

Skill Formation, Demographic Change and Labor Market Volatility

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June 15, 2016

Abstract

We embed different generations into search model to generate cyclical unemployment fluctuation through combinational change among different generations. After we entangle the skill formation with demographic change, we are capable to match labor market volatility with its empirical data, which comoves with demographic transition. Unlike Hagedorn and Manovshii (2008), it doesn't require any strict specification of parameters like value of unemployment benefit or value of worker's bargaining power.

Keywords: Skill Premium, Demographic Transition, Overlapping Generation, Job Search

1 Introduction

Shimer (2005) shows the basic search and matching model's inability in generating labor market volatility to match the empirical data, known as the Shimer's critique. The key lies in the fact that market tightness is only mildly pro-cyclical. If productivity decreases relative to vacancy costs and the value of non-market time, firms will post less job vacancies and workers find the job harder to get. This will pull up the unemployment rate and pull down the market tightness. If this is the only mechanism being working, then market tightness should be pro-cyclical. The problem is that when job finding rate decreases, the outside option for the workers also decreases, which will drive the wage down, then firms will react by posting more vacancies which will mitigate the former vacancies decrease. Finally, vacancies will just mildly move downward and market tightness is only mildly pro-cyclical. Thus, as Shimer (2005) claims, it is impossible to generate enough cyclical unemployment fluctuations with the baseline search models.

Hagedorn and Manovshii (2008) argue that with higher value of unemployment benefit and extremely small value of worker's bargaining power, the second mechanism mentioned above is only going to have a tiny effect as wage barely moves and a pro-cyclical market tightness

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follows up naturally. But their solution relies heavily on the specification of unemployment benefit and worker's bargaining power, which gives rise to unrealistically high opportunity cost of employment, thus workers are motivated only by a tiny piece of profit as criticized by Mortensen and Nagypal (2007).²

The very basic idea behind Hagedorn and Manovshii (2008) is to squeeze worker's profit to a tiny piece to be sensitive to productivity shocks. We can work on the a similar idea which doesn't need strict parameter specification of unemployment benefit and worker's bargaining power: suppose workers have different skill levels and productivity has a multiplicative form by worker's skill level and productivity shock, with this setup, the profit generated by low skill workers can be squeezed to a considerably low level depending on how unskillful these workers are. A natural way to rationalize different skills among workers is to have age heterogeneity and thus life cycle skill formation.

Coincidentally, Jaimovich and Siu (2009)'s empirical work shows that the compositional changes of different age group comove with and also econometrically account for a significant fraction of business cycle volatility. Using panel data methods, they exploit variation in the timing and the magnitude of the demographic change across G7 countries to show that the age distribution has a significant effect on cyclical volatility. To be specific, they provide sufficient evidence that the youth share in the whole labor force is positively correlated with aggregate output volatility in most of G7 countries. It is high likely that the introduction of age heterogeneity and skill formation into the basic search model can help to generate enough cyclical volatility in market tightness and unemployment.

Unfortunately, the introduction of age heterogeneity into search model is not a easy task as demonstrated by Chéron et al.(2013)'s life cycle unemployment model in the basic search framework. The good thing is there is still another way to model age heterogeneity which assume age transition from one level to another just follows a uniform distribution, as in Fujita & Fujiwara(2015) and Farmer (2015). This method greatly simplifies the whole problem because it doesn't need to trace each worker's age for each year.

Another important reason for the introduction of age heterogeneity is due to the fact of rapidly aging in several developed countries, especially Japan and Germany. Fujita & Fujiwara(2015) point out that aging of labor force accounts for 40% of the declines in real interest between 1980-2000 in Japan. We want to explore the role of aging in cyclical volatility by running demographic transition experiments.

To make it clear, the whole process proceeds like this: workers are categorized as young, middle age and old. The demographic transition from one age group to another just follows certain uniform distribution. Learning from Jaimovich and Siu (2009)'s empirical results, we treat young and old age groups as volatile when consider the cyclical variation in unemployment

²The flow surplus enjoyed by employed worker is minuscule, 2.8% to be precise, from the calculation of Mortensen and Nagypal (2007).

and job vacancies in response to productivity shock. We want to come up with a skill evolving mechanism to make sure that middle age group have the easiest access to high skill, while old age group less and young age the least, thus, we entangle the demographic transition together with skill evolving process: young workers are all low skill and they automatically and unconditional become high skill workers once they enter into middle age, with this assumption, one extreme case would be some young unemployment worker might never get a job position and still become high skill worker once up to certain age; middle age unemployment workers have the risk of losing their skill premium and can regain with certain probability once become employed, which will make sure they stay in high skill relatively stably; old unemployment workers also have the same risk of losing their skill premium but can't regain once skill premium is lost.

The paper proceeds as follows. Section 2 gives the literature review of basic search model's inability in generating labor market volatility and its various solutions follow up, of the introduction of skill formation into the search model and of the age heterogeneity into search framework. Section 3 embed the different generations into the model of search model with skill level formation. Section 4 concludes.

2 Literature review

Starting from Shimer (2005)'s argument of the basic search and matching model's inability in generating labor market volatility to match the empirical data, Hall (2005) and Hagedorn and Manovshii (2008) follow up respectively with the introduction of wage stickiness and strict parameter specification of unemployment benefit and worker's bargaining power. Menzio and Shi (2011) develops a directed search model with on the job search and signaling for the true match quality. Their model are capable of generating large fluctuations in worker's transitions, unemployment and vacancies with respect to productivity shocks but only when matches are experience goods, which means observing the signal doesn't provide any useful information at all, and also under a completely different setup comparing with the basic random search model. The biggest contribution of this paper is that it elegantly proves the existence of a block-recursive equilibrium in the direct search model, which is independent of the employment states and can be solved outside of the steady state. Elsby and Michaels (2013) sticks with random matching model while allowing the heterogeneity from firm side. The variation in firm size gives rise to certain combinational effect between the average and marginal product of labor of the average-sized firm, which enables the elasticity of the market tightness to aggregate productivity to have a much wider range.

