The Design of the University System*

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April 11, 2008

Abstract

This paper studies a general equilibrium model suitable to compare the organisation of the university sector under private provision with the structure which would be chosen by a welfare maximising government. To attend university, and earn higher incomes in the labour market, students pay a tuition fee, and each university chooses its tuition fee to maximise the amount of resources that can be devoted to research. Research bestows an externality on society; government intervention increases needs to balance labour market efficiency consideration – which would tend to equalise the number of students attending each university –, with efficiency considerations, which suggest that the most productive universities should teach more students and carry out more research.

Very Preliminary

JEL Numbers: I210, I280, H420.
Keywords: Higher education, The organisation of the university sector.

*An earlier version was presented at the University of Lancaster.
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1 Introduction.

This paper studies the organisation of the university sector. The dearth of theoretical analyses devoted to this topic, at least relative to, say, government procurement, health or primary and secondary school systems, is surprising, in view of the wealth of peculiar features characterising it, as well as, of course, its immediate relevance to the daily life of many researchers in this area.

Universities provide both tertiary education and "blue sky" research. This is often attributed (Becker, 1975**?** and 1979**?**) to the existence of externalities between teaching and research, so that teachers (respectively researchers) result more productive if they also carry out research (respectively teaching). The empirical evidence for this externality is flimsy[1] and we therefore refrain from making our analysis hinge on it: we allow "research only" and "teaching only" universities, and assume that research and teaching are perfect substitutes in the university production function. Conversely, we do not posit any comparative advantage in these two activities: universities that are "better at research" are also (equally) "better at teaching". A university's payoff is (an increasing function of) the amount of research that they do. Moreover, we assume that universities have a monopolistic advantage in teaching, and that they use it to recruit students as a way of raising funds to carry out research.[2] This does find a justification in our paper: among our results, we show that with government provision, teaching and research can be separated, for example by having institutions entirely devoted to teaching only with the unrealistic assumption that the government knows the productivity of each university. In the realistic case of asymmetric informationthe complementarity between teaching and research emerges endogenously as a feature of the equilibrium, rather than being exogenously assumed as a feature of the technology.

We do not restrict research to be "socially desirable": in the private market there can be too much or too little research. There are two aspects of academic research. Some research increases labour productivity, thus bestowing a positive externality on the rest of society: a mathematical theorem may help improve computer software used in design or robots; a chemical discovery may allow the development of more effective drugs, and reduce the number of days lost due to illness; advances in game theory may lead to improved understanding of incentive mechanism used by organizations to select and motivate staff, and so on. Some research can instead be viewed as an end in itself, and evaluated as such by the policy maker, just like artistic creation[3] or considered to have very long term and uncertain benefits, which might benefit only future generations while not affecting the present productivity in society.

[1] For a survey of empirical results see Hattie and Marsh, 1996**?**, and for a detailed discussion on functional forms employed to evaluate universities cost function and teaching/research output measures see Agasisti and Dal Bianco, 2007**?**.

[2] By which we mean "blue sky" research: externally funded research towards a specific project is carried out in universities, as a commercial activity. In this sector they compete with private providers, and therefore make no economic profits.

[3] Some may have commercial value, like a new edition of Shakespeare's sonnets, which of course is captured by the revenues it generates, but there is no externality mediated by prices.

The social desirability of research depends therefore on the subjective preferences of the financing agency, that is, ultimately, the government. A "philistine" government may therefore attribute a negative value to (some) research, which may more than offset the positive income externality: in this case its intervention would attempt to reduce the amount of research. The focus of our paper is on the *distribution* of research and teaching among different institutions, rather than the aggregate amount, and while our approach can be easily extended to include this possibility, we do not study the role of the government's preference for research assuming instead that it designs its policy to maximize the sum of individual utilities.

Students attend university to increase their labour market opportunities; they differ in ability (measured by the disutility cost of attending university), and they are imperfectly mobile in the sense that there are some friction in the students' capacity to choose university. Two universities offering identical services to students need not charge the same price.

Our main result can be summarized by stating that the optimal policy of the government is to concentrate research and teaching in the most productive universities. While the location of research is a matter of indifference, the imperfect geographical mobility of the students implies that the same is not true of teaching: concentrating teaching in some institutions prevents some students to attend university who would otherwise have benefited from doing so. The trade-off between concentrating teaching and research in the most productive universities and ensuring that the most suitable students attend university, irrespective of their location is apparent when the perfect information case, where the government creates "teaching only" universities, concentrating all research in the top group of institutions, which of course also teach their students, and are resourced accordingly. This is however not possible with asymmetric information, and the concentration of research is pushed to a lesser extent. There are more universities in a fully private system, and the top universities do less research than under government intervention.

The paper is organised as follows. We present the model in Section 2. In Section 3 we study a private university system, and in Section 4 derive the government optimal policy, and then compare it with the private system derived in Section 3. Finally, in Section 5 we argue that the analysis is robust to relaxation of some of the assumptions.

2 The model

2.1 Universities.

In the economy we consider there is a continuum of education markets, separated from each other, and a single economy-wide labour market. There are two types of jobs, skilled and unskilled: to be employed in the skilled labour market it is necessary to obtain university education. In each local education market there is a single potential university, which, if it operates, acts as a local monopoly:[4] it is available to all local residents, and only to them. Potential universities differ in the value of a productivity parameter, $\theta \in \left(0, \bar{\theta}\right]$, with $\bar{\theta} > 1$. The distribution of θ in the economy is given by a differentiable function $F(\theta)$, with density $f(\theta) = F'(\theta) > 0$ for $\theta \in \left(0, \bar{\theta}\right)$. The total number of universities is normalised without loss of generality to 1: $F\left(\bar{\theta}\right) = 1$.

Universities can engage in research and teaching, and to do so, they must employ "professors" and build lecture theatres, laboratories, libraries and so on. If $n > 0$ is the number of professors (normalise away all the other costs), then each university has a production function given by $\hat{h}(r, t, n, \theta) = 0$, which, without loss of generality can be written as

$$n = h(t, r, \theta)$$

where research is measured by $r \geqslant 0$, and $t \geqslant 0$ is the number of students taught. In words, universities uses one input, "professors" to produce two outputs, research and teaching,[5] with $h_r(t, n, \theta) > 0$, $h_t(t, n, \theta) > 0$, $h_\theta(t, n, \theta) < 0$: the first two imply that research and teaching both require professors, and the third defines θ as a positive measure of productivity: a university with a higher θ can do the same amount of research and teaching with fewer professors. A convenient functional shape is:

$$n = \frac{t + r}{\theta} \tag{1}$$

That is, we assume that the outputs of the university are perfect substitutes, and in a linear relationship with the number of professors[6]. For a given productivity parameter θ, an increase in teaching (the number of students) must be compensated by an increase in the number of professors if the university wants to maintain the same level of research effort.

Universities receive income from students, who pay a tuition fee of $p \in \mathbb{R}$ per student (not restricted to be non-negative), and possibly from the government, in the form of a grant $g \in \mathbb{R}$ (which again can be negative and therefore a tax). Their costs are the salaries associated with their professors, and so the budget constraint of a university is

$$pt + g - yn = 0 \tag{2}$$

where y is the salary paid to a professor, endogenously determined by a competitive labour market (see below) as a function of the total amount of research in the society $R \geqslant 0$, and of the total number of graduates in society, $T \in [0, 1]$. If we denote by $r(\theta)$ and $t(\theta)$ the *average* amount of research carried out by the universities of type θ, and

[4] This is a simple way of capturing the assumption that students are not infinitely mobile: if this were the case that one university, the best, would attract *all* students, and carry out *all* research. See Section 5.1 on how it can be relaxed.

[5] As recognised by Cohn et al. 1989**?**, they should therefore be viewed as multi-product firms. Further outputs that have been suggested, such as the transfer of knowledge (Johnes et al., 2005)**?**, or the production of human capital (Rothschild and White (1995)**?**), could clearly be incorporated by extending the concept of teaching.

[6] To the extent that scientific research is more and more specifically focused, the alternative hypothesis of complementarity between research and teaching does seem less justified.

the *average*[7] number of students in the universities of type θ, we have:

$$R = \int_0^{\bar{\theta}} r(\theta) f(\theta) \, d\theta \tag{3}$$

$$T = \int_0^{\bar{\theta}} t(\theta) f(\theta) \, d\theta \tag{4}$$

Universities are typically managed by academics whose vocation is research. Its staff are rewarded more for success in research than in teaching: as shown, among others, by Hammond et al. (1969)**?** and Tuckman et al. (1976)**??**. We therefore posit that the objective function of universities is the maximisation of the amount of research they do. They derive no pleasure from teaching, but view it instead merely as a source of income, a necessary way to pay for their research. If, at face value, this may seem too cynical a view, it can be made more palatable by extending, plausibly, the definition of research: think of "research" as any activity which benefit individuals or groups who cannot be made to pay for it. Thus for example universities may subsidise doctoral supervision, or offer scholarship and financial aid to students from deprived backgrounds: these activities are undertaken by universities because they increase their payoff, even though – by definition – they do not generate enough revenue to cover their cost: to the extent that they generate benefits to (parts of) society, for example by increasing future research activities or enhancing diversity and offering role-models to able individuals in deprived neighbourhoods, they fit the revised definition of "research" given above.

2.2 Students and the labour market.

Each local education market serves a population of potential students, with measure normalised to 1. They can choose between basic education, available in all local labour markets at no cost, which guarantees an unskilled job, with income

$$y_U(R, T)$$

R measures the "state of technology in the society", defined as the sum of all research undertaken by the active universities, and T the total number of graduates in the society: the total number of unskilled workers is therefore $1-T$, and demand and supply considerations make it reasonable to impose $\frac{dy_U(R,T)}{dT} > 0$. The positive externality of research implies that $\frac{dy_U(R,T)}{dR} > 0$: workers are more productive if more research is carried out in society.

In each local market the potential university, may choose to become active in teaching, and enrol students. If it does so, then potential students may pay the tuition fee, attend university, obtain a degree and subsequently work in the skilled labour market. All students who attend university do graduate and then, working in the graduate

[7] In the equilibrium we consider, all type by θ universities make identical choices, and so $r(\theta)$ and $t(\theta)$ are the amount of research and the number of students in each type θ university.

labour market, receive income[8]:

$$y(R,T) \tag{5}$$

There are two possible destinations in the graduate labour market: graduate can work as academics in one of the local universities, or they can be employed in as graduate in the "business" sector. Income and job satisfaction are the same, and the "business sector" absorbs all the graduates that are not required to work as academics. Supply and demand considerations determine the salary in this market, and consequently also the academic salary. This implicitly implies that the labour academic market is "small" relative to the rest of the economy. Relaxing this assumption would imply writing the functions y and y_U as $y(R, T-N)$ and $y_U(R, T-N)$ where N is the number of academics, defined, from (1) as

$$N = \int_0^{\bar{\theta}} \frac{r(\theta)+t(\theta)}{\theta} f(\theta)\, d\theta \tag{6}$$

The payoff of a unskilled worker is his income, whereas for a graduate is the difference between the labour market income, net of tuition fees, and the cost of effort while at university. Students are characterised by an exogenous parameter a, which takes values in $[a_{\min}, a_{\max})$, with distribution $\Phi(a)$ and density $\phi(a) = \Phi'(a)$, with monotonic hazard rate $\frac{d}{da}\left(\frac{\Phi(a)}{\phi(a)}\right) > 0$, and captures the cost of the effort exerted while at university.[9] Formally, attending university has an effort cost $\alpha(a)$, with $\alpha'(a) > 0$.

