A DSGE model with unemployment and the role of institutions

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Abstract

During the last years, after the outburst of the global financial crisis and the troubles with EU sovereign debts followed by an increase in the unemployment rate and a drop in the output growth, some authors – among whom Smets, Wouters and Galì (2012) - included the unemployment rate as an observable variable in DSGE NK models. I embed into these kinds of models a proportional taxation, a public debt path and a productive public expenditure. The aim of the paper is to evaluate the effects of a fiscal policy and to capture the role of tax rates. The work has also the purpose to investigate the role of labor market frictions in form of search costs. The taxation has distortionary effects on the framework and a public expenditure shock has positive effects in particular on the unemployment rate and on the output while the absence of search costs has only effects on the demand shocks.

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1 Introduction

During the last years many New Keynesian (below NK) Diagnostic Stochastic General Equilibrium (henceforth DSGE) works with labor market frictions and wage rigidities consider the unemployment as a key variable - among others Galì, Smets and Wouters (2012); Christiano, Trabandt and Walentin(2013); Galì (2010); Trigari (2009) and Blanchard and Galì (2010) - in order to capture what drive the unemployment rate's fluctations. Blanchard and Galì (2010) consider the unemployment rate in a model with labor market frictions, following the search and matching work of Mortensen and Pisarrides (below MP), and real wage rigidities. The two authors conclude that the relation between inflation and unemployment depends on the relation between labor market tightness and unemployment. From a normative point of view, they argue that, in the presence of labor market frictions and real wage rigidities, strict inflation stabilization does not deliver the best monetary policy. This means that a monetary policy rule, set up by a central bank with the goal of stabilizing inflation, can lead to inefficient and persistent movements of the unemployment rate in response to a productivity shock only with staggered prices and real wage rigidities.

Galì, Smets and Wouters (2012) investigate what are the sources of the unemployment rate movements in a framework - following Smets and Wouters (2007) - with nominal and real wage rigidities but without labor market frictions and Nash bargaining of wages. They find that in the short term the fluctuations of the unemployment rate are driven by demand factors and shocks, while in a medium term are supply shocks which determine the unemployment rate. They also consider involuntary unemployment derived by market powers following Galì (2011a) and Galì (2011b).

Galì (2010) describes all the extensions of the NK baseline framework, taking into account the unemployment as an observable variable. Galì finds out that labor market frictions have not a big influence on the equilibrium but are able to explain the link between wage rigidities and unemployment.

In my model I extend the NK models just described embedding in them income taxes, a law of motion of public debt and a productive public expenditure.[1] The aim of the work is to evaluate the role of taxes in affecting the business cycle and the effects of an expansive fiscal policy financed with debt. The model has also the goal to capture the effects of labor market frictions.

The paper is structured in six sections. Section I serves as an introduction.

In Section II, I set up a model with a capital accumulation function, investment adjustment costs, labor market frictions and I embed the labor force in the utility function. In this way, I take into account employed people, people searching for a job and inactive people. Thus, I

[1] Many NK works take into account taxes. For example, Ravenna and Walsh (2010) insert in a Nk model taxes which are able to implement a first best equilibrium. Annichiarico et alt. (2009) make a confront between an active and inactive fiscal policy in a model which includes taxes and a public debt path while Krause and Moyen (2013) study the effects of inflation on the real public debt.

consider the opportunity cost of being in the labor market or not. The opportunity cost is due to the fact being in the labor market has a cost for people: people employed have a disutility from working while people unemployed sustain costs- direct and indirect - to search for a job. I consider a product market with the presence of many firms who can set the prices of their goods. The prices are sticky following the Calvo rule (Calvo 1983). I find out the Nk Philips curve. Firms choose the level of employment while the wage is object of bargaining.

In Section III, I describe the structure of the labor market following a strand of literature, which combines the NK DSGE model with the search and matching framework of Mortensen and Pissarides (1994) and Pissarides (2000)[2]. I make allowance for nominal rigidities - only a fraction of workers are able to negotiate wages during a given period - in the shape of Carlo rule. I assume a labor market with two kinds of frictions: workers and firms bargain the wages in order to catch the wage surplus and the second one is represented by the presence of search costs. Thus, I find a wage curve - or a wage index equation - with a non- competitive labor market.

Nominal and real wage rigidities are embedded in NK models, both with labor market frictions and without, with the purpose to reconcile the framework to what the empirical evidence displays[3].

In Section IV, I briefly describe the monetary policy rule and the fiscal policy rule adopted respectively by the central bank and the government. I allow for a budget constraint of the government not in equilibrium. I assume that government emits bonds in order to finance public expenditure and to pay the interests on past debt. There is room for fiscal policy: the public expenditure, financed with debt and taxes, is productive and it is inside the production function.

The central bank controls the interest rate following a simple Taylor rule[4]. In section V, I calibrate the model in order to obtain the impulse response functions of eight variables - interest rate, inflation rate, employment, unemployment, labor force, consumption, wage and output - referred to a monetary shock, to a productivity shock, to a government expenditure shock and to a supply shock. First, I obtain the IRFS, considering the model with taxation, and then I consider the model with lower tax rates in order to capture the role of an expansive fiscal policy like could be a measure of tax cut. In paragraph three, I take into account the model without search costs with the aim to find out the effects of search costs - labor market frictions - on the business cycle.

Section VI serves as a conclusion to the paper.

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[2] The first author to embed labor market frictions in a dynamic general model was Merz (1995). Her framework has a walrasian product market.

[3] Erceg and Levine (2000) insert in the Nk model without labor market frictions staggered wages. Shimer (2005) and Hall (2005) embed in the NK model labor market frictions and real wage rigidities. Trigari and Gertler (2009) in their work include in a NK model labor market frictions and staggered multi-periods wage. Blanchard and Galì (2010) include real wage rigidities in a framework with Nash bargaining and labor market frictions. Christofell and Linzert (2010) make a confront between right to manage and efficient bargaining.

[4] See Taylor (1993).

2 The households and the firms.

In this section, I set up the model taking into account the households and the firms. In paragraph one, I describe the behavior of the households, which supply labor and they are the owner of the capital stock. I find out the equation for the labor supply and the consumption euler equation. In paragraph two, I explain the features of firms. I consider a product market with sticky prices, following Calvo (1983), and not in perfect competition: firms are able to set prices. I also find the marginal cost to which firms apply a mark-up. In paragraph three, I take into account the maximization of the firms' profit with the purpose to catch on the NK Philips curve.

2.1 Households

In this paragraph, I describe the behavior of households. First, I consider consumption's decision of the households, then I investigate the labor supply.

2.1.1 Consumption

I analyze an economy composed by a large number of homogenous households. I consider full consumption risk sharing within each household[5]. The representative household, or family, has the following utility function:

$$E_0 \sum_{t=0}^{\infty} B^t \, U(C_t, L_t) \qquad 1)$$

The total consumption of the households is defined by a CES - constant elasticity of substitution - function considering a Dixit - Stiglitz aggregator[6]:

$$C_t = \left(\sum_{i=0}^{\infty} {C_{tf}}^{1-\frac{1}{v}} \right)^{\frac{v}{v-1}}$$

where C_t is the total consumption and C_{tf} indicates the consumption for a representative household of a single differentiated good. Indeed, the family chooses the level of total consumption considering every differentiated good. v the elasticity of substitution between the consumption of goods.

In the labor market there is no full participation and as consequence I take into account the existence of the labor force[7]:

[5] Merz (Merz 1995) was the first author to adopt a representative household with a conventional utility function in a search model

[6] See Dixit and Stiglitz (1977).

[7] The labor force coincides with the labor supply.

$L_t = N_t + U_t$

where N_t and U_t denote respectively the fraction of workers, household members employed, and the fraction of households unemployed and looking for a job. These kinds of people are inside the job market. Indeed, I consider people who are not in the job market as inactive people. The unemployment rate is in the utility function of the households. Thus, the decisions of the households are affected both by the employment rate and by the unemployment rate considering the labor force. Gali (2010) uses a similar structure in a DSGE model.

The households buy the goods with the wage earned from their work, thus they have a disutility from working and for searching for a job. I assume - following Galì (2011b) and Galì, Smet and Wouters (2012) - that labor is indivisible: changes in the employment rate are possible at the extensive margin, this is coherent with the data. Movements of the employment and unemployment at the intensive margin are possible but they do not affect the fluctuations of both variables.

Many authors - Smets and Vouters (2003 and 2007), Christiano, Eichebaum and Evans (2005) and Gertler sala and Trigari (2008) - consider a fully inelastic supply of work: in this way changes in the unemployment rate match exactly with changes in the employment rate. Indeed, the behavior of the unemployment is completely explained by the fluctuations of the employment rate. The presence of a fully inelastic labor force does not allow for capturing the sources of the unemployment and its effects on the business cycle. I assume a labor supply not inelastic as in Galì (2010). In this way, I take into account movements in unemployment rate which do not are specular to fluctuations in employment rate.

I make the assumptions of full consumption risk-sharing between the households and as consequence employed people and unemployed people have the same level of consumption[8].

The households are the owners of the capital, they renting it to the firms that operate in the goods sector.

The explicit form of equation 1 is a constant relative risk aversion function (CRRA). This function is divided in two parts: one relative to the utility of consuming the differentiated goods, the other one relative to the disutility of work or from searching for a job with an endogenous preference shifter. The objective of the homogenous households is to maximize their explicit utility function taking into account two constraints, a capital accumulation constrain and a budget one:

$$max\ E\beta \left\{ \sum_{t=0}^{\infty} \frac{1}{1-v} (C_t - hC_{t-1})^{1-v} - \frac{O}{1+l} \phi (L_t)^{1+l} \right\}$$

[8] Christiano, Trabandt and Walentin (2010) do not take into account the assumption of full risk-sharing consumers and in their model they consider that employed people are better off -consume more- than unemployed people. This is in line with the empirical studies about consumption.

s.t:

$$C_t + I_t + \frac{BO_t}{P_t} = (1 - t_y)W_t N_t + \Pi + i_{t-1}\frac{BO_{t-1}}{P_{t-1}} - (1 - t_y)r_t Z_t K_{t-1} + a(Z_t)Z_t K_{t-1}$$

$$K_t = (1 - \delta)K_{t-1} + \left\{1 - S\left(\frac{I_t}{I_{t-1,i}}\right)\right\} I_t.$$

Where C_t is the total consumption for the households, BO is the quantity of public debt bonds emitted - the quantity of public debt -, t_y is an income tax rate paid by households, I_t is the total investment function, h is the habit parameter - an index of the consumption smoothing -, l is the labor supply elasticity between the current period and a future period. W_t is the aggregate nominal wage, S is the adjustment cost function, in steady state S=S'=0 while S'' >0. Z_t is an endogenous parameter representing the capital utilization. The capital utilization transforms the physical capital, K_t, into the effective capital,K_{ts}, which households rent to the firms: $K_{ts} = Z_t K_{t-1}$. The households rent the capital to the firms at rate r_t and their income for renting capital services is: $rZ_t K_{t-1}$. The cost of changing the capital utilization is: $a(Z_t)K_{t-1}$ and δ is the depreciation rate. The disutility of work, the second part of the utility function, has an endogenous shifter preference parameter O, defined as $\frac{F_t}{C_t - hC_{t-1}}$, where $F_t = {F_{t-1}}^{1-vo}(C_t - hC_{t-1})^{vo}$. The endogenous shifter parameter has the role of linking the long-run growth balanced path to a short run wealth effect. The weight, the importance, of the short run effects is indicated by vo. F_t is considered a smooth trend for the aggregate consumption. The consumption's framework, just described, implies that during consumption booms the individual disutility from work decreases.

The endogenous preference shifter plays an important role to capture the common behavior of the consumption, labor force and wage over the business cycle. ϕ represents an AR(1) process which reflects a labor supply shock.

Now I solve the households' maximization problem:

$$\begin{aligned}\ell = E\left(\frac{1}{1-v}(C_t - hC_{t-1})^{1-v} - \frac{O\phi}{1+l}L_t^{1+l}\right)& \\ + \vartheta_t\Big\{C_t + I_t + \frac{BO_t}{P_t} - (1 - t_y)W_t N_t - i_{t-1}\frac{BO_{t-1}}{P_t - 1} + (1 - t_y)r_t Z_t K_{t-1}& \\ - a(Z_t)Z_t K_{t-1}\Big\}\gamma_t\left\{(1-\delta)_{t-1}K_{t-1} + \left\{1 - S\left(\frac{I_t}{I_{t-1} - 1}\right)\right\} I_t - K_{ti}\right\}&\end{aligned}$$

ℓ is the lagrangian. ϑ_t and γ_t are the Lagrange multipliers. These two terms are called shadow prices: the first one indicates how the objective function varies when there is a change in the

consumption constraint and the second one indicates how the objective function changes when there is a variation of the capital constraint.
Now, I find out the first order conditions (FOC) in order to obtain the Euler equation, the Q Tobin's expression and the investment equation.

