Labour Markets in Depression: Worker Flows and Unemployment Dynamics in Greece, Portugal and Spain since the Financial crisis of 2008

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Abstract

In the wake of the Great Recession, a number of advanced economies witnessed a significant rise in their unemployment rates. A growing literature has studied the underlying causes of the cyclical dynamics of unemployment in the largest of these economies, notably the UK and the US. But, so far, no attention has been paid towards the behaviour of worker flows into and out of unemployment in those economies that experienced depression-like labour market conditions such as Greece, Portugal and Spain in the aftermath of the world financial and Euro zone crises. Indeed, we know little about how the behaviour of worker flows into and out of unemployment changes when extraordinary shocks hit the labour market. And our understanding of the propagation mechanisms in depressed labour markets is, at best, incomplete. Using Quarterly Labour Force Survey (LFS) data for Greece, Portugal and Spain, we evaluate the labour market conditions in the aftermath of the extraordinary shocks in comparison with normal recessions. We employ the seminal methodologies of Shimer (2012) and Elsby et al. (2009) to estimate the transition probabilities of worker flows into and out of unemployment and, thus, examine the salient characteristics of labour markets in depression. Our results uncover substantial heterogeneities in the significance of the separation and job finding rates across our sample and between business cycles. In turn, these heterogeneities can be traced to cross-country differences in labor market institutions and call for significant institutional reforms.

Keywords: Unemployment; Worker Flows; Job Finding Rate; Separation Rate.
JEL Classification: E24, E32, J63, J64.

1 Introduction

The Great Recession of 2008-2010 witnessed deterioration in the labour market conditions of most advanced economies. A good deal of research has already studied in a systematic way the dynamics behind the rise in unemployment in the largest of these economies, notably the UK and the USA. Elsby et al. (2010) and Shimer (2012) examine the contribution of the inflow and outflow rates to unemployment dynamics in the United States while Elsby et al. (2011), Smith (2011) and Gomes (2012) do the same for the United Kingdom. But, so far, no attention has been paid towards the behaviour of worker flows into and out of unemployment in those economies that have experienced unusually severe shocks and have endured double-digit and persistent unemployment rates such as Greece, Portugal and Spain in the wake of the world financial and Euro zone crises. How does the cyclical behaviour of worker flows change in times of major economic shocks? How do the separation and job finding rates behave under conditions of extraordinary shocks relative to their behaviour in previous normal recessions? Which labour market institutions govern the disparities in the observed behaviour of the flow rates? Is there room for policy intervention? The goal of this paper is to contribute towards addressing these questions.

To this end, the paper uses Eurostat's Quarterly Labour Force Survey data for Greece, Portugal and Spain from 1998Q1 to 2013Q4 and supplements them with the Spanish National Statistics Labour Force Survey data that stretch back to 1987Q2. It adopts and compares the methodologies of Shimer (2012) and Elsby et al. (2009) and, as in Elsby et al. (2013), it also allows the actual unemployment rate to deviate from its steady state value. The analysis reveals evident heterogeneities in the role of the separation and job finding rates across countries. For Greece, the contribution of the job finding rate is almost twice that of the separation rate. The cumulative contribution of the decline in the job finding rate for Spain has been stable across recessions with varying severity. Moreover, the results for Portugal show that, in contrast to Greece and Spain, the separation rate plays the dominant role in the dynamics of unemployment during the big recession.

The remainder of the paper is as follows. Section 2 offers a concise literature survey. Section 3 discusses the data sources and the methodology. Section 4 discusses and compares the key empirical findings in detail. And Section 5 draws conclusions.

2 Unemployment Dynamics: Empirical evidence from the UK and the US

The question of the cyclical behaviour of worker flows has recently attracted a great deal of attention. In his 2012 seminal paper, Shimer infers the inflow and outflow rates of unemployment through the development of a methodology that exploits the use of publicly available series from the US Current Population Survey for the period 1948 to 2010. He finds that "there are substantial fluctuations in unemployed workers' job finding probability at business cycle frequencies, while the probability a worker exits employment is comparatively acyclic", questioning therefore the conventional wisdom that high outflow rate from employment accompanies periods of economic downturns (see also Hall,2006).

Shimer (2012)'s analysis has been refined and extended by Fujita and Ramey (2009) and Elsby et al. (2009). They confirm the significant contribution of the procyclical job finding rate to the unemployment dynamics, yet in contrast to Shimer (2012)'s findings, their analysis reveals an important role played by the countercyclical inflows into unemployment. To be precise, they find that the separation rate is the main driver to the unemployment dynamics during the first phases of recessions whereas duration takes over in later periods. More importantly, they caution that neglecting the dynamic relationships between the job finding and separation rates results in an underestimation of the contribution of the separation rate, therefore concluding that both rates should be taken into consideration when trying to understand the dynamics behinds the surge in the unemployment rate. Elsby et al. (2010), arrive at the same conclusion in their study on US labour market fluctuations throughout the 2008-2010 recessions (See, among others,Pissarides,2013, for a detailed analysis of the labour market during recessions).