The strand of the literature by introducing skill formation or human capital evolution into search model is quite rich. Ortego-Marti (2012) exploits the role of human capital depreciation in generating of significant wage dispersion in on-the-job search framework. Workers lose human capital during unemployment; therefore they will lower their reservation productivity

and accept lower wages to leave unemployment more quickly. He shows that the addition of unemployment history can explain around one third to one half of the observed residual wage dispersion. Then, he adds on-the-job search to the model with unemployment history. Wage dispersion increases significantly. Carrillo-Tudela (2012) constructs and quantitatively assesses an equilibrium with search model with on-the-job search and general human capital accumulation in the framework of Burdett and Mortensen (1998). The main differences are the introduction of firm productivity differentials and job arrival rates that depend on workers' employment status.³ Tjaden and Wellschmied (2012) use a similar method as Ortego-Marti (2012), the only difference in model structure is the addition of learning by doing together with skill loss during unemployment.⁴

By focusing on the role of demographic structure of the U. S. population in unemployment rate change, Perry (1970) predicted that the entrance of the baby boom would push up the unemployment rate. Further verification were followed by Flaim (1979, 1990) and Shimer (1998, 2001). But very few works have been done to investigate its role in the variation of unemployment rate and the business cycle fluctuation in search friction frameworks. One needs to be specifically mentioned is Jaimovich and Siu (2009). Jaimovich and Siu (2009) present the most important empirical research in this area, which analyzes unemployment spells and business cycle from the angle of demographic change. They exploit variation in the timing and the magnitude of population changes across G7 countries and find that changes in the age composition of the labor force account for about 1/3 of the business cyclical volatility in G7 economies, in which Japan's economy structure is totally different from Euro zone and US and this excludes the possibility of spurious regression. Other similar works also include: Clark and Summers (1981) was the first paper to report employment volatility by age group; Shimer (1998) indicate that compositional change in the labor force due to the baby boom generated a substantial fraction of the rise and fall in U.S. unemployment from the 1960s through to the end of the century; Abraham and Shimer (2001) also use the aging of the baby boom population to

³ Its main idea is similar as Yamaguchi (2010) and Bagger et al. (2013) who use the random search model posit by Postel-Vinay and Robin (2002) and introduce general human capital accumulation with productivity shocks to explain the workers' wage dynamics. Different from them, it does not allow for productivity shocks and assume that firms do not observe workers' reservation wages or react to their employees' outside offers.

⁴ The biggest difference lies in the conclusion: quite contrary with Ortego-Marti (2012)'s findings which show that with the introduction of human capital depreciation and on-the-job search almost all of the residual wage dispersion can be accounted, but Tjaden and Wellschmied (2012) use the same data set, the Survey of Income and Program Participation(SIPP), and find that the search model never accounts for more than twenty percent of the residual wage dispersion within an age cohort after controlling for the channels of skill development, duration dependence in unemployment benefits and on-the-job search.

investigate the role of demographic change in US unemployment. Motivated by their findings, Lugauer (2012) embeds overlapping generations into a discrete time labor search model and theoretical explains the role of demographic change in cyclical unemployment fluctuations. He finds that young workers frequently move in and out of employment because they tend to be in bad matches which last for shorter time and older workers are likely to be employed in good matches which last for longer time as they have more time searching for good matches.

Several research introduces age heterogeneity into search framework, but few model it in a simple way. Fujita & Fujiwara(2015) investigates the role of labor force demographic change in interest rate and inflation in Japan. They simply assume age transition from one level to another just follows a uniform distribution: each period, the young workers give birth and become old workers with the corresponding probability; old workers die with certain probability. This method greatly simplifies the whole problem as it doesn't need to trace each worker's identity and still generate a distribution of labor force at each age group. Similar idea is also used in Ferrero et al.(2015) and Farmer (2015).

3 Search model with different generations

This model builds on Pissarides' (1985) matching model but allows for the coexistence of different age groups with different skill level. Assume time is discrete and agents have infinite time horizons.

3.1 Search model with two generations

Let's start with a search model with two generations. Agents discount their future value with a stochastic discounting factor. The demographic transition follows like this: there are young and old workers in the economy, each period, with a probability ρ^b , a young worker gives birth to a new born who will enter into the labor force as unemployed in the current period; with probability ρ^o , a young becomes old; with probability ρ^d , an old dies. Let's skip skill formation in two generation model and consider the skill difference between groups as a fact. Old workers are considered to be unskilled with productivity $e^{A_t}\varepsilon$ while all young workers are skilled with productivity e^{A_t} .⁵ Technology innovation follows a AR(1) process:

$$A_t = \psi A_{t-1} + e_t$$

where $\psi \in (0, 1)$ and e_t follows a random walk.

The matching market is divided by age, thus young workers only search in labor market for

⁵Whether we assume old workers to be skilled or unskilled doesn't affect our main results, a setup with young workers to be considered as unskilled with productivity $e^{A_t}\varepsilon$ while all old workers as skilled with productivity e^{A_t} is going to generate the same results. As we mentioned early, the skill formation is not a topic in two generations model. The life cycle skill formation will be explicitly displayed in search model with three generations.

young workers while old workers only search in labor market for old. Firm posts vacancies at the cost of ζ^y and ζ^o respectively in each market until the value of posting an additional vacancy is zero, once a vacancy is filled, production happens at the beginning of next period. Workers in both markets face the same exogenous separation shock at the beginning of next period, with probability s of becoming unemployed and getting unemployment benefit z . The probability for a vacancy to be filled in the current period equals $q(\theta_t^i) = m_t^i/v_t^i$ and the probability for an unemployed worker to be matched in the current period equals $f(\theta_t^i) = m_t^i/u_t^i$, where $m_t^i = m^i(v_t^i, u_t^i) = \bar{m}^i(v_t^i)^{1-\mu}(u_t^i)^\mu$ and $i = y, o$.

