DUAL LABOUR MARKET THEORIES AND IRREGULAR JOBS: IS THERE A DUALISM EVEN IN THE IRREGULAR SECTOR?

ELISABETTA MARZANO

elisabetta.marzano@tiscali.it
Università Parthenope, Napoli
Abstract
The research deals with the characteristics of the irregular labour. Current literature considers that irregular labour arises because of the heavy tax burden on labour, or because of the existence of regulations which impose too many constraints on the labour market. In Italy the labour market has some effective constraints, for instance, due to firing regulations, minimum wage legislation, or to regulations on multiple job holdings (for instance it is only since last year that retired people can have new jobs legally), whereas, regarding taxation, we think that it is not always a valid explanation of irregular labour. Actually, during past years there were several facilities to new hiring, especially in Southern Italy. We suggest that irregular jobs can originate from different causes, and, consequently, they can have different characteristics, referred to as good and bad irregular match. As to our opinion, dual labour market theories are the main framework for studying this phenomenon. This theory assesses that there are two tiers in the labour market, therefore, two type of jobs: the jobs in the primary sector and those in the secondary sector; the secondary tier of the market is the one where turnover is more accentuated. This hypothesis is supported by some empirical observation about irregular labour. Actually, data available for Italy, (INPS), tells us that irregular workers, in the most of cases, have been employed very recently. The 85% of irregular workers censored by INPS during 2001 had been working for less than 12 months; the same percentage in 2002 was 0.88. Dual labour market approach has been used by Boeri and Garibaldi (2002), who analyzed irregular labour in depressed areas. We use a similar framework, which is mainly adapted from the research of Acemoglu (2001), where dualism arises as endogenous choice caused by the different technology used in two different sectors.

Keywords: bad irregular match, good irregular match.
JEL code: E26, J31, J42.
1. Introduction

Current literature on underground economy usually assesses that irregular labour arises because of the heavy tax burden on labour (Schneider and Enste 2002), or because of the existence of regulations which impose some constraints either on the supply side or on the demand side (Loayza 1996) of the labour market.

In Italy the labour market has some effective constraints, for instance, due to dismissal regulations, minimum wage legislation, or to regulations on multiple job holdings (it is only since last year that retired people can have new jobs legally), whereas, as to taxation, we think that it is not always a valid explanation of irregular labour. Actually, during past years there were several fiscal facilities for new hiring, especially in Southern Italy.

The aim of this paper is to verify whether it is possible to model different kind of irregular jobs, investigating which determinants, other than taxation, can explain their existence. The analysis will be developed using the standard tools of the dual labour market theories. The debate about labour market regulations and labour market segmentation has been recently afforded by several authors (Acemoglu 2001, Yaniv 2001, Moen 2003). The analogy between bad jobs and irregular jobs has been already exploited in recent literature on underground economy (Kolm and Larsen 2001; Boeri and Garibaldi 2002; Cavalcanti 2002) even though achieving different interpretation of the wage in the irregular sector, which turns out to be higher (Boeri and Garibaldi 2002), or lower than the wage in the regular sector (Cavalcanti 2002). We will show how this apparently incoherence among similar models can be easily traced back to the existence of different typologies of irregular

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1 This paper has benefited by observations from the participants to the XIV AISSEC Conference, and participants to the seminar held at the University of Salerno, Department of Economics. I want to thank Maria Rosaria Carillo, Bruno Chiarini and Erasmo Papagni for some useful comments to a previous version of this paper. I am responsible for any remaining error.

2 We refer to several laws which allowed a lower cost of labour for firms operating in southern regions, using the instrument known as fiscalizzazione degli oneri sociali, in other terms a share of social benefits on labour were paid directly by the government. These incentives were drastically reduced due to EU’s competition regulations, but see section 2 for more details.
labour, as already pointed out by, among others, Camera dei Deputati (1998), Bianco (2002), Carillo and Papagni (2002), CNEL (2000), Lucifora (2003), Marzano (2003), Meldolesi (2000), Svimez (2003). The hypothesis supported in this paper is the existence of two different irregular matches, a good irregular match and a bad irregular match (Marzano, 2003); actually, it will be demonstrated that irregular workers are better paid if involved in good match, while bad match offer low-pay and low capital intensive jobs.

This hypothesis is also supported by empirical investigation, based on inspections done by the Institute for private sector’s social welfare (INPS). In fact, digits available for recent years, suggest the existence of two main categories of irregular workers:

- workers who have a specific and additional gain acting in the irregular sector, because they keep a social insurance (retired or unemployed), or because they evade income taxation (extra hours of work not declared or dependent employee registered as autonomous, or eventually second job holders);
- workers which seem to bear the decision to operate irregularly: minors, immigrants and at least a part of all the other irregular workers unknown to official registers that have no other revenue than the irregular one, which are the main category emerging from the INPS inspections.

To catch this dualism inside the irregular sector, we have developed a model that in a first step excludes taxation on labour, coherently with some existing labour market policies which offset social benefit payments with fiscal incentives, but assuming that irregular jobs, due to fiscal authorities controls, are risky for employers. It is demonstrated that in this situation irregular workers are paid less than regular workers to produce an homogeneous output. In a second step taxation on labour is added to the model, and it is shown that in this case net wages in the irregular sector can be higher than the corresponding remuneration in the regular field. The model thus confirms the existence of two different irregular matches: when taxation on wages is not effective, then the bad irregular match arises, the irregular technology is less productive and irregular workers would prefer to be regularly employed; on the contrary, in absence of fiscal incentives, taxation on wages can generate a second typology of irregular workers, which receive a higher wage, so that we refer to this situation as to the good
match.
The paper is organised as follows. The next section describes some stylized facts about the characteristics of irregular labour in Italy, and also about some labour market policies. Section 3 describes a model able to unify the different visions of irregular labour in a framework of a segmented labour market. Section 4 concludes the paper.