The first-order conditions are:

$$\frac{\partial \ell}{\partial C_t} = (C_t - hC_{t-1})^{-v} + \vartheta_t = 0 \qquad 2)$$

$$\frac{\partial \ell}{\partial BO} = \frac{\vartheta_t}{P_t} - i_t E\left(\frac{\vartheta_{t+1}}{P_{t+1}}\right) = 0 \qquad 3)$$

$$\frac{\partial \ell}{\partial \mathrm{K_t}} = E_t\left(\vartheta_{t+1}\left((1-t_y)r_{t+1}Z_{t+1} - a(Z_{t+1})Z_{t+1}\right) + \gamma(1-\delta)\right) = \gamma_{\ t} \qquad 4)$$

$$\frac{\partial \ell}{\partial I_t} = \vartheta_t = \gamma_{\ t}\left[1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right] + \gamma_{t+1} is E_t\left[S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right] \qquad 5)$$

$$\frac{\partial \ell}{\partial Z_t} = (1-t_y)r_t = a'(Z_t) \qquad 6)$$

From equation 2, I obtain the Lagrange multiplier of the first constraint:

$$\vartheta_t = -(C_t - hC_{t-1})^{-v};\ \vartheta_{t+1} = -(C_{t+1} - hC_t)^{-v} \qquad 7)$$

I log-linearize equations 7 and I find out:

$$\widetilde{\vartheta_t} = -\frac{v(\widetilde{C_t} - h\widetilde{C_{t-1}})}{(1-h)} \qquad \widetilde{\vartheta_{t+1}} = -\frac{v(\widetilde{C_{t+1}} - h\widetilde{C_t})}{(1-h)} \qquad 8)$$

From the log-linearization of expression 3 I obtain:

$$\widetilde{\vartheta_t} = \widetilde{\iota_t} + E_t\widetilde{\vartheta_{t+1}} - E_t\widetilde{\pi_{t+1}} \qquad 9)$$

Combining 8) and 9), I obtain the consumption's Euler equation. From this equation I can argue that the households taken their decisions about good's consumption watching the expected inflation rate, the elasticity of substitution, the interest rate, the past consumption - in a measure indicated by the habit parameter- and the expected future consumption.

Then, I substitute in equation 4 equation 6, then I put the new equation ahead of one period:

$$E_t\big(\vartheta_{t+1}(a'(Z_{t+1})Z_{t+1} - a(Z_{t+1})Z_{t+1} + \gamma_{t+1}(1-\delta)\big) = \gamma_t. \qquad 10)$$

I divide equation 10 for ϑ_t, after I times and divide the right end side of the expression obtained and I log- linearize it:

$$\tilde{q} = \chi\delta E_t\widetilde{Z_{t+1}} + (1-\delta)E_t\widetilde{q_{t+1}} + E_t\widetilde{\vartheta_{t+1}} - \widetilde{\vartheta_t} \qquad 11)$$

Where $q = \frac{\gamma}{\vartheta}$ and is called Tobin's Q. This expression is equal to one when the adjustment costs are considered null.

From equation 5 I obtain the log-linearized investment function:

$$\widetilde{I_t} = \frac{\widetilde{q_t}}{2S''} + \frac{\widetilde{\iota_{t-1}}}{2} + E_t\frac{\widetilde{I_{t+1}}}{2} \qquad 12)$$

Indeed, the investment depends on the Tobin's q, the past investment and the expected future investment decisions. From the capital accumulation constraint, I obtain the log-linearized capital accumulation equation:

$$\widetilde{K_t} = (1-\delta)\widetilde{K_{t-1}} + \delta\widetilde{I_t}; \qquad 13)$$

The capital accumulation depends on the past capital accumulation depreciated - δ is the depreciation rate - plus the investment depreciated which is the current capital accumulation[9].

2.1.2 The labor supply

I Consider the FOC regarding N_t in order to obtain the labor force:

$$\frac{\partial \ell}{\partial N_t} = OL_t^l\phi = -\vartheta_t(1-t_y)W_t$$

Then I log-linearize this function considering the expression for O - the endogenous preference shifter – and I find out the equation which represents the labor force:

[9] In appendix A, I describe all the passages necessary to obtain the log-linearized expression for Tobin's q, the log-linearized investment's function and the log-linearized capital accumulation equation.

$$\widetilde{L_t}l + \phi = -\widetilde{\vartheta}_t - (1 - t_y)\widetilde{W_t} - \widetilde{F}_t \qquad 14)$$

where $\widetilde{F_t}$ is:

$$\widetilde{F_t} = (1 - vo)\widetilde{F_{t-1}} + (vo - 1)\left(\frac{\widetilde{C_t} - h\widetilde{C_{t-1}}}{(1-h)}\right)$$

The labor force, equal to the labor supply, depends on ϑ_t, the wage - considering also the taxes - and F_t plus a labor supply shock. Indeed, wages consumptions, and the labor market productivity - highlighted by the variable F_t - play an important role in determine the level of the labor force.

2.2 Firms

I consider a market in monopolistic competition composed by a large number of firms. Each firm produces a differentiated good and has the power to set the good's price. Factories to produce face costs on which they apply a mark-up.
First, I analyze the production function and the marginal costs of firms, then I describe the process of prices' formation.

2.2.1 Production and costs

The demand for a single differentiated good, deduced from the maximization problem of families, is:

$$C_{ft} = \left(\frac{P_{ft}}{P_t}\right)^{-v} C_t$$

Firms produce their goods using capital, the effective capital K_{ts}, and employing workers. I assume that firms buy the effective capital from the households directly. Many DSGE models[10] allow for a product market divided in two sectors: an intermediate sector and a final sector. In the intermediate sector firms are in perfect competition and produce with capital - if it is considered - and labor an intermediate good. This good is sold to the factories operating in the final goods' sector. This kind of firms produces, in a market of monopolistic competition, differentiated goods. For the sake of simplicity, considering the fact that the wage is bargaining between firms and workers, I assume that in the product market there is only one sector:

[10] See among others Smets and Wouters (2007) and Blanchard and Galì (2010)

households transform physics capital into effective capital and then sell it to the firms. The wage is not offer by an agency[11] but is object of bargaining between representative workers and firms. Firms have a constant return to scale production function and the production technology is the same across firms. I assume a Copp-Douglas production function:

$$Y_t = AG_t N_t^{1-\alpha} K_{ts}^{\alpha} \qquad 15)$$

where A is a factor - which follows an Ar(1) process - describing a factor augmenting technological shock - productivity shock - and G_t is a productive public expenditure - goods and services -. The role of the productive public expenditure is to capture the government contribute to the growth of the gross domestic product. Below in the text I describe the features of this variable[12]. $1-\alpha$ and α are the elasticity of inputs with respect to the income.
I log-linearize equation 15:

$$\widetilde{Y_t} = a + \widetilde{G_t} + (1-\alpha)\widetilde{N_t} + \alpha \widetilde{K}_{ts}, \qquad 16)$$

where a is the log-linearized productivity shock. The level of employment, N_t, is chosen by firms. In many countries, both in Europe and in USA, the employment is not bargained while the wage is bargained.
The employment level follows a particular law of motion equation:

$$N_t = (1-\varphi)N_{t-1} + H_t \qquad 17)$$

where φ is the separation rate and H_t is the hiring rate. The employment level is depends on the flows in employment in the past period, plus the hiring rate - the numbers of jobs created- determined by firms in the current period. Equation 17, expressed in terms of a log-linearized equation is:

$$\widetilde{N_t} = (1-\varphi)\widetilde{N_{t-1}} + \widetilde{H_t} \qquad 18)$$

To produce goods, firms address costs because they have to pay the input used to produce the differentiated good. I consider a plain cost function that includes the price of the capital, - the

[11] In some works - see for example Smet and Wouters (2007), Justiniano Primiceri and Tambalotti (2008) and Erceg and Levine (2000) - there is an agency which offers the labor service, supplied by workers, to the firms. In these models wage is not bargaining but depends on the choice of firms and agencies.

[12] Galì (2008) and Ratto, Roeger and Veld (2009) consider the public expenditure in the production function in works which consider a open economy. It is taken into account a country inside a currency union. The Euro Area is very close to the framework set up by Galì and Ratto..

interest paid by firms to households - the quantity of capital bought by firms, the wage index[13] and the level of employment. All the firms have the same cost function that is:

$$CT_t = W_t N_t + r_t K_{ts}$$

To find marginal costs I minimize the cost function subject to the production function constraint. The result of the minimization process is the marginal costs equation, then I log-linearize this expression. Thus, the log-linearized marginal costs are:

$$\widetilde{mc} = (1-\alpha)\widetilde{W}_t + \alpha \tilde{r}_t - a - \widetilde{G_t}. \qquad 19)$$

The log-linearized marginal costs depend on the price of inputs weighted with their elasticities, on the productivity shock and on the public expenditure[14].

2.2.2 Prices

Every firm - firm f- produces a differentiated good and has the goal to maximize its good's price subject to the total demand of the households for the differentiated good produced. The objective function is represented by the profit's function. The demand of goods is a constraint for firms and this is a typical Keynesian assumption. Prices are sticky, modeled following the Calvo rule: a fraction θ_p of firms in a given period t could not change the price of the differentiated good produced, while a fraction $1-\theta_p$ of firms could change their prices. The maximization problem for firm f is:

$$max_{P_t} \sum_{k=0}^{\infty} \theta_p^K Q\, E_t\{(P_{t*} - mc_{t+k|t})Y_{t+k|t}\} \qquad 20)$$

$$s.t\ \ Y_{t+k|t} = \left(\frac{P_{t*}}{P_{t+k}}\right)^{-v} E_t Y_{t+k} \qquad 21)$$

where β is the stochastic discount factor, $Y_{t+k|t}$ is the total demand for the good produced by f in period $t+k$ conditioned the last time firm changes its price was period t. P_{t*} is the general price level in period t. The marginal costs and the output are looking forward and they are

[13] As explain better below in the text I consider staggered wages. Thus, there are workers who can change their wage and they bargain it with firms and there are workers that earn the wage of the last period they bargain it. The sum of the two wages, considering their relative weight (the degree of rigidity), is the wage index.

[14] In appendix A I describe all the calculus to find the log-linearized employment rate and the log-linearized expression for the marginal costs.

conditioning at the time in which the firm could readjust the price which is period t. Q represents the discount stochastic factor:

$$Q = E_t\beta\left(\frac{C_{t+k}}{C_t}\right)^{-v}\frac{1}{1+\pi_{t+k}}$$

where $\pi_{t+k} = \frac{P_t}{P_{t+k}}$ is the gross inflation rate. Firms that could change their price have the same technology and address the same demand curve. Indeed, they have the same price: $P_{t*} = P_{ft}$.

Thus, Firms are homogenous and maximize the same objective function with the same demand curve as constraint. As consequence, henceforth the maximization process is referred to the bundle of firms.

The first-order condition is:

$$\sum_{k=0}^{\infty}\theta_p^K\beta\, QE_t\left[\left(\frac{P_{t*}}{P_{t+k}}\right)^{-v}Y_{t+k} - vP_{t*}\left(\frac{P_{t*}}{P_{t+k}}\right)^{-v-1}Y_{t+k}\frac{1}{P_{t+k}} + vmc_{t+k|t}\left(\frac{P_t}{P_{t+k}}\right)^{-v-1}Y_{t+k}\frac{1}{P_{t+k}}\right] = 0$$

I multiply and divide both sides of this equation for P_{t*} , obtaining:

$$\sum_{k=0}^{\infty}\theta_p^K\, QY_{t+k|t}E_t\left[(1-v)P_{t*} - P_{t+k}mc_{t+k|t}v\right] = 0$$

I divide this last expression for $(1-v)$ and I find out that:

$$\sum_{k=0}^{\infty}\theta_p^K Q\, E_t\left[\left(P_{t*} - P_{t+k}\mathcal{M}mc_{t+k|t}\right)\right] = 0 \qquad 22)$$

where $\mathcal{M} = \frac{v}{1-v}$ is a gross mark-up applied to the marginal costs.

