{x}]$, that is

$$E(x \mid x \geq R) = \frac{\int_R^{\overline{x}} x\,g(x)\,dx}{G(\overline{x}) - G(R)}$$

In Table 4 we report the equilibrium values of these two measures of aggregate welfare for different levels of firing restrictions. Along with these main measures, we report some other index of well-being in the economy, as the permanent income of unemployed and employed worker in different conditions.

Table 4

Impact of *F* on aggregate welfare

	Y	*Y – RC*	*rW(1)*	*rW(E(x))*	rU	*AWF*
F = 0	1.70	1.62	0.73	0.71	0.69	0.71
F = 1	1.49	1.43	0.66	0.65	0.61	0.64
F = 2	1.30	1.26	0.58	0.59	0.55	0.58
F = 3	1.14	1.11	0.52	0.55	0.50	0.54
F = 4	1.01	0.98	0.48	0.52	0.46	0.50

As standard in the literature, firing restrictions reduce unambiguously all measures of aggregate welfare, regardless we think about well-being in terms of production or utility of agents[8]. This is not surprising, since we know that under restriction $\alpha = \beta$ the *laissez–faire* economy gets the

[8] For what concern the welfare measures in terms of utility we should remember that we have assumed that workers are risk neutral and impatient, which implies zero saving and full consumption. This is usually done in this literature to avoid to solve the consumption problem, so as we can work with the maximized Bellman equation to derive the steady-state equilibrium of the model. Nonetheless, to some extent such limitation should be taken in mind when we think about the policy implications of our results.

constrained efficiency. More interesting is the size of the reduction of production. In particular, a middle level of firing restrictions is sufficient to yields a production lower than 25% respect to the *laissez–faire* case, whereas in the *substantial firing restrictions* case the production is lower than 40%. Indeed, despite the negative impact of EPL on aggregate welfare is well-known (see e.g. Cahuc and Postel-Vinay, 2002), such worrying reduction in production is not standard:

Proposition 3 – Compared to the standard equilibrium with p as a parameter of the model, in the equilibrium with endogenous labour productivity EPL reduces even more the aggregate welfare, regardless we consider the well-being of the economy in terms of production or aggregate utility.

Nonetheless, hidden under this result there is exactly the negative impact of firing restrictions on labour productivity, which not only reduces the total production of the economy, but also the surplus from job matches and, therefore, the utility of agents. Unsurprisingly, the inclusion in the analysis of this element enriches the picture of our model and, certainly, tells us an alarming result we should worry about.

6. Conclusion

The matching model studied in this paper has revealed that, indeed, the level of labour productivity in the economy can be influenced by labour market policies usually implemented by governments. Stimulated by the recent empirical evidence, we have focused on EPL and have shown that a higher level of firing restrictions partially substitute high labour productivity in reducing job destruction and this, consequently, brings down the optimal level of productivity. Furthermore, the response of productivity to EPL reasonably affects the level of production and, in fact, numerical simulation of the model has shown that a higher level of firing costs induces a consistent reduction on production, beyond the standard reduction found in the literature. Moreover, despite the reduction on the disutility of effort, higher EPL reduce unambiguously our measures of aggregate welfare (*AWF*), inducing a worsening on the well-being of both employed and unemployed workers. Therefore, in the light of the predominant role of labour productivity growth in driving the income growth in the last twenty years (OECD, 2003, 2007), the result of this paper bring in a further element in support of the consolidated voice of the literature for a reduction of EPL especially in European countries.

To conclude, the extension of the endogenous labour productivity pursued in this paper allows us to rationalize within the already fruitful matching approach the well-established empirical evidence on the impact on EPL on labour productivity, which indeed assumes the appearance of a macro-stylized fact in the European economies and, thus, should be explained in a macro model of the labour market. On the other hand, the inclusion of the optimal workers' response to political tools should be a positive element for any other policy evaluations. In particular, including both optimal agents' responses and market outcomes, the matching approach might turn out to be an ideal framework to address crucial questions usually analyzed in microeconomic contexts, but that certainly present significant macro implications.