2. Some stylized facts

In Italy the size of the irregular sector has been estimated by several authors, using different methods (ISTAT, 2003; Bovi and Castellucci, 1999; Schneider and Enste, 2000; Zizza, 2002; Chiarini and Marzano, 2004). Nonetheless, only the official estimates allow to distinguish, in the generic aggregate usually defined as underground economy, some more specific measures, such as the size of irregular workers. This is the reason why we will refer to digits available by the National Statistical Office (ISTAT), which are quite detailed about irregular jobs/workers. In table 1 the official estimates realized by ISTAT are shown; these digits refer to the size of the underground economy for Italy as a whole, and they start from 1992.

Table 1: The share of the underground economy in Italy

<table>
<thead>
<tr>
<th>Year</th>
<th>Minimal Hypothesis*</th>
<th>Maximum Hypothesis *</th>
</tr>
</thead>
<tbody>
<tr>
<td>1992</td>
<td>12.9</td>
<td>15.8</td>
</tr>
<tr>
<td>1993</td>
<td>13.9</td>
<td>16.8</td>
</tr>
<tr>
<td>1994</td>
<td>14.5</td>
<td>16.5</td>
</tr>
<tr>
<td>1995</td>
<td>15.8</td>
<td>17.1</td>
</tr>
<tr>
<td>1996</td>
<td>15.9</td>
<td>17.0</td>
</tr>
<tr>
<td>1997</td>
<td>15.9</td>
<td>17.7</td>
</tr>
<tr>
<td>1998</td>
<td>15.8</td>
<td>16.8</td>
</tr>
<tr>
<td>1999</td>
<td>14.9</td>
<td>17.0</td>
</tr>
<tr>
<td>2000</td>
<td>15.2</td>
<td>16.9</td>
</tr>
</tbody>
</table>

* underground economy is calculated as % of total GDP Source: ISTAT

Of course, as the percentage reported are quite high, the underground
economy is a phenomenon which existence can be traced back in the previous years. In table 2 there are more detailed indications about the size of irregular workers at regional level.

Table 2: Percentage of irregular workers on total workers (irregularity ratios), 2001

<table>
<thead>
<tr>
<th></th>
<th>Italy</th>
<th>North west</th>
<th>North east</th>
<th>Centre</th>
<th>South</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture</td>
<td>33,0</td>
<td>21,0</td>
<td>25,8</td>
<td>27,6</td>
<td>41,5</td>
</tr>
<tr>
<td>Industry</td>
<td>8,3</td>
<td>4,0</td>
<td>3,3</td>
<td>9,8</td>
<td>20,3</td>
</tr>
<tr>
<td>Ind. strictly defined</td>
<td>6,0</td>
<td>3,4</td>
<td>2,9</td>
<td>6,8</td>
<td>15,4</td>
</tr>
<tr>
<td>Building</td>
<td>15,7</td>
<td>6,9</td>
<td>5,1</td>
<td>18,5</td>
<td>28,9</td>
</tr>
<tr>
<td>Services</td>
<td>16,8</td>
<td>14,5</td>
<td>14,3</td>
<td>16,4</td>
<td>21,0</td>
</tr>
<tr>
<td>Total economy</td>
<td>15,3</td>
<td>11,1</td>
<td>11,3</td>
<td>15,1</td>
<td>22,9</td>
</tr>
</tbody>
</table>

Source: ISTAT

Digits reported in table 2 point out that the well known Italian economic dualism is confirmed also by the irregular economy: irregularity ratios are always higher in southern regions. Moreover, what is very interesting is the strong presence of irregular workers in the southern industrial sector. In fact, figures reported for northern Italy, ranging from 3.2 to 4.5%, suggest that, in manufacturing, the irregularity is coherent with a structural phenomenon, linked, for example, to the presence of extra-hours of work not declared or to the presence of some immigrants not regularly registered. On the contrary, figures estimated for southern regions, ranging around 19%, necessarily imply a deeper consideration and presumably ask for different explanations.

In table 3 we report figures from inspections held by the Institute for private sector’s social welfare (INPS) for the industrial sector. Not regular workers are recorded in two main different categories: black workers, whose status is completely irregular, and irregular workers, who are only partially out of rules. Numbers in columns represent the share of each typology of non regular workers as % of total non regular workers detected by inspections.
Table 3: Typologies of irregular workers, 2001 (Based on INPS investigation)

<table>
<thead>
<tr>
<th></th>
<th>Italy</th>
<th>North west</th>
<th>North east</th>
<th>Centre</th>
<th>South</th>
</tr>
</thead>
<tbody>
<tr>
<td>CIG* Insurance</td>
<td>0.22</td>
<td>0.08</td>
<td>0.20</td>
<td>0.12</td>
<td>0.42</td>
</tr>
<tr>
<td>Sickness Insurance</td>
<td>0.05</td>
<td>0.04</td>
<td>0.06</td>
<td>0.07</td>
<td>0.03</td>
</tr>
<tr>
<td>Unempl. Insurance</td>
<td>0.9</td>
<td>0.88</td>
<td>1.24</td>
<td>0.44</td>
<td>0.99</td>
</tr>
<tr>
<td>Second job</td>
<td>0.18</td>
<td>0.25</td>
<td>0.29</td>
<td>0.12</td>
<td>0.06</td>
</tr>
<tr>
<td>Minors</td>
<td>0.22</td>
<td>0.32</td>
<td>0.21</td>
<td>0.23</td>
<td>0.15</td>
</tr>
<tr>
<td>Immigrants</td>
<td>9.8</td>
<td>14.37</td>
<td>14.95</td>
<td>10.56</td>
<td>1.36</td>
</tr>
<tr>
<td>Students</td>
<td>0.23</td>
<td>0.15</td>
<td>0.36</td>
<td>0.44</td>
<td>0.05</td>
</tr>
<tr>
<td>Retired</td>
<td>0.67</td>
<td>0.8</td>
<td>1.21</td>
<td>0.82</td>
<td>0.06</td>
</tr>
<tr>
<td>Others</td>
<td>74.78</td>
<td>66.57</td>
<td>63.40</td>
<td>73.87</td>
<td>91.29</td>
</tr>
<tr>
<td>Irregular Workers</td>
<td>12.94</td>
<td>16.54</td>
<td>18.08</td>
<td>13.33</td>
<td>5.59</td>
</tr>
</tbody>
</table>

Source: Author’s elaboration based on INPS data.
*CIG is a social benefit for people who are only temporary out of job.

