Search Equilibrium with Migration:  
the Case of Poland  

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Abstract  

The EU enlargement has facilitated labour force movements between the former EU member countries and the accession countries. Foremost, the outflow of workers from the new member countries to countries which introduced open-door policy has magnified. The aim of the paper is to shed some light on the possible effects of reinforced emigration from Poland on its labour market. In particular, it focuses on the impact of the migration flows on wages. The wage equation derived from the search and matching model augmented with migration flows (emigration and the return migration) was estimated employing Bayesian inference. It allowed calculating an approximate magnitude of emigration of workers and describing the impact the labour movements should have had on the real wage in Poland. From 2002 to 2006 the temporary emigration increased by roughly 5 ppt. of the Polish population whereas the resulting increase in the real wage was only moderate and amounted to around 1.3%. The implied elasticity of wages to reduction of workforce due to emigration between 2003 and 2006 was in a range of 0.2–0.3. Mediocre response of wages to emigration corresponds well with earlier studies on the impact of emigration on the source country wage rate. Yet, the explanation of the limited impact of emigration on wages lies in the adjustment of the demand for labour in the steady-state and substantial intensity of the return migration predominantly to employment found in the data.

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1 Introduction

Impact of migration on the labour market, in particular on the wage setting process has become a subject of concern in Poland at least since the accession to the European Union. Both anecdotal evidence and the data indicate that the number of Polish workers looking for a job abroad, in Western European countries, has substantially increased after the date. This paper focuses on the effect emigration flows have had on wages in Poland. The link between emigration and the dynamics of wages in the source country seems to be much underprivileged subject in the literature. Much more attention was henceforth paid to the relationship between immigration and the wage rate in the host country. Friedberg and Hunt (1995) and recently Blanchflower et al. (2007) presented the current review of the vital empirical evidence on the wage immigration relationship. Longhi et al. (2004), in turn, deliver meta-analytic assessment of the effect of immigration of wages. They find that increase in share of immigrants in the labour market reduces the wage rate (on average in the summarized studies) by around 0.1%. The effect is therefore negative but of moderate magnitude.

A few recent studies investigated the effect of emigration on the source country labour market outcomes. Mishra (2006) and Aydemir and Borjas (2006) employ „area approach” to explore the issue. The idea of the approach used is that emigration is unequally concentrated in different geographical regions as well as skill-, education- or industry-groups. The authors match the information from Population Census in Mexico and the United States to evaluate the repercussions of emigration on wages in Mexico. Results they get are consistent with emigration flows being significant factor pulling up the source country wage rates. Mishra estimates that the elasticity of wages with respect to outflow of labour was 0.4% (between 1970–2000), Aydemir and Borjas confirm this result delivering the estimates of wage elasticity in the range of 0.3–0.4% (for shorter sample span 1980–2000). Hanson (2005) examines Mexican wages in low-migration and high-migration states in 1990s to find evidence that average hourly earnings in high-migration states rose by 6–9% in relation to earnings in low-emigration states.

These estimates are consistent with positive albeit limited impact of the labour force outflows on the wage-setting process. Still, all quoted estimates apply to the Mexican labour market. Longhi et al. (2004) showed that host countries labour market responses to immigration impulse might be diverse for various economies. As long as the same argument works for the source countries consequences of emigrant outflows may as well differ country by country depending on in-country mobility of the work force or general flexibility of
labour market. Second, the authors refer to the population census data. Hence, in fact the estimates of the wage elasticity concern medium- to long-term. Third, either Aydemir and Borjas nor Hanson took account for plausible positive impact of remittances on the growth rates and the real wages. In the latter study Hanson found evidence that the likelihood of receiving remittances from abroad is the higher the greater outmigration rate in previous years. In light of these results, it is likely that states which had had higher emigration rate could have reported larger inflow of remittances and were therefore inclined to have higher growth rates. Hence, Hanson’s results may indicate on significant positive relationship between the wage growth and the emigration rate which is rooted not that much in the labour market interactions as in the growth differentials between states. Finally, the estimates based on Mexican and the United States experience may underestimate the role of the return migration that is relevant phenomena in the European countries when labour movement restrictions are in general less strict (Dustman (1996), Klintheall (1999)). The survey on ex post emigration conducted jointly with the population census in 2002 support the marked role played by the return migration in Poland.

Here, an attempt is made to link and explain the dynamics of wages in Poland with intensity of the labour market flows and the country-emigration labour movements. The approach explicitly refers to the information on the emigration probability conditional on the previous labour market status and to the information on the return migration probability to different labour market states.

Contrary to the standard approach which takes account of flows between labour market states only, four states are distinguished in the paper: employment, unemployment, inactivity and temporary emigration. This proposition lies at heart of the proposed approach to evaluate the relation between the wage-setting mechanisms and emigration. It takes account of dependency of the wage rate not solely on the magnitude of migration outflows but also on relative concentration of migration outflows among employed, unemployed and those out of labour force.

The paper to a great degree relies on the data and findings described by Budnik (2007). Using results of the household survey conducted concurrently with the survey on labour market activity (LFS data) and merging the information from both data sources the author calculated emigration and return migration flows directly corresponding with flows between three standard labour market states. These data were adjusted for missing information bias in Permanent emigration (emigration which coincides with registration of the departure at an administrative unit) constitutes only negligible fraction of the total emigration. Immigration flows and their impact on wages are not analyzed in the paper.
migration flows on the base of the Population Census 2002 information\textsuperscript{2}. The major conclusions from the analyzes may be summarized as follows:

- Emigration transition probabilities from employment, unemployment and out of the labour market were relatively low compared to other escape probabilities throughout the period 1999–2000,

- Propensity to emigrate for employed, unemployed and non-participants was however in an upward trend from 2003 on with sharp increase after the EU accession,

- Propensity to emigrate was generally the highest for unemployed in the period 1995–2006,

- The return migration probability remained approximately stable from 1995 to 2006,

- The return migrants are significantly more likely to enter employment than non-employed workers at the home labour market.

These outcomes condition shall the results described in this paper.

The explicit theoretical search equilibrium model was derived and estimated with Bayesian methods. It pinned down the channels through which emigration may affect wages and allowed to estimate how significant was emigration for dynamics of the wage rate. Basic Pissarides search equilibrium model was augmented for two additional labour market states: non-participation and emigration. Bayesian inference was an attractive alternative to frequentist approach due to generally scarce information concerning structural parameters of the economy contained in the time series data spanning only a decade of history and high non-linearity of the derived wage equation. It offered the ability to explicitly augment the data with a priori information and take account of uncertainty about the parameters value in the estimation process.

Next, the estimated wage equation was employed to run counterfactual simulation of the wage rate under assumption of no change in the intensity of emigration flows. It delivered the estimate of the effect of emigration after 2002 (including the EU enlargement effect) on wage. Yet, the simulation offered insight into forces driving higher wage growth after the accession: changes in the probability of filling vacancy, in the emigration costs, changes in the permanent income of unemployed, inactive and emigrant workers. Last but

\textsuperscript{2}The data might be still severely biased due to missing observation or reporting errors recognized in the earlier literature on flows. However, as no additional source of information existed to correct the flows, reported findings concern flows adjusted only for undercounting problem of emigrants in the sample.
not least, the estimated wage equation supported the hypothesis of an important impact of institutional changes at the home labour market on the wage development in the last few years.

The paper is structured as follows. First section describes the search and matching model with emigration. Next section depicts the data and methodology. Third chapter summarizes the estimation and simulation results. The final section concludes.

2 Search Equilibrium with Migrations

The backbone of the model used in the paper is search and matching model developed in works of Diamond (1982), Mortensten (1984) and Pissarides (2000). The core assumption of the model is that both firms and workers explore labour market to find a partner to set up production process. Search is a time consuming and costly process so once firm and worker are matched and a vacancy filled there is a job surplus which may be bargained over.

The search equilibrium framework was used by Borersma et al. (2004) to derive wage equation which depended exclusively on the labour market flows. Their wage equation did not refer directly to the relationship between unemployment rate and either wage rate or wage growth rate but represented some empirical alternative to the Philips curve or wage curve specifications. Borersma et al. set up the model with inactive workers and presumed that they do not take part in the bargaining process. Still, fallback position of negotiating employed workers and unemployed ones is affected by both transition probabilities to inactivity and expected income of non-participants.

