

Efficiency-Wage Competition and Nonlinear Dynamics*

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Abstract

In this paper we develop a nonlinear version of the efficiency-wage competition model pioneered by Hahn (1987). Under the assumption that the strategic relationship among optimal wage bids put forward by competing firms is non-monotonic, we show that market wage offers can display persistent fluctuations. Thereafter, assuming that employers are never constrained in the labour market, we give evidence that in the parameter region of chaotic dynamics, the model is able to reproduce the business cycle regularity according to which average wages fluctuate less than aggregate employment.

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1 Introduction

The possibility of endogenous fluctuations in macroeconomic model with efficiency-wages has been explored by a number of scholars. For instance, Choi (1995) as well as Manfredi and Fanti (2000) address the dynamic consequences of assuming a link between workers' productivity and wages within the Goodwin's (1967) growth cycle model. Moreover, Coimbra (1999) considers the implications of increasing returns to scale and indivisible labour in an efficiency-wage setting à la Solow (1979) in order to explore the oscillations of employment and wages. In addition, De Palma and Seegmuller (2005) study the impact of wage differentials on macroeconomic fluctuations in an overlapping-generation model with a dual labour market grounded on the

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efficiency-wage apparatus by Shapiro and Stiglitz (1984). More recently, Grandmont (2008) introduces efficiency-wage arguments and involuntary unemployment in a finance-constrained economy with heterogeneous agents and unemployment insurance by deriving the conditions for the local (in)determinacy of the stationary solution.

The dynamic contributions mentioned above incorporate the efficiency-wage hypothesis according to which firms - regardless to the conditions prevailing in the labour market - do not cut wages because this would harm their own productivity. However, the referred works fail to consider the fact that sometimes employers actually do not cut wages because this would enhance the productivity of their output competitors (cf. Hahn, 1987; van de Klundert, 1988; Jellal and Wolff, 2002; Guerrazzi, 2013). Truthfully, the strategic implications of the efficiency-wage theory have been largely neglected by the literature that explored the deterministic dynamics arising from macroeconomic models with non-Walrasian labour markets.

In order to fill this gap, in this paper we develop a nonlinear version of the efficiency-wage competition model pioneered by Hahn (1987) recently revisited by Guerrazzi (2013). Under the assumption that the strategic relationship among optimal wage bids put forward by competing firms is non-monotonic switching from strategic complements to strategic substitutes, we show that market wage offers can actually display persistent fluctuations. The dynamic analysis is carried out by considering the case in which firms are perfectly coordinated as well as the one in which wage competitors act without coordination and, in both cases, we find we find multistability of equilibria and high-periodicity or chaotic attractors.

In addition, assuming that employers are never constrained in the labour market, i.e., assuming that labour supply is never binding for profit maximization, we give evidence that in the parameter region of chaotic dynamics, the model economy is able to reproduce the well-known business cycle regularity according to which average market wages fluctuate less than aggregate employment. Specifically, calibrating the mean of total factor productivity (TFP) in order to match US long-run unemployment, the theoretical model is fairly able to replicate the observed wage stickiness.

The paper is arranged as follows. Section 2 introduces a model of efficiency-wage competition. Section 3 discusses the existence and the strategic characterization of Nash equilibria. Section 4 characterizes some dynamical properties of the model under the assumption that the wage-competition process is played by firms as a sequential game of alternate wage offers. Section 5 takes into consideration the implied macroeconomic dynamics of average market wages and aggregate employment. Finally, section 6 concludes.

2 The model

We consider a model economy in which there are two firms indexed by i , $i = 1, 2$, that compete one another for high-quality workers. The production function of those productive units is

given by

$$Y_i = A (e_i L_i)^\alpha \quad 0 < \alpha < 1 \quad (1)$$

where Y_i is produced output, A is the TFP, e_i is the efficiency - or the working effort - of the labour force while L_i is the number of employed workers by the individual firm.¹

Nonlinearities enter the model through the effort function which is assumed to be the following:

$$e_i(w_i, w_{-i}) = (w_i - w_{-i}(\kappa - w_{-i}))^\beta \quad \kappa > 0, \quad 0 < \beta < 1 \quad (2)$$

where w_i is the wage offer of firm i .

The expression in eq. (2) reveals that for each firm workers' effort depends on its own wage bid as well as on the offer put forward by its opponent. According to the standard theory of efficiency-wages (cf. Solow, 1979), the more the firm pays, the more productive will be its incumbent workforce. Along this dimension - following Hahn (1987) and Guerrazzi (2013) - the effort function is assumed to be strictly concave. As suggested by Wu and Ho (2012), this effort specification implies that there is a real wage offer such that $e_i = 0$ - in the present case $w_i = w_{-i}(\kappa - w_{-i})$ - that can be dubbed as the minimum wage. Such an institutional feature of the labour market is implicitly neglected by contributors of the efficiency-wage theory that modeled workers' productivity in a different manner. For instance, s-shaped effort functions are usually assumed to pass through the origin so that $e_i = 0$ if and only if $w_i = 0$ (e.g. Stiglitz, 1973).

While the postulated link between e_i and w_i is rather standard, the relation between e_i and w_{-i} conveyed by eq. (2) is definitely mould-breaking; indeed, the reactivity of working effort to the wage offer put forward by the opponent firm is assumed to be non-monotonic. Specifically, in adherence with the literature on efficiency-wage competition (cf. Hahn, 1987; Jellal and Wolff, 2002; Guerrazzi, 2013), as long as $w_{-i} < \kappa/2$, effort for the firm i decreases (increases) whenever the firm $-i$ decides to increase (decrease) its wage bid. However, when $w_{-i} > \kappa/2$, the situation is assumed to be the opposite so that an increase (decrease) in the wage offer of the competing firm leads to an increase (decrease) in the efficiency of the employed workforce. A pictorial representation of eq. (2) is given in figure 1.

The negative relation between e_i and w_{-i} should appear quite trivial. Everything else being equal, whenever the firm $-i$ decides to increase its wage bid the workers employed by firm i are likely to perceive that they are not fairly treated by their current employer (cf. Adams, 1963; Kahneman et al., 1986a-b). Therefore, unless firm i decide to follow its opponent by raising wages, workers will be discouraged and they will provide less effort.² The effort function in

¹Equivalent specifications are exploited in Akerlof (1982), Alexopoulos (2004) and Guerrazzi (2013).

²Consequently, $\kappa/2$ is a measure of what workers employed by firm i reckon to be the fair wage offer from firm $-i$ provided that their employer is paying a wage above the minimum level (e.g. Akerlof and Yellen, 1990;

eq. (2) is, however, suited to describe a situation in which this pattern comes to an end and it also reverses its shape. A rationale for this switching behaviour can be given on the account of two important foundations of the efficiency-wage theory. The former is the negative relation between wages and employment conveyed by labour demand.³ The latter is the disciplining effect of unemployment, i.e., the counter-cyclical pattern of effort (e.g. Oster, 1980; Shapiro and Stiglitz, 1984; Rebitzer, 1987, 1988; Green and Weisskopf 1990; Ackum Agell, 1994; Uhlig and Xu, 1996; Guerrazzi, 2007, 2013). In this way, when w_{-i} becomes high enough, i.e., higher than $\kappa/2$, workers might realize that current and future employment opportunities will be rationed.⁴ Consequently, this will lead the employees of firm i to work harder.

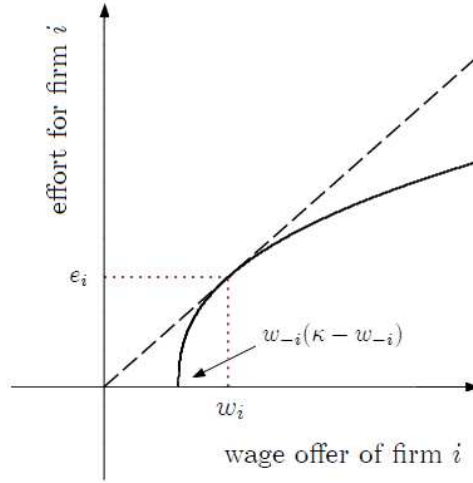


Figure 1: Effort function

As far as eq.s (1) and (2) are concerned, profits of the individual firm are given by

$$\pi_i(w_i, w_{-i}, L_i) = A(e_i(w_i, w_{-i})L_i)^\alpha - w_i L_i \quad (3)$$

Thereafter, the problem of firm i can be conveyed as

$$\max_{L_i \geq 0, w_i \geq w_{-i}(\kappa - w_{-i})} \pi_i(w_i, w_{-i}, L_i) \quad (4)$$

where $w_{-i} \geq 0$ is taken as given by the individual firm.

Let us now assume that $w_{-i} > 0$. Whenever w_i is fixed such that $w_i - w_{-i}(\kappa - w_{-i}) = 0$, i.e., $w_i = w_{-i}(\kappa - w_{-i})$, π_i achieves its maximum for $L_i = 0$ and in this case profit will be equal to zero. Instead, whenever w_i is fixed such that $w_i - w_{-i}(\kappa - w_{-i}) > 0$, the firm will obtain a

Johansen and Strøm, 2001). In fact, if $w_{-i} = \kappa/2$, then $\partial e_i / \partial w_{-i} = 0$.

