

Inefficient Labor-Saving Technological Change due to Frictions in the Labor Market

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Abstract:

Does labor saving technological change generate unemployment? Is it efficient? This paper presents a growth model with R&D and two types of technological change: neutral, which increases the productivity of both capital and labor, and labor saving, which increases the productivity of capital but reduces the productivity of labor. When there is unemployment, labor saving technological change generates unemployment and is inefficiently high due to a negative “external effect” on employment. In a dual labor market environment, without unemployment, labor saving technological change is also inefficiently large due to the negative “external effect” on employment in the high wage sector.

Key words: Growth Theory, R&D, unemployment, labor saving and neutral technological change, Welfare analysis, unions.

JEL: E24, J5, O3, O41.

1. Introduction

Empirical evidence shows the importance of labor saving technological change (see Caselli and Feyrer, 2007; Eden and Gaggl, 2015; Karabarbounis and Neiman, 2014; Levy and Murnane, 2003; Sturgill, 2012; and Zuleta, 2008a). But, is labor saving technological change efficient? Labor saving technological change, by its own definition, involve employment destruction and may generate unemployment. Empirical evidence also suggests that labor saving technological change has contribute to the increasing polarization in the wage distribution due to the destruction of “good jobs” (see Katz and Kearney, 2008; Dustmann, Ludsteck, and Schönberg, 2009; Hussey and Jetter, 2017; Maarten, Manning, and Salomons, 2009, 2014; Schmitt and Jones, 2012a, 2012b). Given that labor saving technological change seems to destruct employment in general, and “good jobs” in particular, it is natural to wonder whether labor saving technological change is efficient, in particular when there are frictions in the labor markets.

The intuition of the inefficiency of labor saving technological change in the case in which there is unemployment is quite obvious: there are R&D resources devoted to labor saving technological change which consist in saving in a resource (labor) that is not fully used and therefore has an opportunity cost equal to zero. To reducing the use of such resource does not imply a reallocation of such resource to another use, it simply reduces even more the used amount of such resource, increasing the waste of such resource for the economy (increasing unemployment). Thus, devoting R&D resources to labor saving technological change is a waste. This does not mean that there is not incentive to devote resource to develop labor-saving technological change. Even if the opportunity cost of labor for the economy is zero (when there is unemployment), the cost of hiring a worker is his wage, which is costly for the firm. Thus, to develop labor saving technological change may be perfectly profitable. The basic problem is that the opportunity cost of labor in the economy do not coincides with the market price of labor (the wage). This is the typical problem when there are external effects: the opportunity cost of the economy does not coincide with the market price. In fact, the behavior of the economy resembles to the one with external effect. Labor saving technological change reduces the demand for labor, generating a “negative external effect” on the amount of employment.

But the problem of the non-coincidence of market prices and opportunity costs, and the existence of “negative external effect” of labor saving technological change is not exclusive of an economy with unemployment, there is other type of frictions in the labor markets that generate the same type of results. More precisely, we develop a model with dual markets in which workers may be hired by a firm in the “high wage sector” or by a firm in the “low wage sector”, which has a lower wage due to the absence of bargaining power of workers. Thus, there is also a difference between the price of labor in the high wage sector and its opportunity costs, which coincides with the productivity of labor in the “low wage sector”, which is inferior to the one in the “high wage sector”. Under these circumstances, labor saving technological change generates also “negative external effects” on the employment of the “high wage sector”, and this external effect generate

an inefficient allocation of investment, due to the overinvestment in the labor saving technological change. Thus, when there are frictions in the labor markets, such as unemployment or dual markets, labor saving technological change worse off the allocation of labor to different activities, generating negative external effects and making the investment in labor saving technological change inefficiently high at the market allocation.

In order to analyze the efficiency of labor saving technological change, we depart from a benchmark growth model with technological change that does not have any frictions in the labor market and that have a quite unusual property in R&D growth model: the market allocation is efficient. Classical R&D growth model, such as Rivera-Batiz and Romer (1991) and Romer (1990), are characterized by the inefficiency of market equilibrium due to the existence of external effects or imperfect competition. Since our goal is to analyze the inefficiencies generated by labor market friction, we pursue to depart from a model without any market inefficiencies. By doing so, the effect of labor frictions on efficiency will be clearer and will not mixed up with other possible sources of inefficiencies. Thus, we design a model to deliver the result of efficiency of market allocation. In order to do that, we avoid friction, presents in other models of the literature, such as imperfect competition or R&D external effects. More precisely, we present a growth model in which there is a R&D firm that decides the amount to invest in the two different types of technological change: i) neutral technological change that increases the productivity of both capital and labor and, ii) labor-saving technological change, that increases the productivity of capital but reduces the marginal product of labor, and therefore reduces the demand for it. The decision of the amount of R&D investment devoted to these two types of technological change depend on the profits that a R&D firm can get from the patent royalties that firms pay for using technology. Firms in the production sector are perfectly competitive in order to ensure that our model delivers the First Welfare Theorem. The model may behave either as a neoclassical model (with steady state) or as an endogenous growth model (with permanent growth) depending on the parameter values. Once the wealth level reaches a certain threshold level, the scarcity of labor and its high price make profitable to invest in labor saving technological change, arising R&D investment in this type of technological change at equilibrium. Most important, the benchmark model delivers the result that is designed for: market equilibrium is efficient.

Once that we analyze the benchmark model, we analyze the effect of labor market frictions by introducing two types of frictions: i) we first analyze the case in which there is unemployment; ii) later, we analyze a dual market in which there are jobs with high wages (“good jobs”) and low wages (“bad jobs”). In order to analyze unemployment, we introduce in the model a bargaining process between a union of insider workers and production firms. This bargaining process involves a wage that is higher than the competitive level; consequently, unemployment arises at equilibrium. It turns out that the bargaining process implies that the employment level depends on the labor intensity of the technology. More precisely, a production function with a higher technological share of labor (more labor intensive) involves a higher level of employment. This implies that

labor saving technological change, which reduces the labor share, reduces the employment level as well, generating unemployment. We analyze the efficiency of technological change in this framework with unemployment. To do that, we compare the market equilibrium with a second-best allocation. Such second best allocation is defined as the allocation that maximizes a benevolent social planner problem, which maximizes the household's utility, subject to the technological constraints and to the "institutional constraint" that unemployment is as determined at the equilibrium. Labor saving technological change is lower at the second-best allocation than at the market equilibrium. That is, there are an inefficient overinvestment in labor saving technology at equilibrium. Furthermore, the combination of technological change at the market allocation is not even efficient from the productive point of view: with the same investments in R&D and capital it is possible to produce more than at the market equilibrium. The results may be interpreted as an external effect: labor saving technological change generates a "negative external effect" on the employment level and, since R&D firms do not take account of such cost in the society, they produce an inefficiently large amount of labor saving technological change.

Finally, we analyze a model with a dual market in which there are two sectors: the high wages sector and the low wages sector. There is a bargaining process between a union of insider workers and production firms in the high wage sector, while workers do not have any bargaining power in the low wage sector. As a result of this bargaining process in the high wage sector, the wages and the labor marginal productivity in the high wage sector are higher than in the low wage sector. As in the model with unemployment, the bargaining process implies that the employment level depends on the labor intensity of the technology. This implies that labor saving technological change reduces the employment level in the high wage. Thus, labor saving technological change involve a reallocation of labor from a sector in which is more productive, the high wage sector, to a sector in which is less productive, the low wage sector. This means that labor saving technological change generates also in this setup a "negative external effect" on the allocation of labor. We analyze the efficiency of the allocation with the same procedure as in the model with unemployment: comparing the market allocation with a social planner allocation in which the misallocation of labor at equilibrium is taken as an "institutional" constraint. The results are alike in both models: there are an inefficiently large investment in the labor saving technological change at the market equilibrium, due to the "negative external effect" that such technological change has on the allocation of labor.

Thus, unemployment is not essential to generate the results of the paper, other type of frictions in the labor market may also generate the same results.

The paper is related with Acemoglu (2010), which differentiate between neutral and labor saving technological change and analyzes the incentive that firms have to develop these two types of technological change. In the Acemoglu's contribution there is neither unemployment nor friction in the labor market. Thus, the former paper does not pursue

the goal of the present one, which focuses on how frictions in the labor markets generate inefficiencies in the technological change process.

Alesina et al (2015), Funk and Vogel (2004), and Lommerud and Rune (2012) analyze how labor regulations affect the incentive to adopt low skill labor saving technologies. These models do not focus either in unemployment or in the efficiency of technological change.

