Equilibria in a model with a search labour market and a matching marriage market.

Roberto Bonilla
Business School, Newcastle University, Newcastle upon Tyne, U.K.

September 8, 2009
Equilibria in a model with a search labour market and a matching marriage market. - Abstract

I analyse an economy where a search labour market with an endogenous wage distribution and a matching marriage market interact. The economy is populated by homogeneous workers, firms and marriage partners (MPs). Workers simultaneously search for firms in order to work and for MPs in order to marry. Firms post wages to attract workers. MPs look for workers in order to marry. Married workers receive a pre-determined flow utility, and married MPs derive flow utility equal to the worker’s earnings. This provides the link between the markets. By interpreting workers and MPs as men and women respectively, I show that the so called married wage premium can arise purely from frictions in both markets. Also, the paper may explain the simultaneous occurrence of three stylised facts: In the model, an increase in the value of women’s option outside marriage leads to a decrease in marriage rates and an increase in the spread of the male wage distribution.

Equilibria in a model with a search labour market and a matching marriage market.

This paper analyses the equilibria in an economy where a search labour market and a matching marriage market interact. The economy is populated by ex-ante homogeneous workers, ex-ante homogenous firms, and ex-ante homogeneous marriage partners (MPs). Workers simultaneously search for firms in order to work and for marriage partners in order to marry. Firms post wages to attract workers; while MPs look for workers in order to marry. I assume that married workers receive a pre-determined flow utility; and that married MPs derive flow utility equal to the worker’s earnings (be it wage or unemployment benefit). This provides the link between the two markets. I use noisy search in the labour market to generate a distribution of wages offered and of wages earned\(^1\). In this set-up, a worker’s search for a firm is analogous to a marriage partner’s search for a worker, and both will use reservation wage strategies in their search efforts\(^2\). The decisions on reservation wages are interdependent: workers decide on their own reservation wage taking as given the marriage partners’ reservation wage and the shape of the wage offer distribution. MPs decide on their own reservation wage using...

\(^1\) The modeling of noisy search is based on Burdett and Judd (1983). I assume that when workers contact firms, they may have contacted one or two firms, with given probability strictly between 0 and 1. When firms are contacted by workers, they do not know if the worker has contacted one or two firms. This gives rise to equilibrium wage dispersion as firms balance the higher probability of a hire when offering higher wages with the lower profit given a hire is made.

\(^2\) A marriage partner may not be willing to marry anybody earning less than a given wage.
wage taking as given the worker’s reservation wage and the shape of the distribution of wages earned.

Gould and Paserman (2003) argue that 25% of marriage rate decline since the 1980s in the US can be explained by the increase in male wage inequality. The argument is that wage inequality increases the option value for women to search longer for a husband. This is interpreted as evidence to support a search model of the marriage market. Loughran (2002) models women’s search for marriageable men in a similar manner as in this paper. Similarly, he finds that rising male wage inequality accounts for 7% to 18% of the decline in the propensity to marry between 1970 and 1990 for white women and well educated black women in the US. Both Loughran (2002) and Gould and Paserman (2003) ignore the equilibrium consequences of modelling a search marriage market in an equilibrium framework encompassing a search labour market. By doing so in this paper, I find a connection between three stylised facts: In the model, increasing the value of women’s option outside marriage leads to a decrease in marriage rates and to an increase in the spread of the male wage distribution. Furthermore, this is true in an environment in which all men are homogenous. Additionally, the paper shows that the so called "married wage premium" (or, more general, a correlation between men’s wages and marital status) can in some circumstances be the equilibrium consequence of search frictions in the two markets. This is completely unrelated to the traditional explanations for a link between wages and marital status based on specialization in

---

3 Ginther and Zavodny (2001) find evidence to support the idea that labour market performance of the prospective partner is considered when making decisions about marriage. They compare the wage premium among men whose marriage was triggered by a pregnancy and was therefore followed by a birth within seven months; with those whose marriage was not followed by birth. They find that "married men with a pre-marital conception generally have a lower return to marriage than other men do". The underlying assumption is that pre-marital conception forces people into marriage.

4 Lundberg (2005) makes a call for research into the interdependence of decisions about work and marriage. The model presented here attempts to be a theoretical contribution to one of the dimensions of such interdependence and its consequences.

5 Gould and Passerman (2003) find that "women are more selective when the value of being single increases (higher female wages)...". Blau, Kahn and Waldfogel (2000) find that better female labour markets have the effect of lowering marriage rates, both for 16-24 year olds and 25-34 year olds. This supports the idea that, to some extent, women view marriage as a detriment to their labour market career, at least for those age groups. According to Fortin and Lemieux, the ratio of female to male median hourly earnings rose from 60% to 75% in the 1980s. Their calculation was done using Current Population Survey data.

6 According to Loughran (2002), 82% of 24 year old women and 71% of 24 year old men had ever been married in 1970. The percentages fell to 58% and 44% respectively in 1990.

7 According to Loughran (2002), male wage inequality rose during the 1980s in the United States. He reports that the difference in log weekly earnings between 90th and 10th percentile of the weekly wage distribution for full-time employed men ages 22-65 rose from 1.38 to 1.69 (19928).
the labour market and on unobserved characteristics that are valuable both in the labour the marriage market. Regarding the former, specialisation in the labour market after marriage is not introduced at all in the paper. Regarding the latter, this would require some kind of heterogeneity, so that some agents have the said unobserved characteristics, while some do not. To my knowledge, there is no other paper that analyses equilibrium in a model with two interacting frictional markets where relationships in both markets are long-term and interdependent decisions are taken by agents in all markets. I believe this to be the main theoretical contribution of this paper. Burdett, Lagos and Wright (2003,2004) present models in which workers in a frictional labour market encounter opportunities to commit a crime at a less than infinite rate, which is eventually endogenised. The workers decide on the reservation wage and on what they call the "crime" wage: workers will not commit a crime if earning more than that. A big difference is that in Burdett, Lagos and Wright (2003) it is the workers who make all the decisions, while in this paper all agents make interdependent decisions.