³This in turn usually follows from the hypothesis of decreasing returns with respect to labour in the production process.

⁴Along these lines, Summers (1988) assumes that workers' outside opportunities are negatively linked to the unemployment rate.

positive profit. This implies that the solution of (4) is interior. Therefore, the conditions that characterize its solution are given by

$$L_i : \Psi(w_i, w_{-i}, L_i) e_i(w_i, w_{-i}) = w_i \quad (5)$$

$$w_i : \Psi(w_i, w_{-i}, L_i) \frac{\partial e_i(w_i, w_{-i})}{\partial w_i} = 1 \quad (6)$$

where $\Psi(w_i, w_{-i}, L_i) := \alpha A (e_i(w_i, w_{-i}) L_i)^{\alpha-1}$.

It is worth noting that whenever $w_{-i} = 0$ the problem in (4) is not well defined because the profit function is unbounded from above; indeed, taking the feasible allocation $w_i = L_i^{-1}$, profit becomes $\pi_i(L_i^{-1}, 0, L_i) = AL_i^{\alpha(1-\beta)} - 1$ and this implies that $\lim_{L_i \rightarrow +\infty} \pi_i(L_i^{-1}, 0, L_i) = +\infty$.

On the one hand, eq. (5) is labour demand from the single firm. On the other hand, eq. (6) is the FOC with respect to w_i . After a trivial manipulation, the two expressions above allow to derive the well-known Solow (1979) condition that - in the present context - reads as

$$\frac{\frac{\partial e_i(w_i, w_{-i})}{\partial w_i} w_i}{e_i(w_i, w_{-i})} = 1 \quad (7)$$

The expression in (7) shows that for each firm the optimal wage bid is such that the elasticity of the effort function with respect to the insider wage is equal to 1. Since the effort attainable by firm i depends on the wage offer put forward by firm $-i$, the two productive units find themselves in a situation of strategic interaction regarding wages that resembles - to some extent - the one described by Cournot (1838) in the context of oligopolistic output competition (cf. Guerrazzi, 2013). Under the assumption that labour supply is never binding so that jobless workers are involuntary unemployed, we will show that such an interaction may produce persistent fluctuations of market wages and aggregate employment.

3 Existence and strategic characterization of Nash equilibria

A Nash equilibrium for the model economy described in section 2 is an allocation such that the Solow (1979) condition in (7) holds simultaneously for the two firms. In order to find the wage strategies that fulfill this requirement, it is necessary to derive the reaction functions of the two productive units and solve the resulting system of equations. Taking into account eq. (2), the equality in (7) implies that optimal wage reply for firm i is given by

$$w_i = f_i(w_{-i}) := \frac{w_{-i}(\kappa - w_{-i})}{1 - \beta} \quad (8)$$

with $w_{-i} > 0$.

The expression in (8) reveals that for each firm the reaction function is a reverse-u-shaped relation with respect to the wage offer put forward by the opponent that drops to zero at the

origin as well as at κ . Such a parabolic relation reaches its maximum at $\kappa/2$ and at that point the reference firm finds profitable to bid a wage equal to $\kappa^2/4(1-\beta)$.

From an economic perspective, the interesting feature of the reaction function in (8) is the non-monotonic pattern of the strategic relationships that holds among the optimal wage offers of the two competing firms. Specifically, as long as the opponent firm bids a wage lower than $\kappa/2$, the optimal wage offers of the reference productive unit are strategic complements. Consequently, whenever the competitor increases (decreases) its wage offer, the optimal wage reply for the reference firm is to rise (decrease) its offer as well. By contrast, taking into account the already discussed properties of the effort function, when the opponent bids a wage higher than $\kappa/2$ the disciplining effect of unemployment is assumed to start to bite. Therefore, in this case optimal wage offers become strategic substitutes so that whenever the competitor decides to increase (decrease) its wage bid, the optimal response for the reference firm is to do the opposite by reducing (increasing) its wage offer.

In this model economy a Nash equilibrium is attained when the expression in (8) holds simultaneously for the two firms. Generally speaking, such a nonlinear relation implies the existence of three distinct Nash equilibria. Those allocations are given by the mutual optimal wage bids of the two firms so that they can be denoted by (w_1, w_2) pairs. Straightforward algebra reveals that Nash equilibria, when they exist, are given by

$$\begin{aligned} E_1 &:= (\kappa + \beta - 1, \kappa + \beta - 1) \\ E_2 &:= \left(\frac{\kappa + 1 - \beta + \sqrt{\Lambda}}{2}, \frac{\kappa + 1 - \beta - \sqrt{\Lambda}}{2} \right) \\ E_3 &:= \left(\frac{\kappa + 1 - \beta - \sqrt{\Lambda}}{2}, \frac{\kappa + 1 - \beta + \sqrt{\Lambda}}{2} \right) \end{aligned} \tag{9}$$

where $\Lambda := (\kappa + 1 - \beta)(\kappa - 3(1 - \beta))$.

The description of the wage strategies in (9) is straightforward. Specifically, E_1 defines the symmetric Nash equilibrium, i.e., an equilibrium in which the two firms offer exactly the same (positive) wage, and it is economically feasible if and only if $\kappa > 1 - \beta$. In the remainder of the paper, we will assume that such a condition is always verified. Namely,

Assumption 1 $\kappa > 1 - \beta$

The two asymmetric Nash equilibria E_2 and E_3 , i.e., the equilibria in which one of the two productive units bids a wage higher than the other, exist if and only if $\Lambda > 0$. That inequality holds whenever $\kappa > 3(1 - \beta)$. Consequently, for $1 - \beta < \kappa < 3(1 - \beta)$ only the symmetric Nash equilibrium exists. The reaction functions of the two firms along with all their possible intersections are illustrated in figure 2 (dashed curves denote the thresholds beyond which the effort for both firms is strictly positive).

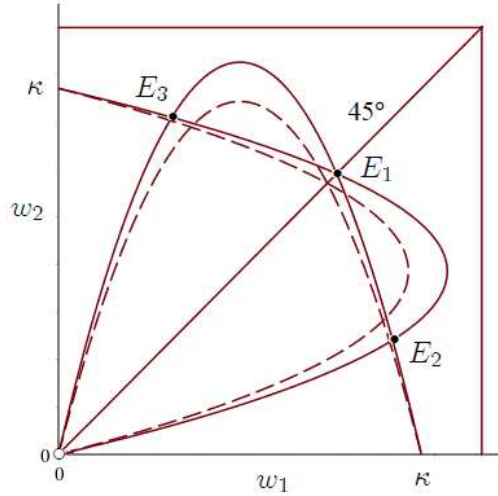


Figure 2: Nash equilibria

The diagram shows that when there exist three distinct Nash equilibria the symmetric equilibrium strategy, i.e., the wage strategy in which the two firms find profitable to bid the same wage, entails another important equivalence. Indeed, since in the neighbourhoods of E_1 both reaction functions are downward-sloped, this means that for the two productive units optimal wage offers are strategic substitutes. This strategic pattern holds as long as the symmetric equilibrium is found beyond the point in which the reaction functions achieve their maximum value, i.e., as long as $\kappa > 2(1 - \beta)$. By contrast, when κ falls in the interval $(1 - \beta, 2(1 - \beta))$ optimal wage offers in the neighbourhoods of (w_1^*, w_2^*) become strategic complements. Obviously, taking into account the condition under which Λ is positive, strategic complementarity is not consistent with the existence of asymmetric Nash equilibria.

4 Market wage dynamics under sequential wage offers

When a limited set of firms compete one another for output supply, individual quantity provisions are often assumed to adjust conjecturing that, in any given period of time, each productive unit observes the other firms' outputs and these quantities will remain unchanged in the next period (cf. Theocharis, 1960).

Translating this Cournotian conjecture into the model of efficiency-wage competition requires the introduction two categories of wages, i.e., the bidden one and the one actually paid to workers, say, respectively, $w_{i,b}$ and $w_{i,m}$. The former is a notional reference, while the latter is the effective value of the wage prevailing on the labour market. Those two variables are

assumed to be determined as follows. First, once $w_{-i,m}$ is known by each firm, $w_{i,b}$ is retrieved from the individual reaction function. Formally speaking, this yields

$$w_{i,b} = \frac{w_{-i,m}(\kappa - w_{-i,m})}{1 - \beta} \quad (10)$$

Thereafter, taking into account (2) and (10), the wage actually paid to workers in the subsequent period is

$$w'_{i,m} = \begin{cases} w_{i,b} & \text{whenever } w_{i,b} - w_{-i,b}(\kappa - w_{-i,b}) \geq 0 \\ w_{-i,b}(\kappa - w_{-i,b}) & \text{whenever } w_{i,b} - w_{-i,b}(\kappa - w_{-i,b}) < 0 \end{cases} \quad (11)$$

where \prime indicates the shift operator.