Some categories reported in the table can be considered less disadvantaged than others; it is the case for those defined as “irregular workers”, which have a regular job but are paid in irregular ways\(^3\), or for some of the black workers, and in particular for people who benefit of several kinds of social insurance (CIG, sickness, unemployment, retirement) or experience irregularity as a second job. The situation is different for some other categories of black workers, such as minors and immigrants. Moreover, it is very remarkable the fact that these “privileged” workers amount only to a minor fraction of total irregular workers, and this is true in particular for southern regions. In fact, in the south of Italy the share of “irregular workers” is very low, 5.59%, and second job-holders are only a residual share, 0.06%, while the largest amount of irregularity stands out in the general category “others”, 91.29%, where there are typologies of workers completely unknown to fiscal authorities, and, presumably, less advantaged. The situation for the north-east is quite different, and the same digits are, respectively: 18.08% for irregular workers, 0.29% for second job-holders, and 63.40% for the residual category “Others”. This evidence seems to

\(^3\) It is the situation of dependent employees which are registered as autonomous workers, or the situation of workers whose remuneration is partially not declared (extra hours not declared).
support the idea to differentiate irregular workers in two categories, which, using the terminology of the model developed in the next section, can be referred to as:

- good irregular matches, for people more privileged, such as regular workers who are paid irregularly only for the extra-hours of work, people who benefit of some kind of welfare benefits (retired, unemployed, CIG), or second job holders;
- bad irregular matches, which comprehends the large majority of irregularity detected by INPS, such as minors, immigrants and the residual category “Others”.

As the usually accepted explanation of the irregular jobs is the too high cost of labour and/or the tax burden, here we report also some data about the size of public incentives to hiring, which were largely present in Italy, and especially in southern Italy, where irregular labour is, as shown, more widespread.

Table 4: Some typologies of incentives to labour

<table>
<thead>
<tr>
<th>Typology</th>
<th>Measure</th>
<th>Period</th>
<th>Areas</th>
</tr>
</thead>
<tbody>
<tr>
<td>Partial exemption social benefits</td>
<td>5-30% of the gross wage</td>
<td>1984-’94</td>
<td>Southern regions</td>
</tr>
<tr>
<td>Total exemption social benefits (for 10 years)</td>
<td>100% of the compulsory social security payment</td>
<td>1976-’94</td>
<td>Southern regions</td>
</tr>
<tr>
<td>Total exemption social benefits (for 1 year)</td>
<td>100% of the compulsory social security payment</td>
<td>1991-97</td>
<td>Southern regions</td>
</tr>
<tr>
<td>Partial exemption social benefits</td>
<td>6-15% of the compulsory social security payment</td>
<td>1994-97</td>
<td>Southern regions</td>
</tr>
<tr>
<td>Tax Credit (l.449/97) for new firms</td>
<td>-</td>
<td>1997-</td>
<td>Less developed areas</td>
</tr>
<tr>
<td>Total exemption social benefits (l.448/98)</td>
<td>-</td>
<td>1998-</td>
<td>Southern regions</td>
</tr>
<tr>
<td>Tax Credit for new hiring</td>
<td>-</td>
<td>2000-</td>
<td>Southern regions</td>
</tr>
</tbody>
</table>

Source: Author’s elaboration based on Contini (1998); Lucifora (2003).

These incentives have had the form of partial or total exemption from
social security payments in order to subsidize employment. Exemption from social security payment was considered a threat for the competitive market by the European Commission, and in 1994 it was decided to eliminate it by 1997\textsuperscript{4}.

Summarizing, empirical evidence just described suggests: the existence of two typologies of irregular workers; a stronger presence of irregular workers, and also a higher percentage of the less advantaged irregular workers in the southern regions of Italy; the availability of strong incentives to hiring –total exemption from the payment of social security benefits– in particular in southern regions of Italy. These facts call for a deeper consideration of the effective role of taxation on labour on the extension of irregularity. The next section develops a model which will try to put together all these facts, showing how it is possible to link the absence of taxation on labour to a strong presence of irregularity, and in particular, to a strong presence of bad irregular jobs, simply introducing the chance to operate using different technologies of production.

3. Modeling the dualism among irregular workers

The present section describes a model able to capture the two different typologies of irregular jobs previously referred to as bad and good matches. The main idea is to model the endogenous decision to enter in the irregular market without introducing necessarily neither institutional nor fiscal burden, but only allowing the firm to use different technologies, regular or irregular, for the production of an intermediate homogenous good. The irregular production is characterised by the use of irregular workers, e.g. workers not officially registered. The use of irregular labour is modelled first simply as a risky option, in absence of taxation on labour, and only in a second step taxation on labour input is introduced. This is an extreme solution which wishes to capture the empirical evidence discussed in section 2 about the presence of strong

\textsuperscript{4} To give an idea of the size of these subsidies, in Bodo and Viesti (1997) is calculated that in 1992 they accounted for the 2.2\% of southern regions’ GDP.
incentives on hiring available in Italy, and in particular in the south of Italy, for a quite long period. Taxation on profits is not explicitly examined, because it is not considered, in this specific model, a source of the irregular employment, but as a cost of production common to all the different typologies of firms. In fact, very often firms use irregular labour but are formally registered, so that they report their production to fiscal authorities. We will show that, in correspondence to the two different models – with and without taxation on labour- two different wages are contracted in the irregular tier of the labour market, and they are responsive to policy parameters.