Similar approach is pursued in this paper. Workers move between four labour market states: employment, unemployment, non-participation and emigration. Thus the main deviation from the standard course lies in treatment emigration as an additional labour market state. Emigrants are perceived as a part of the total labour force but the model still remains closed with regard to total population. The model is tailored to fit the data which are at hand. Information about migration movements is for most of the national labour markets scarce but for Poland the LFS survey data could be used to calculate flows between different labour market states and temporary emigration. No

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\(^{3}\)The distinction between the wage curve specification and the wage equation estimated by Borersma et al. remained however solely technical. If the matching function would be explicitly introduced to their search equilibrium model, the wage rate would be dependent on the unemployment rate.
imigration apart from the return migration is taken into account.\textsuperscript{4} Both emigrant and non-active workers do not participate in the bargaining but their incomes enter assets equations for workers and unemployed and through that channel influence the wage rate in the economy.

Transitions between home labour market states and emigration between each two periods are governed by a Markov matrix $P$ (the transition probabilities matrix) of a form:

\[
\begin{bmatrix}
  p_{EE} & p_{EU} & p_{EN} & p_{EM} \\
  p_{UE} & p_{UU} & p_{UN} & p_{UM} \\
  p_{NE} & p_{NU} & p_{NN} & p_{NM} \\
  p_{ME} & p_{MU} & p_{MN} & p_{MM}
\end{bmatrix}
\]  

First row of the matrix represents transition probabilities from employment to (in sequence order) employment ($E$), unemployment ($U$), non-participation ($N$) and emigration ($M$). Successive rows include transition probabilities form unemployment, out of the labour market and emigration. In particular, the last row gives the return migration probabilities (first three elements). Elements of each row sum up to unity.

2.1 Firms

Let $V_F$ be the present-discounted value of expected profit from an occupied job and $r$ real interest rate. $V_V$ is in turn the present-discounted value of expected profit from the vacant job. Without capital accumulation introduced into the model the one-period profit from the occupied job is equal to the productivity of an employee corrected for the real total labour cost (wage and taxes and quasi-taxes on wages). Let $y$ be the worker’s productivity, $w$ the wage rate and $i^{\text{CORP}}$ the effective social contribution rate paid by employer. The condition saying that return on the asset is equal to the capital cost has the form:

\[
rV_F = y - w(1 + t^F) - (1 - p_{EE})(V_F - V_V) + \dot{V}_F. 
\]

For simplicity all time subscripts were dropped. $p_{EE}$ is the probability that worker stays at the job throughout two consecutive periods and $\dot{V}_F$ stands for a change in asset value of occupied job. The return on the asset consists of one period profit, expected change in the asset value of the occupied job once

\textsuperscript{4}The model can be thought of as a practical device for analyzing the impact of migration on wages as long as intensity of immigration is negligible compared to emigration and the return migration.
the worker quits the job and expected changes in the valuation of job between periods.

The asset equation for vacant job takes into account hiring costs associated with filling the vacancy. They are assumed to be proportional to the productivity $cy$ where $c > 0$:

$$rV = -cy + p^V (V_F - V) + \dot{V}_V. \quad (3)$$

The vacant job yields loss of $cy$ each period and is filled with probability $p^V$. It is worth to have a closer look at $p^V$ already at this point. The probability of filling the vacancy is bound to the vacancy rate $\vartheta$ defined as the number of generated vacancies in the period related to the level of employment and the transition probabilities: from unemployment to employment $p_{UE}$, from non-participation to employment $p_{IE}$ and from emigration to employment (at home market) $p_{ME}$. These probabilities are „weighted” by respective shares of unemployed, non-participants and emigrants in the population and normalized by the inverse of the employment rate (so that they correspond with the vacancy rate):

$$p^V = \vartheta \left( p_{UE} \frac{U}{P} + p_{NE} \frac{I}{P} + p_{ME} \frac{M}{P} \right) \left( \frac{E}{P} \right)^{-1} \quad (4)$$

where here $U$ is the number of unemployed in the economy, $N$ the number of inactive, $M$ the number of emigrants and $E$ stands for the number of employed. $P$ is the total population, namely $P = E + U + N + M$. The probability of filling the vacancy is a decreasing function of the vacancy rate and an increasing function of flows into employment.

In the steady-state both $V_F$ and $V$ are constant which means that $\dot{V}_F = \dot{V} = 0$. Moreover, value of the vacant job has to be equal to zero so that no new jobs are created in the equilibrium. Putting these conditions together with 2 and 3 the steady-state value of filled job is:

$$V_F = \frac{c \cdot y}{p^V}. \quad (5)$$

Here, the value of the filled job is equal to the cost of filling a vacancy multiplied by the expected duration of a vacancy.
2.2 Workers

There are four asset equations for workers corresponding with four labour market states distinguished in the model. Let $V_E$, $V_U$, $V_N$ and $V_M$ be the present-discounted value of the expected income stream of employed, unemployed, non-participant and emigrant respectively. $I_E$, $I_U$, $I_N$, $I_M$ stand for instantaneous incomes of workers in different labour market states. The asset equations are of the form:

$$
 rV_E = I_E + p_{EU}(V_U - V_E) + p_{EI}(V_N - V_E) + p_{EM}(V_M - V_E) + \dot{V}_E \\
 rV_U = I_U + p_{UE}(V_E - V_U) + p_{UI}(V_N - V_U) + p_{UM}(V_M - V_U) + \dot{V}_U \\
 rV_N = I_N + p_{IE}(V_E - V_N) + p_{IU}(V_U - V_N) + p_{IM}(V_M - V_N) + \dot{V}_N \\
 rV_M = I_M + p_{ME}(V_E - V_M) + p_{MU}(V_U - V_M) + p_{MI}(V_N - V_M) + \dot{V}_M 
$$

Equation 6 says that permanent income of employed worker is equal to the instantaneous income when employed corrected for expected changes in the value of job tied to transitions to other labour market states: unemployment (with probability $p_{EU}$), non-participation (with probability $p_{EN}$) and emigration (with probability $p_{EM}$). In equilibrium the permanent income is constant so $V_E$ is equal to zero. Equations 7–9 shall be interpreted likewise. In the steady-state $\dot{V}_E = \dot{V}_U = \dot{V}_N = \dot{V}_M = 0$.

Further I assume that the current income of workers $I_E$ is proportional the real wage corrected for the social contribution levied on employees and income tax rates, namely $w(1 - t^{EMP})$.

2.3 Wage Bargaining

Wage bargaining takes place between firms and employed workers. The negotiated wage contract concerns only one job denoted by $i$ and the wage rate prevalent at the market is perceived by both firm as an employee as given. A surplus from a job match is distributed between an employer and an employee according to the Nash solution with $\beta$ being the parameter of the relative bargaining power of employed workers:

$$
 w^i = \arg\max (V^i_E - V^i_U)^\beta (V^i_F - V^i_V)^{1-\beta}.
$$

Equations 5-10 let express the negotiated wage rate $w^i$ in the steady-state as a function of a permanent income of unemployed, inactive and emigrant workers, tax and quasi-tax rates, the discount or real interest rate, the labour productivity and the bargaining parameter:
\[ w_i = \frac{rV_U + \beta(y - rV_U) + (1 - \beta)(p_{EN}(V_U - V_N) + p_{EM}(V_U - V_M))}{(1 - t^{EMP}) + \beta(t^{EMP} + t^{CORP})}. \] (11)

The higher the total tax wedge \( t^{EMP} + t^{CORP} \) the lower is the wage rate in the equilibrium. The relationship between wages and the tax wedge reflects negative impact of the tax wedge on the level of rents: the surplus from a job match is lower the higher the tax burden. The higher share of rents is appropriated by workers the stronger the steady-state pass-through of tax wedge into lower wages due to stronger disincentives to post new vacancies. If the government collects higher taxes only on employees the negotiated gross real wage goes up. Workers demand higher real wages to remain possibly close to the level of permanent income prior to the tax rate hikes and their share in rents shifts upwards. Still the net wage falls but excessive tax burden is shared by employers and employees.