The latter (former) case describes the situation in which the minimum wage is (not) binding for the single firm. This kind of dynamic adjustment allows to disentangle the dynamics of notional and effective - or market - wage offers. For each firm, the two coincides when the two players bid wages that deliver a feasible level of effort, i.e., $e_i > 0$. When feasibility fails, each firm is assumed to adjust its wage offer to the minimum level in order to accommodate a shirking allocation in which $e_i = L_i = 0$. The underlying hypothesis of this adjustment mechanism is that in the background there is an employment agency - or a union - that collects and transmits the wages bids of competing firms to workers. On the basis of this information, the production possibilities of employers are binded by workers' effort decisions and market wages have to be set accordingly.

By introducing the functions $F(z) := \frac{z(\kappa-z)}{1-\beta}$ and $G(z) := \frac{z(\kappa-z)}{1-\beta} \left(\kappa - \frac{z(\kappa-z)}{1-\beta} \right)$ the dynamics of market wages obtained by the iteration of the two-dimensional piecewise differentiable map $T : R^2 \rightarrow R^2$ defined as

$$T := \begin{cases} x' = \begin{cases} F(y) & \text{if } F(y) - G(x) \geq 0 \\ G(x) & \text{if } F(y) - G(x) < 0 \end{cases} \\ y' = \begin{cases} F(x) & \text{if } F(x) - G(y) \geq 0 \\ G(y) & \text{if } F(x) - G(y) < 0 \end{cases} \end{cases} \quad (12)$$

The map T is symmetric, i.e., it does not change if variables x and y are swapped. In other words, $T \circ S = S \circ T$, where $S : (x, y) \rightarrow (y, x)$. This implies that the diagonal defined as $\Delta = \{(x, y) : x = y\}$ is an invariant manifold, i.e., the dynamics of market wages lie on Δ for every t by starting with $x(0) = y(0)$. Obviously, this means that the dynamics of $w_{i,m}$ and $w_{-i,m}$ is synchronized. Specifically, when an initial condition is taken on the diagonal, the dynamics of market wages is described by the restriction T_Δ of map T on Δ , say $q(x) = T_\Delta : \Delta \rightarrow \Delta$, where the map q results from setting $y = x$ in (12), and is given by

$$T_\Delta : x' = q(x) := \max \{F(x), G(x)\} \quad (13)$$

In this case, there is a perfect coordination in the wage setting behaviour of the two firms. In other words, consistently with a one-dimensional efficiency-wage model, the map (13) describes

the dynamics of the market wage offers put forward by only one representative firm who interacts with the employment agency mentioned above.

Now we shortly state some properties of the map T_Δ in (13). Specifically, F is an unimodal map conjugate to the logistic map $z' = \mu z(1 - z)$ through the change of variables $x = \mu z$ and $\mu = \kappa/(1 - \beta)$ with a maximum point at $\kappa/2$. The function G describes an unimodal (respectively, bimodal) map if $\kappa \in (1 - \beta, 2(1 - \beta))$ (respectively, $\kappa > 2(1 - \beta)$) with a maximum point at $\kappa/2$, (respectively, with a minimum point at $x = \kappa/2$ and two maxima at $x = x_{M_1} := \left(\kappa - \sqrt{\kappa(\kappa - 2(1 - \beta))}\right)/2$ and $x = x_{M_2} := \left(\kappa + \sqrt{\kappa(\kappa - 2(1 - \beta))}\right)/2$). In addition, $F(0) = G(0) = F(\kappa) = G(\kappa) = 0$ and if $\kappa < 1$, then $F(x) > G(x) \forall x \in (0, \kappa)$, whereas if $\kappa > 1$, then there exist \bar{x} and $\bar{\bar{x}}$ such that $F(x) < G(x)$ (respectively, $F(x) > G(x)$) for $x \in (0, \bar{x})$ or $x \in (\bar{\bar{x}}, 0)$ (respectively, for $x \in (\bar{x}, \bar{\bar{x}})$).

Having said that, it is easy to state the following results:

Lemma 1 *The map T_Δ always admits the fixed point $x^* = \kappa + \beta - 1$ and it is the unique fixed of F .*

The proof of Lemma 1 immediately follows from Assumption 1. Moreover, we have

Proposition 2 *If $\kappa < 3(1 - \beta)$, then x^* is locally asymptotically stable. If $3(1 - \beta) < \kappa < 4(1 - \beta)$, then x^* is unstable and the set $[0, \kappa]$ is trapping. For $\kappa = 3(1 - \beta)$ the map T_Δ undergoes a flip bifurcation.*

Lemma 3 *If $\kappa < 1$, then no fixed points exist for G .*

Proof. *If $\kappa < 1$, then $F(x) > G(x) \forall x \in (0, \kappa)$. Therefore, any solution of $G(x) = x$ is not feasible. ■*

Lemma 4 *For any $\beta \in (0, 1)$ there exists a value $\bar{\kappa} > 1$ such that $\forall \kappa < \bar{\kappa}$, no fixed points exist for G . In addition, for $\kappa = \bar{\kappa}$, G undergoes a saddle node bifurcation and $\forall \kappa > \bar{\kappa}$, there are two fixed points x^{**} and x^{***} with $x^{**} < x^{***}$.*

Proposition 5 *If x^{**} and x^{***} exist, then x^{**} is unstable and there exists a value $\bar{\bar{\kappa}}$ such that x^{***} is stable $\kappa \in (\bar{\bar{\kappa}}, \bar{\kappa})$. Moreover, for $\kappa = \bar{\bar{\kappa}}$, G undergoes a flip bifurcation whereas $\forall \kappa > \bar{\bar{\kappa}}$, x^{***} is unstable.*

Remark 6 *Notice that the equation $x = G(x)$ can have three solutions x_1^* , x^{**} and x^{***} . However, since $F(x_1^*) > G(x_1^*)$, x_1^* is not a feasible fixed point for the map T_Δ in (13).*

The three panels of figure 3 show the evolution of dynamics on the diagonal when κ varies and β is set at 0.23. Consistently to Proposition (2), for κ sufficiently low ($\kappa < 2.31$), the fixed point x^* is locally asymptotically stable whereas for $\kappa = 2.31$, x^* loses its stability and it undergoes a supercritical flip bifurcation generating an attracting two-period cycle. We can

note that for $\kappa = \tilde{\kappa} \simeq 2.365$ the system undergoes a border collision (see the bifurcation diagram in figure 3.a). Increasing the value of κ we have a sequence of period doubling bifurcations and the dynamics is characterized by a chaotic regime for $\kappa > 2.85$ spaced out by some windows of stability (see also figure 3.c where the Lyapunov exponent is plotted in blue). The red area in figure 3.a shows the birth and the evolution of a second attractor on the diagonal. Moreover, the figure 3.b depicts the coexistence of two chaotic attractors for $\kappa = 2.94$ (see also the values of Lyapunov exponents in figure 3.c). In addition, it is worth noting that for both attractors the two pieces of map (13) are involved.

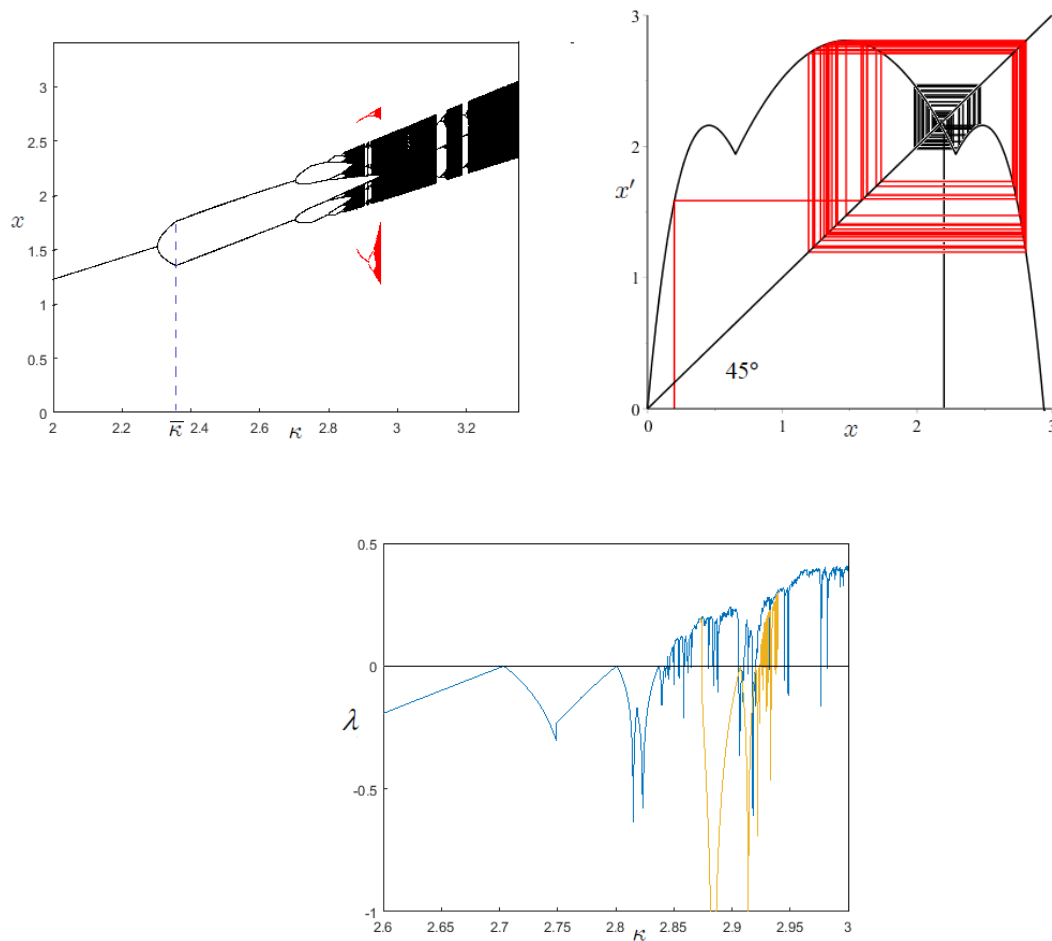


Figure 3: (a) Bifurcation diagram for κ ($\beta = 0.23$). The black area is obtained by varying the initial condition in order to display the evolution of the attractor when κ is changed. The red part shows a coexisting attractor on the diagonal for $\kappa \in (2.88, 2.95)$; (b) Two coexisting chaotic attractors on the diagonal; (c) Lyapunov exponents for the two attractors