The timing of the events follows the following order: production happens in the very beginning, then demographic transitions and exogenous separation shocks occur, finally job search takes place.

3.1.1 Stock flow of labor force

Here we don't allow simultaneous search for newly separated workers in the current period, besides, assume that production happens in the very beginning of each period. We can write the laws of motion for each type of work in the following:

$$u_{t+1}^y = \rho^b(u_t^y + n_t^y) + (1 - \rho^o)[(1 - f^y(\theta_t^y))u_t^y + sn_t^y] \quad (1a)$$

$$n_{t+1}^y = (1 - \rho^o)[f^y(\theta_t^y)u_t^y + (1 - s)n_t^y] \quad (1b)$$

$$u_{t+1}^y + n_{t+1}^y = (1 + \rho^b - \rho^o)(u_t^y + n_t^y) \quad (1c)$$

$$u_{t+1}^o = (1 - \rho^d)[(1 - f^y(\theta_t^o))u_t^o + sn_t^o] + \rho^o[(1 - f^y(\theta_t^o))u_t^y + sn_t^y] \quad (1d)$$

$$n_{t+1}^o = (1 - \rho^d)[f^y(\theta_t^o)u_t^o + (1 - s)n_t^o] + \rho^o[f^y(\theta_t^o)u_t^y + (1 - s)n_t^y] \quad (1e)$$

$$u_{t+1}^o + n_{t+1}^o = (1 - \rho^d)(u_t^o + n_t^o) + \rho^o(u_t^y + n_t^y) \quad (1f)$$

Equation 1a gives the transition of young unemployed, which is comprised of the new born, ratio ρ^b of the young labor force, freshly separated young workers and former unemployed young workers without finding a job in this period. Equation 1c and 1f are the summation of the corresponding labor force and gives the transition for young and old labor force. They are useful to deal with population growth:

With equation 1c and 1f, let $l_t^y \equiv u_t^y + n_t^y$ and $l_t^o \equiv u_t^o + n_t^o$, we have:

$$l_t^y/l_t^o = \frac{1 + \rho^b - \rho^o}{(1 - \rho^d)(l_{t-1}^o/l_{t-1}^y) + \rho^o} \quad (2)$$

which means l_t^y/l_t^o is an exogenous process depending only on demographic transition parameters. The limit of this series is $l_\infty^y/l_\infty^o = \frac{\rho^b + \rho^d - \rho^o}{\rho^o}$. In order to get rid of the effect of l^y/l^o 's initial value, it's unharmed to assume its value to be at its limit where the growth

rates of the two cohorts are also going to be the same. As the growth rate of young workers is: $g_t^y = \frac{l_{t-1}^y(\rho^b - \rho^o)}{l_{t-1}^y} = \rho^b - \rho^o$, the growth rate of old workers is: $g_t^o = \frac{l_{t-1}^o \rho^o - l_{t-1}^o \rho^d}{l_{t-1}^o} = (l_{t-1}^y/l_{t-1}^o)\rho^o - \rho^d$, and their limits $g_\infty^y = g_\infty^o = \rho^b - \rho^o$. Note that in this limit, the growth rate is irrelevant with ρ^d . This doesn't necessarily mean ρ^d doesn't play any role in the growth rate. As ρ^d changes, l^y/l^o is going to be off the limit and $g_t^o = (l_{t-1}^y/l_{t-1}^o)\rho^o - \rho^d \neq \rho^b - \rho^o$. The direct effect of ρ^d is in l_∞^y/l_∞^o .

With equation 1c, 1f and 2, we can simplify the above transition equations in level into the following transition equations in rate:

$$\begin{aligned} \hat{u}_{t+1}^y(1 + \rho^b - \rho^o) &= \rho^b + (1 - \rho^o)[(1 - f^y(\theta_t^y))\hat{u}_t^y + s(1 - \hat{u}_t^y)] & (3a) \\ \hat{u}_{t+1}^o(1 + \rho^o \frac{l_t^y}{l_t} - \rho^d) &= (1 - \rho^d)[(1 - f^y(\theta_t^o))\hat{u}_t^o + s(1 - \hat{u}_t^o)] + \rho^o \frac{l_t^y}{l_t} [(1 - f^y(\theta_t^o))\hat{u}_t^y + s(1 - \hat{u}_t^o)] & (3b) \end{aligned}$$

where \hat{u}_t^i stands for the unemployment rate of group i and $i = y, o$.

3.1.2 Equilibrium

An old worker receives wage payment when production takes place at the beginning of the period along with the future value of W_{t+1}^o with probability $1 - s$ and U_{t+1}^o with probability s conditional on that he doesn't die. Thus the assert value of an employed old worker is given by:

$$W_t^o = \omega_t^o + E_t \beta_{t+1} ((1 - s)(1 - \rho^d)W_{t+1}^o + s(1 - \rho^d)U_{t+1}^o)$$

where $E_t \beta_{t+1}$ is the stochastic discounting factor.

Suppose we have pooling assumption for all workers' consumption following Merz (1995), which will enable us to use a representative household's problem to equally distribute consumption and ensure all labor force participate in the labor market. Suppose the utility function of worker is of the following form: $u(c_t) = \frac{c_t^{1-\tau} - 1}{1-\tau}$, then the problem of each worker can be written as:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\tau} - 1}{1-\tau}$$

subject to: $c_t = y_t - v_t^o \zeta^o - v_t^y \zeta^y$, where $y_t = e^{A_t}(\varepsilon n_t^o + n_t^y)$. This implies a stochastic discount factor: $E_t \beta_{t+1} = \beta E_t c_{t+1}^{-\tau} / c_t^{-\tau}$.