APPENDIX

Properties of the matching function

Proof of $\frac{\partial q(\theta)}{\partial \theta} < 0$.

$$q(\theta) = \frac{m(v,u)}{v} = \frac{m(v,u)/u}{v/u} = \frac{m(\theta,1)}{\theta}$$

$$\frac{\partial q(\theta)}{\partial \theta} = \frac{\partial\left[\frac{m(\theta,1)}{\theta}\right]}{\partial \theta} = \frac{\frac{\partial m(\theta,1)}{\partial \theta}\theta - m(\theta,1)}{\theta^2} = \frac{m(\theta,1)}{\theta^2}\left[\frac{\partial m(\theta,1)}{\partial \theta}\frac{\theta}{m(\theta,1)} - 1\right] = q(\theta)\left(\frac{\xi-1}{\theta}\right)$$

therefore, we have $\frac{\partial q(\theta)}{\partial \theta} < 0$ because $0 < \xi < 1$. ■

Proof of $\frac{\partial \theta q(\theta)}{\partial \theta} > 0$.

$$\theta q(\theta) = \frac{m(v,u)}{u} = m(\theta, 1)$$

$$\frac{\partial \theta q(\theta)}{\partial \theta} = \frac{\partial m(\theta,1)}{\partial \theta}$$

therefore, we have $\frac{\partial \theta q(\theta)}{\partial \theta} > 0$ because the matching function is increasing in both arguments. ■

Proof of $\eta = \xi - 1$.

$$\frac{\partial q(\theta)}{\partial \theta} = q(\theta)\left(\frac{\xi-1}{\theta}\right) \quad \Rightarrow \quad \frac{\partial q(\theta)}{\partial \theta}\frac{\theta}{q(\theta)} = \xi - 1 \quad \Rightarrow \quad \eta = \xi - 1.$$ ■

Outside wage equation (7)

The sharing rule implies that in equilibrium the outside wage solves

$$W(\bar{x}) - U = \beta[J(\bar{x}) - V + W(\bar{x}) - U]$$

which gives us

$$J(\bar{x}) = \frac{(1-\beta)}{\beta}[W(\bar{x}) - U] \tag{33}$$

or

$$\beta r J(\bar{x}) - (1-\beta) r W(\bar{x}) + (1-\beta) r U = 0 \tag{34}$$

where we have used the equilibrium zero–profit condition (3).

Similarly, the sharing rule states that inside a match in equilibrium has to hold

$$W(x) - U = \beta[J(x) - V + pF + W(x) - U]$$

which gives us

$$J(x) = \frac{(1-\beta)}{\beta}[W(x) - U] - pF \tag{35}$$

From the asset value of a filled job (2) we have that

$$\beta r J(\bar{x}) = \beta p\bar{x} - \beta w_0 + \beta\lambda \int_R^{\bar{x}} \frac{(1-\beta)}{\beta}[W(s) - U]\, dG(s) - \beta\lambda \int_R^{\bar{x}} pF\, dG(s) - \beta\lambda G(R) pF -$$

$$\beta\lambda \frac{(1-\beta)}{\beta}[W(\bar{x}) - U] \tag{36}$$

where we have used (33) and (35).

Similarly, from the asset value of employed worker (5) we have that

$$(1-\beta) r W(\bar{x}) = (1-\beta) w_0 - (1-\beta)\frac{1}{2}\gamma p^2 + (1-\beta)\lambda \int_R^{\bar{x}} W(s)\, dG(s) + (1-\beta)$$

$$\lambda G(R) U - (1-\beta)\lambda W(\bar{x}) \tag{37}$$

where we have used the productivity relation $p = e$.