To a great degree equation 11 echoes of the standard bargaining model result. The wage rate is equal to the reservation wage and the share in rents proportional to the bargaining power of workers. The most pronounced difference is the need to adjust the expression for the plausible change of the unemployed status into non-participation or emigration. Principally, the reservation wage amounts to permanent income when unemployed \( rV_U \) corrected for expected difference in the income if the worker changes her status to non-participation or to emigration \( p_{EI}(V_U - V_N) + p_{EM}(V_U - V_M) \).

### 2.4 Aggregate Wage Equation

The negotiated wage rate in 11 does not depend on any job specific variable. When all firms act symmetrically then for each \( i \)
\[ V^i_F = V_F \]
\[ V^i_E = V_E \].
Moreover, the system of four equations 6–9 allows describe the wage rate as a function of the structural parameters and contemporary incomes of workers in distinct labour market states. The bargaining condition detailed in 5 and 10 is in force for the whole economy:

\[ (1 - \beta)(V_E - V_U) = \beta \left( \frac{cy}{pV} \right). \] (12)

In the equilibrium all firms set the same wage rate so for each firm \( i \) \( w^i \) is equal to \( w \). To arrive at the aggregate wage equation the wage equation 11 was combined with the solution of the system of four equations 12 and 6–9:
\[ w = \frac{\beta \left( 1 + \frac{e^\theta}{p^r} \right) y}{\beta (t_{EMP} + t_{CORP}) + (1 - (1 - \beta)t_{EMP})(1 - \alpha_{UR}rr_U - \alpha_{NR}rr_N - \alpha_{MR}rr_M)}. \tag{13} \]

The replacement rate for unemployed \( r_{rrU} \) is defined as the instantaneous income of unemployed divided by the real net wage rate: \( I_U/w(1 - t_{EMP}) \). The replacement rates for non-participants \( r_{rrN} \) and emigrants \( r_{rrM} \) are defined respectively. \( \theta, \alpha_U, \alpha_N \) and \( \alpha_M \) are functions of the elements of the probability matrix \( P \) and the discount rate \( r \). Detailed derivation of the wage equation is given in the Appendix.

The wage rate in the economy is positively related to the bargaining power of workers. It depends negatively on the total tax wedge. The negative impact on wages of higher tax burden is even more robust once the bargaining power of workers is high. Increase in the replacement rate of either unemployed, non-participants or emigrants raises the level of reservation wage and therefore puts upward pressure on wages in the economy. And finally, the wage rate is an increasing function of the average time of filling the vacancy and cost of posting the vacancy.

3 Data and Methodology

The sample period spans between 1995 and 2006. Detailed description of the variables and data sources is given in a table at the end of the paper. Here, only labour market gross flows data are discussed more thoroughly to outline the key advantages of the data and problems associated with their use.

3.1 Labour Market Gross Flows

The key variables in the wage equation are transition probabilities between different labour market states and emigration. Polish LFS data allow for straightforward computation of gross flows between employment, unemployment and non-participation. To augment the available data with emigration and return migration flows I referred to the information contained in the household questionnaire. This questionnaire is filled for each household in the sample concurrently with the questionnaire on economic activity of household members. It contains the question about household members who stay abroad at the time of inquiry for more than two months. This information was matched with the data from the questionnaire on economic activity of individuals on
the ground of common characteristics: household identification variables, gender, date of birth. The procedure was run for each two consecutive quarters and allowed calculate the gross flows between distinct labour market states and emigration. The final gross flows matrix $F$ has a form:

\[
\begin{bmatrix}
F_{EE,t} & F_{EU,t} & F_{EN,t} & F_{EM,t} \\
F_{UE,t} & F_{UU,t} & F_{UN,t} & F_{UM,t} \\
F_{NE,t} & F_{NU,t} & F_{NN,t} & F_{NM,t} \\
F_{ME,t} & F_{MU,t} & F_{MN,t} & F_{MM,t}
\end{bmatrix}
\] (14)

where $F_{ij,t}$ denotes number of individuals changing their state from $i$ to $j$ and $i, j \in \{E, U, N, M\}$ between period $t - 1$ and $t$.

As recognized by Abowd and Zellner (1985), Summers and Poterba (1986) and later Blachard and Diamond (1990) estimated gross flows might be biased due to classification and missing observation error. Summers and Poterba used Reinterview Survey results to correct the gross flows calculated on the base on the Current Population Survey in the U.S. for classification error and showed that adjusted data indicate at much lower escape probabilities from labour market states, most of all from unemployment. However, even if the problem might appear in the Polish data no similar follow-up survey procedure is pursued by Polish Statistical Office. Neither was it possible to use the popular raking method which assures consistency of gross flows with stocks of workers in different labour market states because no data on emigrants stocks in the sample period were available.

Still, gross flows to emigration and outflows form emigration could be approximately adjusted for missing observation error. The missing observation error is likely to be strongly correlated with emigrant status due to the known undercounting problem of some groups of emigrants in the data. Emigrant category covers individuals who stay abroad for more than two months, are still counted as the permanent residents of Poland and are related to household where the LFS survey is conducted. In practice, the data gathering method which bases on surveying households in sampled dwellings and imprecise principle of counting the individual as emigrant are responsible for missing observation biases which are presumably more pronounced in the emigration and return migration gross flows than in flows between home labour market states.

\footnote{Popularity of raking method is connected with easiness of its implementation and low information requirements. Even if it does not necessarily correct the data for both types of errors the argument for its use lies in the expectation that both errors bias stocks more heavily than gross flows.}
First, outflows of workers from employment, unemployment and out of the labour force are likely to be underrated on account of the fact that emigration of all households is rarely registered in the data. It concerns foremost singles and emigration of whole families. Second, the number of person who stay abroad throughout two consecutive periods is subject to downward bias. What follows return migration probability may be overestimated. The presumption is mainly tied to the fact that long-term emigrants are undercounted in the sample of emigrants as they are less probable than short-term emigrants to be still related to households in the source country. Long-term emigrants have in general lower individual return probability (Klinthall (1999)) therefore their sparsity in the sample may lead to too high estimates of the average probability of return migration.

Under simplifying assumption missing observation bias of emigration and return migration flows was roughly constant throughout the period under observation the gross flows matrix for each $t$ was adjusted with a set of rescaling parameters grouped in two vectors $k = (k_{EM}, k_{UM}, k_{NM})$ and $l = (l_{ME}, l_{MU}, l_{ME})$ in a way presented below:

$$
\begin{bmatrix}
F_{EE,t} & F_{EU,t} & F_{EN,t} & k_{EM}F_{EM,t} \\
F_{UE,t} & F_{UU,t} & F_{UN,t} & k_{UM}F_{UM,t} \\
F_{NE,t} & F_{NU,t} & F_{NN,t} & k_{NM}F_{NM,t} \\
l_{ME}F_{ME,t} & l_{MU}F_{MU,t} & l_{MN}F_{MN,t} & F_{MM,t}
\end{bmatrix}
$$

Parameters contained in $k$ and $l$ are estimated together with other statistical parameters of the empirical wage equation and elements of the empirical transition probability matrix corresponding with $l$ are for each period computed as $\hat{F}_{i,t} = \sum_j \hat{F}_{ij,t}$ so that $P_t \equiv P(F_t; k, l).

### 3.2 Steady-State Solution

As showed by Budnik (2007) the LFS aggregate data on the actual level employment, unemployment and non-participation might to be significantly biased upward after 2003. The problem results from the difficulty Polish Statistical Office has to correctly estimate the LFS population in the face of significant intensification of temporary migration movements. It might impede the results as the use of the data would lead to underestimation of the level of labour productivity at the end of the sample period. Moreover, stocks of unemployed,

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6Similar argument could be made in respect to presumed abundance of records of families’ emigration. Families are thought to be less mobile than individuals and in particular to have lower return probability.
non-participants and emigrants are required to calculate the probability of filling the vacancy. When the two former are available, the data on the level of temporary emigration would have to be measured as the number of emigrants in the sample which would lead to meaningful undercounting of the stock of emigrants owing to the missing observation problem.