Then, I divide equation 22) for P_{t-1}:

$$\sum_{K=0}^{\infty}\theta_p^K\, QE_t\left\{\left(\frac{P_{t*}}{P_{t-1}} - \pi_{t+k;t-1}\mathcal{M}mc_{t+k|t}\right)\right\} = 0 \qquad 23)$$

where $\pi_{t+k;t-1}$ is the inflation rate looking forward - at time $t+k$ – referred to the last period which some firms could re-optimize their good's prices - period $t-1$ - .

Equation 17 makes clear that in case of flexible prices firms do not produce at a pareto efficient level: they fix the price applying a markup on marginal costs.

Now I log-linearize equation 23 and after many passages, described in appendix B, I find out the New Keynesian Philips Curve:

$$\widetilde{\pi_t} = \beta E_t\widetilde{\pi_{t+1}} + \lambda E_t\widetilde{mc_{t+k}} \qquad 24)$$

where $\lambda = \frac{(1-\beta\theta_p)(1-\theta_p)}{\theta_p}$

3 The labor market

In section one and two I have defined the behavior of households and firms. In this section, I describe the labor market in detail. In paragraph one, I analyze the Nash Bargaining process. In paragraph two, I find out the expression for the unemployment rate and I take into account the costs faced by firms in order to catch the suitable workers Then, in paragraph three, I add in the model nominal wage rigidities following the Calvo rule.

3.1 The bargaining process

In this paragraph I describe the wage bargaining which occurs between a representative firms - or an association of firms - and a workers' union. The bargaining process involves only the workers which are able to modify the wages. People hired between one negotiation and another earn an average salary. The bargaining process take into account search costs and the hiring rate: thus, there are workers that do not match with firms. In the next Paragraph, I describe in detail search costs and the role of the hiring rate. To identify the number of match and to capture labor market frictions I consider a match function. In some models - for example Blanchard Gali (2010) and Galì (2010) - the identification of the matching process occurs only using the cost function.

The match function is $M(v_t; U_t^0)$, where v_t represents the aggregate vacancies and U_t^0 is the initial unemployment rate. I divide the match function for ν_t:

$$\frac{M(v_t;U_t^0)}{\nu_t} = p\left(\frac{v_t}{U_t^0}\right)$$

where $p = \frac{M(v_t;U_t^0)}{\nu_t}$ and $\frac{v_t}{U_t^0}$ is the job finding rate expressed henceforth with x_t .

I assume that over time all the vacancies are posted, in order that the firms- or the representative firm, -are on their labor demand curve. Now I describe the Nash bargaining process. The representative firm, when posts a vacancy, has a value of:

$$V_t = -ygco + E_t\rho(x)(J - V) \qquad 25)$$

where gco are the search costs and J is the value of the firm when the match is reached:

$$J_t = y - w - E_t\varphi(J - V) \qquad 26)$$

$J - V$ is the loss of utility subsequent to the end of the match.

I assume free entry in the market, thus when a vacancy is open $V = 0$. This happens because in a model, which allows for no- participation in the labor market, it is indifferent for the households entry in the market with an additional unit or not. With $V = 0$ equation 25 becomes:

$$J = \frac{y_t gco_t}{\rho(x)}. \qquad 27)$$

The bundle of employed workers, gathered in a union of workers, has a value, a utility stemming from working, indicated with:

$$WI_t = (1 - ty)w_t + E_t\varphi(U - W) \qquad 28)$$

where w is the nominal wage bargained considering the income tax. During the periods, a worker could become an unemployed person: U-W represents the loss of utility derived from being fired. U indicates the value of being an unemployed worker:

$$U_t = b + x_t\rho(x)E_t(WI - U) \qquad 29)$$

where b is the government's unemployment benefit given to unemployed workers, whereas $WI - U$ indicates the gain of utility from being employed.

During the bargaining process both the workers' union and the representative firm have the purpose to maximize their rents, $W - U$ for the union and $J - V$ for the association of firms, in order to gain the majority of the surplus trough the bargaining[15]. The process could be expressed by the Nash rule:

$$w_t = arg\max_{w}(WI_t - U_t)^{1-\omega} \ (J_t - V_t)^{\omega}. \qquad 30)$$

Both firms and workers have a bargaining power, proxy of the capacity to gain a surplus during the negotiation, ω for the former and $1 - \omega$ for the latter.

Now I maximize equation 30 substituting in it equations 25,26,28 and 29. After some algebra[16] I find out the log- linearized wage bargained:

$$\widetilde{w_t} = \frac{\widetilde{y_t}}{(1+\overline{gco}\,\overline{x})} + \frac{\widetilde{y_t}\overline{x}\,\overline{gco}}{(1+\overline{gco}\,\overline{x})} + \frac{\overline{gco}\,\overline{x}\widetilde{x_t}}{(1+\overline{gco}\,\overline{x})} + \frac{\overline{x}\,\overline{gco}\widetilde{gco}}{(1+\overline{gco}\,\overline{x})}. \qquad 31)$$

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[15] Wage bargaining brings rise to the battle for the wage surplus because firms and workers negotiate a wage which is not a salary of Walrasian equilibrium, and thus create a wage surplus which is divided after a negotiation (the battle).

[16] All the passages are described in appendix C

The wage bargained relies on the firm's search cost function, on the output and on the hiring rate. In particular, as mentioned before, this equation makes clear that the bargaining is possible for the presence of frictions in labor market: firms search the suitable workers, the match function display the matching process, and to do it sustain costs represented by the function gco.

3.2 Unemployment and search costs.

I follow a strand of literature, which combines the DSGE NK model with the DP model. Thus, the labor market is not in perfect competition: labor market frictions are present in form of search costs addressed by firms in order to find the correct match with workers. Many authors combine the key features of the NK model with the DP framework. For example, Walsh (2003, 2005) evaluates the effects of a monetary shock on various variables in a framework with labor market friction, flexible wages and sticky prices. In Walsh's works, wages are flexible but the labor market is not in full equilibrium. Shimer (2005) assesses that the NK model, with inside the MP framework, do not replicate the observed magnitude of the flows in labor market in response to a technological shock[17]. Shimer finds out that the NK model integrated with the MP framework displays an excessive volatility in the movements of the wages and too little fluctuations of the employment rate respect to the empirical evidence. In these kinds of models wages are choose after a bargaining process in form of Nash bargaining: The bargaining process occurs between a representative firm and a representative worker's union as mentioned in the previous paragraph. The framework just described has some shortcomings. As pointed out by Hall (2005) there is not empirical evidence of wages bargained every period. Then, in NK models with flexible wages and search costs, wages change in response to movements in the unemployment rate, in the prices level, in the consumption level and in response to movements in the productivity rate and There is not empirical evidence about this statement. Thus, to overcome these troubles, many authors embed in the model wages rigidities. In paragraph three, I describe the role of wage rigidities and I embed in my work nominal rigidities in form of staggered wages.

Firms need to search for workers to fill a vacancy; as soon as a firm finds the appropriate worker the vacancy is filled: the firm matches with workers as highlighted in paragraph 3.1.

In order to fill vacancies and to post them, firms sustain endogenous search costs[18] described by the following cost function:

$$gco_t = cox_t^{\tau} \qquad 32)$$

where co is the cost to post a single vacancy .

[17] This is the so called Shimer puzzle.

[18] In some models search costs are exogenous in others are endogenous (for example Blanchard Galì 2010)

At the end of the period considered (period t) not all the people unemployed at the beginning of the period - their number is equal to the initial unemployment - or people who enter in the job market are hired during the same period. Thus, at the end of the time considered there are people who remain unemployed, and they are searching for a job: I allow for involuntary unemployment. I take into account endogenous labor force in order to consider people inside the job market and outside it. The initial unemployment, the difference between the labor force and the flows in the employment rate that occurs in period $t-1$, is:

$$U_t^0 = L_{t-1} - (1-\varphi)N_{t-1} \tag{33)}$$

while, the unemployment rate is:

$$\boldsymbol{U_t = (1 - x_t)U_t^0} \tag{34)}$$

where $\mathrm{x_t} = \frac{H_t}{U_t^0}$ is the job finding rate, the probability for a workers to find a job[19]. Indeed, the unemployment rate is considered as the unemployment at the beginning of the period considered minus the hiring rate which could be written as $H_t = \mathrm{x_t U_t^0}$. In this framework the unemployment depends on the people who entry and exit in the labor market - described by the hiring rate and by the destruction rate - , on the condition of the aggregate demand - indicated by the level of prices, by the level of output and by the household's consumption decisions - and on the wages. Thus, the determinants of the labor force play an important role in affecting the fluctuations of the unemployment rate.

3.3 The aggregate wage

The NK baseline framework, both with labor market frictions and without them, is not able to indicate the inflation inertia. These kinds of models are also unable to capture the trade-off, the problem faced by monetary authorities between stabilizing the output-gap -or the output- and stabilizing the inflation rate. In the baseline model stabilizing inflation is considered as equivalent to stabilizing the welfare relevant output gap[20]. Besides, in NK models with labor market frictions there is evidence of the Shimer puzzle. Many authors, in order to overcome the shortcomings of the baseline model and to find a solution for the Shimer puzzle, introduce nominal and real rigidities in the labor market. For example, Erceg and Levin (2000) in their work display that in a framework without labor market frictions and with staggered prices and

[19] I make the assumption that in every period firms hire all the workers they are searching for. Thus, the vacancies present in the match function are equal to the hiring rate and are all covered.
[20] This phenomenon is called divine coincidence and is not displayed by the data.

wages the monetary policy cannot reach the pareto-optimum. In this model, each household supplies its work - the labor service - to the production sector. Nominal rigidities are introduced in the form of staggered wages, following the Calvo rule, meaning that only a fraction of workers could renegotiate the contract during each period. Christofell and Linzert (2005) draw attention to real wage rigidities. In their model, the labor market is linked directly to inflation: the authors argue that real wage rigidities can explain the inflation persistence. Christofell and Linzert (2010) consider the direct and indirect effects of real wage rigidities on inflation and others variables taking into account both a right to manage bargaining and an efficient bargaining. Trigari (2009) sets up a model considering endogenous destruction of work, search frictions, matching frictions and Efficient Nash Bargaining[21]. Her work allows for movements in the employment rate both at the intensive and at the extensive margin, thus changes in the number of workers employed are taken into account. In Trigari's work, the presence of search and matching frictions in the labor market generates a lower elasticity of marginal costs respect to output. Trigari and Gertler (2009) build a NK DSGE model with real wage rigidities and staggered multi-period contracts, whereby for each period only a subset of firms and workers negotiate a wage and each wage bargaining takes place between a firm and its existing workforce. The two authors introduce nominal wage rigidities in the baseline search and matching model of Pissarides and Mortensen in order to avoid the high volatility of wages not being captured by the data.

In my work, I assume nominal wages rigidities because they highlight better than real wage rigidities the empirical evidence especially when the economy is hit by a technological shock[22]. The assumption of staggered wages is also coherent with the data: wage contracts may last many years and may not expire at the same moment. Taylor (1999) found that the mean duration of a wage contract is about one year.

Thus, I consider sticky wages following the Calvo rule[23]: in a given period t only a random fraction of workers, indicated with $1-\theta_w$, bargain the wage with the representative firm while the rest of the workers, θ_w , cannot renovate the wage in the period considered. θ_w does not depend on the time elapses since the last bargaining and the new employees hire between one negotiation an another receive an average wage[24].

I assume the aggregate wage as measure for the total wage earned by the households minus the income tx. This variable, is equal to:

$$(1-t_y)W_t = \theta_w\,(1-t_y)W_{t-1} + (1-\theta_w)\,w_t \qquad 35)$$

[21] A representative firm and union bargain both the wage and the employment level.
[22] See Riggi (2010).
[23] See Galì (2011b)
[24] Gertler and Trigari (2009) assume the hypothesis of multi-periods staggered wages.

Where w_t is the wage bargaining by workers who last re-optimized their wage in period t, θ_w is the rigidity parameter and t_y is an income tax.

Now I log-linearize 35:

$$\left(1-t_y\right)\overline{W}\widetilde{W_t} = \theta_w \overline{W}\widetilde{W_{t-1}}\left(1-t_y\right) + (1-\theta_w)\overline{w}\widetilde{w_t}. \qquad 36)$$

I write equation 35 in steady state after some algebra:

$$\left(1-t_y\right)\overline{W} = \overline{w}$$

I substitute this expression in 36 and I find the log-linearized aggregate wage :

$$\widetilde{W_t} = \theta_w \widetilde{W_{t-1}} + (1-\theta_w)\widetilde{w_{\cdot t}} \qquad 37)$$

This is the nominal aggregate wage.