Using (36) and (37) we have that

$$\beta r J(\bar{x}) - (1-\beta) r W(\bar{x}) = \beta p\bar{x} - w_0 + (1-\beta)\frac{1}{2}\gamma p^2 - \beta\lambda pF$$

and knowing that in equilibrium (34) has to hold, we have that the outside wage solves

$$\beta p\bar{x} \ - w_0 \ + (1 - \ \beta)\frac{1}{2}\gamma p^2 - \beta\lambda pF + (1 - \ \beta)rU = 0 \tag{38}$$

From the asset value of unemployed worker (4) we have that

$$rU = z + \theta q(\theta)[W(\bar{x}) - \ U] = z + \frac{\beta}{(1-\beta)}\theta q(\theta)J(\bar{x}) = z + \frac{\beta}{(1-\beta)}pc\theta \tag{39}$$

where we have used first (33) and then the zero–profit condition (3).

Finally, we substitute (39) in (38) and get the outside wage equation (7)

$$w_0 = (1 - \ \beta)\left(z + \frac{1}{2}\gamma p^2\right) + \beta p(\bar{x} \ + c\theta \ - \lambda F) \qquad \blacksquare$$

Starting from the sharing rule inside a match, same calculation gives the inside equation (8).

Worker permanent income at the upper support of the price distribution (17)

There are different ways in which the permanent income equation (17) can be derived using the equilibrium conditions, here we show one of these which allow us to establish different interesting relations.

First from the asset value of unemployed worker (4) we have that

$$U = \frac{z}{r + \theta q(\theta)} + \frac{\theta q(\theta)}{r + \theta q(\theta)} \ W(\bar{x}) \tag{40}$$

From the asset value of employed worker (5) we have that

$$(r + \ \lambda)W(x) = \ w(x) - \frac{1}{2}\gamma p^2 + \ \lambda \int_R^{\bar{x}} W(s)dG(s) + \ \lambda G(R)U \tag{41}$$

Evaluating (41) at the upper support of the price distribution and at the reservation productivity

$$(r + \ \lambda)W(\bar{x}) = \ w_0 - \frac{1}{2}\gamma p^2 + \ \lambda \int_R^{\bar{x}} W(s)dG(s) + \ \lambda G(R)U \tag{42}$$

$$(r + \ \lambda)W(R) = \ w(R) - \frac{1}{2}\gamma p^2 + \ \lambda \int_R^{\bar{x}} W(s)dG(s) + \ \lambda G(R)U \tag{43}$$

Now subtracting (43) from (42) and using the reservation property *W(R) = U* we get

$$(r + \ \lambda)[W(\bar{x}) - \ W(R)] = \ \beta p(1 - R) - \beta pF(r + \ \lambda)$$

$$W(\bar{x}) - U = \beta p\left[\frac{(1-R)}{(r+\lambda)} - F\right] \tag{44}$$

Substituting (40) in (44) we obtain

$$rW(\bar{x}) = z + \left(r + \theta q(\theta)\right)\beta p\left[\frac{(1-R)}{(r+\lambda)} - F\right] \tag{45}$$

Similarly, subtract (43) from (41) to get

$$W(x) - U = \beta p\frac{(x-R)}{(r+\lambda)}$$

and now substitute (40) and use (45) to obtain

$$rW(x) = rW(\bar{x}) - r\beta p\left[\frac{(1-x)}{(r+\lambda)} - F\right] \tag{46}$$

This expression is extremely interesting because establishes the relation between the permanent income of a new worker at the upper support of the price distribution and that of a generic employed worker. In particular, it says that when firing costs are low the permanent income of a generic worker is always lower than that of a new worker, being the difference due to the different level of the idiosyncratic productivity. However, when firing costs are high the advantage of being already inside a match, which leads to a higher wage (see (7) and (8)), overturns the relation in favour of the generic worker. Indeed, this is exactly what we observe with the numerical simulation of the model in Table 4.