Petrongolo and Pissarides (2008), on the other hand, examine the contribution of inflows and outflows to the dynamics of unemployment in the United Kingdom, France and Spain by exploiting quarterly labour force surveys in addition to administrative data that record all workers who join or leave the unemployment register during a time period. Their findings show substantial difference in the contribution of the job finding and separation rates for the three economies. These disparities are stemming from the fact that the labour market for each country is governed by different institutions.

Elsby et al. (2011) and Smith (2011) have examined the contribution of the inflow and outflow rates to unemployment dynamics in the United Kingdom. The former uses data on recalled labour market status available from the Labour Force Survey microdata while the latter uses data from the British Household Panel Survey. Their results show that the rise in unemployment during the previous UK recessions was mainly caused by the separation rate. This conclusion is consistent with Elsby and Smith (2010) in their study of the 2008-10 recession. More recently, Elsby et al. (2013) make use of publicly available yearly data to infer measures of the job finding and separation rate for a total of fourteen countries. They reveal stark differences in the flow rates into and out of unemployment between the Anglo-Saxon, Nordic and European economies with these rates being significantly lower in Europe than those for the other part of the world. Specifically, they conclude that both hazard rates contribute evenly in explaining unemployment fluctuation within European and Nordic countries, claiming therefore that a complete examination of the underlying determinants behind these flow rates is required for a full understanding of the unemployment dynamics.

3 Data and Methodology

The paper uses the quarterly Labour Force Survey (LFS) data of the Eurostat for the three economies, which covers the period from 1998Q1 to 2013Q4. The data for Spain, however, extends back to 1987Q2 due to its availability from the National Statistical Service of Spain (INE). This includes data on the total number unemployed, employed and those less than three months unemployed. The series have been seasonally adjusted using Census X-12 subsequent to the estimation of the flow rates. The recession dates used later in the analysis are based on the classical business cycle method given that there is no reference chronology for the Euro zone countries business cycles.[1]

As with regards to the methodology, we closely follow that of Shimer (2012) in the computation of the separation and the job finding rate. The starting point is to assume that the number of unemployed evolve according to the subsequent identity

$$\frac{dU}{dt} = s_t E_t - f_t U_t \tag{1}$$

[1]The recession dates stressed in the text are insensitive to the choice of procedure used in the identification of the business cycle turning points. Similar to the Classical Business Cycle method using the ‘Quarters of Growth’ technique, Hamilton’s Markov Regime Switching method signals analogous recessions with minor differences pertaining to the starting quarter of each recession.

In the above equation, U_t denotes the stock of unemployment and equivalently E_t denotes the stock of employment, f_t represents the job finding rate and s_t the separation rate. We further assume a constant labour force (L). Specifically, we ignore non-participation, thus by substituting $E_t = L_t - U_t$ in the previous equation, we get

$$\frac{dU}{dt} + (s_t + f_t)U_t = s_t L_t \tag{2}$$

The first step would be to calculate the job finding rate, we assume that the stock of jobless people at period t is the sum of those who were previously classified as unemployed and could not find a job and short term unemployed (less than 3 months)

$$U_{t+1} = U_t - F_t U_t + U^s_{t+1} \tag{3}$$

Observing the above equation, it can be rewritten as

$$F_t = 1 - \frac{U_{t+1} - U^s_{t+1}}{U_t} \tag{4}$$

With this job finding probability, we can compute the associated job finding rate, assuming it follows a Poisson distribution

$$f_t = -\ln(1 - F_t) \tag{5}$$

Calculating the separation rate, on the other hand, is more complicated. Workers transition quickly between labour market states, a person may be reported as employed at time t and t+1 while he/she looses a job but manages to find another one in between. An issue well documented in the literature as 'Time aggregation bias'.[2] To correct this, we solve the first order differential equation (2)by taking U_t as an initial condition

$$U_{t+1} = (1 - e^{-(s_t+f_t)})\frac{s_t}{s_t + f_t}L_t + U_t e^{-(s_t+f_t)} \tag{6}$$

With the available data on employment and unemployment from the LFS and the job finding rate obtained above, s_t can be retrieved through solving equation (6).