4 The monetary policy rule and the government

In this section, I define the monetary policy rule, the structure of the fiscal policy and the public debt path. I assume a government which produces goods and services in order to augment the output and finance the expenditure with debt and taxes. In the first two paragraphs, I describe the monetary policy rule and the public expenditure equation while in paragraph 3 I analyze the public debt path.

4.1 The monetary policy rule

The central bank determines the nominal interest rate by changing the interest rate in response to movements in the output and in the inflation rate. I consider as monetary policy rule a simple Taylor rule with inside an interest rate smoothing. Thus, the log-linearized monetary policy rule is:

$$\widetilde{\imath_t} = p_r\, \widetilde{\imath_{t-1}} + (1-p_r)\left[\emptyset_\pi \widetilde{\pi_t} + \emptyset_y \widetilde{y_t}\right] + m. \qquad 38)$$

where $\widetilde{\imath_t}$ is the interest rate, $\widetilde{y_t}$ is the total output, $\widetilde{\pi_t}$ is the inflation rate, p_r determines the degree of the interest rate smoothing, and $\emptyset_y\ and\ \emptyset_\pi$ are the weights for the stabilization of output and interest rate decided by the central bank. Indeed, a central bank which considers more important stabilizing inflation has a $\emptyset_\pi$ higher than $\emptyset_y$, and whereas, vice versa. m is a monetary policy shock which follows an AR(1) process.

4.2 Fiscal policy

The government produces goods and services, thus it creates a public expenditure. It is an endogenous variable inside the production function and is considered product augmenting[25]:

$$\widetilde{G_t} = \psi\widetilde{G_{t-1}} + gg. \qquad 39)$$

The public expenditure on services and goods depends, in a measure indicated by ψ, from the past expenditure plus an AR(1) process, which reflects a public expenditure shock, gg.

4.3 Public debt

Many authors investigate the role of taxes and public policies in DSGE models especially in works, which take into account an open-economy considering a currency union[26]. These works are the aim to capture the best fiscal and monetary policies considering sticky prices and a simple structure of the labor market. Galì, Lopez-Salido and Valdes (2007) investigate the effects of a public spending policy in a model with sticky prices and with the presence of Ricardian and no - Ricardian households[27]. They display the effects of a public expenditure shock on various variables considering a competitive labor market and then a no competitive labor market. Schmitt - Grohe and Uribe (2007) introduce fiscal policy in a NK model with nominal rigidities in the product sector and in the labor market. The aim of this work is to evaluate the effects, in terms of welfare loss or gain, of a stabilizing fiscal and monetary policy. These works embed in the NK framework fiscal policies and government expenditure but do not take into account a public debt path. In the last Years, after the global crisis and the high debt growth in Euro Area countries, many authors take into account in NK DSGE models the role of the public debt and the effects of policies based on the growth or decrease of public debt and public debt output ratio. Ratto, Roeger and Veld (2009) consider, in their work, an open economy and the presence of a debt path. They estimated the model for the Euro Area using Bayesian techniques. Veld et al (2012) evaluate the effects of an unsustainable public debt growth of a country in a currency union. They take into account for the analysis the data of Spain. Villaverde (2010) evaluates the effects of fiscal policies considering the presence of financial frictions. He finds that taking into account financial market frictions a raise of public

[25] Some authors consider a productive public expenditure in the production function, for example Irmel and Kuehnel (2008) and Ratto, Roeger and Veld (2009).

[26] See for example Galì Monacelli (2008).

[27] Not Ricardian's consumers are those who consume all the current income soon and thus they don't save or borrow nothing. The not Ricardian households do not save money because they have fear to save money, they do not own capital and because they don't consider the opportunity of inter-temporal changes. Ricardian's consumers are those who save and have access to capital market in order to rent the capital to the firms. This framework is based on rules of Thumb consumers model of Campbell and Mankiw (1989).

debt is better for the economic growth than a reduction in tax rates. Krause and Moyen (2013) investigate in what extent a higher inflation can reduce the public debt's growth considering short term maturity bond and long term maturity bond. Gali (2014) demonstrates that a money financed fiscal stimulus has very strong effects on the economy growth with respect to a fiscal stimulus financed by public debt. Galì highlights, also, the role of nominal rigidities in shaping those effects.

I embed in my work a public debt path in order to investigate the effects of the public debt on the unemployment rate and on the others variables taking into account. As said before, it is important, now-days, consider the public debt as a variable. To find the public debt path I write the budget constraint of the government:

$$G_t + \beta \frac{BO_{t-1}}{P_{t-1}}(1 + i_{t-1}) = t_y Y_t + \beta \frac{BO_t}{P_t}. \qquad 40)$$

On the left side of the equation, there is the public expenditure and the payment of the past public debt. On the right side there are the government's assets: the income tax and the new public debt. As mentioned in the text the income's taxes are paid by the households and are referred to the output – equal to the income-.

I times equation 40 for P_t and considering that $R_t = \frac{P_t}{P_{t-1}}(1 + i_{t-1})$ I obtain:

$$P_t G_t + \beta R_t BO_{t-1} = P_t t_y Y_t + \beta BO_t.$$

Then I log-linearize this last expression and considering that in steady state $\beta R_t = 1$ I find out:

$$\widetilde{BO}_t b = \widetilde{G}_t gi + b\widetilde{R}_t + b\widetilde{BO}_{t-1} - \widetilde{Y}_t t_y \qquad 41)$$

Where $b = \frac{\frac{\bar{B}}{\bar{P}}}{\bar{Y}}$ and $gi = \frac{\bar{G}}{\bar{Y}}$. These two values are set respectively at 2.4 and 0.2 following many authors like Galì (2014), Annichiarico et alt (2009).

The value of 2.4 is the annual growth rate of public debt which corresponds to the average growth of the last forty years for US(60% quarterly).

5 Calibration and Results

In this section, I calibrate the model, using values from the literature for the Us, in order to find the impulse response functions referred to four shocks: two demand shocks - monetary shock and public expenditure shock - and two supply shocks - productivity shock and labor supply shock-. All the variables considered[28] are log- linearized around

[28] The variables are employment rate, unemployment rate, labor force, wage, output, inflation rate, interest rate, public expenditure and public debt.

their zero steady state level- the calculus are all described in appendix A,B and C -. I treat first the baseline calibration and then in paragraph 2 I illustrate the results of the model with an expansive policy implemented by the government through a reduction in tax rates. In order to capture the role of the labor market frictions in paragraph 3 I calibrate the model without search costs. Indeed, I allow for a more flexible labor market but not for flexible wages, wages are bargaining and they continue to be staggered.

5.1 Calibration

The calibration of the parameters is taken from the literature and is referred to USA. The habit in consumption, h is fixed at 0.7, this value is calibrated using Bayesian techniques on Usa data by among others Galì, Smets and Wouters (2012) and Smets and Wouters (2007)[29]. α is equal to 0.3, β is fixed at 0.9965, these values are commonly used in literature - among others Blanchard and Galì (2010), Galì (2010), Galì (2014), Annichiarico et alt (2009) and Erceg and Levine (2000). δ is equal to 0.025 and this value is considered fixed in various works but it is estimated by Annichiarico et al (2009) applying the Klein Algorithm using UE data[30]. I assume θ_p and θ_w both equal to 0.66, $S'' = 4, p_r = 0.6$ and $\chi = 5$. Those last values are taken from Justiniano, Primiceri and Tambalotti (2008) which estimate them for USA using Bayesian methods. v is equal to 1.5 Galì, Smets and Wouters (2012); vo is equal to 0.5 Galì, Smets and Wouters (2012). These two parameters are estimated by the authors using USA data and Bayesian estimation techniques. l is set at 2 (Galì, Smets and Wouters 2012). I fix the weight for the output at 0.15 and the weight for the inflation rate at 0.125[31]. I set the labor force steady state at 0.634, the employment rate steady state at 0.57 and the unemployment at 0.065. These values are valid for the USA and they are referred to the mean of the employment rate, the unemployment and the labor force rate for the last 40 years[32]. The steady state market labor tightness is fixed at 0.7 (Blanchard et Galì 2010) and this value is coherent for USA data. The participation rate- the job destruction rate- is equal to 0.15[33] and the value of the cost of posting a single vacancy is post at 0.1[34]. The value of ψ is set to 0.85. The empirical evidence and the data display that in many countries the public expenditure depends in large part from the past expenditure. The tax rate is fixed at 0.24, which is the average personal tax rate of the last

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[29] Justiniano, Primiceri and Tambalotti (2008) find a value of 0.5 using Bayesian techniques considering USA data from 1954 to 2004.

[30] See Klein (2000)

[31] These values are very commonly in literature: they are estimated using Bayesian techniques by Smets and Wouters (2007) and Galì, Smets and Wouters (2012) among others.

[32] Source: National Bureau of Labor Statistics Usa.

[33] This parameter is found following Blanchard and Galì (2010) which determine the separation rate through this relation: $\varphi = \frac{Ux}{(1-u)(1-x)}$ which in this case is equal to 0.15.

[34] This number is the result of a equation taken from Blanchard Galì (2010): $co = \frac{0.01}{\varphi * x}$.

twenty years[35]. I set the parameter of the AR(1) shocks - a_m for the monetary shock, a_g for the public expenditure shock, a_a for the productivity shock and a_l for the labor supply shock - all equal to 0.5. Table 1 summarizes the calibration of the parameter considered.

5.2 Baseline model

In this paragraph, I analyze the IRFs referred to the baseline model. In figure 1 are shown the curves of the two demand shocks: the plain line represents the impulse response functions referred to the monetary shock and the dashed line describes the IRFs referred to a public expenditure shock. Figure 2 displays the impulse response functions referred to a technological shock - square dotted line - and the IRFs referred to a labor supply shock - dotted and dashed line-.

The impulse response function referred to a monetary shock of the output first goes up of about 1%, then after ten quarters returns to its steady state level. Wages have the same movement of output's impulse response function. Employment and labor force IRFs raise , in particular the employment rate rises of 1.20 %, then turn down and final they come back to their steady state level. Instead, the unemployment rate soon after the shock goes up then, turns down and final returns to the steady state after seven quarters. The fluctuations of the unemployment rate are particularly volatile and they reflect the greatest flexibility of the USA' Labor market. Indeed, I can argue that a positive monetary shock draws people in the job market. Then, Some people leave the job market because they do not find a job. As Consequence, the job finding rate first raises then declines (not shown in graphs).

The public debt has a downturn movement and does not return to its steady state value. The public debt declines because the revenues for the government are higher than the government's expenditure: with a monetary shock, the public expenditure has too small movements to be relevant and thus the government has a surplus in its balance sheet.

The dashed line displays the effects of a positive public expenditure shock. After this shock, the public debt goes up, then the public debt's trend becomes negative. The public debt has an upward movement because with a raise of the public expenditure taxes are not enough to guarantee a government surplus. The interest rate has first a negative movement then a positive one. The employment and the labor force have the same trend: after the shock, they raise and then go down. At the end of the period considered both the employment rate and the labor force return to their steady state level. The unemployment rate, after the shock, has a little downward movement then he turns just above the steady state before coming back to zero. Indeed, after a public expenditure shock people are attracted in the job market. Then, after some terms they

[35] The average is based on Oecd data.

leave the job market because they have not succeeded in finding work[36], as consequence, the labor force and the employment diminish when the unemployment rises.

In figure two are represented the IRFs referred to supply shocks. The impulse response functions referred to a productivity shock show a raise in the output after the shock and then a return to the steady state level after five quarters. Wages, after the shock, have a positive trend then, they return to the steady level after seven quarters. The employment rate goes down and then comes backs to the steady state value. The labor force has the same movement of the employment rate. The unemployment rate after the shock has a little raise and after ten quarters moves to its steady state value. The unemployment rate has a positive trend because the higher wage and the higher output are an incentive to enter in the job market but the employment goes down. Thus, in my model there is evidence of the productivity employment puzzle: after a technological shock, firms do not employ more workers but less although the output has an upward movement, while wages go up. Indeed, firms produce more with a better technology and less workers, but with higher wages. These results are in contrast with some search and match models - for example Gertler and Trigari (2009) - which highlighted the fact that after a positive productivity shock there is a positive movement in the employment rate and of the labor force but are in line with many DSGE works and with the empirical evidence[37].