If we let κ increase further, then for $\kappa \in (\kappa^*, \bar{\kappa})$, where $\kappa^* = 3.08$ and $\bar{\kappa} \simeq 3.465$, we have $F(\kappa/2) > \kappa$ and there exists the set with positive measure $S_1 := \bigcup_{n=0}^{-\infty} (T_\Delta)^{-n}(x_1, x_2)$ such that a generic initial condition starting from S_1 generates a trajectory that becomes negative, that is

unfeasible, after a finite number of iterations. Nonetheless, an attractor Ψ survives and given that $F^2(\bar{x}) > x_{M_2}$ it lies in the interval bounded by $F(\bar{x})$ and $F^2(\bar{x})$. Figure 4.a, depicted for $\kappa = 3.15$, also illustrates the second-iterate map that shows the existence of a 2-period repulsive cycle for T_Δ which in turn defines the immediate basin of attraction of Ψ , say B . Because of the non invertibility of the map (13) and the shape of the graphs of F and G , the whole basin of attraction of Ψ , say \mathbf{B} , is formed by an infinite number of non-connected portions given by the preimages of B , i.e., $\mathbf{B} = \bigcup_{n=0}^{-\infty} (T_\Delta)^{-n}(B)$. The grey segments on the 45-line in figure 4.a show its largest portions.

For $\kappa > \bar{\kappa}$, i.e., after the saddle node bifurcation ($\bar{\kappa} \simeq 3.465$), a new attractor emerges for the system (13) as illustrated in figure 4.b depicted for $\kappa = 3.67$. Since this attractor is born in an area of the state-space in which - before the bifurcation - Ψ existed, its birth causes the death of Ψ and almost the trajectories converge to x^{***} or to the new attractor around x^{***} . By contrast, for $\kappa > 3.685$ almost all trajectories diverge.

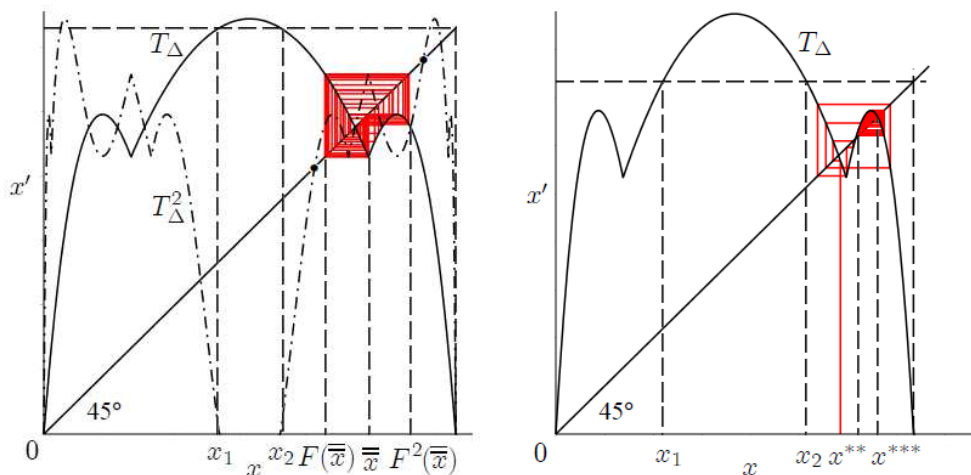


Figure 4: Dynamics on the diagonal, respectively, for $\kappa = 3.15$ (a) and $\kappa = 3.67$ (b)

Selecting a lower value for β it becomes possible to obtain the coexistence of three fixed points even with a map onto itself. An example in which almost all the trajectories are caught by an attractor born around x^{***} is illustrated in figure 5 for $\beta = 0.023$ and $\kappa = 3.82$.

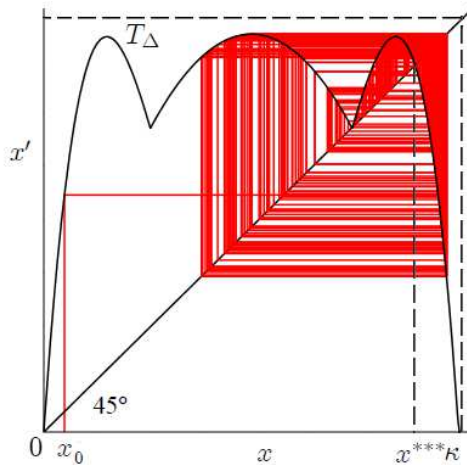


Figure 5: Dynamics on the diagonal for $\beta = 0.023$ and $\kappa = 3.82$

We can now start to address the two dimensional map. To begin with, we notice that the map (12) has a peculiar property; indeed, for each iteration, the state of any variable is determined by the lagged state of the variable itself or by the other one. More precisely, we have three possible cases:

- When the iterate of the two variables entering the dynamic system in (12) is driven by F , the state of each variable is determined by the lagged state of the other variable. In this case, the market wage offers of the two firms obey their respective reaction functions. From an analytic point of view, following Bischi et al. (2000) and by considering the symmetry of the map we can state the next properties. Let $H(z) := F \circ F(z)$, then we have

Proposition 7 *Let (x_0, y_0) be an initial condition such that the trajectory is described by F for any iteration $n \geq 1$. Then $T^{2n}(x_0, y_0) = (H^n(x_0), H^n(y_0))$, and $T^{2n-1}(x_0, y_0) = (F^{2n-1}(x_0), F^{2n-1}(y_0))$.*

This property follows from the fact that the square map T^2 , i.e., the second iterate of T , is a decoupled map. This implies that the value taken by the odd iterate of each variable depends on the initial condition of the other variable and its point value is determined by applying only F . Moreover, the value taken by the even iterate depends on the initial condition of the variable itself by applying only G . Additional properties of similar maps are deepened in Bischi et al. (2000).

- When the iterate of x and the iterate of y are both calculated by applying G , the map is decoupled. This implies that the evolution of x is not affected by y and viceversa. Consequently, from an economic point of view, the wage behaviour of each firm does not depend on the lagged choices of its competitor.

- When the iterates of x and y are driven, respectively, by F and G , the system becomes triangular. In this case, the strategic relationship among the two firms becomes somehow asymmetric. Specifically, the wage paid by firm 1 (x') is determined by the lagged choices of firm 2 (y). By contrast, taking into account the information conveyed by the employment agency, firm 2 is required to pay the minimum wage. Similar arguments hold by swapping x with y .

Proposition 8 *Under Assumption 1, we have that E_1 is a fixed point for (12). If $\kappa > 3(1 - \beta)$, then we have that E_2 and E_3 are fixed points for (12). There may exist other fixed points: a) $\tilde{E}_i = (z_r, z_s)$, $r = 1, \dots, 3$, $s = 1, \dots, 3$ where z_p is a solution of $z_p = G(z_p)$ such that $F(z_p) < G(z_p)$, $p = r, s$; b) $\bar{E}_j = (m, n)$ and $\hat{E}_j = (n, m)$, $j = 1, \dots, 3$, where (m, n) are solutions of $\begin{cases} m = F(n) \\ n = G(m) \end{cases}$ such that $G(m) \leq F(n) < G(n)$.*

In order to exemplify the action of the piece-wise map under scrutiny, figure 6 shows the areas Ω_i - with $i = 1, \dots, 4$ - in which the four distinct specifications in (12) apply and the respective fixed points (see the associated caption for details).