Since the steady state definition implies a zero change between two consecutive peri-

[2]There is no equivalent time aggregation bias for the job finding rate mainly due to the fact that short term unemployed absorbs such thing.

ods; the steady state unemployment is obtained by equating dU/dt to zero in equation (2) and recalling the conventional unemployment formula $u_t = \frac{U_t}{L_t}$,

$$u_t^* = \frac{s_t}{s_t + f_t} \tag{7}$$

Assuming that the steady state unemployment rate is a close estimate to the actual rate of unemployment, we follow Elsby et al. (2009) in the decomposition of unemployment variations to those caused by changes in the outflow rate and those by changes in the inflow rate. To be specific, we log-linearize equation (7) to get

$$d\ln u_t^* = (1 - u_{t-1}^*)[d\ln s_t - d\ln f_t] \tag{8}$$

Let Σ_t^f denote $-(1 - u_{t-1}^*)d\ln f_t$ and Σ_t^s denote $(1 - u_{t-1}^*)d\ln s_t$ in which $\Sigma_t^f(\Sigma_t^f)$ represents the contribution of the job finding (separation rate) to the changes in the steady state rate of unemployment expressed in logarithmic terms. Now to measure these contributions, we adapt Fujita and Ramey (2009)'s methodology by dividing the covariance of the logarithmic changes in the steady state unemployment rate with its variance.

$$\beta^f = \frac{Cov(d\ln u_t^*, \Sigma_t^f)}{Var(d\ln u_t^*)} \tag{9}$$

$$\beta^s = \frac{Cov(d\ln u_t^*, \Sigma_t^s)}{Var(d\ln u_t^*)} \tag{10}$$

However, the assumptions underlying Elsby et al. (2009)'s methodology may not hold in practice. Specifically, the steady state unemployment rate may not provide a good approximation to the actual rate of unemployment. We therefore implement Elsby et al. (2013)'s approach because it presents a more comprehensive description of the labour markets at hand. To start with, a log-linearisation of equation (6) yields

$$d\ln u_t = \lambda_{t-1}\left\{(1 - u_{t-1}^*)[d\ln s_t - d\ln f_t] + \frac{1-\lambda_{t-2}}{\lambda_{t-2}}d\ln u_{t-1}\right\} \tag{11}$$

with $\lambda_{t-1} = 1 - e^{-(s_{t-1}+f_{t-1})}$. The cumulative contribution of the inflow rate, outflow rate, and the initial divergence of the actual unemployment rate from its steady state value are

$$\overline{\Sigma_t^f} = \lambda_{t-1}[-(1-u_{t-1}^*)d\ln f_t + \frac{1-\lambda_{t-2}}{\lambda_{t-2}}\overline{\Sigma_{t-1}^f}] \quad (12)$$

$$\overline{\Sigma_t^s} = \lambda_{t-1}[(1-u_{t-1}^*)d\ln s_t + \frac{1-\lambda_{t-2}}{\lambda_{t-2}}\overline{\Sigma_{t-1}^s}] \quad (13)$$

$$\overline{\Sigma_t^0} = \lambda_{t-1}\frac{1-\lambda_{t-2}}{\lambda_{t-2}}\overline{\Sigma_{t-1}^0} \quad (14)$$

where $\overline{\Sigma_0^f}$ and $\overline{\Sigma_0^s}$ are set to zero and $\overline{\Sigma_0^0} = \ln u_0 - \ln u_0^*$ documents the logarithmic deviation of the actual unemployment rate from the steady state unemployment rate at $t = 0$. Similar to equation (9) and (10), we quantify each contribution by computing the respective betas

$$\beta^f = \frac{Cov(d\ln u_t, \overline{\Sigma_t^f})}{Var(d\ln u_t)} \quad (15)$$

$$\beta^s = \frac{Cov(d\ln u_t, \overline{\Sigma_t^s})}{Var(d\ln u_t)} \quad (16)$$

$$\beta^0 = \frac{Cov(d\ln u_t, \overline{\Sigma_t^0})}{Var(d\ln u_t)} \quad (17)$$

4 Discussion of Empirical Findings

Analysis of Figure 1 reveals three insightful facts about the transition rates in the three economies. To begin with, there have been noticeable heterogeneities in the transition rates across the countries prior to the 2008 recession. More precisely, the job finding rate in Spain is significantly higher than that of Greece and Portugal, while both - separation and job finding rate-showed substantial fluctuations throughout the same period.

The second important note to make is the apparent behaviour of the transition rates during recessions and recoveries. Throughout normal economic downturns, workers enter the pool of unemployment at an accelerating rate followed by a persistent and steady decline during recoveries. These results are consistent in Portugal and Spain and are in line with the findings of Elsby and Smith (2010) on major economies, notably the UK and the US. Similarly, workers flow out of unemployment at a decreasing rate in Spain but the same is not evident in Portugal.

