A structural model of life-cycle decisions of (Italian) households*

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Work in progress. Very preliminary and incomplete!

Abstract

This paper analyzes the life-cycle decisions of Italian men and women from the end of compulsory schooling until their mid-forties. We build a discrete-choice dynamic programming model where, following the Eckstein et al. (2016) framework, individuals make decisions about education, work, marriage/divorce and fertility. We estimate the parameters of the model using Italian data for the 1970 cohort from the Survey of Household Income and Wealth and the Simulated Method of Moments. The model is able to explain quite well the behavior of men and women in the cohort. Then, we use the model to study the effects of a childbirth transfer on fertility and labor supply. Preliminary results show that the permanent childbirth transfer is successful in increasing the total fertility rate of married women, even if it has a negative effect on female employment.

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1 Introduction

Over the last decades the gender gap in terms of education disappeared in many developed countries and now women are on average more educated than men. Despite this improvement, married women are still working much less and earning lower wages than their husbands, even if there are large differences between countries (Arulampalam et al., 2007). At the same time fertility decreased sharply and in some European countries it reached such low levels (Kohler et al., 2002) that governments are starting to study subsidies to boost the number of childbirths without pushing women out of the labor force.

In this paper we build and estimate a discrete-choice stochastic dynamic programming model in order to study the life-cycle labor supply and fertility decisions of Italian married men and women. For the estimation we use the Survey of Household Income and Wealth (SHIW) data from 1984 to 2014. We focus on the behavior of the 1960 cohort, composed of people born between 1957 and 1963.

We start from a simplified version of the Eckstein et al. (2016) model, which extends the Keane and Wolpin (1997) model for men and Keane and Wolpin (2010) for women. Men and women make their decisions as individuals from age 17 to age 45, and they interact with others through the marriage market. The model produces endogenous individual choices about education, marriage, individual and household labor supply, wages and fertility.

The model is estimated on repeated cross-sections holding preference parameters fixed.

Structural estimation offers some important advantages with respect to reduced form approaches. First, it allows to model different sources of endogeneity (ex. self-selection into labor market participation). Second, it provides parameters from a theoretical model that can be used to simulate the effects of policy experiments. We can perform an ex-ante evaluation of policies before implementation, studying both the short term an long term effects of a given set of policies.

In this case we use the estimated model to run the following policy experiment: we simulate the impact of a childbirth transfer similar to the 'bonus bebe' recently introduced by the Italian government. We look at the long run effects of the subsidy in terms of total fertility, wages and labor market experience. The only structural paper making a similar evaluation, up to our knowledge, is Adda et al. (2015).

Italy is an interesting case study because presents an employment rate of women significantly below the European average and, at the same time, a very low fertility rate. In fact, the employment rate of women aged 25-54 was 57.6% in Italy in 2014, around 20 percentage points lower than in France, Germany and the United Kingdom (Marino et al., 2016). The coexistence of very low fertility and low participation to the labor market, makes it necessary to find policies that stimulate fertility without harming female employment. Many women in Italy exit the labor market after the birth of their first child and never go back to work. One possible explanation is the lack of adequate social policies (Del Boca and Sauer, 2009).

Our model builds on the structural literature about labor supply and fertility. The first contribution to this literature (Eckstein and Wolpin, 1989) focuses on the employment choice of married women in their post-fertility stage of life and consider the labor supply behavior of husbands as exogenous. Van der Klaauw (1996) expands the model making the marital status decision of the woman endogenous, while Keane and Wolpin (1997) model the career decisions of young men. In Francesconi (2002) the joint modeling of fertility and labor supply of women is introduced as well as the distinction between part-time and full-time jobs. Keane and Wolpin (2010) estimate a model in which women make sequential joint decisions about school attendance, work, marriage, fertility and welfare participation.
Some recent contributions (Eckstein and Lifshitz, 2015) take a game-theoretic approach and explain the heterogeneity in married women participation to the labor market as the result of different games played within the household. A strand of the literature (Attanasio et al., 2008; Eckstein and Lifshitz, 2011; Eckstein et al., 2016) takes a cohort perspective to explain some long term trends, like the increased labor market participation of married women.

The paper makes three relevant contributions to the existing literature. First, it provides and estimates a more complete model of economic behavior with respect to past structural studies on Italian data (Del Boca and Sauer, 2009; Marino et al., 2016). Second, this is one of the very few studies to assess dynamic effects of subsidies to fertility with a structural model. Finally, the model provides a useful framework for future research on differences in behavior across cohorts both at the individual and the household level.

The main findings of our paper can be summarized as follows. First, we show that the estimated model is able to replicate quite well the behavior of households in real data. Estimates of the main parameters are in line with the previous literature. Second, the simulated childbirth transfer is successful in increasing fertility. However, especially for couples that are young when the policy is introduced, the increase in total fertility comes at the expense of a slight decrease in the time spent on the labor market for women.

The paper proceeds as follows. Section 2 describes the facts that motivate our paper, in particular the labor supply behavior of married men and women in different Italian cohorts. We also describe the policy that is used as a benchmark for the experiment. Section 3 presents the model. The following Section, Section 4, explains the methods used to solve and estimate the model. Section 5 describes the data used in the empirical analysis, whose results are commented in Section 6. Finally, Section 7 summarizes the main findings and concludes.