Finally, insert (46) in the integral expression of the asset value of a new worker to get

$$rW(\bar{x}) = w_0 - \frac{1}{2}\gamma p^2 + \lambda\int_R^{\bar{x}}\left\{W(\bar{x}) - \beta p\left[\frac{(1-s)}{(r+\lambda)} - F\right]\right\}dG(s) + \lambda G(R)U - \lambda W(\bar{x})$$

$$= w_0 - \frac{1}{2}\gamma p^2 - \lambda G(R)[W(\bar{x}) - U] + \frac{\lambda\beta p}{(r+\lambda)}\left[\int_R^{\bar{x}} s\, dG(s) - \left(1 - G(R)\right)\right] + \lambda\beta pF\left(1 - G(R)\right)$$

and now using (44) and substituting the outside wage equation (7) gives us (17)

$$rW(\bar{x}) = (1-\beta)z + \beta p\left(\bar{x} + c\theta - \frac{1}{2}\gamma p\right) + \frac{\lambda\beta p}{r+\lambda}\left[G\left(R(p)\right)R(p) + \int_{R(p)}^{\bar{x}} s\, dG(s) - \bar{x}\right] \quad \blacksquare$$

Similarly, inserting (46) in the asset value of the generic worker gives his permanent income.

Total production (31) and aggregate welfare function (32)

In equilibrium there are *(1 – u)* producing workers, who differ only for the level of the idiosyncratic productivity *x*. Among these $u\theta q(\theta)$ workers are in the first period of employment, therefore produce at the upper support of the price distribution $\bar{x}$. Instead, the other $(1 - u - u\theta q(\theta))$ workers were employed already the previous period and indeed their level of *x* is not the same for all of them. In particular, a fraction λ faced a productivity shock and changed the level of *x* in a new value between $\bar{x}$ and *R*, whereas the complement $(1 - \lambda)$ maintained the same level of the previous period. In turn, among these old workers maintaining the level of *x*, a fraction $u\theta q(\theta)$ entered two period ago and therefore produce at the upper support of the price distribution $\bar{x}$, whereas the others $(1 - u\theta q(\theta))$ entered more than two period ago and indeed we should distinguish again between those who faced a productivity shock and those who not and so forth. Therefore, the total production is

$$Y = u\,\theta q(\theta)\,p\bar{x} + \big(1 - u - u\,\theta q(\theta)\big)\big\{\lambda\,pE(x \mid x \geq R) + (1 - \lambda)\big[u\,\theta q(\theta)\,p\bar{x} +$$
$$1{-}u\,\theta q\theta\,\lambda\,pEx\ \ x{\geq}R + 1{-}\lambda\,u\,\theta q\theta\,px + 1{-}u\,\theta q\theta\ldots\ldots\ldots$$

As intuitive, the precise computation of the level of idiosyncratic productivity of producing workers in steady state is troubling, due to the recursive computation. Nonetheless, given that our aim is to evaluate the impact of firing restrictions on total production, it would be harmless to make an assumption to simplify the computation which affects in the same way the value of production between the *laissez–faire* and the *substantial firing restriction* case. Obviously, more an employed worker is old higher is the probability that he faced a productivity shock and changed his level of *x*. For simplicity, in (31) we assume that all workers older than two periods faced a productivity shock. Therefore, our measure of total production is

$$Y = u\,\theta q(\theta)\,p\bar{x} + \big(1 - u - u\,\theta q(\theta)\big)\big\{\lambda\,pE(x \mid x \geq R) + (1 - \lambda)\big[u\,\theta q(\theta)\,p\bar{x} +$$
$$1{-}u\,\theta q\theta\,pEx\ \ x{\geq}R$$

which after some easy algebra gives us (31). ■

Moreover, to check if our assumption is really harmless for our purpose, we repeated a similar numerical exercise of Table 4 when we derived the total production assuming that all workers older than three periods faced a productivity shock. In this case the total production is

$$Y = u\,\theta q(\theta)\,p\bar{x} + \left(1 - u - u\,\theta q(\theta)\right)\left\{\lambda\,pE(x \mid x \geq R) + (1 - \lambda)\left[u\,\theta q(\theta)\,p\bar{x} + \right.\right.$$
$$1{-}u\,\theta q\theta\,\lambda\,pEx\;\;x{\geq}R + 1{-}\,\lambda\,u\,\theta q\theta\,px + 1{-}u\,\theta q\theta\,pEx\;\;x{\geq}R$$

The conclusion was that as intuitive the value of production was slightly higher, but there was no difference on the impact of firing restrictions on total output, which led us to assess our assumption as innocuous for our purpose.