The framework of the analysis is a standard search model, in which there are two different varieties of jobs: regular and irregular. Irregular jobs are first modelled simply as secondary and risky jobs (no taxation on wage/Bad match). In a second step we will introduce taxation and verify its implications (taxation on wage/Good match). The model is mainly adapted on the model of dual jobs suggested by Acemoglu (2001).

3.1 The production function

Each firm produces a unique final good, using three different intermediate goods/inputs, whose aggregate production are defined as: $Y_{Reg}$ for the regular and primary input, $Y_{BadR}$ for the regular but secondary input, and $Y_{BadIRR}$ for the irregular and secondary input$^5$. Intermediate goods are immediately transformed into the final consumption good, which enter in the utility function of the agents. The final good production function is:

$$Y = \left\{ \alpha Y_{Reg} + (1 - \alpha) \left[ Y_{BadR} + Y_{BadIRR} \right] \right\} Y_{Bad}$$

Intermediate goods are produced using labour, $L$, and capital, $K$, and the technology of production is Leontief: when a worker matches with a firm with the necessary capital equipment, he produces one unit of the regular (primary and secondary) intermediate good, and a share $\gamma$ of the irregular intermediate good. Following Acemoglu (2001), the cost of equipment is a creation cost which is incurred when opening a

$^5$ The choice to keep three different intermediate productions is useful to stress the complete homogeneity between the regular and the irregular intermediate goods.
vacancy, in other words before the firm meets its employees. The amount of the equipment per worker required to produce the regular, the secondary regular and the irregular goods are, respectively:

\[ k_{\text{REG}}; k_{\text{BadR}}; k_{\text{BadIRR}} \]

And the technologies are:

\[
\begin{align*}
Y_{\text{REG}} & = \min\{K / k_{\text{REG}}; L\} \\
Y_{\text{BadR}} & = \min\{K / k_{\text{BadR}}; L\} \\
Y_{\text{BadIRR}} & = \gamma \min\{K / k_{\text{BadIRR}}; L\}
\end{align*}
\]

With the assumption on the parameter \( \gamma \) in the irregular Leontief technology:

\[ \gamma < 1 \]

This assumption wants to capture a lower total factor productivity in the irregular technology, which could originate from several causes: lower productive scale due to fiscal controls, worse entrepreneur ability, negative externalities\(^6\). Intermediate goods are sold in competitive markets, so that:

\[
\begin{align*}
Y_{\text{REG}} & = \alpha Y_{\text{REG}}^{\rho-1} Y^{1-\rho} \\
Y_{\text{BadR}} & = p_{\text{BadIRR}} = (1 - \alpha) Y_{\text{Bad}}^{\rho-1} Y^{1-\rho} \equiv p_{\text{Bad}}
\end{align*}
\]

The only necessary assumption, for the subsequent analysis, is that the market value of the intermediate primary good is higher than the price of the secondary intermediate good, so that\(^7\):

\[ p_{\text{REG}} > p_{\text{Bad}} \]

3.2 No taxation on wages
As in standard searching models, the process of matching in the labour market is represented by a matching function \( M(u, v) \) which is twice differentiable, increasing in its arguments, and presents constant returns to scale. The ratio \( v/u \) represents the market tightness, \( \theta \), while the flow rate of match for a vacancy is \( M(u, v)/v \), which is referred to as

\(^6\) The main results showed in the paper do not rest substantially on this assumption, even though they are strengthened.

\(^7\) As we are assuming competitive markets the price of each intermediate good is equal to its marginal productivity.
Finally, we denote the share of jobs in the primary sector as $\phi$. As in standard models of search, we define the value of employment, unemployment, vacancies and filled jobs.

### 3.2.1 Workers

In each instant of time workers can be: unemployed and looking for a job, employed in primary/regular jobs, employed in secondary regular jobs, or in secondary irregular jobs; the present discounted value for each employment state is defined, respectively, as: $J^U$, $J^E_{Reg}$, $J^E_{BadR}$, $J^E_{BadIRR}$. The simplifying assumption is that there is not searching on the job. In steady state the probability of finding a job when unemployed is $\theta q(\theta)$. Then, the value function of the unemployment is:

$$V^U = z + \theta q(\theta)\left\{\delta J^E_{BadR} + (1 - \delta)J^E_{BadIRR} - J^U\right\}$$  \hspace{1cm} (5)

Where $z$ represents an exogenous income for unemployed people, such as social security benefits, and $\delta$ represents the share of secondary regular jobs.

The value functions for employment in the three sectors are:

$$rJ^E_{Reg} = w_{Reg} + s(J^U - J^E_{Reg})$$

$$rJ^E_{BadR} = w_{BadR} + s(J^U - J^E_{BadR})$$

$$rJ^E_{BadIRR} = w_{BadIRR} + (s + p)(J^U - J^E_{BadIRR})$$  \hspace{1cm} (6)

Equations 6 accounts for the possibility of job destruction, $s$, which is common to all jobs and is exogenous; besides, in the irregular sector there is another possible cause of matching dissolution, that is the probability to be detected by fiscal authorities, $p$.

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8 The main aim of this paper is to show how the introduction of some non competitive market characteristics can cause a dualism in the labour market. This is why we do not model explicitly all the parameters of the model, while we concentrate the attention on the wages’ equations considering the parameters as exogenous.