2 Motivation

2.1 Facts to explain

Italy is still lagging behind most other developed countries in terms of married women participation to the labor force. Surprisingly Italian women also experience one of the lowest fertility rates in the world. In this section we describe, using the SHIW dataset, some changes in the behavior of Italian married people over time. We define here 3 cohorts of married men and women, the 1950 cohort (born between 1947 and 1953), the 1960 cohort (1957-1963) and the 1970 cohort (1967-1973). In figure 1a we look at the employment rate for married women in the 3 cohorts between age 28 and 57. Since we have repeated cross-sections for 30 years we are unable to compare the complete life-cycle labor supply behavior of different cohorts. Indeed, we have data covering the whole time interval just for the 1960 cohort. However, it is possible to detect some clear trends. The more striking fact is the large increase in the labor force participation of women above age 50 between the 1950 and 1960 cohorts, with employment growing from less than 40% to almost 50% for women aged 57 [1]. We also observe a higher labor supply of women below age 40 in the 1970 cohort with respect to the 2 older cohorts.

In figure 1b we look at real wages for women in the 3 cohorts. Wages increase significantly moving from the 1950 cohort to the younger cohorts. The increase in the average level of education is partly responsible for this trend. Higher wages offered on the market contribute to explain the increase in the employ-

[1] The observed change in employment is mostly due to the recent pension reforms enacted by the Italian governments.

ment rate between cohorts.
We observe the labor supply of men in figure 2a. The employment rate of married men is well above 90% until the late 40ies. Labor market participation, as for women, decreases faster after age 50 for the 1950 cohort than for the 1960 cohort. Real wages, in figure 2b, follow the same trend seen for women.

2.2 The policy experiment

Our goal is to study the effects of a childbirth transfer on household behavior. We simulate the introduction of a subsidy to fertility very similar to the one introduced by the Italian government at the end of 2014 [2].
All couples with children born starting from January 1st, 2015 and with household income below a given threshold [3] are eligible for the transfer. The bonus consists of a payment of 80 euros per month for the first 3 years of the baby's life.
In order to avoid an excessive level of computational complexity we evaluate the effect of a policy that provides the entire cash transfer at birth. We use our model to simulate the effects on women of different ages (28, 33 and 38) when the policy starts.

3 Model

In our model, men and women start out as single individuals at age 17 in school, who make private decisions about school continuation and work. The men and women in the model also interact in a marriage market, so they can choose to form (and later dissolve) couples. Once a couple is formed, decisions about labor supply and fertility are made jointly. In order to make marriage decisions, individuals must compare the value of remaining single vs. the value of being married. So for us to model the marriage decision, we must first obtain the values of the married and single states. We describe the problem of married couples and single individuals below.

[2]The policy, called Bonus Bebe, was introduced in December 2014, by law 190/2014, that is the law regulating the public budget for year 2015. 500 millions have been allocated to finance the policy for the first year, 2015, increasing to a peak of 1.5 billions in the following years.

[3]Bonus bebe' is directed to all households with a measure of equivalent household income, called ISEE, below 25000. The threshold is not low for the Italian standards, making a large majority of couples eligible. The bonus doubles if ISEE is below 7000.

The problem of a married couple

We describe here the optimization problem of a married couple. Let t denote the annual time period and $j = f, m$ denote gender. Individuals have 1 unit of time per period. The time is split between work on the market, h, and time at home, l, so:

$$h_t^j + l_t^j = 1 \qquad j = f, m$$

In each period an individual can work full-time, part-time or stay out of the labor market spending all the time endowment at home. Thus:

$$h_t^j \in \{0, 0.5, 1\} \qquad l_t^j \in \{0, 0.5, 1\}$$

Married couples have three choice variables: $\{l_t^m, l_t^f\}$ and pregnancy p_t. We assume that the decision to have a pregnancy leads to childbirth in the next period with probability one.

Let X_t^j denote work experience, and N_t denote the number of children under 18. The laws of motion for the two variables are:

$$X_{t+1}^j = X_t^j + h_t^j \qquad N_{t+1}^j = N_t + p_t - p_{t-18}$$

We focus here on the joint decisions of couples conditional on marriage.

Preferences and constraints

Married couples have total income :

$$Y_t^M = w_t^m h_t^m + w_t^f h_t^f + b_m I[h_t^m = 0] + b_f I[h_t^f = 0]$$

where w_t^j are wage rates and b_j is the unemployment benefit plus the value of home production. The M superscript indicates values for married individuals. The household budget constraint is:

$$C_t^M = (1 - \theta(N_t))Y_t^M$$

where C_t^M is the total couple's expenditure on consumption, N_t is the number of children (under 18) and $\theta(N_t)$ is the fraction of household income spent on children [4]. In every period, the utility of a married individual of age t and gender j is:

$$U_t^{jM}(\Omega_{jt}) = \frac{1}{\alpha}(\psi C_t^M)^\alpha + L_j(l_t^j) + \theta_t + \pi_t^M p_t + A_j^M Q(l_t^f, l_t^m, Y_t^M, N_t)$$

[4] As in Eckstein et al. (2016) we use the square root equivalence scale to determine $\theta(N_t)$.

where:

$$L_j(l_t^j) = \frac{\beta_{jt}}{\gamma}(l_t^j)^\gamma + \mu_{jt} l_t^j \qquad \gamma < 1, \alpha < 1$$