Similarly, the aggregate welfare function is the weighted sum of utility of the different workers in steady state, knowing that the utility of worker depends on the idiosyncratic component of productivity. Following the identical argument of before, in equilibrium there are u unemployed worker, $u\theta q(\theta)$ workers in the first period of employment enjoying the utility at the upper support of the price distribution $\bar{x}$, $(1 - u - u\theta q(\theta))$ old workers. Among these, a fraction λ faced a productivity shock and enjoys the utility between $\bar{x}$ and R, whereas the complement $(1 - \lambda)$ maintained the same utility of the previous period and, in particular, a fraction $u\theta q(\theta)$ entered two period ago and enjoys the utility at $\bar{x}$, whereas the others ($1 - u\theta q(\theta)$) entered more than two period ago and so forth. As the total production, in (32) we maintain the assumption that all workers older than two periods faced a productivity shock. Therefore, the aggregate welfare function is

$$AWF = u\,rU + u\,\theta q(\theta)\,rW(\bar{x}) + \left(1 - u - u\,\theta q(\theta)\right)\left\{\lambda\,rW\left(E(x \mid x \geq R)\right) + \right.$$
$$1{-}\,\lambda\,u\,\theta q\theta\,rWx + 1{-}u\,\theta q\theta\,rWEx\;\;x{\geq}R$$

which after some easy algebra gives us (32). ■

BIBLIOGRAPHY

Autor, D., Donohue, J. and Schwab, S.J. (2006), 'The Costs of Wrongful-Discharge Laws', *Review of Economics and Statistics*, vol. 88 (2), pp. 211-231.

Autor, D., Kerr, W. and Kugler, A. (2007), 'Does employment protection reduce productivity? Evidence from US states', *Economic Journal*, vol. 117, pp. F189–F117.

Bartelsman, E.J. and Hinloopen, J. (2005), 'Unleashing animal spirits: investment in ICT and economic growth', in L. Soete and B. ter Weel (eds.), *The Economics of the Digital Society*, Cheltenham, UK and Northampton, MA, USA Edward Elgar.

Bassanini, A., Nunziata, L. and Venn, D. (2009), 'Job protection legislation and productivity growth in OECD countries', *Economic Policy*, vol. 24 (58), pp. 349–402.

Bassanini, A. and Venn, D. (2007), 'Assessing the Impact of Labour Market Policies on Productivity: A Difference-in-Differences Approach', *OECD Social Employment and Migration Working Papers*, No. 54, OECD Publishing.

Bentolila, S. and Bertola, G. (1990), 'Firing costs and labour demand: How bad is eurosclerosis', *Review of Economic Studies*, vol. 57 (3), pp. 381–402.

Bentolila, S. and Dolado, J. (1994), 'Labour flexibility and wages: lessons from Spain', *Economic Policy*, vol. 18, pp. 55-99.

Bentolila, S. and Saint-Paul, G. (1992), 'The macroeconomic impact of flexible labor contracts, with an application to Spain', *European Economic Review*, vol. 36, pp. 1013– 1053.

Bertola, G. (1990), 'Job security, employment and wages', *European Economic Review*, vol. 34 (4), pp. 851–886.

Blanchard, O.J. and Diamond, P.A. (1989). 'The Beveridge curve', *Brookings Papers on Economic Activity*, vol. 1, pp. 1 – 76.

Boeri, T. and Garibaldi, P. (2007), 'Two Tier Reforms of Employment Protection Legislation. A Honeymoon Effects', *The Economic Journal*, vol. 117, pp. 357–385.

