On the efficiency properties of the Roy's model under asymmetric information

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Abstract

We consider Roy's economies with perfectly competitive labor markets and asymmetric information. Firms choose their investments in physical capital before observing the characteristics of the labor markets they will face. We provide conditions under which equilibrium allocations are constrained Pareto efficient, i.e., such that it is impossible to improve upon the equilibrium allocation by changing agents' investments and letting the other endogenous variables adjust to restore market clearing. We also provide a robust example of a class of economies where these conditions fail and where equilibria are characterized by overinvestments in high skills.

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JEL classification: D60, D82, J24

1 Introduction

The Roy's model provides a natural setting for the analysis of many labor market phenomena. Its key feature is the emphasis on the role of workers' comparative advantages in different jobs. This allows for a richer set of implications, compared to the ones obtainable in pure efficiency unit models. Additionally, and closer to the issue discussed in this paper, as soon as we move outside the class of perfect market economies, the Roy's model may have welfare properties, and - consequently - policy implications, which are sharply different from the ones obtained in pure efficiency units economies. From this viewpoint, the key question is how the equilibrium choices at the extensive margin are determined, and how they interact with the optimal choices at the intensive margin in delivering the welfare properties of equilibria.

This issue has been studied in several papers. To relate it to the framework considered here, it is convenient to focus on the simplest example. Let's first look at a pure efficiency unit model. Consider a two-period economy where firms choose their investments in physical capital ex-ante, without knowing exactly the wage rate that will prevail in the labor market that they will face in the next period. This may happen, for instance, because investments in human capital (HC from now on) depend upon the realization of some random variable not perfectly observed by the firms when they choose their investments. In the second period, perfectly competitive labor markets open and clear at the equilibrium wages. It is easy to see that, in a pure efficiency unit model, the equilibrium allocation is constrained Pareto optimal. By this, we mean that it is impossible to improve upon this allocation by choosing appropriately the individual investments and letting the endogenous variables adjust to restore market clearing. Consider now a similar set-up in a Roy's model, where there are two industries. In one, firms use high skill labor. In the other, they use low skill workers. As before, investments in physical capital are selected ex-ante, and depend upon the distribution of equilibrium wages. Also, assume that profits are increasing in the average level of HC of the two types of workers. In this set up, it is easy to construct examples where equilibria are constrained inefficient. The argument goes as follows: at the equilibrium, workers are endogenously partitioned into two subsets, defined by the type of HC they have invested in. Suppose that the agents investing in high skills are the ones with a relatively low cost of their effort in acquiring HC. Then, a marginal change in the partition, reducing the *size* of the set of agents investing in high skills, simultaneously increases the *average level* of HC of both low and high skilled workers. This increases the optimal level of the investments in physical capital in both industries and it may very well be Pareto improving. Since welfare increases shrinking the size of the set of agents with high skills, inefficiency is due to overeducation. An example of this sort is analyzed in detail in Mendolicchio, Paolini, and Pietra (2012b). Results which are, essentially, in the same spirit arise in several, different set-ups. For instance, Charlot and Decreuse (2005) consider a two-sector economy with matching frictions. Firms create vacancies for jobs using either high or low skill labor. Workers optimally choose to enter one of the two labor markets. Under the assumption of complementarity between innate ability and education, high ability workers are the ones investing in high skills at the equilibrium. The authors show that equilibria are characterized by overinvestment in high skills. Their analysis is generalized in Mendolicchio, Paolini, and Pietra (2012a). In this framework, the creation of vacancies in the two sectors plays essentially the same role of the investments in physical capital in the previous example with frictionless labor markets. Again, the key feature of the economy is that the distribution of HC in the two labor markets matters. If the threshold defining the partition of the workers into the two skills moves up, i.e., if some agents switch from the high to the low skill labor market, this increases the expected HC of workers in both markets and, consequently, the vacancy creation. This, in turn, has a positive welfare effect. The different welfare properties of pure efficiency units and Roy's models can also be verified in the set-up proposed by Acemoglu (1996). He considers a model with full employment and where wages are determined by bargaining between workers and firms, using as equilibrium concept the Nash bargaining solution with exogenous weights. In a pure efficiency unit model, he shows that, at the equilibrium, undereducation always holds. As in the previous, perfectly competitive, example, Mendolicchio, Paolini, and Pietra

(2014) establish that the nature of the inefficiency can be reversed, once workers' choices at the extensive margin are also taken into account.

These results are obtained in different classes of economies, but they all share two basic features: First, some variables - investments in physical capital or vacancies - are selected ex-ante by the firms considering the equilibrium distribution of some variables related to the labor supply in the two markets. Workers self-select into one of two labor markets by investing in HC. A change in the equilibrium threshold modifies, at the same time, the distribution of the labor supply in both labor markets. This affects the optimal value of the predetermined variable, e.g., firms' investments, and it may induce a welfare improvement. Another framework where, essentially, the same phenomenon may take place is given by economies where peer effects are relevant for the investments in HC.

In this paper, we extend the analysis of the efficiency properties of the Roy's model of investments in HC asking the following question. Consider an economy where labor markets are perfectly competitive, but investments in physical capital are selected ex-ante, before the random variable affecting investments in HC realizes. Let's define an equilibrium allocation to be constrained efficient if it is impossible to improve welfare by changing the profile of the investments at either margins and letting the other equilibrium variables adjust to restore market clearing. Under which conditions the equilibrium allocation is constrained Pareto optimal? The bottom line is that this property is guaranteed provided that the equilibrium partition of workers is state-contingent. As soon as we depart from this property, we can construct robust examples of economies such that constrained efficiency fails.

The results proposed in this paper are, we believe, interesting for at least two different reasons. First, they identify the basic features of the economy determining its constrained efficiency properties. This helps to put in a proper perspective the previous results obtained in the literature. Moreover, they can be immediately applied to many other classes of economies with similar structures and properties. Secondly, they can contribute indirectly to the literature on optimal taxation in Roy's models, which has known important developments in the last few years (see, Saez (2004), and Rothschild and Scheuer (2012, 2014)).

The structure of the paper is the following. Next section presents the main, common features of the two classes of economies that we are going to study. Section 3 introduces conditions which are sufficient to guarantee that each equilibrium allocation is constrained Pareto optimal. Section 4 specifies a set of conditions under which equilibrium allocations are typically constrained inefficient. Some conclusions follow in Section 5.

2 General framework

We start with an outline of the economies that we are going to analyze. The simplest way to capture their essential features is to consider a collection of labor markets. Each "local" labor market can be thought of as an island composed by a continuum of workers. They are heterogeneous according to some parameter δ affecting their optimal investments in HC. By assumption, the higher δ, the lower their utility cost of acquiring HC. Hence, we will occasionally refer to δ as an index of innate ability. In the sequel, we will use a superscript $s = e$ to refer to workers investing in high skills, a superscript $s = ne$ to refer to low skill workers.

The characteristics of these islands will change during the discussion. Indeed, we will consider two cases. In the first, the distribution of the parameter δ will be identical across islands, while, potentially, the individual choices will differ across them due to the island-specific realization of some random variable, specifically, the direct costs of education, described by a r.v. $\widetilde{T}$ uniformly distributed on $[\underline{T}, \overline{T}]$. In the second case, workers will be identical in each island (i.e., they will have the same δ), but differ across islands, that will then be indexed by the associated value $\delta \in [\underline{d}, \overline{d}]$. Hence, islands will differ for two possible reasons: (a) the island-specific realization of some r.v. affecting the investments in HC; (b) the innate ability of their workers. As we will see, equilibria of these two types of economies have completely different efficiency properties.

In both cases, on the producers' side, there is a large number of perfectly competitive firms, endowed with the same technology.[1]

All sets of agents are endowed with the Lebesgue measure and, whenever we state that a set is measurable, we mean that it is measurable with respect to this measure.[2]

The economy lasts two periods. In the first, each firm is matched with some labor markets, i.e., with one, or two, islands. At this stage, firms also choose their investments in physical capital, knowing the distribution of HC, but without knowing their precise realizations in the labor markets they are facing. To be more explicit, in case (a), they do not know the realization of the r.v. $\widetilde{T}$ and, therefore, the partition of the workers. If (b), firms do not know the realizations of δ for the two islands corresponding to the two labor markets they will deal with.[3] Firms choose their investments maximizing their expected profits.

In the second period, the actual HC of each worker becomes observable, competitive spot labor markets open and clear, and production takes place. Ex-post, HC is perfectly observable, and, therefore, there is no signalling component in the workers' behavior. This allows us to focus on a two-period model without any loss of generality.

Given that the distinction between the two classes of economies is crucial, it may be worthwhile to reformulate the point once again: first, firms choose their investments in physical capital. For the second stage, we consider two

[1] We introduce a continuum of identical firms matched with each pair (high and low skills) of labor markets only to justify the assumption of perfect competition and because it is important to keep clear the distinction between individual and aggregate investments in physical capital.

[2] Most of our statements should be qualified, specifying that they hold *a.e.*, i.e., for all the agents, but, possibly, for some set of Lebesgue measure zero. In most instances, we will omit this qualification to streamline the presentation.

[3] The full description of the economy considered under case (b) is postponed to Section 4.

possibilities:

(a) Each firm is matched with a single island, whose population is endogenously partitioned into two measurable subsets of agents, $\{\Delta^{ne}(T), \Delta^{e}(T)\} \equiv \Delta(T)$. Islands are different because the realizations of the r.v. $\widetilde{T}$ are so.

(b) Firms are matched with a pair of labor markets, each one of them characterized by a value δ^s, $s = ne, e$, invariant across workers of the same island. Firms know the equilibrium partition, but they do not know the actual realization $(\delta^{ne}, \delta^{e})$.

In both cases, workers observe the investments in physical capital before choosing type and level of their own HC.

In (a), we use $\widetilde{T}$ as the r.v. whose realizations differ across islands. To introduce other sources of uncertainty, related to preferences, technology, or to the support of δ, would require a heavier notation without adding any substantive insight. In fact, it is fairly intuitive, and it can be formally shown, that, even with these sources of uncertainty, all the results concerning efficiency of equilibria would be identical to the ones established below.

Concerning the distinction between high and low skill HC, from the point of view of workers, they differ because of the possible differences in their wages and in their direct cost. From the point of view of the firms, the two skills are simply different inputs in the production process. For some parametric classes of production processes, the distinction between them translates immediately into properties of the production function.[4] This may not be true for general production function. The fundamental feature is that the two skills are not perfectly fungible, i.e., that they enter the production function as different inputs. We may actually define the high skill HC as the one with the higher expected wage per efficiency unit of labor, because, at equilibrium, the expected unit wage of the costly skill must always be higher than the one of the cost-free skill, otherwise no worker would acquire it.

2.1 Individual behavior

Each worker is endowed with one unit of time that he/she inelastically supplies. This unit of time is converted into h^s units of HC of skill s, where h^s depends upon the worker's effort. Once acquired, HC of type s converts 1-to-1 into efficiency units of labor supply of type s. Hence, workers make a choice at both margins, intensive and extensive. To invest in high skills also entails a fixed cost T, perfectly observed by the workers when they choose their investments.

In general, their preferences are described by a utility function $u\left(c, h; \delta\right)$ where c is consumption and h is the amount of HC, or, more properly, the effort applied to acquire HC. The proofs of our efficiency results, the core of the paper, are drastically simplified by imposing some very standard, but strong, restrictions on the utility functions.

[4] Later on we will consider the production function $F_j(.) = Ak_j^{\alpha}\{\phi^{ne}\ell_j^{ne\,\theta} + \phi^{e}\ell_j^{e\,\theta}\}^{\frac{1-\alpha}{\theta}}$. Here, it is natural to assume that $\phi^e > \phi^{ne}$, so that, at each $\ell_j^e = \ell_j^{ne}$, the marginal product of high skill labor is higher than the one of low skilled labor.

Assumption U: For each δ, preferences are described by a strictly concave, C^2 utility function $u(c,h;\delta) \equiv v(c) - \frac{g(h)}{f(\delta)}$. $f(\delta)$ is strictly increasing. Moreover, $-\frac{\frac{\partial^2 v(.)}{\partial c^2}c}{\frac{\partial v(.)}{\partial c}}$ is "sufficiently" small.

With separability, the meaning of the parameter δ is very transparent: $f(\delta)$ determines the marginal rate of substitution between consumption and effort, and, therefore, the optimal investment in HC. Given the wage rate, since $f(\delta)$ is increasing in δ, the higher δ, the higher the investment in HC.[5]

Assumption (U) is standard and it guarantees that the labor supply is increasing in the wage rate. To obtain this result for unskilled labor, the measure of the curvature of $v(.)$ (in fact, its Arrow-Pratt measure of relative "risk-aversion") must be below 1.[6] A slightly stronger condition is necessary to get the same property for skilled labor.[7]

We also assume that, in case (a), i.e., when workers are partitioned into the two skill types on each island, there is a continuum of identical individuals, denoted by $i \in [0,1]$, for each δ. Hence, the set of agents will be described by a square $[0,1] \times [\underline{d}, \overline{d}]$, endowed with the Lebesgue measure.[8] Each agent will be identified with a pair (δ, i). This is because, in Roy's models, the choice at the extensive margin induces a lack of convexity of the individual demand correspondence at the critical wage configurations where a worker switches from one type of skill to the other. To consider a continuum of identical workers for each type δ allows us to deal in a straightforward way with this lack of convexity.