I describe in detail two pure strategy equilibria that can obtain in the environment generally described so far. In what I term the Very Picky (VP) Equilibrium, MPs reject marriage to unemployed workers and to low earners, and only marry employed, high earning workers. In what I term the Picky Equilibrium (P), MPs reject marriage to unemployed workers, but accept marriage to all employed workers, regardless of the wage earned. In the VP equilibrium, the utility workers derive from marriage is particularly relevant. It affects workers' reservation wage, since the reservation wage must compensate workers for the loss of marriageability in addition to the option of continued search for better wages. This affects the distributions of wages offered and of wages earned, which in turn are crucial in the MPs decision to accept low earners for marriage or not. Something analogue is true for the utility MPs enjoy while single. As a result, there is a two way equilibrium relationship between workers reservation wage and MPs behaviour. Further, there is an endogenous correlation between wages and marital status. It is in this context that the link emerges between the three stylised facts referred to above. In the P equilibrium, the worker's problem has a corner solution: they always find it optimal to increase their reservation wage in just the amount required to become marriageable. In this case, there is a one way relationship, as the MPs reservation wage affects the workers reservation wage; and there is no correlation between wages.

---

8 See Becker (1991) and Nakosteen and Zimmer (1987) respectively.
9 In Burdett, Lagos and Wright (2004), firms post wages, but it is still true that workers decide on both the reservation wage and the crime wage; while in here workers must take as given the reservation wage used by MPs.
10 In Section 4 I briefly describe another pure strategy equilibrium and a mixed strategy equilibrium, the detailed derivation of which are available from the author.
and marital status.
I use some simplifying assumptions, the removal of which is the basis for further or ongoing research. In particular, I assume that divorce is infinitely costly and therefore never occurs. When agents accept marriage, they do so knowing that they will never get divorced. This allows me to solve the problem analytically and to obtain interesting results about the marriage problem. In the conclusion, I discuss the consequences of allowing for divorce and preliminary results of ongoing research. A further assumption is that single MPs enjoy a predetermined flow utility, which I call $X$. In this set-up, $X$ can have many interpretations, like the value of living with parents, the value of accessing a the labour market, or the possibility of marrying differently skilled workers. I discuss in the conclusion consequences of modeling $X$ in more detail and preliminary results.

Section 1 below sets up the model and the strategies for the firms, the workers and the MPs. Sections 2 and 3 present the pure strategy equilibria briefly described above taking arrival rates as parametric. Section 4 briefly addresses other equilibria not fully described in the model. Section 5 endogenises the arrival rates and separates the parameter space into the two pure strategy equilibria described above. Section 6 concludes.

1 The Model.

In this section I set up the model and the assumptions relevant to the three types of agents: Firms, Workers and MPs.

Firms. Individual firms post wages and contact workers who are either single or married. Firms can wage discriminate according to the workers’ marital status.\footnote{At the end of both Sections 2 and 3, I argue that the corresponding equilibrium outcome looks exactly the same if firms are not allowed to wage discriminate in this manner.} Consider the problem vis-a-vis single workers first. Each firm takes as given the reservation wage of unemployed-single workers ($R$) and the distributions of wages for single workers offered in the market $G(w)$. The same is true about unemployed-married workers whose reservation wage I denote $R_m$, and the distribution of wages for married workers is $I(w)$.

When an individual firm contacts a worker, the worker may have contacted only her with probability $g$ or one other firm with probability $1 - g$, where $0 < g < 1$. If a firm offers wage $w$, and worker accepts, flow productivity is $p$. The match destroys if the worker dies, at an exogenous rate $\delta$. I model the labour market as a pure search market, hence firms can absorb all workers that contact her and accept her wage offer.

Workers. Workers take as given the reservation wage of MPs ($T$), and the distribution of wages offered ($G(w)$ for singles and $I(w)$ for marrieds).

\footnote{This means marriage partners will not marry a worker earning $w < T$.}
Unemployed workers decide their reservation wage (singles decide on \( R \) and marrieds decide on \( R_m \)). When workers make a contact, with probability \( g \) they contact only one firm and with probability \( 1 - g \) they contact two firms. Hence, the distribution of wages faced by single or married workers in their search effort is different from that respective distribution of wages offered. I denote by \( F(w) \) the distribution of wages faced by single workers and by \( H(w) \) the distribution faced by married workers. All workers, regardless of their marital status, receive unemployment benefit \( b \) while unemployed. When workers are married, they enjoy flow value \( m \); regardless of their labour market status (in addition to their wage if they are employed, and to the unemployment benefit if they are unemployed). Workers contact firms at rate \( \lambda_0 \) when single and there is no on-the-job search. They contact MPs at rate \( \lambda_m \) when single. They die at rate \( \delta \) whatever their employment status. For simplicity, the total number of workers is normalised to 1.

**Marriage Partners (MPs).** MPs take as given the distribution of wages earned (by single workers, who got their job while single), \( F(w) \)\(^{13}\) (including the reservation wage \( R \)); they decide on their own reservation wage, \( T \), i.e., they will not marry anybody earning less than \( T \). When they are married to an employed worker earning \( w \), they enjoy flow value \( w \). When they are married to an unemployed worker receiving unemployment benefit \( b \), they enjoy flow value \( b \)\(^{14}\). MPs enjoy flow value \( X \) while single. MPs contact single employed workers earning a wage \( w \geq T \) at rate \( e_s \), employed workers earning a wage \( w < T \) at rate \( e_{nm} \), and single unemployed workers at rate \( u_s \), they die at rate \( \delta \) when single. I assume that married MPs die if their partner dies, and this happens at rate \( \delta \). I normalise the number of single marriage partners to \( \lambda_m \). Notice that the marriage market is a matching market with non-transferable utility and that the worker's wage is a public good within a marriage.\(^{15}\)

To keep things simple, I use quadratic matching in the marriage market and cloning of single MPs. I assume that a new marriage partner comes into the market every time one gets married or dies, so as to maintain that stock constant. Regarding workers, I assume that everytime a worker dies, another worker enters the market as single and unemployed. I assume that divorce is infinitely costly and therefore never occurs. In the model, time is continuous and I only analyse steady state equilibria.

**Wage Distributions**

\(^{13}\)Since there is no on the job search, the distribution of wages faced by workers in their search effort is the same as the distribution of wages earned.

\(^{14}\)MPs do not participate in the labour market. It is not an unrealistic assumption to think of agents that do not engage in the labour market. Even today, this is the case for women in most developing countries.

\(^{15}\)The analysis does not change if it is assumed that only a fraction of the worker's wage is a public good.
In Section 2, 3 and 4, I characterise the possible equilibria in this model. Here I derive results about the wage distributions that are valid for all those equilibria.