The dotted and dashed lines in figure 2 show the IRFs of the variables considered referred to a labor supply shock. The labor force and the interest rate after the shock go down and then come back to their steady state. Wages and the Inflation rate first have a negative movement then, before turning to zero, have a little raise. The public debt first has an upward movement then comes to a positive trend – it has a downturn - before returning to its steady state level. The employment rate first drops and after has a volatile trend. The unemployment rate first has an upturn then fluctuates around its steady state. Both the variables return to their steady state level. The public expenditure is about zero and is not shown in the graphs. Thus, a labor supply has a negative effect on labor market's variables and an uncertainty effect, in some quarters positive in others negative, on output and wages.

5.3 The role of taxes

In this paragraph, I analyze the model after a reduction in tax rates implemented by the government. This is of course an expansive fiscal policy. I consider a tax rate of about 0.1, thus the tax reduction is about 0.15. The aim of modifying the taxation is twofold: it allows capturing the role of taxes in the business cycle and permitting to examine the effects of an expansive fiscal policy on the dynamic of the variables taken into account. In figure three are displayed the

[36] The fluctuations of the labor market tightness, not shown in the graphs, have first a positive trend then a negative trend .

[37] For example Galì (1999). Francis and Ramey (2005). Basu, fernald and Kimball (2006), Galì Smets and Wouters (2012) and Galì (2010).

impulse response functions referred to the two demand shocks. As in figure 1, the black line represents the monetary shock and the dashed line represents the public expenditure shock.
The output respect to the baseline model has a more volatile trend but the values are the same: in both the calibration the peak is about 0.8 %. The fluctuations of the employment rate are more volatile and higher than the baseline model. The unemployment rate has first a strong downturn - a drop of about 1 % - then it has movements around the steady state. The IRF of the labor force is the same of the baseline calibration. Wages have a positive trend.
Thus, with a monetary shock, lower taxes produce a gain in the wellness of worker respect to the model with a higher taxation. The IRFs referred to the model with tax rates at 0.1 are more volatile and with higher values respect to the IRFs referred to the baseline calibration.
The public debt has a lower reduction than in the baseline calibration: the government receives fewer taxes and as consequence, the public debt has a minor reduction. This is important to underline that the public debt do not increase. The public expenditure has very little fluctuations and is not represented in the graphs.
After a government expenditure shock, the public expenditure do not change respect to the baseline calibration. Wages are higher in this calibration than in the baseline one: the households, with lower tax rates, obtain higher net payrolls. The labor market variables first raise - the unemployment has a downward movement - then go down - the unemployment goes up -. In the baseline model they come back to zero while in the model with reduced tax rates they do not return to their steady state value. The output has the same movements of the calibration with tax rate at 0.24, but at the end of the time taking into account it returns to its steady state level, while in the baseline calibration the IRF stays below the steady state. The IRFs of the inflation rate and the interest rate are negative. Thus, there is no evidence of the crowding out effect of the public expenditure.
The public debt, after an expenditure shock, has an upward movement and this curve is different of that found in the baseline calibration. A Tax rate of 0.1 does not guarantee a reduction in public debt after an expenditure public shock: a tax reduction produces a raise of the public debt. Indeed, after an expenditure shock a less strong taxation generates a sort of trade-off for the government: if it applies a tax cut the public debt raises as the wages and the unemployment is lower. Thus, the government has to choice between less unemployment, higher net wages and a growth in public debt and higher unemployment, lower net wages and a drop in public debt. A government which takes care mostly to public debt movements do not apply a tax cut while it implements a reduction in taxes if takes care more to fluctuations of wages and of the unemployment rate.
In figure 4, are displayed the impulse response functions referred to the two supply shocks. After a productivity shock, wages have higher fluctuations compared to those in the baseline calibration. The public debt goes up, indeed when taxes are low in case of a productivity shock the government liabilities are higher than taxes. The productivity employment puzzle is

confirmed while the inflation rate and the interest rate after have positive movements before turning to their steady state value.

The dashed and dotted lines represent the IRFs referred to a labor supply shock.

The lower tax rate does not affect the movements of the IRFs referred to a labor supply shock, the only difference regards the wages: with a minor taxation they raise more than in the baseline model. The curves have little higher or lower , if negative, values.

Thus, tax rates play an important role and are an important tool for the government to implement fiscal policies. The Baseline calibration suggests that a public expenditure policy, expressed in a model by a shock, could be sustained by taxes if they are high. With lower taxes, the expansive policy adopted by the government brings to a raise in the public debt.

Taxes are an important factor in the decision of workers and employers: the effects of a tax reduction are very high and remarkable on the wages and on the labor market variables, in particular when are taken into account demand shocks.

I embed in my work income's taxes paid by the households. This is a realistic hypothesis because in many countries the majority of government's revenues derived from taxes on employee's payroll taxes on capital and on firms' income which in this framework are owned by the households.

5.4 The role of the labor market frictions

Search costs are proxy of a labor market with frictions. Indeed, I consider the model without search costs in order to capture the effects of a more flexible labor market[38]. Figure 5 and 6 show the impulse response functions referred to the model calibrated without search costs. The output, after a monetary shock, growths more compared with the baseline model as shown in figure 1 and 5: the IRF reaches a peak at 1 % respect the 0.8 % of the baseline calibration. The labor market variables have higher values with respect to their IRFs in the baseline model. As displayed in figure 1 and 5 with a more flexible labor market the wage has a significantly raise: in the baseline calibration wages raise of a 0.13 % while in the calibration without search costs wages raises of about 0.41%.

All the IRFS referred to a monetary shock without search cost are not smooth and the IRFS of the output, employment, unemployment and wages fluctuate - except for the first quarters - around their steady state level.

In the presence of a public expenditure shock, the impulse response functions are the same of those shown in figure 1 for the interest rate, the inflation rate and the public expenditure. The labor market variables do not change between the two models taking into account.

[38] I do not consider a completely flexible wage model in order to take into account the fact that the firms do not employ all the works. Inserting flexible wages in these kind of models is equivalent to consider a market in which all the workers in the job market and the end of a given period find a work. This hypothesis is unrealistic and in contrast with the empirical evidence.

The public debt has a minor growth than in the calibration with search costs. The output has higher values in the model without search costs than in the baseline model.

Figure 6 shows the IRFS of the supply shocks in absence of search costs. First, I analyze the productivity shock. The public debt, the wage, the output, the interest rate, the unemployment and the inflation rate have the same trend as the IRFS shown in figure 2. The employment rate and the labor have little downward movement not much different from those found in the baseline model. Thus, it is confirmed the evidence about the existence of the employment productivity puzzle. Considering the labor supply shock - the dotted and dashed line - the IRFS are similar to those found in figure 1. Only the wages and the public debt have higher values in confront with the baseline model

A more flexible labor market has relevant effects only on the demand shocks. In particular, after a monetary shock the absence of search costs entails a gain of wellness: a higher output, higher wages and a minor unemployment. The effects are the same but less evident in the presence of a public expenditure shock.

6 Conclusion.

I set up a rich NK model, which is able to capture what affects the public debt fluctuations and the unemployment one. It is also able to underline the role of an expansive fiscal policy. In the framework, I embed a public debt equation, a tax proportional to debt and a public expenditure shock. I find that and expansive public policy has positive effects on the wellness and welfare of people and on the output growth. I also display that if tax rates are high, the public debt as not a big raise but under certain circumstances a reduction. My work could be a contribution to the debate about the role of the fiscal policy in affecting the growth and the welfare of people. Many models investigate the effects of fiscal stimulus and relief[39]. My work differ from these kind of models for two reasons: First, this paper considers a simple monetary rule while after the crisis many works take into account an interest rate at its zero lower bound.

Then many DSGE models, which have the attainment to investigate the role of a fiscal policy and to evaluate the fluctuations of the public debt, do not consider the labor market with frictions and the Nash bargaining process. My framework highlights the role of the labor market frictions and nominal rigidities in affecting the growth and the wellness of the economic system following the NK DSGE models which take into account the monetary policy[40] taking into account a fiscal policy rule, a public expenditure shock and taxes.

[39] Eggertson (2010) and Woodford (2011) affirm that an expansive fiscal policy has better result in terms of GDP growth when the interest rate is at the zero lower bound. Galì, Lopez Salido and Valdes assess that in a framework with labor and product market rigidities and with financed constrained consumers an higher public expenditure stimulate the output's growth.

[40] Blanchard and Galì (2010), Galì, Smets and Wouters (2012) and Christiano, Trabandt and Valentin (2013).

I find out that while taxes are at their average level ,for the US, a public expenditure shock has positive effects on the output, on the employment rate and on the unemployment rate and does not have particular negative effects on public debt. While when the government cuts taxes the public debt, after a public expenditure shock, raises. In general, a tax reduction enhances the movements of the output, the employment rate the unemployment rate, the wages in particular in the presence of demand shocks. The public debt trend is affecting negatively by the downturn of tax rates. Thus, for the fiscal authorities arises a trade-off: more taxes, less GDP growth but and a smooth raise of the public debt or lower tax rates, more   GDP growth, and a relevant of the public debt.

Appendix A: Log linearized equilibrium variables

Capital accumulation equation

Considering that S=S'=0 the log-linearized capital equation is:

$$\bar{K}\widetilde{K_t} = (1-\delta)\bar{K}\widetilde{K_{t-1}} + \bar{I}\widetilde{I_t} \qquad \text{1A)}$$

I consider 1A in steady state:

$$\bar{K} = (1-\delta)\bar{K} + \bar{I};$$

From this equation, I derive the expression from $\bar{K}$

$$\bar{K} = \frac{\bar{I}}{\delta}$$

I substitute this value in expression 1A) and I obtain:

$$\widetilde{\mathrm{K_t}} = (1-\delta)\widetilde{\mathrm{K_{t-1}}} + \delta\widetilde{\mathrm{I_t}};$$

This is the log-linearized expression for the capital accumulation.

The value of Tobin's q

I write the expression for $_t$:

$$\vartheta_{t+1}(a'(Z_{t+1})Z_{t+1} - a(Z_{t+1})Z_{t+1}) + \gamma_{t+1}(1-\delta) = \gamma_t \qquad \text{2A)}$$

I divide equation 2A) for ϑ_{t+1}:

$$a'(Z_{t+1})Z_t - a(Z_t) + E_t q_{t+1}(1-\delta) = \frac{\gamma_t}{\vartheta_{t+1}}$$

Where $q_t = \frac{\gamma_t}{\vartheta_t}$ is the Tobin's q.

I times and divide the right hand side of the last equation for ϑ_t and I find:

$$a'(Z_{t+1})E_t Z_{t+1} - a(Z_{t+1})E_t Z_{t+1} + E_t q_{t+1}(1-\delta) = \frac{\vartheta_t}{\vartheta_{t+1}}\, q_t \qquad \text{3A)}$$

I log-linearize equation 3A) considering that in steady state $Z_t = 1$
And $a(1) = 0$

$$a''(1)E_t\widetilde{Z_{t+1}} + (1-\delta)\bar{q}\,\widetilde{E_t q_{t+1}} = \widetilde{\vartheta_t}\bar{q} + \bar{q}\widetilde{q_t} - E_t\widetilde{\vartheta_{t+1}}\bar{q} \qquad 4A)$$

I write expression 3A in steady state considering that $Z_t = 1$
and $a(1) = 0$:

$$\bar{q} = \frac{a'(1)}{\delta}$$

I substitute this equation in 4A:

$$\frac{a'(1)}{\delta}\tilde{q} = a''(1)E_t\widetilde{Z_{t+1}} + (1-\delta)\frac{a'(1)}{\delta}\widetilde{E_t q_{t+1}} + \frac{a'(1)}{\delta}\widetilde{E_t\vartheta_{t+1}} - \frac{a'(1)}{\delta}\widetilde{\vartheta_t}$$

Dividing this expression for $a'(1)$ and times for δ I obtain:

$$\tilde{q} = \frac{a''(1)}{a'(1)}E_t\widetilde{Z_{t+1}}\delta + (1-\delta)E_t\widetilde{q_{t+1}} + E_t\widetilde{\vartheta_{t+1}} - \widetilde{\vartheta_t}$$

Calling $\frac{\left(a''(1)\right)}{a'(1)}\ \chi$:

$$\tilde{q} = \chi\delta\widetilde{Z_{t+1}} + (1-\delta)E_t\widetilde{q_{t+1}} + E_t\,\widetilde{\vartheta_{t+1}} - \widetilde{\vartheta_t} \qquad 5A)$$

This is the expression for the log-linearized value of Tobin's q.