Let $w(.) = \{w^{ne}(.), w^e(.)\}$ be a wage map and $\Delta(.)$ be a measurable partition of the set of workers.

We can describe workers' behavior as follows. First, given the realization T and the wage map, each worker solves, for each s, the optimization problem

$$\max_{h^s} u(c^s, h^s; \delta), \qquad (U^s)$$

with $c^{ne} = w^{ne}h^{ne}$ and $c^e = w^e h^e(.) - T$.

Given s, let $\widetilde{h}^s(w^s;\delta,T)$ be the supply of HC of agent i with skill s, and $V^s(w^s;\delta,T)$ be the value function of problem (U^s) for agent δ with skill s. Evidently, a worker invests in skill e only if $V^e(w^e;\delta,T) \geq V^{ne}(w^{ne};\delta)$.

The key properties of workers' behavior are summarized in the following Lemma, whose proof is in Appendix.

[5] It will become clear in the sequel that the time-costs of the investment in HC would have no relevant implications. Therefore, we ignore them.

[6] To use the notion of A-P measure here could be a little misleading. The point is that the restriction on $v(c)$ is defined in terms of the measure proposed by A-P, in a different context, to measure relative risk aversion.

[7] The condition is $\frac{\partial^2 v(.)}{\partial c^{s2}} / \frac{\partial v(.)}{\partial c^s}|_{c^e} w^e h^e > -1$, where the A-P measure is evaluated at $c^e < w^e h^e(.)$, due to the fixed cost of the investment in human capital.

[8] As we will show, each equilibrium partition can be defined by a threshold $\overline{\delta} \in [\underline{d}, \overline{d}]$. Each worker with ability δ will invest in low skills if $\delta < \overline{\delta}$, in high skills if $\delta > \overline{\delta}$. Agents with $\delta = \overline{\delta}$ will be indifferent. Since the equilibrium partition is defined up to zero measure subsets, we do not need to be too precise about the actual behavior of agents with $\delta = \overline{\delta}$, because they are a set of measure zero.

Lemma 1 *Under assumption (U), for each* δ *and each* s, *(i)* $\frac{\partial \widetilde{h}^s(.)}{\partial w^s} > 0$ *and (ii)* $\frac{\partial \widetilde{h}^e(.)}{\partial T} > 0$. *Moreover, (iii) if* $w^e > w^{ne}$, $\frac{\partial V^e(.)}{\partial \delta} > \frac{\partial V^{ne}(.)}{\partial \delta}$.

As already pointed out, (i) and (ii) are standard results and are reported here for completeness. Bear in mind that these properties hold for the *notional* supply functions $\{\widetilde{h}^{ne}(.), \widetilde{h}^e(.)\}$. Obviously the *actual* demand correspondences $\{h^{ne}(.), h^e(.)\}$ are not continuous functions at the critical set of wage profiles where a worker switches from one skill to the other.

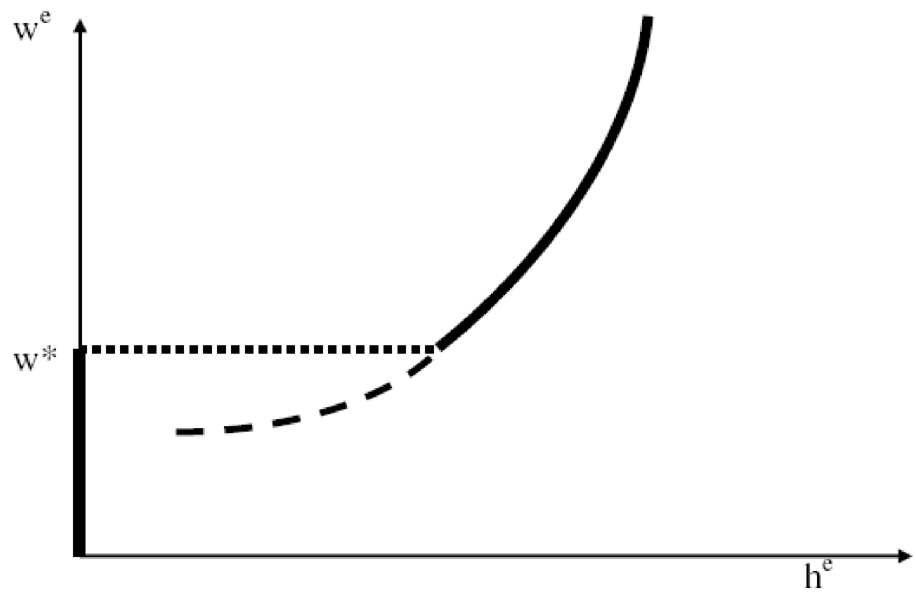

Figure 1

Figure 1 describes the typical individual supply curve for high skill labor. Let w^* be the threshold for w^e. For $w^e < w^*$, the actual supply of skilled labor is nil. For $w^e > w^*$, it is described by the thick curve. The notional supply curve (which coincides with the actual one at $w^e > w^*$) is described by the dashed curve. Evidently, at the threshold, $h^e(.)$ is a (non convex-valued) correspondence. The properties of the low skill labor supply correspondence are similar.

Firms are endowed with the same concave, C^2 production function $F_j(k_j, \ell_j^{ne}, \ell_j^e)$, with $(k_j, \ell_j^{ne}, \ell_j^e) \in \mathbb{R}_+^3$. Returns to scale are constant. Production requires a positive amount of capital and of, at least, one type of labor, i.e., $F_j(k_j, \ell_j^{ne}, \ell_j^e) = 0$ for each $k_j = 0$ and/or any $(\ell_j^{ne}, \ell_j^e) = 0$.. Moreover, $\frac{\partial F_j(.)}{\partial k_j} > 0$ and $\frac{\partial F_j(.)}{\partial \ell_j^s} > 0$, for each s, whenever $(k_j, \ell_j^{ne}, \ell_j^e) >> 0$. These properties are either required or, at least, convenient for our efficiency results. More stringent assumptions are required to establish the existence of equilibria. We will come back to this issue later on.

Without any essential loss of generality, we assume that all the commodity prices (for outputs and investments) are equal to 1. As common in the literature, this can be rationalized by making appeal to a "small open economy" assumption.

To summarize, given a wage profile $\{w^{ne}(.), w^{e}(.)\}$ and a measurable partition $\Delta(.)$ of the set of workers, each firm chooses its investment profile and its labor demand solving the optimization problem

$$\max_{(k_j, \ell_j^{ne}(.), \ell_j^{e}(.))} E\left[F_j(k_j, \ell_j^{ne}(.), \ell_j^{e}(.)) - w^{ne}(.)\ell_j^{ne}(.) - w^{e}(.)\ell_j^{e}(.)|\Delta(.)\right] - k_j. \quad (\Pi)$$

2.2 Equilibrium

Define an allocation as $\chi \equiv \left\{(k_j, \ell_j^{ne}(.), \ell_j^{e}(.)), (c_i^s(.), h_i^s(.)), s = ne, e\right\}$. Equilibrium is defined by market clearing and individual optimization, under the assumption that expectations are rational. Formally,

Definition 2 *An equilibrium is a wage map* $\{\overline{w}^{ne}(.), \overline{w}^{e}(.)\}$ *with associated measurable partition* $\overline{\Delta}(.)$ *and allocation* $\overline{\chi}$ *such that*[9]

i. *for each* i, δ *and* s, $(c^s(.), h^s(.))$ *solves* (U^s),

ii. $V^e(\overline{w}^e(.); \delta, T) - V^{ne}(\overline{w}^{ne}(.); \delta) > 0$ *only if* $\delta \in \overline{\Delta}^e(.)$,

iii. $\left\{k_j, \ell_j^{ne}(.), \ell_j^{e}(.)\right\}$ *solves* (Π),

iv. *labor markets are in equilibrium in each spot market.*

The four conditions require that, conditional on the information available, all the agents choose their optimal investments $(i - iii)$, and labor markets clear at the given wages (iv). This, together with the fact that agents make their choices conditional on the equilibrium partition map and on the additional information available to them, if any, implies rational expectations.

The essential difference between the economies described above as (a) and (b) is in the definition of the partition $\overline{\Delta}(.)$. Under (a), a state is defined by a realization of the r.v. T and $\overline{\Delta}(T)$ is a T-contingent partition of the set of agents, $[0,1] \times [\underline{d}, \overline{d}]$. Under (b), a state is defined by a realization $\{\delta^{ne}, \delta^{e}\}$ and $\overline{\Delta}$ partitions the set of islands, $[\underline{d}, \overline{d}]$. Consequently, condition (iv) takes a different form in case (a) and (b). If (a), an interval $[0,1]$ of identical firms demands labor of the two skills. Thus, the market clearing conditions are

iv.a. $\int_{\overline{\Delta}^s(T)} \left(\int_0^1 h_i^s(.)di\right) d\delta = \int_0^1 \ell_j^s(.)dj$, for each s and T.

In case (b), an interval $[0,1]$ of identical firms demands labor of the two skills, too. However, in each island, agents are identical and labor of skill s is supplied by an interval $[0,1]$ of identical workers with $\delta = \delta^s$. Hence, the market clearing conditions are

[9] As already mentioned, the individual optimality conditions must hold *a.e.*, however it would just be pedantic to restate this fact over and over again.

iv.b. $\int_0^1 h_i^s(.;\delta^s)di = \int_0^1 \ell_j^s(.)dj$, for each s and $\{\delta^e, \delta^{ne}\}$.

In both set-ups, existence of equilibria is not a trivial issue. As we will see later on, it may require additional restrictions on utility and production functions. We postpone the discussion of this question, since, as we will argue, it is somewhat peripheral to our main interest.

2.3 Constrained Pareto optimality (CPO)

Our main interest is in the efficiency properties of equilibria. Given that there are no full insurance opportunities, full Pareto efficiency is obviously out of reach. Lack of insurance markets and $T-$invariance of the investment profile $\{k_j\}$ are the only sources of inefficiency. Since spot labor markets are perfectly competitive, the resulting spot allocations are always Pareto efficient, *conditional* on the profile $\left\{\overline{k}_j\right\}$ and $\overline{\Delta}(.)$.

We adopt a notion of CPO based on the comparison of the utilities obtained at the equilibrium with the ones that individuals could obtain at a *conditional* equilibrium associated with some alternative profile $\{\widetilde{k}_j\}$ and $\widetilde{\Delta}(.)$. This concept of CPO is obviously related to the notion exploited in the GE literature for economies with incomplete markets (see Geanakoplos and Polemarchakis (1986)). In our context, it presents two key advantages. First, the problem of CPO can be easily converted into an appropriate planner's optimization problem. This allows us to discuss the efficiency properties of equilibria in a straightforward way, by comparing the first order conditions of the planner's problem with the ones defining the equilibrium. This will be very convenient in Section 3. With respect to the properties that we will discuss in Section 4, the more restricted the planner's policy tools are, the stronger the inefficiency results. With the proposed notion of CPO, the instruments are just the profile of investments in physical capital and the partition of the workers. Hence, they are actually fairly weak. This strengthens the inefficiency result, since a Pareto improvement could, in principle, be implemented fairly easily (see, in particular, Proposition 6 below). Finally, to restrict the analysis to allocations that are *conditional* equilibrium allocations entails no substantive loss of generality because, given $\left\{\overline{k}_j\right\}$ and $\overline{\Delta}(.)$, the standard version of both fundamental theorems of welfare economics holds. The precise notion of CPO adopted is the following:

Definition 3 *An equilibrium $\{\overline{w}^{ne}(.), \overline{w}^e(.)\}$ with associated measurable partition $\overline{\Delta}(.)$ and allocation $\overline{\chi}$ is constrained Pareto optimal if and only if there is no alternative wage map $\{\widetilde{w}^{ne}(.), \widetilde{w}^e(.)\}$ with associated measurable partition $\widetilde{\Delta}(.)$ and allocation $\widetilde{\chi}$ such that:*

1. *$\{\widetilde{k}_j\}$ is state invariant,*
2. *$\{\widetilde{w}^{ne}(.), \widetilde{w}^e(.)\}$ with associated partition $\widetilde{\Delta}(.)$ and allocation $\widetilde{\chi}$ is an equilibrium conditional on $\{\widetilde{k}_j\}$ and $\widetilde{\Delta}(.)$,*
3. *a.e., $u(\widetilde{c}_i^s(.), \widetilde{h}_i^s(.);\delta,T) \geq u(\overline{c}_i^s(.), \overline{h}_i^s(.);\delta,T)$, with $u(\widetilde{c}_i^s(.), \widetilde{h}_i^s(.);\delta,T) > u(\overline{c}_i^s(.), \overline{h}_i^s(.);\delta,T)$ for some set of agents of positive Lebesgue measure,*

4. *expected profits are nonnegative:* $E\left[\left(F_j\left(\widetilde{k}_j,\widetilde{\ell}_j^{ne},\widetilde{\ell}_j^{e}\right)-\sum_s \widetilde{w}^s\widetilde{\ell}_j^s\right)|\widetilde{\Delta}(.)\right]-\widetilde{k}_j\geq 0.$

(1) describes the fundamental constraint (and possible sources of inefficiency) for our economy. (2) restricts the welfare comparison to conditional equilibria. (3) is the usual definition of Pareto optimality. Less obvious is the last condition. It can be rationalized in two different ways. First, and directly, as a feasibility constraint. Alternatively, we may assume that firms are owned by an additional class of agents, "rentiers", with linear utility functions. At time 0, they have some large initial endowment that can be either consumed or invested as physical capital of the firms they own. With this second interpretation, our economy is embedded into a fully specified general equilibrium model and our notion of CPO allocation essentially coincides with the one of Pareto optimal allocation constrained by (1). Given that returns to scale are constant, both interpretations can be adopted.