It is convenient to assume that the externality is the same for both markets, and that the earnings function is separable.

Assumption 1 *$y(R,T) = x(T) + w(R)$, and $y_U(R,T) = x_U(T) + w(R)$.*

Therefore, a potential student of type a in a local market where the university of type θ charges a tuition fee $p(\theta)$ takes this tuition fee, and labour market earning in the graduate, $y(R,T)$ and unskilled labour market $y_U(R,T)$ as fixed, and chooses to attend university and work in the skilled labour market if

$$x(T) + w(R) - p(\theta) - \alpha(a) > x_U(T) + w(R)$$

Conversely, a potential student prefers to work in the unskilled labour market if the reversed above inequality holds, and is indifferent if the two sides the inequality are equal. We maintain through, for the sake of definiteness, the assumption that indifferent students attend university: since they have measure 0 in $[a_{\min}, a_{\max}]$, this entails no loss of generality.

Note that, again for simplicity, wages are uncorrelated with a (this assumption can be replaced by the assumption that x_U decreases with a, at a lower rate than $\alpha(a)$

[8] While there is some evidence of different external effects on the earnings of skilled and unskilled workers (eg, Moretti, 2004**?**), the small size of this difference justifies the simplication of the algebra which can be achieved by positing the same function $w(R)$ in the two labour markets.

[9] In models of this type, the parameter differentiating the individuals (their type) can denote either the cost of effort or the labour market earnings: it will become evident that the qualitative features of the formal analysis analysis of the paper would not change if the alternative set-up were considered.

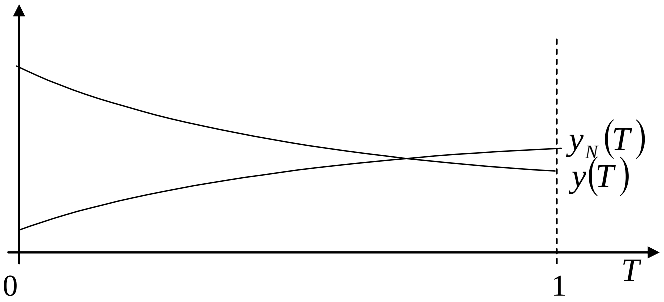

Figure 1: Earnings in the labour markets.

increases with x. In order to ensure that some - but not all - potential students do in fact choose to attend university (and graduate), we impose some natural assumptions on $x(T)$ and $x_U(T)$.

Assumption 2 *(i)* $x(1) - \alpha(a_{\max}) < x_U(1)$*;*
(ii) $x(0) - \alpha(a_{\min}) > x_U(0)$
(iii) $x'_U(T) > 0$*.*
(iv) $x'(T) < 0$

That is, (i) if there are very few unskilled workers, they can obtain a higher utility than the students with the highest cost of effort would obtain if they attended university for free, and (ii) if there are very few graduates, there are at least some potential students whose cost of effort would make prefer to attend university, if the tuition fee is 0; (iii) and (iv) say that the earnings in each market increase when the supply shrinks. Figure 1 illustrate an example of two functions $x(T)$ and $x_U(T)$ satisfying Assumption 2.

Define $\Delta(T)$ as $x(T) - x_U(T) > 0$, and so we have that the students who are indifferent between going to university and not going have type

$$\alpha^{-1}(\Delta(T) - p(\theta)) \tag{7}$$

Individuals characterised by type below (7) attend university and work in the graduate labour market. For fixed T and R, the horizontal line is the payoff $y_U(R,T)$ for unskilled workers. The dashed line is the payoff for attending university and then working in the skilled labour market.

The model is essentially static. Current students pay for their current university education with their future income. This implies that they can borrow against this income, which in turn is possible if there are perfect capital markets, or that, if tuition is financed by parents, there are no systematic differences between the ability of parents in different households to fund their children's tuition. The existence of some liquidity constrained households can be accomodated by assuming that, in each labour market, only a fraction of the young individual is a potential students; the other are liquidity constrained. Alternatively, the cost of attending university, a, can include a component which is the interest that needs to be paid for financing university attendance via borrowing.

3 Equilibrium with no government

We assume in this section that the university sector is private: in each local education market, the local potential university decides whether to operate or not, and if so, the tuition fee it charges. The number of students[10] taught by a university of type θ which chooses price $p(\theta)$ is:

$$t(\theta) = \Phi\left(\alpha^{-1}\left(\Delta(T) - p(\theta)\right)\right) \tag{8}$$

Universities charge the same price to all students, for example, because they cannot observe their type, and have no instruments to provide incentive for truth-telling. It is helpful to define the function $z_k : [0,1] \longrightarrow \mathbb{R}$, for $k \in (0,1]$, which will play a key role in the rest of the analysis:

$$z_k(t) = \alpha\left(\Phi^{-1}(t)\right) + kt\frac{\alpha'\left(\Phi^{-1}(t)\right)}{\phi\left(\Phi^{-1}(t)\right)} \tag{9}$$

Assumption 3 *For every $a \in (a_{\min}, a_{\max})$, $-\frac{\phi(a)}{\Phi(a)} < \frac{\alpha''(a)}{\alpha'(a)}$.*

This is true if $\alpha''(a) \geqslant 0$, and, if $\alpha''(a) < 0$, that is if $\alpha'(a)$ is either increasing, or, if not, it decreases at a rate which is bounded from below.

Lemma 1 *If Assumption 3 holds, $z_k(t)$ is monotonically strictly increasing for every $k \in (0,1]$.*

Proof. Differentiate (9), writing $(\cdot)$ for $\left(\Phi^{-1}(t)\right)$:

$$\begin{aligned} z_k'(t) = \frac{dz_k}{dt} &= \frac{\alpha'(\cdot)}{\phi(\cdot)} + kt\frac{\alpha''(\cdot) - \alpha'(\cdot)\frac{\phi'(\cdot)}{\phi(\cdot)}}{\phi(\cdot)^2} + k\frac{\alpha'(\cdot)}{\phi(\cdot)} \\ &= \frac{k\alpha'(\cdot)\,t}{\phi(\cdot)^2}\left(\frac{\phi(\cdot)}{t}\frac{1+k}{k} + \left(\frac{\alpha''(\cdot)}{\alpha'(\cdot)} - \frac{\phi'(\cdot)}{\phi(\cdot)}\right)\right) \end{aligned}$$

This is positive if

$$kt\frac{\alpha'(\cdot)}{\phi(\cdot)^2}\left(\frac{\frac{1+k}{k}\phi(\cdot)}{t} + \left(\frac{\alpha''(\cdot)}{\alpha'(\cdot)} - \frac{\phi'(\cdot)}{\phi(\cdot)}\right)\right) > 0$$

that is if (recall that $\alpha'(\cdot)$ is positive):

$$\frac{\alpha''(\cdot)}{\alpha'(\cdot)} > \frac{\phi'(\cdot)}{\phi(\cdot)} - \frac{\frac{1+k}{k}\phi(\cdot)}{t} \tag{10}$$

[10] In this set up, where students have perfect information about thier type, there is no scope for selection or screening (that is excluding students who would be willing to enrol at the current price). The analysis of this case is in Gary-Bobo and Trannoy 2004.?

Now notice that, given the assumption of a monotonic hazard rate for $\Phi(\cdot)$, we have

$$\frac{d}{dx}\left(\frac{\Phi(x)}{\phi(x)}\right)=1-\frac{\Phi(x)\phi'(x)}{\phi(x)^2}>0$$

and therefore:

$$\frac{\phi'(x)}{\phi(x)}<\frac{\phi(x)}{\Phi(x)}$$

when evaluated at $x=\Phi^{-1}(t)$, this becomes:

$$\frac{\phi'\left(\Phi^{-1}(t)\right)}{\phi\left(\Phi^{-1}(t)\right)}<\frac{\phi\left(\Phi^{-1}(t)\right)}{t}$$

or

$$\frac{\phi'\left(\Phi^{-1}(t)\right)}{\phi(\Phi^{-1}(t))}-\frac{\frac{1+k}{k}\phi\left(\Phi^{-1}(t)\right)}{t}<\frac{\phi\left(\Phi^{-1}(t)\right)}{t}-\frac{\frac{1+k}{k}\phi\left(\Phi^{-1}(t)\right)}{t}=-\frac{\phi\left(\Phi^{-1}(t)\right)}{kt}<\frac{\alpha''\left(\Phi^{-1}(t)\right)}{\alpha'(\Phi^{-1}(t))} \quad (11)$$

The last inequality sign follows from Assumption 3. Thus (10) holds and the Lemma is established. ■

We are now in the position to establish our first result.

Proposition 1 *If Assumption 3 holds, a university will enrol a number of students given by:*

$$t(\theta)=z_1^{-1}\left(\Delta(T)-\frac{y(R,T)}{\theta}\right) \quad (12)$$

and therefore set a tuition fee:

$$p(\theta)=\Delta(T)-\alpha\left(\Phi^{-1}\left(z_1^{-1}\left(\Delta(T)-\frac{y(R,T)}{\theta}\right)\right)\right) \quad (13)$$

Proof. Substituting (1) into (2), we can write:

$$pt+g-y(R,T)\frac{t+r}{\theta}=0$$

That is

$$r=(pt+g)\frac{\theta}{y(R,T)}-t$$

Inverting (8), we obtain the price that a university must charge in order to enrol $t(\theta)$ students:

$$p(t)=\Delta(T)-\alpha\left(\Phi^{-1}(t)\right) \quad (14)$$

A university chooses t to maximise the amount of research it does, and therefore, if an internal solution exists, it satisfies the first order condition

$$r'(t)=(p'(t)t+p(t))\frac{\theta}{y(R,T)}-1=0 \quad (15)$$

Where the dependance of p and r on t is made explicit. From (14) we get:

$$\frac{dp}{dt}=-\frac{\alpha'\left(\Phi^{-1}(t)\right)}{\phi\left(\Phi^{-1}(t)\right)}<0$$

And so (15) is

$$\left(-\frac{\alpha'\left(\Phi^{-1}(t)\right)}{\phi\left(\Phi^{-1}(t)\right)}t+\Delta(T)-\alpha\left(\Phi^{-1}(t)\right)\right)\frac{\theta}{y(R,T)}-1=\left(\Delta(T)-z_1(t)\right)\frac{\theta}{y(R,T)}-1=0$$

The second order condition is

$$r''(t)=-\frac{\theta}{y(R,T)}z_1'(t)<0$$

which, as shown in the Lemma, is satisfied if Assumption 3 holds. By (15) At the optimum choice of r:

$$z_1(t)=\Delta(T)-\frac{y(R,T)}{\theta} \tag{16}$$

%which gives:

$$\%z_1(t)=\Delta(T)-\frac{y(R,T)}{\theta}$$

from which (12) is derived. (13) is immediate from (14), and the proof of the proposition is complete. ■

The next result gives the relationship between productivity and size and fees.