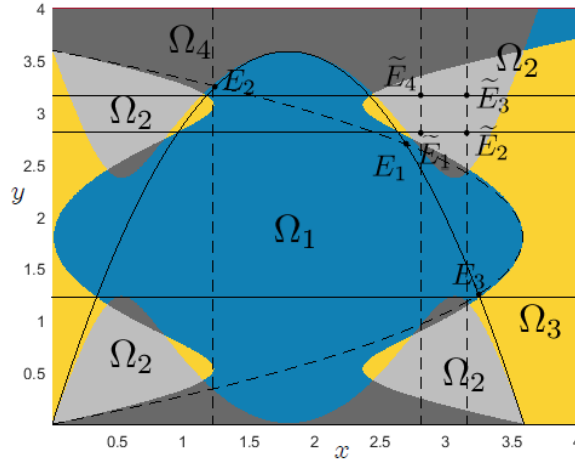


Figure 6: Regions Ω_i ($i = 1, \dots, 4$) of the phase plane where different pieces of the map (12) work are evidenced by different colors ($\beta = 0.1$ and $\kappa = 3.59$). The blue region is $\Omega_1 := \{(x, y) \in \mathfrak{R}_{++}^2 : F(y) - G(x) \geq 0 \text{ and } F(x) - G(y) \geq 0\}$, the light-grey region is $\Omega_2 := \{(x, y) \in \mathfrak{R}_{++}^2 : F(y) - G(x) < 0 \text{ and } F(x) - G(y) < 0\}$, the yellow region is $\Omega_3 := \{(x, y) \in \mathfrak{R}_{++}^2 : F(y) - G(x) \geq 0 \text{ and } F(x) - G(y) < 0\}$ and the dark-grey region is $\Omega_4 := \{(x, y) \in \mathfrak{R}_{++}^2 : F(y) - G(x) < 0 \text{ and } F(x) - G(y) \geq 0\}$. Only the largest portions of Ω_i ($i = 1, \dots, 4$) are labelled. Dashed (solid) lines depict loci such that $x = F(y)$ or $x = G(x)$ ($y = F(x)$ or $y = G(y)$)

It is worth noting that despite the properties just described the map can not be decoupled because of the dynamic constraints implied by the existence of a minimum wage (see the definitions of F and G in map (12)); indeed, the constraints involve both variables at the same time. This, in turn, implies that the trajectories generated by map (12) can switch persistently among the different regions Ω_i ($i = 1, \dots, 4$).

Taking into account the stability properties of the fixed points, the following results hold:

Theorem 9 *If $\kappa < 3(1 - \beta)$, then E_1 is locally asymptotically stable. If $\kappa = 3(1 - \beta)$, then E_1 undergoes a flip bifurcation.*

In order to characterize the action of the map, it is useful to introduce the diagram in figure 5 that depicts the graph of the functions $F(z)$ and $G(z)$.

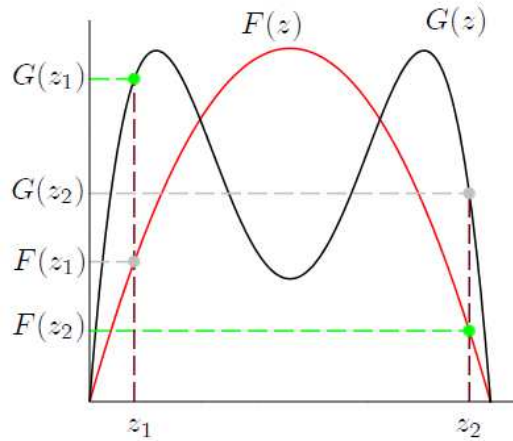


Figure 7: Geometrical properties of the action of the map in (12)

Given a generic couple $(x, y) = (z_1, z_2)$, by considering the values attained by F and G in correspondence of such a couple, it is possible to find the piece of map that is actually working. For instance, in the case illustrated in figure 7 we have that $F(z_1) < G(z_2)$, therefore $x' = G(z_2)$. Moreover, since $F(z_2) < G(z_1)$, it follows that $y' = G(z_1)$.

It is worth noting that the diagram in figure 7 can be taken as a starting point for the analysis of successive iterates of the map on the plane. With this aim in mind, we consider the graph of $H(z)$, i.e., $F^2(z)$, and we implement the following graphical rule for the dynamics of x :

Let (x, y) , (x', y') and (x'', y'') be three iterates of the map in (12). Thereafter,

1) If (x', y') is such that $F(y) > G(x)$ and $F(y') > G(x')$, then we draw three segments: *i*) a segment starting from the point (y, x') that belongs to the graph of $F(z)$ and ending to the point $(x, 0)$; *ii*) a vertical segment starting from the point $(x, 0)$ and ending to the point (x, x'') that belongs to the graph of $H(z)$; *iii*) a horizontal segment starting from the point (x, x'') and ending to the point (y', x'') that belongs to the graph of $F(z)$.

2) If (x', y') is such that $F(y) > G(x)$ and $F(y') < G(x')$, then we draw two segments: *i*) a horizontal segment starting from the point (y, x') and ending to the point (x', x') ; *ii*) a vertical segment starting from the point (x', x') and ending to the point (x', x'') .

3) If (x', y') is such that $F(y) < G(x)$ and $F(y') > G(x')$, then we draw a segment starting from the point (x, x') and ending to the point (y', x'') .

4) If (x', y') is such that $F(y) < G(x)$ and $F(y') < G(x')$, then we draw two segments: *i*) a horizontal segment starting from the point (x, x') and ending to the point (x', x') ; *ii*) a vertical segment starting from the point (x', x') and ending to the point (x', x'') .

By considering the evolution of the ordinates of the graph we can infer about the fate of one of the coordinates of the trajectory.⁵ Specifically, if the ordinates of the graph converge to a unique value, then the associated coordinate stabilizes around a stationary value; if the ordinates of the graph oscillate, then the associated coordinate displays a cyclical or a chaotic behaviour. Moreover, by looking at which graphs are involved in the iterates (F , F^2 or G) we can understand which pieces of map act.

In order to appraise different possible varieties of dynamic behaviour, let us consider some numerical examples. We start with $\beta = 0.213$ and we let κ vary. For $\kappa = 2.3$ a unique fixed point exists and it corresponds to the symmetric Nash equilibrium defined in (9). Its basin of attraction is depicted in figure 8.a where the white area describes a portion of the set of initial conditions generating unfeasible trajectories, i.e., initial conditions such that one or both wage offers becomes definitely negative. In this case, the long run dynamics are governed by the F -components of map (12) as illustrated in figure 8.b. Given the piecewise definition of the map, it is worth noting that even if one of the two coordinates of the initial condition is higher than κ , provided that the other coordinate is such that G is acting, we have that the generated trajectory is feasible for any iterate and it converges towards the Nash equilibrium.

⁵For readability reasons we apply the procedure only for the x -coordinate.

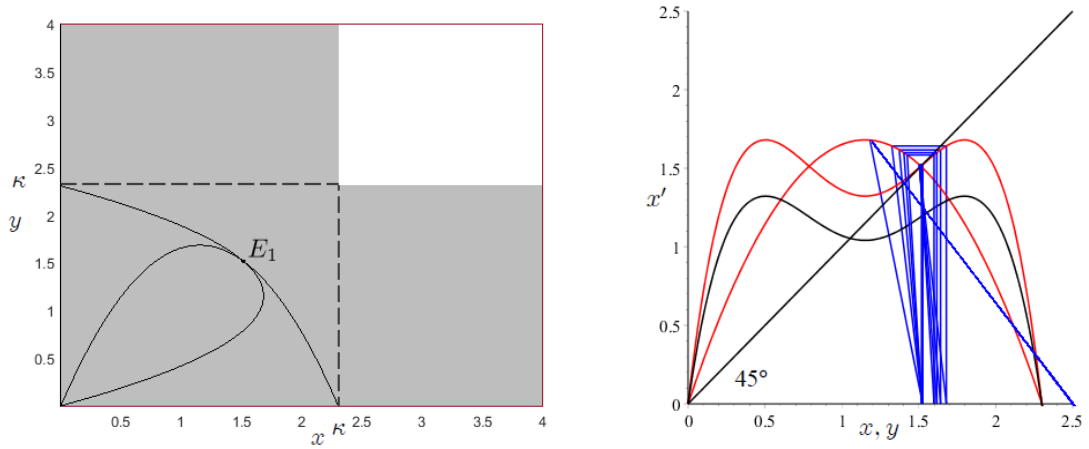


Figure 8: (a) Basin of attraction of the symmetric Nash equilibrium; (b) Dynamics of x . We can see that after the first iteration, the iterates are alternatively given by the two red curves in figure and converge to the Nash equilibrium.

If we let κ increase, then E_1 undergoes a degenerate flip bifurcation and an attractor on the diagonal coexists with two attracting symmetric fixed points (see figure 9.a depicted for $\kappa = 2.37$). At this stage of analysis, we can notice that the attractors are all generated by the action of the F components of the map (12). Moreover, for $\kappa \simeq 2.41$ the cycle on the diagonal undergoes a border collision bifurcation. This causes the loss of symmetry of the coordinates of cycles (see figure 9.b, 9.c and 9.d depicted for $\kappa = 2.6$).