However, the variations in the mechanism of the flow rates across the three economies fade away when we target the Great Recession, the sovereign debt crisis and their aftermath. During this period, these countries underwent a period of chronic and steep increase in the rate of unemployment. This was accompanied by a comparable rise in the separation rate and a sharp decline in the job finding rate in Greece and Spain and to a lesser extent in Portugal.

Despite the obvious evidence shown earlier of the substantial role played by the separation rate in accompanying the rate of unemployment, a decomposition of the logarithmic flow steady state unemployment unveils a much greater role for the job finding rate. Table 1 shows that over 80 per cent of the fluctuations over the business cycle are driven by variations in the job finding rate compared with a relatively negligible 10-15 percent variation by its counterpart.

Now, to fully understand the dynamics behind the persistent increase in the rate of unemployment in times of normal and severe shocks, we turn our focus to Figure 2. Starting with Greece, the cumulative contribution of the decline in the job finding rate accounted for 81 percent of the increase in the rate of unemployment during the 2008 financial crisis compared with over two thirds during the sovereign debt crisis. In contrast, the contributions of the rise in the separation rate are relatively humble.

But can we generalise these conclusions to Spain and Portugal? The story for Spain is fairly similar, yet the magnitude of the contributions is comparatively higher. The decrease in the rate at which workers flow out of unemployment explained half of the variation during the early 1990s recession and even higher (approx. 88 per cent) during the 2008 economic downturn. The contributions of the outflow rate remained the same throughout the sovereign debt crisis. Given the input from Figure 2, it is safe to say that the contribution of the declined job finding are noteworthy in the early phases of recessions but it decreases as the effect of the shock fades away.

The case of Portugal however, is rather intriguing. Unlike Greece and Spain, the separation rate plays the dominant role in not only one but all recessions. Taking a closer inspection reveals an almost 60-80 percent contribution during the last shock.

It is important at this instance to stress the assumptions behind the decomposition explored above. The methodology is built on the assumption that the actual rate of unemployment is strongly approximated by its steady state value. This identity does not hold in our analysis. This can be depicted in Figure 3. The correlation between the former and the latter is 89, 93 and 94 per cent for Greece, Portugal and Spain, respectively. We therefore resort to a more representative method that would take into account this conclusion. The findings tabulated in the second part of Table 1 suggest a partial role for the separation rate, implying hence that both transition rates are equally important in explaining the persistent increase in unemployment during crises.

5 Concluding Remarks

The goal of this paper is to study the behaviour of labour markets in depression, as were Greece, Spain and Portugal in the wake of the world financial and Euro zone crises. In particular, the paper attempts to identify the proximate causes of the cyclical dynamics of worker flows when severe shocks hit the labour market in comparison to previous normal recessions.

In Greece, most of the dramatic increase in unemployment during the recent big recession is caused by a collapse in the job finding rate. In Spain, and in comparison with previous normal recessions, the contribution of the job finding rate during the recent crisis is reasonably similar. On the other hand, unemployment dynamics in Portugal during the recent crisis are primarily driven by the separation rate. The heterogeneity in the flow rates between the three economies hint at cross-country differences in labour market institutions and requires further careful scrutiny. Further research is also needed in shedding light on the gender and skills dimension of these unemployment dynamics.