Corollary 1 $\frac{dp}{d\theta}<0$, $\frac{dt}{d\theta}>0, \frac{dr}{d\theta}>0$, *and* $\frac{dn}{d\theta}>0$.

Proof. The first two assertions have been established in the proof of Proposition 1. For the third:

$$\frac{dr}{d\theta}=\left(\left(\Delta(T)-z_1(t)\right)\frac{\theta}{y(R,T)}-1\right)\frac{dt}{d\theta}+\frac{p(t(\theta))\,t(\theta)+g}{y(R,T)}>0 \tag{17}$$

since the first term vanishes by the first order condition on the choice of r, (15). Finally,

$$\frac{dn}{d\theta}=\frac{\frac{dt}{d\theta}+\frac{dr}{d\theta}}{\theta}-\frac{t(\theta)+r(\theta)}{\theta^2}$$

using (17) and $t(\theta)+r(\theta)=\frac{p(t(\theta))t(\theta)+g}{y(R,T)}\theta$ this simplifies to $\frac{1}{\theta}\frac{dt}{d\theta}>0$. ■

More productive universities do therefore teach more students, charge a lower price to attract them, employ more academics and carry out more research. This relationship between size and efficiency is often observed in empirical studies,[11] and our analysis indicates that it need not necessarily imply that there are economies of scale and scope. Corollary 1 shows that universities which employ more staff have lower unit costs, both in teaching and in research,**?**, even though each individual university employs the same linear technology with no economies of scale and scope.

[11] Cohn et al (1989)**?** study a sample of 1,887 US colleges and universities in 1981-82, through a multi-product cost approach: they find economies of scale and scope. De Groot, et al (1991)**?** and Glass, et al (1995)**?** extended Cohn's research focusing respectively on the sensitivity of the cost functions estimates to different output measures and on flexible cost function. See also, for a stochastic frontier analysis and DEA, Johnes and Johnes,(1995) **?** and McMillan and Chan (2006)**?**.

The above analysis clearly holds if the university of type θ is able to operate. We turn next of which universities are in fact active. For university of type θ to conduct research, it must be that, at its optimal number of students, it can make positive revenues to pay for its research.

$$(p(t(\theta))t(\theta)+g)\frac{\theta}{y(R,T)}-t(\theta)>0 \tag{18}$$

Note that the optimal number of students is independent of the grant g. If $g>0$, then a university can always do some research by enrolling no students and spending all its grant on research. Let therefore consider the case where $g=0$. In this case we have the following.

Corollary 2 *If $g=0$, a university of type θ enrols students and carries out research if and only if*

$$\theta>\frac{y(R,T)}{\Delta(T)-\alpha(a_{\min})}$$

Proof. If $g=0$, then (18) reduces to

$$\left(p(t(\theta))\frac{\theta}{y(R,T)}-1\right)t(\theta)>0$$

The preferred value of t is given by the intersection of the increasing function $z_1(t)$ with the horizontal line $\Delta(T)-\frac{y(R,T)}{\theta}$ (see (16)). This intersection occurs for a positive value of t if $z_1(0)<\Delta(T)-\frac{y(R,T)}{\theta}$:

$$\alpha\left(\Phi^{-1}(0)\right)=\alpha(a_{\min})<\Delta(T)-\frac{y(R,T)}{\theta}$$

This establishes the Corollary. ■

The interpretation is natural: the teaching cost borne to prepare a student to the labour market is $\frac{y(R,T)}{\theta}$. For the university to want to teach at least one student it must be worth for at least one student to pay for this cost, and the willingness to pay for tuition of the ablest student is the increase in her labour market earnings as a consequence of her having a degree, reduced by the utility cost of attending university.

This is illustrated in Figure 2. The increasing line is $z_1(t)$. The abscissa of its intersection with the horizontal line $\Delta(T)-\frac{y(R,T)}{\theta}$ gives, as shown in (16), the number of students. For lower values of θ, the horizontal curve is also lower, implying that fewer students are enrolled. For values of θ such that the horizontal line intersects the curve $z_1(t)$ at $t=0$, or lower, the university is unable to recruit any students at a price which covers its cost: this value of θ is given in Corollary 2.

To close the model, we need to derive R and T. They are obtained from the definition of T and R given in (4) and (3) as follows:

$$R=\int_{\frac{y(R,T)}{\Delta(T)-\alpha(a_{\min})}}^{\bar{\theta}}\left((p(t(\theta))t(\theta)+g)\frac{\theta}{y(R,T)}-t(\theta)\right)f(\theta)\,d\theta \tag{19}$$

$$T=\int_{\frac{y(R,T)}{\Delta(T)-\alpha(a_{\min})}}^{\bar{\theta}}t(\theta)f(\theta)\,d\theta \tag{20}$$

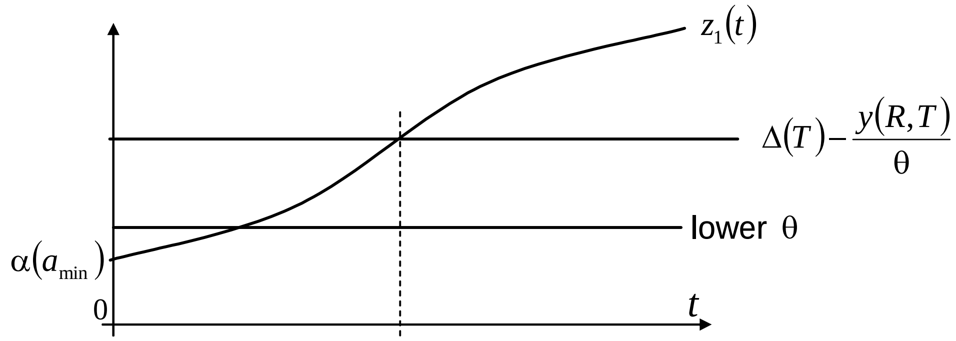

Figure 2: The determination of the number of students in a university.

where of course $p(\theta)$ and $t(\theta)$ are given in Proposition 1. The simultaneous solution of (19)-(20) in T and R closes the model.

4 Government intervention.

We now assume that the government runs the higher education sector. Because of their specialised nature, tertiary education and research must be undertaken by the universities in the local market. We assume that the objective of the government and of universities are not aligned, and, moreover, that universities have an informational advantage over the government: they know their own θ and obviously know how much r they carry out, while the government cannot observe either. We also assume that the university can increase its n, the number of academics it employs, without the government noticing.[12] If the government could observe employment, then it could infer the university's productivity from the observation of the number of students enrolled, and the university would have no information advantage.

The government payoff function is the total after tax income of the population, net of the disutility costs borne by those who attend university. We do not include the utility of universities in the government payoff.[13]

[12] There are many ways in which a university can "hide" research activities, that is do a different amount of researh than what stipulated in an agreement with or directive from the government. from funding research collaborations, to hiring temporary teachers, to buyout to lower teaching loads, to establishing visiting positions, to funding PhD scholarships, research centres, and so on. In general it is difficult for a third party (the government) to stipulate how two contracting parties (the university and its staff) should contract and enforce the government preferred agreement if they prefer something different.

[13] While this may seems arbitrary, and to contrast, for example with the practice to include (a share of) a regulated firm's profit in the computation of social welfare, these have a monetary correspondent, and the share that is not taken abroad by the firms' foreign shareholders, does benefit some individuals under the government's jurisdiction; this is not the case for the utility of the university, which, by its very nature, does not have a profit dimension, and is therefore not directly comparable to monetary income. At any rate, the university payoff could be included in the payoff of the government, with no qualitative change in the results.

A university policy is a pair of functions that the government commits to: $\{p(t), g(t)\}$, where t is the number of students enrolled, $p(t)$ the tuition fee charged for enroling t students, and $g(t)$ the –possibly negative– lump sum grant awarded, also depending on the number of students. Faced with this policy, each university can choose freely the number of students it enrols, and receives the grant from the government and the fees from its students accordingly.

To determine the government's preferred policy, we take a standard revelation approach, whereby the government asks each university to report its own type, and commits to a vector of variables, which is a function of the reported type, and which the university must choose: by the revelation principle, the government cannot obtain a better payoff than the highest payoff that can be obtained by restricting attention to mechanisms with the property that no university has an incentive to mis-report its type.

With this perspective, a policy is a triple, $\{t(\theta), p(\theta), g(\theta)\}$, where t, p, and g, are functions of the reported type. We include *both* t and p as policy variables, thus allowing, potentially, for the number of students enrolled in a university to be different from the number of individuals who, given the tuition fee, would prefer to graduate. Clearly it cannot exceed it, and so we impose the constraint

$$\Phi^{-1}(t(\theta)) \leqslant \alpha^{-1}(\Delta(T) - p(\theta)) \tag{21}$$

That is, the type of the marginal student must be no greater than the type of the student who is indifferent between going and not going to university. The analysis below illustrates that (21) is in fact binding at the government's optimal policy. This is due to the fact that, because of the shadow cost of public funds, it is always preferable for the government to raise funds through tuition fees than through taxes. This avoids the potential difficulty of determining a method to ration the number of university places among the individuals willing to attend.