9 The implicit assumption about the unemployment insurance is its universality. We are conscious that it is not the real situation, because in Italy only workers fired in the regular sector can accede to unemployment insurance. This consideration would imply two different Bellman equations for unemployment: the first one including unemployment insurance for regular workers, and the second one for irregular workers without unemployment benefits. The main consequence would be a lower value for the outside option in the wages equations for irregular workers, which would reduce the remuneration of irregular workers.
3.2.2 Firms

On the firm side, the value function of filled jobs is distinct for the regular and non-regular jobs:

\[
\begin{align*}
\text{rJ}^F_{\text{REG}} &= p_{\text{REG}} - w_{\text{REG}} + s(J^V_{\text{REG}} - J^F_{\text{REG}}) \\
\text{rJ}^F_{\text{BadR}} &= p_{\text{BadR}} - w_{\text{BadR}} + s(J^V_{\text{BadR}} - J^F_{\text{BadR}}) \\
\text{rJ}^F_{\text{BadIRR}} &= \gamma p_{\text{Bad}} - w_{\text{BadIRR}} + s(J^V_{\text{BadIRR}} - J^F_{\text{BadIRR}}) + p(J^D_{\text{BadIRR}} - J^F_{\text{BadIRR}})
\end{align*}
\]

\(J^D = C < 0\)  

Equations 7 shows the value of filled jobs for firms; each regular job produces one unit of regular intermediate output, while an irregular job yields \(\gamma\) units of the irregular intermediate good. The net benefit for firms is the difference between the market value of the output and the cost of labour. But, it is also possible that workers quit their labour, the quit rate \(s\) is fixed and exogenous. In this case firms will lose the difference between the expected value of a filled job, \(J^F_i\), and the expected value of a vacancy, \(J^V_i\). When firms hire irregular workers, they also support the risk of an imposed job destruction, \(J^D\), due to the intervention of the fiscal authority. The probability of being caught, \(p\), is exogenous. The dead-weight loss of a forced destruction is set simply equal to a negative constant (equation 8).

The value function for vacancies is:

\[
\text{rJ}^V_i = q(\theta)(J^F_i - J^V_i) \quad i = \text{REG, BadR, BadIRR}
\]

In fact, vacancies have a specific creation cost but there are no costs due to hiring activities, in contrast to what assumed in Pissarides (2000), so that the rate of return on the asset is simply the net return \((J^F_i - J^V_i)\) yielded by the change of state occurring at the rate \(q(\theta)\).  

Introducing the hypothesis that wage are set up on a Nash bargaining, and that workers have the same market power, \(\beta\), in both sectors, we have:\(^\text{11}\):

\(^\text{10}\) Fiscal authorities’ controls only apply to filled jobs, not to vacancies.
\(^\text{11}\) These rules of rent sharing can be easily derived by the maximization process:

\[
\max_{J^F_i, J^V_i, J^F_i, J^V_i} \left( J^F_i - J^V_i \right)^\theta \left( J^F_i - J^V_i \right)^{1-\theta}
\]
As there is free entry on the firm side, in equilibrium it should not be possible for an additional vacancy to be posted and make expected net profits: the present-discounted value of expected profit from a vacancy has to be equal to the creation cost specific of each kind of vacancy. Hence the zero profit conditions are described as follows:

\[(1 - \beta) (J^E_{\text{REG}} - J^U) = \beta (J^F_{\text{REG}} - J^V_{\text{REG}})\]
\[(1 - \beta) (J^E_{\text{Bad}R} - J^U) = \beta (J^F_{\text{Bad}R} - J^V_{\text{Bad}R})\]
\[(1 - \beta) (J^E_{\text{Bad}IRR} - J^U) = \beta (J^F_{\text{Bad}IRR} - J^V_{\text{Bad}IRR})\]

(10)

3.2.3 Wages equations

In steady state, both types of vacancies meet workers at the same rate, and in equilibrium workers accept both types of jobs. Solving the system of equation (6)-(10), and using conditions (11), we get the wages equations for the three different jobs (see appendix A1):

\[w^R = \beta (p^R - rk^R) + (1 - \beta)rJ^U\]  \[(12)\]
\[w^R = \beta (p^R - rk^R) + (1 - \beta)rJ^U\]  \[(13)\]
\[w^R = \beta [(p^R - rk^R) + p(C - k^R)] + (1 - \beta)rJ^U\]  \[(14)\]

Workers get a share \(\beta\), e.g. their bargaining power, of the firms’ surplus, and a share \((1-\beta)\) of their outside option \(rJ^U\). If we consider the wage equation for secondary irregular workers, equation 14, we observe that:

1. irregular employers want to obtain a value of output which must also cover the expected sunk cost of being detected, that is \(p(C - k^R)\);
2. the value attached to the outside option is unchanged.

\[12\] This is another difference with Pissarides (2000) resulting from the specific interpretation about the creation cost \(k_i\) assumed following Acemoglu (2001).
Three important results are achieved at this point:

**Proposition 1**: there is a precise range for the optimal capital intensity in the three technologies:

\[ k_{REG} > k_{BadIR>IR} > k_{BadIRR} \]  \hfill (15)

**Proof**: see appendix A2

The different cost/intensity of equipment implies a different technological content of the three intermediate productions. The equipment per worker is lower in the secondary irregular production, but this convenience is offset by the value, lower than unity, for the parameter $\gamma$. These results are coherent with an empirical experience that highlights the existence of irregularity in less innovative and more labour intensive productions. The technological gap between primary and secondary production (both of them regular) is clearly explained by the higher market value of the primary intermediate good. The technological gap which endogenously arises inside the secondary sector is due to the risk inherent the use of irregular workers. This simple range does not apply for wages, in fact:

**Proposition 2**: the wage paid in the primary regular sector is higher than the wage paid in the secondary regular sector.

**Proof**: see appendix A3

This result is very interesting, because it implies a strong relationship between capital investment and workers' remuneration in the regular sector, and it is absolutely coherent with Acemoglu (2001).

**Proposition 3**: the relative wage in the secondary sector, in absence of effective taxation on labour, is indeterminate, and it depends upon the technology as well as enforcement policy parameters.