**Equal profits condition and distributions of wages offered: \( G(w) \) and \( I(w) \).** Assume a firm offers wage \( w \geq R \) and consider the firm’s problem vis-a-vis single workers. Given a worker accepts the job offer, the firms discounted profits from employing that worker are \( \pi(w) = \frac{p-w}{1+g} \). Given that a worker has been contacted, and wage \( w \geq R \) is offered, the expected profits are

\[
\Pi(w) = (1 - g)G(w)\pi(w) + g\pi(w).
\]

since one has to consider the probability that the worker opts for the other firm if two firms were met. Denote the lowest and highest wages offered by \( w \) and \( \bar{w} \) respectively. For any wage \( w \) such that \( w \leq w \leq \bar{w} \), in equilibrium it must be true that \( \Pi(w) = \Pi(w) \). Assuming \( R = \bar{w} \) (which below I argue to be true in equilibrium, for the standard arguments), then \( \Pi(w) = g\pi(w) \) since \( G(w) = 0 \). Then:

\[
\begin{align*}
\Pi(w) &= \Pi(w) \Rightarrow g\pi(w) = (1 - g)G(w)\pi(w) + g\pi(w) \\
G(w) &= \frac{1 - g}{g} \frac{w - w}{p - w}, \\
G(w) &= 1 \Rightarrow \bar{w} = gp + (1 - g)R
\end{align*}
\]

Notice, \( G(w) \) is continuous along its support. \( R = \bar{w} \) is therefore true, for the standard reasons: i) A wage \( w < R \) will not be offered by any firm because no worker will accept it. ii) Assume \( w > R \), then \( F(w) = F(R) = 0 \). Then any firm offering \( w \) can reduce its wage offer all the way to \( R \) and increase its expected profits.

The problem vis-a-vis married workers is analogue to the above, with the difference that the minimum and maximum wages need not be the same as for single workers. Denote by \( w_m \) and \( \bar{w}_m \) the minimum and maximum wage respectively in the wage distribution offered to married workers. Then it is given (analogously) by \( I(w) \) where:

\[
I(w) = \frac{1 - g}{g} \frac{w - w_m}{p - w}, \quad \bar{w}_m = gp + (1 - g)w_m
\]

where \( w_m = R_m \).

**Search-relevant wage distributions: \( F(w) \) and \( H(w) \).** Given that workers contact one firm or two firms with the respective probabilities \( g \) and \( 1 - g \), the distribution of wages faced by single [married] workers in their search...
effort is given by:
\[ F(w) = gG(w) + (1 - g)G(w)^2 \Rightarrow F(w) = \frac{(1 - g)^2 (w - w)(p - w)}{g} \]
\[ H(w) = gI(w) + (1 - g)I(w)^2 \Rightarrow H(w) = \frac{(1 - g)^2 (w - w_m)(p - w_m)}{g} \]

In Sections 2 and 3, I study two possible equilibria in this model. I term them the Very Picky Equilibrium (VP)\(^{16}\) and the Picky Equilibrium (P)\(^{17}\). When the subscripts \(vp\) or \(p\) appear on a variable, this denotes that the variable takes the value corresponding to the VP or P equilibrium respectively.

2 The Very Picky Equilibrium (VP).

In the VP equilibrium, MPs reject marriage to some low-wage employed workers and with unemployed workers\(^{18}\). This means that, in equilibrium, unemployed workers are willing to accept wages that make them unmarriageable. I first analyse the worker’s problem and then go on to analyse the MP’s problem. Then I show when the equilibrium obtains.

**Workers.** Assume single workers decide on a reservation wage \(R = R_w\). Then, following the desired properties of the VP equilibrium, I require that

\[ i) R_w < T < \tilde{w}, \quad ii) R_w > b \]

Condition \(i\) ensures that unemployed workers are willing to accept wages that make them unmarriageable. Condition \(ii\) ensures that the minimum wage accepted by unemployed workers is strictly higher than their unemployment benefit \(b\)\(^{19}\). If working at a wage \(x < T\), the worker’s payoff is given by \(V_1(x)\) defined by \(rV_1(x) = x - \delta V_1(x)\), since there is no expectation of marrying. If working at a wage \(x \geq T\), the worker’s payoff is given by \(V_2(x)\) where \(rV_2(x) = x + \lambda_m (V_3(x) - V_2(x)) - \delta V_2(x)\) (since \(\lambda_m\) is the rate at which marriageable workers meet MPs), where \(V_3(x)\) is the payoff of being married and working at wage \(x\). If working at a wage \(x\) and married, the workers payoff is given by \(V_3(x)\), where \(rV_3(x) = x + m - \delta V_3(x)\). The payoff of being single is given by

\(^{16}\)Because MPs reservation wage \(T\) is higher than the workers reservation wage \(R\), which means that some employed workers, those earning \(w\) such that \(R < w < T\), are unmarriageable.

\(^{17}\)Because MPs reject marriage to unemployed workers but accept all employed workers.

\(^{18}\)This is not necessarily true always. There may be reasons why a MP could prefer marriage to an unemployed worker over marriage to a low earner, but these are not built into this model. Uncertainty over the unemployed workers productivity is an example, as this would imply uncertainty over the workers expected performance in the labour market.

\(^{19}\)\(R_w < b\) is not rational from the unemployed worker’s point of view, and \(R = b\) would lead to a qualitatively different type of equilibrium as will become clear later.
\[ rV_0 = b + \lambda_0 \int_T^w [\max(V_1(x), V_0) - V_0] f(x)dx + \lambda_0 \int_T^{\bar{w}} [\max(V_2(x), V_0) - V_0] f(x)dx - \delta V_0 \]  

(1)  

In equation (1), a worker faces a wage offer distribution \( F(w) \). He receives \( b \) while unemployed. He contacts firms at rate \( \lambda_0 \). If the contacted firm offers a wage \( x \) such that \( R_w < x < T \), then he must choose between accepting the job which makes him unmarriageable with payoff \( V_1(x) \) or remaining single. If the firm offers a wage \( x \) such that \( T \leq x < \bar{w} \) then the worker must choose between accepting the job which makes him marriageable with payoff \( V_2(x) \) or remaining single. The worker dies at rate \( \delta \). Given a wage offer \( w \) has been received by a worker, \( \frac{\partial V_1(w)}{\partial w} > 0 \) and \( \frac{\partial V_0}{\partial w} = 0 \). Then, the standard definition of a reservation wage implies \( V_1(R_w) = V_0 \), \( w \geq R_w \), then the worker accepts any wage \( w \geq R_w \), and (1) can be written  

\[ rV_0 = b + \lambda_0 \int_{R_w}^T [V_1(x) - V_0] f(x)dx + \lambda_0 \int_T^{\bar{w}} [V_2(x) - V_0] f(x)dx - \delta V_0 \]  