The parameter $\psi \in (\frac{1}{2}, 1)$ captures economies of scale in consumption and $\beta_{jt} > 0$ shifts tastes for leisure. For women β_{jt} depends on p_t, for both partners it depends on education and health status. Stochastic variation in the marginal utility of l is captured by $\mu_{jt} l_t^j$. The third term, θ_t is the utility the individual derives from marriage itself, the fourth term $\pi_t^M p_t$ captures the utility from pregnancy and the fifth term $A_j^M Q(l_t^f, l_t^m, Y_t^M, N_t)$ the utility a couple receives from quality and quantity of children. We assume that the stochastic process for tastes for leisure is:

$$ln(\mu_{jt}) = \tau_{0j} + \tau_{1j} ln(\mu_{j,t-1}) + \tau_{2j} p_{t-1} + \epsilon_{jt}^l l_t^j$$

where $\epsilon_{jt}^l \sim N(0, \sigma_\epsilon^l)$ and $0 < \tau_{1j} < 1$. We expect that the marginal utility of home time will go up when a newborn arrives, especially for women (i.e. $\tau_{2f} > 0$). The utility from marriage (i.e. the match quality) takes the form:

$$\theta_t = d_1(E^m - E^f = 0) + d_2(E^m - E^f > 0) + d_3(E^f - E^m > 0) + d_4(H^m - H^f)^2 + \mu^M + \epsilon_t^M$$

where $\epsilon_t^M \sim N(0, \sigma_\epsilon^M)$. The first four terms capture assortative mating based on education and health. μ^M is a match quality draw and ϵ_t^M are transitory shocks to match quality. Now consider the utility from pregnancy:

$$\pi_t = \pi_1 m_t + \pi_2 H_{ft} + \pi_3 N_t + \pi_4 p_{t-1} + \epsilon_t^p$$

where $\epsilon_t^p \sim N(0, \sigma_\epsilon^p)$ is a stochastic shock to tastes for pregnancy. The value of π_t is a function of marital status, women's health, the number of children and lagged pregnancy. Finally, consider the function Q that gives the utility a couple get from the quality and quantity of children:

$$Q(l_t^f, l_t^m, Y_t^M, N_t) = (a_f(l_t^f)^\rho + a_m(l_t^m)^\rho + a_g(\theta(N_t)Y_t^M)^\rho + (1 - a_f - a_m - a_g)N_t^\rho)^{\frac{1}{\rho}}$$

The first three inputs increase child quality, while A_j^M is a scale parameter for Q.

We can now write the choice-specific value functions for married individuals:

$$\begin{aligned} V_t^{jM}(l_t^m, l_t^f, p_t | \Omega_{mt}, \Omega_{ft}) = & \frac{1}{\alpha}(\psi C_t^M)^\alpha + L(l_t^j) + \theta_t + \pi_t p_t + A_j^M Q(l_t^f, l_t^m, Y_t^M, N_t) \\ & + \delta E(m_{t+1} V_{t+1}^{jM}(\Omega_{m,t+1}, \Omega_{f,t+1}) + (1 - m_{t+1}) V_{t+1}^j(\Omega_{j,t+1})) \end{aligned}$$

The future component of the value function consists of two parts, corresponding to whether the marriage continues at $t + 1$ or not.

The term $V_{t+1}^{jM}(\Omega_{m,t+1}, \Omega_{f,t+1})$ is the value of next periods state for partner j given that the marriage continues. The term $V_{t+1}^{j}(\Omega_{j,t+1})$ is the value of next periods state for partner j if he/she becomes single (i.e., a divorce occurs). Next period state depends on the current state $(\Omega_{mt}, \Omega_{ft})$ and current choices (l_t^m, l_t^f, p_t) through the law of motion of the state variables. The parameter δ is the discount rate.

Household decision making

The partners choose leisure and fertility to maximize:

$$V_t^M(l_t^m, l_t^f, p_t|\Omega_{mt}, \Omega_{ft}) = \lambda V_t^{fM}(l_t^m, l_t^f, p_t|\Omega_{mt}, \Omega_{ft}) + (1-\lambda) V_t^{mM}(l_t^m, l_t^f, p_t|\Omega_{mt}, \Omega_{ft})$$

Here λ and $1-\lambda$ are Pareto weights. Couples seek a choice vector $\{l_t^m, l_t^f, p_t\}$ that maximize $V_t^M(l_t^m, l_t^f, p_t|\Omega_{mt}, \Omega_{ft})$ subject to the constraint that both individuals must prefer marriage. Let $V_t^m(\Omega_{mt})$ and $V_t^f(\Omega_{ft})$ denote the maximized value functions of single males and females in period t. We assume that a divorce occurs if the value of the outside (single) option exceeds the value of marriage for either party. The set of feasible choices is:

$$\mathcal{F} = \{l_t^m, l_t^f, p_t | V_t^M(l_t^m, l_t^f, p_t|\Omega_{mt}, \Omega_{ft}) \geq V_t^j(\Omega_{jt}) - \Delta_{jt}\}$$

where Δ_{jt} is the cost of divorce. The vector of household choices that maximize $V_t^M(l_t^m, l_t^f, p_t|\Omega_{mt}, \Omega_{ft})$ is:

$$\{l_t^{m*}, l_t^{f*}, p_t^*\} = \begin{cases} argmax_{\{l_t^m, l_t^f, p_t\}} V_t^M(l_t^m, l_t^f, p_t|\Omega_{mt}, \Omega_{ft}) & \text{if} \quad \mathcal{F} \neq \emptyset \\ \emptyset \quad \text{if} \quad \mathcal{F} = \emptyset \end{cases}$$

We define the maximized value function of a married individual in state Ω_{jt}:

$$V_t^{jM}(\Omega_{mt}, \Omega_{ft}) = \begin{cases} V_t^{jM}(l_t^{m*}, l_t^{f*}, p_t^*|\Omega_{mt}, \Omega_{ft}) & \text{if} \quad \mathcal{F} \neq \emptyset \\ -\infty \quad \text{if} \quad \mathcal{F} = \emptyset \end{cases}$$

If $\mathcal{F} = \emptyset$ no action exists such that individual j can be married at time t. The behavior is governed only by the properties of the single value function.
A divorce occurs in period t if:

$$V_t^f(\Omega_{ft}) - \Delta_{ft} > V_t^{fM}(\Omega_{mt}, \Omega_{ft})$$

or

$$V_t^m(\Omega_{mt}) - \Delta_{mt} > V_t^{mM}(\Omega_{mt}, \Omega_{ft})$$

where Δ_{jt} is the cost of divorce.