Another natural constraint is that the government's total grant must be funded through taxation: to keep things simple, we model taxation as a lump sum tax τ, the same for all individuals. As customary, raising taxes has an administrative and distortionary cost captured by a parameter λ:

$$\tau = (1+\lambda) \int_0^{\bar{\theta}} g(\theta) f(\theta) \, d\theta$$

We also impose a cap on the amount of taxation that can be imposed on each individual: at the very least is seems natural to require that no one is required to pay more than their maximum potential income, net of tuition fees

$$\tau \leqslant \max\{y(R,T) - p(\theta), y_U(R,T)\} = y_U(R,T)$$

The last equality follows from the fact that, since $\alpha(a) \geqslant 0$ for $a \in [a_{\min}, a_{\max}]$, a university which charged more than $\Delta(T)$ would not enrol any students. Combining the above relations, we get

$$(1+\lambda) \int_0^{\bar{\theta}} g(\theta) f(\theta) \, d\theta \leqslant y_U(R,T) \tag{22}$$

The determination of the government's optimal policy uses optimal control techniques (Leonard and van Long, 1992). To set up the problem in a suitable way, we introduce auxiliary variables R, the total amount of research, T, the total number of students, subject to definitional constraints (4) and (3). $\underline{\theta}$, the cut-off point type of university, such that those above operate, those below do not, is determined endogenously, as the "initial time" (Leonard and van Long, 1992, p 222 ff). Finally, $r(\theta)$ is also treated as a variables chosen by the government, but it is of course subject, as explained above, to the incentive compatibility constraint, that is that the university reveal their type truthfully. We derive this constraint in Proposition 2. Note first that the utility of a university who has reported θ is

$$r(\theta) = [p(\theta)t(\theta) + g(\theta)]\frac{\theta}{y(R,T)} - t(\theta) \tag{23}$$

Proposition 2 *The incentive compatibility constraint is given, for* $\theta \in [\underline{\theta}, \overline{\theta}]$*, by:*

$$\dot{r}(\theta) = \frac{p(\theta)t(\theta) + g(\theta)}{y(R,T)} \tag{24}$$

$$\dot{t}(\theta) \geqslant 0 \tag{25}$$

Proof. Let the government policy be $\{t(\theta), p(\theta), g(\theta)\}$. By choosing to report type $\hat{\theta} \in [0,1]$, university of type θ receives a price for tuition $p\left(\hat{\theta}\right)$, a grant $g\left(\hat{\theta}\right)$, and is required to teach $t\left(\hat{\theta}\right)$ students. Given the market salary for its staff, $y(R,T)$, it employs:

$$\frac{p\left(\hat{\theta}\right)t\left(\hat{\theta}\right) + g\left(\hat{\theta}\right)}{y(R,T)} \tag{26}$$

academics, which will enable it to carry out an amount of research u such that:

$$\frac{p\left(\hat{\theta}\right)t\left(\hat{\theta}\right) + g\left(\hat{\theta}\right)}{y(R,T)} = \frac{u + t\left(\hat{\theta}\right)}{\theta}$$

Hence the utility of a university of type θ for reporting $\hat{\theta}$ is

$$\varphi\left(\theta, \hat{\theta}\right) = \frac{p\left(\hat{\theta}\right)t\left(\hat{\theta}\right) + g\left(\hat{\theta}\right)}{y(R,T)}\theta - t\left(\hat{\theta}\right)$$

The revelation principle requires that the above is maximised at $\hat{\theta} = \theta$. The first order condition for the choice of $\hat{\theta}$ is:

$$\left.\frac{\partial\varphi\left(\theta,\hat{\theta}\right)}{\partial\hat{\theta}}\right|_{\hat{\theta}=\theta} = \left.\frac{\partial\left(\frac{p(\hat{\theta})t(\hat{\theta})+g(\hat{\theta})}{y(R,T)}\theta - t\left(\hat{\theta}\right)\right)}{\partial\hat{\theta}}\right|_{\hat{\theta}=\theta} = 0 \tag{27}$$

which gives:

$$\left(p(\theta)\dot{t}(\theta) + \dot{p}(\theta)t(\theta) + \dot{g}(\theta)\right)\frac{\theta}{y(R,T)} - \dot{t}(\theta) = 0 \tag{28}$$

Next differentiate $r(\theta)$ given in (23),

$$\dot{r}(\theta) = \left[p(\theta)\dot{t}(\theta) + \dot{p}(\theta)t(\theta) + \dot{g}(\theta)\right]\frac{\theta}{y(R,T)} + \frac{p(\theta)t(\theta) + g(\theta)}{y(R,T)} - \dot{t}(\theta)$$

and substitute (28) in it to obtain (24). Now (25): following Laffont and Tirole (1993, p 121)**?**, for a policy to be incentive compatible it must be that

$$\frac{\partial^2 \varphi\left(\theta, \hat{\theta}\right)}{\partial\theta\partial\hat{\theta}} \geqslant 0$$

We have

$$\frac{\partial^2 \varphi\left(\theta, \hat{\theta}\right)}{\partial\theta\partial\hat{\theta}} = \frac{\partial\left(\frac{p(\hat{\theta})t(\hat{\theta})+g(\hat{\theta})}{y(R,T)}\right)}{\partial\hat{\theta}} = \frac{\partial\left(\frac{r(\hat{\theta})+t(\hat{\theta})}{\hat{\theta}}\right)}{\partial\hat{\theta}} \geqslant 0$$

$$\frac{1}{\hat{\theta}}\left(\dot{r}\left(\hat{\theta}\right) + \dot{t}\left(\hat{\theta}\right) - \frac{r\left(\hat{\theta}\right) + t\left(\hat{\theta}\right)}{\hat{\theta}}\right) \geqslant 0$$

substitute $\frac{r(\hat{\theta})+t(\hat{\theta})}{\hat{\theta}} = \frac{p(\hat{\theta})t(\hat{\theta})+g(\hat{\theta})}{y(R,T)} = \dot{r}\left(\hat{\theta}\right)$ from (24),to obtain (25). ■

It must also be the case that each university satisfies its budget constraint:

$$\frac{y(R,T)}{\theta}\left(r(\theta) + t(\theta)\right) - g(\theta) - p(\theta)t(\theta) = 0 \qquad \theta \in \left[\underline{\theta}, \bar{\theta}\right]$$

Moreover, the number of students must be non-negative, and cannot exceed 1. With regard to the latter, note that, by virtue of Assumption 2, if the number of students from a local education market were 1, then the total utility form individuals in that market could be increased simply by stopping the students with the highest cost of effort a, from attending university, and so a situation were there are some $\theta \in \left[0, \bar{\theta}\right]$ were $t(\theta) = 1$ cannot be an equilibrium, and the constraint $t(\theta) \leqslant 1$ can be omitted.

Having described the instruments and derived the constraints, to complete the statement of the government problem, we present formally the government's objective function.

Proposition 3 *The government's objective function can be written as:*

$$\int_{\underline{\theta}}^{\bar{\theta}}\left(\left(\Delta(T) - p(\theta)\right)t(\theta) - \int_{a_{\min}}^{\Phi^{-1}(t(\theta))}\alpha(a)\phi(a)\,da - (1+\lambda)g(\theta)\right)f(\theta)\,d\theta \qquad (29)$$
$$+ w(R) + x_U(T)$$

Proof. Consider local labour market θ. The total pre-tax utility is:

$$\int_{a_{\min}}^{\Phi^{-1}(t(\theta))}\left(x(T) + w(R) - p(\theta) - \alpha(a) - \tau\right)\phi(a)\,da + \left(1 - t(\theta)\right)\left(x_U(T) + w(R) - \tau\right)$$

where the first term is the total utility of the individuals who go to university, and the second the total utility of those who work in the unskilled labour market. Rearrange to obtain

$$(\Delta(T) - p(\theta))\, t(\theta) - \int_{a_{\min}}^{\Phi^{-1}(t(\theta))} \alpha(a)\, \phi(a)\, da + w(R) + x_U(T) - \tau \tag{30}$$

integrating for $\theta > \underline{\theta}$, using the fact that $(1+\lambda)\int_{\underline{\theta}}^{\bar{\theta}} g(\theta)\, f(\theta)\, d\theta = \tau$ (the total tax paid equals the total value of the subsidies given by the government to the university sector increased by the deadweight loss costs of taxation per unit of tax raised) and rearranging gives (29). ■

The aggregate income from university of type θ is given by (30) in the proof and has a natural interpretation: before tax, all potential students receive utility $y_U(R,T)$ at least; of the potential students $t(\theta)$ do go to university, and receive an *additional* income, net of tuition fee, equal to $\Delta(T) - p(\theta)$: this is the second term in (30). In addition, those whose type is a incur a disutility of effort $\alpha(a)$, and so the aggregate disutility is given by the third term in (30).

In the language of optimal control analysis, the problem can be written as a free initial time optimal control problem with R and T as parameters; the integral constraint (3), (4) and (22) are re-written as state constraints (Leonard and van Long, 1992, p 190), with $r_0(\theta)$, $t_0(\theta)$ and $g_0(\theta)$ as auxiliary variables.

$$\max_{\substack{p(\theta),t(\theta),r(\theta),\\ g(\theta),R,T,\underline{\theta}}} \int_{\underline{\theta}}^{\bar{\theta}} \left((\Delta(T) - p(\theta))\, t(\theta) - \int_{a_{\min}}^{\Phi^{-1}(t(\theta))} \alpha(a)\, \phi(a)\, da - (1+\lambda)\, g(\theta) \right) f(\theta)\, d\theta \tag{31}$$

$$+ w(R) + x_U(T)$$

$$\text{s.t.: } \dot{r}(\theta) = \frac{p(\theta)\, t(\theta) + g(\theta)}{y(R,T)} \qquad r(\underline{\theta}) = 0 \qquad r(\bar{\theta}) \text{ free} \tag{32}$$

$$\frac{y(R,T)}{\theta}(r(\theta) + t(\theta)) - g(\theta) - p(\theta)\, t(\theta) = 0 \qquad \theta \in [\underline{\theta}, \bar{\theta}] \tag{33}$$

$$\Delta(T) - p(\theta) - \alpha\left(\Phi^{-1}(t(\theta))\right) \geqslant 0 \qquad \theta \in [\underline{\theta}, \bar{\theta}] \tag{34}$$

$$\dot{r}_0(\theta) = r(\theta)\, f(\theta); \qquad r_0(\underline{\theta}) = 0 \qquad r_0(\bar{\theta}) = R \tag{35}$$

$$t(\theta) \geqslant 0 \tag{36}$$

$$\dot{g}_0(\theta) = g(\theta)\, f(\theta); \qquad g_0(\underline{\theta}) = 0 \qquad y_U(R,T) - g_0(\bar{\theta}) \geqslant 0 \tag{37}$$

$$\dot{t}_0(\theta) = t(\theta)\, f(\theta); \qquad t_0(\underline{\theta}) = 0 \qquad t_0(\bar{\theta}) = T \tag{38}$$

The next proposition describes the government's optimal policy. Let

$$\sigma(R,T,N;\theta) = \Delta(T) - \frac{y(R,T)}{\theta}\left(1 - \frac{(1-F(\theta))}{f(\theta)\,\theta}\right) - w'(R)\left(\frac{1}{1+\lambda} - N\right)\frac{\int_{\theta}^{\bar{\theta}} \tilde{\theta} f(\tilde{\theta})\, d\tilde{\theta}}{f(\theta)\,\theta^2}$$
$$+ x_U'(T)\left(\frac{1}{1+\lambda} - T\right) - x'(T)(T-N) \tag{39}$$

and let the vector $(R, T, \underline{\theta}, N)$ simultaneously satisfy the following conditions

$$T = \int_{\underline{\theta}}^{\bar{\theta}} z_{\frac{\lambda}{1+\lambda}}^{-1} \left(\sigma\left(R, T, N; \theta\right)\right) f\left(\theta\right) d\theta \tag{40}$$

$$\alpha\left(a_{\min}\right) = \sigma\left(R, T, N; \underline{\theta}\right) \tag{41}$$

$$R = \int_{\underline{\theta}}^{\bar{\theta}} \theta \int_{\underline{\theta}}^{\theta} \frac{z_{\frac{\lambda}{1+\lambda}}^{-1}\left(\sigma\left(R, T, N; \tilde{\theta}\right)\right)}{\tilde{\theta}^2} d\tilde{\theta} f\left(\theta\right) d\theta \tag{42}$$

$$N = \int_{\underline{\theta}}^{\bar{\theta}} \left(\frac{z_{\frac{\lambda}{1+\lambda}}^{-1}\left(\sigma\left(R, T, N; \theta\right)\right)}{\theta} + \int_{\underline{\theta}}^{\theta} \frac{z_{\frac{\lambda}{1+\lambda}}^{-1}\left(\sigma\left(R, T, N; \tilde{\theta}\right)\right)}{\tilde{\theta}^2} d\tilde{\theta} \right) f\left(\theta\right) d\theta \tag{43}$$