3 A sufficient condition for the CPO of equilibrium allocations

In this section, we focus on the properties which are sufficient to guarantee that equilibria are CPO. We restrict the analysis to the class of economies previously described as (a), i.e., the ones with a $T-$conditional equilibrium partition. For them, equilibrium allocations are always CPO. This is established in the next Proposition, whose proof is in Appendix.

Proposition 4 *Consider the class of economies described under (a), with a T-conditional partition. Then, each equilibrium allocation is CPO.*

This result is in the spirit of the first fundamental theorem of welfare economics. Hence, the issue of the existence of an equilibrium is beside the point. Obviously, an equilibrium exists for some set of appropriate restrictions on the fundamentals. No matter what these restrictions are, the equilibrium allocation is CPO.

Firms are identical and their optimal investment levels are always equal. Thus, we will refer to equilibrium allocations conditional on an investment profile $\{k_j\}$ simply as $K-$conditional equilibrium allocations, for $K=k_j$, *a.e.*.

Since our notion of CPO is based on the comparison of the welfare properties of ($K-$conditional) equilibrium allocations associated with different values of K, as a preliminary result we report two properties of these equilibria, which will be handy in the discussion of Prop. 4.

Lemma 5 *Let $\mathcal{E}$ be any open set of economies such that an equilibrium exists. Then: (i) At each equilibrium, $\Delta(T)\equiv\{[\underline{d},\delta(T)),[\delta(T),\overline{d}]\}\times[0,1]$, (ii) The equilibrium allocation is locally described by a collection of C^1 functions.*

The proof of the Lemma is in Appendix. Bear in mind that, under the maintained assumptions, the two properties hold true for every open subset of economies such that an equilibrium exists. We are a little sloppy here, because we do not provide a precise parameterization of the space of economies, i.e., we do not formally define what an open set of economies is in this context. This could be easily done, but here it suffices to say that the Lemma holds whenever the properties stated in Section 2.1 for economies of type (a) are satisfied.

To interpret the implications of Proposition 4, it is convenient to recast the issue in terms of maximization of an appropriate welfare function. We adopt now the usual fiction of a benevolent planner choosing the allocation to maximize welfare under the constraints imposed on its choices by the distortions at play in the economy.

Given K, define, for each realization T, the function

$$\begin{aligned} W(.;T,K) \equiv & \sum_s \int_{\Delta^s(T)} \left(\int_0^1 \phi^s(\delta;T)\, u(c_i^s, h_i^s;\delta) di \right) d\delta \\ & + \int_0^1 \left(F_j\left(k_j, \ell_j^{ne}, \ell_j^e\right) dj - \sum_k k_j \right) dj \\ & - \sum_s \int_{\Delta^s(T)} \left(\int_0^1 c_i^s(.) di \right) d\delta - T \int_{\Delta^e(T)} d\delta \end{aligned} \tag{1}$$

$$\text{subject to } 0 = \int_{\Delta^s(T)} \left(\int_0^1 h_i^s di \right) d\delta - \int_0^1 \ell_j^s dj, \text{ for each } s,$$

for some map $\phi^s(\delta;T) > 0$, for each $(\delta;T)$.

Now, define the ex-ante planner's optimization problem,

$$\max_{(K,\ell,h,c)} E(W(.;T,K)) \text{ subject to } \int_{\Delta^s(T)} \left(\int_0^1 h_i^s di \right) d\delta - \int_0^1 \ell_j^s dj = 0, \text{ for each } s \text{ and } T. \tag{2}$$

At an equilibrium, given the individual budget constraints, the sum of the last two terms in $W(.;T,K)$ is the total producers' surplus in state T. Since, at each equilibrium, expected profits are zero, $E(W(.,T;K))$ is a standard welfare function, i.e., the sum of individual expected utilities weighted by some collection of functions $\{\phi^s(\delta;T)\}$.

For quasi-linear utility function, and given $\phi^s(\delta;T) = 1$, for each $(\delta;T)$, $E(W(.;T,K))$ is the total expected surplus. For general utility functions, we can provide two alternative interpretations of $E(W(.;T,K))$. First, we can look at it as the (normalized) Lagrangian of an optimization problem having as objective function the weighted sum of individual utilities, and, as a constraint, the condition that expected profits must be non-negative. Alternatively, we can take $E(W(.;T,K))$ as a standard social welfare function for the completely specified general equilibrium economy outlined above.

We can discuss CPO simply comparing the equilibrium conditions with the ones characterizing the optimal solution to the planner's problem (2). The main advantage of this reformulation is that it makes transparent the key properties leading to CPO. Bear in mind that, by Lemma 5, the equilibrium map is (locally) continuously differentiable, and the partition is defined by a threshold value.[10] Fix investments in physical capital. Since each K-contingent equilibrium allocation is PO, it is also an optimal solution to the optimization problem (1), given the functional $\phi^s(.) = \frac{1}{\frac{\partial v}{\partial c^s}}$.[11] Then, let's start considering problem (1). We are considering (conditional) equilibria and we have established in Lemma 5 that each equilibrium partition is defined by a threshold value $\delta(T;K)$. Therefore, we can simply identify the partition with this value. Optimization problem (1) can be rewritten as a two stage problem: first, given $\{\delta(T;K), K\}$,

$$\text{choose } (c^s(.), h^s(.)) \in \arg\max W(.;\delta(T;K), K).$$

Given that spot markets are perfectly competitive, the FOCs of this planner's problem coincide with the equilibrium conditions (individual optimality and market clearing). Let $W(\delta(T;K), K)$ be the state T optimal value of the welfare function. The (conditional on K) CPO partition $\widehat{\delta}(T;K)$ can then be obtained solving

$$\max_{\delta(T;K)} W(\delta(T;K), K).$$

Once again, this follows by the fact that, for each T, the $K-$conditional allocation is Pareto optimal, by the first fundamental theorem of welfare. Hence, by the envelope theorem, the FOC of this problem is

$$\begin{aligned} 0 &= -\phi(\delta(T;K))\left[u(c^e(\delta(.)), h^e(\delta(.)); \delta(.)) - u(c^{ne}(\delta(.)), h^{ne}(\delta(.); \delta(.))\right] \\ &\quad - \left[\frac{\partial F}{\partial \ell^e}\ell^e(\delta(.)) - \frac{\partial F}{\partial \ell^{ne}}\ell^{ne}(\delta(.)) + c^{ne}(\delta(.)) - c^e(\delta(.)) - T\right]. \end{aligned}$$

The first term in square brackets is zero by definition of equilibrium threshold.[12] The second is zero at each equilibrium, because of the budget constraints of the marginal agents (i.e., the ones with δ equal to the threshold value). This FOC is necessary and sufficient to guarantee that, conditional on K, the equilibrium

[10] A *caveat* is required here. We have defined equilibrium in terms of wages and of the threshold $\delta(K)$. We have shown that these endogenous variables are, locally, a C^1 function of the investment profile K. However, the map $\alpha(\delta, i)$ assigning agents of the same type δ to skill ne is clearly not a continuous function, since it is equal to 1 for each $\delta < \delta(K)$, and equal to 0 for $\delta > \delta(K)$. Still, this is immaterial for our argument, since the set of agents such that $\delta = \delta(K)$ has always measure zero.

[11] At the threshold value of δ, typically $\phi^{ne}(\delta) \neq \phi^e(\delta)$. The argument requires us to define $\phi(\delta) = \phi^{ne}(\delta)$ for $\delta \leq \delta(T;K)$, $\phi(\delta) = \phi^e(\delta)$ for $\delta > \delta(T;K)$. Bear in mind that the discontinuity in the value of $\phi(\delta)$ at $\delta = \delta(T;K)$ is irrelevant in our proof, because the set of agents such that $\delta = \delta(T;K)$ has zero measure.

[12] Again, to differentiate between workers with $\delta = \widehat{\delta}(T;K)$ choosing high vs. low skills would not affect the result, since the values of the utility function associated with the two choices are identical.

allocation solves (1). Let $\overline{W}(T;K)$ be the value function of the previous problem and, finally, consider the ex-ante problem

$$\max_K E(\overline{W}(.,T;K)).$$

By the envelope theorem, its FOCs are $E(\frac{\partial F_j}{\partial k_j})-1=0$, and, by expected profits maximization, they must be satisfied at each equilibrium.

To conclude, the conditions required for the CPO of equilibrium allocations to hold in general are: first, at each T, the equilibrium is PO, contingent on the K profile. This allows us to exploit the envelope theorem and, therefore, to ignore the second order effects of marginal changes in K. Second, firms must be maximizing expected profits (i.e., firms' owners must be risk-neutral). In general, state-contingency of the partition is crucial, because it allows us to apply the envelope theorem state by state. The logic of the argument breaks down as soon as the theorem does not apply at some stage.

We have focussed on the case where the r.v. is the cost of education. For this specific structure, it is actually straightforward to implement the full Pareto optimal allocation. It suffices to allow workers to insure against the variability of T, either introducing a T-contingent policy of taxes and subsidies or modifying the labor contracts, so that the (risk-neutral) firms are actually bearing the risk. However, one could write down economies which are analytically equivalent, but for which there are no obvious insurance possibilities. For instance, randomness in the marginal utility of consumption may have, analytically, the same effects of randomness of T. Evidently, for this kind of uncertainty, implementation of the full Pareto optimum would be problematic, to say the least.

4 A class of economies with constrained inefficient equilibria

In this section, we analyze in detail the class of economies described above as (b), i.e., such that the equilibrium partition refers to the set of islands. There are several possible specifications of this class of economies. First, one could assume that there are two possible production processes, each one using just one type of skills. Firms have access to both technologies and, once matched with one, or more, "local" labor market they adopt the appropriate production process. The analysis of a parametric example of this polar type is in Mendolicchio, Paolini, and Pietra (2012b). Here, we want to allow for more general production functions, using jointly both types of HC. Then, some care is required to properly specify the features of the economy. To keep the details manageable, we assume that effort supply is perfectly inelastic, meaning that each agent of type δ supplies δ units of HC. This entails no substantive loss of generality: different values of the elasticity of the effort supply may change the quantitative results, but they cannot change the qualitative ones we are interested in.

Our main result is that equilibria may not be CPO. In Proposition 6, we show that constrained Pareto inefficiency occurs if all inputs are Edgeworth complements (*E-complements* in the sequel), i.e., if $\left(\frac{\partial^2 F}{\partial \ell^e \partial K}, \frac{\partial^2 F}{\partial \ell^{ne} \partial K}, \frac{\partial^2 F}{\partial \ell^e \partial \ell^{ne}}\right) > 0$, and if an additional, fairly mild, restriction holds. When this is the case, equilibria are always characterized by overeducation: a decrease in the size of agents acquiring high skills leads to a Pareto improvement. This result is of some interest only if there are economies fitting this set-up and such that an equilibrium exists. Thus, in Proposition 7, we establish existence of equilibria for a subset of economies satisfying the assumptions of Proposition 6.