Cahuc, P. and Postel-Vinay, F. (2002), 'Temporary jobs, employment protection and labour market performance', *Labour Economics*, vol. 9(1), pp. 63 – 91.

Davis, S.J. and Haltiwanger, J.C. (1992), ‘Gross Job Creation, Gross Job Destruction and Employment Reallocation’, *The Quarterly Journal of Economics*, August, vol. 107 (3), pp. 819–863.

DeFreitas, G. and Marshall, A. (1998), ‘Labour Surplus, Worker Rights and Productivity Growth: A Comparative Analysis of Asia and Latin America’, *Labour*, vol. 12 (3), pp. 515–539.

Garibaldi, P. (1998), ‘Job Flow Dynamics and Firing Restrictions’, *European Economic Review*, vol. 42 (2), pp. 245–275.

Garibaldi, P. (2006), ‘*Personnel Economics in Imperfect Labour Markets*’, Oxford: Oxford University Press.

Hopenhayn, H. and Rogerson, R. (1993), ‘Job Turnover and Policy Evaluation: A General Equilibrium Analysis’, *The Journal of Political Economy*, vol. 101 (5), pp. 915–938.

Ichino, A. and Riphahn, R.T. (2005), ‘The Effect of Employment Protection on Worker Effort: A Comparison of Absenteeism During and After Probation’, *Journal of the European Economic Association*, vol. 3 (1), pp. 120–143.

Layard, R., Nickell, S. and Jackman, R. (1991), ‘*Unemployment: Macroeconomic Performance of the Labour Market*’, Oxford: Oxford University Press.

Lilien, D. (1982), ‘Sectoral shifts and sectoral unemployment’, *Journal of Political Economy*, vol. 90, pp. 777–793.

Lisi, D. (2009), ‘The Impact of Temporary Employment on Labour Productivity: Evidence from an Industry–Level Panel of EU Countries’, *mimeo*.

Micco, A. and Pages, C. (2006), ‘The Economic Effects of Employment Protection: Evidence from International Industry-Level Data’, IZA Discussion Paper No. 2433.

Mortensen, D.T. and Pissarides, C.A. (1994), ‘Job creation and job destruction in the theory of unemployment’, *Review of Economic Studies*, vol. 61, pp. 397–415.

Mortensen, D.T. and Pissarides, C.A. (1999a), ‘New developments in models of search in the labour market’, *Handbook of Labour Economics*, eds. O. Ashenfelter and D. Card, Amsterdam: North–Holland.

Mortensen, D.T. and Pissarides, C.A. (1999b), ‘Unemployment responses to “skill-biased” shocks: The role of labour market policy’, *Economic Journal*, vol. 109, pp. 242–265.

OECD (2003), *The Sources of Economic Growth in OECD Countries*, OECD, Paris.

OECD (2007), *OECD Employment Outlook*, OECD, Paris.

Pissarides, C.A. (1985), ‘Short-run equilibrium dynamics of unemployment, vacancies and real wages’, *American Economic Review*, vol. 75, pp. 676–690.

Pissarides, C.A. (1990), ‘*Equilibrium Unemployment Theory’*, Oxford: Basil Blackwell.

Pissarides, C.A. (2000), ‘*Equilibrium Unemployment Theory’*, 2nd edn, The Mit Press, Cambridge, MA.

Riphahn, R.T. (2004), ‘Employment Protection and Effort Among German Employees’, *Economics Letters*, vol. 85 (3), pp. 353–357.

Saint-Paul, G. (1997), ‘Is Labour Rigidity Harming Europe’s Competitiveness? The Effect of Job Protection on the Pattern of Trade and Welfare’, *European Economic Review*, vol. 41 (3–5), pp. 499–506.

Saint-Paul, G. (2002), ‘Employment Protection, International Specialization and Innovation’, *European Economic Review*, vol. 46 (2), pp. 375–395.

Shapiro, C. and Stiglitz, J.E. (1984), ‘Equilibrium Unemployment as a Worker Discipline Device’, *The American Economic Review*, vol. 74 (3), pp. 433–444.