Proposition 4 *Let*

$$\frac{w'\left(R\right)}{y\left(R, T\right)} \left(\frac{1}{1+\lambda} - N\right) > \frac{1 - F\left(\theta\right)}{\int_{\theta}^{\bar{\theta}} \tilde{\theta} f\left(\tilde{\theta}\right) d\tilde{\theta}} \tag{44}$$

for every $\theta \in \left[\underline{\theta}, \bar{\theta}\right]$. *If, at the solution to problem (31)-(38) constraint (22) is slack, then we have:*

$$t\left(\theta\right) = z_{\frac{\lambda}{1+\lambda}}^{-1} \left(\sigma\left(R, T, N; \theta\right)\right) \tag{45}$$

$$p\left(\theta\right) = \Delta(T) - \alpha\left(\Phi^{-1}\left(z_{\frac{\lambda}{1+\lambda}}^{-1}\left(\sigma\left(R, T, N; \theta\right)\right)\right)\right) \tag{46}$$

$$r\left(\theta\right) = \theta \int_{\underline{\theta}}^{\theta} \frac{z_{\frac{\lambda}{1+\lambda}}^{-1}\left(\sigma\left(R, T, N; \tilde{\theta}\right)\right)}{\tilde{\theta}^2} d\tilde{\theta} \tag{47}$$

Proof. Begin by constructing the Lagrangean for (31)-(38):

$$\mathcal{L} = \left(\left(\Delta(T) - p\left(\theta\right)\right) t\left(\theta\right) - \int_{a_{\min}}^{\Phi^{-1}(t(\theta))} \alpha\left(a\right) \phi\left(a\right) da - \left(1+\lambda\right) g\left(\theta\right) \right) f\left(\theta\right) \tag{48}$$
$$+ \mu\left(\theta\right) \delta \frac{p\left(\theta\right) t\left(\theta\right) + g\left(\theta\right)}{y\left(R, T\right)} + \beta\left(\theta\right) \left[\frac{y\left(R, T\right)}{\theta} \left(r\left(\theta\right) + t\left(\theta\right)\right) - g\left(\theta\right) - p\left(\theta\right) t\left(\theta\right)\right]$$
$$+ \tau\left(\theta\right) \left[\Delta(T) - \alpha\left(\Phi^{-1}\left(t\left(\theta\right)\right)\right) - p\left(\theta\right)\right] + \eta\left(\theta\right) t\left(\theta\right) + \left(\gamma g\left(\theta\right) + \rho r\left(\theta\right) + \xi t\left(\theta\right)\right) f\left(\theta\right)$$

Where $\beta\left(\theta\right)$, $\tau\left(\theta\right)$, $\eta\left(\theta\right)$, are the Lagrange multipliers for constraints (33), (34), (36), respectively and $\mu\left(\theta\right)$, γ, ρ, and ξ are the Pontagryin multipliers for the state variables in constraints (32), (37), (35), (38) respectively. To simplify the analysis of the perfect information case, we have multiplied the incentive compatibility constraint (32) by an indicator $\delta \in \{0, 1\}$, with $\delta = 1$ for the imperfect information case, and $\delta = 0$ for the case in which the government can costlessly observe the type of the university. We have the following first order conditions:

$$-\frac{\partial \mathcal{L}}{\partial r(\theta)} = \delta \dot{\mu}(\theta) = -\beta(\theta)\frac{y(R,T)}{\theta} - \rho f(\theta) \tag{49}$$

$$\frac{\partial \mathcal{L}}{\partial g(\theta)} = \frac{\delta \mu(\theta)}{y(R,T)} - \beta(\theta) - (1+\lambda-\gamma) f(\theta) = 0 \tag{50}$$

$$\frac{\partial \mathcal{L}}{\partial p(\theta)} = -t(\theta) f(\theta) + \frac{\delta \mu(\theta) t(\theta)}{y(R,T)} - \beta(\theta) t(\theta) - \tau(\theta) = 0 \tag{51}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial t(\theta)} &= \left[\Delta(T) - p(\theta) - \alpha\left(\Phi^{-1}(t(\theta))\right)\right] f(\theta) + \frac{\delta \mu(\theta) p(\theta)}{y(R,T)} \\ &\quad + \beta(\theta)\left(\frac{y(R,T)}{\theta} - p(\theta)\right) - \tau(\theta)\frac{\alpha'\left(\Phi^{-1}(t(\theta))\right)}{\phi\left(\Phi^{-1}(t(\theta))\right)} + \xi f(\theta) + \eta(\theta) = 0\end{aligned} \tag{52}$$

and, for T, R, and $\underline{\theta}$ (Leonard and van Long, 1992, Theorem 7.11.1, p 255):

$$\xi = x'_U(T) + \int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial \mathcal{L}(\theta)}{\partial T} d\theta \tag{53}$$

$$\rho = w'(R) + \int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial \mathcal{L}(\theta)}{\partial R} d\theta \tag{54}$$

$$\mathcal{L}(\underline{\theta}) = 0 \tag{55}$$

Derive $\beta(\theta)$ from (50):

$$\beta(\theta) = \frac{\delta \mu(\theta)}{y(R,T)} - (1+\lambda-\gamma) f(\theta) \tag{56}$$

into (49):

$$\delta \dot{\mu}(\theta) = -\left(\frac{\delta \mu(\theta)}{y(R,T)} - (1+\lambda-\gamma) f(\theta)\right)\frac{y(R,T)}{\theta} - \rho f(\theta)$$

$$\delta \dot{\mu}(\theta) = -\frac{\delta \mu(\theta)}{\theta} + (1+\lambda-\gamma) y(R,T)\frac{f(\theta)}{\theta} - \rho f(\theta)$$

and so, for $\delta = 1$, we get the two differential equations:

$$\dot{\mu}(\theta) = -\frac{\mu(\theta)}{\theta} + (1+\lambda) y(R,T)\frac{f(\theta)}{\theta} - \rho f(\theta) \qquad \mu(\underline{\theta}) \text{ free} \qquad \mu\left(\bar{\theta}\right) = 0$$

$$\dot{r}(\theta) = \frac{p(\theta) t(\theta) + g(\theta)}{y(R,T)} \qquad r(\underline{\theta}) = 0 \qquad r\left(\bar{\theta}\right) \text{ free}$$

the first has solution

$$\mu(\theta) = -\frac{\rho \int_{\theta}^{\bar{\theta}} \tilde{\theta} f\left(\tilde{\theta}\right) d\tilde{\theta} - (1-F(\theta))(1+\lambda-\gamma) y(R,T)}{\theta}$$

next substitute (56) into (51), to obtain:

$$\tau(\theta) = (\lambda-\gamma) t(\theta) f(\theta) \tag{57}$$

If constraint (22) is slack, then constraint (37) is satisfied at the solution found for the problem without it, and γ can be set to 0 (if this were not the case, then γ would be positive, and it would correspond to an increase in the shadow cost of public funds). Then this implies that $\tau(\theta) > 0$ if $\lambda > 0$ and $t(\theta) > 0$, and so (34) holds as an equality. We can substitute $p(\theta) = \Delta(T) - \alpha\left(\Phi^{-1}(t(\theta))\right)$, $\beta(\theta)$, $\gamma = 0$ and $\tau(\theta)$ from (57) into (52) and re-arrange:

$$\frac{\partial \mathcal{L}}{\partial t(\theta)} = \frac{\delta\mu(\theta)\, p(\theta)}{y(R,T)} + \left(\frac{\delta\mu(\theta)}{y(R,T)} - (1+\lambda) f(\theta)\right)\frac{y(R,T)}{\theta} -$$

$$\left(\frac{\delta\mu(\theta)}{y(R,T)} - (1+\lambda) f(\theta)\right) p(\theta) - \lambda t(\theta) f(\theta) \frac{\alpha'\left(\Phi^{-1}(t(\theta))\right)}{\phi\left(\Phi^{-1}(t(\theta))\right)} + \xi f(\theta) + \eta(\theta) = 0$$

$$\alpha\left(\Phi^{-1}(t(\theta))\right) = \frac{\delta\mu(\theta)}{(1+\lambda) f(\theta)\,\theta} - \frac{y(R,T)}{\theta} + \Delta(T) - \frac{\lambda}{1+\lambda}\frac{\alpha'\left(\Phi^{-1}(t(\theta))\right) t(\theta)}{\phi\left(\Phi^{-1}(t(\theta))\right)} + \frac{\xi}{1+\lambda} + \eta(\theta) \tag{58}$$

substitute $\mu(\theta)$ to obtain:

$$z_{\frac{\lambda}{1+\lambda}}(t(\theta)) = \Delta(T) - \frac{y(R,T)}{\theta} + \delta\left(\frac{-\frac{\rho\int_{\theta}^{\bar{\theta}} \tilde{\theta} f(\tilde{\theta})\, d\tilde{\theta} - (1-F(\theta))(1+\lambda) y(R,T)}{\theta}}{(1+\lambda) f(\theta)\,\theta}\right) + \frac{\xi}{1+\lambda} + \eta(\theta) \tag{59}$$

Lemma 2 *Let*

$$\frac{\rho}{(1+\lambda)\, y(R,T)} > \frac{1-F(\theta)}{\int_{\theta}^{\bar{\theta}} \tilde{\theta} f\left(\tilde{\theta}\right) d\tilde{\theta}}$$

for every $\theta \in \left[\underline{\theta}, \bar{\theta}\right]$. *Then the RHS of (59) is increasing in* θ.