(2)  

Integration by parts of (2) using \( V_1(x), V_2(x), V_3(x) \) and \( V_1(R_w) = V_0 \) yields  

\[ R_w = b + \frac{\lambda_0 \lambda_m m (1 - F(T))}{(r + \partial + \lambda_m)(r + \delta)} + \lambda_0 \int_{R_w}^{\bar{w}} \frac{1 - F(x)}{r + \delta} dx \]  

In the above equation, the first and third elements of the right hand side are standard: the reservation wage must compensate the worker for the loss of unemployment benefit and for the option of continued search for better wages. The second term relates to the marriage option. If the workers accept wages that make them unmarriageable, they are giving up the expected utility attached to marriageability. The reservation wages must compensate them for this loss: the probability of contacting a firm (at rate \( \lambda_0 \)) that offers a marriageable wage (with probability \( 1 - F(T) \)), and then contacting a marriage partner (with probability \( \lambda_m \)) would leave the worker enjoying flow value \( m \). In the limit as \( r \to 0 \), and using \( F(w) \) as in Section 1 with \( R_w = \bar{w} \) and \( \bar{w} = gp + (1 - g) \), this yields  

\[ R_w = b + \frac{k_0 k_m m \left[ 1 - \frac{(1 - g)^2(R_w - T)(-p + R_w)}{g(p - T)^2} \right]}{1 + k_m} - k_0 \Lambda(-p + R_w) \]  

(3)  

\(^{20}\)Considering as well that \( V_2(w) > V_1(w) \)
where \( \Lambda = \frac{g(2g-1) - \ln(1-g)(1-g)^2}{g} > 0 \) and \( k_i = \frac{\lambda_i}{g} \).

From (3), it is possible to derive the necessary results to characterise the behaviour of \( R_w \) in the range \( R_w < T < \bar{w} \).

To avoid technical complications, I will assume \( m < m_a \), where
\[
m_a = \frac{(1+k_m)(p-b)g}{(g+1+k_0)k_m k_0}.
\]
The intuition behind this condition is easier to explain after stating and explaining Proposition 1 below. In Proposition 1, I evaluate \( R_w \) in the two extremes: when \( T = R_w \) (as low as it can be) and when \( T = \bar{w} \) (as high as it can be); and I characterise the behaviour of \( R_w(T) \) in the region \( R_w(T) < T < \bar{w} \). The main message of Proposition 1 states that \( R_w(T) \) is a downward sloping concave curve. The intuition is provided after the Proposition.

**Proposition 1.**

i) \( T = R_w(T) \) implies \( R_w(T) = R_1 \) and \( T = T_1 \) where
\[
R_1 = T_1 = \frac{(1+k_m)(b+k_0\Lambda p) + k_0 k_m m}{(1+k_m)(1+k_0\Lambda)}
\]

ii) \( T = \bar{w} \) implies \( R_w(T) = R_2 \) and \( T = T_2 \) where
\[
R_2 = \frac{b+k_0\Lambda p}{1+k_0\Lambda} < R_1, \quad T_2 = \frac{p(g+k_0\Lambda) + b(1-g)}{1+k_0\Lambda} > T_1
\]

iii) \( T_2 > T_1 \) and (3) represents a downward sloping curve in \( R_w, T \) space in the range \( R_w < T < \bar{w} \).

**Proof.** See appendix.

Figure 1 exemplifies the situation. The intuition for Proposition 1 and Figure 1 is as follows:

i) If \( m \) is very high, marriage is too valuable for workers. They would never be willing to accept an unmarriageable wage, as that would mean giving up the prospect of enjoying \( m \) altogether.

ii) Assume \( m \) is high but not so high (\( m < m_a \) satisfies "\( m \) is not so high"). Hence, workers could be willing to accept a non-marriageable wage under certain conditions. Assume as well that \( R_w(T) = T \). As \( T \) goes up, workers have less incentive to reject any wage \( x < T \), since further search is less likely to produce a marriageable wage. This implies \( R_w(T) \) goes down.

iii) If \( m \) is very high, the effect of an increasing \( T \) on \( R_w(T) \) is very high. Hence, as \( T \) goes up, \( R_w(T) \) falls very fast. If \( m \geq m_a \) as defined above, then \( R_w(T) \) falls below \( b \) before \( T = \bar{w} \). From then on, even as \( T \) continues increasing, equation (3) no longer describes the behaviour of \( R_w \), as it would be irrational for workers to accept a lower reservation wage. I am avoiding this last complication by assuming \( m < m_a \).

**Marriage Partners.** Assume \( MP \)s know workers use reservation wage \( R = R_{mp} \). The properties of the \( VP \) equilibrium require

i) \( R_{mp} < T < \bar{w}, \quad ii) R_{mp} > b \)
MPs enjoy \( X \) while single \(^{21}\), and they contact employed marriageable workers at rate \( e_{vp} \). If they only accept marriage with employed workers earning \( w \geq T > R_{mp} \) then their payoff is given by

\[
rM_1 = X + es_{vp} \int \frac{M_2(x) - M_1}{1 - F(T)} dF(x) - \delta M_1
\]

since the distribution of wages among marriageable workers is conditional on \( w \geq T \). Section 5 below deals with the steady states values and shows that MPs meet workers earning a wage \( w > T \) at rate \( e_{s,vp} = \frac{\delta}{\sigma + \lambda_0} \lambda_0 (1 - F(T)). \)

Hence, using \( \eta = \frac{\delta}{\sigma + \lambda_0} \lambda_0 \), the above equation can be written

\[
rM_1 = X + \eta \int \frac{M_2(x) - M_1}{1 - F(T)} dF(x) - \delta M_1 \tag{4}
\]

In (4), given a contact with a single-employed worker earning \( T < w < \bar{w} \), marriage occurs yielding payoff \( M_2(x) \) (the payoff of an MP married to an employed worker earning wage \( x \)). The worker’s wage is a random draw from \( G(w) \). The MP dies at rate \( \delta \). The value \( M_2(x) \) is given by

\[
rM_2(x) = x - \delta M_2(x) \tag{5}
\]

In (5) above, if the MP is married to an employed worker, then its status will only change if death arrives, which happens at rate \( \delta \). Given a contact with a worker earning \( w \), notice that \( \frac{\delta M_2(w)}{\delta w} > 0 \) and \( \frac{\delta M_1}{\delta w} = 0 \). Then \( M_2(T) = M_1, M_2(T) \geq M_1 \) if \( w \leq T \). Integration by parts of (4) using (5), \( M_2(T) = M_1 \), and evaluating in the limit as \( r \rightarrow 0 \) implies

\[
T = X + \rho \int \frac{M_2(x) - M_1}{1 - F(x)} dx \tag{6}
\]

where \( \rho = \frac{\eta}{\eta} \).