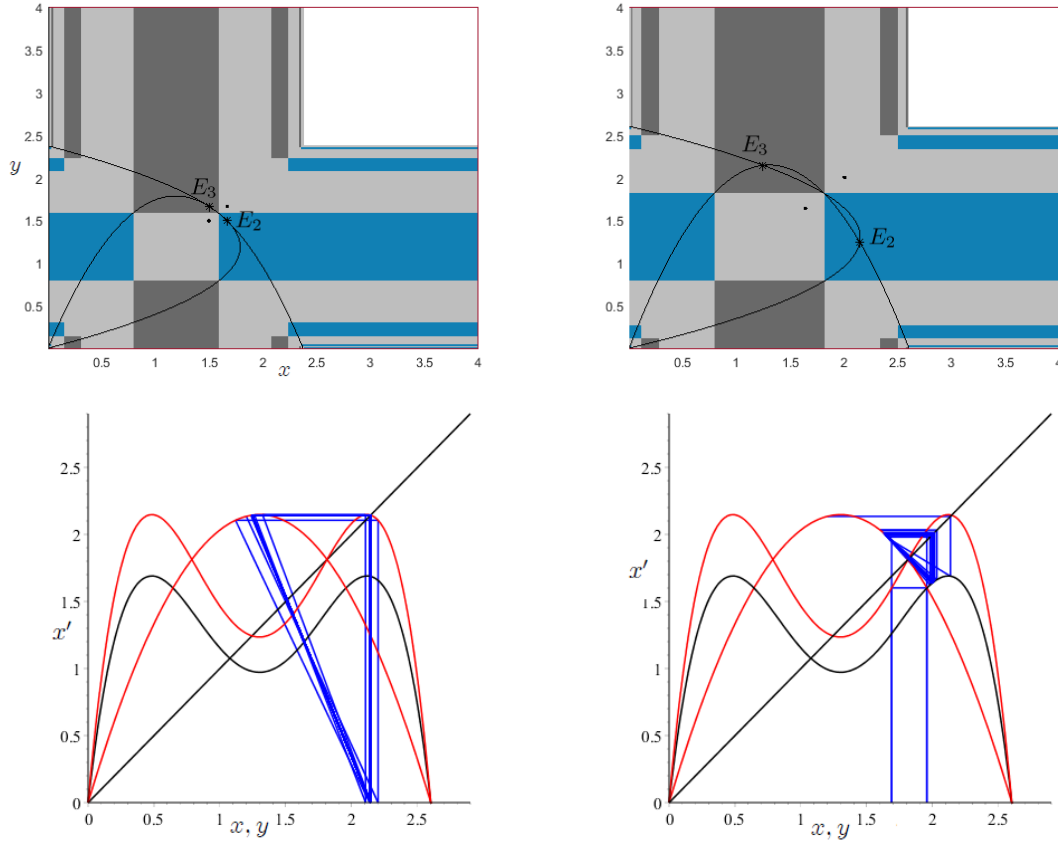


Figure 9: (a) A flip bifurcation produced a two-period cycle along the diagonal and two attractors E_2 , E_3 outside of the diagonal ($\kappa = 2.37$); (b) Evolution of the attractors after the border collision bifurcation ($\kappa = 2.6$); (c) Dynamics of x starting from an initial condition in the blue region. In this case the graph shows the convergence to a fixed point which is not the Nash equilibrium, but it is given by the intersection of the second iterate of F and the diagonal ($\kappa = 2.6$); (d) Dynamics of x starting from an initial condition in the light-grey region ($\kappa = 2.6$). In this case the graph shows that x converges to a two-period cycle (the same applies for y).

For larger values of κ new attractors emerge. Figure 10.a shows a pair of two-period cycles close to the diagonal ($\kappa = 2.71$). If we let κ increase further, then new interesting dynamic phenomena appear. The asymmetric Nash equilibria lose their stability through a flip bifurcation (see figure 10.b depicted for $\kappa = 2.72$). This is the first step towards the emergence of new kinds of ω -limit sets. After a new flip bifurcation (not shown) a global bifurcation causes the birth of two symmetric strange attractors (C and D in figure 10.c with $\kappa = 2.791$). We note that for this value of κ no symmetric attractors exist. For bigger values of κ , by focusing on the areas of the phase plane depicted in blue and dark-grey, several stability windows are intermingled with chaotic regimes (see the bifurcation diagram in figure 11.a). The rise in κ leads to the increase of the periodicity of the attracting cycles close to the diagonal and for $\kappa = 2.88$ four chaotic attractors exist.

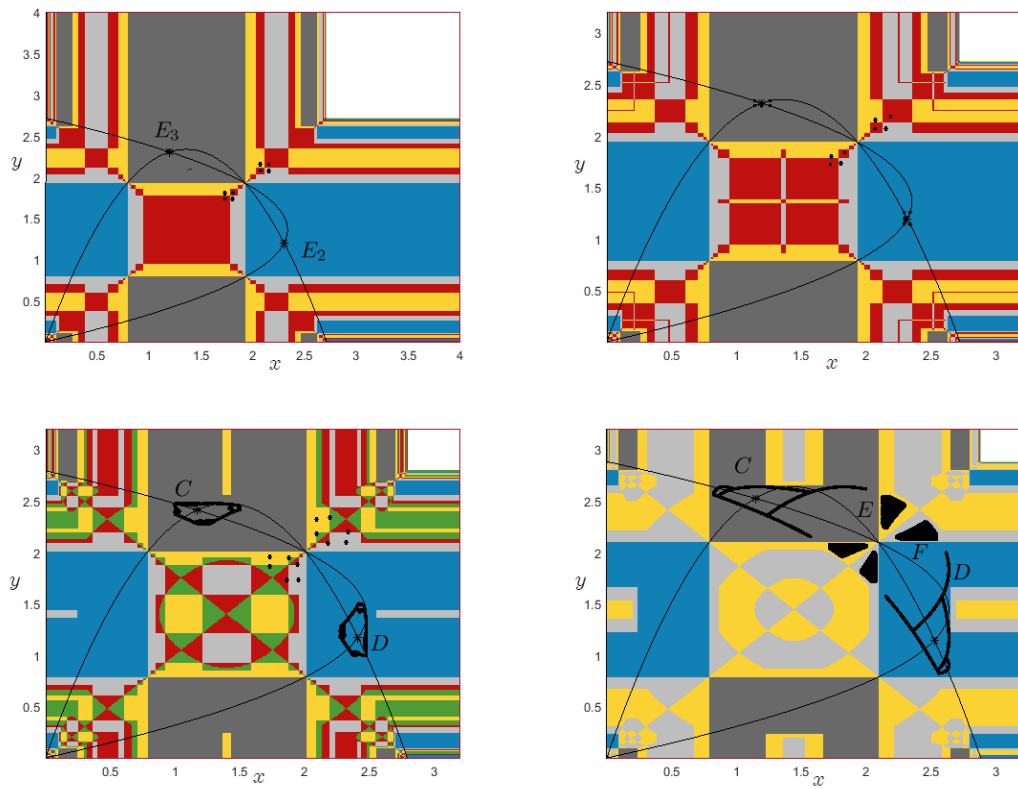


Figure 10: Evolution of the phase plane respectively, for (a) $\kappa = 2.71$; (b) $\kappa = 2.72$; (c) $\kappa = 2.791$; (d) $\kappa = 2.88$

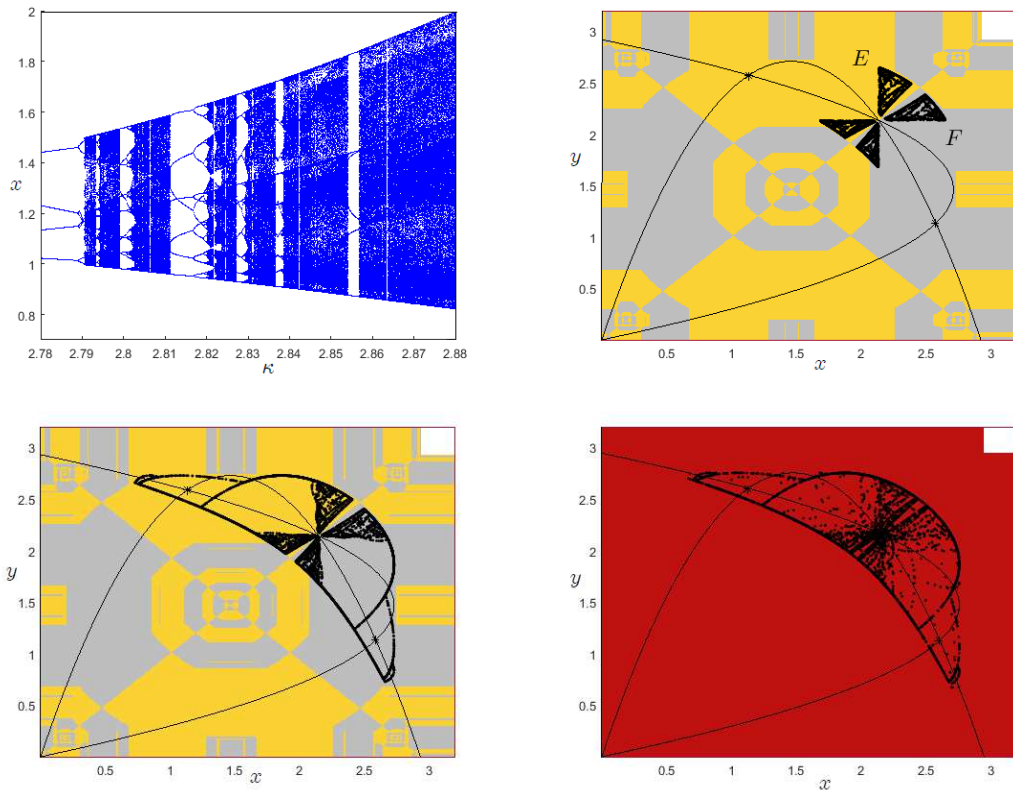


Figure 11: (a) Bifurcation diagram obtained by starting from an initial condition in the dark-grey region ($\beta = 0.213$). Evolution of the phase plane respectively, for (b) $\kappa = 2.92$; (c) $\kappa = 2.93$; (d) $\kappa = 2.944$

For $\kappa = 2.915$ a new global bifurcation occurs given by the collision of the stable manifolds that delimit the basins of attraction of the attractors C , D , E and F and the attractors C and D . This causes the death of the attractors C and D that become repellers and all the feasible trajectories are captured by the attractors E and F . A new contact between the attractors E and F and the repeller sets C and D respectively causes the union of the set.