Let's fully describe our set-up. As already explained, each firm chooses its investments in physical capital without knowing the actual realization $\{\delta^{ne}, \delta^e\}$ for the labor markets it is matched with. Hence, each firm is uncertain about the wage rates it will face. The measures of the set of islands with low and high skills labor is endogenous. Therefore, to close the model, it is necessary to specify three technologies: the one using both kinds of labor and the ones using HC of just one type. To simplify, let's focus on the case where the equilibrium threshold $\overline{\delta}$ is larger than (or equal to) $\frac{\overline{d}+\underline{d}}{2}$, so that $\left(\overline{\delta}-\underline{d}\right) \geq \left(\overline{d}-\overline{\delta}\right)$.[13] This means that each island where workers have high skills is matched with one island with low skill workers. On the other hand, an island with low skill workers is matched to a high skill labor market with probability $\frac{\overline{d}-\overline{\delta}}{\overline{\delta}-\underline{d}}$, while, with probability $\left(1-\frac{\overline{d}-\overline{\delta}}{\overline{\delta}-\underline{d}}\right)$, these agents will work in firms with a technology described by a production function using just low skill labor, $f_j(\underline{k}_j, \underline{\ell}_j^{ne})$.

We assume that $f_j(\underline{k}_j, \underline{\ell}_j^{ne})$ satisfies the standard assumptions and exhibits constant returns to scale. We will denote inputs of firms using just low skill workers by underlining the associated variables.

Workers choose their skill level knowing their own innate ability and, hence, the labor supply of all workers of their own type. On the other hand, they do not know the labor supply of the workers of the different skill active in the labor market they will be matched with. Therefore, they face uncertainty on their actual wage.[14]

Assume that an equilibrium, $\left\{\left(w^e(\delta^{ne}, \delta^e; \overline{\delta}), w^{ne}(\delta^{ne}, \delta^e; \overline{\delta})\right), \underline{w}(\delta^{ne}; \overline{\delta})\right\}$, exists, where $\underline{w}(\delta^{ne}; \overline{\delta})$ is the wage rate in the industries using just low skill labor. We start presenting our (in)efficiency result.

Proposition 6 *In addition to the maintained assumptions, (i)* $\overline{\Delta} \equiv \left[[\underline{d}, \overline{\delta}), \left[\overline{\delta}, \overline{d}\right]\right]$, *(ii) all inputs are Edgeworth complements and (iii) the expected utility gain for low skill workers of being active in the sector employing both skills is sufficiently small. Then, equilibrium allocations are not CPO and equilibria are characterized by overeducation.*

The proof is in Appendix. The logic of the argument is transparent. An increase in the value of the threshold $\overline{\delta}$ induces, as a first order effect, a decrease in both expected wages from the point of view of the firms. This induces an

[13] To consider explicitely also the alternative case will not change the main result.

[14] In fact, if low skill, workers do not know if they will get a job in an industry using both types of skills or just low skilled individuals. It is possible to show that the precise specification of the information of workers when choosing their skill is irrelevant for our results.

increase in investments in physical capital that, in turn, pushes up the wage rates in all the labor markets, since inputs are *E-complements*. This has a positive effect on welfare. At the equilibrium threshold, the expected utility of a worker is the same as skilled or unskilled, by definition. Hence, the change in the value of the threshold has zero direct impact on welfare. There is an additional component due to the change in the probability for an unskilled worker to be employed in the two industries. Given that, by assumption, the expected utility gain of switching from one industry to the other is, for each unskilled worker, small, its negative impact on social welfare is dominated by the first effect.

We still have to show that there are classes of economies such that an equilibrium exists. To do that, let's consider an economy with production function

$$F_j(.) = Ak_j^{\alpha}\left[\psi^{ne}\ell_j^{ne\theta} + \psi^{e}\ell_j^{e\theta}\right]^{\frac{1-\alpha}{\theta}}, \text{ with } \psi^{ne} < \psi^{e}.$$

Assume that the alternative production function using just low skill labor is

$$f_j(.) = \psi^{ne\frac{\theta}{1-\alpha}} A\underline{k}_j^{\alpha}\underline{\ell}_j^{ne1-\alpha}.$$

If $(1-\alpha) \geq \theta$, all inputs are (weakly) *E-complements*.

Proposition 7 *Under the maintained assumptions, there is an open set of economies such that an equilibrium exists.*

The proof (in Appendix) applies to a specific set of economies, with inelastic supply of labor and utility for consumption sufficiently close to linearity. These restrictions could be suitably relaxed. However, since the aim of Proposition 7 is just to show that there are some sets of economies for which equilibria exist, the additional technical troubles required to establish a more general existence result would not be justified.

An heuristic existence argument is summarized in the following two figures. Figure 2 shows the expected marginal product of capital, $E(MP_k(.))$, for a typical firm, as a function of *aggregate* investments, given several arbitrary values of the threshold $\{\delta^1, \delta^2, \delta^3\}$. These curves are obtained exploiting the labor market clearing conditions in the different islands. Since they are decreasing, the individual firm's profit maximizing condition identifies the unique levels of the equilibrium aggregate investments associated with the different thresholds, so that K($\overline{\delta}$) is a well-defined, increasing, function.

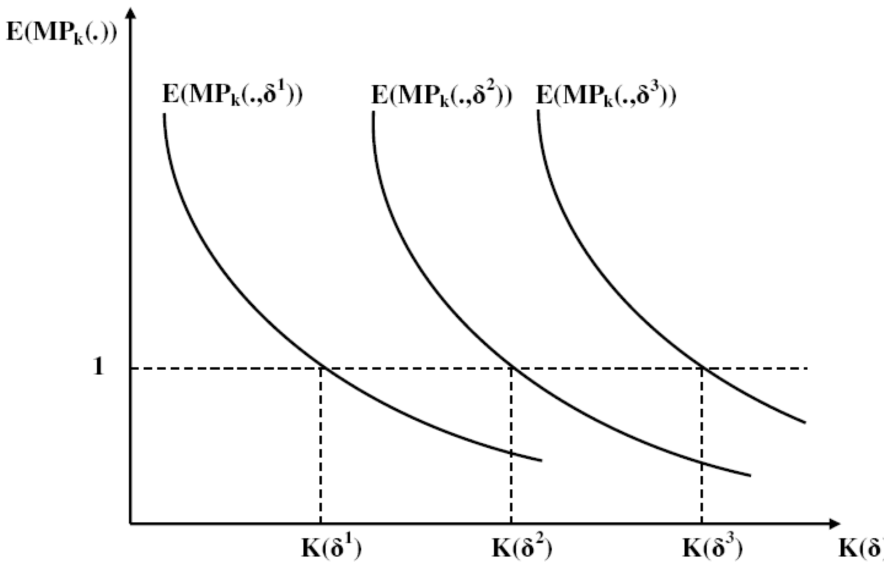

Figure 2

Figure 3 shows the difference between the expected utilities of an individual with innate ability δ with high or low skills. Each curve is associated with a different level of the threshold and takes into account the market clearing wages corresponding to the value of aggregate investments $K(\overline{\delta})$. The curves are drawn assuming $T = 0$. Consider, for instance, the one associated with δ^1. We can clearly pick an appropriate value of T, T^1, such that $E(V^e(w^e(.); T = T^1); \delta^1) - E(V^{ne}(w^{ne}(.); \delta^1)) = 0$. As long as this function is increasing in δ, it is easy to see that δ^1, with associated $K(\delta^1)$, define an equilibrium given T^1. It is also straightforward to see that, for some open set of values of T, an equilibrium exists. The proof in Appendix spells out the details of the argument, showing that the functions described in the two figures do actually have the stated properties.

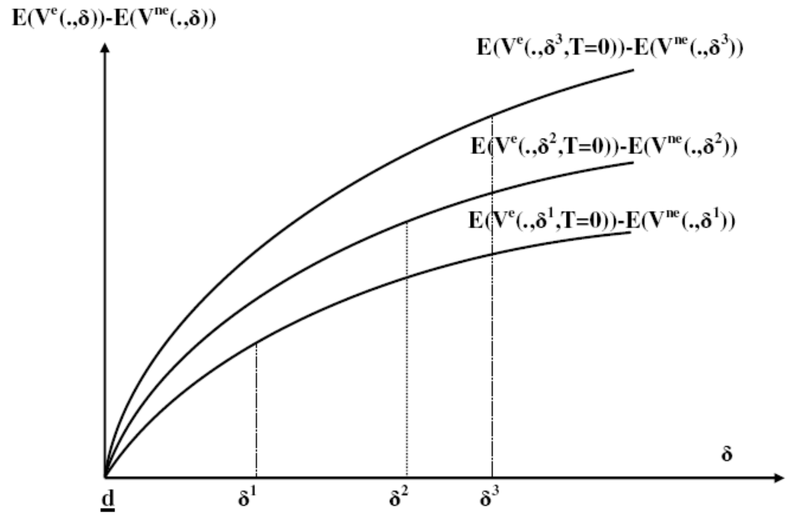

Figure 3

We conclude with a parametric example, illustrating the characteristics of the inefficiency of the equilibrium allocations for this kind of economies.

Example 8 *Let* $F_j(.) \equiv Ak_j^{\alpha}\left[\psi^{ne}\ell_j^{ne\theta}+\psi^{e}\ell_j^{e\theta}\right]^{\frac{1-\alpha}{\theta}}$. *Set* $(1-\alpha)=\theta=\frac{2}{3}$. *The first is the usual estimate for the income share of labor. The value of* θ *implies an elasticity of substitution between skilled and unskilled labor equal to 3, which is somewhat large compared to standard estimates. Given that this example has a purely illustrative purpose, the advantage in terms of computational tractability justifies the choice of this value. Since* $\frac{1-\alpha}{\theta}=1$, *the production functions reduce to* $F_j(.)=k_j^{\frac{1}{3}}(\psi^{ne}\ell_j^{ne\frac{2}{3}}+\psi^{e}\ell_j^{e\frac{2}{3}})$ *and* $f_j(.)=\psi^{ne}\underline{k}_j^{\frac{1}{3}}\underline{\ell}_j^{ne\frac{2}{3}}$. *By direct computation, the (K-conditional) equilibrium wages are* $w^s(\delta^s;K)=\frac{2\psi^s}{3}\left(\frac{K}{\delta^s}\right)^{\frac{1}{3}}$ *and* $\underline{w}(\delta^{ne};\underline{K})=\frac{2\psi^{ne}}{3}\left(\frac{\underline{K}}{\delta^{ne}}\right)^{\frac{1}{3}}$. *Replacing the (K-conditional) demand functions into the conditions for expected profit maximization, we obtain*

$$\frac{\partial E(\Pi(.))}{\partial k_j} = \frac{\int_{\overline{\delta}}^{\overline{d}}\frac{\psi^e}{3}\left(\frac{2}{3}\frac{\psi^e}{w^e(\delta^e;K)}\right)^2 d\delta^e}{\overline{d}-\overline{\delta}} + \frac{\int_{\underline{d}}^{\overline{\delta}}\frac{\psi^{ne}}{3}\left(\frac{2}{3}\frac{\psi^{ne}}{w^{ne}(\delta^{ne};K)}\right)^2 d\delta^{ne}}{\overline{\delta}-\underline{d}} - 1 = 0,$$

$$\frac{\partial E(\underline{\Pi}(.))}{\partial \underline{k}_j} = \frac{\int_{\underline{d}}^{\overline{\delta}}\frac{\psi^{ne}}{3}\left(\frac{2}{3}\frac{\psi^{ne}}{\underline{w}^{ne}(\delta^{ne};\underline{K})}\right)^2 d\delta^{ne}}{\overline{\delta}-\underline{d}} - 1 = 0.$$

Replacing into these eqs. the (K-conditional) equilibrium wages, we obtain

$$K(\overline{\delta}) = \left(\frac{\psi^e}{5}\frac{\overline{d}^{\frac{5}{3}}-\overline{\delta}^{\frac{5}{3}}}{\overline{d}-\overline{\delta}} + \frac{\psi^{ne}}{5}\frac{\overline{\delta}^{\frac{5}{3}}-\underline{d}}{\overline{\delta}-\underline{d}}\right)^{\frac{3}{2}}, \qquad \underline{K}(\overline{\delta}) = \left(\frac{\psi^{ne}}{5}\frac{\overline{\delta}^{\frac{5}{3}}-\underline{d}^{\frac{5}{3}}}{\overline{\delta}-\underline{d}}\right)^{\frac{3}{2}}.$$

Assume that preferences in consumption are linear, and to fix ideas, set $\overline{d} = 10$, $\underline{d} = 1$, $\psi = \{1, 1.5\}$.[15] *The equilibrium threshold is then obtained setting* $M(\overline{\delta}, T) = 0$, *where* $M(\overline{\delta}, T)$ *defines the difference between the expected utilities of a worker with innate ability* $\overline{\delta}$ *investing in high vs. low skills. Fix* $T = 4.6867$, *then* $M(\overline{\delta}, T) = 0$ *at* $\overline{\delta} = 7.5$. *Figure 4 shows the expected surplus map as a function of the threshold.*[16] *Since the map is increasing at* $\overline{\delta} = 7.5$, *the equilibrium is characterized by overeducation.*

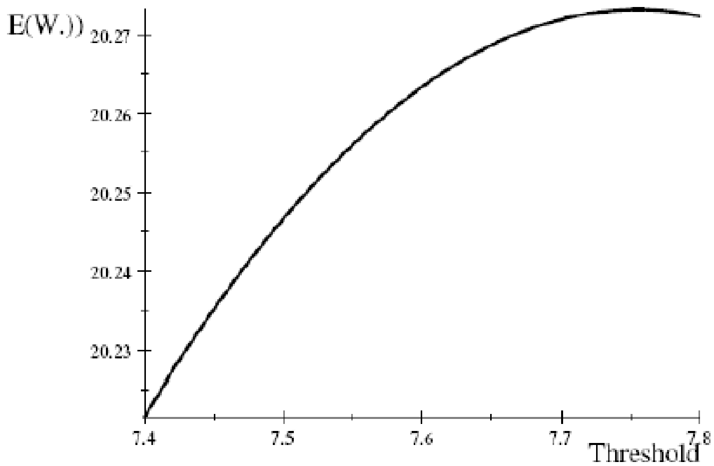

Figure 4

If, as in the previous example, the production function is a Cobb-Douglas with labor input defined by a CES aggregator, the proof of Proposition 7 requires $(1-\alpha) \geq \theta$. This assumption is sufficient to deliver the property that the cross derivatives of the marginal product of the two types of labor are positive, i.e., they are *E-complements*. Using the standard empirical estimate of $(1-\alpha) \backsimeq \frac{2}{3}$, this requires that, in absolute value, the elasticity of substitution between the two types of labor is, at most, 3. The restriction is plausible from an empirical viewpoint, since most estimates are significantly lower.[17] We have also required

[15] The linearity assumption obviously simplify the computations, but plays no substantive role in the argument. Once we obtain an equilibrium, we can perturb preferences, introducing strict concavity in consumption. The new equilibrium has the same efficiency properties of the original one.