I now characterise the behaviour of (6), using \( F(x) \) and \( \bar{w} \) as above, when \( R_{mp}(T) < T < \bar{w} \). Proposition 2 below lists all required information to sketch the graph of (6) in \( R_{mp}(T), T \) space. Such a graph is depicted in Figure 2. To state Proposition 2, I follow the same strategy used for Proposition 1: I evaluate \( R_{mp}(T) \) in the extremes where \( T = R_{mp}(T) \) and when \( T = \bar{w} \), and I characterise \( R_{mp}(T) \) when \( T \) satisfies \( R_{mp}(T) < T < \bar{w} \).

\(^{21}\)There are many possible interpretations for \( X \): Living at home, working in a low wage competitive labour market, the possibility of marrying differently skilled workers, etc. A more detailed characterisation of \( X \) is discussed in the conclusion.
The main message of Proposition 2 is that $R_{mp}(T)$ is an upward sloping concave curve. The intuition is provided after the Proposition.

**Proposition 2.** If $p > X$, then

1. $T = R_{mp}(T)$ implies $T = T_3$, $R_{mp}(T) = R_3$, where

$$R_3 = T_3 = \frac{\rho p \Lambda + X}{1 + \rho \Lambda} < \bar{w}$$

2. $T = \bar{w}$ implies $T = T_4$ and $R_{mp}(T) = R_4$, where

$$R_4 = \frac{X - gp}{1 - g} < \bar{w}, \quad T_4 = X$$

3. $R_3 > R_4, T_3 > T_4$ and (6) represents an upward sloping curve in $R_{mp}(T), T$ space for $R_{mp}(T) < T < \bar{w}$.

**Proof.** See appendix.

Following the results in **Proposition 2**, one can sketch the graph of (6) as in Figure 2. The intuition is as follows: As the worker’s reservation wage ($R_{mp}$, which is taken as given by $MP$s) increases, the distribution of wages offered shifts up. This is because firms respond to worker’s reservation wage, i.e. $G(w)$ is endogenous. Given a better distribution of wages earned, $MP$s become pickier in accepting marriage, since the option of continued search for a higher earner is more valuable. This describes a positive relationship between $MP$’s reservation wage ($T$), and worker’s reservation wage, ($R_{mp}$). An equilibrium exists if the functions $R_w(T)$ and $R_{mp}(T)$ cross while $R_w(T) < T < \bar{w}$ and $R_{mp}(T) < T < \bar{w}$. In order to state Lemma 1 below, I first define

$$X_a = \frac{p(g + k_0 \Lambda) + (1 - g)b}{1 + k_0 \Lambda} < p$$

**Lemma 1.** $X = X_a$ implies $R_4 = R_2$ and $T_4 = T_2$. This implies a situation as depicted in Figure 3.

**Proof.** See appendix.

**Lemma 2.** As $X$ decreases, $R_w(T)$ remains unchanged and $R_{mp}(T)$ shifts to the left. If $X = X_a - \epsilon, \epsilon > 0$ then the VP Equilibrium obtains. The situation is as depicted in Figure 4.

**Proof.** See appendix.

The intuition for Lemma 2 is as follows: As $X$ decreases, $MP$s utility when single decreases. This implies that they give up less when accepting marriage to an $MP$, hence they become less picky in accepting marriage. This implies that, ceteris paribus, their reservation wage, $T$, decreases. Graphically, this means $R_{mp}(T)$ shifts to the left.
Lemma 3. $T_1 \geq T_3$ if and only if $X \leq X_b$, where

$$X_b = \frac{b(1 + \rho \Lambda)}{(1 + k_0 \Lambda)} + \frac{k_0 k_m m [1 + \Lambda \rho]}{(1 + k_0 \Lambda)(1 + k_m)} + \frac{(k_0 - \rho) p \Lambda}{(1 + k_0 \Lambda)}$$

and $m < m_a \Rightarrow X_b < X_a$. In this case, the situation is as depicted in Figure 5, and the VP equilibrium does not obtain if this is the case: $X$ is so low that $R_{mp}(T)$ has shifted too much to the left.

Proof. See appendix.

Proposition 3. The VP Equilibrium obtains if $X_b < X < X_a$.

Proof. See appendix.

Following Lemmas 1-3 and by inspection of the associated Figures, if the condition in Proposition 3 hold, the situation is as depicted in Figure 4.

Notice,

i) When the VP equilibrium obtains, there is a correlation between wages and marital status. Married workers are necessarily employed at a wage $w \geq T$, while single workers can be unemployed or employed at a wage $w \geq R$, where $R < T$. This can be a source for the so called married wage premium prevalent in the empirical literature.

ii) An increase in the value of MP s option out of marriage $X$ (that does not take $X$ above $X_a$) will make MP s more picky and they will increase their reservation wage $T$ (as stated in Proposition 3). As a result, workers find it optimal to reduce their reservation wage, as they now have a smaller incentive to wait for (less probable) marriageable wages. Hence, there is a negative relationship between $X$ and $R$. As the MP s flow utility when single increases, they will increase their reservation wage $T$, causing a decrease in workers reservation wage $R$. As a consequence, the endogenous link between wages and marital status becomes stronger. In addition, the spread of the wage distribution increases since $0 < \frac{\partial w}{\partial T} < 1$. Notice this also implies a decrease in marriage rates as $T$ is now higher and $R$ is smaller.

iii) As the value workers give to marriage increases (an increase in $m$ that keeps $X_b < X < X_a$) this will increase their reservation wage $R_w$, as it must compensate them for a bigger loss since marriageability is now more valuable. It is easy to show formally that this occurs through an upshift of $R_w$ and this causes an increase in the equilibrium $T$ as well.