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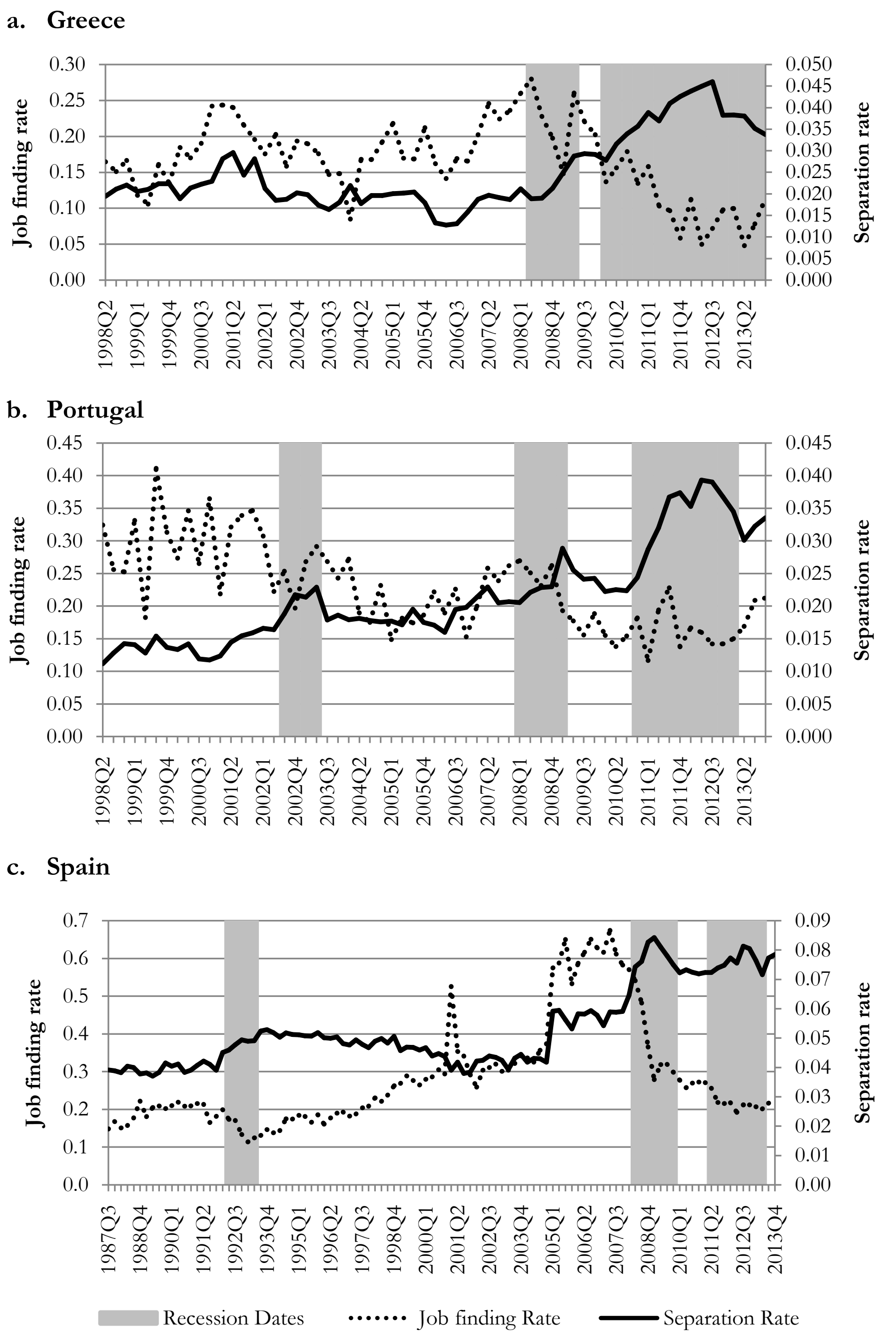
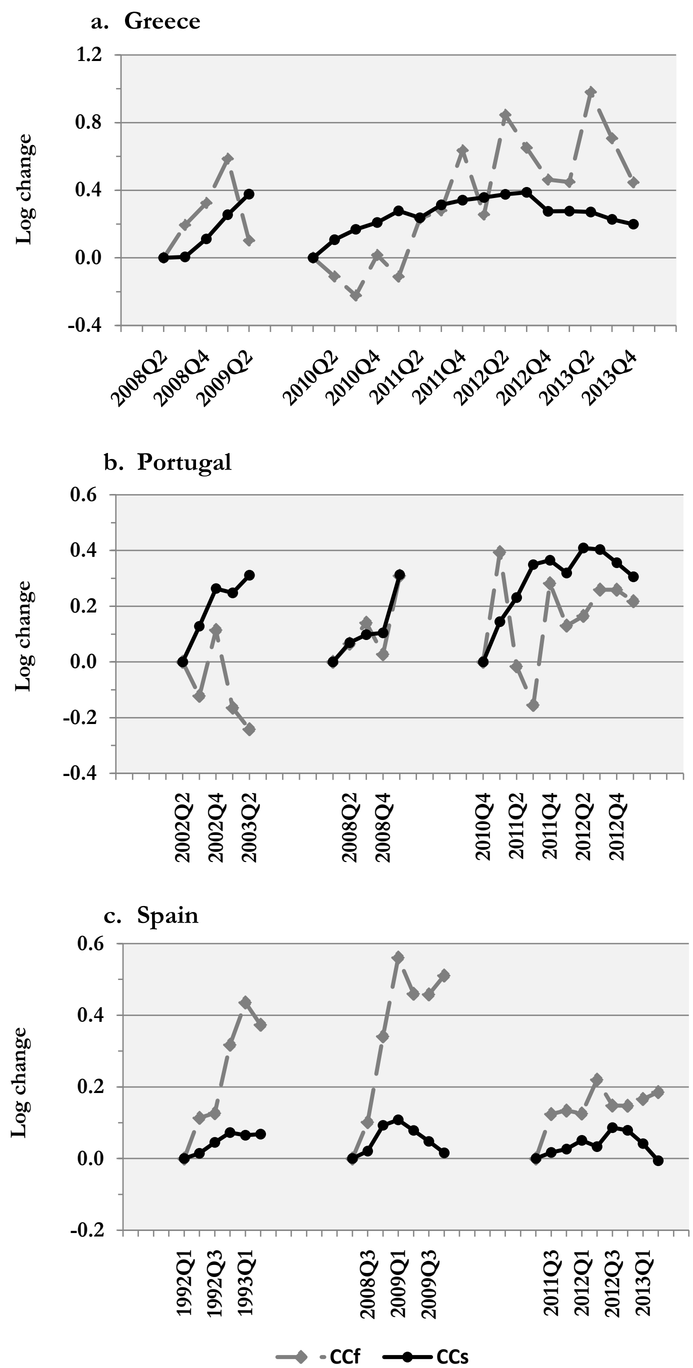
Figure 1: Job finding and Separation Rates for Greece, Portugal and Spain