Proof. The statement is true for $\delta = 0$. Let therefore $\delta = 1$. The derivative of the RHS of (59) is given by:

$$\frac{1}{\theta}\left(\frac{\rho}{(1+\lambda)} + \frac{f'(\theta)\,\theta + 2f(\theta)}{f(\theta)^2\,\theta^2}\left(\frac{\rho}{1+\lambda}\int_{\theta}^{\bar{\theta}} \tilde{\theta} f\left(\tilde{\theta}\right) d\tilde{\theta} - y(R,T)(1-F(\theta))\right)\right)$$

with the assumption that $\frac{d}{d\theta}\left(\frac{1-F(\theta)}{f(\theta)}\right) < 0$, the term $f'(\theta)\,\theta + 2f(\theta)$ is positive: to see this write $\frac{d}{d\theta}\left(\frac{1-F(\theta)}{f(\theta)}\right) > 0$ as $f'(\theta) > \frac{f(\theta)^2}{1-F(\theta)}$ and we have $f'(\theta)\,\theta + 2f(\theta) > \frac{f(\theta)^2\theta}{1-F(\theta)} + 2f(\theta) > 0$. The assumption in the statement of the Lemma implies that the term that $\left(\frac{\rho}{1+\lambda}\int_{\theta}^{\bar{\theta}} \tilde{\theta} f\left(\tilde{\theta}\right) d\tilde{\theta} - y(R,T)(1-F(\theta))\right)$ is positive. ■

Because of the complementarity slackness, if $t(\theta)$ is positive, then $\eta(\theta) = 0$. Since z_k is increasing, given that the expression on the RHS of (59) is increasing in θ, then $t(\theta)$ is itself increasing, which implies that (25) holds, and moreover that there is a threshold value of θ, call it θ_0, such that $t(\theta) > 0$ if and only if $\theta > \theta_0$.The threshold θ_0 is given by the solution in θ of

$$\alpha(a_{\min}) = \Delta(T) - \frac{y(R,T)}{\theta} - \frac{\frac{\int_{\theta}^{\bar{\theta}} \tilde{\theta} f(\tilde{\theta})\, d\tilde{\theta}}{f(\theta)\theta^2}\rho - \xi}{1+\lambda} + \frac{(1-F(\theta))}{f(\theta)\,\theta^2} y(R,T)$$

To continue with the proof, we need to derive the values of the multipliers ξ and ρ. Expand the first order condition for ξ, (59):

$$\xi = x_U'(T) - (1+\lambda)\int_{\underline{\theta}}^{\bar{\theta}} \left(\frac{r(\theta)+t(\theta)}{\theta}\right) f(\theta)\, d\theta y_T(R,T) + \Delta'(T)(1+\lambda)\int_{\underline{\theta}}^{\bar{\theta}} t(\theta) f(\theta)\, d\theta$$

now recall the definition of N given in (6), and we can write:

$$\frac{\xi}{1+\lambda} = x_U'(T)\left(\frac{1}{1+\lambda} - T\right) + x'(T)(T-N) \tag{60}$$

Similarly for R: expanding (54) we have

$$\rho = w'(R)\left(1 - (1+\lambda)\int_{\underline{\theta}}^{\bar{\theta}} \frac{r(\theta)+t(\theta)}{\theta} f(\theta)\, d\theta\right)$$

$$\frac{\rho}{1+\lambda} = w'(R)\left(\frac{1}{1+\lambda} - N\right) \tag{61}$$

Now substitute (60) and (61) into (59), to obtain:

$$z_{\frac{\lambda}{1+\lambda}}(t(\theta)) = \Delta(T) - \frac{y(R,T)}{\theta}\left(1 - \delta\frac{(1-F(\theta))}{f(\theta)\theta}\right) -$$

$$\delta w'(R)\left(\frac{1}{1+\lambda} - N\right)\frac{\int_{\theta}^{\bar{\theta}} \tilde{\theta} f\left(\tilde{\theta}\right) d\tilde{\theta}}{f(\theta)\theta^2} + x_U'(T)\left(\frac{1}{1+\lambda} - T\right) + x'(T)(T-N)$$

Which, for $\delta = 1$ is (39). Now we want to establish that the lowest θ given in (41) is the also the value of θ such that $t > (\theta)$ (in a right neighbourhood). Expand (55). At $\underline{\theta}$, the terms in the square brackets in (48) and the term $\eta(\underline{\theta})\, t(\underline{\theta})$ are all 0 because of the slackness complementarity constraints. Also 0 is the term $\rho r(\underline{\theta}) f(\underline{\theta})$, because $r(\underline{\theta}) = 0$, And so:

$$\mathcal{L} = \left((\Delta(T) - p(\underline{\theta}))\, t(\underline{\theta}) - \int_{a_{\min}}^{\Phi^{-1}(t(\underline{\theta}))} \alpha(a)\phi(a)\, da - (1+\lambda)\, g(\underline{\theta})\right) f(\underline{\theta})$$

$$+\delta\mu(\underline{\theta})\frac{p(\underline{\theta})\, t(\underline{\theta}) + g(\underline{\theta})}{y(R,T)} + \xi t(\underline{\theta}) f(\underline{\theta}) = 0$$

since $r(\underline{\theta}) = 0$,

$$g(\underline{\theta}) = \left(\frac{y(R,T)}{\underline{\theta}} - p(\underline{\theta})\right) t(\underline{\theta})$$

and so:

$$\mathcal{L} = \left((\Delta(T) - p(\underline{\theta}))\, t(\underline{\theta}) - \int_{a_{\min}}^{\Phi^{-1}(t(\underline{\theta}))} \alpha(a)\phi(a)\, da - (1+\lambda)\left(\frac{y(R,T)}{\underline{\theta}} - p(\underline{\theta})\right) t(\underline{\theta})\right) f(\underline{\theta})$$

$$+\frac{\delta\mu(\underline{\theta})}{\underline{\theta}} t(\underline{\theta}) + \xi t(\underline{\theta}) f(\underline{\theta}) = 0$$

$$\mathcal{L} = \left[\Delta(T) - p(\underline{\theta}) - \frac{\int_{a_{\min}}^{\Phi^{-1}(t(\underline{\theta}))} \alpha(a)\phi(a)\, da}{t(\underline{\theta})} - (1+\lambda)\left(\frac{y(R,T)}{\underline{\theta}} - p(\underline{\theta})\right) + \frac{\delta\mu(\underline{\theta})}{f(\underline{\theta})\underline{\theta}} + \xi\right] t(\underline{\theta}) f(\underline{\theta}) = 0 \tag{62}$$

write (58) (with $\eta(\theta)=0$) as

$$\frac{\delta\mu(\theta)}{f(\theta)\theta}+\xi=\left(z_{\frac{\lambda}{1+\lambda}}(t(\theta))+\frac{y(R,T)}{\theta}-\Delta(T)\right)(1+\lambda)$$

and so (62) becomes:

$$\mathcal{L}=\left(z_{\lambda}(t(\underline{\theta}))\,t(\underline{\theta})-\int_{a_{\min}}^{\Phi^{-1}(t(\underline{\theta}))}\alpha(a)\,\phi(a)\,da\right)f(\underline{\theta})=0$$

This is 0 at $t(\underline{\theta})=0$, moreover

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial t(\underline{\theta})}&=\left(z_{\lambda}'(t(\underline{\theta}))\,t(\underline{\theta})+z_{\lambda}(t(\underline{\theta}))-\alpha\left(\Phi^{-1}(t(\underline{\theta}))\right)\right)f(\underline{\theta})\\&=\left[z_{\lambda}'(t(\underline{\theta}))\,t(\underline{\theta})+t(\underline{\theta})\,\lambda\frac{\alpha'\left(\Phi^{-1}(t(\underline{\theta}))\right)}{\phi\left(\Phi^{-1}(t(\underline{\theta}))\right)}\right]f(\underline{\theta})\end{aligned}$$

is positive: therefore, whenever $\mathcal{L}(t(\theta))=0$, it is increasing in $t(\theta)$, and therefore there can only be one value where this happens, which is $t(\underline{\theta})=0$. What remains to be established is (46) and (47). The first follows from (34), for the second, start from the following equality:

$$r(\theta)=[p(\theta)\,t(\theta)+g(\theta)]\,\frac{\theta}{y(R,T)}-t(\theta)=\int_{\underline{\theta}}^{\theta}\frac{p\left(\tilde{\theta}\right)t\left(\tilde{\theta}\right)+g\left(\tilde{\theta}\right)}{y(R,T)}d\tilde{\theta}$$

and erite it as:

$$g(\theta)\,\theta=\int_{\underline{\theta}}^{\theta}g\left(\tilde{\theta}\right)d\tilde{\theta}+\int_{\underline{\theta}}^{\theta}p\left(\tilde{\theta}\right)t\left(\tilde{\theta}\right)d\tilde{\theta}+t(\theta)\,y(R,T)-p(\theta)\,t(\theta)\,\theta$$

differentiate both sides with respect to θ, and divide by θ:

$$\dot{g}(\theta)=\frac{\dot{t}(\theta)}{\theta}y(R,T)-\frac{d(p(\theta)\,t(\theta))}{d\theta}$$

Integrate both side in the above to get

$$g(\theta)=\left(\frac{t(\theta)}{\theta}+\int_{\underline{\theta}}^{\theta}\frac{t\left(\tilde{\theta}\right)}{\tilde{\theta}^2}d\tilde{\theta}\right)y(R,T)-p(\theta)\,t(\theta)$$

and therefore:

$$\begin{aligned}r(\theta)&=\left[p(\theta)\,t(\theta)+\left(\frac{t(\theta)}{\theta}+\int_{\underline{\theta}}^{\theta}\frac{t\left(\tilde{\theta}\right)}{\tilde{\theta}^2}d\tilde{\theta}\right)y(R,T)-p(\theta)\,t(\theta)\right]\frac{\theta}{y(R,T)}-t(\theta)\\&=\theta\int_{\underline{\theta}}^{\theta}\frac{t\left(\tilde{\theta}\right)}{\tilde{\theta}^2}d\tilde{\theta}\end{aligned}$$

From which (47) is derived, which, integrated gives (42). Similarly, integration of $t(\theta)$ and $\frac{r(\theta)+t(\theta)}{\theta}$ gives the values of T and N in (40) and (43); this completes the proof. ■

The requirement that constraint (22) is slack implies that the total tax needed to finance the preferred level of tertiary education is not so high as to require more than the total income that the economy can obtain with no university sector. This is a plausible requirement.