The investment function

From the investment Foc I have the equation:

$$\vartheta_t = \gamma_{\ t}\left[1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right] + E_t\left[\gamma_{t+1}S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right] \qquad 6A)$$

I divide this expression for ϑ_t and I find:

$$1 = q_t\left[1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right] + E_t\left[\frac{\gamma_{\ t+1}}{\vartheta_t}S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right]$$

I times and divide the second term of the right hand side for ϑ_{t+1} and I obtain:

$$1 = q_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right] + E_t\left[\frac{\vartheta_{\ t+1}}{\vartheta_t} q_{t+1} S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right] \qquad 7A)$$

Then I log-linearized 7A considering that S=S'=0 :

$$\widetilde{q_t}\bar{q} - \bar{q}S''\frac{1}{\bar{I}}\bar{I}\widetilde{I_t} + \bar{q}S''\frac{\bar{I}}{\bar{I}*\bar{I}}\bar{I}\widetilde{I_{t-1}} + \bar{q}S''\frac{\bar{I}}{\bar{I}}\widetilde{I_{t+1}} - \bar{q}S''\frac{\bar{I}}{\bar{I}*\bar{I}}\bar{I}\widetilde{I_t = 0}$$

simplifying I find:

$$\widetilde{I_t} = \frac{\widetilde{q_t}}{2S''} + \frac{\widetilde{I_{t-1}}}{2} + E_t\frac{\widetilde{I_{t+1}}}{2} \qquad 8A)$$

This is the log-linearized investment function.

The marginal costs

I minimize the cost function taking into account a constraint represented by the production function:

$\min N_t W_t + r_t K_{ts}$

s.t: $Y_t = AGN_t^{1-\alpha}K_{ts}^{a}$

$$\ell = N_t W_t + r_t K_{ts} + \Psi(Y_t - AGN_t^{1-\alpha}K_{ts}^{a})$$

$$\frac{\partial \ell}{\partial N_t} = W_t = (1-\alpha)\Psi\, AGN_t^{-\alpha}K_{ts}^{a} \qquad 9A)$$

$$\frac{\partial \ell}{\partial K_{ts}} = r_t = \alpha\, \Psi AGN_t^{1-\alpha}K_{ts}^{a-1}$$

From 9A I find out:

$$\frac{K_{ts}}{N_t} = \frac{\alpha}{1-\alpha}\frac{W_t}{r_t} \qquad 10A)$$

Solving this equation for r_t and log-linearizing it I obtain:

$$\widetilde{r_t} = \frac{\alpha}{1-\alpha}\left(\widetilde{N_t} + \widetilde{W_t} - \widetilde{K_{ts}}\right) \qquad 11A)$$

Substituting the value of W_t and r_t, derived by 9A, in the cost function: $N_t W_t + r_t K_{ts}$, I obtain the equation for the marginal cost:

$$mc = \left(\alpha^{-\alpha} * ((1-\alpha)^{-1+\alpha})\right)({w_t}^{1-\alpha}{r_t}^{\alpha})(AG)^{-1} \qquad 12A)$$

Now I log-linearized this function and I find out the log-linearized marginal costs which is:

$$\widetilde{mc} = \left((1-\alpha)w_t + \alpha r_t\right) - a - g \qquad 13A)$$

The employment rate

$$N_{tf} = (1-\varphi)N_{t-1,f} + H_{tf} \qquad 14A)$$

I log-linearized this equation:

$$\overline{N}\widetilde{N_t} = (1-\varphi)\overline{N}\widetilde{N_{t-1}} + \overline{H}\widetilde{H_t}$$

I write equation 14A) in steady state:

$$\overline{N} = (1-\varphi)\overline{N} + \overline{H}$$

which is equal to:

$$\overline{N}\varphi = \overline{H}$$

I substitute the expression in equation 14A and simplifying I find the log-linearized employment rate:

$$\widetilde{N_t} = (1-\varphi)\widetilde{N_{t-1}} + \varphi\widetilde{H}_t \qquad 15A)$$

The hiring rate

From the labor market tightness: $x_t = \frac{H_t}{U_t^0}$ I derive the expression for the hiring rate:

$H_t = x_t U_t^0$

Log-linearizing this equation I find the log-linearezed value for H_t :

$$\widetilde{H_t} = \widetilde{x_t} + \widetilde{U_t^0} \qquad \text{16A)}$$

Initial unemployment rate

The initial unemployment rate is::

$$\boldsymbol{U_t^0 = L_t - (1-\varphi)N_{t-1}} \qquad \text{17A)}$$

I log-linearized equation 17A :

$$\widetilde{\mathrm{U_t^0}}\overline{U^0} = \widetilde{\mathrm{L_t}}\bar{L} - (1-\varphi)\ \widetilde{\mathrm{N_{t-1}}}\bar{N} \qquad \text{18A)}$$

In steady state equation 17A is:

$$\overline{U^0} = \bar{L} - (1-\varphi)\ \bar{N}$$

I insert this expression in equation 18A and i have the log-linearized unemployment rate at the beginning of the period considered:

$$\widetilde{\mathrm{U_t^0}} = \frac{\widetilde{\mathrm{L_t}}\bar{L}}{\bar{L}-(1-\varphi)\ \bar{N}} - \frac{(1-\varphi)\ \widetilde{\mathrm{N_{t-1}}}\bar{N}}{\bar{L}-(1-\varphi)\ \bar{N}} \qquad \text{19A)}$$

The labor market tightness

I can write the unemployment rate as:

$$U_t = (1-x_t)U^0 \qquad \text{20A)}$$

I log-linearized 20A):

$$\bar{U}\widetilde{U_t} = -\bar{x}\overline{U^0}\widetilde{x_t} + (1-\bar{x})\overline{U^0}\widetilde{U_t^0}$$

Managing this expression and considering equation 20A in steady state I find out:

$$\widetilde{x_t} = \frac{(1-\bar{x})\widetilde{U_t^0}}{\bar{x}} - \frac{(1-\bar{x})\widetilde{U_t}}{\bar{x}}$$

The unemployment rate

As in recent literature [41] I consider the unemployed as the difference in log between the labor force and employment:

$$\widetilde{U_t} = \widetilde{L_t} - \widetilde{N_t} \qquad \text{21A)}$$

[41] See Galì (2010) and Galì, Smets and Wouters (2012):

Appendix B: finding the New Keynesian Philips curve

I log-linearize expression 23 in the text considering that in steady state:

$$\overline{\pi_{t+k;t-1}} = 1$$

$$p_{t*} = p_{t-1}$$

$$\overline{mc} = \frac{1}{\mathcal{M}}$$

$$\sum_{k=0}^{\infty} \theta_p^K E_t \beta \bar{Y} (\widetilde{p_{t*}} - \widetilde{p_{t-1}}) = \sum_{k=0}^{\infty} \theta_p^K E_t \beta \bar{Y} \left(\mathcal{M}\overline{mc}\, \widetilde{mc_{t+k|t}} + \mathcal{M}\overline{mc} (\widetilde{p_{t+k}} - \widetilde{p_{t-1}}) \right)$$

rearranging and considering that $\mathcal{M}\overline{mc} = 1$ and $\sum_{k=0}^{\infty} \theta_p^K \beta = \frac{1}{1-\beta\theta_p}$ I obtain:

$$\widetilde{p_{t*}} - \widetilde{p_{t-1}} = (1 - \beta\theta_p) \sum_{k=0}^{\infty} \theta_p^K \beta E_t (\widetilde{mc_{t+k|t}} + \widetilde{p_{t+k}} - \widetilde{p_{t-1}}) \qquad 1B)$$

Through iteration's process, I find:

$$\sum_{k=0}^{\infty} \theta_p^K \beta E_t (\widetilde{p_{t+k}} - \widetilde{p_{t-1}}) = \sum_{k=0}^{\infty} \theta_p^K \beta E_t \pi_{t+k}$$

where π_{t+k} is the inflation rate. I substitute this expression in equation 1B which becomes:

$$\widetilde{p_{t*}} - \widetilde{p_{t-1}} = (1 - \beta\theta_p) \sum_{k=0}^{\infty} \theta_p^K \beta E_t (\widetilde{mc_{t+k}}) + \sum_{k=0}^{\infty} \theta_p^K \beta E_t \pi_{t+k} \qquad 2B)$$

Expression 2B is the result of a stochastic difference equation:

$$\widetilde{p_{t*}} - \widetilde{p_{t-1}} = (1 - \beta\theta_p) \sum_{k=0}^{\infty} \theta_p^K \beta E_t (\widetilde{mc_{t+k}}) + \sum_{k=0}^{\infty} \theta_p^K \beta E_t \pi_{t+k};$$

$$\widetilde{p_{t*}} - \widetilde{p_{t-1}} = \beta\theta E_t (\widetilde{p_{t+1*}} - \widetilde{p}_t) + (1 - \beta\theta_p)\, \widetilde{m}c_t + \pi_t \,;$$

$$(1 - \beta\theta_p L e^{-1})(\widetilde{p_{t*}} - \widetilde{p_{t-1}}) = (1 - \beta\theta_p)\, \widetilde{m}c_t + \pi_t \,;$$

$$(\widetilde{p_{t*}} - \widetilde{p_{t-1}}) = \frac{1}{(1-\beta\theta_p L e^{-1})} \left((1 - \beta\theta_p)\, \widetilde{m}c_t + \pi_t \right) \qquad 3B)$$

where Le^{-1} is a lead operator. 3B is the dynamic stochastic equation, which is equal to 1B. Now I write the expression for the Price index:

$$P_t = \left[\int_0^1 {P_{tf}}^{1-\epsilon}\, dj\right]^{\frac{1}{1-\epsilon}}$$

which becomes:

$$P_t^{1-\epsilon} = \left(1-\theta_p\right)P_{t*}^{1-\epsilon} + \theta_p P_{t-1}^{1-\epsilon}$$

I divide both side of this equation for $P_{t-1}^{1-\epsilon}$:

$$\pi_t^{1-\varepsilon} = \left(1-\theta_p\right)\left(\frac{P_{t*}}{P_{t-1}}\right)^{1-\epsilon} + \theta_p \qquad \text{4B)}$$

I log-linearized equation 4B:

$$\tilde{\pi}_t = \left(1-\theta_p\right)(\widetilde{p_{t*}} - \widetilde{p_{t-1}})$$

This is the log-linearized price index. I substitute this equation in expression 3B and I find equation 24 in the text, which is the New Keynesian Philips Curve.

Appendix C: The bargained wage

I maximize equation 27 in the text considered the value of the firm with posted vacancies, the value of employed people and the value of unemployed people. The result of the maximization process is:

$$(1-\omega)\frac{1+\varphi}{(y_t-w_t)} = \omega\frac{1+\varphi+x_t\rho(x)}{w_t-b} \qquad \text{1C)}$$

Then, from equation 24 I find out:

$$\rho(x) = \frac{y_t gco}{J_t}$$

I substitute in this expression the value of J and I obtain:

$$\rho(x) = \frac{y_t gco_t(1+\varphi)}{(y_t-w_t)} \qquad \text{2C)}$$

I combine expression 1C and 2C:

$$(1-\omega)\frac{1+\varphi}{(y_t-w_t)} = \omega\frac{1+\varphi+x_t\frac{y_t gco_t(1+\varphi)}{(y_t-w_t)}}{w_t(1-t_y)-b} \qquad \text{3C)}$$

I rewrite 3C simplifying :

$$(1-\omega)\left(w_t(1-t_y)-b\right) = \omega(y_t-w_t) + \omega x_t y_t gco_t$$

Then I times the term in parenthesis:

$$w_t - t_y w_t - w_t\omega + \omega t_y w_t - b\,(1-\omega) = \omega y_t - \omega w_t + \omega x_t y_t gco_t \qquad \text{4C)}$$

Then I rearranging equation 4C):

$$w_t\,\left(1+t_y(\omega-1)\right) = b\,(1-\omega) + \omega y_t + \omega x_t y_t gco_t$$

I log-linearized 4C in order to find the value of the wage bargaining:

$$\overline{w}\widetilde{w_t} = \overline{w}\widetilde{w}\omega\widetilde{y}_t\overline{y} + \omega\overline{y}\widetilde{y}_t\overline{x}\overline{gco} + \omega\overline{y}\,\overline{gco}\overline{x}\widetilde{x}_t + \omega\overline{y}\overline{x}\widetilde{x}_t\overline{gco}\widetilde{gco_t} \qquad \text{5C)}$$

I write equation 5C in steady state:

$$\bar{w}\left(1+t_y(\omega-1)\right)=\omega\bar{y}(1+\overline{gco}\bar{x})$$

Substituting this equation in expression 5C:

$$\tilde{w}=\frac{\bar{y}\widetilde{y_t\omega}}{\omega\bar{y}(1+\overline{gco}\bar{x})}+\frac{\omega\bar{y}\widetilde{y_t}\bar{x}\,\overline{gco}}{\omega\bar{y}(1+\overline{gco}\bar{x})}+\frac{\omega\bar{y}\,\overline{gco}\bar{x}\widetilde{x_t}}{\omega\bar{y}(1+\overline{gco}\bar{x})}+\frac{\omega\bar{y}\bar{x}\overline{gco}\widetilde{gco_t}}{\omega\bar{y}(1+\overline{gco}\bar{x})}$$

Simplifying I obtain:

$$\tilde{w}=\frac{\widetilde{y_t}}{(1+\overline{gco}\bar{x})}+\frac{\widetilde{y_t}\bar{x}\,\overline{gco}}{(1+\overline{gco}\bar{x})}+\frac{\overline{gco}\bar{x}\widetilde{x_t}}{(1+\overline{gco}\bar{x})}+\frac{\bar{x}\,\overline{gco}\widetilde{gco}}{(1+\overline{gco}\bar{x})} \qquad 6C)$$

This is the wage bargained by the representative firm and the union of workers.