**Proof**: see appendix A3

In fact, if we write the wage gap in the secondary sector:
\[ \Delta w_{Bad} = w_{BadR} - w_{BadIRR} = \beta(p_{BAD} - rk_{BADR}) - \beta[p_{Bad} - rk_{BadIRR} + p(C - k_{BadIRR})] \] (16)

We see that the gap has two different components: a first one, which sign is ambiguous, referable to technological differences between the two productions, and a second one, always positive\(^{13}\), affected by the enforcement policy parameter \(p\), representing the risk of the irregularity. Two different situations can occur:

i. wages are higher in the regular sector: a condition that would always ensure a positive wage gap, even assuming a weak enforcement, is:

\[ p_{BAD} \frac{1 - \gamma}{r} \geq k_{BadR} - k_{BadIRR} \] (17)

I will refer to this condition as a technological premium for regularity, because it depends on the parameters of the production function.

ii. wages are higher in the irregular sector: only the existence of a technological premium for irregularity and a weak enforcement:

\[ p_{BAD} \frac{1 - \gamma}{r} < k_{BadR} - k_{BadIRR}; \quad E(p) \approx 0 \]

could cause a negative wage gap or a higher wage for irregular workers.

Even though it is plausible to assume a quite low enforcement policy, the idea of a technological premium for irregularity is not easily defendable, so it is plausible to presume that equation 16 has a positive sign. It is possible to classify the irregular match here sketched as the bad irregular match, because the sign of expression 16 states the relative convenience to work in the irregular sector\(^{14}\) for secondary

\(^{13}\) It is useful to remember that the penalty for irregularity, \(C\), is a negative constant by definition.

\(^{14}\) This model is not able to catch other important aspects of jobs’ remuneration, which would need to be characterised using an utility function. I refer, for example, to social security benefits, which are modelled as a tax on gross wage, but have also a positive impact on the worker’s utility. Here I simply assume that jobs better paid, in terms of
workers.

These results draw from the assumption that there is no effective taxation on wages\(^{15}\), but the firm is allowed to use a combination of two different technologies, one of which is risk free (the regular) while the other is risky.

### 3.3 Taxation on wages

It is possible to investigate whether, in presence of taxation on wages, the relative convenience to act in the irregular labour market changes for one or both of the parties involved in the match. The framework does not need strong modifications, and I will discuss only the necessary adjustments. As in the irregular market taxation is completely evaded, no changes are needed to get the wage equation for the secondary irregular jobs. On the contrary, introducing taxation on wage, \(\tau\), implies to change equations 6 to obtain the wage equations in the regular sector:

\[
\begin{align*}
    rJ^{F}_{REG} &= p_{REG} - w_{REG}(1 + \tau) + s(J^{V}_{REG} - J^{F}_{REG}) \\
    rJ^{F}_{BadR} &= p_{BadR} - w_{BadR}(1 + \tau) + s(J^{V}_{BadR} - J^{F}_{BadR}) 
\end{align*}
\]

(6')

In the appendix (A1) it is shown how to get the wage equation for the secondary regular jobs in presence of taxation:

\[
w_{BadR_{\tau}} = \frac{1}{1 + \beta \tau} \left[ \beta (p_{Bad} - r k_{BadR}) + (1 - \beta) r J^{U} \right]
\]

(13')

As the derivation of the capital intensity in the different sectors was founded on the wage equations, proposition 1 no longer applies. In particular:

**Proposition 4**: in presence of taxation on wages, capital intensity in the regular production decreases. In equilibrium capital intensity is still higher for the primary intermediate good compared to the secondary regular one, but in the secondary sector the relative capital intensity (regular versus irregular) is a function of fiscal policy parameters (\(\tau;p;C\)).

---

\(^{15}\) Our assumption is that taxation is compensated by incentives to hiring.
Proof: see appendix A2

Taxation on wages alters the relative convenience to operate in the two secondary sectors (regular and irregular), and capital allocation is immediately affected. In fact, introducing taxation, capital invested in the secondary regular sector certainly decreases, even though it is not clear if this fall is so strong as to cause higher capital intensity in the secondary and irregular production. As capital intensity in the irregular sector is a decreasing function of the expected penalty rate, then the relative capital intensity in the secondary sector (regular versus irregular) is a function of all the fiscal policy parameters ($\tau; p; C$): the highest is the level of taxation, and the weakest is the enforcement, the highest will be the relative capital intensity in the irregular production.

**Proposition 5**: the introduction of taxation on wages reduces the net wage paid to regular secondary workers and can cause a positive wage gap for irregular workers.

Proof: see appendix A3

Taxation on wages has a direct effect on wages, due to discount factor of equation 13', but also an indirect effect through the equilibrium level of capital intensity of the regular production. The incentive to act in the irregular sector seems to operate through both the inputs: firms are willing to pay a lower wage to regular workers and, coherently, they reduce the amount of capital they want to invest in the regular production. The loss of wage due to taxation for regular secondary workers is:

$$\Delta w_{\beta\alpha R\tau} \equiv w_{\beta\alpha R\tau} - w_{\beta\alpha R\tau} = \frac{\beta\tau}{1 + \beta\tau} w_{\beta\alpha R\tau} > 0 \quad (18)$$

while, the wage gap equation in the secondary sector becomes:

$$w_{\beta\alpha R\tau} - w_{\beta\alpha R\tau} = \frac{\beta(p_{\beta\alpha R} - r k_{\beta\alpha R\tau})}{1 + \beta\tau} - \frac{(1 - \beta) r \beta w^{U}}{1 + \beta\tau} - \beta p(C - k_{\beta\alpha R\tau})$$

$$- \beta p(C - k_{\beta\alpha R\tau}) \quad (16')$$
The introduction of taxation on wages acts as a discounting factor, reducing the net wage paid to regular secondary workers, eq. 18, and can cause a positive gap for irregular workers, eq 16'. The necessary conditions to have a higher net wage in the regular secondary tier of the market are not very straightforward, and the sign of the relevant mathematical relation is ambiguous, depending on fiscal policy parameters as well as on technological parameters. Nonetheless, looking at equation 16', some considerations apply:

i. the effect of the technological premium for regularity

\[ \frac{1 - \gamma}{\beta} k_{BadR} - k_{BadIRR} \]