This paper is also related with the literature on factor saving innovation such as Boldrin and Levine (2002), Givon (2006), Perera-Tallo (2017), Peretto and Seater (2013), Zeira (1998, 2006), Zuleta (2008b, 2015), Zuleta and Young (2013). Technological change reduces the needs of labor in production in this literature, generating endogenous growth. The previous contributions do not consider unemployment, which is the focus of the present paper.

The paper is also related with the large literature in labor economics, which analyzes the effect of unions on technological change from a microeconomic perspective. See for instant Bárcena-Ruiz and Campo (2005), Calabuig and González-Maestre (2002), Dobson (1994), Fallick and Assett (1999), Starr and Biloski (1985), Tauman and Weiss (1987), Ulph and Ulph (1998) and Van der Ploeg, F. (1987).

2. The Benchmark Model:

The goal of this section is to build a R&D growth model without frictions in the labor market that generate an efficient market allocation. Once that such model is constructed, we will introduce frictions in the labor market (in next section) to be able to determine whether such frictions may generate inefficiency in the technological change process. Since the goal of this section is to build a model in which market allocation is efficient, we will avoid the sources of inefficient common in the R&D growth literature. That is, we will consider that there is perfect competition in the production sector and we will not introduce any external effect in the model. Finally, to make sure that the market allocation is efficient, we will consider that the number of firms in the production sector exogenous. The reason for this last feature is that we have a model in which production firms pay royalties to the R&D firm. Such royalties constitute a sunk cost to pay a non-rival factor, and consequently would generate inefficiently large firms (that is, an inefficiently small number of firms) in the case in which free entry would be allowed (in which the number of firm were endogenous). To avoid this possible source of inefficiency, we will consider the number of firm as an exogenous variable.

2.1 The Benchmark Model

Time is discrete and endless and it is indexed by $t \in \{0,1,2,\dots\}$.

2.1.1 Production Technology

We consider that each agent has one efficiency unit of labor. We assume that a portion n of agents has a firm (they are entrepreneur). Each firm has the following technology:

$$y_t^f = \begin{cases} A_t (k_t^f)^\alpha (\ln l_t^f + Z_t)^\beta & \text{if } l_t^f \geq e^{-Z_t} \text{ and } l_t^e \geq 1 \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

where $y_t^f \in \mathfrak{R}_+$ is the production by the firm, $l_t^f \in \mathfrak{R}_+$ is the number of worker in the firm, and $l_t^e \in \mathfrak{R}_+$ is the number of entrepreneurs. Each firm needs an entrepreneur. $A_t \in \mathfrak{R}_+$ is an index of the neutral technological change, $Z_t \in \mathfrak{R}_+$ is an index of the labor saving technological change. $\alpha, \beta, \gamma, \delta$ are parameters that satisfy the following constraints: $\alpha, \beta, \gamma, \delta \in (0,1)$, $\alpha + \beta + \gamma + \delta \leq 1$. This production function implies an average cost function with U form. Note that the share of labor in income (or elasticity of production with respect to labor) in this production function is a decreasing function of both the amount of labor, l^f , and the index of labor saving technological change, Z :

$$\lambda = \frac{\partial y^f}{\partial l^f} \frac{l^f}{y^f} = \frac{\beta}{\ln l^f + Z} \Rightarrow \frac{\partial \lambda}{\partial l^f} < 0; \quad \frac{\partial \lambda}{\partial Z} < 0$$

where λ is the share of labor in income. As we will see, the key features in the production function that is needed to deliver the results of the paper is that the share of labor on income is a decreasing function of both labor, l^f , and the index of labor saving technology, Z . We have chosen this particular functional form for the production function because allow us to generate a simple close form solution in the case of the benchmark model.

We will consider that the capital accumulates according with the following equation:

$$k_{t+1} = k_t + i_t^k \quad (2)$$

where i_t^k is the gross investment on capital. Notice that, in order to simplify, we have assumed that the depreciation rate is zero.

2.1.2 R&D Technology

The technological parameters A_t and Z_t are chose by the R&D firm to maximize revenues from the patent of the technology according to the following investment technology:

$$\begin{aligned} A_{t+1} &= A_t + \Gamma_A i_t^A \\ Z_{t+1} &= Z_t + \Gamma_Z i_t^Z \end{aligned} \quad (3)$$

where i_t^A is the per capita investment in the neutral technological change and i_t^Z is the per capita investment in the labor saving technological change.

2.1.3 Households:

Agent belong to a representative household with infinite life and a continuum of agents (this feature of the model does not play any role in the benchmark model). Population is constant and indexed in the interval $[0,1]$. The utility of the representative household is the following time separable utility function:

$$\sum_{t=0}^{+\infty} \left[\int_0^1 u(c_t(j)) dj \right] \left(\frac{1}{1 + \dots} \right)^t \quad (4)$$

where “ $c_t(j)$ ” is the consumption of the member “ j ” of the household and $u(\cdot)$ is the instantaneous felicity function of each agent that is given by the CES utility function:

$$u(c) = \begin{cases} \frac{c^{1-\dagger}}{1-\dagger} & \text{if } \dagger \in (0,1) \cup (1,+\infty) \\ \ln(c) & \text{if } \dagger = 1 \end{cases} \quad (5)$$

The concavity of the utility function implies that at any optimal path the consumption of every member of households should be equal, thus the utility function may be rewritten as follows:

$$\sum_{t=0}^{+\infty} u(c_t) \left(\frac{1}{1 + \dots} \right)^t \quad (6)$$

where c_t is the average consumption of the households’ member.

2.2 Agent’s decisions:

2.2.1 The production firms

The sequence of decisions of the potential entrepreneurs is as follows: at the end of the period before the production takes place the n (in per capita terms) agents that have the opportunity of becoming entrepreneur decide whether to create a firm or not. Then, they invest in the capital of the firm. In the period in which production takes place, entrepreneur hire workers first, then production takes place, and factors are payed. To analyze the firm’s decisions, we proceed backward.

Once entrepreneurs have decided to create a firm, he maximizes benefits:

$$\max_{l_t^f, k_t^f} A_t (k_t^f)^r (\ln l_t^f + Z_t)^s - w_t l_t^f - r_t k_t^f - P_t \quad (7)$$

where w_t is the payment to workers (the wage), r_t is the price of use of the capital and P_t is the patent royalty that firms should pay to the R&D firm for using the technology.

The first order conditions are:

$$sA_t^* (k_t^f)^r (\ln l_t^f + Z_t)^{s-1} \frac{1}{l_t^f} = w_t \quad (8)$$

$$rA_t^* (k_t^f)^{r-1} (\ln l_t^f + Z_t)^s = r_t \quad (9)$$

That is, firms hire factor up to the point in which its marginal productivity equalize its price. We define the before pattern profit function as the maximum profit of the firm before paying the patent royalty (where the superscript “bp” means before patent royalty):

$$f^{bp}(w_t, r_t, A_t, Z_t) = \max_{l_t^f, k_t^f} A_t^* (k_t^f)^r (\ln l_t^f + Z_t)^s - w_t l_t^f - r_t k_t^f \quad (10)$$

The opportunity cost of entrepreneur is to work in another firm. Thus, in order that agents that may become entrepreneur decide to do so, it is necessary that the payment that they receive from entrepreneurial activities exceed the payment that they would receive as a worker¹:

$$f^{bp}(w_t, r_t, A_t, Z_t) - P_t \geq w_t \Leftrightarrow P_t \leq f(w_t, r_t, A_t, Z_t) - w_t \quad (11)$$

2.2.2 The R&D Firms

It follows from equation (11) and the fact that there are n entrepreneurs (and firms) in per capita terms that the per capita demand for patterns is as follows:

$$\begin{cases} n & \text{if } P_t \leq f^{bp}(w_t, r_t, A_t, Z_t) - w_t \\ 0 & \text{otherwise} \end{cases}$$

Thus, the profit maximization problem of the R&D is as follows:

$$\max_{P_t, i_{t-1}^A, i_{t-1}^B} \sum_{t=0}^{+\infty} \frac{\int_0^n P_t dj - i_t^A - i_t^Z}{\prod_{j=1}^t (1 + r_j)} \quad (12)$$

$$s.t.: P_t \leq f^{bp}(w_t, r_t, A_t, Z_t) - w_t$$

$$A_t = A_{t-1} + \Gamma_A i_{t-1}^A$$

$$Z_t = Z_{t-1} + \Gamma_Z i_{t-1}^Z$$

¹ Notice that if the per capita number of firms, n , were endogenous, expression (11) would determine the size of the firm. Due to the fact that firms should pay the sunk cost of the patent royalty and the fact that the technological knowledge is a non-rival “good”, the firm size of firms would be inefficiently high and the number of firms would be inefficiently low. Since the goal of this section is to build a model with technological change in which the market allocation is efficient, we introduced the assumption that the per capita number of firms is exogenous to avoid the existence of an inefficiently low number of firms.