Note: Author's calculations, Eurostat's LFS data for Greece, Portugal and Spain (1998Q1- 2013Q4), Spanish National Statistics LFS data (1987Q2-1997Q4).

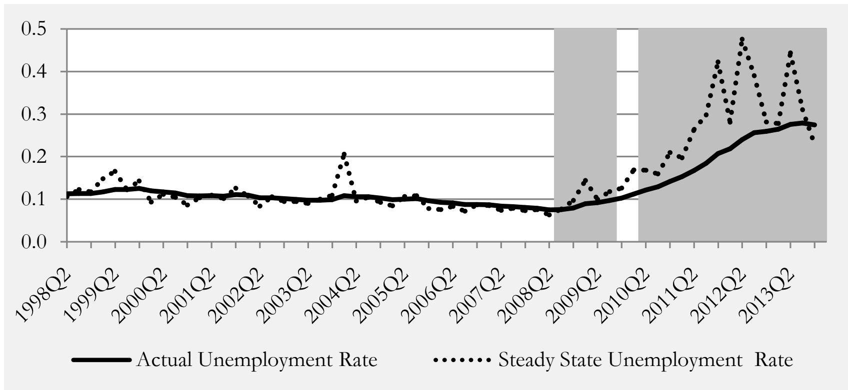
Figure 2: Cumulative Contribution of the Separation and job finding Rates by Recession.



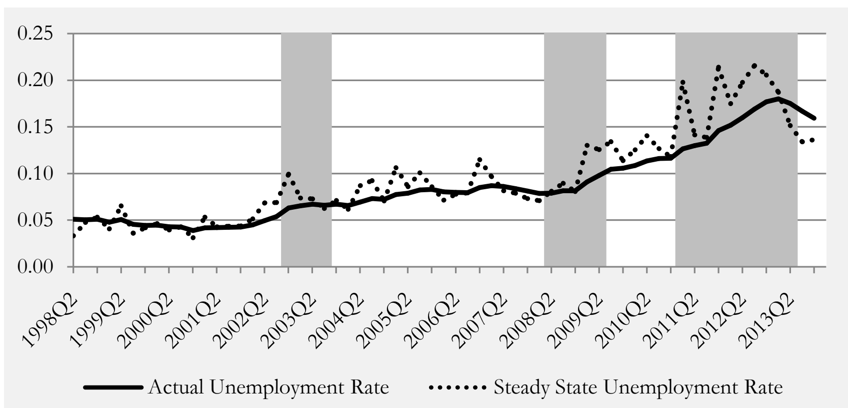
Note: Authors calculations, Eurostats LFS data for Greece, Portugal and Spain (1998Q1- 2013Q4), Spanish National Statistics LFS data (1987Q2-1997Q4). Recession dates were calculated based on the Classical Business Cycle Method. CCf denotes the cumulative contribution of the job finding rate while CCs denotes the cumulative contribution of the separation rate. CCf and CCs lines sum to the cumulative change in the steady state unemployment rate (Δu_t^*).