The solution used in the paper was to calculate steady-state shares of employed, unemployed, non-participants and temporary emigrants in the total population on the base of adjusted transition probability matrix. Next, these figures were combined with the data on the population of Poland (which were beforehand adjusted to be consistent with the population covered by the LFS) to compute the steady-state levels of employment, unemployment, inactivity and emigration. For each period $t$ the following steady-state problem was solved:

$$p_{EU,t}^{ESS} + p_{NE,t}^{ESS} + p_{ME,t}^{ESS} = (p_{EU,t} + p_{EN,t} + p_{EM,t})E_t^{SS}$$  
$$p_{EU,t}^{USS} + p_{NU,t}^{USS} + p_{MU,t}^{USS} = (p_{UE,t} + p_{UN,t} + p_{UM,t})U_t^{SS}$$  
$$p_{EN,t}^{USS} + p_{NU,t}^{USS} + p_{MN,t}^{USS} = (p_{NE,t} + p_{Nt} + p_{NM,t})I_t^{SS}$$  
$$p_{EM,t}^{USS} + p_{UM,t}^{USS} + p_{NM,t}^{USS} = (p_{ME,t} + p_{MU,t} + p_{MN,t})M_t^{SS}$$

delivering the estimates of fractions of workers in four labour market states as a functions of the elements of the empirical transition probability matrix $P_t$:

$$E_t^{SS} = \frac{E_t^{SS} + U_t^{SS} + N_t^{SS} + M_t^{SS}}{E_t^{SS} + U_t^{SS} + N_t^{SS} + M_t^{SS}}, \quad U_t^{SS} = \frac{E_t^{SS} + U_t^{SS} + N_t^{SS} + M_t^{SS}}{E_t^{SS} + U_t^{SS} + N_t^{SS} + M_t^{SS}}, \quad N_t^{SS} = \frac{E_t^{SS} + U_t^{SS} + N_t^{SS} + M_t^{SS}}{E_t^{SS} + U_t^{SS} + N_t^{SS} + M_t^{SS}}, \quad M_t^{SS} = \frac{E_t^{SS} + U_t^{SS} + N_t^{SS} + M_t^{SS}}{E_t^{SS} + U_t^{SS} + N_t^{SS} + M_t^{SS}}. $$

These shares multiplied by the total population of Poland $pop_t$ replaced the numbers of employed, unemployed, non-participants and emigrants in the empirical wage equation. Hence, the levels of employment, unemployment, inactivity and temporary emigration are treated as functions of the gross flow matrix, the population variable and model parameters. Namely, $E_t = E(F_t, pop_t; k, l)$, $U_t = U(F_t, pop_t; k, l)$, $N_t = N(F_t, pop_t; k, l)$, $M_t = M(F_t, pop_t; k, l)$.

### 3.3 Further Model Assumptions

The bargaining power structural parameter $\beta$ was treated as variable in time due to the fact that in the sample period Polish economy experienced far-reaching structural changes which could have abated the share of monopolistic rents appropriated by workers. No perfect indicator variable, which could range over all dimensions of these developments, was available. Thus, the share of those employed on the base of fixed term contracts to all employed $f_t$ according to the LFS was used to capture at least part of the shift in the bargaining power.
of workers. The bargaining power of workers parameter $\beta$ was assumed to be a linear function of the indicator variable $\beta_t = \beta_0 + \beta_1 f_t$ and two statistical parameters $\beta_0$ and $\beta_1$.

The vacancy rate is ordinarily defined as the ratio of the stock of unfilled vacancies to the level of employment. In the model described in the previous section, the vacancy rate is however thought of as the number of generated vacancies in the period to the level of employment. The available vacancy data which covers whole sample period and is of monthly frequency is the number of unfilled vacancies registered in the job centers $vac_t$. These data reflect changes only in unfilled vacancies (hence include no direct information about job creation rate) and might overstate cyclical swings in the economy wide vacancy rate\(^7\). With notice at the limited ability of the job centers data to fully reflect the average level and variance of the aggregate vacancy rate it was assumed that the model consistent vacancy rate in a period $t \vartheta_t$ is linear function of an observed time series of the unfilled vacancy rate:

$$\vartheta_t = \vartheta_0 + \vartheta_1 \frac{vac_t}{E(F_t, pop_t; k, l)}$$ (20)

with two parameters to be estimated: $\vartheta_0$ and $\vartheta_1$. Probability of filling the vacancy set forth in an expression (4) for each $t$ was calculated as:

$$p_t^V = (\vartheta_0 + \vartheta_1 vac_t) \cdot \left( p_{UE}(F_t; k, l) \cdot \frac{U(F_t, pop_t; k, l)}{pop_t} + p_{NE}(F_t; k, l) \cdot \frac{N(F_t, pop_t; k, l)}{pop_t} + p_{ME}(F_t; k, l) \cdot \frac{M(F_t, pop_t; k, l)}{pop_t} \right) \left( \frac{E(F_t, pop_t; k, l)}{pop_t} \right)^{-1}.$$ (21)

In short $p_t^V = p^V(F_t, pop_t, vac_t; k, l, \vartheta_0, \vartheta_1)$.

The productivity of employee $y_t$ was defined as the value added $va_t$ to the level of employment:

$$y_t = \frac{va_t}{E(F_t, pop_t; k, l)}$$ (22)

\(^7\)Job offers registered in the job centers rarely address high-skilled workers. If demand for high-skill workers is less elastic that the demand for low-skill workers, the vacancy rate which takes account of changes in the demand only for the latter category of workers might suggest misleadingly high fluctuations throughout the business cycle.
3.4 Empirical Wage Equation

\[ w_t = \ln \left\{ \left( \beta_0 + \beta_1 f_t \right) \left( 1 + \frac{c\theta(F_t; k, l, r)}{p^V(F_t; \text{pop}_t, \text{vac}_t; k, l, \vartheta_0, \vartheta_1)} \left( \frac{v_{at}}{E(F_t; \text{pop}_t; k, l)} \right) \right) / \left( \beta_0 + \beta_1 f_t \right) \left( t_t^{\text{EMP}} + t_t^{\text{CORP}} \right) + \left( 1 - (\beta_0 + \beta_1 f_t) \right) t_t^{\text{EMP}} \right\} \]

\[ + \left( 1 - \alpha_U(F_t; k, l, r) \rho_{Urr,U,t} - \alpha_N(F_t; k, l, r) \rho_{Nrr,N,t} - \alpha_M(F_t; k, l, r) \rho_{M(1 + \tau e_{ut})rr_M,t} \right) \] + \varepsilon_t, \quad \text{(23)}

where \( \varepsilon_t \sim N(0, \sigma) \).

Three replacement rates variables corresponding with non-working status and emigration were rescaled in the empirical wage equation with parameters \( \rho_U, \rho_N \) and \( \rho_M \). The replacement rates for unemployed and those out of labour market summarize information about changes in the relative income of non-working population taking account of institutional changes in the generosity of unemployment insurance and social assistance systems. Yet, I allowed for some deviations from the postulated elasticity of the wage rate to the replacement rates to correct the model responses for plausible non-income effects of staying in non-working labour market states (leisure time, social distress).

“Replacement rate” of emigrants measures the expected income of a worker abroad compared with the average wage rate at the home labour market. No adjustment for emigration cost was pursued at the stage of calculating the variable. Hence, the role of the parameter \( \rho_M \) is to adjust the emigrants replacement rate so that it corresponds with the actual ratio of expected income abroad to income at the home labour market after deduction of emigration cost. Moreover, the wage rate model 23 allows for a jump change in emigration cost after the EU enlargement. Dummy variable \( e_{ut} \) is equal to one from the second quarter 2004 on and the parameter \( \tau \) reflects the percentage change in the emigrants replacement rate after the EU accession.

The empirical wage equation includes 13 statistical parameters: \( \beta_0, \beta_1, k, l, \vartheta_0, \vartheta_1, \rho_U, \rho_N, \rho_M, \tau, r \) and \( \sigma \) and 22 independent variables.