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Figure 1: dynamic responses to demand shocks

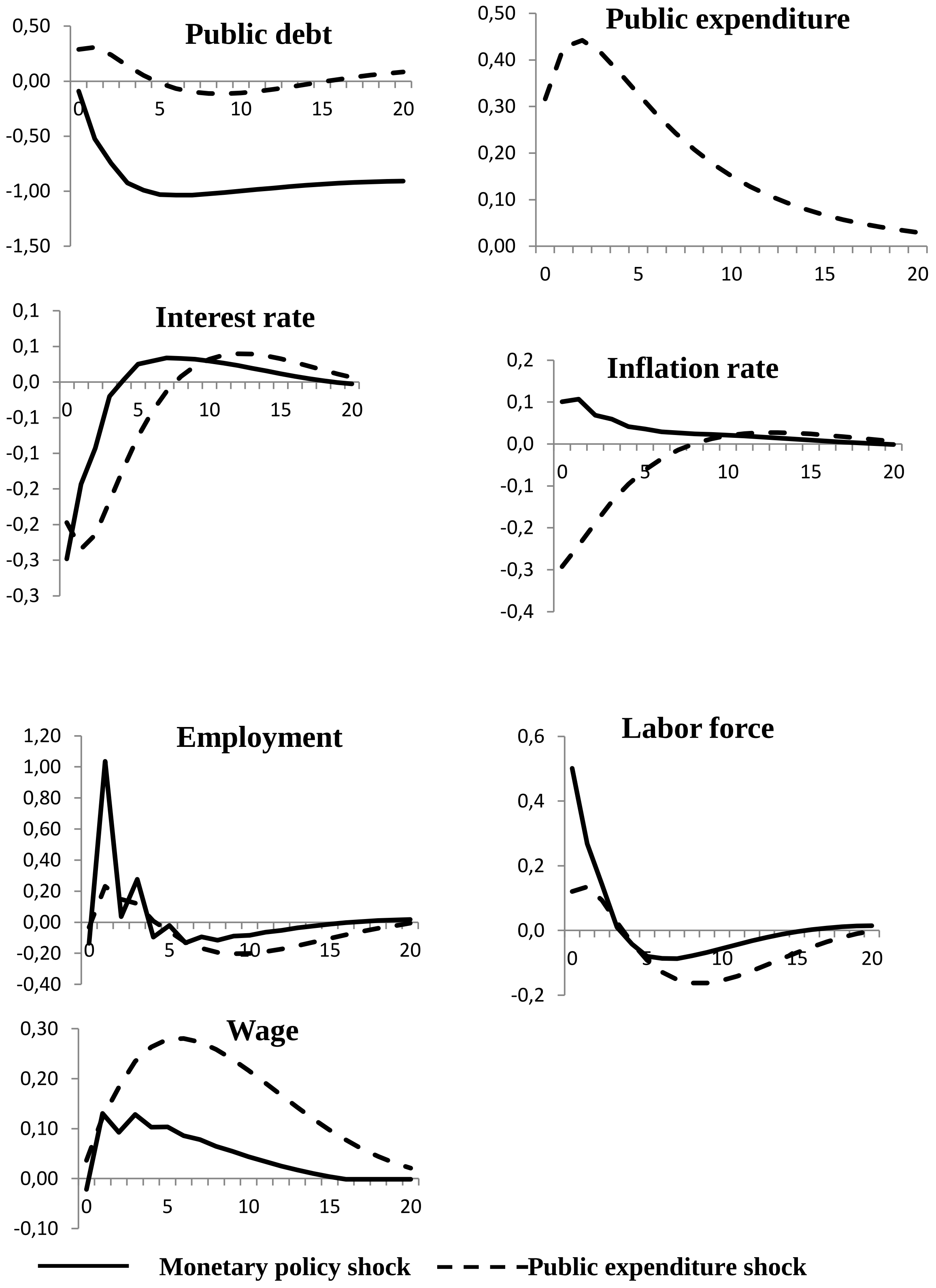

Figure 2: dynamic responses to supply shocks

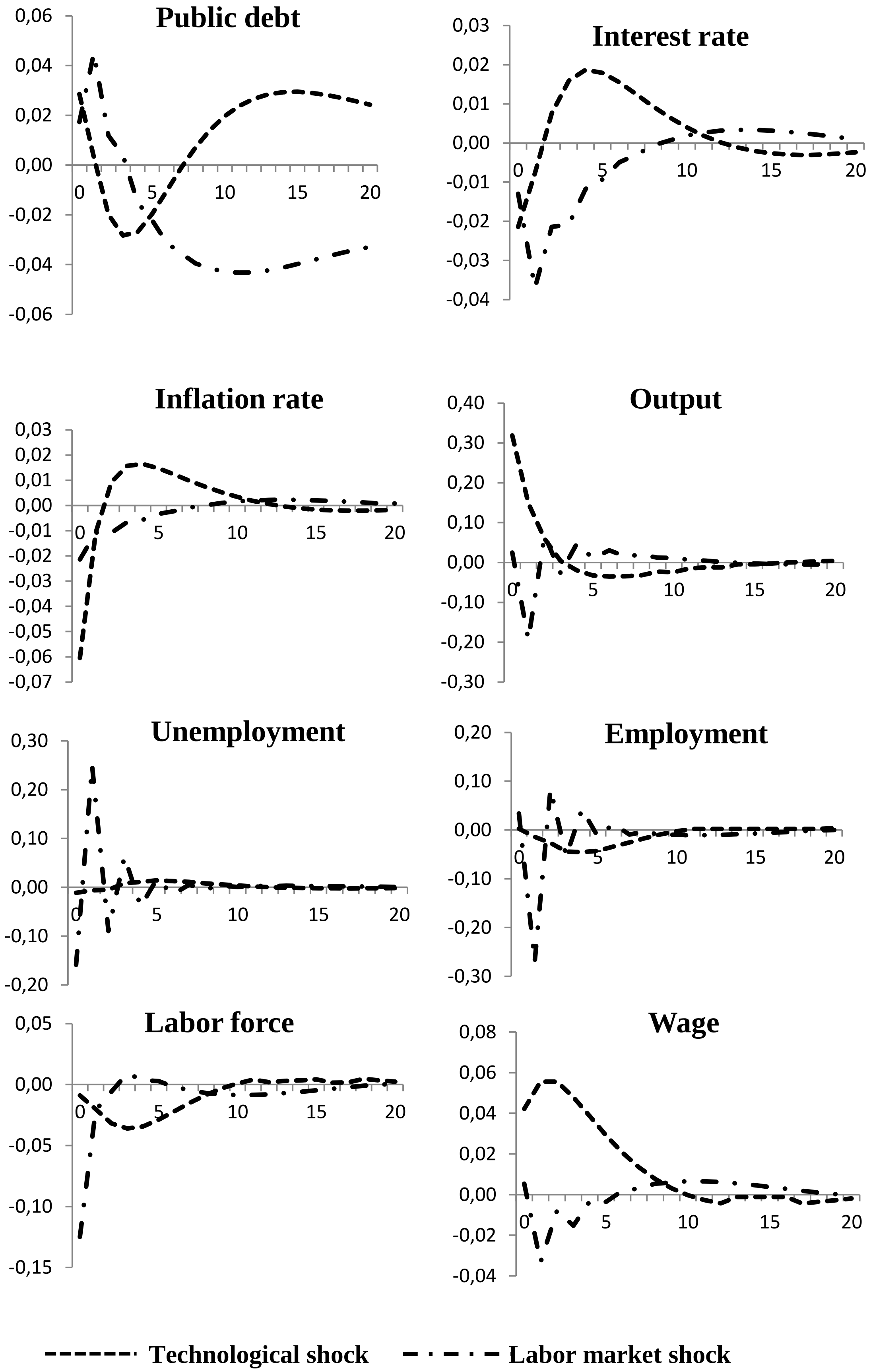

Figure 3: dynamic responses to demand shocks with tax reduction

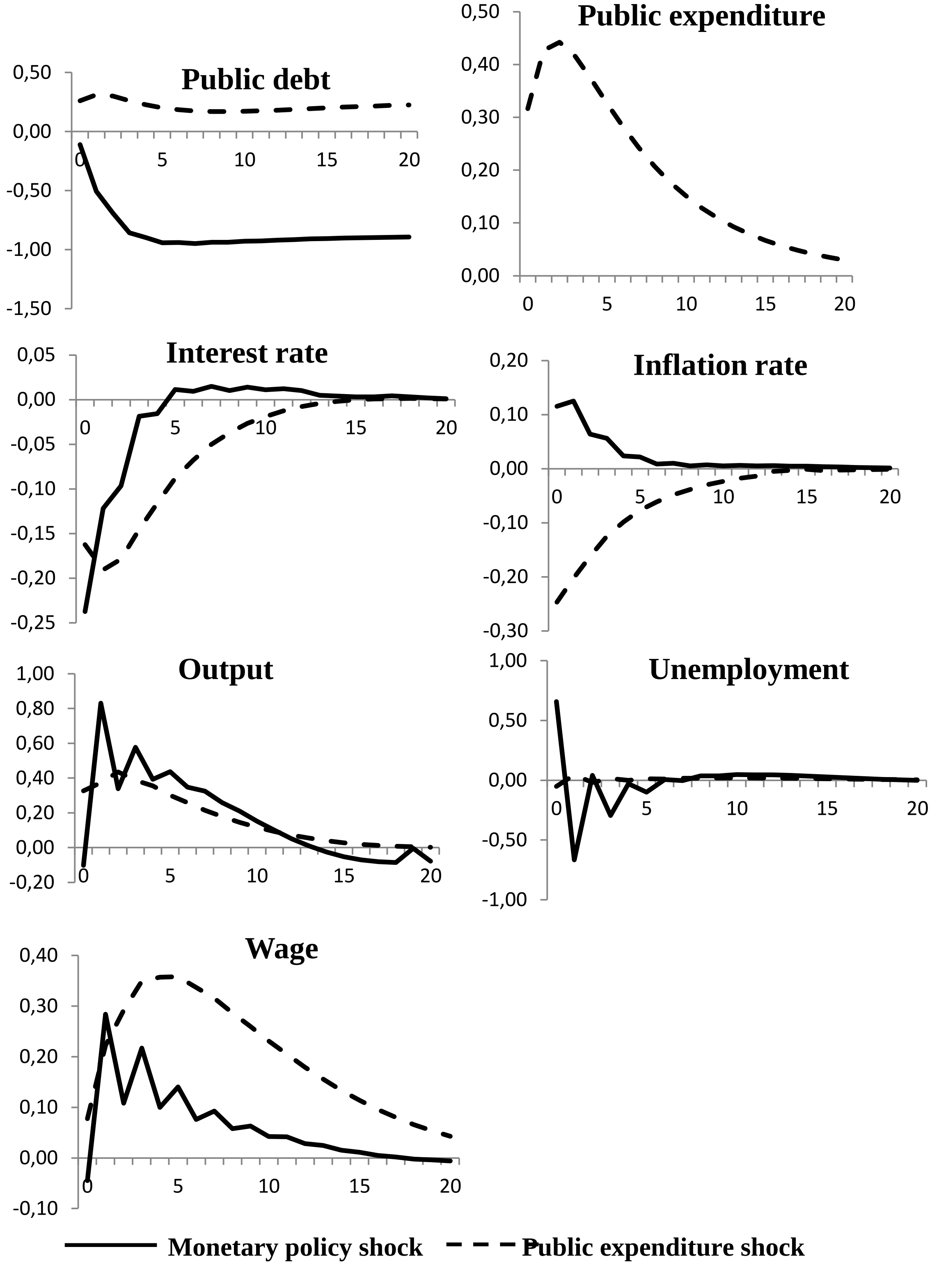

Figure 4 dynamic responses to supply shocks whit tax reduction

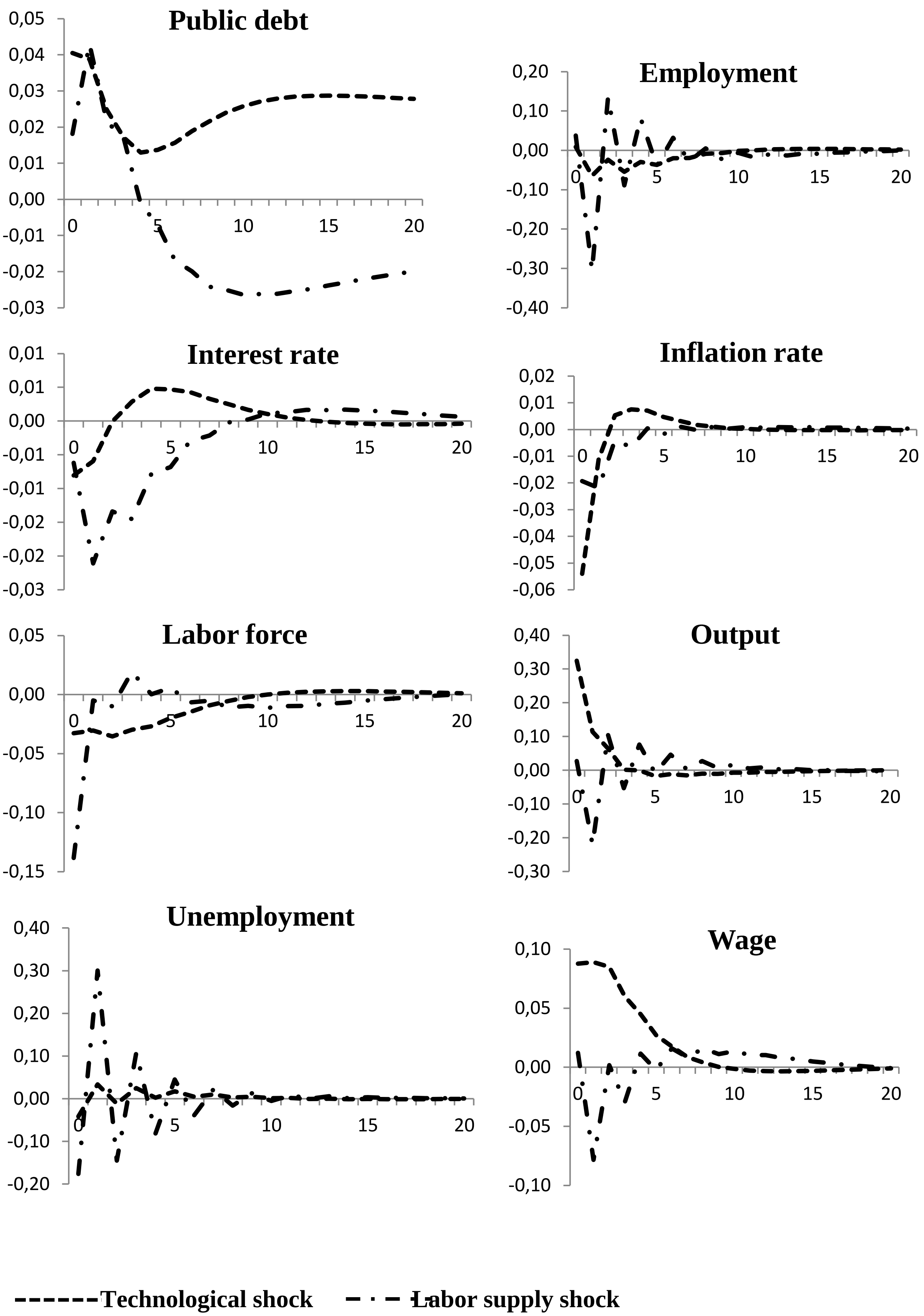