5 Macroeconomic dynamics

If firms are not rationed in the labour market, i.e., if current labour supply is never binding in the process of profit maximization, then actual employment is determined by the labour demand schedules of the two productive units. Plugging eq. (2) into the FOCs in (4), those schedules can be written as

$$L_{i,m} = A \left(\frac{\alpha}{w_{i,m}} \right)^{\frac{1}{1-\alpha}} (w_{i,m} - w_{-i,m} (\kappa - w_{-i,m}))^{\frac{\alpha\beta}{1-\alpha}}, \forall t \quad (14)$$

Thereafter, once $w_{1,m}$ and $w_{2,m}$ are given by the trajectories generated by the dynamic system in (11), the evolution of aggregate employment (L_m) and the average wage (w_m) can be retrieved, respectively, from

$$L_m = \sum_{i=1}^2 L_{i,m}, \forall t \quad (15)$$

$$w_m = \frac{\sum_{i=1}^2 w_{i,m} L_{i,m}}{L_m}, \forall t \quad (16)$$

It is worth noting that eq. (16) entails a straightforward source of stickiness for the average wage prevailing in the economy; indeed, the negative slope of labour demand schedules imply that the productive units that pays the highest (lowest) wage will have the lowest (highest) employment. Consequently, there will be an automatic tendency of individual wage offer fluctuations to be somehow balanced out by employment fluctuations. Obviously, in comparison with employment dynamics, this pattern will smooth the path of the average wage prevailing on the market.

At this stage of analysis, aiming at providing a quantitative assessment of the macroeconomic dynamics generated by the model under investigation, it is necessary to have an idea about the magnitude of the involved set of parameters, i.e., say $\Theta \equiv \{\alpha, \beta, \kappa, A\}$. The literature

on real business cycles is very helpful in that direction; indeed, as far as the US economy is concerned, there are two parameters out of four for which there exist reliable point estimates. Specifically, starting a honoured tradition of macro computational contributions, Kydland and Prescott (1982) fix the value of α at 0.64. In addition, de la Croix et al. (2000), estimating the effort function in a panel of countries, provide a value of β equal to 0.0063. This rather low value β hasn't been considered in the previous section for the sake of expositional clarity; indeed, when β is close to zero, the reaction functions of the two firm are close to the dynamic constraints implied by the existence of the minimum wage.

Given the value of β , i.e., the concavity of the effort function, it becomes possible to evaluate the amplitude response of the individual market wage generated by different values of κ over the region for which feasible trajectories exist. The corresponding bifurcation diagram is illustrated in figure 12.a.

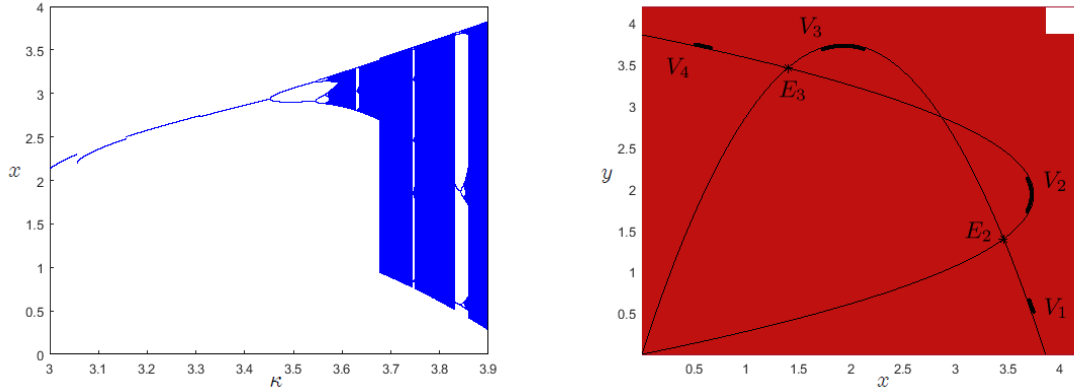


Figure 12: (a) Bifurcation diagram for κ with $\beta = 0.0063$; (b) Phase plane for $\kappa = 3.858$

The graph clearly shows that initially the attractor takes a small portion in phase plane and for a value of κ around 3.677 it undergoes a sudden increase. Consequently, fixing values of κ in the right-hand part of the bifurcation diagram, the system in (12) can actually produce chaotic fluctuations. Taking this finding into account, we set κ at 3.858. In this way, we capture a wage pattern in which the two firm pay quite different wages (V_1 and V_4) as well as similar wages (V_2 and V_3). See the phase plane in figure 12.b.

In the remainder of the paper, we test the predicted wage stickiness of the theoretical model by imposing that the TFP entering the production function in eq. (1) delivers an artificial value for the average employment, say \bar{L} , consistent with the average unemployment rate observed over the last fifty years. Specifically, retrieving quarterly data from the US Bureau of Labor Statistics for the period 1959-2003, the average unemployment rate turns out equal to 5.9%. Simulating the model according to (12) and taking into account the expression in eq. (15), the theoretical value of average aggregate employment all over a corresponding time span (180 quarters) is consistent with this figure by setting A equal to 1.3573; indeed, in this case

$\bar{L} = 0.941$.⁶ Thereafter, on account of eq.s (15) and (16), the aggregate wage-employment fluctuations generated by the model economy are illustrated in the two panels of figure 13.⁷

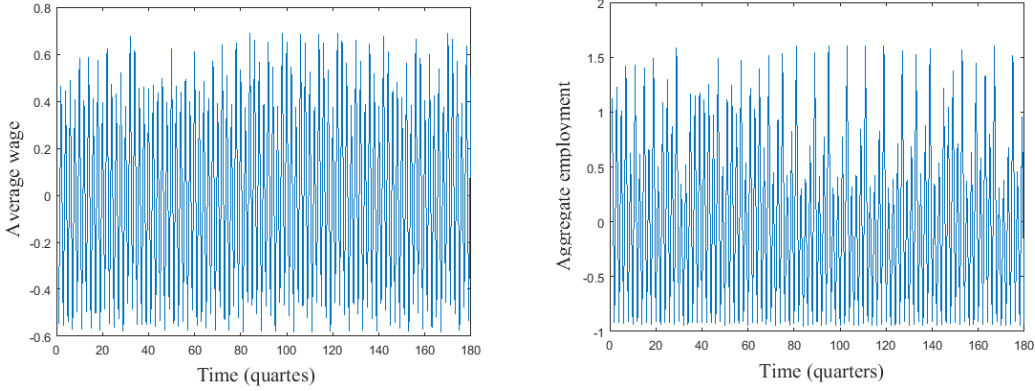


Figure 13: Wage-employment fluctuations (deviations from the mean)
 $\alpha = 0.64, \beta = 0.0063, \kappa = 3.858, A = 1.3573$

The simulated series shows that the average wage fluctuates much less than aggregate employment. Specifically, as far as artificial time series are concerned, the ratio between standard deviation of employment and the corresponding measure for average wages amounts to 1.89. In other words, the theoretical model predicts that employment is about 90% more volatile than wages, a figure of the same order of magnitude observed in the US labour market; indeed, all over time horizon exploited to retrieve the average unemployment rate, Ravn and Simonelli (2007) estimate that the ratio between the standard deviation of actual employment and the corresponding figure for wages amounts to 1.69. Consequently, this simple and parsimonious theoretical model appears actually able to reproduce a degree of wage stickiness close to the available empirical evidence.⁸

6 Concluding remarks

This paper develops a nonlinear model of efficiency-wage competition build along the lines of Hahn (1987) and Guerrazzi (2013). Specifically, assuming that the strategic relationship among optimal wage bids put forward by competing firms is non-monotonic, we show that market wage offers can actually display endogenous fluctuations. Specifically, taking into account the case

⁶Implicitly, we are normalizing to 1 aggregate labour supply.

⁷Initial conditions are set at $(w_{1,m} = x^* + 0.01, w_{2,m} = x^* - 0.01)$. In this way, the two firms put forward different wage offers. Moreover, in order to mitigate the butterfly effect, we do not consider the first 1,000 replications.

⁸Given the peculiar form of the attractor in figure 12.b, the model fails to match the volatility of the single involved series as well as their joint correlation. The derivation of a chaotic attractor able to replicate in a deterministic manner all the business cycle properties of employment and wage is beyond the scope of the present work.

of perfect coordination among wage competitors as well as the case of no coordination, we find multistability of equilibria and high-periodicity or chaotic attractors.