It is not yet a sufficient condition to have a positive wage gap for regular workers: fiscal policy parameters must be accounted for, because they can cause a different sign on the wage gap. In particular, if enforcement is weak (p approaches to zero), and taxation is sufficiently high, irregular workers could be better paid than regular ones, even in presence of the technological premium for regularity. In this case it is reasonable to assume that the decision to operate in the irregular sector is shared by employers and employees;

ii. if technologies are the same both in the regular and in the irregular production:

\[ \gamma = 1; k_{BadR} = k_{BadIRR} \]

and the expected penalty is low, then wages are always higher for irregular workers;

iii. finally, the existence of a technological premium for irregularity and a weak enforcement cause a negative wage gap or a higher wage for irregular workers.

The analysis of secondary job wages just described shows how taxation can cause a strong incentive, for workers, to operate in the irregular sector, as irregular jobs can be better paid\(^\text{16}\), giving rise to the good irregular match. Conversely, the first model, which supposes that incentives to hiring offset taxation on wages induce an opposite

\(^{16}\) See note 14.
conclusion, wages are higher in the regular field (bad irregular match). These two extreme situations give a possible interpretation of the reason why, in reality and in theoretical literature, it has often verified the existence of a dualism even among irregular workers. Taxation in some economic context is not the only relevant motivation to the existence of the underground sector, and empirical evidence about the size of the underground economy in southern regions of Italy seems to be a robust confirmation of this hypothesis.

4. Conclusions

This paper has shown how it is possible to model different typologies of irregular jobs, based on the different relative wage, considering different regimes of fiscal policy. A first typology of irregular job, arising when taxation is high and enforcement is weak, is characterised by a higher wage compared to an equivalent job in the regular sector, and presumably, by a shared decision to go idle by employer and employee (good irregular match). A second kind of irregular job, arising when taxation is offset by incentives and irregular technology is less capital intensive, pays a lower relative wage, and, presumably, is a second best solution for workers (bad irregular match).

Empirical observation, and in particular digits from INPS inspections, seem to support the existence of a dualism in the irregular sector of labour market, based on a different convenience/motivation of the workers.

The innovative aspect of the paper lies in a specific model which is able to model both kind of irregular labour, and suggesting that they refer to different motivations: evading tax burden in the first case, and non-fiscal motivations in the second.

A very interesting result of the model is the endogenous derivation of the capital intensity in each of the productive field modelled and the relationship between capital intensity and the decision to act in the irregular sector, which will need a deeper consideration. This allows to improve characterising irregular jobs, which, in absence of taxation on wages, are not only low-pay jobs, but also low-capital intensive. This
seems to be coherent with some of the results shown in Carillo and Papagni (2002), about the less innovative content of the irregular production. Some possible hypothesis to explain this peculiar typology of irregular jobs are: the quality of human capital, as assessed in Carillo and Papagni (2002), strong negative externalities, cultural attitude, attitude toward risk and intensity of fiscal controls, abundance of unemployment and availability of technologies with different labour intensity.
APPENDIX

A1) Wages Equations
No taxation on wages

To obtain equation 13 it is necessary to solve the system:

\[
(1 - \beta)(J_{\text{BadR}}^E - J_{\text{BadR}}^U) = \beta(J_{\text{BadR}}^F - J_{\text{BadR}}^E)
\]
\[
rJ_{\text{BadR}}^F = p_{\text{Bad}} - w_{\text{BadR}} + s(J_{\text{BadR}}^V - J_{\text{BadR}}^F)
\]
\[
rJ_{\text{BadR}}^E = w_{\text{BadR}} + s(J_{\text{BadR}}^U - J_{\text{BadR}}^E)
\]

And the analogous system needs to be solved to get equation 12.

To obtain equation 14 it is necessary to solve the system:

\[
(1 - \beta)(J_{\text{BadIRR}}^E - J_{\text{BadIRR}}^U) = \beta(J_{\text{BadIRR}}^F - J_{\text{BadIRR}}^E)
\]
\[
rJ_{\text{BadIRR}}^F = p_{\text{Bad}} - w_{\text{BadIRR}} + s(J_{\text{BadIRR}}^V - J_{\text{BadIRR}}^F) + p(D^D_{\text{IRR}} - J_{\text{BadIRR}}^E)
\]
\[
rJ_{\text{BadIRR}}^E = w_{\text{BadIRR}} + (s + p)(J_{\text{BadIRR}}^U - J_{\text{BadIRR}}^E)
\]

Taxation on wages
The introduction of taxation changes the starting system to solve to obtain bad regular wages:

\[
(1 - \beta)(J_{\text{BadR}}^E - J_{\text{BadR}}^U) = \beta(J_{\text{BadR}}^F - J_{\text{BadR}}^V)
\]
\[
rJ_{\text{BadR}}^F = p_{\text{Bad}} - w_{\text{BadR}}(1 + \tau) + s(J_{\text{BadR}}^V - J_{\text{BadR}}^F)
\]
\[
rJ_{\text{BadR}}^E = w_{\text{BadR}} + s(J_{\text{BadR}}^U - J_{\text{BadR}}^E)
\]
A2) Capital intensities

No taxation on wages

To derive the range of capital intensities in the three technologies we explicitly write the zero profit condition using the system:

\[
\begin{align*}
\langle rJ^V_{REG} &= p_{REG} - w_{REG} + s(J^V_{REG} - J^F_{REG}) \\
\langle rJ^V_{REG} &= q(\theta)(J^V_{REG} - J^V_{REG}) \\
J^V_{REG} &= k_{REG}
\end{align*}
\]

and with some simple algebra we get:

\[
\begin{align*}
J^F_{REG} &= \frac{1}{r+s}(p_{REG} - w_{REG} + sJ^V_{REG}) \\
J^V_{REG} &= \frac{q(\theta)(p_{REG} - w_{REG})}{r(s + q)} \\
q(\theta)\left[p_{REG} - (\beta(p_{REG} - k_{REG}) + (1 - \beta)rJ^U_{REG})\right] &= k_{REG}
\end{align*}
\]