Where we have included the constraint (11) in the above problem, since if such constraint is not satisfied agents that may become entrepreneurs would not choose to do so, and the demand for the patent would be zero. The profits of the R&D firm consist in the revenues from the patent royalties that firms pay minus the R&D costs of developing new technologies. The above maximization problem may be rewritten as follows:

$$\max_{A_t, B_t} \sum_{t=0}^{+\infty} \frac{n \left[A_t^* (k_t^f)^\Gamma (\ln l_t^f + Z_t)^S - w_t (l_t^f + 1) - r_t k_t^f \right] - \frac{A_{t+1} - A_t}{\Gamma_A} - \frac{Z_{t+1} - Z_t}{\Gamma_Z}}{\prod_{j=1}^t (1 + r_j)} \quad (13)$$

where k_t^f and l_t^f are the optimal levels of capital and labor chosen by the firm. The first order conditions for the case of interior solution² are:

$$n_n A_t^{* -1} (k_t^f)^\Gamma (\ln l_t^f + Z_t)^S = \frac{r_t}{\Gamma_A} \quad (14)$$

$$n S A_t^* (k_t^f)^\Gamma (\ln l_t^f + Z_t)^{S-1} = \frac{r_t}{\Gamma_Z} \quad (15)$$

The above equations mean that the marginal revenues of the investment in both neutral and labor saving technological change should be equal to its marginal cost.

We will assume that n is low enough to guaranty that profits of R&D firms are positive:

Assumption 1:³ $1 \leq (1-n) \ln \left(\frac{1-n}{n} \right) \Leftrightarrow n \leq 0.2178117..$

2.2.3 Households

The maximization problem of households is rather conventional: they maximize their utility subject to their budget constraint:

² As we will see later on, it may be the case that at equilibrium there is not interior solution for the R&D firm, more precisely, the labor saving technological change Z may be zero at equilibrium.

³ This assumption means that the profits of the R&D firms are positive:

$$\frac{f_t^{R\&D}}{y_t^f} = n \left[\frac{y_t^f}{y_t^f} - \frac{w_t (l_t^f + 1)}{y_t^f} - \frac{r_t k_t^f}{y_t^f} \right] - \frac{r_t A_t}{\Gamma_A y_t^f} - \frac{r_t Z_t}{\Gamma_Z y_t^f} = n \left[1 - r - n - s \frac{\left(\frac{1}{1-n} + Z_t \right)}{\left(\ln \left(\frac{1-n}{n} \right) + Z_t \right)} \right] \geq 0.$$

Since $r + n + s \leq 0$, a sufficient condition in order that the above equation holds is.:

$$\frac{\left(\frac{1}{1-n} + Z_t \right)}{\left(\ln \left(\frac{1-n}{n} \right) + Z_t \right)} \leq 1 \Leftrightarrow (1-n) \ln \left(\frac{1-n}{n} \right) \geq 1 \Leftrightarrow n \leq 0.2178117..$$

$$\max_{\{b_{t+1}, c_t\}_{t=0}^{+\infty}} \sum_{t=0}^{+\infty} u(c_t) \left(\frac{1}{1 + \dots} \right)^t \quad (16)$$

$$s.t. : b_{t+1} = f_t^f n + w_t(1-n) + (1+r_t)b_t + f_t^{R\&D} - c_t$$

where b_t is the per capita amount of assets of the household, n is the per capita number of agents in the households that become entrepreneurs, each of them earn the profit of their firm f_t^f (where f_t^f is the profits of firms after paying the pattern: $f_t^f = f_t^{bp} - P_t$). $w_t(1-n)$ is the per capita labor earning of the $(1-n)$ per capita workers of the household. We assume that each household has the right of a share of the profits of the R&D firm, $f_t^{R\&D}$. In order to simplify, we assume that the shares in the profits of the R&D firm are not tradable. In case that the profits of the R&D firm were negative, then $f_t^{R\&D}$ is interpreted as a lump sum tax that is collected in order to subsidy the R&D firm (assumption 1 guaranties that these profits are positive in the benchmark model).

The household maximization problem implies the following necessary conditions (for interior solution):

$$u'(c_t) = \frac{u'(c_{t+1})(1+r_{t+1})}{1 + \dots} \quad (17)$$

$$\lim_{t \rightarrow \infty} \frac{u'(c_t)}{(1 + \dots)^t} = 0 \quad (18)$$

Equation (17) is the well-known Euler Equation, which means that the marginal disutility that one unit of saving generate due to the reduction of consumption in the present $u'(c_t)$, should be equal to the marginal utility that such saving generates in the future, $\frac{u'(c_{t+1})(1+r_{t+1})}{1 + \dots}$. One unit of saving in the present generates $(1+r_{t+1})$ units of consumption in the future, and the marginal utility of one unit of consumption in the future is equal to $\frac{u'(c_{t+1})}{1 + \dots}$, therefore $\frac{u'(c_{t+1})(1+r_{t+1})}{1 + \dots}$ is the marginal utility of one unit of saving generate in the future. Equation (18) is the transversality condition.

2.3 Definition of Equilibrium

Definition 1: Equilibrium is an allocation $\{c_t, b_t, y_t^f, k_t^f, l_t^f, i_t^K, i_t^A, i_t^Z, k_t, A_t, Z_t\}$ and a sequence of prices $\{w_t, r_t, P_t\}$ such that:

- Households maximize their utility: $\{c_t, b_{t+1}\}_{t=0}^{+\infty}$ are the solution of household's maximization problem (16).
- Production Firms maximize profits: $\{k_t^f, l_t^f\}_{t=0}^{+\infty}$ is the solution of the maximization problem of the firms (7) and entrepreneurs get as benefits at least their opportunity cost (11). The production of each firm y_t^f is given by production function (1).
- The R&D firm maximizes profits: $\{P_t, i_{t-1}^A, i_{t-1}^Z, A_t, Z_t\}_{t=0}^{+\infty}$ is the solution of the maximization problem of the R&D (12).
- Capital accumulation: capital accumulates according with equation (2).
- Financial markets clear: $b_{t+1} - b_t = i_t^K + i_t^A + i_t^Z$ (19)
- Labor market clears: $1 - n = nl_t^f$ (20)
- Capital market clear: $k_t = nk_t^f$ (21)
- Good market clears: $ny_t^f = i_t^K + i_t^A + i_t^Z + c_t$ (22)

We assume that $b_0 = k_0 + \frac{A_0}{\Gamma_A} + \frac{Z_0}{\Gamma_Z}$. This assumption means that the wealth of households coincides with the value⁴ of different types of “capital”. This assumption implies that in every period the wealth of households coincides with the wealth of the economy $b_t = k_t + \frac{A_t}{\Gamma_A} + \frac{Z_t}{\Gamma_Z}$.

2.4 Dynamic Behavior

Using the definition of equilibrium, it follows that the behavior of the economy is given by the following dynamic system (see the Appendix for details):

$$b_{t+1} = y^{bm}(b_t) - c_t \quad (22)$$

$$\frac{1}{u'(c_{t+1})} = \frac{1 + r^{bm}(y^{bm}(b_t) - c_t + b_t)}{1 + \dots} \frac{1}{u'(c_t)} \quad (23)$$

⁴ Note that the cost of creating one unit of capital in term of the consumption good is one. Thus, the marginal cost and the price of capital is one (see 2). The price (the marginal cost) of technological knowledge, A and Z, are equal to $1/\Gamma_A$ and $1/\Gamma_A$ respectively. Thus, the value of the stock of different types of capital (physical and technological) is equal to $k_t + \frac{A_t}{\Gamma_A} + \frac{Z_t}{\Gamma_Z}$.