An old unemployed worker receives unemployment benefit z each period along with the future value of W_{t+1}^o with probability f_t^o and U_{t+1}^o with probability $1 - f_t^o$ conditional on that

he doesn't die. Thus the asset value of an unemployed old worker is given by:

$$U_t^o = z + E_t \beta_{t+1} (f_t^o (1 - \rho^d) W_{t+1}^o + (1 - f_t^o) (1 - \rho^d) U_{t+1}^o)$$

For the firms operating in the old labor force market, because of free entry into the vacancy pool, the free entry condition and the asset value of a filled job with respect to the firm can be simply written as:

$$\frac{s_t^o}{q_t^o} = E_t \beta_{t+1} J_{t+1}^o$$

and

$$J_t^o = e^{A_t} \varepsilon - \omega_t^o + (1 - s)(1 - \rho^d) E_t \beta_{t+1} J_{t+1}^o$$

With Nash bargaining, which implies $(1 - \eta)(W_t^o - U_t^o) = \eta J_t^o$, we can get the explicit expression for wage:

$$\omega_t^o = \eta \varepsilon e^{A_t} + (1 - \eta)z + \eta f_t^o (1 - \rho^d) E_t \beta_{t+1} J_{t+1}^o$$

Plug this wage equation into the asset value of a filled job, together with free entry condition, we can get a implicit function for market tightness in old labor force market θ_t^o :

$$\frac{s_t^o}{q_t^o} = (1 - \eta)(e^{A_t} \varepsilon - z) + (1 - \rho^d)(1 - s - \eta p_t^o) E_t \beta_{t+1} \left(\frac{s_t^o}{q_{t+1}^o} \right) \quad (4)$$

Similarly, we can also get the implicit function for market tightness in the young labor force

market θ_t^y :⁶

$$\frac{\zeta^y}{q_t^y} = (1 - \eta)(e^{A_t} - z) + (1 - s - \eta f_t^y) E_t \beta_{t+1} \left((1 - \rho^o) \frac{\zeta^y}{q_{t+1}^y} + \rho^o \frac{\zeta^o}{q_{t+1}^o} \right) \quad (5)$$

Equations 3a, 3b, 4 and 5 depict the dynamic of the whole system.

3.1.3 Propagation of shocks

With equations 4 and 5, we can get the following proposition:

Proposition 1. If we denote the elasticity of θ_t^o with respect to productivity as $e_{\theta,A}^o$ and the elasticity of θ_t^y with respect to productivity as $e_{\theta,A}^y$, then we can get:

$$e_{\theta,A}^o = \frac{A}{A - \frac{z}{\varepsilon}} \frac{\alpha_1 + \alpha_2 p^o(\theta^o)}{\alpha_1(1 - \Delta^o) + \alpha_2 p^o(\theta^o)}$$

where $\alpha_1 = 1 - (1 - \rho^d)\beta$, $\alpha_2 = (1 - \rho^d)s\eta\beta$, $\Delta^o \equiv e_{p^o,\theta^o}^o$ and

$$e_{\theta,A}^y = \frac{A}{A - z} \frac{\alpha_3 + \alpha_4 p^y(\theta^y) - \alpha_5 \frac{\zeta^o}{\zeta^y} \left(\frac{\theta^o}{\theta^y}\right)^\mu + \alpha_6 \frac{\theta^o}{\theta^y}}{\alpha_3(1 - \Delta^y) + \alpha_4 p^y(\theta^y) + \alpha_6 \frac{\theta^o}{\theta^y} \Delta^y} \left(1 + \frac{\varepsilon \theta^o (1 - \Delta^o) (\alpha_5 - \alpha_6 p^y(\theta^y))}{\alpha_1(1 - \Delta^o) + \alpha_2 p^o(\theta^o)} \right)$$

where $\alpha_3 = 1 - (1 - s)(1 - \rho^o)\beta$, $\alpha_4 = (1 - \rho^o)s\eta\beta$, $\alpha_5 = (1 - s)\rho^o\beta$, $\alpha_6 = \eta\rho^o\beta$, $\Delta^y \equiv e_{p^y,\theta^y}^y$, $\Delta^o \equiv e_{p^o,\theta^o}^o$

The importance of proposition 1 lies in the fact that $e_{\theta,A}^o$ is proportional to $\frac{A}{A - \frac{z}{\varepsilon}}$ while $e_{\theta,A}^y$ is proportional to $\frac{A}{A - z}$. For reasonable specification of the parameters, $\frac{A}{A - \frac{z}{\varepsilon}}$ equals close to 2 and the rest of the equation for $e_{\theta,A}^y$ is going to be close to 1, while for $e_{\theta,A}^o$, $\frac{A}{A - \frac{z}{\varepsilon}}$ has a much

⁶ Free entry condition and asset equations for young workers are:

$$\frac{\zeta^y}{q_t^y} = E_t \beta_{t+1} J_{t+1}^y$$

$$J_t^y = e^{A_t} - \omega_t^y + (1 - s) E_t \beta_{t+1} [(1 - \rho^o) J_{t+1}^y + \rho^o J_{t+1}^o]$$

$$W_t^y = \omega_t^y + E_t \beta_{t+1} ((1 - s)((1 - \rho^o) W_{t+1}^y + \rho^o W_{t+1}^o) + s((1 - \rho^o) U_{t+1}^y + \rho^o U_{t+1}^o))$$

$$U_t^y = z + E_t \beta_{t+1} (f_t^y ((1 - \rho^o) W_{t+1}^y + \rho^o W_{t+1}^o) + (1 - f_t^y) ((1 - \rho^o) U_{t+1}^y + \rho^o U_{t+1}^o))$$

with Nash bargaining,

$$\omega_t^y = \eta e^{A_t} + (1 - \eta)z + \eta f_t^y E_t \beta_{t+1} [(1 - \rho^o) J_{t+1}^y + \rho^o J_{t+1}^o]$$

which can be combined into:

$$\frac{\zeta^y}{q_t^y} = (1 - \eta)(e^{A_t} - z) + (1 - s - \eta f_t^y) E_t \beta_{t+1} \left((1 - \rho^o) \frac{\zeta^y}{q_{t+1}^y} + \rho^o \frac{\zeta^o}{q_{t+1}^o} \right)$$