No wage discrimination. Here I analyse the consequence of removing the assumption that firms can wage discriminate across marital status. First consider the reservation wage of a married-unemployed worker (should one exist), $R_m$. As divorce is not possible, he can ignore the marriage problem. Intuitively, his reservation wage comes from the solution to (3) with $R_w = R_m, F(.) = H(.)$ and $\lambda_m = 0$. This results in $R_m = \frac{b + k_0 A p^{22}}{1 + k_0 \Delta}$. In

\[\text{Formally, this is easy to derive by noting that the bellman equation of an unemployed}\]
any VP equilibrium, $R > R_m$. This implies that all wages paid in the VP equilibrium are above $R_m$, and are paid to single unemployed workers. Firms can (and do) offer $R_m$ and wages in $I(w)$, but this is irrelevant in practice since married unemployed workers do not exist. Without wage discrimination, any individual firm must now offer one wage only, payable both to single and married workers, rather than one wage for each. The issue is if any given firm is willing to offer $R_m$ only rather than $R$ only (which is what in practice she is doing as married unemployed workers do not exist). It turns out no firm would be willing to do this, because single unemployed workers would not accept $R_m$ and married unemployed workers do not exist. The equilibrium looks the same with the only difference that now firms offer only one wage distribution rather than two: They offer $R$ and the wage distribution is $F(.)$ to any worker that contacts her, but these are all singles, just as before. They do not offer $R_m$ nor $H(w)$, but this does not make any tangible difference as these wages were not paid before because married unemployed workers did not exist either.

3 The Picky Equilibrium (P).

In the $P$ equilibrium, MPs marry workers if and only if these are employed. I construct this equilibrium by proposing that all workers and MPs use the same reservation wage, so $T = R = T_p = \frac{\theta \lambda \nu \alpha - X}{1 + \rho a}$. If this is true in equilibrium, then it implies $w = T_p$. The proof of Proposition 4 below shows when no individual worker or MP has an incentive to deviate from this strategy. The problem for an MP who is single is therefore to choose a reservation wage $T$, assuming that the minimum wage in $F(w)$ is $\bar{w} = T_p$. For any $T$, the problem of the MPs can be described as in the VP equilibrium, assuming assuming $w = T_p$. Hence, $T$ satisfies

$$T_{W=T_p} = X + \rho \int_{T_{W=T_p}}^{\bar{w}} [1 - F(x, w = T_p)] dx$$

where $F(x)$ is as given in Section 1 but using $w = T_p$. It is easy to show that $T(w = T_p) = T_p$.

An unemployed worker’s problem in this environment can be presented in a way more convenient for my purposes in this section, as it is more familiar to the concept of a corner solution. The payoff of an unemployed worker is described by $V_0$ as in (2) but with $w = T_p$. An individual worker takes married worker is given by $rV_0' = b + m + \lambda_0 \int \bar{w}_m \max(V_3(x), V_0) h(x) dx - \delta V_0$ and solving for the reservation wage in the standard way.

23 Except in the limiting case when $X = X_0$, when $R = R_2 = R_m$. 

13
as given that $w = T_p$. If the worker decides to accept any offer with wage $x \geq R$, then, the worker’s problem is $\max_R V_0$ subject to:

1) $F(w)$ as given in Section 1

2) $V_1(x), V_2(x)$ and $V_3(x)$ as given in Section 2

3) $R \leq T_p, w = T_p, w = T_p(1 - g) + gp$

This problem is analogue to the one solved in Section 1, and therefore yields an analogue solution:

$$R^*(X) = b + \frac{k_0k_mm}{1+k_m} \left[ 1 - \frac{(1-g)^2(w-T)(-p+w)}{g(p-T)^2} \right] - k_0\Lambda(-p + w)$$

Because in the $P$ equilibrium $w = T = T_p$, I impose this to get

$$R^*(X, w = T = T_p) = b + \frac{k_0k_mm}{1+k_m} + \frac{k_0\Lambda(p - X)}{1 + \rho \Lambda} = R^e(X)$$

**Proposition 4.** Assume $X_{b'} < X < X_b$, where $X_b$ has been defined above and

$$X_{b'} = \frac{b(1 + \rho \Lambda)}{(1 + k_0\Lambda)} + \frac{(k_0 - \rho)\Lambda p}{(1 + k_0\Lambda)} = X_b(m = 0)$$

Then an equilibrium obtains where $R = T = T_p$.

**Proof.** See appendix.

In the Picky Equilibrium, $X$ is high but not too high, hence so is $MP$'s reservation wage, $T$. Workers always find it optimal to increase their reservation wage just the required amount in order to be marriageable when employed, i.e. until $R = T$. Alternatively, the value workers attach to marriage, $m$, is relatively high enough, so they always increase their reservation wage to ensure marriageability when employed. Opposite to the $VP$ equilibrium

1) When the $P$ equilibrium obtains, there is no correlation between wages and marital status, there is only a correlation between employment status and marital status: if a worker is a married then he is employed.

2) An increase in $X$ (that does not take $X$ above $X_b$) will make $MP$'s more picky and they will increase their reservation wage $T$. As a result, workers will increase the reservation wage in the same amount in order to become marriageable when employed. This is clear by just looking at the equilibrium $T = R = T_p$

3) As the value workers give to marriage increases (an increase in $m$ that keeps $X_{b'} < X < X_{b'}$) this has no effect on worker’s reservation wage. This is clear by just looking at the equilibrium $T = R = T_p$. This occurs because workers are marriageable in equilibrium anyway.
No wage discrimination. Given that the equilibrium wage in the $P$ equilibrium is $R = \frac{\rho_1 \Delta + X}{1 + \rho_1} > R_m = \frac{b + k_0 \Delta p}{1 + k_0 \Delta}$, it is again true that the equilibrium outcome looks the same if firms cannot wage discriminate according to marital status, for reasons analogue to those exposed in the $VP$ equilibrium section.

4 Other Equilibria

For completeness, this section describes another pure strategy equilibrium and a mixed strategy equilibrium, the derivation of which is not included in the paper but is available from the author.