The problem of single households

In this Section we describe the optimization problem of single men and women. The income of a single person is:

$$Y_t^j = w_t^j h_t^j + b_j I[h_t^j = 0]$$

The budget constraint for a single person is:

$$C_t^j = (1 - \theta(N_t))Y_t^j$$

Both single men and women may have children from a previous marriage or, in the case of women, born outside of marriage.

The utility function for singles is very similar to that of married people. Consider the utility function of a single female:

$$U_t^f(\Omega_{ft}) = \frac{1}{\alpha}(C_t)^\alpha + L_f(l_t) + \theta_{ft}S_t + \pi_t p_t + A_f^s Q(l_t, 0, Y_t, N_t)$$

where S_t is an indicator of school attendance and θ_{ft} are tastes for school. Consumption is now individual specific ($\psi = 1$) and the home-time of the husband is set to zero in the Q function. We make the assumption that only single people can attend school. Tastes for school attendance, θ_{ft}, have the form:

$$\theta_{ft} = \theta_{0f} + TI(E_t > HSG) + \theta_{1f}PE + \theta_{2f}\mu_f^w$$

where TI are tuition costs in higher education, PE is parents' education and μ_f^w is the skill endowment. Choice-specific value functions for single females are:

$$V_t^f(l_t, p_t, s_t|\Omega_{ft}) = \frac{1}{\alpha}(C_t)^\alpha + L_f(l_t) + \theta_{ft}S_t + A_f^s Q(l_t, 0, Y_t, N_t) + \delta E[V(\Omega_{f,t+1})]$$

where $E[V(\Omega_{f,t+1})] = E(m_{t+1}V_{t+1}^{fM}(\Omega_{m,t+1}, \Omega_{f,t+1}) + (1 - m_{t+1})V_{t+1}^f(\Omega_{f,t+1}))$.

For single males the problem is symmetric (without pregnancy):

$$V_t^m(l_t, s_t|\Omega_{mt}) = \frac{1}{\alpha}(C_t)^\alpha + L_m(l_t) + \theta_{mt}S_t + A_m^s Q(0, l_t, Y_t, N_t) + \delta E[V(\Omega_{m,t+1})]$$

Now we consider the optimization problem of singles.

For women we have:

$$V_t^f(\Omega_{ft}) = maxV_t^f(l_t, p_t, s_t|\Omega_{ft})$$

and for men:

$$V_t^m(\Omega_{mt}) = maxV_t^m(l_t, s_t|\Omega_{mt})$$

These value functions can be used in divorce and marriage decisions.

The labor market
The wage equations have the standard form:

$$ln(w_t^j) = \omega_{0j} + \omega_{1j}E_t + \omega_{2j}X_t + \omega_{3j}X_t^2 + \epsilon_{jt}^w$$

where E_t is education, X_t is work experience and $\epsilon_{jt}^w \sim$ N(0,σ_ϵ^w). An unemployed receives at most one job offer per period (Full-Time or Part-Time). In each period an individual have three possible choice sets for hours: D = {0},{0,0.5},{0,1}. The probability that each of the three choice sets is offered is determined by a trinomial logit:

$$P_j(D_{kt}) = \begin{cases} \exp(\rho_{jk0} + \rho_{jk1}E_t + \rho_{jk2}X_t + \rho_{jk3}H_t)/IV_{jt} & \text{if} \quad k = 2, 3 \\ 1/IV_{jt} \quad \text{if} \quad k = 1 \end{cases}$$

where $IV_{jt} \equiv 1 + \sum_{k=2}^{3} \exp(\rho_{jk0} + \rho_{jk1}E_t + \rho_{jk2}X_t + \rho_{jk3}H_t)$. The probability of each choice set depends on education, work experience and health. The probability of a job loss is a logit function of the same three variables. In each period a person may be unemployed because he/she draws the empty set D = {0} or because has a part-time or full-time offer and rejects it.

Health status
Health status is modeled as a three-state Markov-chain, where $H_{jt} \in \{1, 2, 3\}$ indicates poor, fair and good health. The transition probabilities differ by age, and, as the process is exogenous, the parameters of the health transition matrix are estimated outside the model.
The health transition probability is a multinomial logit:

$$P(H_{jt} = k) = \begin{cases} \exp(\sum_{q=1}^{3} \chi_{jkq} I[H_{j,t-1} = q])/IV_{jt}^H & \text{if} \quad k = 2, 3 \\ 1/IV_{jt}^H \quad \text{if} \quad k = 1 \end{cases}$$

where $IV_{jt}^H \equiv 1 + \sum_{k=2}^{3} \exp(\sum_{q=1}^{3} \chi_{jkq} I[H_{j,t-1} = q])$. Health status affects tastes for leisure and the job offer probability. The assumption that health evolves exogenously means that it generates exogenous variation in these decisions.