3.5 Estimation Strategy

Estimation of the parameters of the wage equation for a transition economy poses certain difficulties tied to a highly non-stationary character of the converging economies. Additional impediment is the length of the available time series which cover only a decade — still relatively short time period to draw
conclusion about the structural parameters of the economy. These premises
and the character of the wage equation (high degree of nonlinearity, over-
parameterization) made it desirable to employ Bayesian methods. Bayesian
inference lets consolidate a priori information about the parameters values
with the information contained in the data. In that way it allows bridge gaps
in the data with theoretical underpinnings. As long as the wage equation has
sound economic footing imposed a priori parameter distributions reflect rese-
archer’s „beliefs” in the value of the parameters which in this case are often
based on standard results for developed economies.

A posteriori parameter distributions were determined using Random Walk
Chain Metropolis-Hastings algorithm with normally distributed increment ran-
dom variable. The algorithm was run with 600 thousands drawings out of
which first 20% was ignored when plotting the a posteriori distributions. Co-
vergence was checked using CUSUM criteria.

3.6 A Priori Parameter Distributions

The expected value of $\beta_0$ was fixed at 0.5 which is the standard assumption
about the value of the bargaining parameter in the literature. The parameter
reflecting marginal change of bargaining power coupled with a change in the
share of fix-term employed in the total population of employed had triangular
a priori distribution so that the probability that the bargaining power rema-
ined constant between 1995 and 2006 was a priori postulated to be significantly
higher than the probability of a structural shift.

To set a priori distribution for $\vartheta_0$ and $\vartheta_1$ I referred to empirical evidence
on the probability of filling a vacancy. Den Haan et al. (2000) calculated the
respective probability on the base of the quarterly job creation rate in the
manufacturing sector in the U.S. (5.2%) and the ratio at which separations are
reposted. Next, he resorted to the steady-state condition saying the number of
closed jobs has to be equal to the number of jobs created to derive the estimate
of the probability of filling the vacancy of 71%. In the estimates of den Haan
no account was taken of the problem of lapsed vacancies. Van Ours and Ridder
(2001) showed that roughly 4% of vacancies were closed before being filled.
Andrews et al. (2005) who concentrated on the youth labour market indicated
that the ratio of lapsed vacancies might be notably higher — 34%. Furthermore
duration of lapsed vacancies in their data was 25% longer than that of filled
vacancies. It suggests that once the problem of lapsed vacancies is explicitly
considered the estimates of the probability of filing the vacancy might be lower.

The data on job creation were collected by Polish Statistical Office in
2004 and 2005. Referring to this evidence the average job creation rate was approximated as the ratio of new hires to the average employment plus the ratio of unfilled vacancies to the number of employed at the end of the period under consideration. The resulting estimate of the job creation rate in 2005 for all sectors of economy was 2% and for manufacturing sector under 7%. Both figures are significantly lower than those used by den Haan. Anyhow Polish data could point to relatively low job creation rates partly due to the fact that they cover only full-time salaried workers.

The result of den Haan was chosen as a benchmark for setting the a priori distributions of the parameters of the vacancy function in the sample period. The expected values of the parameters of the vacancy rate function were fitted so that the expected probability of filling the vacancy in a quarter (based on a priori distributions) equals 60%.

The a priori distribution of the flow cost parameter of opening the vacancy $c$ assures that the expected value of the flow cost of posting the vacancy is 2.5%. The reference data for setting the expected value of $c$ was found in the study of Barron et al. (1997). Based on survey evidence from 1982 and 1992 on the time and cost of recruitment Hagedorn and Manovski (2006) concluded that the average cost of time spent hiring one worker is 2.2% to 3.2% of quarterly hours. Taking into account the a priori average duration of the vacancy, the flow cost of posting the vacancy shall amount to 2% of the average quarterly product per worker. Slightly higher estimate was finally chosen to allow for possibly higher productivity of management who usually recruits the stuff (similar line of arguments was presented by Hagedorn and Manovski).

The a priori expected values of $\rho_U$ and $\rho_N$ parameters were set to one. As far as the “emigrants’ replacement rate” is concerned, the expected value of the rescaling parameter is fixed at 0.3. The value was chosen under assumption that in 2002 the labour market was in the steady-state. Therefore, share of those who emigrated and those who stayed in the total population should have corresponded with relative expected income abroad and at the home market. On similar foundation the upper bound of the $\rho_M$ parameter distribution was fixed on 0.6 assure that income (after deduction of social and other costs) abroad was not higher that income of employed workers at the home market. Additionally, the model allows for a level shift in emigration costs after the EU enlargement. These costs dropped due to at least two factors: introduction of the open-door policy by some of the EU countries and concurrent reduction of traveling costs (a few cheap airlines entered the Polish market around the accession date). Relatively low informative prior distribution of $\tau$ was set a priori. The positive shift in effective “emigrants’ replacement rate”
had uniform a priori distribution in the range from zero to 100%.

Migration flows were rescaled with two parameters only. It was assumed that all elements of vectors \( k \) and \( l \) are equal as no additional information was at hand about differences in the magnitude of possible downward (in case of emigration outflows) and upward (in case of return migration flows) biases between separate outflows or inflows of workers at home labour market (depending on earlier or current labour market state of emigrating or returning workers). A priori expected values of these parameters as well as their lower and upper bounds were set in accordance with calibration applied by Budnik (2007). The calibration referred to the Population Census data which was conducted in 2002 (PC2002). In case of \( k \) parameters based on the ratio of one-person households and families emigration in the total number of emigrants in the census year. The information on the duration of emigration experience of ex post emigrants in 2002 from the survey accompanying the PC2002 was used jointly with the census data to fix the distributions of the latter parameters.

Finally, additional restrictions were imposed on the parameters’ values. First, linear function approximating the bargaining power of workers structural parameters could take valued between zero and one. Similar restriction concerned the probability of filling the vacancy. Additionally, emigration rate defined as the number of temporary emigrants to the labour force in Poland in three first quarters of 2002 had to be closed to the emigration rate calculated on the base of the PC2004 (2.4%).

4 Results

This section summarizes the main results and outlines model simulations. First, the a posteriori model parameters are described. Second, the wage rate responses to changes in the values of model variables and structural parameters are discussed. Next, the structural model parameters and dynamics of key variables between 1995 and 2006 are interpreted in the light of previous results. It allows some insight into evolution of Polish labour market from the transition years in the first half of nineties to the mid of the present decade. Finally, the effect of an increase in emigration propensity on the wage rate and related labour market variables are analyzed.

4.1 A Posteriori Parameter Distributions

The expected a posteriori value of the parameter \( \beta_0 \) is lower than it was assumed a priori (Figure 1). This result was accompanied by sound reduction
of variance of the parameter indicating that the data contained significant information about the parameter. Moreover, a posteriori probability that the institutional changes played a role in shaping the process of wage-setting throughout the period (parameter $\beta_1$) was slightly higher than postulated a priori. Although, a posteriori distribution remained close to a priori distribution prior sensitivity analysis, namely imposing uniform a priori distribution on the parameter, confirmed that the data supported the hypothesis about the shift in the bargaining power of workers. A posteriori distribution in the sensitivity analysis was cumulated close to the upper bound of the parameter distribution. It suggested, that a graduate weakening of the bargaining power of workers could have moderately reduced the wage rate foremost at the end of the observation period.

A posteriori distribution of a cost parameter $c$ did not noticeably differ from a priori distribution. The density functions of both vacancy rate parameters $\vartheta_0$ and $\vartheta_1$, in turn, were shifted to the right compared with the respective a priori density functions. It indicated that the vacancy rate could have been moderately higher and demonstrated more significant cyclical changes than expected a priori.

A priori and a posteriori distributions of the parameters which modify the elasticity of the wage rate to the replacement rates of unemployed and out of the labour market did not noticeably differ. The a posteriori values of the $\rho_M$ parameter were concentrated on the left from the mode of a priori values. The a posteriori distribution of the shift parameter $\tau$ was concentrated around lower values and the expected change in the effective emigrants’ “replacement rate” is around 40%.