where $y^{bm}(b)$ is an increasing continuous function that describes the relationship between per capita production and per capita wealth, b (the superscript “ bm ” means benchmark model), and $r^{bm}(b)$ is a decreasing continuous function that describes the relationship between per the return on investment and per capita wealth, b :

$$y^{bm}(b) = \begin{cases} n^{1-r} \frac{r^r ({}_n\Gamma_A)^r \left(\ln\left(\frac{1-n}{n}\right) \right)^s}{(r+{}_n)^{r+{}_s}} b^{r+{}_s} & \text{if } b \leq \tilde{b}^{bm} \\ n^{1-r} \frac{r^r ({}_n\Gamma_A)^r (s\Gamma_z)^s}{(r+{}_n+s)^{r+{}_s+s}} \left(b + \frac{\ln\left(\frac{1-n}{n}\right)}{\Gamma_z} \right)^{r+{}_s+s} & \text{if } b \geq \tilde{b}^{bm} \end{cases} \quad (24)$$

$$r^{bm}(b) = \begin{cases} \frac{n^{1-r} (r+{}_n)^{1-r-{}_s} r^r ({}_n\Gamma_A)^r \left(\ln\left(\frac{1-n}{n}\right) \right)^s}{b^{1-r-{}_s}} & \text{if } b \leq \tilde{b}^{bm} \\ \frac{n^{1-r} r^r ({}_n\Gamma_A)^r (s\Gamma_z)^s (r+{}_n+s)^{1-r-{}_s-s}}{\left(b + \frac{\ln\left(\frac{1-n}{n}\right)}{\Gamma_z} \right)^{1-r-{}_s-s}} & \text{if } b \geq \tilde{b}^{bm} \end{cases} \quad (25)$$

where $\tilde{b}^{bm} \equiv \frac{(r+{}_n) \ln\left(\frac{1-n}{n}\right)}{s \Gamma_z}$ is the threshold level of wealth such that labor saving technological change arise (see equation 28 below). The relationship between the amount of capital and different types of technological change and per capita wealth (b) is as follows:

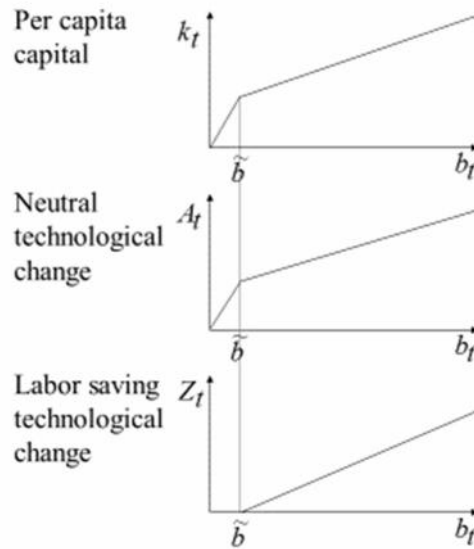
$$k^{bm}(b) = \begin{cases} \frac{r}{r+{}_n} b & \text{if } b \leq \tilde{b}^{bm} \\ \frac{r}{r+{}_n+s} \left(b + \frac{\ln\left(\frac{1-n}{n}\right)}{\Gamma_z} \right) & \text{if } b \geq \tilde{b}^{bm} \end{cases} \quad (26)$$

$$A^{bm}(b) = \begin{cases} \frac{{}_n\Gamma_A}{r+{}_n} b & \text{if } b \leq \tilde{b}^{bm} \\ \frac{{}_n\Gamma_A}{r+{}_n+s} \left(b + \frac{\ln\left(\frac{1-n}{n}\right)}{\Gamma_z} \right) & \text{if } b \geq \tilde{b}^{bm} \end{cases} \quad (27)$$

$$Z^{bm}(b) = \begin{cases} 0 & \text{if } b \leq \tilde{b}^{bm} \\ \frac{s\Gamma_z}{r + n + s} \left(b - \frac{(r + n)}{s} \frac{\ln\left(\frac{1-n}{n}\right)}{\Gamma_z} \right) & \text{if } b \geq \tilde{b}^{bm} \end{cases} \quad (28)$$

The above equations are displayed in figure 1. When wealth is not large enough, labor is relatively abundant (and cheap) and consequently there are not incentives to generate labor saving technological change, there is only investment on capital and neutral technological change, which rises with wealth (b). Once that a certain threshold level of wealth \tilde{b}^{bm} is reached, labor becomes relatively scarcer and more expensive and this makes investment on labor saving technological change profitable. Thus, once the threshold of wealth \tilde{b} is reached, the per capita capital and both the neutral and labor saving technological changes increase with wealth.

Figure 1: Relationship between per capita capital, neutral and labor saving technological change and wealth

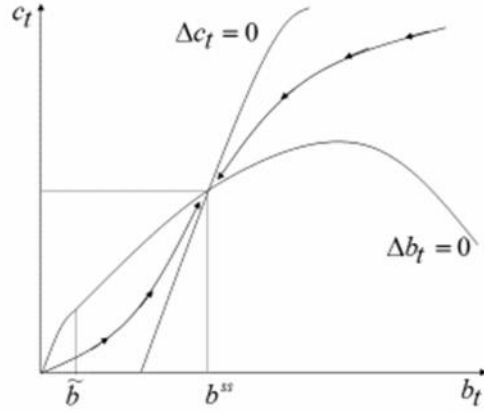


It follows from equations (24) and (25) that the behavior of the model may be of two types: i) when $\Gamma + n + S < 1$, there is not endogenous growth and the model behaves as the Ramsey's model; ii) when $\Gamma + n + S = 1$, there is endogenous growth and the model behaves very similar to a AK model

Dynamic when there is not endogenous growth ($\Gamma + n + S < 1$):

When $\Gamma + n + S < 1$, it follows from equations (24) and (25) that there is not endogenous growth and the model behaves as the Ramsey's model, with the typical saddle point dynamic, as figure 2 below displays:

Figure 2: Dynamic Behavior when $\alpha + \beta + \theta < 1$



Since this paper is about labor saving technological change, we will introduce the assumption below in order to guaranty that there is labor saving technological change at the steady state. In fact, we will concentrate in the case in which there is labor saving technological change (when $b_t > \tilde{b}^{bm}$) from now on.

Assumption 2: $r^{bm}(\tilde{b}^{bm}) = \frac{n^{1-\Gamma} r^{\Gamma} (\Gamma_A)^{\Gamma} (S\Gamma_z)^{1-\Gamma-\sigma}}{\left(\ln\left(\frac{1-n}{n}\right)\right)^{1-\Gamma-\sigma-S}} > \dots$

Dynamic when there is endogenous growth ($\Gamma + \sigma + S = 1$):

If $\Gamma + \sigma + S = 1$, once that the per capita wealth reaches the threshold level of wealth \tilde{b}^{bm} in which there is labor saving technological change ($b_t > \tilde{b}^{bm}$), the dynamic behavior of the model resembles a AK model with endogenous growth, in which the growth rate of consumption and production is the same and constant, while the growth rate of wealth tends to the same rate. It is shown in the Appendix that consumptions and wealth behave according with the following equations:

$$c_t = \left(n^{1-\Gamma} r^{\Gamma} (\Gamma_A)^{\Gamma} (S\Gamma_z)^S - g_c \right) \left(b_t + \frac{\ln\left(\frac{1-n}{n}\right)}{\Gamma_z} \right)$$

$$b_{t+1} = g_c \frac{\ln\left(\frac{1-n}{n}\right)}{\Gamma_z} + (1 + g_c) b_t$$

where $g_c \equiv \left[\frac{1 + n^{1-r} r^\Gamma (\Gamma_A)^r (\Gamma_z)^s}{1 + \dots} \right]^{\frac{1}{\dagger}} - 1$ is the growth rate of consumption, which is constant⁵. The growth rate of production is also constant and equal to the growth rate of consumption, while the growth rate of wealth is decreasing and tends to be the same as the consumption and production:

$$\frac{c_{t+1}}{c_t} = (1 + g_c) \equiv \left[\frac{1 + n^{1-r} r^\Gamma (\Gamma_A)^r (\Gamma_z)^s}{1 + \dots} \right]^{\frac{1}{\dagger}} = \frac{y_{t+1}}{y_t}; \quad \frac{b_{t+1}}{b_t} \equiv \frac{g_c \ln\left(\frac{1-n}{n}\right)}{\Gamma_z b_t} + (1 + g_c)$$

2.5 Efficient allocation

The social planner problem maximizes the representative household's utility subject to feasibility constraints:

$$\begin{aligned} & \max_{\{c_t, k_t^f, l_t^f, i_t^f, i_t^A, i_t^B, k_{t+1}, A_{t+1}, B_{t+1}\}_{t=0}^{+\infty}} \sum_{t=0}^{+\infty} u(c_t) \left(\frac{1}{1 + \dots} \right)^t \\ \text{s.t. : } & i_t^K + i_t^A + i_t^Z = n A_t^* (k_t^f)^r (\ln l_t^f + Z_t)^s - c_t \\ & 1 - n = n l_t^f \\ & k_t = n k_t^f \\ & k_{t+1} = i_t^K + k_t \\ & A_{t+1} = \Gamma_A i_t^A + A_t \\ & Z_{t+1} = \Gamma_Z i_t^Z + Z_t \end{aligned} \tag{29}$$