[16] Under the stated assumptions on preferences, an allocation is CPO only if it maximizes expected surplus. We use expected surplus as a measure of welfare just for expositional convenience.

[17] For instance, Katz and Autor (1999) indicate as most plausible range of values 1-2. Krusell et al. (2000) use as preferred estimate 1.67. Autor, Katz and Kearney (2008) and Goldin and Katz (2008) obtain estimates around 1.64. Ciccone and Peri (2005) restrict the range to 1.2-2 and indicates as preferred estimate 1.5. All these values are mostly referred to the US experience. Some of the estimates for other countries are larger, in fact larger than the

(as a sufficient condition), $\theta > 0$. This is also in line with the empirical estimates, since the values of the elasticity of substitution are typically larger than 1.

The specific production function has one capital goods exhibiting the same elasticity of substitution with the two types of labor. Similar results hold for production functions which do not satisfy this restriction. In particular, assume that $F_j(.) = A\left[\psi^{ne}\ell_j^{ne\theta} + (1-\psi^{ne})\left(\lambda k_j^{\rho} + (1-\lambda)\,\ell_j^{e\rho}\right)^{\frac{\theta}{\rho}}\right]^{\frac{1}{\theta}}$. Then, given K, it is still possible to exploit the first order condition for profit maximization (evaluated at the conditional equilibrium) to determine the equilibrium pair $(w^{ne}, w^{e})(\delta, K)$, as in the proof of Proposition 7. Then, one can move backward to compute the equilibrium value $K(\overline{\delta})$. There is no substantive qualitative change in the results. There is also no qualitative change in the results switching the "position" of ℓ^e and ℓ^{ne}.[18] Finally, we could introduce an additional capital good entering the production function in a multiplicative fashion, along the lines of Krusell *et al.* (2000). Under suitable (and empirically plausible) restrictions on the parameters, a more elaborate argument would deliver a similar existence result. Hence, our main results are, qualitatively, very robust to alternative specification of the production function.

5 Conclusions

The paper considers the welfare effects of the interaction between self-selection of workers into different labor markets, segmented by skill levels, and investments in physical capital. The key assumption is that firms, when choosing their investments, are uncertain about the characteristics of the labor markets they will actually face when hiring will take place. The distribution of future equilibrium wages and the partition of workers into the distinct labor markets are endogenously determined at the equilibrium. To evaluate welfare, we consider the effects of changes in the investments in physical capital and in the partition of workers across skills. An allocation is constrained Pareto optimal if it cannot be improved upon by changing these variables and letting the other endogenous variables to adjust to restore the (conditional) equilibrium conditions. Assume that workers investments in skills depend upon the realization of some random variable such as the cost of the investment in HC, T. Then, the equilibrium allocation is constrained PO if the equilibrium partition is state contingent. For each realization of T, given the predetermined level of the investments in physical capital, the conditional equilibrium is fully Pareto efficient and it can be expressed as the optimal solution to the standard planner's problem. By, essentially, the envelope theorem, we can ignore the second order effects on welfare of changes in the level of the investments in physical capital. It follows that the first order conditions of the firms' expected profit optimization problems

value of 3 used in the example above. For instance, Mello (2008) reports values up to 3.3 (Switzerland) and 5 (Canada).

[18] Evidently, by switching these positions, we are imposing different restrictions on the values of the elasticities of substitution across inputs.

coincide with the FOCs of the (ex-ante) planner optimization problem, so that equilibria, if they exist, are always constrained Pareto optimal. This property is the key for CPO. It requires that the partition of workers is Pareto efficient (given K) in each state of the world. If this property does not hold, CPO may fail at the equilibrium. To show that, we consider a class of economies with a somewhat similar time and market structure, but where equilibria are characterized by overinvestment in high skills, meaning that, by restricting the set of workers acquiring high skills, we can actually implement a Pareto improvement.

The paper presents two contributions: first, it gives precise conditions under which, in perfect labor markets, informational asymmetries on the actual quality of prospective hirings may, or may not, induce constrained inefficiency, and, specifically, inefficient overeducation. As mentioned in the introduction, we also believe that this type of results can be of some interest for the literature on taxation in Roy's economies. Abstracting from the details, assume that a policy vector ξ is selected to maximize some welfare function $E(W(\xi))$ at the associated market equilibrium. For instance, assume that ξ is an optimal linear tax profile. Several contributions in the literature have established that the classical Diamond and Mirlees (1971) results concerning the main, general features of optimal linear taxation break down when labor inputs are not perfectly substitutable and the partition of the agents is exogenously given. Specifically, in the Diamond and Mirlees (1971) framework, production is on the efficient frontier and it is possible, essentially, to ignore the equilibrium price adjustment effects of tax changes, so that optimal tax formulas are the same if prices are treated as fixed, or as derived at the equilibrium. When labor inputs are not perfectly substitutable in production and the partition of workers are fixed, both results breaks down. Stiglitz (1982) shows that with two types of skills, the effects of equilibrium price adjustments cannot be ignored. Naito (1999) shows that efficiency in production also fails. Saez (2004) analyzes the properties of optimal taxation in an economy with several types of labor, imperfectly substitutable in the production function. He establishes that both properties (efficiency in production and irrelevance of the effects of price adjustments) are restored once one considers a long run model, where the partition of workers across skills is selected optimally at the equilibrium, and labor supply is inelastic. In all these papers, different types of labor are the only inputs in production. More recently, Gahvari (2014) considers economies with both labor (with exogenous partition) and capital, and shows that introducing capital as an input may play an important role, reversing some of the results obtained in Stiglitz (1982). One way to interpret our results is as a positive contribution to this literature. Ignoring the technicalities, the key issue for the Saez (2004) result is that the partition of the workers optimally adjust to changes in the policy parameters. This is exactly what will happen in any economy with the properties sketched in model (a), i.e., as long as the partition is state contingent. Therefore, we conjecture that results analogous to the ones of Saez (2004) hold for this class of economies. On the other hand, they are bound to fail whenever the workers partition is not state contingent, as in the class of economies described under (b).

6 Appendix

Proof of Lemma 1. *(i)* By the implicit function theorem *(IFT)*,

$$\frac{\partial \widetilde{h}^s(.)}{\partial w^s} = -\frac{\frac{\partial v(.)}{\partial c^s} + \frac{\partial^2 v(.)}{\partial c^{s2}} w^s \widetilde{h}^s}{\frac{\partial^2 v(.)}{\partial c^{s2}} w^{s2} - \frac{\partial^2 g(.)}{\partial h^{s2}} \frac{1}{f(\delta)}} = \frac{\partial v(.)}{\partial c^s} \frac{1 + \left(\frac{\partial^2 v(.)}{\partial c^{s2}} / \frac{\partial v(.)}{\partial c^s} \right) w^s \widetilde{h}^s}{\frac{\partial^2 g(.)}{\partial h^{s2}} \frac{1}{f(\delta)} - \frac{\partial^2 v(.)}{\partial c^{s2}} w^{s2}}.$$

Concavity of $v(.)$ and strict convexity of $g(.)$ imply that the denominator is strictly positive. For $s = ne$, if the Arrow-Pratt measure of relative risk aversion for the utility index $v(.)$ is larger than -1, i.e., if $\left(\frac{\partial^2 v(.)}{\partial c^{ne2}} / \frac{\partial v(.)}{\partial c^{ne}} \right) w^{ne} \widetilde{h}^{ne} > -1$, the numerator is strictly positive. For $s = e$, the same measure must be sufficiently greater than -1, so that that $\left(\frac{\partial^2 v(.)}{\partial c^{e2}} / \frac{\partial v(.)}{\partial c^e} \right) |_{c^e} w^e \widetilde{h}^e \equiv \left(\frac{\partial^2 v(.)}{\partial c^{e2}} / \frac{\partial v(.)}{\partial c^e} \right) |_{c^e} (c^e + T) > -1$.

(ii) By the IFT, $\frac{\partial \widetilde{h}^e(.)}{\partial T} = \frac{-\frac{\partial^2 v(.)}{\partial c^{s2}} w^s}{\frac{\partial^2 g(.)}{\partial h^{s2}} \frac{1}{f(\delta)} - \frac{\partial^2 v(.)}{\partial c^{s2}} w^{s2}} > 0$.

(iii) At $w^e = w^{ne}$ and $T = 0$, $\widetilde{h}^e(w^e, T = 0; \delta) = \widetilde{h}^{ne}(w^{ne}; \delta)$. Since $\frac{\partial \widetilde{h}^e(.)}{\partial T} > 0$, and $\frac{\partial \widetilde{h}^e(.)}{\partial w^e} > 0$, $\widetilde{h}^e(w^e, T; \delta) > \widetilde{h}^{ne}(w^{ne}; \delta)$ at each $(w^e, T) >> (w^{ne}, 0)$. Define $G(.) \equiv u(\widetilde{c}^e(.), \widetilde{h}^e(.); \delta, T) - u(\widetilde{c}^{ne}(.), \widetilde{h}^{ne}(.); \delta, T)$. Then, by the envelope thm, $\frac{\partial G(.)}{\partial \delta} \equiv \left[\frac{g(\widetilde{h}^e)}{f(\delta)^2} - \frac{g(\widetilde{h}^{ne})}{f(\delta)^2} \right] \frac{\partial f}{\partial \delta}$. Since $\widetilde{h}^e(w^e; T, \delta) > \widetilde{h}^{ne}(w^{ne}; \delta)$ (clearly, here we are considering the notional labor supplies), the term in square brackets is positive. By assumption $\frac{\partial f}{\partial \delta} > 0$. Hence, $\frac{\partial G(.)}{\partial \delta} > 0$. ■

Proof of Proposition 4. The easiest way to establish the result is by playing with the equivalence between equilibria of the actual economy and equilibria of an economy with an additional class of risk-neutral agents, rentiers, owning the firms. Consider an equilibrium allocation $\overline{\chi}$ with measurable partition $\overline{\Delta}(T)$. Assume that there exists another $\widetilde{K}-$conditional equilibrium $\{\widetilde{w}^{ne}(.), w^e(.)\}$ with allocation $\widetilde{\chi}$ and measurable partition $\widetilde{\Delta}(T)$ which Pareto dominates $\overline{\chi}$ in the artificial economy with rentiers, i.e., such that $u(\widetilde{c}^s(.), \widetilde{h}^s(.)) \geq u(\overline{c}^s(.), \overline{h}^s(.))$ *a.e.,* with strict inequality for some positive measure subset of agents and such that a similar property holds for the set of rentiers. In view of Lemma 5, both partitions are defined by a threshold, $\overline{\delta}$ and $\widetilde{\delta}$, respectively. Without any loss of generality, assume that $\widetilde{\delta} \geq \overline{\delta}$. Consider the measurable set of agents choosing $s = e$ at both allocations. Then, $u(\widetilde{c}^e(.), \widetilde{h}^e(.)) \geq u(\overline{c}^e(.), \overline{h}^e(.))$ implies $\widetilde{c}^e(.) + T \geq \overline{w}^e \widetilde{h}^e(.)$ *a.e.*, with strict inequality, *a.e.*, for the agents such that the first inequality holds strictly. Similarly, for the agents choosing $s = ne$ at each allocation.