Finally, the density functions of the parameters correcting emigration flows and the flow cost parameter were shifted to the right and left respectively as compared with the a priori assumptions. Next, the model was estimated without equality constraint imposed on the rescaling factors (two parameters $k$ and $l$ were replaced by six separate parameters $k_{EM}$, $k_{UM}$, $k_{NM}$, $l_{ME}$, $l_{MU}$ and $l_{MU}$ in the estimation process). The distinguished return migration rescaling parameters a posteriori distributions did not significantly differ from the estimates of $l$ distribution in the constrained version of the model. However, the data suggested that outflow of inactive workers to emigration might have been more strongly downward biased than the other two emigration flows. The expected value of the a posteriori parameter correcting outmigratio of non-participants was around 2 ppt. higher than the expected values of the parameters scaling up outmigration of employed and unemployed. Still, these differences were too little to change the results presented in further sections.
4.2 Elasticities of the Wage Rate to Model Variables

In the framework, an elasticity of the wage rate to any of factors shall depend on the combination of values of the whole set of other parameters and variables. In particular, these elasticities are not constant in time. Table 3 briefs estimates (the expected values and medians) of the elasticities of the wage rate to the tax rates, the bargaining parameter, the cost of posting the vacancy parameter, the vacancy rate, the interest rate (discount rate), the labour productivity and the replacement rates in the last quarter of the sample period (4th quarter of 2006).

A weaker position of workers in the wage negotiations (1 ppt. change in the bargaining parameter) results in a drop of wages by 2.2%. A change in the vacancy rate, other things equal, drives growth of wages of 0.05%. It is tied to the fact, that the higher the vacancy rate, the lower the probability of filling a vacancy and the higher the average cost of posting a vacancy. The value of a filled vacancy is therefore higher which raises the job surplus. Similarly, an upward pressure on wages is exerted by an increase in the flow cost of posting a vacancy (1 ppt. change shifts the wage rate up by 0.15%).

The pass-through of taxes and social contributions into wages is partial. Over 70% of the increase in taxes and social contributions levied on employees is reflected in higher wages. Around 30% of the tax burden is incurred on the net earnings of employees. An increase of effective rate of social contributions paid by employers by 1 ppt. results in a reduction of the wage rate by 0.54%.

An upward shift in any of the replacement rates drives wages up. The elasticity of the wage rate to replacement rates distinguished in the model can be magnified or dampen depending on the transition probabilities. For instance, strong response of wages to variation of the replacement rate of those out of the labour market in 2006 may be in part explained by significant outflows of the labour force into inactivity.

Still, the interpreted elasticities reflect the steady-state response of wages to different impulses. They do not inform about adjustment path the wage rate. Moreover, they do not account for the thinkable adjustment in the employment or unemployment figures as the labour market flows are treated as exogenous in the search model depicted in the previous chapter.

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8The simulation abstracts from the influence of changes in the effective tax and social contribution rates on the replacement rates. If they were taken into account the positive effect on wages could be expected to be stronger
4.3 Labour Market in Poland 1995–2006

The estimated parameters of the model allow for derivation of the set of critical model variables and structural parameters. Furthermore, on the base of the above results some story can be told about the development of Polish labour market throughout the last decade.

The unemployment rate (Figure 3) and vacancy rate reflect changes at the labour market coupled with the business cycle (Figure 6). The slowdown following the Russian crisis in 1998 reduced the labour demand and the vacancy rate dropped from around 6% to around 5%. The revival at the labour market after 2003 initiated gradual growth in the vacancy rate to over 7% in 2006.

The probability of separation (Figure 7) and probability of filing the vacancy (Figure 8) were in the downward trend in the period under consideration reflecting among other decreasing mobility between labour market states. In the second half of the nineties flows between employment and unemployment or between employment and out of the labour market were high to a great degree echoing transitional changes. From the end of the nineties the key motor of changes in the intensity of flows between the labour market states was the business cycle.

From 2003 on, there was significant increase in the number of temporary emigrants as compared to the population at the home labour market (Figure 2). The outflow of workers was triggered by the EU accession in 2004 and erosion of immigration restriction in some of the EU countries including introduction of open-door policy in the UK, Ireland, Sweden and later Spain and the Netherlands.

An increased propensity to emigrate of those employed contributed to a stabilization of the probability of separation when home labour market forces (reduction in the transition probability from employment to unemployment) were driving the separation probability down. Besides, relatively high probability of transition from emigration to employment and unemployment with raising share of the total population of those who stay abroad upheld the probability of filling the vacancy at a high level\(^9\).

\(^9\)The transition probability from unemployment to employment was increasing in line with the revival at the labour market after 2004. Therefore, those emigrants who joined the pool of unemployed still had high probability to move to employment in the successive quarter
4.4 Effects of Emigration on Wages

The estimated steady-state emigration rate calculated on the base of the flows data and a posteriori model parameters increased from around 2.1% in 2002 to almost 7% in 2006. One of the key questions the article addresses is the impact of intensified migration movements on wages. To get an insight into this issue, a scenario which assumed stabilization of the emigration rate at levels observed in 2002 was compared with the model prediction of the wage rate between 2003 and 2006. The deviation of the wage rate in the counterfactual scenario from the baseline path was interpreted as the effect of build-up of emigration on wages.

The counterfactual scenario was constructed on the base of technical criterion of stabilization of the emigration rate at the level close to that observed in 2002. Therefore, it does not directly answer the question about the effects of post-accession emigration. The main rational for using the criterion was the difficulty to empirically establish the exact contribution of the EU enlargement to aggregate emigration flows. It would have required setting additional assumptions to justify pursued decomposition of the emigration flows. Instead, the simpler approach was taken, which assumed that intensity of the outflow of workers abroad from each of the labour market states between 2003 and 2006 remained constant and close in the magnitude to intensity registered in 2002. The return migration probabilities were in turn equal in both scenarios\textsuperscript{10}.

There are certain short-comings of the analysis tied to character of the model, mostly to the assumed exogeneity of the transition probabilities. No potential impact of the emigration outflows on the home country labour market flows could have been explicitly taken into account. In line with the reduction of the outflow probability from a given labour market state the persistency this market state was proportionately raised. Hence, it was assumed that emigrants who had left country in particular period would have otherwise not changed their labour market state on that date (they would have remained employed, unemployed or inactive depending on their original labour market status)\textsuperscript{11}.

In both scenarios the vacancy rate and the labour productivity paths were

\textsuperscript{10}No statistically significant shift in the return migration probabilities calculated on the base of the flows data throughout the sample period could have been identified. Accordingly, there was no evidence of a change in the return migration probabilities around the EU accession date.

\textsuperscript{11}The scenario in which reduction in the outflow probability was compensated by the increment of the other transition probabilities from a given labour market state proportional to their respective values (the relative home labour market transition rates were kept unchanged) halved the final effect of the emigration flows on the wage rate. However, works and direction of the impulse were similar.
assumed equal. No effort was made to analyze the impact of compositional changes in the labour force on the average labour productivity and through that channel on wages. The vacancy rate was kept constant in both scenarios exemplifying the assumption that the demand for labour remained unaffected by the sharp increase in the emigration rate around the accession date.

Finally, the replacement rates were same in both scenarios. It was founded on the assumption that generosity of the unemployment, social assistance, retirement or disability benefits would be similar if there were no changes in the emigration propensity of workers. The $\tau$ parameter was in the counterfactual scenario set to zero to reflect lack of one-off reduction in emigration cost after the EU accession.

The experiment showed that around 5 ppt. increase in the emigration rate as compared to the level from 2002 led to moderate growth of the wage rate of around 1.3% in 2006. This result is consistent with earlier studies which indicated at rather low elasticity of real wages to changes in population caused by migrations. Due to high variance of the estimates of the model parameters the 90% interval for the growth of wages initiated by more intensive migration movements is between 0% and 3%.

Higher transition probabilities to emigration from 2003 on together with a decline of emigration cost induced an increase in the permanent income of emigrants and non-working population. In particular, the expected stream of income of unemployed went up improving their fallback position in the wage bargaining. In the result, workers were able to appropriate a higher share of the monopolistic rent and imposed an upward pressure on wages.

However, two other factors were at work which attenuated the final effect of the emigration on the wage rate. First, the bargaining power of workers abated in the years preceding the EU enlargement (Figure 5). Second, the surplus to be divided between firms and workers shrank in line with increasing share of the total population abroad.