The first order conditions for interior solution imply (see Appendix):

$$u'(c_t) = \frac{u'(c_{t+1})}{1 + \dots} \left[1 + r n^{1-r} A_{t+1}^* (k_{t+1})^{r-1} (\ln(l_{t+1}^f) + Z_{t+1})^s \right] \tag{30}$$

$$\begin{aligned} r n^{1-r} A_{t+1}^* (k_{t+1})^{r-1} (\ln(l_{t+1}^f) + Z_{t+1})^s &= \Gamma_A n^{1-r} A_{t+1}^{*-1} (k_{t+1})^r (\ln(l_{t+1}^f) + Z_{t+1})^s = \\ \Gamma_Z n^{1-r} A_{t+1}^* (k_{t+1})^r (\ln(l_{t+1}^f) + Z_{t+1})^{s-1} & \end{aligned} \tag{31}$$

⁵ In the case of endogenous growth and $\dagger < 1$, we need to impose the following assumption in order that the household' problem has solution: $1 + r > 1 + g_c \Leftrightarrow$

$$\left[1 + n^{1-r} r^\Gamma (\Gamma_A)^r (\Gamma_z)^s \right]^{1-\dagger} < 1 + \dots \Leftrightarrow \dagger > 1 - \frac{\ln(1 + \dots)}{\ln\left(1 + n^{1-r} r^\Gamma (\Gamma_A)^r (\Gamma_z)^s\right)}$$

where $l_t^f = \frac{1-n}{n}$. These equations may be obtained from the equilibrium conditions (9), (14), (15) and (17). Thus, the solution of the social planner problem is exactly the same as the equilibrium allocation. Thus, first welfare theorem applies: the Walrasian equilibrium is efficient.

Remark: The equilibrium is efficient.

Thus, we have reached the goal of this section: we have built a R&D model in which market allocation is efficient. Now that we have constructed our benchmark model, we proceed to introduce frictions in the labor market to be able to answer the question pose in the introduction: Is technological change efficient?

3. A model with unemployment

3.1 Modification of the benchmark model

The technology and the preferences of the model are the same. The only change in the model is that in this section we introduce a bargaining process between entrepreneur and workers. In order to avoid some technical problems, we will concentrate in the case in which there is not endogenous growth. That is, in this and the following section we assume that $\Gamma + n + S < 1$.

In order to introduce a bargaining process between entrepreneur and workers, we will consider from now on that individual agents like to contribute to the household with their labor earnings. More precisely, individual workers (unemployed workers included) and entrepreneurs have each period the following utility function respectively:

$$\begin{aligned}
 U^e(j) &= u(c_t(j)) + v^e(f_t(j)) - \frac{\int_0^n v(f_t(\tilde{j})) d\tilde{j}}{n} & \text{if } j \in [0, n] \\
 U^w(j) &= u(c_t(j)) + v^w(w_t(j)) - \frac{\int_n^1 v^w(w_t(\tilde{j})) d\tilde{j}}{1-n} & \text{if } j \in [n, 1]
 \end{aligned} \tag{32}$$

where $U^e(j)$ and $U^w(j)$ are respectively the utility of entrepreneurs and workers. We indexed entrepreneurs as the “first n agents”, $j \in [0, n]$, and workers as the “last $n-1$ agents”, $j \in [n, 1]$. $w_t(j)$ is labor earning and $f_t(j)$ the earning from entrepreneurial activities. Thus, individual agents care about two things: their consumption $c_t(j)$ (which in any optimal solution of the household is equal to the average consumption of the household: $c_t(j) = c_t$) and their contribution to the household with their labor earnings

$w_i(j)$ or entrepreneurial earning in relation with the average contribution of other household's members of the same type (workers and entrepreneurs respectively). We detract from the utility of contributing of each agent $v(w_i(j))$ the average utility of the other agents $\int_n^1 v(w_i(\tilde{j}))d\tilde{j} / (1-n)$ in order that the utility of the household to be the same as in the benchmark model (equation 8). Following the labor literature, we will assume that workers are risk averse and entrepreneurs risk neutral. Thus, we assume that the "love for contributing with earnings to the household" is represented by the constant elasticity of substitution utility function in the case of workers:

$$v^w(w) = w^x \quad (33)$$

"Love for contributing with earnings to the household" is represented by the lineal utility function in the case of entrepreneurs:

$$v^e(f) = f \quad (34)$$

3.2 The bargaining process between entrepreneurs and workers

We will consider that there are two types of workers: insiders and outsiders. Insiders are the workers that negotiate with entrepreneurs and they do not suffer unemployment at equilibrium. Outsiders are hired by the firms once that insiders have been hired and that insiders have negotiated with entrepreneurs.

The sequence of decisions of an entrepreneur is as follows: at the end of the period before the production takes place the n (in per capita terms) agents that have the opportunity to become entrepreneur decides where to create a firm or not. Then they invest in the capital of the firm. In the period in which production takes place, entrepreneur hire insiders first, then insiders and the entrepreneur negotiate the wage, after that, the entrepreneur hires outsiders and production takes place. To analyze the firm's decisions, we proceed backward.

The bargaining process between insider workers and entrepreneurs is given the Nash bargaining solution, which consist in maximizing the product of the objective function of both agents (entrepreneur and insiders) minus the value of their function in case of no agreement:

$$\max_{w_t, l_t^f} [l_t^{i,f} (v^w(w_t) - \bar{v}^w)] [v^e(A_t^e (k_t^f)^\Gamma (\ln l_t^f + Z_t)^\delta - w_t l_t^f - (1+r_t)k_t^f - P_t) - \bar{v}^e]$$

where:

$$\bar{v}^w = p_t^o v^w(\tilde{w}_t) + (1-p_t^o) v^w(0) \quad (35)$$

$$\bar{v}^e = v^e(-(1+r_t)k_t^f - P_t)$$

where $l_t^f = l_t^{i,f} + l_t^{o,f}$ is the total amount of labor hired by the firm, which includes insider workers, $l_t^{i,f}$, and outsiders $l_t^{o,f}$. \bar{v}^w and \bar{v}^e is the reserve utility of respectively workers and entrepreneurs in case that they do not reach an agreement. We assume that if insiders and the entrepreneur do not reach an agreement, production does not place. In this case insiders do not receive anything from the firm and they become outsiders, which receive a wage of the market \tilde{w}_t with probability p_t^o , which is the probability that an outsider is hired, and zero if they do not find a job as an outsider. Thus, the reserve utility of insider worker in case of no agreement, \bar{v}^w , is equal to the utility of outsider $p_t^o v^w(\tilde{w}_t) + (1 - p_t^o)v^w(0)$. If insider workers and the entrepreneur do not reach an agreement, production does not take place and the entrepreneur will have the loses associated to not producing anything, which consist in the fixed costs ($rk_t^f + P_t$), which includes the costs associated with the capital rk_t^f and the payment of the patent P_t . Thus, the reserve utility of the entrepreneur in case of no agreement is $v^e(-rk_t^f - \Phi)$. The maximization problem (35) may be rewritten as follows:

$$\max_{w_t, l_t^f} \left[l_t^{i,f} (v^w(w_t) - p_t^o v^w(\tilde{w}_t)) \right] \left[A_t^r (k_t^f)^r (\ln l_t^f + Z_t)^S - w_t l_t^f \right] \quad (36)$$

Note that \tilde{w}_t is the wage that insider workers may get in other firms, and therefore is not a variable chosen in the bargaining process of the firm. The first order conditions for the bargaining problem in the case of interior solution may be written as follows:

$$\frac{v^{w'}(w_t)}{v^w(w_t) - p_t^o v^w(\tilde{w}_t)} = \frac{l_t^f}{A_t^r (k_t^f)^r (\ln l_t^f + Z_t)^S - w_t l_t^f} \quad (37)$$

$$S A_t^r (k_t^f)^r (\ln l_t^f + Z_t)^{S-1} \frac{1}{l_t^f} = w_t \quad (38)$$

Equation (38) is the first order condition with respect to the wage. It means that in marginal increase in the utility of insider worker due to an increase in the wage, $v^{w'}(w_t)$, weighted by $1/v^w(w_t) - p_t^o v^w(\tilde{w}_t)$ should be equal to the marginal decrease in the profit of the entrepreneur due to an increase in the wage, l_t^f , weighted by $1/[nA_t^r (k_t^f)^r (\ln l_t^f + Z_t)^S - w_t l_t^f]$. A key feature of this first order condition is that the higher is the reserve utility of the insider workers in case of no agreement, the higher is the weight that those agents have in the objective function. In other world, the higher the reserve utility of insider workers, the higher its bargaining power. Since the reserve utility of insider workers is equal to the expected utility of outsider workers, and such utility depend positively on the probability that outside workers to be hired, the bargaining power of insider workers decreases with the unemployment rate and increases with the

employment rate. Equation (38) is the conventional condition of the equalization between the marginal product of labor and its unit cost (the wage).