Consider now the set of agents such that $\overline{h}^{ne}(.) = 0$, while $\widetilde{h}^e(.) = 0$, i.e., the one switching from $s = e$ to $s = ne$. Still, we must have $\widetilde{c}^{ne}(.) \geq \overline{w}^{ne} \widetilde{h}^{ne}(.)$, because $u(\widetilde{c}^{ne}(.), \widetilde{h}^{ne}(.)) \geq u(\overline{c}^e(.), \overline{h}^e(.)) \geq u(\overline{c}^{ne}(.), \overline{h}^{ne}(.))$. Finally, for the (risk-neutral) rentiers, it must be

$$\int_0^1 \widetilde{c}_j(.) dj \geq \int_0^1 F_j(\widetilde{k}_j, \widetilde{\ell}^e_j, \widetilde{\ell}^{ne}_j) dj - \int_0^1 \widetilde{k}_j dj - \int_0^1 \overline{w}^e \widetilde{\ell}^e_j dj - \int_0^1 \overline{w}^{ne} \widetilde{\ell}^{ne}_j dj.$$

Integrating over the set of agents, we obtain

$$\int_{\underline{d}}^{\widetilde{\delta}} \int_0^1 \widetilde{c}^{ne}(.)did\delta^{ne} + \int_{\widetilde{\delta}}^{\overline{d}} \int_0^1 \widetilde{c}^{e}(.)did\delta^{e} + \int_{\widetilde{\delta}}^{\overline{d}} \int_0^1 Tdid\delta^{e} + \int_0^1 \widetilde{c}_j(.)dj$$

$$> \int_0^1 F_j(\widetilde{k}_j, \widetilde{\ell}_j^e, \widetilde{\ell}_j^{ne})dj - \int_0^1 \widetilde{k}_j dj,$$

which implies that the allocation $\widetilde{\chi}$ is not feasible. ■

Proof of Lemma 5. Consider any equilibrium allocation contingent on the profile of investments in physical capital. Fix $k_j = \overline{K} >> 0$, for each j. Let $\chi(T, \overline{K})$ denote any T-contingent allocation, $\{\Delta(.), (c^s(.), h^s(.))\}$.

First, let's show that $\Delta(T, \overline{K}) \equiv \{[\underline{d}, \delta(T, \overline{K})), [\delta(T, \overline{K}), \overline{d}]\} \times [0, 1]$. Consider the ($\overline{K}-$contingent) equilibrium allocation. If $\Delta^e(T, \overline{K}) = \emptyset$, we can set $\delta(T, \overline{K}) = \overline{d}$ and there is nothing to prove. Similarly if $\Delta^{ne}(T, \overline{K}) = \emptyset$. Hence, assume that $\Delta^e(T, \overline{K}) \neq \emptyset$ and $\Delta^{ne}(T, \overline{K}) \neq \emptyset$. By Lemma 1, at each δ,

$$\frac{\partial u(\widetilde{c}^e(.), \widetilde{h}^e(.), \delta; T)}{\partial \delta} - \frac{\partial u(\widetilde{c}^{ne}(.), \widetilde{h}^{ne}(.), \delta; T)}{\partial \delta} > 0,$$

where "$\widetilde{.}$" denotes notional consumption and labor supply. Uniqueness of the threshold follows immediately.

To show the second part of the Lemma, fix $k_j = \overline{K} >> 0$, for each j and any realization T. Consider the equilibrium conditions:

$$\Phi(w, \overline{\delta}; T, \overline{K}) \equiv \begin{bmatrix} \Phi^{ne}(w, \overline{\delta}; T, \overline{K}) \\ \\ \Phi^{e}(w, \overline{\delta}; T, \overline{K}) \\ \\ \Phi^{\delta}(w, \overline{\delta}; T, \overline{K}) \end{bmatrix} = \begin{bmatrix} \int_{\underline{d}}^{\overline{\delta}} \int_0^1 h_i^{ne}(.)did\delta^{ne} - \int_0^1 \ell_j^{ne}(.)dj \\ \\ \int_{\overline{\delta}}^{\overline{d}} \int_0^1 h_i^{e}(.)did\delta^{e} - \int_0^1 \ell_j^{e}(.)dj \\ \\ V^e(., \overline{\delta}) - V^{ne}(., \overline{\delta}) \end{bmatrix} = 0,$$

where $\overline{\delta}$ is the threshold defining the equilibrium partition. Hence,

$$D_{(w,\overline{\delta})}\Phi(w, \overline{\delta}; T, \overline{K}) \equiv \begin{bmatrix} \frac{\partial \Phi^{ne}(w,\overline{\delta};T,\overline{K})}{\partial w^{ne}} & 0 & \int_0^1 h_i^{ne}(.)di \\ \\ 0 & \frac{\partial \Phi^{e}(w,\overline{\delta};T,\overline{K})}{\partial w^{e}} & -\int_0^1 h_i^{e}(.)di \\ \\ -\frac{\partial v(.)}{\partial c^{ne}} h^{ne}(.) & \frac{\partial v(.)}{\partial c^{e}} h^{e}(.) & (g(h^e(.)) - g(h^{ne}(.))) \frac{\frac{\partial f(\delta)}{\partial \delta}}{f(\delta)^2}|_{\overline{\delta}} \end{bmatrix},$$

since $\frac{\partial V^s(.,\delta)}{\partial w^s} = \frac{\partial v(.)}{\partial c^s} h^s(.) + \left(\frac{\partial v(.)}{\partial c^s} w^s - \frac{\partial g(.)}{\partial h^s} \frac{1}{f(\overline{\delta})} \right) \frac{\partial h^s(.)}{\partial w^s} = \frac{\partial v(.)}{\partial c^s} h^s(.)$, at each optimal solution to the individual optimization problem, while $\frac{\partial V^s(.,\delta)}{\partial \delta} = \frac{\frac{\partial f(\delta)}{\partial \delta} g(h^s(.))}{f(\delta)^2}$, by the envelope theorem.

We now show that $\det D_{(w,\overline{\delta})}\Phi(.) \neq 0$. At an equilibrium, since $T > 0$, it must be $w^e > w^{ne}$. Under the maintained assumptions, $\frac{\partial \Phi^s(w,\overline{\delta};T,\overline{K})}{\partial w^s} > 0$ for each s. Consider the system of eqs. $D_{(w,\overline{\delta})}\Phi(.)\,[a,b,c]^T = 0$. Given the structure of $D_{(w,\overline{\delta})}\Phi(.)$, if a non trivial solution exists, it must be $c \neq 0$, so that we can set $c = 1$, which implies that it must be $a < 0$ and $b > 0$. Given that $\left[g(h^e(w^e,\overline{\delta})) - g(h^{ne}(w^{ne},\overline{\delta}))\right]\left(\frac{\partial f(\delta)}{\partial \delta}/f(\delta)^2\right) > 0$, for all the possible triples $(a,b,1)$ with these properties,

$$-a\frac{\partial v(.)}{\partial c^{ne}}h^{ne}(w^{ne},\overline{\delta})+b\frac{\partial v(.)}{\partial c^{e}}h^{e}(w^{e},\overline{\delta})+\left[g(h^e(w^e,\overline{\delta})) - g(h^{ne}(w^{ne},\overline{\delta}))\right]\left(\frac{\partial f(\delta)}{\partial \delta}/f(\delta)^2\right) > 0.$$

Hence, $D_{(w,\overline{\delta})}\Phi(.)$ has rank 3. By the IFT, locally, the solution to $\Phi(w,\overline{\delta};T,\overline{K}) = 0$ is a C^1 function. Since the set of equilibria, $\left(\{w^{ne}(.),w^e(.)\},\overline{\delta}(T,K)\right)$, coincides with the set of solutions to $\Phi(w,\overline{\delta};T,\overline{K}) = 0$, the result follows immediately. ■

Proof of Proposition 6. It suffices to show that, by imposing a small increase in the value of the equilibrium threshold $\overline{\delta}$, we can implement a Pareto improvement of the allocation. Since equilibrium expected profits are nil for each possible value of $\overline{\delta}$, we may ignore them. The expected utility of a generic high skilled worker with $\delta = \overline{\delta}^e$ is

$$E(V^e(w^e(\delta^{ne},\overline{\delta}^e;K(\overline{\delta}));\overline{\delta}^e)) = \int_{\underline{d}}^{\overline{\delta}} \frac{V^e(w^e(\delta^{ne},\overline{\delta}^e;K(\overline{\delta}));\overline{\delta}^e)d\delta^{ne}}{\overline{\delta}-\underline{d}},$$

so that

$$\begin{aligned}\frac{\partial E\left(V^e\left(.\right)\right)}{\partial\overline{\delta}} &= \frac{V^e(w^e(\delta^{ne},\overline{\delta}^e;K(\overline{\delta}));\overline{\delta}^e) - E(V^e(w^e(\delta^{ne},\overline{\delta}^e;K(\overline{\delta}));\overline{\delta}^e))}{\overline{\delta}-\underline{d}} \\ &\quad + \int_{\underline{d}}^{\overline{\delta}}\left(\frac{\frac{\partial V^e(.)}{\partial w^e}\frac{\partial w^e}{\partial K}d\delta^{ne}}{\overline{\delta}-\underline{d}}\right)\frac{\partial K}{\partial\overline{\delta}}.\end{aligned}$$

Similarly, assuming that $\overline{\delta}-\underline{d} > \overline{d}-\overline{\delta}$, the expected utility of a generic low skilled worker with $\delta = \overline{\delta}^{ne}$ is

$$\begin{aligned}E(V^{ne}(w^{ne}(\delta^{ne},\overline{\delta}^e;K(\overline{\delta})),\underline{w}^{ne}(\delta^{ne});\overline{\delta}^{ne})) &= \frac{\overline{d}-\overline{\delta}}{\overline{\delta}-\underline{d}}\int_{\overline{\delta}}^{\overline{d}}\frac{V^{ne}(w^{ne}(\delta^{ne},\overline{\delta}^e;K(\overline{\delta}));\overline{\delta}^{ne})d\delta^e}{\overline{d}-\overline{\delta}} \\ &\quad + \frac{2\overline{\delta}-(\overline{d}+\underline{d})}{\overline{\delta}-\underline{d}}V^{ne}(\underline{w}^{ne}(\delta^{ne};\underline{K}(\overline{\delta}));\overline{\delta}^{ne})\end{aligned}$$

and

$$\begin{aligned}\frac{\partial E(V^{ne}(.))}{\partial \overline{\delta}} \quad = \quad & \frac{\overline{d}-\underline{d}}{(\overline{\delta}-\underline{d})^2}\left(V^{ne}(\underline{w}^{ne}(\delta^{ne};\underline{K}(\overline{\delta}));\overline{\delta}^{ne}) - \int_{\overline{\delta}}^{\overline{d}} \frac{V^{ne}(.)d\delta^e}{\overline{\delta}-\underline{d}}\right) \\ & + \frac{\overline{d}-\overline{\delta}}{\overline{\delta}-\underline{d}} \frac{-V^{ne}(w^{ne}(\delta^{ne},\overline{\delta}^e;K(\overline{\delta}));\overline{\delta}^{ne}) + E(V^{ne}(w^{ne}(\delta^{ne},\overline{\delta}^e;K(\overline{\delta}));\overline{\delta}^{ne}))}{\overline{d}-\overline{\delta}} \\ & + \frac{\overline{d}-\overline{\delta}}{\overline{\delta}-\underline{d}}\int_{\overline{\delta}}^{\overline{d}} \frac{\frac{\partial V^{ne}(.)}{\partial w^{ne}}\frac{\partial w^{ne}}{\partial K} d\delta^e}{\overline{d}-\overline{\delta}}\frac{\partial K}{\partial\overline{\delta}} \\ & + \frac{2\overline{\delta}-(\overline{d}+\underline{d})}{\overline{\delta}-\underline{d}} \frac{\partial V^{ne}(\underline{w}^{ne}(\delta^{ne};\underline{K}(\overline{\delta}));\overline{\delta}^{ne})}{\partial \underline{w}}\frac{\partial \underline{w}}{\partial \underline{K}}\frac{\partial \underline{K}}{\partial \overline{\delta}}.\end{aligned}$$

Lemma A1 below establishes that, under the maintained assumptions,

$$\begin{aligned} 0 \quad \leq \quad & \left(\frac{V^e(w^e(\delta^{ne},\overline{\delta}^e;K(\overline{\delta}));\overline{\delta}^e) - E(V^e\left(w^e(\delta^{ne},\overline{\delta}^e;K(\overline{\delta}));\overline{\delta}^e\right))}{\overline{\delta}-\underline{d}}, \frac{\partial w^e}{\partial K}, \frac{\partial K}{\partial \overline{\delta}}\right) \\ & \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (A) \\ 0 \quad \leq \quad & \left(\frac{-V^{ne}(w^{ne}(\delta^{ne},\overline{\delta}^e;K(\overline{\delta}));\overline{\delta}^{ne}) + E(V^{ne}(w^{ne}(\delta^{ne},\overline{\delta}^e;K(\overline{\delta}));\overline{\delta}^{ne}))}{\overline{d}-\overline{\delta}}, \frac{\partial w^{ne}}{\partial K}, \frac{\partial \underline{w}^{ne}}{\partial \underline{K}}, \frac{\partial \underline{K}}{\partial \overline{\delta}}\right).\end{aligned}$$

Hence, for each high skill worker, $\frac{\partial E(V^e(.))}{\partial \overline{\delta}} > 0$. For low skill workers, $\frac{\partial E(V^{ne}(.))}{\partial \overline{\delta}} > 0$, provided that the expected utility gain of working jointly with high skill workers, $\left(V^{ne}(.);\overline{\delta}^{ne}) - \int_{\overline{\delta}}^{\overline{d}} \frac{V^{ne}(w^{ne}(.);\overline{\delta}^{ne})d\delta^e}{\overline{d}-\overline{\delta}}\right)$, is sufficiently small. ■

Lemma A1. Under the assumption of Proposition 6, at each equilibrium, inequalities (A) hold.