The marriage market
At the start of a period a single individual may receive a marriage offer. Denote the probability of receiving an offer $p_j^H(\Omega_{jt})$. The probability depends on

gender, age (above or below 18), and whether a person is in or out of school. A marriage offer is characterized by a vector of attributes of a potential spouse (M_{jt}). A marriage offer can be constructed in three steps:
First, marriage offers always come from potential spouses of the same age (t). This assumption will not have a great effect on the results since a large majority of married couples are close in age.
Second, we draw the education level of the potential spouse. We assume potential spouses have three possible education levels: less than high school, high school and university. The probability of receiving an offer from a potential spouse of each type depends on a person's own education. If the individual gets a marriage offer, we draw the potential partner's education using a multinomial logit with latent indices:

$$v_{jt}^{C} = \eta_{0j}^{C} + \eta_{1j}^{C} I[E^m - E^f = 2] + \eta_{2j}^{C} I[E^m - E^f = 1] + \epsilon_{jt}^{C}$$

$$v_{jt}^{SC} = \eta_{0j}^{SC} + \eta_{1j}^{SC} I[E^m - E^f = 1] + \epsilon_{jt}^{SC}$$

and $v_{jt}^{HS} = 0$ is the baseline. η_{1j}^{C}, η_{2j}^{C}, η_{1j}^{SC} govern the extent to which a person is more or less likely to receive offers from potential partners whose education differ from their own.
Third, we draw the other characteristics of the potential spouse (i.e. the elements of M_{jt}) from the population distribution of all potential partners within the person's age-education cell. We obtain these distributions from the data. Specifically, we draw the potential partners: health status, potential work experience, number of children, and work status in the previous period. We also draw unobservables from their population distribution: the potential partner's tastes for leisure μ_{jt}, labor market ability μ_j^w, transitory parts of wage $\epsilon_{jt}^{\tilde{w}}$ and taste for marriage ϵ_t^M. We assume that the 4 stochastic terms above are observed by both parties as part of the marriage offer. Both parties understand which terms are permanent and which transitory. The marriage offer for a single female consists of the vector:
$M_{jt} = (E^m, H^m, X^m, N^m, h_{t-1}^m, \mu_m^l, \mu_m^w, \epsilon_{mt}^{\tilde{w}}, \epsilon_t^M)$.
Offers for males are the same. The 3-step procedure generates a distribution of marriage offers conditional on a person's state. Given a marriage offer M_{jt}, the single person has enough information to construct the vector $(\Omega_{ft}, \Omega_{mt})$ that will be state of the couple if they do marry. Both parties calculate the value of marriage $V_t^{jM}(\Omega_{ft}, \Omega_{mt})$. A marriage is formed if and only if: $V_t^{fM}(\Omega_{ft}, \Omega_{mt}) > V_t^{f}(\Omega_{ft})$ and $V_t^{mM}(\Omega_{ft}, \Omega_{mt}) > V_t^{m}(\Omega_{mt})$. If the pair decides to marry they proceed to make decisions about work and fertility. If

they decide to remain single they make decisions about work, school and (for women) fertility.

4 Solution and estimation

We solve the model backwards from a final age (we assume 45) to age 17.
We list here all the state variables with the number of possible values for each. The variables for a married individual are gender ($j = m, f$); age ($t = 28, ..., 57$); education (3 levels: less than high school; high school and university); experience with 5 levels (0-1,2-4,5-9,10-16,17+); children with 4 levels (0,1,2,3+); health with 3 levels (poor, fair and good); taste for leisure with 3 levels; lagged pregnancy (2 levels) plus education, experience, health and taste for leisure of the spouse (with the same number of levels). For example, an individual aged 35 of gender j has 145800 points in the state space. In order to reduce the computational burden we exclude childbirth after age 40, that is quite rare in the data. We estimate the model using data from the Survey of Household Income and Wealth (SHIW) for the period 1984 to 2014. The sample is composed of single and married people aged 17 to 45. We focus our attention on the 1970 cohort, i.e. people born between 1967 and 1973, since in the SHIW we have data covering all the ages in the model for this cohort. For the estimation of the model's parameters we use the Method of Simulated Moments, as in McFadden (1989) and Pakes and Pollard (1989), implemented by minimizing the distance between actual data and simulated data from our model. The moments used to fit the model are listed in Table 1. After the backward solution of the model, we simulate forward to generate life-cycle choices from the initial age until the terminal period (age 45). We need to draw initial conditions for each person's education and taste for leisure. We assume for now that initial experience equals zero. Then we must draw, for each individual i in each period t, a job offer, a health realization and the shocks.
Conditional on these realizations the model generates simulated choices and outcomes for the endogenous variables: employment, children and wages. We simulate data for 1000 men and women. We construct a set of statistics for both the simulated and the actual data summarizing key predictions of the model. The statistics include employment rates and wages for men and women. Let d_j denote a statistic from the actual data, and let $d_j^s(\theta)$ be the corresponding statistic calculated from the simulated data. We construct moments of the

form:

$$m_j^s(\theta) = [d_j - d_j^s(\theta)] \quad for \quad j = 1, ..., J$$

The vector of simulated moments is given by $g'(\theta) = [m_1^s(\theta)...m_J^s(\theta)]$. We minimize the objective function $G(\theta) = g'(\theta)Wg(\theta)$ with respect to θ, where the weighting matrix W is a diagonal matrix consisting of the inverse of the estimated variance of each moment. The variance of the estimator is:

$$\hat{V} = (1 + \frac{1}{NS})(\hat{G}'W\hat{G})^{-1}$$

where NS is the number of households times the number of simulations. $\hat{G}$ is the matrix of the first derivative of every moment with respect to every parameter. The derivatives are approximated numerically. We estimate 57 parameters using 145 moments.

5 Data

Our analysis is mainly based on data from the SHIW, a survey administered by the Bank of Italy (usually) every two years to a sample of approximately 8000 households (20000 individuals) per year. The survey collects detailed information on education, labor market, consumption and income-related issues. Historical data start from year 1977, however exact values for the age of individuals are available just from 1984. For this reason we will focus our attention on waves from 1984 to 2014. In order to be coherent with the specified model we restrict our sample to single and married or cohabiting individuals aged 17 to 45. We then use this sample to compute the empirical moments defined in the previous section for the 1970 cohort.