To get a handle on the solution, begin by considering the perfect information case: we set $\delta = 0$ and $\eta(\theta) = 0$ in (59). As will be apparent from the proof, in the perfect information case, it is necessary to impose a limit to the amount of research that can be carried out in each university; let $r_{\max}$ be such maximum; moreover, let it be "high". The next proposition describes the government's optimal policy if it knew perfectly the type of each university. Let

$$\sigma_p(R,T,N;\theta) = \Delta(T) - \frac{y(R,T)}{\theta} + x_U'(T)\left(\frac{1}{1+\lambda} - T\right) + x'(T)(T-N) \tag{63}$$

and let the vector $(R,T,\underline{\theta},N)$ simultaneously satisfy the following conditions

$$T = \int_{\underline{\theta}}^{\bar{\theta}} z^{-1}_{\frac{\lambda}{1+\lambda}}\left(\sigma_p(R,T,N;\theta)\right) f(\theta)\, d\theta \tag{64}$$

$$\alpha(a_{\min}) = z^{-1}_{\frac{\lambda}{1+\lambda}}\left(\sigma_p(R,T,N;\theta)\right) \tag{65}$$

$$N = r_{\max}\int_{\frac{1+\lambda}{1-(1+\lambda)N}\frac{y(R,T)}{w'(R)}}^{\bar{\theta}} \frac{f(\theta)}{\theta} d\theta + \int_{\underline{\theta}}^{\bar{\theta}} \frac{z^{-1}_{\frac{\lambda}{1+\lambda}}\left(\sigma_p(R,T,N;\theta)\right)}{\theta^2} f(\theta)\, d\theta \tag{66}$$

$$R = \left(1 - F\left(\frac{1+\lambda}{1-(1+\lambda)N}\frac{y(R,T)}{w'(R)}\right)\right) r_{\max} \tag{67}$$

Proposition 5 *The optimal policy of the government with perfect information satisfies:*

$$t(\theta) = z^{-1}_{\frac{\lambda}{1+\lambda}}\left(\sigma_p(R,T,N;\theta)\right) \tag{68}$$

$$p(\theta) = \Delta(T) - \alpha\left(\Phi^{-1}\left(z^{-1}_{\frac{\lambda}{1+\lambda}}\left(\sigma_p(R,T,N;\theta)\right)\right)\right) \tag{69}$$

$$r(\theta) = \begin{cases} r_{\max} & \textit{for} \quad \theta \geqslant \frac{1+\lambda}{1-(1+\lambda)N}\frac{y(R,T)}{w'(R)} \\ 0 & \textit{for} \quad \theta \in \left[\underline{\theta}, \frac{1+\lambda}{1-(1+\lambda)N}\frac{y(R,T)}{w'(R)}\right) \end{cases} \tag{70}$$

Proof. Impose $\delta = 0$ in the proof of Proposition 4. (50) becomes:

$$-\beta(\theta) - (1+\lambda) f(\theta) = 0$$

As before, we have set $\gamma = 0$. Because of the possibility that the optimum is a corner solution, (49) must be replaced by

$$r = 0 \text{ if } \frac{\partial \mathcal{L}}{\partial g(\theta)} < 0$$
$$\frac{\partial \mathcal{L}}{\partial g(\theta)} = 0 \text{ for } r \in (0, r_{\max})$$
$$r = r_{\max} \text{ if } \frac{\partial \mathcal{L}}{\partial g(\theta)} = 0$$

This implies that

$$
\begin{aligned}
r = 0 \text{ if } \theta &< \frac{1+\lambda}{\rho} y\left(R, T\right) \\
r = r_{\max} \text{ if } \theta &> \frac{1+\lambda}{\rho} y\left(R, T\right)
\end{aligned}
$$

that is the solution is "bang-bang". (51) becomes

$$
\begin{aligned}
-t\left(\theta\right) f\left(\theta\right) - \beta\left(\theta\right) t\left(\theta\right) - \tau\left(\theta\right) &= 0 \\
\lambda f\left(\theta\right) t\left(\theta\right) &= \tau\left(\theta\right)
\end{aligned}
$$

as before. (59) in turn becomes:

$$
z_{\frac{\lambda}{1+\lambda}}\left(t\left(\theta\right)\right) = \Delta(T) - \frac{y\left(R, T\right)}{\theta} + \frac{\xi}{1+\lambda}
$$

and the last university active is given by the solution in θ of

$$
\alpha\left(a_{\min}\right) = \Delta(T) - \frac{y\left(R, T\right)}{\theta} + \frac{\xi}{1+\lambda}
$$

The multipliers ρ and ξ are still given by (61) and (60), and substituting the value of ρ into the above, the Proposition is obtained. ∎

The proviso made above that $r_{\max}$ is high ensures that, given that all the research that the government wants to carry out can be carried out by fewer universities than $(1 - F(\underline{\theta}))$. Notice that the hypothesis of an exogenous upper bound on research expenditure, implies a cost function where the marginal cost is 0 up to this bound, and increases to $+\infty$ beyond; with less extreme forms of decreasing returns to research expenditure, the principle would remain that research is concentrated in the most productive universities.

We begin the comparison by noting that unlike the private market case, and the case of imperfect information, with perfect information there are "teaching only" universities. Attendance to these universities, moreover, is charged more than the cost, so that more research can be carried out in the universities that do do research. This follows from the determination of the government subsidy $g\left(\theta\right)$, which is given by

$$
g\left(\theta\right) = \begin{cases} \frac{r_{\max}}{\theta} y\left(R, t\right) - \frac{\alpha'\left(\Phi^{-1}(t)\right)}{\phi\left(\Phi^{-1}(t)\right)} t\left(\theta\right)^2 & \text{for} \quad \theta \geqslant \frac{1+\lambda}{1-(1+\lambda)N} \frac{y(R,T)}{w'(R)} \\ -\frac{\alpha'\left(\Phi^{-1}(t)\right)}{\phi\left(\Phi^{-1}(t)\right)} t\left(\theta\right)^2 & \text{for} \quad \theta \in \left[\underline{\theta}, \frac{1+\lambda}{1-(1+\lambda)N} \frac{y(R,T)}{w'(R)}\right) \end{cases}
$$

The situation is sketched in Figure 3. The solid curve indicates the amount of research with private provision; the dashed curve with government intervention with imperfect information; as shown below, if the total research is the same across regimes, the dashed line is above the solid one for high θ and vice versa as depicted. They are both 0 only for the least productive university that recruits any students. By contrast, the dotted line plots the amount of research carried out by universities in a regime

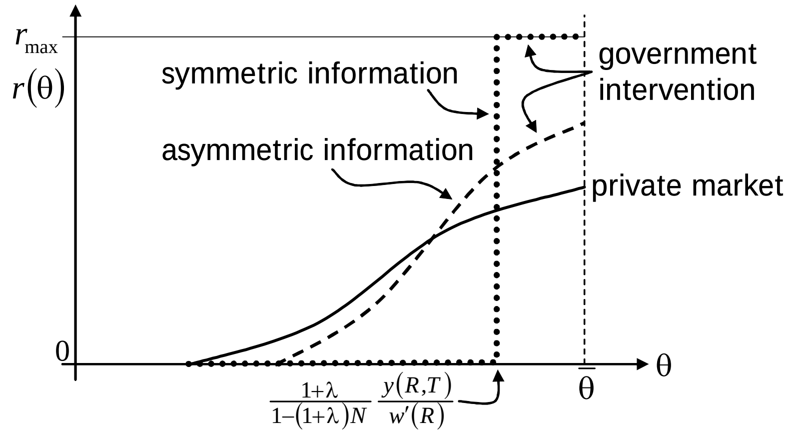

Figure 3: The amount of research with private and government provision.

where the government can observe their different productivity: there is a threshold value of θ above which all universities carry out the maximum amount of research technological feasible, and below which carry out no research, and are "teaching only" institutions. With perfect information, the government can separate teaching and research: the former is chosen on the basis of efficiency consideration alone, in each local market to the point where marginal benefits equal marginal costs, adjusted by the externality that graduates in one market bestow on the global market captured by the last two terms in (63). On the other hand, total research, R, is chosen at the optimally global (see (67)), given by the amount such that global marginal benefits equal global marginal costs, and then this amount of research is allocated to the university in the most cost effective way.

Consider next the number of students. The following result is helpful in the comparison between the two regimes.

Lemma 3 *For every $k \in [0,1]$, $z_k(0) = \alpha(a_{\min})$. Let $k_1 > k_0$; hen, for every $t > 0$, we have $z_{k_1}(t) > z_{k_0}(t)$. If moreover we have $\alpha''(a) \geqslant 0$, we also have $z'_{k_1}(t) > z'_{k_0}(t)$.*

Proof. The first statement follows immediately from (16). Next, take $z_{k_1}(t) - z_{k_0}(t) = (k_1 - k_0)\, t \frac{\alpha'(\Phi^{-1}(t))}{\phi(\Phi^{-1}(t))}$. Consider now the last statement. We can write

$$\begin{aligned}\frac{dz'_k(t)}{dk} &= \frac{\alpha'(\cdot)\,t}{\phi(\cdot)^2}\left(\frac{\phi(\cdot)}{t}\frac{1+k}{k} + \left(\frac{\alpha''(\cdot)}{\alpha'(\cdot)} - \frac{\phi'(\cdot)}{\phi(\cdot)}\right)\right) - \frac{k\alpha'(\cdot)\,t}{\phi(\cdot)^2}\frac{\phi(\cdot)}{t}\frac{1}{k^2} \\ &= \frac{\alpha'(\cdot)\,t}{\phi(\cdot)^2}\left(\frac{\phi(\cdot)}{t} + \left(\frac{\alpha''(\cdot)}{\alpha'(\cdot)} - \frac{\phi'(\cdot)}{\phi(\cdot)}\right)\right) > 0\end{aligned}$$

This is equivalent to (11), evaluated when $\frac{1+k}{k} = 1$, which implies $k \to +\infty$, and the penultimate term in (11) becomes 0. ■

We begin with the following (we use the superscript *pr* and *pu* in the obvious way).

Corollary 3 *If $x'_U(T) > 0$ or $x'(T) < 0$, there exists a threshold value of λ, λ_0 such that for $\lambda > \lambda_0$ we have $t^{pr}(\theta) > t^{pu}(\theta)$, and $T^{pr} > T^{pu}$*

Proof. If $\lambda \to +\infty$, then $z_{\frac{\lambda}{1+\lambda}}(t)$ tens to $z_1(t)$, that is, the increasing functions on the LHS of (16) and (59) coincide with private and government provision. The RHS, however, is lower with government provision: therefore, for every active university, there are fewer students under public control. Moreover, in this case, the same argument used for the private provision case illustrates that the "last" university in the market has higher type with government intervention, and the statement follows. ■

The intuition is straightforward: when the shadow cost of public funds is sufficiently high, the government *taxes* university attendance. Notice that this is independent of the extent of the research externality: this is fully captured by the private market, in the sense that given the high cost of public funds, the government does not want to push provision beyond the private level, even though this would increase aggregate income, as to do so would also increase the deadweight loss cost of taxation.

While this effect is interesting, the aim of the paper is not to study the potential benefits of taxing education provision, but to determine the balance between teaching and research, and to concentrate on the latter, we isolate the former by ensuring that, with perfect information and infinite shadow cost of public funds, when $\lambda \to +\infty$ private and government provision are exactly equivalent. A simple way of ensuring this is by imposing

Assumption 4 *For every $T \in [0, 1]$, $x'_U(T) = x'(T) = 0$.*

Clearly, the amount of research could be different in the three regimes, and it is not necessarily the case that relative amount of research in the three cases can be determined a priori. To see this, consider that relative to benchmark case of the social optimum obtianed when the government has perfect information, the research in the private market can be higher or lower. Research carries both a benefit, the increase in labour market incomes, and a cost, the portion of the salary of the academic working in universities that is devoted to research activities, that is subtracted to the income enhancing teaching. In general there is no presumption that a private university system will carry out too much or too little research: for $w'(R)$ sufficiently small, that is when research has no effect on aggregate income, but is purely a "hobby" for universities and their staff, it has only the function of inducing universities to carry out their teaching, and if the government had perfect information, so that it could choose the research level of each university, would choose an aggregate (and therefore each university's) level of research equal to 0.[14]

The comparison between regimes, the public and private, with the perfect information as the benchmark, can therefore conceptually be divided into two parts: the

[14] With the assumption that $w''(R) < 0$, which implies that $\frac{y(R,T)}{w'(R)}$ is increasing, the optimal R is 0 if $\frac{1+\lambda}{1-(1+\lambda)N} \frac{y(0,T)}{w'(0)} > 1$, see (67).

difference in the global level of research and teaching, and the difference in the distribution of these activities. To concentrate on the second, more microeconomic, aspect, we compare, in the rest of the section, the situations where the total amount of research, R is the same across regimes.