Figure 3: Actual Versus Steady State Unemployment

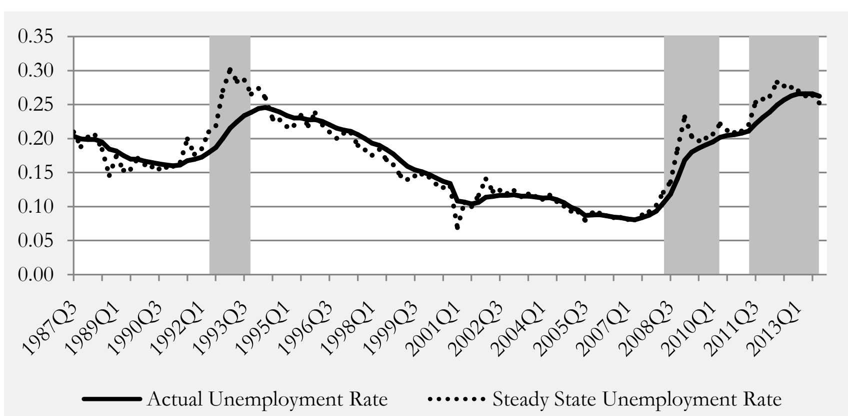
a. Greece



b. Portugal



c. Spain



Note: Author's calculations, Eurostat's LFS data for Greece, Portugal and Spain (1998Q1- 2013Q4), Spanish National Statistics LFS data (1987Q2-1997Q4).

Table 1: Decomposing Unemployment Variations, by method, for Greece, Portugal and Spain

	Steady State Decomposition			Non-Steady State Decomposition			
	Bf	**Bs**	**Total**	**Bf**	**Bs**	**B0**	**Total**
Greece	0.82	0.15	0.97	0.64	0.33	0.00	0.97
Portugal	0.91	0.08	0.99	0.73	0.23	0.03	0.99
Spain	0.83	0.15	0.98	0.81	0.17	-0.02	0.96

Note: Decomposition for the entire period of (1998Q1- 2013Q4) for Greece and Portugal and (1987Q2-2013Q4) for Spain

Appendix A Mathematical Derivations

- Derivation of equation (6)

The derivation closely follows the techniques proposed by Shone (2002) and Boyce and DiPrima (2001) in solving differential equations.

Since equation (2) is non-autonomous (time variant), we need to multiply by a specific function $\mu(t)$ so that the resulting equation becomes integrable

$$\mu(t)\frac{dU}{dt} + (s_t + f_t)\mu(t)U_t = s_t L_t \mu(t) \tag{A.1}$$

we need to choose $\mu(t)$ for the left side of equation (A.1) to be the derivative of the function of familiar form.

$$\frac{d}{dt}\left[\mu(t)U\right] = \mu(t)\frac{dU}{dt} + \frac{d\mu(t)}{dt}U \tag{A.2}$$

the first term in the right hand side is equivalent and the second term matches as well, given that we select $\mu(t)$ that would satisfy the following equation

$$\frac{d\mu(t)}{dt} = (s_t + f_t)\mu(t) \tag{A.3}$$

provided that $\mu(t)$ is positive, a simple rearrangement of the above equation gives

$$\frac{d\mu(t)/dt}{\mu(t)} = (s_t + f_t) \tag{A.4}$$

integrating equation (A.4) yields

$$\ln \mu(t) = (s_t + f_t)dt + k \tag{A.5}$$

assuming k is zero and taking the exponential of both sides

$$\mu(t) = e^{\int (s_t + f_t)dt} = e^{s_t + f_t} \tag{A.6}$$

substituting $\mu(t)$ back in equation (A.1) we get

$$e^{(s_t+f_t)}\frac{dU}{dt} + (s_t + f_t)e^{(s_t+f_t)}U_t = e^{(s_t+f_t)}s_t L_t \tag{A.7}$$

because of the choice we made earlier about $\mu(t)$, equation (A.7) can be expressed as

$$\frac{d}{dt}e^{(s_t+f_t)}U_t = e^{(s_t+f_t)}s_tL_t \tag{A.8}$$

integrating both sides, gives

$$\begin{aligned}
\int \frac{d}{dt}e^{(s_t+f_t)}U_t dt &= \int e^{(s_t+f_t)}s_tL_t dt + c \\
e^{(s_t+f_t)}U_t &= \frac{e^{(s_t+f_t)}}{s_t+f_t}s_tL_t + c \\
U_t &= \frac{s_tL_t}{s_t+f_t} + ce^{-(s_t+f_t)} \qquad \text{(A.9)}
\end{aligned}$$

when $c = 0$, the equation above corresponds to the steady state solution

$$U_t^* = \frac{s_tL_t}{s_t+f_t} \tag{A.10}$$

Therefore, equation (A.9) can be written as

$$U_t = U_t^* + ce^{-(s_t+f_t)} \tag{A.11}$$

if we assumed that the initial condition $U(0) = U_t$ we can solve for c and substitute back into equation (A.9) to obtain

$$\begin{aligned}
U_{t+1} &= U_t^* + (U_t - U_t^*)e^{-(s_t+f_t)} \\
U_{t+1} &= (1 - e^{-(s_t+f_t)})\frac{s_t}{s_t+f_t}L_t + U_te^{-(s_t+f_t)} \qquad \text{(A.12)}
\end{aligned}$$