The latter outcome is connected with the fact that, as the data suggests, high share of the emigrants eventually come back to the source country. Moreover, the return migrants had significantly higher probability to be employed than those unemployed or out of the labour market. As the number of vacancies in the reference scenario is lower than in counterfactual scenario, the higher temporary emigration rate and relatively high employment probability of the return migrants shifted up the probability of filling a vacancy\textsuperscript{12}. Higher

\textsuperscript{12}Two simulation assumptions shall be summoned here. The vacancy rate and the labour productivity remained unchanged between scenarios. Newly open jobs are proportional to the employment level in both scenarios. Therefore, the higher emigration rate reduces both
probability of filling a vacancy compressed the value of the filled vacancy and the surplus from the job match.

5 Conclusions

The aim of the article was to build in the migration flows into the wage model and use it to explain wage dynamics in Poland after 1995. The search and matching model of Diamod (1982), Mortensten (1984) and Pissarides (2000) was used as a core model and augmented with emigration and return migration movements. The estimation of the wage equation delivered some evidence for gradual liberalization of the labour market in Poland and confirmed gradual fading of the transition factors throughout the last decade.

The framework chosen offered not only a fresh insight into labour market development in Poland but also challenged commonly shared view about large effects of the post-accession emigration on wages in Poland and served as a springboard to explain moderate wage effects of migration on wages found in earlier studies.

The estimated steady-state impact of an outflow of around 5% of workers from Poland between 2002 and 2006 on the wage rate was 1.3%. Sound theoretical underpinnings of the estimated let unravel the factors which were at work throughout the period. However the intuition behind this result seems clear. In the long run the wage rate is anchored at the labour productivity. As long as the intensified emigration does not affect the productivity level wages in the economy may raise owing to reshuffle of the division of monopolistic rents that favor workers. The shift in rents shares initiated by improvement of fallback position of negotiating workers set off mechanisms on the labour demand side which hamper the wage pressure. In response to higher separation rate and wage demands less jobs are created. Besides, those which are created in the steady-state might be easier to fill due to high employability of the return migrants. In fact, job surplus may shrink and counteract against workers interests.

The model did not account for immigration flows which were likely to reduce the wage pressure or remittances which might have influenced the relative income of unemployed and inactive and through this channel triggered the wage growth in the period under consideration. Most importantly, no adjustment path was depicted in the analysis. In the short-run the effect of emigration on the wage rate might have been significantly different and stronger than the product of the economy and the number of created jobs.
gested by the steady-state solution.

Still, sharp increase in the emigration rate in Poland around the EU accession date studied in the formal model framework provided an illustration of mechanisms which might be at work at home or source country labour markets when migration movements intensify. Modest wages response to large emigration flows in Mexico or limited impact on wages of immigration in the host countries found in former studies on the subject, especially those which refer to cross-section variation of migration intensity to detect relation between migration and wages, might be tied to the long-term perspective taken by researchers (most frequently the perspective is imposed by the data at disposal). If the demand for labour adjusts, the wage rate changes activate firms closures or openings, and the initial impulse may attenuated. The wage convergence set off by migration movements across countries or regions shall be curtailed by the convergence of the labour productivity.
Appendix A

Let $A$ be a matrix of a form $A = (1+r)I - P$ (where $I$ is unity matrix and $r$ the real interest rate) where the first row has been replaced by a vector $[1 \quad -1 \quad 0 \quad 0]$. $A_{ij}$ where $i, j \in \{1, 2, 3, 4\}$ shall be a 3x3 matrix which includes all elements of $A$ after deletion of the $i$ – th row and $j$ – th column. Vector $V$ contains the present-discounted values of income stream of workers $[\text{VE} \quad \text{VU} \quad \text{VN} \quad \text{VM}]$ and vector $I$ instantaneous incomes of workers $[\text{IE} \quad \text{IU} \quad \text{IN} \quad \text{IM}]$ with the first element replaced by $(\frac{\beta}{1-\beta}) cy$. Then the problem from (2.4) can be written down in the matrix form:

$$AV = I$$  \hspace{1cm} (24)

Further let $a_{i,j} = detA_{i,j}/detA$. Using Kramer’s rule the system can be solved for elements of $U$:

$$V_U = -a_{1,2}((\frac{\beta}{1-\beta}) \frac{cy}{pV}) + a_{2,2}I_U - a_{3,2}I_N + a_{4,2}I_M$$  \hspace{1cm} (25)

$$V_N = a_{1,3}((\frac{\beta}{1-\beta}) \frac{cy}{pV}) - a_{2,3}I_U + a_{3,3}I_N - a_{4,3}I_M$$  \hspace{1cm} (26)

$$V_M = -a_{1,4}((\frac{\beta}{1-\beta}) \frac{cy}{pV}) + a_{2,4}I_U - a_{3,4}I_N + a_{4,4}I_M$$  \hspace{1cm} (27)

Using the equations above and 10 and introducing the notation used in the text:

$$\theta = -a_{1,2}(r + p_{EN} + p_{EM}) - a_{1,3}p_{EN} + a_{1,4}p_{EM}$$  \hspace{1cm} (28)

$$\alpha_U = a_{2,2}(r + p_{EN} + p_{EM}) + a_{2,3}p_{EN} - a_{2,4}p_{EM}$$  \hspace{1cm} (29)

$$\alpha_N = -a_{3,2}(r + p_{EN} + p_{EM}) - a_{3,3}p_{EN} + a_{3,4}p_{EM}$$  \hspace{1cm} (30)

$$\alpha_M = a_{4,2}(r + p_{EN} + p_{EM}) + a_{4,3}p_{EN} - a_{4,4}p_{EM}$$  \hspace{1cm} (31)
Appendix B

Relative income of emigrants’ versus employed

A relative income of emigrants to the wage rate at the home labour market is used as an approximation of the expected economic betterment of a worker following emigration. Comparable data on the average wage, unemployment rate and the prices level for a large set of developed countries are collected and made available in AMRO database. The expected income of an emigrant was defined as the weighted average of the expected income of an emigrant in individual countries with weights corresponding to the probability to leave for a given country.

The expected income of worker in a country was settled as the nominal compensation per employee (in the total economy) multiplied by the employment rate. Therefore, all other differences between countries which may have an impact on the income of an emigrant, for instance generosity of unemployment benefits or social assistance systems, were ignored. The value was divided by the nominal compensation of employees in the total economy for Poland. Next, the ratio was corrected for the relative purchasing power parities. In the AMRO database only GDP purchasing power parties can be found so the ratios of GDP deflator to private consumption deflator for each country were used to adjust the final result.

The country weights were fixed based on information about emigrants from the household survey accompanying the LFS. The weights corresponded with shares of emigrants who left for different destinations in the period from the second quarter of 1993 to third quarter 2006 (with two middle quarters gap in 1999). These shares were changing throughout the period considered and these changes might have been at least partly coupled with shifts in relative income and migration costs in distinguished host countries. The “replacement rate” was expected to reflect variability of relative emigrants’ income without taking account of changes in migration costs. Accordingly constant weights based on the full sample were calculated. The group of destination countries were restricted to countries which hosted at least 1% of Polish emigrants (according to the data) and shares were normalized so that the weights summed up to unity: Germany (34.6%), the United States (29.3%), Italy (10%), UK (5.4%), France (4.2%), Canada (3.4%), Austria (3.2%), Belgium (3.1%), Greece (2.6%), Netherlands (2.4%), Spain (2.3%).