Thereafter, under the assumption that firms are never constrained in the labour market, we give evidence that in the parameter region of chaotic dynamics, the model economy calibrated in order to mimic the first-moment of US long-run unemployment is able to reproduce a cyclical wage stickiness consistent with the empirical evidence.

It is well-known that there is a tremendous amount of works that in the context of the Cournot model of output competition study the nonlinear dynamics of prices and quantities in different oligopoly settings (e.g. Bischi et al., 2010). Taking this growing and evergreen literature into account, our auspice is that the present work may be further extended in order to derive a deeper theoretical and numerical understanding of the deterministic dynamics of wages and (un)employment at the micro and macro level.

References

- [1] ADAMS, J.S. (1963), Wage Inequities, Productivity and Work Quality, *Industrial Relations: A Journal of Economy and Society*, Vol. 3, No. 1, pp. 9-16.
- [2] ACKUM AGELL, S. (1994), Swedish Evidence on the Efficiency Wage Hypothesis, *Labour Economics*, Vol. 1, No. 2, pp. 129-150.
- [3] AKERLOF, G.A., YELLEN, J.L. (1990), The Fair Wage-Effort Hypothesis and Unemployment, *Quarterly Journal of Economics*, Vol. 105, No. 2, pp. 255-283.
- [4] AKERLOF, G.A. (1984), Gift Exchange and Efficiency-Wage Theory: Four Views, *American Economic Review*, Vol. 74, No. 2, pp. 79-83.
- [5] ALEXOPOULOS, M. (2004), Unemployment and the Business Cycle, *Journal of Monetary Economics*, Vol. 51, No. 2, pp. 277-298.
- [6] BACHARACH, M. (1976), *Economics and the Theory of Games*, London: Macmillan Press.
- [7] BISCHI, G.I., CHIARELLA, C., KOPEL, M., SZIDAROVSKY, F. (2010), *Nonlinear Oligopolies: Stability and Bifurcations*, New York, Springer-Verlag.
- [8] BISCHI, G.I., MAMMANA, C., GARDINI, L. (2000), Multistability and Cyclic Attractors in Duopoly Games, *Chaos, Solitons & Fractals*, Vol. 11, No. 4, pp. 543-564.
- [9] CHOI, H. (1995), Goodwin's Growth Cycle and the Efficiency Wage Hypothesis, *Journal of Economic Behaviour & Organization*, Vol. 27, No. 2, pp. 223-235.
- [10] COIMBRA, R. (1999), Efficiency Wages, Increasing Returns and Endogenous Fluctuations, *Discussion Papers in Economics*, No. 6, University of York.

- [11] COURNOT, A. (1838), *Recherces sur les Principes Mathematiques de la Theorie de la Richesse*, Paris: L. Hachette.
- [12] DE LA CROIX, D., PALM, F., URBAION, J-P. (2000), Labor Market Dynamics When Effort Depends on Wage Growth Comparison, *Empirical Economics*, Vol. 25, No. 3, pp. 393-419.
- [13] DE PALMA, F., SEEGMULLER, T. (2005), Dual Labor Market and Endogenous Fluctuations, *Macroeconomic Dynamics*, Vol. 9, No. 3, pp. 398-411.
- [14] GOODWIN, R.M. (1967), A Growth Cycle, in FEINSTEIN, C.H. (ed.), *Socialism, Capitalism and Economic Growth, Essays presented to Maurice Dobb*, Cambridge: Cambridge University Press, pp. 54-58.
- [15] GRANDMONT, J-M. (2008), Negishi-Solow Efficiency Wages, Unemployment Insurance and Dynamic Deterministic Indeterminacy, *International Journal of Economic Theory*, Vol. 4, No. 2, pp. 247-272.
- [16] GREEN, F., WEISSKOPF, T.E. (1990), The Worker Discipline Effect: A Disaggregative Analysis, *Review of Economics and Statistics*, Vol. 72, No. 2, pp. 241-49.
- [17] GUERRAZZI, M. (2008), A Dynamic Efficiency-Wage Model with Continuous Effort and Externalities, *Economic Issues*, Vol. 13, No. 2, pp. 37-58.
- [18] GUERRAZZI, M. (2013), Involuntary Unemployment and Efficiency-Wage Competition, in POLACHEK, S.W., TATSIRAMOS, K. (ed.s), *Research in Labor Economics: New Analyses in Worker Well-Being*, Vol. 38, pp. 193-210.
- [19] HAHN, F.H. (1987), On Involuntary Unemployment, *Economic Journal*, Supplement: Conference Papers, pp. 1-16.
- [20] JELLAL, M., WOLFF, F-C. (2002), Dual Labor Market and Strategic Efficiency Wage, *International Economic Journal*, Vol. 17, No. 3, pp. 99-112.
- [21] JOHANSEN, K., STROM, B. (2001), Efficiency Wages, Interfirm Comparison, and Unemployment, *Southern Economic Journal*, Vol. 68, No. 2, pp. 387-402.
- [22] KAHNEMAN, D., KNETSCH, J.L., THALER, R.H. (1986a), Fairness as a Constraint on Profit Seeking: Entitlements in the Market, *American Economic Review*, Vol. 76, No. 4, pp. 728-741.
- [23] KAHNEMAN, D., KNETSCH, J.L., THALER, R.H. (1986b), Fairness and the Assumption of Economics, *Journal of Business*, Vol. 59, No. 4, pp. S285-S300.

- [24] KOPEL, M. (1996), Simple and Complex Adjustment Dynamics in Cournot Duopoly Models, *Chaos, Solitons & Fractals*, Vol. 7, No. 12, pp. 2031-2048.
- [25] KYDLAND, F.E., PRESCOTT, E.C. (1982), Time to Build and Aggregate Fluctuations, *Econometrica*, Vol. 50, No. 6, pp. 1345-1370.
- [26] MANFREDI, P., FANTI, L. (2000), Long-Term Effects of the Efficiency Wage Hypothesis in Goodwin-Type Economies, *Metroeconomica*, Vol. 51, No. 4, pp. 454-481.
- [27] OSTER, G. (1980), Labour Relations and Demand Relations: A Case Study of the 'Unemployment Effect', *Cambridge Journal of Economics*, Vol. 4, No. 4, pp. 337-348.
- [28] PUU, T. (1991), Chaos in Duopoly Pricing, *Chaos, Solitons & Fractals*, Vol. 1, No. 6, pp. 573-581.
- [29] RAVN, M.O., SIMONELLI, S. (2007), Labor Market Dynamics and the Business Cycle: Structural Evidence from the United States, *Scandinavian Journal of Economics*, Vol. 109, No. 4, pp. 743-777.
- [30] REBITZER, J.B. (1987), Unemployment, Long-Term Employment Relations, and Productivity Growth, *Review of Economics and Statistics*, Vol. 69, No. 4, pp. 627-635.
- [31] REBITZER, J.B. (1988), Unemployment, Labor Relations, and Unit Labor Costs, *American Economic Review*, Vol. 78, No. 2, pp. 389-394.
- [32] SALANT, SW., SWITZER, S., REYNOLDS, R.J. (1983), Losses from Horizontal Merger: The Effects of an Exogenous Change in Industry Sector on Cournot-Nash Equilibrium, *Quarterly Journal of Economics*, Vol. 98, No. 2, pp. 185-199.
- [33] SHAPIRO, C., STIGLITZ, J.E. (1984), Equilibrium Unemployment as a Worker Discipline Device, *American Economic Review*, Vol. 74, No. 3, pp. 433-444.
- [34] SOLOW, R.M. (1979), Another Possible Source for Wage Stickiness, *Journal of Macroeconomics*, Vol. 1, No. 1, pp. 79-82.
- [35] STIGLITZ, J.E. (1973), Alternative Theories of Wage Determination and Unemployment in L.D.C.'s: The Efficiency Wage Model, *Cowles Foundation Discussion Papers*, No. 357.
- [36] SUMMERS, L.H. (1988), Relative Wages, Efficiency Wages, and Keynesian Unemployment, *American Economic Review*, Vol. 78, No. 2, pp. 383-388.
- [37] THEOCHARIS, R.D. (1960), On the Stability of the Cournot Solution on the Oligopoly Problem, *Review of Economic Studies*, Vol. 27, No. 2, pp. 133-134.

- [38] UHLIG, H. AND XU, Y. (1996), Effort and the Cycle: Cyclical Implications of Efficiency Wages, *Tilburg University Discussion Paper*, No. 49.
- [39] VAN DE KLUNDERT, T.C.M.J. (1988), Wage Differentials and Employment in a Two-Sector Model with a Dual Labour Market, *Research Memorandum*, Tilburg University, Department of Economics, Vol. FEW 314.
- [40] VARIAN, H.R. (1992), *Microeconomic Analysis*, 3rd Edition, New York: W.W. Norton & Company.
- [41] WU, J.S.K., HO, C.P. (2012), Towards a More Complete Efficiency Wage Theory, *Pacific Economic Review*, Vol. 17, No. 5, pp. 660-676.