In Table 2 we present summary statistics for the main variables used in the analysis. The employment rate is 12.4 percentage point higher for married women without children than for wives with one or more children. The average number of children per married couple is around 1.6. While more than one out of five women work part-time, almost all the men in the sample have a full-time job. Married men tend to earn much more than married women, with a gender gap above 20%. Finally, wages and employment of wives increase dramatically with the level of education. The average wage for a woman with a university degree is more than 50% higher than the wage for an high-school dropout. The same happens for the probability of being employed: while just 35% of women with a low level of education work, the share of university graduates working is close to 83%. In order to compute transition matrices for the

Table 1: List of moments used in the analysis

Moment	Number of moments 1960 cohort
married men Full Time	10
unmarried men Full Time	10
married women Full Time	10
unmarried women Full Time	10
married men Part Time	10
unmarried men Part Time	10
married women Part Time	10
unmarried women Part Time	10
married women with children employment	10
married women no children employment	10
unmarried women with children employment	10
unmarried women no children employment	10
married women children by age	5
unmarried women children by age	5
marriage rate	10
divorce rate	10
married men wage	10
unmarried men wage	10
married women wage	10
unmarried women wage	10
wage by education - women	3x10
employment by education - women	3x10

health function we use health status information from EU-SILC.
In Table 3 we define the values of some parameters that we keep constant in the estimation. The discount factor δ is set at 0.95, very close to values estimated in the previous literature. The Pareto weights λ, reflecting the bargaining power of each of the two spouses, are fixed at 0.5. Some of the recent literature (Browning et al., 2013) found higher weights for women in a marriage, however the evidence is based on Canadian data, for Italy we think that a 50-50 split might be more appropriate. The value for economies of scale in household consumption, ψ, comes from Eckstein et al. (2016).

6 Results

In this section we present the main estimation results. The estimates for the model parameters are shown in Tables 4-9. Table 10 presents some measures of model fit and Table 11 shows the results of the policy experiment.

6.1 Parameter estimates

The estimates for the wage equations are shown in Table 6. The return from completing an additional level of education is 0.41 for men and 0.38 for women. This is approximately equivalent to a return from an additional year of education of 8%, a figure in line with the findings of the literature (Belzil, 2007). Experience, as expected, has a positive effect on wages for both men and women. One additional year on the job increases earnings by about 5%, however the effect is decreasing over time. This value is slightly larger than other similar estimates in the recent literature (Marino et al., 2016).

In Table 7 we present the estimated parameters of the home time (leisure) equations. We find that the tastes for leisure μ_j are more persistent for women than for men. Furthermore, having a pregnancy in $t-1$ have a much stronger effect on tastes for leisure of wives than husbands. This last finding is not surprising since in Italy many women, especially the low skilled, decide to exit the labor market at childbirth. On the other hand the labor supply of men tend to be independent of the birth of children.

Table 8 shows the parameters estimated from the utility from pregnancy and utility from children equations. The utility from a pregnancy in t is negatively affected by the number of children in the household and by having had a pregnancy in $t-1$.

6.2 Model fit

In Table 10 we present the model fit for employment, wages and fertility in different age groups. The model is able to predict wages quite well for both men and women. However, simulated wages are higher than real wages for older male workers and lower for older employed women. This fact is also evident in Figures 3a and 3b, where we show the dynamics of wages for women and men respectively. The gap for women in the last years is probably due to the fact that in Italy low productivity women exit the labor market in their early fifties, leaving just the higher ability women on the job. The model is not able to fully account for this positive self-selection. Employment is underestimated for both men and women at younger ages. For elderly workers the model underestimates employment rates for husbands but overestimates them for wives. These moments confirm that the model fails to explain the large rate of early retirement among women. The low estimates for employment rates at young ages is partly due to the fact that we set the years of experience to zero in period 1 as an initial condition. This assumption might be reasonable for university graduates but it is too restrictive for workers with a lower level of education. Finally, due to the fact that we assume women get married when they reach age 28, childbirth happens at older ages in the model than in the data. This explains the slightly lower employment rate for women aged 37-45 with respect to younger women in the model and the fact that fertility is higher than in real data for women between 34 and 40.

6.3 Policy experiments

In Table 11 we show the results of the experiment. The goal is to study the effect of the permanent implementation of a childbirth transfer on household behavior. We simulated the effect of a subsidy worth 2880 euros given in the first year after a child is born. The evaluation is performed for households aged 28, 33 and 38 when the policy starts. As expected, the effect is largest when the transfer is implemented at the beginning of the household's fertility period (Column 1). We find a 4.8% increase in total fertility in this group, as well as a decrease in both Full Time and Part Time employment of 0.21 years and 0.04 years respectively. In Column 2 we present the results of the experiment for the households aged 33 when the policy starts. The increase in total fertility is still quite large, at 3.7%, and the decrease in employment is lower than before. We observe a reduction of 0.13 years in full time employment and 0.01 years in part time employment. Finally, if the subsidy is implemented when the household's members are 38 years old, i.e. almost at the end of their fertility period, the effect on total fertility is close to zero, and the same is true for the effect on the time spent in employment.