As the calculated emigrants’ “replacement rate” had yearly frequency, the index was decomposed into quarterly frequency using quadratic average-match method.
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<td>Population $pop_t$</td>
<td>Population 15+ in households</td>
<td>Own calculations</td>
<td>Based on the population data and the population and households forecasts of the Polish Statistical Office</td>
<td></td>
</tr>
<tr>
<td>Variable</td>
<td>Data</td>
<td>Source</td>
<td>Seasonal adjustment</td>
<td>Remarks</td>
</tr>
<tr>
<td>-------------------------------</td>
<td>-------------------------------------------</td>
<td>------------------------------------------------------------------------</td>
<td>---------------------</td>
<td>--------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Replacement rate for unemployed $r_{tU,t}$</td>
<td>Own calculations based on LFS individual data and the Ministry of Finance and Central Statistical Office data</td>
<td>Own calculations based on LFS individual data and the Ministry of Finance and Central Statistical Office data</td>
<td>Details in the Appendix</td>
<td>Details in the Appendix</td>
</tr>
<tr>
<td>Replacement rate for inactive $r_{tI,t}$</td>
<td>Own calculations</td>
<td>Own calculations based on LFS individual data and the Ministry of Finance and Central Statistical Office data</td>
<td>Details in the Appendix</td>
<td>Details in the Appendix</td>
</tr>
<tr>
<td>Replacement rate for emigrants $r_{tM,t}$</td>
<td>Own calculations</td>
<td>Own calculations based on LFS individual data and the Ministry of Finance and Central Statistical Office data</td>
<td>Details in the Appendix</td>
<td>Details in the Appendix</td>
</tr>
<tr>
<td>Unfilled job vacancies $\text{vac}_t$</td>
<td>Number of unfilled job vacancies in the national economy</td>
<td>OECD seasonally adjusted (Teramo-Seats)</td>
<td>Vacancies registered in the job centers</td>
<td>Vacancies registered in the job centers</td>
</tr>
<tr>
<td>Share of temporary employed $f_t$</td>
<td>Ratio of fix-term employed to the total employment</td>
<td>LFS</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 2: Parametr values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Range</th>
<th>A priori Expected value</th>
<th>A priori Standard deviation</th>
<th>Mode</th>
<th>A posteriori Expected value</th>
<th>A posteriori Standard deviation</th>
<th>Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_0$</td>
<td>Bargaining parameter (constant)</td>
<td>[0, 1]</td>
<td>0.50</td>
<td>0.22</td>
<td>0.50</td>
<td>0.43</td>
<td>0.09</td>
<td>0.40</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>Bargaining parameter (marginal change)</td>
<td>[0, 0.5]</td>
<td>0.17</td>
<td>0.12</td>
<td>0.00</td>
<td>0.18</td>
<td>0.12</td>
<td>0.00</td>
</tr>
<tr>
<td>$c$</td>
<td>Flow cost of posting the vacancy (in %)</td>
<td>[0, 10]</td>
<td>2.50</td>
<td>1.44</td>
<td>1.67</td>
<td>2.49</td>
<td>1.46</td>
<td>1.65</td>
</tr>
<tr>
<td>$\vartheta_0$</td>
<td>Vacancy rate (constant) (in %)</td>
<td>[2, 8]</td>
<td>4.40</td>
<td>1.44</td>
<td>4.00</td>
<td>4.81</td>
<td>1.04</td>
<td>4.38</td>
</tr>
<tr>
<td>$\vartheta_1$</td>
<td>Vacancy rate (mariginal change) (in %)</td>
<td>[0, 1.5]</td>
<td>0.50</td>
<td>0.27</td>
<td>0.38</td>
<td>0.56</td>
<td>0.27</td>
<td>0.47</td>
</tr>
<tr>
<td>$\rho_U$</td>
<td>Replacement rate of unemployed „elasticity”</td>
<td>[0.5, 1.5]</td>
<td>1.00</td>
<td>0.22</td>
<td>1</td>
<td>1.00</td>
<td>0.22</td>
<td>1.00</td>
</tr>
<tr>
<td>$\rho_I$</td>
<td>Replacement rate of out of the labour market „elasticity”</td>
<td>[0.5, 1.5]</td>
<td>1.00</td>
<td>0.22</td>
<td>1</td>
<td>1.00</td>
<td>0.22</td>
<td>0.95</td>
</tr>
<tr>
<td>$\rho_M$</td>
<td>Relative income of emigrants „elasticity”</td>
<td>[0, 0.55]</td>
<td>0.33</td>
<td>0.11</td>
<td>0.37</td>
<td>0.27</td>
<td>0.10</td>
<td>0.29</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Change in the „replacement rate” of emigrants after the EU accession (in %)</td>
<td>[0, 1]</td>
<td>0.50</td>
<td>0.29</td>
<td>-</td>
<td>0.36</td>
<td>0.28</td>
<td>0.09</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Discount rate</td>
<td>[0, 0.1]</td>
<td>0.50</td>
<td>0.24</td>
<td>0.50</td>
<td>0.58</td>
<td>0.22</td>
<td>0.54</td>
</tr>
<tr>
<td>$k$</td>
<td>Rescalling LFS parameter (outflow migration)</td>
<td>[1, 1.7]</td>
<td>1.31</td>
<td>0.15</td>
<td>1.28</td>
<td>0.36</td>
<td>0.14</td>
<td>1.38</td>
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<tr>
<td>$l$</td>
<td>Rescalling LFS parameter (return migration)</td>
<td>[0.4, 0.7]</td>
<td>0.58</td>
<td>0.06</td>
<td>0.60</td>
<td>0.58</td>
<td>0.06</td>
<td>0.53</td>
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<tr>
<td>$\sigma$</td>
<td>Error variance</td>
<td>[0, \infty]</td>
<td>0.50</td>
<td>0.35</td>
<td>0.25</td>
<td>0.91</td>
<td>0.42</td>
<td>0.60</td>
</tr>
<tr>
<td>Parameter / variable</td>
<td>Description</td>
<td>Impulse</td>
<td>Expected wage growth</td>
<td>Median wage growth</td>
<td>5 perc.</td>
<td>95 perc.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>----------------------</td>
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<td>----------------------</td>
<td>--------------------</td>
<td>---------</td>
<td>---------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \beta )</td>
<td>Increase in bargaining power of workers</td>
<td>1ppt.</td>
<td>2.19%</td>
<td>2.18%</td>
<td>2.01%</td>
<td>2.38%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \vartheta )</td>
<td>Increase in vacancy rate</td>
<td>1ppt.</td>
<td>0.05%</td>
<td>0.05%</td>
<td>0.01%</td>
<td>0.11%</td>
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<td></td>
</tr>
<tr>
<td>( c )</td>
<td>Increase in flow cost of posting the vacancy</td>
<td>1ppt.</td>
<td>0.15%</td>
<td>0.15%</td>
<td>0.11%</td>
<td>0.21%</td>
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<td></td>
</tr>
<tr>
<td>( r )</td>
<td>Increase of interest rate (discount rate)</td>
<td>1ppt.</td>
<td>-0.23%</td>
<td>-0.21%</td>
<td>-0.37%</td>
<td>-0.10%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \tau U )</td>
<td>Increase in replacement rate of unemployed</td>
<td>10%</td>
<td>0.16%</td>
<td>0.16%</td>
<td>0.10%</td>
<td>0.22%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \tau I )</td>
<td>Increase in replacement rate of inactive</td>
<td>10%</td>
<td>1.19%</td>
<td>1.16%</td>
<td>0.69%</td>
<td>1.76%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \tau M )</td>
<td>Increase in relative income of emigrants</td>
<td>10%</td>
<td>0.26%</td>
<td>0.23%</td>
<td>0.05%</td>
<td>0.59%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( t^{\text{CORP}} )</td>
<td>Raise of effective tax rate paid by employers</td>
<td>1ppt.</td>
<td>-0.54%</td>
<td>-0.54%</td>
<td>-0.64%</td>
<td>-0.43%</td>
<td></td>
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</tr>
<tr>
<td>( t^{\text{EMP}} )</td>
<td>Raise of effective tax rate paid by employees</td>
<td>1ppt.</td>
<td>0.76%</td>
<td>0.76%</td>
<td>0.74%</td>
<td>0.77%</td>
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<td></td>
</tr>
<tr>
<td>( y )</td>
<td>Change in the labour productivity</td>
<td>1%</td>
<td>1.11%</td>
<td>1.11%</td>
<td>0.99%</td>
<td>1.21%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figures 1: A priori and a posteriori parameters distributions
Figure 2: Steady-state emigration rate

Figure 3: Steady-state unemployment rate

Figure 4: Steady-state activity rate
Figure 5: Bargaining power of workers parameter

Figure 6: Vacancy rate
Figure 7: Probability of separation

Figure 8: Probability of filling a vacancy
Figure 9: Wage growth tied to an increase in the emigration rate between 2003 and 2006

Figure 10: Wage growth versus change in the emigration rate in 2006