We assume that the portion μ of workers of each firm are insiders:

$$l_t^{i,f} = \sim l_t^f \quad (39)$$

At aggregate level:

$$l_t = n l_t^f \quad (40)$$

$$l_t^i = n l_t^{i,f} = \sim n l_t^f = \sim l_t \quad (41)$$

Thus, the probability that an outsider gets a job is as follows:

$$p_t^o = \frac{l_t - l_t^i}{1 - n - l_t^i} = \frac{(1 - \sim) l_t}{1 - n - \sim l_t} \quad (42)$$

The numerator of the above fraction is the amount of outsider that get a job, which is the total number of employment minus the number of insiders, that always are employed. The numerator is the total amount of outsiders, which consist in the total number of workers $1 - n$ (the number of agents minus the number of entrepreneurs) minus the number of insider workers.

Using (37) and (42), it yields:

$$\} _t = \frac{x}{x + 1 - p_t^o} = \frac{x}{x + 1 - \frac{(1 - \sim) l_t}{1 - n - \sim l_t}} \quad (43)$$

where $\} _t$ is the share of labor income in the production of the firm:

$$\} _t = \frac{w_t l_t^f}{y_t^f} \quad (44)$$

Equation (43) means that insider workers have a higher bargaining power when employment is high and therefore they can get a higher share of production. The reason is that if insiders do not reach an agreement with entrepreneurs, they would become to an outsider. Thus, its ‘‘opportunity cost’’ in case of no agreement is the expected utility of an outsider worker. The expected utility of an outsider workers rises with the employment level, since in the employment level raises the probability of find a job by the outsider. Thus, higher employment level means higher bargaining power for insider workers and therefore a higher labor share of the production of the firm. Equation (43) is displayed in figure 2 and called negotiation labor share. Such curve has positive slope due to the fact that the higher the employment, level the higher the share of labor income in production.

Using (38) and (40) it yields:

$$l_t = \frac{S}{\ln\left(\frac{l_t}{n}\right) + Z_t} \quad (45)$$

The above equation means that the technology of the firm implies that the labor share decreases with the number of workers of the firm, which is a decreasing function of the level of employment. Obviously, the share of labor in production is also decreasing in the labor saving technological index Z_t . Figure 2 displays equation (45) and is called technological labor share. Figure 2 also displays the effect of a labor saving technological change in production: it reduces the “technological” labor share, diminishing the level of employment and the equilibrium share of labor on income. Thus, labor saving technological change reduces the employment level, this is the key feature that will make labor saving technological change inefficient.

Using (43) and (44) it is possible to obtain the amount of employment in equilibrium:

$$\frac{x}{x+1 - \frac{(1-\alpha)l_t}{1-n-\alpha l_t}} = \frac{S}{\ln\left(\frac{l_t}{n}\right) + Z_t} \quad (46)$$

We denote as $l^{um}(Z_t)$ the employment level that is the above equation, which according with the Implicit Function Theorem is a decreasing function of the labor saving technological change (where the superscript “*um*” means unemployment model):

$$l^{um}(Z) \stackrel{Def}{\Leftrightarrow} \frac{x}{x+1 - \frac{(1-\alpha)l(Z)}{1-n-\alpha l(Z)}} - \frac{S}{\ln\left(\frac{l^{um}(Z)}{n}\right) + Z} = 0 \Leftrightarrow$$

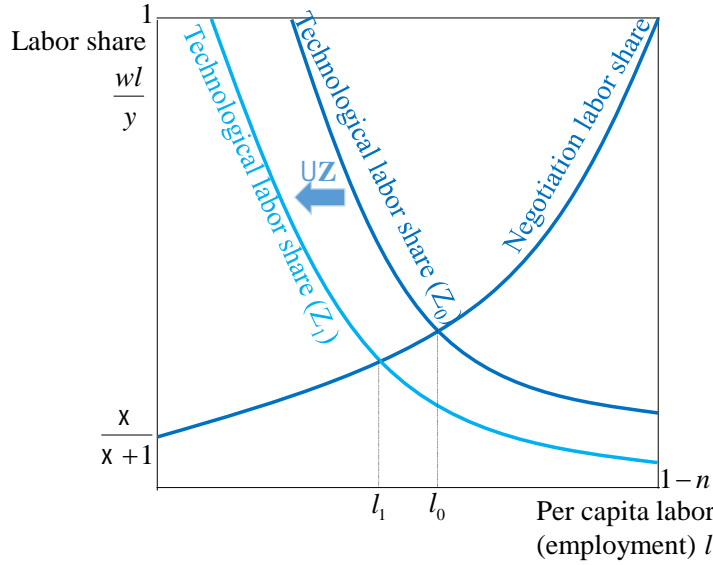
$$x \left[\ln\left(\frac{l^{um}(Z)}{n}\right) + Z \right] + S \left[- (x+1) + \frac{(1-\alpha)l^{um}(Z)}{1-n-\alpha l^{um}(Z)} \right] = 0 \quad (47)$$

$$\frac{\partial l^{um}(Z)}{\partial Z} = - \frac{x}{\frac{x}{l^{um}(Z)} + S \frac{(1-\alpha)(1-n)}{[1-n-\alpha l^{um}(Z)]^2}} < 0$$

The wage that results from this bargaining process is as follows:

$$w_t = SA_t^* (k_t^f)^r \left(\ln\left(\frac{l^{um}(Z)}{n}\right) + Z_t \right)^{s-1} \frac{n}{l^{um}(Z)} \quad (48)$$

Figure 3: The effect of labor saving technological change on employment



3.3 Agents decisions and Dynamic Behavior

The behavior of agents in the economy does not change much besides the bargaining process between workers and firms. Once that wage is determined in the bargaining process (equation 48), the decisions of households, production firms and R&D firms do not change with respect to the benchmark model. Using the same procedures as in the last sections, the maximization problem of the production firms and the R&D firm imply that the interest rate at equilibrium would equalize the private return of investment in capital and neutral and labor saving technological change:

$$r_t = r_t^{priv,k,um} = r_t^{priv,A,um} = r_t^{priv,Z,um} \quad (49)$$

where $r_t^{priv,k,um}$, $r_t^{priv,A,um}$, $r_t^{priv,Z,um}$ are the private returns to invest in respectively capital, neutral and labor saving technological change defined as follows:

$$r_t^{priv,k,um} = \Gamma n^{1-\Gamma} A_t^* (k_t)^\Gamma \left(\ln \left(\frac{l^{um}(Z_t)}{n} \right) + Z_t \right)^s \quad (50)$$

$$r_t^{priv,A,um} = \Gamma^A n^{1-\Gamma} A_t^{*-1} (k_t)^\Gamma \left(\ln \left(\frac{l^{um}(Z_t)}{n} \right) + Z_t \right)^s \quad (51)$$

$$r_t^{priv,Z,um} = \Gamma^Z S n^{1-\Gamma} A_t^* (k_t)^\Gamma \left(\ln \left(\frac{l^{um}(Z_t)}{n} \right) + Z_t \right)^{s-1} \quad (52)$$

Using the same procedures as in the last sections, the household's Euler equation would be as follows:

$$u'(c_t) = \frac{u'(c_{t+1})}{1 + \dots} [1 + r_{t+1}] \quad (53)$$

The dynamic behavior of the economy does not change much either. The dynamic behavior is given by a dynamic system very similar to the one presented in the benchmark model:

$$b_{t+1} = y^{me,um}(b_t) - c_t \quad (54)$$

$$\frac{1}{u'(c_{t+1})} = \frac{1 + r^{me,um}(y^{me,um}(b_t) - c_t)}{1 + \dots} \frac{1}{u'(c_t)} \quad (55)$$

where $y^{me,um}(b)$ and $r^{me,um}(b)$ are functions that describes the relationship between respectively per capita income and interest rate, and the wealth level (these functions are defined in the appendix). The superscript “*me*” means market equilibrium (to distinguish the market allocation from the second best explained below) and the superscript “*um*” means unemployment model. The function $y^{me,um}(b)$ is an increasing function, while $r^{me,um}(b)$ is decreasing. Thus, the dynamic behavior of the economy is qualitatively alike the one in the benchmark model: it behaves as the standard Ramsey’s model (remember that we have assumed that $\Gamma + n + S < 1$). The unique substantial difference is that when the economy reaches the threshold level of wealth \tilde{b}^{um} in which labor saving technological change arises and the per capita amount of labor decreases along the transition due to the labor saving technological change. Along the transition there is labor saving technological change which reduces the demand for labor, the labor share and increases unemployment, as figure 3 displays. Thus, along the transition employment decreases while unemployment increases, due to labor saving technological change.