Where we have used the expression for wage (12) in the last equation, from which we obtain the zero profit condition:

\[
q(\theta)\left[(1 - \beta)(p_{REG} - rJ^U_{REG})\right] = r_{REG}
\]

Using the same approach we get a similar result for the regular secondary sector:

\[
q(\theta)\left[(1 - \beta)(p_{BadR} - rJ^U_{REG})\right] = r_{BadR}
\]

For the irregular secondary sector we start form the system:
The relevant expressions for optimal capital intensities are:

\[
\begin{align*}
  rJ^E_{\text{BadIRR}} &= \mathcal{P}_{\text{Bad}} - w_{\text{BadIRR}} + s(J^V_{\text{BadIRR}} - J^F_{\text{BadIRR}}) + p(J^D_{\text{BadIRR}} - J^F_{\text{BadIRR}}) \\
  rJ^V_{\text{BadIRR}} &= q(\theta)(J^F_{\text{BadIRR}} - J^V_{\text{BadIRR}}) \\
  J^V_{\text{BadIRR}} &= k_{\text{BadIRR}}
\end{align*}
\]

Using the assumption that prices are higher in the primary sector, and equal in the secondary sector, it is immediate to verify that

\[
k_{\text{REG}} > k_{\text{BadR}} > k_{\text{BadIRR}}
\]

Taxation on wages

Considering the presence of taxation on wages causes a change in the equations for regular sectors:

\[
\begin{align*}
  q(\theta)\left[ (1-\beta)(p_{\text{REG}} - rJ^U) \right] &= r k_{\text{REG}} \\
  q(\theta)\left[ (1-\beta)(p_{\text{BadR}} - rJ^U) \right] &= r k_{\text{BadR}} \\
  q(\theta)\left[ (1-\beta)(p_{\text{Bad}} + pC - rJ^U) \right] &= r k_{\text{BadIRR}}
\end{align*}
\]

So that
While the relative capital intensity in the secondary sector is no longer defined to begin with, but depends upon taxation:

\[
\frac{\partial k_{BadR}}{\partial \tau} < 0; \quad \frac{\partial k_{BadIRR}}{\partial p} < 0; \quad \frac{\partial k_{BadIRR}}{\partial C} > 0.
\]

In fact, as C increases the penalty lowers (C<0), so that optimal capital allocated in the irregular sector increases.

A3) Relative wages

No taxation on wages

To investigate the size of the relative wages it is necessary to use the relevant zero profit conditions, as expressed in appendix A2, in the respective wages’ equations, to get:

\[
w_{REG} = \beta(p_{REG} - rJ^U) \left[ 1 - \frac{q(\theta)(1 - \beta)}{(r + s) + q(1 - \beta)} \right] + rJ^U
\]

\[
w_{BadR} = \beta(p_{Bad} - rJ^U) \left[ 1 - \frac{q(\theta)(1 - \beta)}{(r + s) + q(1 - \beta)} \right] + rJ^U
\]

\[
w_{BadIRR} = \beta(p_{Bad} + pC - rJ^U) \left[ 1 - \frac{q(\theta)(1 - \beta)}{(r + s + p) + q(1 - \beta) \frac{r + p}{r}} \right] + rJ^U
\]

Then it is easy to show that

\[
w_{REG} - w_{BadR} = \beta(p_{REG} - p_{Bad}) \left[ 1 - \frac{q(\theta)(1 - \beta)}{(r + s) + q(1 - \beta)} \right] > 0
\]
While:

\[ w_{BadR} - w_{BadIRR} = \beta(p_{Bad} - rJ^U) \left[ 1 - \frac{q(\theta)(1 - \beta)}{(r + s) + q(1 - \beta)} \right] + \]

\[ - \beta(qp_{Bad} + pC - rJ^U) \left[ 1 - \frac{q(\theta)(1 - \beta)}{(r + s + p) + q(1 - \beta)\frac{r + p}{r}} \right] \]

Whose sign is not immediate. It is possible to have a more clear interpretation of the wage gap in the secondary sector using expression found in the appendix A1. The difference between equations 13 and 14 is:

\[ w_{BadR} - w_{Badr} = \beta(p_{Bad} - rk_{Badr}) + (1 - \beta)rJ^U - \beta[qp_{Bad} - rk_{BadIRR}] + \]

\[ - \beta(p(C - k_{BadIRR}) - (1 - \beta)rJ^U = \]

\[ = \beta(p_{Bad} - rk_{Badr}) - \beta[qp_{Bad} - rk_{BadIRR} + p(C - k_{BadIRR})] \]

Taxation on wages

Relative secondary wages

Using some algebra it is possible to show that the difference between the two wages is given by:

\[ w_{BadR_T} - w_{Badr_T} = \frac{\beta(p_{Bad} - rk_{BAD})}{1 + \beta \tau} + \frac{(1 - \beta)rJ^U}{1 + \beta \tau} - r(1 - \beta)J^U + \]

\[ - \beta[qp_{Bad} - rk_{IRR} + p(C - k_{BadIRR})] \]

Or:

\[ w_{BadR_T} - w_{Badr_T} = \frac{\beta(p_{Bad} - rk_{BAD}) - \beta(1 + \beta \tau)[qp_{Bad} - rk_{IRR}]}{1 + \beta \tau} + \]

\[ - \frac{(1 - \beta)rJ^U}{1 + \beta \tau} - \beta(p(C - k_{BadIRR}) \]

Which sign from the beginning is quite difficult to define.
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