The smitten equilibrium. It is easy to show that an equilibrium can obtain where worker’s are marriageable both when employed and when unemployed, which means that their decision on the reservation wage ignores the existence of the marriage market. In this case, their reservation wage is given by $R = R_m = \frac{b + k_0 \Delta p}{1 + k_0 \Delta}$ which is the same as solving for $R_w$ from (3) with $\lambda_m = 0$. This equilibrium obtains for $X < X_c$ where $X_c = \frac{b(1 + \rho_1 \Delta)}{1 + k_0 \Delta} + \frac{(k_0 \Delta - \rho_1 \Delta)p}{1 + k_0 \Delta} = X'$. 

5 Matching and Steady State.

I start this Section by Proposition 5 below, which fully separate the parameter range in the $VP$ and $P$ equilibrium. The derivation of the steady state values that follows Proposition 5 serves as proof of Proposition 5.

Proposition 5. For $0 < m < m_a$
If $X_b < X < X_a$, an equilibrium obtains where $R < T$, and $R_1 < R \leq R_2$, $T_3 < T \leq T_4$
If $X_b' < X < X_b$, an equilibrium obtains where $R = T = T_p$.

To keep things simple, I use quadratic matching in the marriage market and cloning of single MPs. I normalise the number of single MPs to $\lambda_m$, and assume that a new marriage partner comes into the market every time one gets married or dies, so as to maintain that stock constant.

Workers can be in either of fives states: $u_s$ is the total number of workers who are single and unemployed, $e_s$ are single and employed earning a marriageable wage $w \geq T$, $u_m$ are married and unemployed; $e_m$ are married and employed and $e_{nm}$ are employed and not marriageable. I assume that a worker comes into the market as single and unemployed every time a worker dies, whatever its state, and normalise so that $u_{s,i} + e_{s,i} + u_{m,i} + e_{m,i} + e_{nm,i} = 1$, where $i = vp, p$.

Very Picky Equilibrium.
In the $VP$ equilibrium, unemployed workers cannot get married, and not all employed workers can get married, but only those that earn $R \geq T$. 

15
Unemployed single workers. The flow in is given by those who replace dead workers. The flow out is given by those workers in this stock who die or find a job. Hence, steady state requires \( \delta = u_{s, vp}(\delta + \lambda_0) \Rightarrow u_{s, vp} = \frac{\delta}{\delta + \lambda_0} \).

Employed single marriageable workers. The flow in is given by those workers who are unemployed and single and find a job with a marriageable wage. The flow out is given by those workers in this stock who die or marry after contacting a MP. Hence, steady state requires \( u_{s, vp}\lambda_0(1 - F(T)) = e_{s, vp}(\lambda_m + \delta) \) which substituting out \( u_{s, vp} \) implies \( e_{s, vp} = \frac{\delta - \lambda_0(1 - F(T))}{\delta + \lambda_0} \). Given the quadratic meeting technology, this is the rate at which MPs meet single marriageable men.

Employed married workers. The flow in is given by those workers who are single and employed at a marriageable wage who marry after contacting a MP. The flow out is given by those in this stock who die. Hence, steady state requires \( e_{s, vp}\lambda_m = e_{m, vp}\delta \Rightarrow e_{m, vp} = e_{s, vp}\frac{\lambda_m}{\delta} \Rightarrow e_{m, vp} = \frac{\delta}{\delta + \lambda_0} \frac{\lambda_0(1 - F(T))\lambda_m}{\lambda_m + \delta} \).

Unemployed married workers. Unemployed workers are not marriageable in this equilibrium so \( u_{m, vp} = 0 \).

Employed non marriageable workers. The flow in is given by those workers who are unemployed and single and accept an job with a unmarriageable wage. The flow out is given by those workers in this stock who die. Hence, steady state requires \( u_{s, vp}\lambda_0 F(T) = e_{nm, vp}\delta \), which implies \( e_{nm, vp} = \frac{\lambda_0 F(T)}{\delta + \lambda_0} \). Because of quadratic matching, the rate at which MPs meet workers earning \( w \geq T \) is \( e_{s, vp} = \frac{\delta}{\delta + \lambda_0} \frac{\lambda_0(1 - F(T))}{\lambda_m + \delta} \).

6 Conclusion.

I obtain the equilibria in a model in which a search labour market and a matching marriage market interact. The economy is populated by ex-ante homogeneous workers, ex-ante homogenous firms, and ex-ante homogeneous marriage partners. Workers simultaneously search for firms in order to work and for marriage partners in order to marry. Firms post wages to attract workers; and marriage partners look for workers in order to marry. I assume that married workers receive a pre-determined flow utility, and that marriage partners derive utility equal to the worker’s wage. I show that the so called "married wage premium" or, more generally, a correlation between men’s wages and marital status, can emerge as an equilibrium result of having search frictions both in the labour and the marriage market. In general, I explore the endogenous relationships that can arise between the marriage market and the labour market. The paper may explain the simultaneous
occurrence of three stylised facts: In the model, an increase in the value of
women’s option outside marriage leads to a decrease in marriage rates and
an increase in the spread of the male wage distribution. I do not know of
another model that analyses the equilibrium interaction of a search market
(the labour market) and a matching market (the marriage market), in which
both models give rise to long term relationships. I see this as the main
theoretical contribution of the paper.
In order to obtain clean analytical results, I use some assumptions the re-
moval of which seems interesting and is the basis of current research. For
example, if divorce is allowed, the model seems to yields empirically valid
predictions not only about the married wage premium, but also about the
"divorced wage premium". Namely, that divorce men enjoy a wage premium
smaller than married men, but still positive over never married men. When
an unmarried and unemployed worker accepts an unmarriageable wage he
looses the option to get married in the future (or what I have termed "mar-
riageability"). When a married and unemployed worker accepts an unmar-
riageable wage he is divorced by his partner, thereby loosing marriage itself,
which is more valuable than the option of a future marriage. Hence, pro-
vided both have a reservation wage lower than that of marriage partners,
the reservation wage of married workers is higher than the reservation wage
of unmarried workers, as they loose more when accepting an unmarriageable
wage.
I assume that single marriage partners enjoy a predetermined flow utility,
which I call $X$. In the paper I have used the interpretation that $X$ can
be a measure of women’s option outside marriage, for example her labour
market opportunities. Amongst other things, $X$ could be interpreted as the
option of marrying differently skilled workers. Preliminary research using
this interpretation yields interesting insights on which type of workers should
enjoy higher married wage premia. In particular, in a situation where there
are differently skilled workers and high skilled workers are more likely to earn
high wages, a marriage partner could accept marriage to unemployed high
skill workers (expecting a high wage when the worker finds a job); but not
to low skill workers employed at a wage in the low end of the distribution.
Hence, a correlation exists between wages and marital status for low skill
workers, but not for high skill workers.