3.4 Second Best Allocation

The social planner problem in this case maximizes the households’ utility subject to feasibility constraint and subject also to the restriction that the employment is the one in the equilibrium. That is, the result of the bargaining process given in equation (46) is introduced as a constraint. Thus, the second best takes account of the bargaining process and takes the equilibrium labor as given by the “labor market institutional technology”. Given this “labor market institutional technology”, the second-best allocation is the one that maximizes the utility of households taking account not only the pure technological constraints of production functions and accumulation equations, but also the “institutional” constraint of the labor market:

$$\begin{aligned}
& \max_{\{c_t, k_t^f, l_t^f, i_t^A, i_t^B, k_{t+1}, A_{t+1}, B_{t+1}\}_{t=0}^{+\infty}} \sum_{t=0}^{+\infty} u(c_t) \left(\frac{1}{1 + \dots} \right)^t \\
& s.t.: i_t^K + i_t^A + i_t^B = nA_t^* (k_t^f)^\Gamma (\ln l_t^f + Z_t)^S - c_t \\
& l_t = nl_t^f \\
& \frac{\chi}{\chi + 1 - \frac{(1-\tilde{\alpha})l_t}{1-n-\tilde{\alpha}l_t}} = \frac{S}{\ln\left(\frac{l_t}{n}\right) + Z_t} \\
& k_t = nk_t^f \\
& k_t = i_{t-1}^K + k_{t-1} \\
& A_t = \Gamma_A i_{t-1}^A + A_{t-1} \\
& B_t = \Gamma_B i_{t-1}^B + B_{t-1}
\end{aligned} \tag{56}$$

The above maximization problem implies the following necessary conditions:

$$u'(c_t) = \frac{u'(c_{t+1})}{1 + \dots} [1 + r_{t+1}^{soc,k,um}] \tag{57}$$

$$r_{t+1}^{soc,k,um} = r_{t+1}^{soc,A,um} = r_{t+1}^{soc,Z,um} \tag{58}$$

where $r_t^{socv,k,dm}$, $r_t^{soc,A,dm}$, $r_t^{soc,Z,dm}$ are the social returns to invest in respectively capital, neutral and labor saving technological change defined as follows:

$$r_{t+1}^{soc,k,um} = r_{t+1}^{priv,k,um} \tag{59}$$

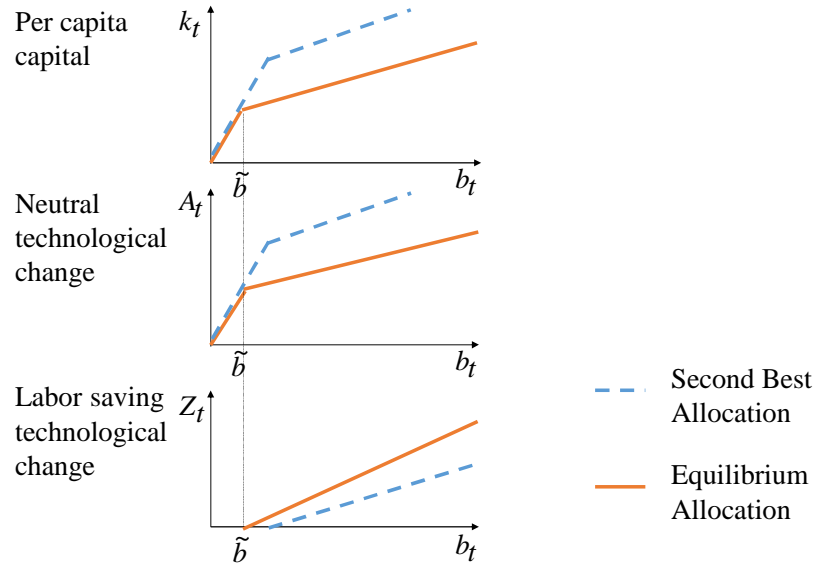
$$r_{t+1}^{soc,A,um} = r_{t+1}^{priv,A,um} \tag{60}$$

$$r_{t+1}^{soc,Z,um} = r_{t+1}^{priv,Z,um} \left(1 + \frac{\overline{l^{um'}(Z_{t+1})}}{l^{um}(Z_{t+1})} \right) < r_{t+1}^{priv,Z,um} \tag{61}$$

If we compare equation (58) and (61) with equation (49), it follows that the allocation of investment to different types of technological change is different in the second-best solution than at equilibrium. The “social return” of labor saving technological change is lower than its private return due to the fact that labor saving technological change reduces the amount of labor at equilibrium $l^{um'}(Z_t)$ and this generate a kind of “negative external effect” on the labor market institutional technology. That is, the amount of labor saving technological change is larger at the equilibrium than at the second-best allocation. Thus, the equilibrium amount of investment devoted to labor saving technological change is inefficiently large, as figure 4 displays. This statement is formalized in the following proposition (proven in the appendix):

Proposition 1: Let's define $k^{me,um}(b_t), A^{me,um}(b_t), Z^{me,um}(b_t)$ and $k^{sb,um}(b_t), A^{sb,um}(b_t), Z^{sb,um}(b_t)$ the amount of capital, neutral technological change and labor saving technological change for an amount of total investment in respectively the market allocation and the second-best allocation (in the unemployment model "um"). Then: $k^{me,um}(b_t) < k^{sb,um}(b_t), A^{me,um}(b_t) < A^{sb,um}(b_t), Z^{me,um}(b_t) > Z^{sb,um}(b_t)$.

Figure 4: Equilibrium Allocation of Investment versus Second Best Allocation



That is, there is an inefficiently high amount of investment devoted to labor saving technological change in the market economy, as figure 4 displays. This excessively high amount of labor saving technological change reduces the amount of employment and therefore the production. In fact, the inefficiency generated is a productive inefficiency: the allocation of investment to different types of technological change and capital does not maximize production. To see this, consider the maximization problem of the production at equilibrium:

$$\begin{aligned}
& \max_{k_t^f, l_t^f, i_{t-1}^A, i_{t-1}^B, k_t, A_t, Z_t} nA_t^* (k_t^f)^r (\ln l_t^f + Z_t)^s \\
s.t.: & ni_{t-1}^f + i_{t-1}^A + i_{t-1}^B = b_t - k_{t-1} - \frac{A_{t-1}}{\Gamma_A} - \frac{Z_{t-1}}{\Gamma_Z} \\
& \frac{x}{x+1 - \frac{(1-\sim)l_t}{1-n-\sim l_t}} = \frac{s}{\ln\left(\frac{l_t}{n}\right) + Z_t} \\
& l_t = nl_t^f \\
& k_t^f = \frac{k_t}{n} \\
& k_t = k_{t-1} + i_{t-1}^K \\
& A_t = A_{t-1} + \Gamma_A i_{t-1}^A \\
& Z_t = Z_{t-1} + \Gamma_Z i_{t-1}^Z
\end{aligned} \tag{62}$$

It is easy to check that the first order conditions of the above problem imply (58). That is, the social returns of different types of capital and technological change should be equal in order to maximize production. Thus, there is productive inefficiency in the market allocation: the investment is not allocated to different types of technological change and capital efficiently, it is possible to increase production by changing this allocation. More precisely, allocating less investment than at equilibrium to the labor saving technological change and more investment to capital and neutral technological change it is possible to increase production (see proposition 1). This inefficiency is generated by the negative external effect of labor saving technological change on employment (on the labor market institutional technology).

4. A model with Dual Labor Market

4.1 Modification of the benchmark model

In this section, we consider that there are two different technologies for production: the one that we presented in the benchmark model, and the following one:

$$\mathbb{E}L_t \tag{63}$$

The above technology presents constant return to scale and consequently it is not possible to get benefits from a firm with this technology. This means that workers do not have any bargaining power: they should accept as a wage the marginal productivity of labor which is equal to \mathbb{E} . We will refer to the firms that use this technology as low wage sector, and the firms that use the technology of the benchmark model as high wage sector.