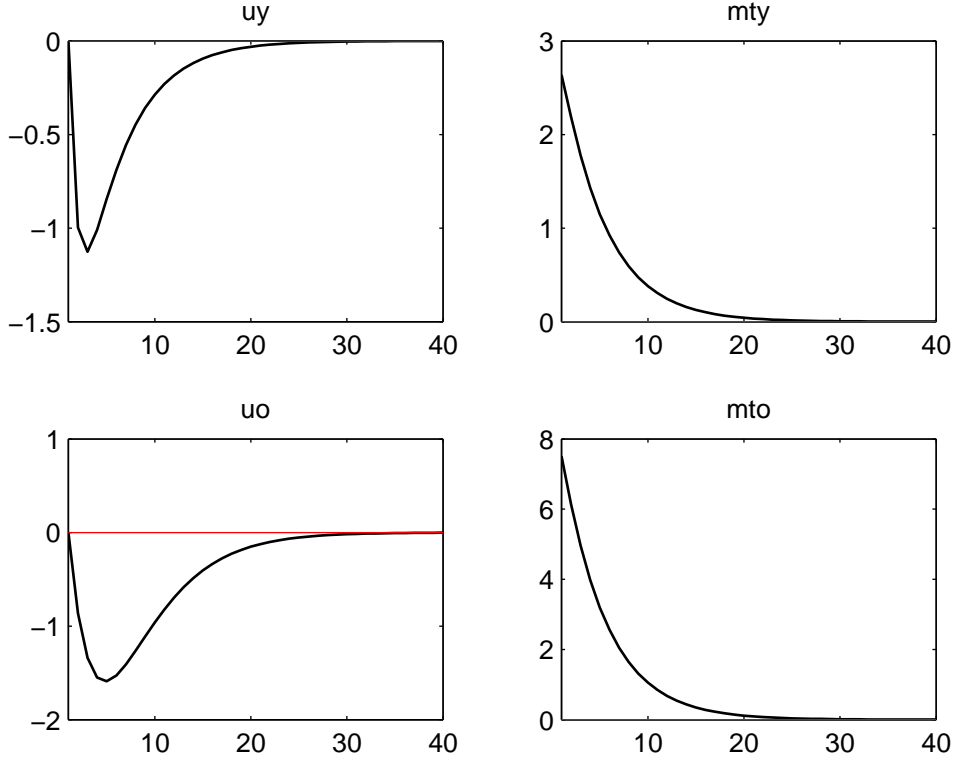


Figure 1: IRFs to a technology shock
(uy stands for unemployment rate for young, uo stands for unemployment rate for old, mty stands for market tightness for young and mto stands for market tightness for old)

wider range as $\varepsilon \in (z, 1)$ and can be close to z as much as possible, which will generate much bigger value for $e_{\theta,A}^o$.

The intuition of this proposition is that for unskilled workers, the profit space can be squeezed to any small number depending on the how unskillful these workers are. Thus, combi-national change of workers with different skill level is going to generate different labor market volatility. To put it in another form, when we entangle the age level with skill level, we can generate different labor market volatility with demographic change. The most important is it doesn't require any strict specification of parameters like value of unemployment benefit or value of worker's bargaining power.

Figure1⁷ gives the IRFs of the system⁸, which illustrates the response of the system to a standard-deviation shock. With a one percent positive technological change in the first period,

⁷Specification of the parameters can be found in the appendix.

⁸ Here we don't allow simultaneous search for newly separated workers in the current period, the results for the case with simultaneous search can be found in the appendix, which doesn't change the main results.

market tightness in both markets respond instantaneously, with much higher value for the unskilled worker, which is the old worker here, and much lower value for the skilled worker, which is the young worker here. This results completely fit with the prediction of proposition 1.

3.1.4 The demographic transition experiment

To be added

3.2 Search model with three generations

3.2.1 Stock flow of labor force

Suppose now we have 3 generations: young, middle age, old. Learning from Jaimovich and Siu (2009)'s empirical results, we treat young and old age groups as volatile when consider the cyclical variation in unemployment and job vacancies in response to productivity shock.

With skill premium and what we learn from the two generation model, it will be reasonable to come up with a skill evolving mechanism to make sure that middle age group have the easiest access to high skill, while old age group less and young age the least, thus, we entangle the demographic transition together with skill evolving process: young workers are all low skill and they automatically and unconditional become high skill workers once they enter into middle age, with this assumption, one extreme case would be some young unemployment worker might never get a job position and still become high skill worker once up to certain age; middle age unemployment workers have the risk of losing their skill premium (δ^d) and can regain with certain probability (δ^u) once become employed, which will make sure they stay in high skill relatively stably; old unemployment workers also have the same risk of losing their skill premium (δ^d) but can't regain once skill premium is lost.

Besides, consistent with the assumptions in the two generation model: young and middle age workers give birth and only old workers leave labor force and don't allow simultaneous search for newly separated workers in the current period, besides, production happens in the very beginning of each period.