7 Concluding remarks

In this work we built and estimated a dynamic life-cycle model with endogenous wages, employment and fertility. Decisions are taken jointly by husbands and wives at the household level. We are able to account for several sources of heterogeneity across individuals and families. The model is estimated using

a repeated cross-section of Italian households and the Method of Simulated Moments.
The model can fit the observed behavior of agents quite well. We then use the estimated model to simulate the outcomes of a policy: a childbirth transfer very similar to the one implemented by the Italian government starting from 2015. The results show that the subsidy, if permanent, is successful in increasing fertility. However, especially for young couples, the increase in total fertility comes at the expense of a decrease in the time spent on the labor market for women.

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Table 2: Summary statistics

Variable	**Mean**	**Std. Dev.**	**Min.**	**Max.**
Share wives with children employed	0.47	0.499	0	1
Share wives no children employed	0.594	0.491	0	1
Number of children per couple	1.595	1.006	0	9
Share of employed wives working FT	0.792	0.406	0	1
Share of employed wives working PT	0.208	0.406	0	1
Share of employed husbands working FT	0.973	0.162	0	1
Share of employed husbands working PT	0.027	0.162	0	1
Married women wage	11790	6180	90	95000
Married men wage	15083	7893	100	120000
Married women wage - low education	9669	4752	150	70000
Married women wage - avg education	12275	5317	90	60000
Married women wage - high education	15155	8478	300	95000
Married women employment - low educ.	0.35	0.477	0	1
Married women employment - avg educ.	0.619	0.486	0	1
Married women employment - high educ.	0.828	0.378	0	1

Table 3: Fixed parameters (from the literature)

	Fixed Value Husband	Fixed value Wife
Discount factor	0.95	
Pareto weight (λ)	0.5	
Economies of scale in HH cons. (ψ)	0.707	
Unemployment benefit + housework (b_j)	2000	3000

In the new version b_j are estimated.

Table 4: Job offers parameters: husbands

	Estimated Value	Standard Error
Job offer parameter - Full Time		
constant	1.5943	0.649
experience	0.2148	0.097
education	0.1518	0.110
health	-0.4147	0.099
Job offer parameter - Part Time		
constant	-0.2857	0.350
experience	0.0806	0.105
education	0.0202	0.119
health	-0.4312	0.256
Job offer parameter - No job		
constant	-0.2860	3.185
experience	0.1621	0.195
education	0.1501	0.361
health	-0.4166	0.345

Table 5: Job offers parameters: wives

	Estimated Value	Standard Error
Job offer parameter - Full Time		
constant	1.4325	0.741
experience	0.2009	0.101
education	0.1502	0.125
health	-0.4070	0.135
Job offer parameter - Part Time		
constant	0.1020	0.337
experience	0.1215	0.095
education	0.0809	0.062
health	-0.4076	0.276
Job offer parameter - No job		
constant	-0.5061	0.493
experience	0.1403	0.246
education	0.1498	0.268
health	-0.4218	0.171

Table 6: Estimated parameters: wage equations

	Estimated Value	Standard Error
Husband's wage equation		
HS dropout	8.795	0.858
HS graduate	9.119	0.907
University graduate	9.334	0.983
Experience (years)	0.0496	0.059
Experience^2	-0.001	0.002
Wage error variance	0.468	0.714
Wife wage equation		
HS dropout	8.635	0.344
HS graduate	8.957	0.339
University graduate	9.216	0.351
Experience (years)	0.0493	0.049
Experience^2	-0.001	0.003
Wage error variance	0.476	0.799

Table 7: Estimated parameters: home time (leisure) equations

	Estimated Value	Standard Error
Husband home time equation		
Constant	0.0010	0.026
AR coefficient	0.6537	0.152
Pregnancy in $t-1$	0.1475	0.335
Home time shock variance	0.2468	1.447
Wife home time equation		
Constant	0.0010	0.008
AR coefficient	0.8442	0.111
Pregnancy in $t-1$	0.7344	0.326
Home time shock variance	0.2469	0.773

Table 8: Estimated parameters: fertility equations

	Estimated Value	Standard Error
Utility from pregnancy		
Health	-0.1460	0.079
Number of kids in the HH	-0.9093	0.828
Pregnancy in $t-1$	-3.1594	4.309
Pregnancy shock variance	0.8936	0.826
Utility from quality-quantity of children		
CES function's parameter (ρ)	-0.8508	0.578
wife leisure (a_f)	0.5623	0.282
husband leisure (a_m)	0.3780	0.285
spending per child (a_g)	0.0004	0.001
Scale parameter A_m	0.251	0.153
Scale parameter A_f	0.398	0.176

Table 9: Estimated parameters: utility function

	Estimated Value	Standard Error
CRRA consumption parameter (α)	0.1449	0.044
CRRA leisure parameter (γ)	0.9151	0.961
leisure when pregnant (wives only)	0.0507	0.255
leisure by education (wives)	0.0399	0.176
leisure by education (husbands)	0.0204	0.141
leisure by health (wives)	0.0506	0.179
leisure by health (husbands)	0.0503	0.300

Table 10: Model fit

	Simulated Data	Real Data
employment 28-36 males	0.730	0.936
employment 28-36 females	0.483	0.495
employment 37-45 males	0.791	0.949
employment 37-45 females	0.503	0.512
employment 46-57 males	0.782	0.874
employment 46-57 females	0.527	0.519
wages 28-35 males	12634	12780
wages 28-35 females	10556	10163
wages 36-45 males	15759	15700
wages 36-45 females	12041	12178
wages 46-57 males	18306	17039
wages 46-57 females	13695	14327
Number of children 29-33	1.18	1.44
Number of children 34+	1.81	1.65