- Derivation of equation (7) with log-linearization [3]

The starting point is the steady state unemployment

$$u_t^* = \frac{s_t}{s_t+f_t} \tag{A.13}$$

Taking the natural log of (A.13)

$$\ln u_t^* = \ln s_t - \ln(s_t + f_t) \tag{A.14}$$

[3]Equivalent derivation using first differencing and log differentiation are seen in Smith (2011) and Karamessini and Koutentakis (2014), respectively.

A first order Taylor series expansion around a steady state yields

$$\ln \overline{u_t^*} + \frac{1}{\overline{u_t^*}}(u_t^* - \overline{u_t^*}) = \ln \overline{s_t} + \frac{1}{\overline{s_t}}(s_t - \overline{s_t}) - \ln(\overline{s_t} + \overline{f_t}) - \frac{1}{\overline{s_t} + \overline{f_t}}(s_t - \overline{s_t}) - \frac{1}{\overline{s_t} + \overline{f_t}}(f_t - \overline{f_t}) \tag{A.15}$$

Note first that $\ln \overline{u_t^*} = \ln \overline{s_t} - \ln(\overline{s_t} + \overline{f_t})$, so the terms will cancel out and we are left with

$$\frac{1}{\overline{u_t^*}}(u_t^* - \overline{u_t^*}) = \frac{1}{\overline{s_t}}(s_t - \overline{s_t}) - \frac{1}{\overline{s_t} + \overline{f_t}}(s_t - \overline{s_t}) - \frac{1}{\overline{s_t} + \overline{f_t}}(f_t - \overline{f_t}) \tag{A.16}$$

$$\frac{(u_t^* - \overline{u_t^*})}{\overline{u_t^*}} = \frac{(s_t - \overline{s_t})}{\overline{s_t}} - \frac{\overline{\boldsymbol{s_t}}}{\overline{s_t} + \overline{f_t}} \frac{s_t - \overline{s_t}}{\overline{\boldsymbol{s_t}}} - \frac{\overline{\boldsymbol{f_t}}}{\overline{s_t} + \overline{f_t}} \frac{f_t - \overline{f_t}}{\overline{\boldsymbol{f_t}}} \tag{A.17}$$

Given the fact that $\ln \frac{x_t}{\overline{x}} = \ln(1 + \frac{x_t - \overline{x}}{\overline{x}}) \cong \frac{x_t - \overline{x}}{\overline{x}}$

$$\ln \frac{u_t^*}{\overline{u_t^*}} = \ln \frac{s_t}{\overline{s_t}} - \overline{u_t^*} \ln \frac{s_t}{\overline{s_t}} - \frac{\overline{f_t}}{\overline{s_t} + \overline{f_t}} \ln \frac{f_t}{\overline{f_t}} \tag{A.18}$$

$$\ln \frac{u_t^*}{\overline{u_t^*}} = (1 - \overline{u_t^*}) \ln \frac{s_t}{\overline{s_t}} - \frac{\overline{f_t} + \overline{\boldsymbol{s_t}} - \overline{\boldsymbol{s_t}}}{\overline{s_t} + \overline{f_t}} \ln \frac{f_t}{\overline{f_t}} \tag{A.19}$$

$$\ln \frac{u_t^*}{\overline{u_t^*}} = (1 - \overline{u_t^*}) \ln \frac{s_t}{\overline{s_t}} - \left[\frac{\overline{f_t} + \overline{s_t}}{\overline{s_t} + \overline{f_t}} - \frac{\overline{s_t}}{\overline{s_t} + \overline{f_t}} \right] \ln \frac{f_t}{\overline{f_t}} \tag{A.20}$$

$$\ln \frac{u_t^*}{\overline{u_t^*}} = (1 - \overline{u_t^*}) \ln \frac{s_t}{\overline{s_t}} - (1 - \overline{u_t^*}) \ln \frac{f_t}{\overline{f_t}} \tag{A.21}$$

Assume further that $\overline{x_t} = x_{t-1}$

$$\ln \frac{u_t^*}{u_{t-1}^*} = (1 - u_{t-1}^*) \ln \frac{s_t}{s_{t-1}} - (1 - u_{t-1}^*) \ln \frac{f_t}{f_{t-1}} \tag{A.22}$$

$$d \ln u_t^* = (1 - u_{t-1}^*) \left[d \ln s_t - d \ln f_t \right] \tag{A.23}$$