Proof of Lemma A1. Let's first consider the industries where both low and high skill workers are active. The equilibrium conditions are described by

$$\Psi(.) \equiv \begin{bmatrix} \ell^e(w^{ne},w^e;K) - h^e(w^e;\delta^e) \\ \\ \ell^{ne}(w^{ne},w^e;K) - h^{ne}(w^{ne};\delta^{ne}) \end{bmatrix} = 0,$$

where, for economy of notation, we omit the integration of the labor supply over the interval [0,1] of identical workers. By the IFT,

$$\begin{bmatrix} D_{(K,\delta^e,\delta^{ne})}w^e \\ \\ D_{(K,\delta^e,\delta^{ne})}w^{ne} \end{bmatrix} = \frac{1}{\det D_{(w^e,w^{ne})}\Psi(.)}\times$$

$$\left[\begin{array}{c|c|c} \frac{\partial \ell^e}{\partial w^{ne}}\frac{\partial \ell^{ne}}{\partial K} - \left(\frac{\partial \ell^{ne}}{\partial w^{ne}} - \frac{\partial h^{ne}}{\partial w^{ne}}\right)\frac{\partial \ell^e}{\partial K} & \left(\frac{\partial \ell^{ne}}{\partial w^{ne}} - \frac{\partial h^{ne}}{\partial w^{ne}}\right)\frac{\partial h^e}{\partial \delta^{ne}} & -\frac{\partial h^{ne}}{\partial \delta^{ne}}\frac{\partial \ell^{ne}}{\partial w^e} \\ \hline \frac{\partial \ell^{ne}}{\partial w^e}\frac{\partial \ell^e}{\partial K} - \left(\frac{\partial \ell^e}{\partial w^e} - \frac{\partial h^e}{\partial w^e}\right)\frac{\partial \ell^{ne}}{\partial K} & -\frac{\partial h^e}{\partial \delta^e}\frac{\partial \ell^e}{\partial w^{ne}} & \left(\frac{\partial \ell^e}{\partial w^e} - \frac{\partial h^e}{\partial w^e}\right)\frac{\partial h^{ne}}{\partial \delta^{ne}} \end{array}\right]$$

Consider first the determinant,

$$\det D_{(w^e,w^{ne})}\Psi(.) = \left(\frac{\partial \ell^e}{\partial w^e}\frac{\partial \ell^{ne}}{\partial w^{ne}} - \frac{\partial \ell^e}{\partial w^{ne}}\frac{\partial \ell^{ne}}{\partial w^e}\right) + \frac{\partial h^e}{\partial w^e}\frac{\partial h^{ne}}{\partial w^{ne}} - \frac{\partial \ell^e}{\partial w^e}\frac{\partial h^{ne}}{\partial w^{ne}} - \frac{\partial h^e}{\partial w^e}\frac{\partial \ell^{ne}}{\partial w^{ne}}.$$

The term in brackets is positive, since $\left(\frac{\partial \ell^e}{\partial w^e}\frac{\partial \ell^{ne}}{\partial w^{ne}} - \frac{\partial \ell^e}{\partial w^{ne}}\frac{\partial \ell^{ne}}{\partial w^e}\right)$ is the determinant of the negative definite square matrix $D_w\ell(.)$. The signs of the other three terms are obvious and it is clear that $\det D_{(w^e,w^{ne})}\Psi(.) > 0$.

By the FOCs of the firm (ex-post) optimization problem and the IFT,

$$\begin{bmatrix} \frac{\partial \ell^e}{\partial K} \\ \\ \frac{\partial \ell^{ne}}{\partial K} \end{bmatrix} = \frac{1}{\frac{\partial^2 F_j}{\partial \ell^{e2}}\frac{\partial^2 F_j}{\partial \ell^{ne2}} - \left(\frac{\partial^2 F_j}{\partial \ell^e \partial \ell^{ne}}\right)^2} \begin{bmatrix} \frac{\partial^2 F_j}{\partial \ell^{ne}\partial K}\frac{\partial^2 F_j}{\partial \ell^e \partial \ell^{ne}} - \frac{\partial^2 F_j}{\partial \ell^{ne2}}\frac{\partial^2 F_j}{\partial \ell^e \partial K} \\ \\ \frac{\partial^2 F_j}{\partial \ell^e \partial \ell^{ne}}\frac{\partial^2 F_j}{\partial \ell^e \partial K} - \frac{\partial^2 F_j}{\partial \ell^{e2}}\frac{\partial^2 F_j}{\partial \ell^{ne}\partial K} \end{bmatrix}.$$

Concavity of $F_j(.)$ implies that $\frac{\partial^2 F_j}{\partial \ell^{e2}}\frac{\partial^2 F_j}{\partial \ell^{ne2}} - \left(\frac{\partial^2 F_j}{\partial \ell^e \partial \ell^{ne}}\right)^2 > 0$. Hence, under *E-complementarity,* $\left(\frac{\partial \ell^{ne}}{\partial K}, \frac{\partial \ell^e}{\partial K}\right) \gg 0$.

Similarly, applying the IFT to the FOCs of the firms' optimization problem, one can easily derive the explicit formulas for the matrix $D_w\ell(.)$. Replacing these values into the vector $\left(\frac{\partial \ell^{ne}}{\partial K}, \frac{\partial \ell^e}{\partial K}\right)$, a simple computation shows that

$$\begin{bmatrix} D_K w^e \\ \\ D_K w^{ne} \end{bmatrix} = \begin{bmatrix} \frac{\frac{\partial^2 F_j}{\partial \ell^e \partial K}}{\frac{\partial^2 F_j}{\partial \ell^{e2}}\frac{\partial^2 F_j}{\partial \ell^{ne2}} - \left(\frac{\partial^2 F_j}{\partial \ell^e \partial \ell^{ne}}\right)^2} + \frac{\partial \ell^e}{\partial K}\frac{\partial h^{ne}}{\partial w^{ne}} \\ \\ \frac{\frac{\partial^2 F_j}{\partial \ell^{ne} \partial K}}{\frac{\partial^2 F_j}{\partial \ell^{e2}}\frac{\partial^2 F_j}{\partial \ell^{ne2}} - \left(\frac{\partial^2 F_j}{\partial \ell^e \partial \ell^{ne}}\right)^2} + \frac{\partial \ell^{ne}}{\partial K}\frac{\partial h^e}{\partial w^e} \end{bmatrix} >> \begin{bmatrix} 0 \\ \\ 0 \end{bmatrix}.$$

Hence, under *E-complementarity*, the pattern of signs of the matrix $D_{(K,\delta^e,\delta^{ne})}w$ is

$$\begin{bmatrix} + & - & + \\ + & + & - \end{bmatrix}.$$

Consider now an arbitrary, high skill agent $\overline{\delta}^e$. Under *E-complementarity,* $\frac{\partial w^e}{\partial \delta^{ne}} > 0$ and, therefore, $\frac{V^e(w^e(\overline{\delta},\overline{\delta}^e;K(\overline{\delta}))) - E(V^e(w^e(\delta^{ne},\overline{\delta}^e;K(\overline{\delta})))}{\overline{\delta}-\underline{d}} > 0$. The same argument implies that $\frac{V^{ne}(w^{ne}(\overline{\delta}^{ne},\overline{\delta};K(\overline{\delta}))) - E(V^{ne}(w^{ne}(\overline{\delta}^{ne},\delta^e;K(\overline{\delta})))}{\overline{\delta}-\underline{d}} < 0$.

Let $\Pi(K;\delta^{ne},\delta^e,\overline{\delta})$ be the producers' surplus in state (δ^{ne},δ^e) and let

$$E(\Pi(K;\delta^{ne},\delta^e,\overline{\delta})) \equiv \frac{1}{\overline{\delta}-\underline{d}}\int_{\underline{d}}^{\overline{\delta}}\left(\frac{\int_{\overline{\delta}}^{\overline{d}}\Pi(K;\delta^{ne},\delta^e,\overline{\delta})d\delta^e}{\overline{d}-\overline{\delta}}\right)d\delta^{ne}.$$

Evidently, at each equilibrium, $E\left(\Pi\left(K;\delta^{ne},\delta^e,\overline{\delta}\right)\right) = 0$. Consider its derivative with respect to the threshold $\overline{\delta}$:

$$\frac{\partial E(\Pi(K;\delta^{ne},\delta^{e},\overline{\delta}))}{\partial\overline{\delta}} = \frac{\frac{\int_{\overline{\delta}}^{\overline{d}}\Pi(K;\delta^{ne}=\overline{\delta},\delta^{e},\overline{\delta})d\delta^{e}}{\overline{d}-\overline{\delta}} - E(\Pi(K;\delta^{ne},\delta^{e},\overline{\delta}))}{\overline{\delta}-\underline{d}}$$

$$-\frac{1}{\overline{\delta}-\underline{d}}\int_{\underline{d}}^{\overline{\delta}}\frac{\Pi(K;\delta^{ne},\delta^{e}=\overline{\delta},\overline{\delta}) - \frac{\int_{\overline{\delta}}^{\overline{d}}\Pi(K;\delta^{ne},\delta^{e},\overline{\delta})}{\overline{d}-\overline{\delta}}}{\overline{d}-\overline{\delta}}d\delta^{ne}.$$

Given (K,δ^{e}) , under *E-complementarity,*

$$\frac{\partial\Pi(K;\delta^{ne},\delta^{e},\overline{\delta})}{\partial\delta^{ne}} = \left[\frac{\partial h^{e}}{\partial w^{e}}\ell^{ne}(.) + \frac{\partial\ell^{e}}{\partial w^{ne}}\ell^{e}(.) - \frac{\partial\ell^{e}}{\partial w^{e}}\ell^{ne}(.)\right]\frac{\partial h^{ne}}{\partial\delta^{ne}} > 0.$$

Since $E(\Pi(K;\delta^{ne},\delta^{e},\overline{\delta})) = 0$, this implies $\frac{\int_{\overline{\delta}}^{\overline{d}}\Pi(K;\delta^{ne}=\overline{\delta},\delta^{e},\overline{\delta})d\delta^{e}}{\overline{d}-\overline{\delta}} > 0$. Essentially the same argument implies that $\int_{\underline{d}}^{\overline{\delta}}\frac{\Pi(K;\delta^{ne},\delta^{e}=\overline{\delta},\overline{\delta})-\frac{\int_{\overline{\delta}}^{\overline{d}}\Pi(K;\delta^{ne},\delta^{e},\overline{\delta})d\delta^{e}}{\overline{d}-\overline{\delta}}}{\overline{d}-\overline{\delta}}d\delta^{ne} < 0$. Hence, $\frac{\partial E(\Pi(K;\delta^{ne},\delta^{e},\overline{\delta}))}{\partial\overline{\delta}} > 0$.

The effect of a change in the aggregate investments is

$$\frac{\partial E(\Pi(K;\delta^{ne},\delta^{e},\overline{\delta}))}{\partial K} = \frac{1}{\overline{\delta}-\underline{d}}\int_{\underline{d}}^{\overline{\delta}}\frac{\int_{\overline{\delta}}^{\overline{d}}\frac{\partial\Pi(K;\delta^{ne},\delta^{e},\overline{\delta})}{\partial K}d\delta^{e}}{\overline{d}-\overline{\delta}}d\delta^{ne},$$

with $\frac{\partial\Pi(K;\delta^{ne},\delta^{e},\overline{\delta})}{\partial K} = -\left[\frac{\partial w^{e}}{\partial K}\ell^{e}(.) + \frac{\partial w^{ne}}{\partial K}\ell^{ne}(.)\right] < 0$, by *E-complementarity.*

Hence, for the equilibrium value of $K(\delta)$, it must be $\frac{\partial K(\overline{\delta})}{\partial\overline{\delta}} > 0$.