With these setups, workers' transition will be:

$$u_{t+1}^{yi} = \rho^b(u_t^{mi} + n_t^{mi} + u_t^{me} + n_t^{me} + u_t^{yi} + n_t^{yi}) + (1 - \rho^m)[(1 - f(\theta_t^y))u_t^{yi} + sn_t^{yi}] \quad (6a)$$

$$n_{t+1}^{yi} = (1 - \rho^m)[f(\theta_t^y)u_t^{yi} + (1 - s)n_t^{yi}] \quad (6b)$$

$$l_{t+1}^y = \rho^b(u_t^{mi} + n_t^{mi} + u_t^{me} + n_t^{me} + u_t^{yi} + n_t^{yi}) + (1 - \rho^m)l_t^y \quad (6c)$$

$$u_{t+1}^{mi} = (1 - \rho^o)[(1 - f(\theta_t^m))(u_t^{mi} + \delta^d u_t^{me}) + s(1 - \delta^u)n_t^{mi}] \quad (7a)$$

$$n_{t+1}^{mi} = (1 - \rho^o)[f(\theta_t^m)(u_t^{mi} + \delta^d u_t^{me}) + (1 - s)(1 - \delta^u)n_t^{mi}] \quad (7b)$$

$$u_{t+1}^{me} = (1 - \rho^o)[(1 - f(\theta_t^m))(1 - \delta^d)u_t^{me} + s(n_t^{me} + \delta^u n_t^{mi})] \\ + \rho^m[(1 - f(\theta_t^m))u_t^{yi} + s n_t^{yi}] \quad (7c)$$

$$n_{t+1}^{me} = (1 - \rho^o)[f(\theta_t^m)(1 - \delta^d)u_t^{me} + (1 - s)(n_t^{me} + \delta^u n_t^{mi})] \\ + \rho^m[f(\theta_t^m)u_t^{yi} + (1 - s)n_t^{yi}] \quad (7d)$$

$$l_{t+1}^m = (1 - \rho^o)l_t^m + \rho^m l_t^y \quad (7e)$$

$$u_{t+1}^{oi} = (1 - \rho^d)[(1 - f(\theta_t^o))(u_t^{oi} + \delta^d u_t^{oe}) + s n_t^{oi}] \quad (8a)$$

$$+ \rho^o[(1 - f(\theta_t^o))(u_t^{mi} + \delta^d u_t^{me}) + s(1 - \delta^u)n_t^{mi}]$$

$$n_{t+1}^{oi} = (1 - \rho^d)[f(\theta_t^o)(u_t^{oi} + \delta^d u_t^{oe}) + (1 - s)n_t^{oi}] \quad (8b)$$

$$+ \rho^o[f(\theta_t^o)(u_t^{mi} + \delta^d u_t^{me}) + (1 - s)(1 - \delta^u)n_t^{mi}]$$

$$u_{t+1}^{oe} = (1 - \rho^d)[(1 - f(\theta_t^o))(1 - \delta^d)u_t^{oe} + s n_t^{oe}] \quad (8c)$$

$$+ \rho^o[(1 - f(\theta_t^o))(1 - \delta^d)u_t^{me} + s(n_t^{me} + \delta^u n_t^{mi})]$$

$$n_{t+1}^{oe} = (1 - \rho^d)[f(\theta_t^o)(1 - \delta^d)u_t^{oe} + (1 - s)n_t^{oe}] \quad (8d)$$

$$+ \rho^o[(1 - f(\theta_t^o))(1 - \delta^d)u_t^{me} + s(n_t^{me} + \delta^u n_t^{mi})]$$

$$l_{t+1}^o = (1 - \rho^d)l_t^o + \rho^o l_t^m \quad (8e)$$

Worker's transition in rate:

Method 1: divide the equations in level correspondingly with l_t^y , l_t^m and l_t^o . We will need the expressions for l_{t+1}^y/l_t^y , l_{t+1}^m/l_t^m and l_{t+1}^o/l_t^o , which can be easily obtained with the overall labor force flow equation for each age group. This method is much simpler, but the underlying assumption for steady state is the ratios l_t^{mi}/l_t^{me} and l_t^{oi}/l_t^{oe} are also going to be constant for steady state.

Method 2: divide the equations in level correspondingly with l_t^y , l_t^{mi} , l_t^{me} , l_t^{oi} and l_t^{oe} , which will allow us to get rid of the equations for employment and simplify all the equations into 5 equations and allow the variation of l_t^{mi}/l_t^{me} and l_t^{oi}/l_t^{oe} in steady state, but we will need five extra ratios: l_t^{mi}/l_t^{me} , l_t^{oi}/l_t^{oe} , l_t^{mi}/l_t^{oi} , l_t^{me}/l_t^{oe} and l_t^{yi}/l_t^{me} , which don't seem easy to calculate.

We have 5 types of labor force: young inexperienced, middle age inexperienced, middle age experienced, old inexperienced and old experienced. Assume the labor market is also divided into these 5 separated markets. The intuition behind this is both age and skill level are easy to verify with worker's ID card and skill level certificates. This assumption also tremendously simplifies our problem.⁹

⁹ Without this assumption and working with the labor market only divided by age, the skill level of any

3.2.2 Equilibrium

to be added

3.2.3 Propagation of shocks

to be added

3.2.4 The demographic transition experiment

To be added

4 Final Remarks

This research tries to solve cyclical unemployment fluctuation puzzle from the angle of the demographic change. We embed different generations into search model to generate cyclical unemployment fluctuation through combinational change among different generations. After we entangle the skill formation with demographic change, we are capable to match labor market volatility with its empirical data, which comoves with demographic transition. Unlike Hagedorn and Manovshii (2008), it doesn't require any strict specification of parameters like value of unemployment benefit or value of worker's bargaining power.

5 Appendix

5.1 Proof of Proposition 1

In steady state, 4 and 5 can be written as:

$$\frac{\zeta^o}{q^o} = (1-s)(1-\rho^d)\beta[A\varepsilon - \eta(A\varepsilon + \theta^o \frac{\zeta^o}{1-s}) - (1-\eta)z + \frac{\zeta^o}{q^o}] \quad (9)$$

newly hired middle or old age worker is going to be unknown, thus firm's hiring decision can only based on expectation of this new worker's skill level. The free entry condition for middle age labor market follows is:

$$\frac{\zeta^m}{q_t^m} = E_t \beta_{t+1} (s_t^{mi} J_{t+1}^{mi} + (1-s_t^{mi}) J_{t+1}^{me})$$

where $s_t^{mi} = \frac{(1-\rho^o)(u_t^{mi} + \delta^d u_t^{me})}{(1-\rho^o)u_t^m + \rho^m u_t^y}$ is the share of low skill middle age job searcher in period t before search market opens. It is impossible to plug this free entry condition back to wage's equation and get a clear expression of wage without worker's asset Bellman equations.