The following summarises the comparison.

Corollary 4 *Let Assumption 4 hold. If the total amount of research carried out is the same,then*

1. *the universities active with private provision are the same the a perfectly informed government would allow to operate;*
2. *with private provision each active university has fewer students than with a perfectly informed government;*
3. *the government information disadvantage reduces the number of active universities;*
4. *the government information disadvantage reduces the number of student at each university except the most productive.*
5. *compared with private provison, government intervention concentrates students in the most productive institutions: the higher (lower) productivity institutions have more (fewer) students than they would in a private system.*

The diagram of Figure 4 illustrates the situation. The horizontal axis measures the number of students. This, for university θ, is given by the intersection of the increasing line $z_{\frac{\lambda}{1+\lambda}}(t)$, with under government provision, and $z_1(t)$ for the private market, with the appropriate horizontal line:

$$\Delta(T) - \frac{y(R,T)}{\theta} \quad \text{for the private market or with perfect government information}$$

$$\Delta(T) - \frac{y(R,T)}{\theta} + \left(\frac{y(R,T)(1-F(\theta)) - w'(R)\left(\frac{1}{1+\lambda} - N\right)\int_{\theta}^{\bar{\theta}} \tilde{\theta} f(\tilde{\theta}) d\tilde{\theta}}{f(\theta)\theta^2}\right) \quad \text{with imperfect government information}$$

Since the amount of research is the same in the three regimes, the expression $\Delta(T) - \frac{y(R,T)}{\theta}$ is independent of T.

The term in the round brackets in the latter expression, call it $\zeta(\theta)$, is 0 at $\theta = \bar{\theta}$ and increases in θ. At the highest possible θ, the situation is represented on the LHS of Figure 4: the horizontal curve is the same in all three regimes, and therefore the number of students is lower with private provision than with government intervention, irrespective of whether the government has perfect or imperfect information (this is the standard "efficiency at the top" result). By Lemma 3, the curve z swings anticlockwise around the point $(0, \alpha(a_{\min}))$, and since $1 = \lim_{\lambda \to +\infty}$, it is lower under government provision for every finite value of λ. The number of students, with obvious mnemonics,

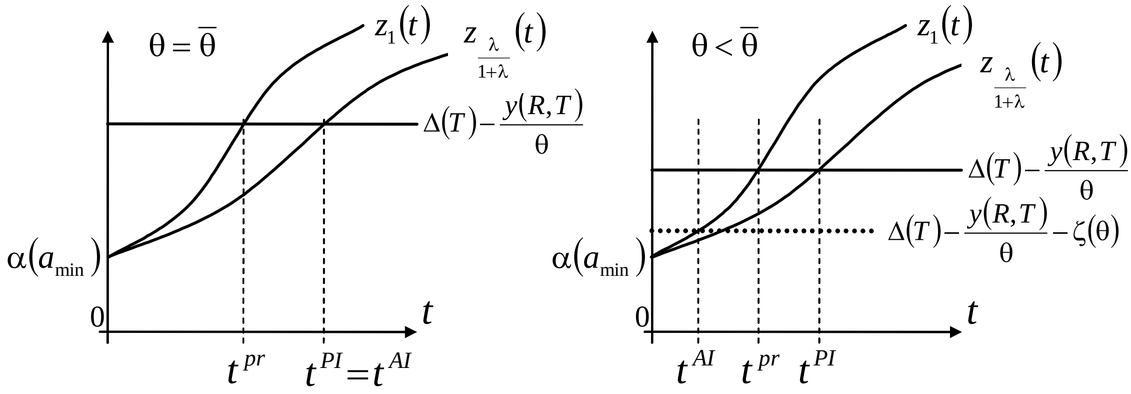

Figure 4: The Determination of the number of students.

is shown in the diagram as t^{pr} with private provision, by t^{PI} (respectively p^{AI}) with government provision under condition of perfect (respectively asymmetric) information. Point 2 in the Corollary is illustrated by the observation that $t^{pr} < t^{PI}$ in the LHS diagram in Figure 4. The RHS considers a lower θ. With lower θ the horizontal curve is lower, for all three regimes, than with $\theta = \bar{\theta}$, but is "more lower", as it were, when the government has imperfect information, as depicted by the dotted horizontal line. As the RHS diagram shows, for sufficiently low θ, we have that $t^{AI} < t^{pr}$ (with both obviously smaller than t^{PI}). As θ decreases further, the horizontal lines intersection with the curves $z_1(t)$ and $z_{\frac{\lambda}{1+\lambda}}(t)$ near the vertical axis, and the value of θ for which it reaches it is the type of the least productive university that does any teaching: clearly it happens for a lower level of θ when the government has imperfect information but for the same value of θ with the private market and with the first best government design.

Figure 5 present a sketch of the possible relationship between θ and the number of students, in the three regimes, analogously to Figure 3 for research. The dotted line, depicting the perfect information case, coincides with the dashed one, the asymmetric information case, at $\theta = \bar{\theta}$, and with the solid one, the private market case, at $\theta = \underline{\theta}$.

This is an efficiency results, teaching should be concentrated in the most efficient universities. The picture for research would be similar, given that the total amount of research is (by hypothesis) the same in the two regimes.

If the total amount of research R is different, then the horizontal curve in Figure 4 would have a different position in the three regimes, and the comparison would have to be made taking into account of the different position of this curve. In general, however, note that this horizontal curve shifts *down* when *more research* is carried out: there are fewer students when research is more valuable illustrating the trade-off between the two activities.

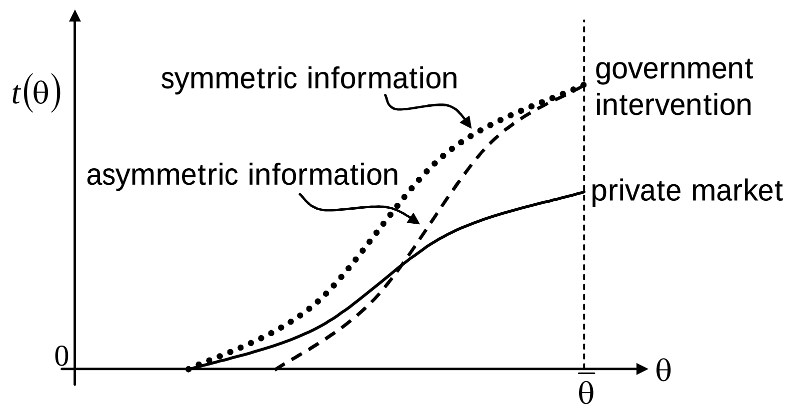

Figure 5: The number of students with private and government provision.

5 Extensions of the model.

We sketch here some possible extensions of the basic model presented above.

5.1 Demand variability

We have so far assumed that potential demand is fixed, i.e. that students do not transfer to an institution in a different local education market. This is clearly unrealistic, and in this section we show that little is gained, other than further complicating the algebra, by relacxing it assuming that students can move from their local education market to attend a university in another education market. A general and tractable way of allowing for this is to assume that the distribution of students potentially attending the university in a local education market depends on the tuition fee charged by the university: there is no objective measure of quality of the teaching, and therefore a student motivation will be determined by price only. Let therefore the distribution of potential students of each university be given by

$$\Phi_c\left(a;p\right)$$

where p is the tuition fee they have to pay. $\Phi_c\left(a;p\right)$ has density $\phi_c\left(a;p\right) = \Phi'_c\left(a;p\right)$, with monotonic hazard rate $\frac{\partial}{\partial a}\left(\frac{\Phi_c(a;p)}{\phi_c(a;p)}\right) > 0$. That students prefer lower fees can be captured by the assumption that if $p_1 > p_0$, then $\Phi_c\left(a;p_1\right)$ first order stochastically dominates $\Phi_c\left(a;p_0\right)$. Then repeating for the present case the analysis carried out in the paper, we have that case a university of type θ which chooses price $p\left(\theta\right)$ will enrol a number of student $t\left(\theta\right)$ satisfying:

$$t\left(\theta\right) = \Phi_c\left(\alpha^{-1}\left(\Delta(T) - p\left(\theta\right)\right), p\left(\theta\right)\right)$$

In this case we have that the relationship between t and p is given by (denoting by $\Phi_c^{-1}(t;p)$ the inverse of $\Phi_c(a;p)$ for given p:

$$\frac{dp}{dt} = -\frac{1}{\frac{\phi\left(\Phi_c^{-1}(t;p)\right)}{\alpha'\left(\Phi_c^{-1}(t;p)\right)} - \frac{\partial\Phi_c(t;p)}{\partial p}} < 0$$

since $\frac{\partial\Phi_c(t;p)}{\partial p} < 0$. The convenience of being able to define the function z_k independently of p is now lost, as the first order condition for the choice of t becomes:

$$\left(-\frac{t}{\frac{\phi\left(\Phi_c^{-1}(t;p)\right)}{\alpha'\left(\Phi_c^{-1}(t;p)\right)} - \frac{\partial\Phi_c(t;p)}{\partial p}} + \Delta(T) - \alpha\left(\Phi_c^{-1}(t;p)\right)\right)\frac{\theta}{y(R,T)} = 1$$

The analysis would yield similar results as obtained before; moreover, the link between number of students and ability level of the lowest ability student in a given university having being broken, it might yield the result that universities that have more students also have more able students. Since the focus here is on the link between research and teaching rather than on the structure of the university student body, we do not pursue this line of research.

5.2 Quality dependant earnings.

We have assumed so far that the type of the university attended by a student does not affect her earnings in the skilled labour market. In this section we sketch how the alternative assumption, that a graduate's income is given by $y(\theta, T, R)$ can be incorporated into the model without any qualitative change in the results.

6 Concluding remarks

This paper studies how a benevolent government should intervene in the university sector. Intervention is required by the existence of an externality in research, and by the fact that students are not perfectly mobile, and so it may happen that a fully private market misallocates university places to students and determines a suboptimal amount of research. We note in passing that we have not signed the externality: the model applies equally when research increases the productivity of workers, and when research diverts resources from more productive uses. We show that the private market would tends to have less concentration of resources: there are fewer and larger universities under the optimal public provision system. The information disadvantage of the government vis-à-vis the universities entails that the public provision system is nearer to the private market than the optimal (symmetry information) system would be. It is only in the latter that research only universities exist: both with private provision and in the optimal second best public university system all universities do both research and teaching.

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