Figure 5 dynamic responses to demand shocks in absence of search costs.

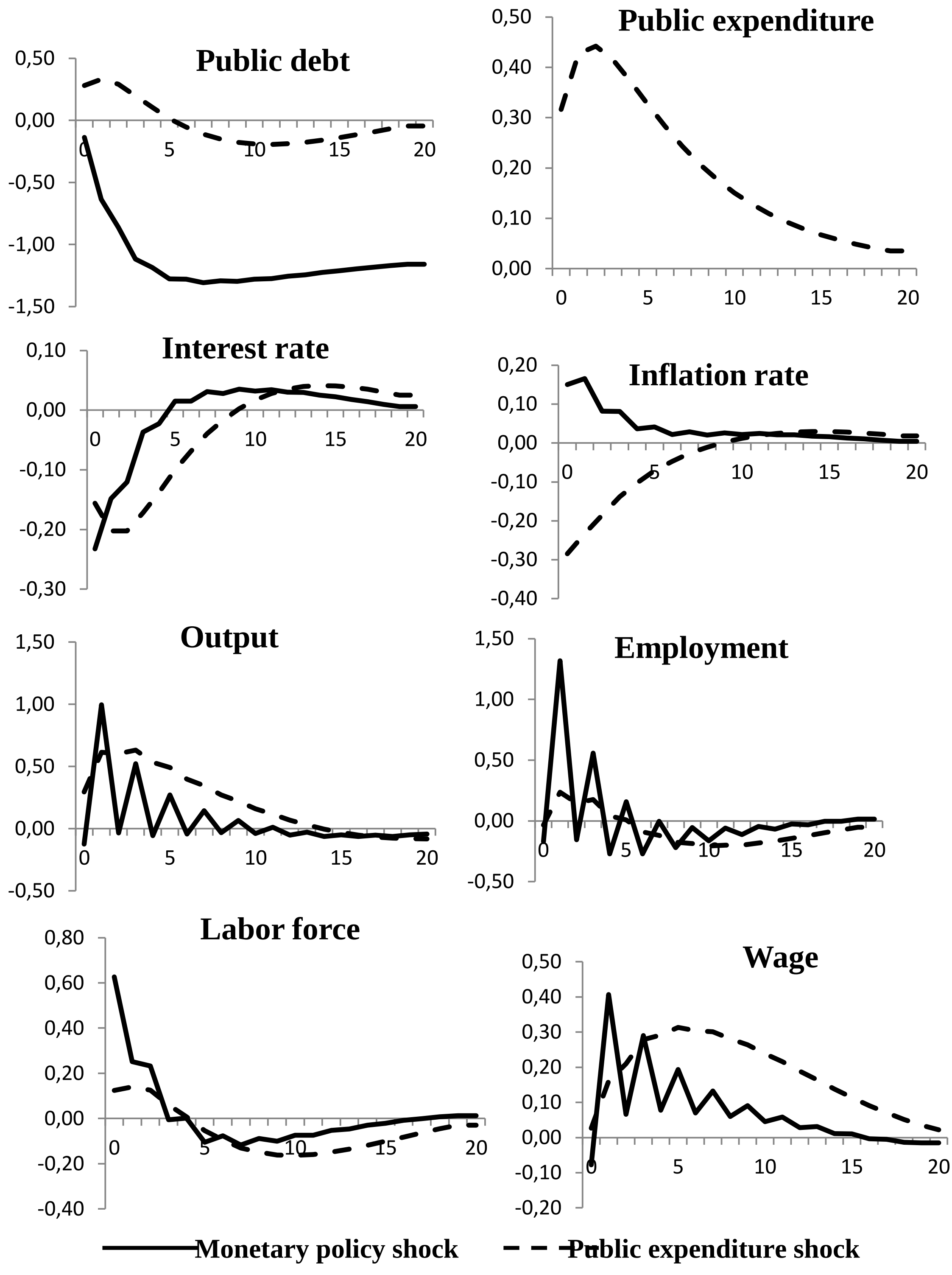

Figure 6: dynamic responses to labor supply shock in absence of search costs

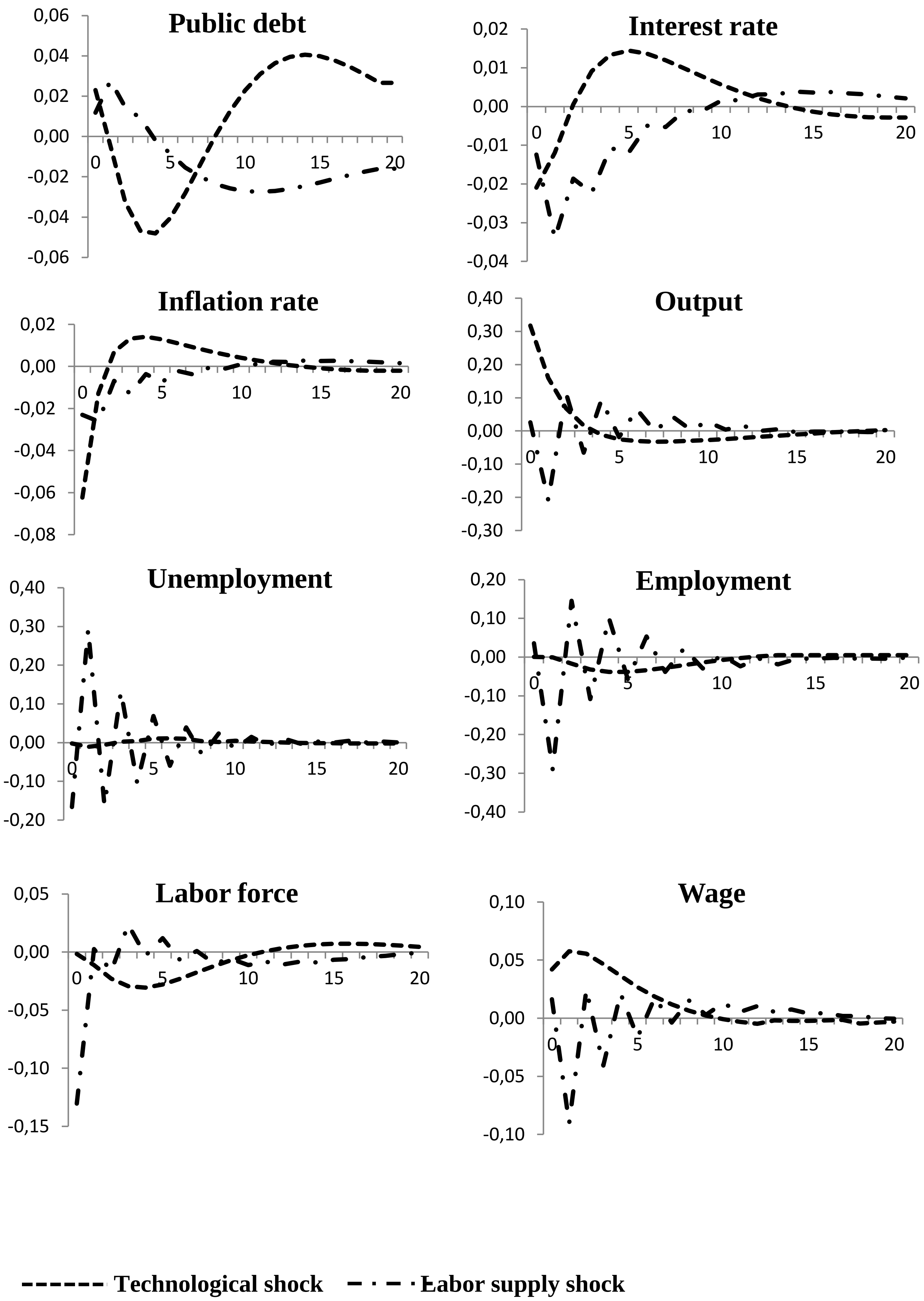

Table 1 : Parameters

Parameter	*value*
h	**0.7**
t_y	**0.24**
$\emptyset_\pi$	**1.5**
$\emptyset_y$	**0.125**
p_r	**0.6**
b	**2.4**
gi	**0.2**
a_m	**0.5**

a_g	**0.5**
a_a	**0.5**
a_l	**0.5**