$\left(\frac{\partial w^{ne}}{\partial\underline{K}^{ne}}, \frac{\partial\underline{K}^{ne}}{\partial\overline{\delta}}\right) > 0$ can be established with a similar argument. ■

Proof of Proposition 7. Here, it is crucial the distinction between investments of an individual firm, k_j, and aggregate investments, K. Since at each equilibrium, $k_j = K$, we will just consider pairs satisfying this restriction. However, keep in mind that changes in k_j, for some j, do not have any effect on the endogenous variables (but the ones referred to firm j).

Consider the subeconomies where firms use both types of labor inputs. Let $w(\delta^{ne},\delta^{e},K;\overline{\delta}) \equiv \{w^{ne}(\delta^{ne},\delta^{e},K;\overline{\delta}), w^{e}(\delta^{ne},\delta^{e},K;\overline{\delta})\}$ be the associated equilibrium pair in state (δ^{ne},δ^{e}). Under the maintained assumptions, it is easy to check that $w(\delta^{ne},\delta^{e},K;\overline{\delta})$ is a C^{1} function of $(\delta^{ne},\delta^{e};K)$. Moreover, since all inputs are *E-complements,* $\nabla_{K}w >> 0$ at each $\{\delta^{ne},\delta^{e}\}$, as established in the proof of Lemma A1.

Given $\overline{\delta}$, let $E(\Pi_j(w,k_j,K;\overline{\delta}))$ be the expected value of profits, which depends directly on the investments of the single firm, k_j, and, indirectly, on the aggregate investments, K, because of their effect on the equilibrium wage map $w(\delta^{ne},\delta^{e},K;\overline{\delta})$.

At the market clearing wages, by the envelope theorem, $\frac{\partial E(\Pi_j(w,k_j,K;\overline{\delta}))}{\partial k_j} = \frac{\partial E(F_j(k_j,\delta^{ne},\delta^{e}))}{\partial k_j} - 1$.

Since, at each conditional equilibrium, labor markets clear,

$$\frac{\partial E(F_j(.))}{\partial k_j} - 1 = \frac{1}{\overline{\delta} - \underline{d}} \int_{\underline{d}}^{\overline{\delta}} \left(\frac{\int_{\overline{\delta}}^{\overline{d}} \frac{\partial F_j(k_j, \delta^{ne}, \delta^e)}{\partial k_j} d\delta^e}{\overline{d} - \overline{\delta}} \right) d\delta^{ne} - 1.$$

Under the maintained assumptions on $F_j(.)$, and given that labor is inelastically supplied,

$$\lim_{k_j \to 0} \frac{1}{\frac{\partial E(F_j(.))}{\partial k_j}} = 0, \quad \text{and} \quad \lim_{k_j \to \infty} \frac{\partial E(F_j(.))}{\partial k_j} = 0.$$

Hence, for each $\overline{\delta} \in (\underline{d}, \overline{d})$, there is a k_j such that $\frac{\partial E(F(.))}{\partial k_j} = 1$. As returns to scale are constant, $E(\Pi_j(w, k_j, K; \overline{\delta}))$ is a linear function of k_j. Therefore, $\frac{\partial E(F_j(.))}{\partial k_j}$ just depends upon aggregate investments, K, because of their effects on equilibrium wages. Since $w^s(\delta^{ne}, \delta^e, K; \overline{\delta})$ is increasing in K, for each s,

$$\frac{\partial^2 E(\Pi_j(.))}{\partial k_j \partial K} = \frac{\partial^2 E(\Pi_j(.))}{\partial k_j \partial w^{ne}} \frac{\partial w^{ne}}{\partial K} + \frac{\partial^2 E(\Pi_j(.))}{\partial k_j \partial w^e} \frac{\partial w^e}{\partial K} < 0.$$

Hence, for each $\overline{\delta}$, there is at most a unique value $K(\overline{\delta})$ such that $\frac{\partial E(F_j(.))}{\partial k_j} = 1$. This implies that $K(\overline{\delta})$ is a function on its domain of definition. By the *IFT* applied to the eq. $\frac{\partial E(F_j(.))}{\partial k_j} = 1$, $\frac{\partial K(\overline{\delta})}{\partial \overline{\delta}} = -\frac{\frac{\partial^2 E(F_j(.))}{\partial k_j \partial \overline{\delta}}}{\frac{\partial^2 E(F_j(.))}{\partial k_j \partial K}}$. By direct computation,

$$\frac{\partial^2 E(F_j(.))}{\partial k_j \partial \overline{\delta}} = \frac{\frac{\int_{\overline{\delta}}^{\overline{d}} \frac{\partial F(k_j, \delta^{ne} = \overline{\delta}, \delta^e)}{\partial k_j} d\delta^e}{\overline{d} - \overline{\delta}} - \frac{\partial E(F_j(.))}{\partial k_j}}{\overline{\delta} - \underline{d}} - \frac{\frac{\int_{\underline{d}}^{\overline{\delta}} \frac{\partial F_j(k_j, \delta^{ne}, \delta^e = \overline{\delta})}{\partial k_j} d\delta^{ne}}{\overline{\delta} - \underline{d}} - \frac{\partial E(F_j(.))}{\partial k_j}}{\overline{d} - \overline{\delta}} > 0.$$

This inequality holds because, under *E-complementarity*, $\frac{\partial^2 F_j(k_j, \delta^{ne} = \overline{\delta}, \delta^e)}{\partial k_j \partial \ell^s} > 0$, for each s, which implies

$$\frac{\int_{\overline{\delta}}^{\overline{d}} \frac{\partial F_j(k_j, \delta^{ne} = \overline{\delta}, \delta^e)}{\partial k_j} d\delta^e}{\overline{d} - \overline{\delta}} > \frac{\partial E(F_j(.))}{\partial k_j},$$

and

$$\int_{\underline{d}}^{\overline{\delta}} \frac{\partial F_j(k_j, \delta^{ne}, \delta^e = \overline{\delta})}{\partial k_j} d\delta^e < \frac{\partial E(F_j(.))}{\partial k_j}.$$

As already established, $\frac{\partial^2 E(F_j(.))}{\partial k_j \partial K} < 0$. Thus, $\frac{\partial K(\overline{\delta})}{\partial \overline{\delta}} > 0$. These results basically translate into Figure 3 in the main text.

Analogous properties can be established for the investments and the equilibrium wages in the industry using just unskilled labor.

Consider any $\overline{\delta}-$conditional equilibrium. Given the properties of the technologies, it is easy to check that, for each threshold $\overline{\delta}$, $K(\overline{\delta}) > \underline{K}(\overline{\delta})$, and, for each (δ^{ne}, δ^e), $w^{ne}(\delta^{ne}, \delta^e, K(\overline{\delta})) > \underline{w}(\delta^{ne}, \underline{K}(\overline{\delta}))$.

The strategy of our proof is now to construct explicitly an equilibrium for an appropriately selected value of T. Given our approach, we can pick arbitrarily the threshold value. To streamline the argument, we choose $\overline{\delta}=\frac{\underline{d}+\overline{d}}{2}$, so that only the industry using both skills is active. This entails no loss of generality whatsoever.

Define the map $M(T):\mathbb{R}_+\to\mathbb{R}$,

$$M(T)=E(u(w^e(\delta^{ne},\delta^e=\overline{\delta},K(\overline{\delta}));\overline{\delta}))-E(u(w^{ne}(\delta^{ne}=\overline{\delta},\delta^e,K(\overline{\delta}));\overline{\delta})).$$

Given the production function specified above, for each $K>0$, $w^e(\delta^{ne},\overline{\delta},K(\overline{\delta}))\geq w^e(\underline{d},\overline{\delta},K(\overline{\delta}))$ and $w^{ne}(\overline{\delta},\overline{d},K(\overline{\delta}))\geq w^{ne}(\overline{\delta},\delta^e,K(\overline{\delta}))$. Clearly,

$$\frac{w^e(\underline{d},\overline{\delta},K(\overline{\delta}))\overline{\delta}}{w^{ne}(\overline{\delta},\overline{d},K(\overline{\delta}))\overline{\delta}}=\left(\frac{\psi^{ne}\underline{d}^{\theta}+\psi^e\overline{\delta}^{\theta}}{\psi^{ne}\overline{\delta}^{\theta}+\psi^e\overline{d}^{\theta}}\right)^{\frac{1-\alpha-\theta}{\theta}}\frac{\psi^e}{\psi^{ne}}>1 \qquad (B)$$

for $\frac{\psi^e}{\psi^{ne}}$ large enough. Therefore, for an appropriate set of values of the parameters, $w^e(\delta^{ne},\overline{\delta},K(\overline{\delta}))\overline{\delta}>w^{ne}(\overline{\delta},\delta^e,K(\overline{\delta}))\overline{\delta}$ for each pair (δ^{ne},δ^e). This immediately implies that $M(0)>0$. By continuity of $M(T)$, there is some value $\overline{T}$ such that $M(\overline{T})=0$. Therefore, the selected $\overline{\delta}$ is a threshold for $T=\overline{T}$.

To verify that $\overline{\delta}$ is the only equilibrium threshold, given $\overline{T}$, it has to be proved that, given $K(\overline{\delta})$, $M(\delta,\overline{T})\equiv E(u(w^e(.))-E(u(w^{ne}(.)))>0$ if and only if $\delta>\overline{\delta}(T)$. This immediately follows if $\frac{\partial M}{\partial\delta}>0$.

Given that $u(c^s,h^s)=v(c^s)$, by direct computation,

$$\begin{aligned}\frac{\partial M}{\partial\delta} &= \int_{\underline{d}}^{\overline{\delta}}\frac{\frac{\partial v}{\partial c^e}\left(w^e(.)+\frac{\partial w^e(.)}{\partial\delta}\delta\right)d\delta^{ne}}{\overline{\delta}-\underline{d}}-\int_{\overline{\delta}}^{\overline{d}}\frac{\frac{\partial v}{\partial c^{ne}}\left(w^{ne}(.)+\frac{\partial w^{ne}(.)}{\partial\delta}\delta\right)}{\overline{d}-\overline{\delta}}d\delta^e\\ &= \int_{\underline{d}}^{\overline{\delta}}\frac{\frac{\partial v}{\partial c^e}w^e(\delta^{ne},\delta,K(\overline{\delta}))}{\overline{\delta}-\underline{d}}\frac{\theta\psi^{ne}\delta^{ne\theta}+(1-\alpha)\psi^e\delta^{\theta}}{\psi^{ne}\delta^{ne\theta}+\psi^e\delta^{\theta}}d\delta^{ne}\\ &\quad-\int_{\overline{\delta}}^{\overline{d}}\frac{\frac{\partial v}{\partial c^{ne}}w^{ne}(\delta,\delta^e,K(\overline{\delta}))}{\overline{d}-\overline{\delta}}\frac{\theta\psi^e\delta^{e\theta}+(1-\alpha)\psi^{ne}\delta^{\theta}}{\psi^{ne}\delta^{\theta}+\psi^e\delta^{e\theta}}d\delta^e.\end{aligned}$$

For $\frac{\partial^2u}{\partial c^2}$ sufficiently close to 0, and given that $(1-\alpha)\geq\theta$,

$$\begin{aligned}\frac{\partial M}{\partial\delta} &> \theta\int_{\underline{d}}^{\overline{\delta}}\frac{w^e(\delta^{ne},\delta,K(\overline{\delta}))d\delta^{ne}}{\overline{\delta}-\underline{d}}-(1-\alpha)\int_{\overline{\delta}}^{\overline{d}}\frac{w^{ne}(\delta,\delta^e,K(\overline{\delta}))d\delta^e}{\overline{d}-\overline{\delta}}\\ \frac{\partial M}{\partial\delta} &> \int_{\underline{d}}^{\overline{\delta}}\frac{w^e(\delta^{ne},\delta,K(\overline{\delta}))d\delta^{ne}}{\overline{\delta}-\underline{d}}-(1-\alpha)\int_{\overline{\delta}}^{\overline{d}}\frac{w^{ne}(\delta,\delta^e,K(\overline{\delta}))d\delta^e}{\overline{d}-\overline{\delta}}>0\\ &> \theta w^e(\underline{d},\delta,K(\overline{\delta}))\overline{\delta}-(1-\alpha)w^{ne}(\delta,\overline{d},K(\overline{\delta}))\overline{\delta}>0,\end{aligned}$$

where the last inequality follows, for $\frac{\psi^e}{\psi^{ne}}$ large enough, by the same argument used to establish (B) above. This concludes the proof.

To summarize, provided that the two types of labor are *E-complements*, that the second order derivative of the uttility function is sufficiently small (in absolute value), and that $\frac{\psi^e}{\psi^{ne}}$ is large enough, an equilibrium exists. Since it is, locally, defined by a collection of C^1 functions, it is easy to check that the set of economies such that an equilibrium exists is open. $\blacksquare$

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