7 Appendix.

Proof of Proposition 1. Taken together, statements a) – e) below imply
that $R_w(T)$ is always i) higher than $b$ and ii) downward sloping, continuous
and concave for $R_w < T < T_2$ when $m < m_a$. 

17
a) From (3), it follows that \( R_w = b \) if \( T = T_a \), where

\[
T_a = \frac{-2kmmb + (2\Lambda gp(1 + k_m) + km(1 + g^2))(-p + b) + (-p + b)\sqrt{T}}{2g(\Lambda(1 + k_m)(-p + b) - km)}
\]

\[
\Gamma = (-mk_m(-1 + g^2) + 4\Lambda g(1 + k_m)(-p + b) - (g^2 + 1)k_m) > 0
\]

Further, \( T_a \geq T_2 \) iff \( m < m_a \) as in the body of the paper.

Implicitly differentiating (3) it is easy to show that

b) \( \frac{\delta R_w}{\delta T} \) when \( R_w = T = R_1 \) is \( \frac{\delta R_w}{\delta T} (R_w = T = R_1) = \frac{(1 - g)k_m}{k_0k_m(m(1 - g + g^2)g(1 + k_m)(-p + b)} \)

< 0 when \( m < m_a \).

c) \( \frac{\delta R_w}{\delta T} = 0 \) if \( T = 2R_w - p > \bar{w} \). Therefore \( \frac{\delta R_w}{\delta T} \neq 0 \) when \( R_w < T < \bar{w} \).

d) \( \frac{\delta R_w}{\delta T} \) exists iff \( m \neq m_b = \frac{g(T - p)(1 + km)(1 + m)}{km(1 - g)k_0(1 - g + g^2)(T + p - 2R_w)} \). It is easy to show that \( m_a < m_b \) when \( R_w < T < T_2 \), so \( R_w \) is a smooth function in that range when \( m < m_a \).

**Proof of Proposition 2.** Items i) and ii) in Proposition 2 follow directly from (6). Item iii) is the consequence of a) – b) below:

a) From (??). knowing that \( \bar{w} = gp + (1 - g)R_{mp} \), one can use implicit differentiation to show that

\[
\frac{dR_{mp}}{dT} = \frac{(1 + \rho(1 - F(T))}{\rho \int_{\bar{w}}^{T} \frac{\delta F(x)}{\delta R}(x)\,dx} \geq 0
\]

which is positive in the relevant region because \( \frac{\delta F(x)}{\delta R} < 0 \) for \( T < x < \bar{w} \).

**Proof of Lemma 1.** Follows immediately from solving the equations \( R_4 = R_2 \) and \( T_4 = T_2 \).

**Proof of Lemma 2.** From inspection (3) it follows that \( \frac{\partial R_w}{\partial X} = 0 \). From implicit differentiation of (6) it follows that

(a) \( \frac{\partial R_{mp}}{\partial X} = \frac{\rho \rho_{mp} \ln\left(\frac{\rho_{mp} + \rho \rho_{mp}((1 - g)}{p - T}\right)}{(1 - g)} \geq 0 \) if \( R_{mp} < T < \bar{w} \). (b) \( \frac{\partial T}{\partial X} = \frac{(p - T)g}{g(p - T) + \rho \rho_{mp}(w_{\text{max}} - T)} \geq 0 \) if \( R_{mp} < T < \bar{w} \). (c) \( \frac{\partial T}{\partial X} = 1 > 0 \) and \( \frac{\partial T}{\partial X} = \frac{1}{1 - g} > 0 \).

Statements (a) – (c) above imply that as \( X \) declines, the graph of \( R_{mp}(T) \) in the \( R_{mp}, T \) space shifts to the left. Starting at \( X = X_a \), a small enough decline in \( X \) yields a situation as depicted in Figure 4.

**Proof of Lemma 3.** Follows directly by using \( T_1 \) and \( T_3 \).

**Proof of Proposition 3.** Follows immediately from Lemmas 1-3.

**Proof of Proposition 4.** It is straightforward to show that the optimal reservation wage chosen by an MP is \( T(w = T_p) = T_p \). I must also show that MPs do not have an incentive to marry unemployed workers. Because the relevant distribution of wages faced married unemployed workers is \( H(x) \) the value of marriage to an unemployed worker is given by
\[ rM_0 = b + \lambda_0 \int \left[ M_2(x) - M_0 \right] h(x)dx - \delta M_0, \text{ where } R_m = \frac{b+k_0\lambda p}{1+k_0}. \]

Simple manipulation of \( M_{1,w=T_p} \) and of \( M_0 \) shows that \( M_{1,w=T_p} \geq M_0 \) if and only if \( X \geq X_{\nu} \) as in Proposition 4. Now consider the problem of an unemployed worker as described in this subsection. I first obtain \( R^* \) and evaluate it when \( w = T = T_p \) to obtain \( R^*(X, w = T = T_p) = R^e(X) \). It is easy to show that \( R^e(X) \) is downward sloping and continuous in the range \( X_{\nu} \leq X < p \). Also, one can show that \( R^e(X_b) = T_p \). Hence, for \( X \geq X_b \) we have \( R = R^e(X) \leq T_p \), and the equilibrium breaks. For \( X < X_b \), then \( R^e(X) > T_p \), so workers reach a corner solution where \( R = T_p \).

8 References


\[^{24} \text{As argued in the last paragraph of Section 2.} \]
The concavity in Figure 1 is not proven but not at all relevant for the results presented in this paper. Several numerical exercises yield a concave curve.

The concavity in Figure 2 is not proven but not at all relevant for the results presented in this paper. Several numerical exercises yield a concave curve.

Figure 1: $R_w$ when $R_w < T < \bar{w}$

Figure 2: $R_{mp}$ when $R_{mp} < T < \bar{w}$

Figure 3: $R_w$ and $R_{mp}$ when $X = X_a$

Figure 4: $R_w$ and $R_{mp}$ when $X = X_a - \epsilon$
Figure 5: $R_w$ and $R_{mp}$ when $X = X_b < X_a$