Table 11: Experimental results: effects of the fertility subsidy

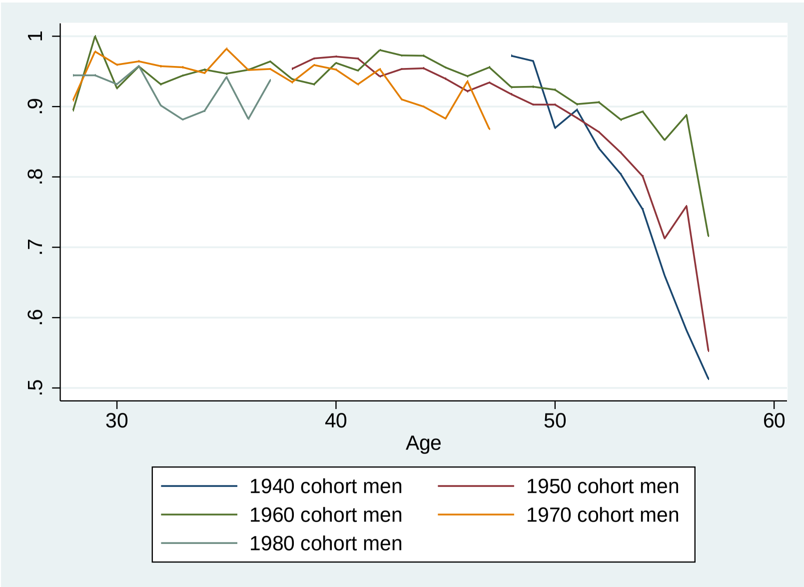
	Age at start of policy		
	27	32	37
Change in total fertility	3.8%	2.7%	0.5%
Change in female years working FT	-0.158	-0.102	-0.030
Change in female years working PT	-0.041	-0.011	-0.003

Figure 1: Employment rate by cohort

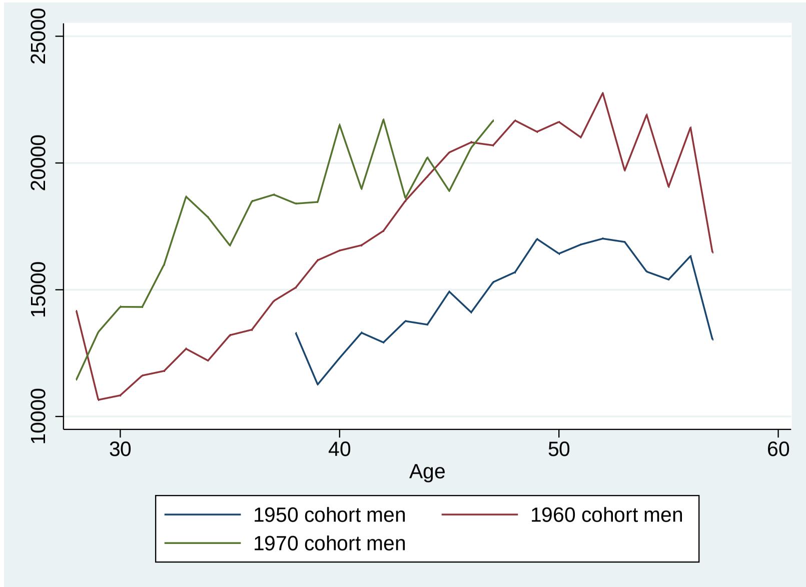
(a) Employment rate women

(b) Real wage women

Figure 2: Real wage by cohort

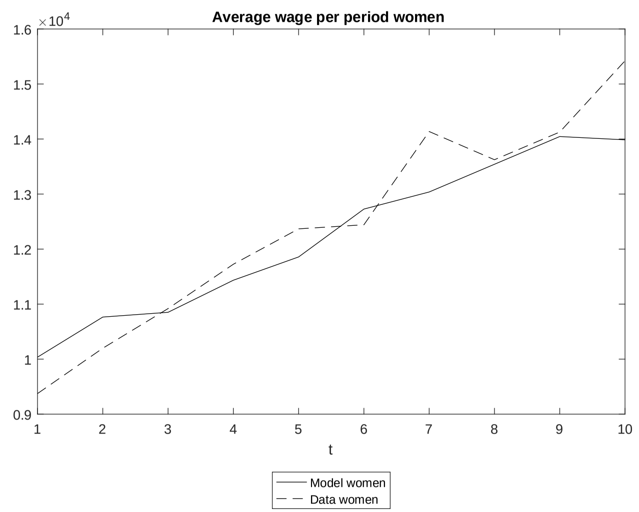


(a) Employment rate men

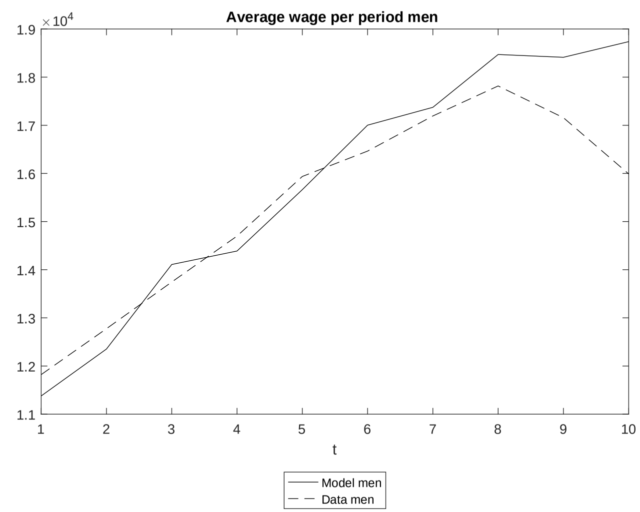


(b) Real wage men

Figure 3: Model fit: wages



(a) Model fit: women wages



(b) Model fit: men wages