and

$$\frac{\varsigma^y}{q^y} = (1-s)(1-\rho^o)\beta[A - \eta(A + \theta^y \frac{\varsigma^y}{1-s}) - (1-\eta)z + \frac{\varsigma^y}{q^y}] + \frac{\rho^o}{1-\rho^d} \frac{\varsigma^o}{q^o} \quad (10)$$

From equation (6), we get:

$$\left[\frac{1}{(1-s)(1-\rho^d)\beta} - 1 \right] \frac{\varsigma^o}{q^o} + \eta \frac{\varsigma^o}{1-s} \theta^o = (1-\eta)\varepsilon(A - \frac{z}{\varepsilon})$$

Denote $\alpha = \frac{1}{(1-s)(1-\rho^d)\beta} - 1$ and totally differentiate the above equation wrt. θ^o and $(A - \frac{z}{\varepsilon})$ to get:

$$-\alpha \varsigma^o \frac{[q^o(\theta^o)]'}{[q^o(\theta^o)]^2} d\theta^o + \eta \frac{\varsigma^o}{1-s} d\theta^o = (1-\eta)\varepsilon d(A - \frac{z}{\varepsilon})$$

then, we can get:

$$\begin{aligned} e_{\theta^o, A}^o &= \frac{d\theta^o}{dA} \frac{A}{\theta^o} \\ &= \frac{A}{A - \frac{z}{\varepsilon}} \frac{d(A - \frac{z}{\varepsilon})}{dA} \frac{d\theta^o}{d(A - \frac{z}{\varepsilon})} \frac{(A - \frac{z}{\varepsilon})}{\theta^o} \\ &= \frac{A}{A - \frac{z}{\varepsilon}} \frac{(1-\eta)\varepsilon}{\eta \frac{\varsigma^o}{1-s} - \alpha \varsigma^o \frac{[q^o(\theta^o)]'}{[q^o(\theta^o)]^2}} \frac{\alpha \frac{\varsigma^o}{q^o} + \eta \frac{\varsigma^o}{1-s} \theta^o}{(1-\eta)\varepsilon \theta^o} \\ &= \frac{A}{A - \frac{z}{\varepsilon}} \underbrace{\frac{\alpha + \frac{\eta p^o(\theta^o)}{1-s}}{\alpha(1 - \Delta^o) + \frac{\eta p^o(\theta^o)}{1-s}}}_{1.24} \end{aligned}$$

where $\Delta^o \equiv \frac{[p^o(\theta^o)]'\theta^o}{p^o(\theta^o)}$ is the elasticity of $p^o(\theta^o)$ wrt. θ^o , $\alpha = \frac{1}{(1-s)(1-\rho^d)\beta} - 1 = 0.14$, $1 - \Delta^o = 0.6$, $\frac{\eta p^o(\theta^o)}{1-s} \approx 0.14$ in our experiment and the result is 8.55.

From equation (7), we get:

$$\left[\frac{1}{(1-s)(1-\rho^o)\beta} - 1 \right] \frac{\varsigma^y}{q^y} - \frac{\rho^o}{1-\rho^d} \frac{1}{(1-s)(1-\rho^o)\beta} \frac{\varsigma^o}{q^o} + \eta \frac{\varsigma^y}{1-s} \theta^y = (1-\eta)(A - z)$$

Denote $\alpha_1 = \left[\frac{1}{(1-s)(1-\rho^o)\beta} - 1 \right]$, $\alpha_2 = \frac{\rho^o}{1-\rho^d} \frac{1}{(1-s)(1-\rho^o)\beta}$ and totally differentiate the above equation wrt. θ^o , θ^y and $(A - z)$ to get:

$$(-\alpha_1 \varsigma^y \frac{[q^y(\theta^y)]'}{[q^y(\theta^y)]^2} + \eta \frac{\varsigma^y}{1-s}) d\theta^y + \alpha_2 \varsigma^o \frac{[q^o(\theta^o)]'}{[q^o(\theta^o)]^2} d\theta^o = (1-\eta) d(A - z)$$

$$\frac{A-z}{\theta^y} \frac{\theta^y}{A-z} \left(-\alpha_1 \varsigma^y \frac{[q^y(\theta^y)]'}{[q^y(\theta^y)]^2} + \eta \frac{\varsigma^y}{1-s} \right) \frac{d\theta^y}{d(A-z)} + \frac{(A-\frac{z}{\varepsilon})}{\theta^o} \frac{\theta^o}{(A-\frac{z}{\varepsilon})} \alpha_2 \varsigma^o \frac{[q^o(\theta^o)]'}{[q^o(\theta^o)]^2} \frac{d\theta^o}{d(A-z)} = (1-\eta)$$

$$\frac{A-z}{\theta^y} \frac{(-\alpha_1 \theta^y \frac{[q^y(\theta^y)]'}{[q^y(\theta^y)]^2} + \frac{\eta}{1-s} \theta^y)}{\frac{\alpha_1}{q^y} - \frac{\varsigma^o}{\varsigma^y} \frac{\alpha_2}{q^o} + \frac{\eta}{1-s} \theta^y} \frac{d\theta^y}{d(A-z)} + \frac{(A-\frac{z}{\varepsilon})}{\theta^o} \frac{\theta^o \varepsilon \alpha_2 \frac{[q^o(\theta^o)]'}{[q^o(\theta^o)]^2}}{\frac{\alpha}{q^o} + \frac{\eta}{1-s} \theta^o} \frac{d\theta^o}{d(A-z)} = 1$$

$$\frac{d\theta^y}{d(A-z)} \frac{A-z}{\theta^y} \frac{\alpha_1(1-\Delta^y) + \frac{\eta}{1-s} p^y(\theta^y)}{\alpha_1 + \frac{\eta}{1-s} p^y(\theta^y) - \alpha_2 \frac{\varsigma^o}{\varsigma^y} \frac{q^y}{q^o}} - \frac{d\theta^o}{d(A-z)} \frac{(A-\frac{z}{\varepsilon})}{\theta^o} \frac{\alpha_2 \varepsilon (1-\Delta^o)}{\alpha + \frac{\eta}{1-s} p^o(\theta^o)} = 1$$

Thus, we have:

$$\begin{aligned} e_{\theta,A}^y &= \frac{d\theta^y}{dA} \frac{A}{\theta^y} \\ &= \frac{A}{A-z} \frac{d(A-z)}{dA} \frac{d\theta^y}{d(A-z)} \frac{(A-z)}{\theta^y} \\ &= \frac{A}{A-z} \frac{\alpha_1 + \frac{\eta}{1-s} p^y(\theta^y) - \alpha_2 \frac{\varsigma^o}{\varsigma^y} \frac{\phi^y}{\phi^o} \left(\frac{\theta^o}{\theta^y}\right)^\mu}{\alpha_1(1-\Delta^y) + \frac{\eta}{1-s} p^y(\theta^y)} \left(1 + \frac{d\theta^o}{d(A-z)} \frac{(A-\frac{z}{\varepsilon})}{\theta^o} \frac{\alpha_2 \varepsilon (1-\Delta^o)}{\alpha + \frac{\eta}{1-s} p^o(\theta^o)}\right) \\ &= \underbrace{\frac{A}{A-z}}_2 \underbrace{\frac{\alpha_1 + \frac{\eta}{1-s} p^y(\theta^y) - \alpha_2 \frac{\varsigma^o}{\varsigma^y} \left(\frac{\theta^o}{\theta^y}\right)^\mu}{\alpha_1(1-\Delta^y) + \frac{\eta}{1-s} p^y(\theta^y)}}_{1.12} \underbrace{\left(1 + \frac{\alpha_2 \varepsilon (1-\Delta^o)}{\alpha(1-\Delta^o) + \frac{\eta}{1-s} p^o(\theta^o)}\right)}_{1.01} \end{aligned} \quad (11)$$

where $\Delta^y \equiv \frac{[p^y(\theta^y)]' \theta^y}{p^y(\theta^y)}$ is the elasticity of $p^y(\theta^y)$ wrt. θ^y , Δ^o is the elasticity of $p^o(\theta^o)$ wrt. θ^o and the result is 2.26.

5.2 Specification of parameters in our experiment

If we allow simultaneous search for newly separated workers in the current period, the IRFs will be changed with much less difference in unemployment's response between groups.

5.3 Impulse response function for the case with simultaneous search for newly separated

If we allow simultaneous search for newly separated workers in the current period, the IRFs will be changed with much less difference in unemployment's response between groups.

Table 1: Parameter values used in the experiment

Parameter	Value	Description
ρ^b	$1-1.085^{(-.25)}$	Birth rate
ρ^o	$1-1.023^{(-.25)}$	Probability of getting old
ρ^d	$1-1.1^{(-.25)}$	Probability of leaving the labor force
ε	0.7	Productivity efficiency of the old
z	0.6	Worker's bargaining weight
η	0.4	Value of non-market activity
s	0.05	Separation rate
ζ^o	0.4	Vacancy cost for old labor market
ζ^y	0.4	Vacancy cost for young labor market

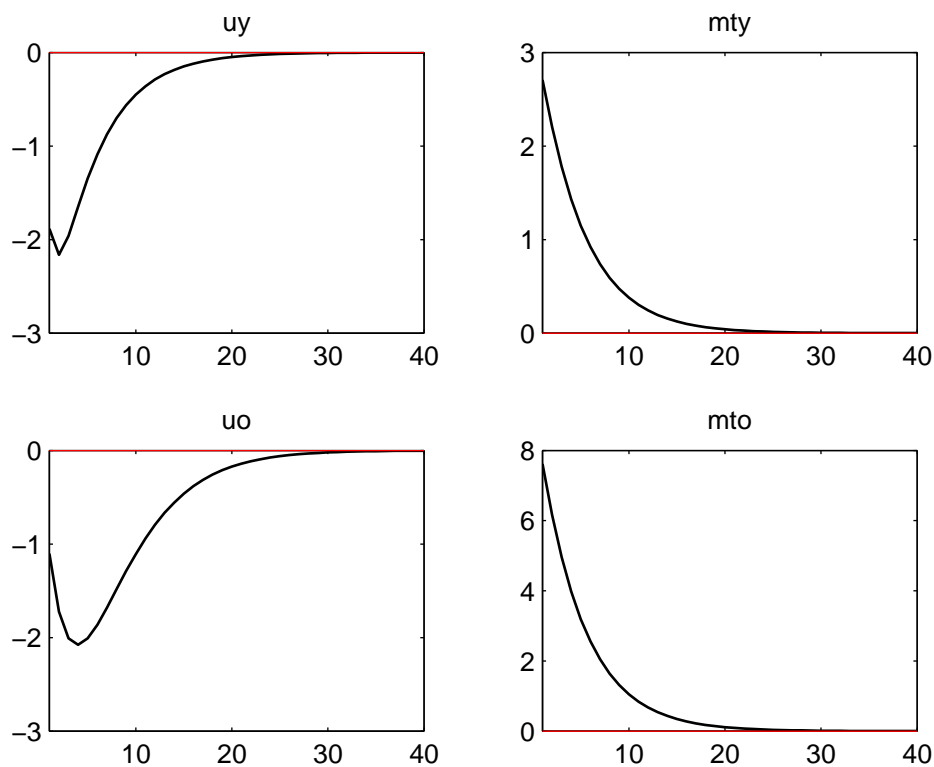


Figure 2: IRFs to a technology shock with simultaneous search

(uy stands for unemployment rate for young, uoy stands for unemployment rate for old, mty stands for market tightness for young and mo stands for market tightness for old)

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