We consider that in this model that unemployment does not arise, if a worker does not find an employment in the high wage sector, she can find it in the low wage sector. This

will affect to the utility of outsiders and therefore to the bargaining power of insiders. More precisely, the bargaining Nash solution of a firm in the high wage sector would be as follows:

$$\max_{w_t, l_t^f} \left[l_t^{i,f} (v^w(w_t) - \bar{v}^w) \right] \left[v^e (A_t (k_t^f)^\gamma (\ln l_t^f + Z_t)^\delta - w_t l_t^f - (1+r_t)k_t^f - P_t) - \bar{v}^e \right]$$

where:

$$\bar{v}^w = p_t^o v^w(\tilde{w}_t) + (1-p_t^o) v^w(\mathbb{E})$$

$$\bar{v}^e = v^e(-(1+r_t)k_t^f - P_t)$$

(64)

If we compare with (35), the difference is that now the reserve utility of insider workers in case of no agreement, takes into account that in case that outsider workers do not get a job in the high wage sector (which occurs with probability $1-p_t^o$), they can work in the low wage sector and get a wage equal to ψ .

Solving the model in the same way as in the last section, we obtain the “negotiation labor share” in this model is as follows:

$$\lambda = \frac{x}{x + (1-p_t^o) \left[1 - \left(\frac{w_t}{\mathbb{E}} \right)^{-x} \right]}$$

(65)

The negotiation labor share depends on two things:

i) The probability that outsiders get employed in the high wage sector (p_t^o): when such a probability rises, the reserve utility in case of no agreement of insider workers increases, raising its bargaining power. Thus, the higher the employment in the high wage sector, the higher the bargaining power of insiders and the higher the negotiation share that these obtain. This feature is reflected in the positive slope of the negotiation labor share in figure 5.

ii) The ratio wage in the high wage sector/ wage in the low wage sector (w_t/\mathbb{E}): the higher the ratio wage in the high wage sector/wage in the low wage sector, the lower the reserve utility of insiders in case of no agreement relative with the utility in case of agreement. That is, the higher w_t/\mathbb{E} , the higher the cost of not reaching an agreement for insiders. Thus, the higher the ratio wage in the high wage sector/wage in the low wage sector, the lower the bargaining power of insiders.

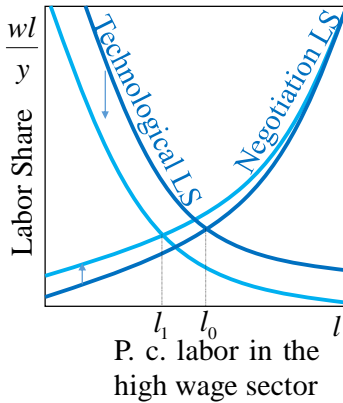
Thus, when per capita capital k_t or neutral technological change A_t increases, wage in the high wage sector rises, and this increases the ratio w_t/\mathbb{E} , raising the cost of no agreement for insider workers. Thus, when per capita capital k_t or neutral technological

change A_t increases, the bargaining power of insider worker falls, making the negotiation labor share to go down, as figures 5.b and 5.c display. It turns out that this undermining in the bargaining power of insider workers results in an increase in the employment in the high wage sector, as figures 5.b and 5.c show.

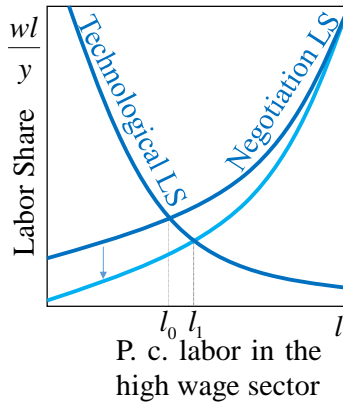
When labor saving technological change Z_t increases, the effect on the negotiation labor share is just the opposite: labor saving technological change reduces the wage in the high wage sector, reducing the ratio w_t/\bar{E} and the cost of no agreement for insider workers, increasing its bargaining power. This strengthen of the bargaining power of insider workers, makes the negotiation labor share to go up, as figure 5.a displays. Furthermore, labor saving technological change also reduces the technological labor share. Thus, since labor saving technological change reduces the technological labor share and increases the negation labor share, it shrinks the employment in the high wage sector, as figure 5a shows.

Figure 5: Labor Share and Employment in the High Wage Sector

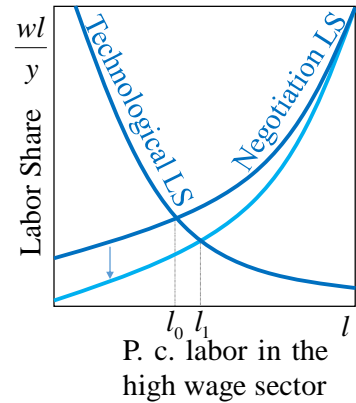
5. a) Effect of Labor saving Technological Change UZ



5. b) Effect of Neutral Technological Change UA



5. c) Effect of increment in per capita capital Uk



The effects of different types of technological change and capital on employment in the high wage sector explained above are formalized in the following proposition:

Proposition 2: The equilibrium amount of employment in the high wage sector is a continuous differentiable function $l^{dm}(Z, A, k)$, where $\frac{\partial l^{dm}(Z, A, k)}{\partial Z} < 0$, $\frac{\partial l^{dm}(Z, A, k)}{\partial A} > 0$, and $\frac{\partial l^{dm}(Z, A, k)}{\partial k} > 0$ (where the superscript “*dm*” means dual market model).

Using the same procedures as in the last sections, the maximization problem of the production firms and the R&D firm imply that the interest rate at equilibrium would

equalize the private return of investment in capital and neutral and labor saving technological change:

$$r_t = r_t^{priv,k,dm} = r_t^{priv,A,dm} = r_t^{priv,Z,dm} \quad (66)$$

where $r_t^{priv,k,dm}$, $r_t^{priv,A,dm}$, $r_t^{priv,Z,dm}$ are the private returns to invest in respectively capital, neutral and labor saving technological change defined as follows:

$$r_t^{priv,k,dm} = r n^{1-\gamma} A_t^* (k_t)^{\gamma-1} \left(\ln \left(\frac{l^{dm}(Z_t, A_t, k_t)}{n} \right) + Z_t \right)^s \quad (67)$$

$$r_t^{priv,A,dm} = \Gamma^A n^{1-\gamma} A_t^{*-\gamma} (k_t)^{\gamma} \left(\ln \left(\frac{l^{dm}(Z_t, A_t, k_t)}{n} \right) + Z_t \right)^s \quad (68)$$

$$r_t^{priv,Z,dm} = \Gamma^Z S n^{1-\gamma} A_t^* (k_t)^{\gamma} \left(\ln \left(\frac{l^{dm}(Z_t, A_t, k_t)}{n} \right) + Z_t \right)^{s-1} \quad (69)$$

Using the same procedures as in the last sections, the household's Euler equation would be as follows:

$$u'(c_t) = \frac{u'(c_{t+1})}{1 + \dots} [1 + r_{t+1}] \quad (70)$$

The dynamic system that defines the equilibrium behavior is very similar to the one in the benchmark model.

The second-best problem in this model consists, as in the last section, in maximizing the households' utility subject to feasibility constraint plus the constraint that the amount of labor used in the high wage sector is the same as in the equilibrium. That is, there are not only technological constraints (given by the production functions and the accumulation equations) but also institutional constraints (given by the bargaining process between insiders and entrepreneurs in the high wage sector):

$$\begin{aligned}
& \max_{\{c_t, k_t^f, l_t^f, i_t^A, i_t^B, k_{t+1}, A_{t+1}, B_{t+1}\}_{t=0}^{+\infty}} \sum_{t=0}^{+\infty} u(c_t) \left(\frac{1}{1+\dots} \right)^t \\
& s.t.: i_t^K + i_t^A + i_t^B = nA_t (k_t^f)^r (\ln l_t^f + Z_t)^s + \mathbb{E}(1-l_t) - c_t \\
& \quad l_t = nl_t^f \\
& \quad l_t = l^{dm}(Z_t, A_t, k_t) \\
& \quad k_t = nk_t^f \\
& \quad k_{t+1} = i_t^K + k_t \\
& \quad A_{t+1} = \Gamma_A i_t^A + A_t \\
& \quad B_{t+1} = \Gamma_B i_t^B + B_t
\end{aligned} \tag{71}$$

The first order conditions are:

$$u'(c_t) = \frac{u'(c_{t+1})}{1+\dots} [1 + r_{t+1}^{soc,k,dm}] \tag{72}$$

$$r_{t+1}^{soc,k,dm} = r_{t+1}^{soc,A,dm} = r_{t+1}^{soc,Z,dm} \tag{73}$$

where $r_t^{socv,k,dm}$, $r_t^{soc,A,dm}$, $r_t^{soc,Z,dm}$ are the social returns to invest in respectively capital, neutral and labor saving technological change defined as follows:

$$r_{t+1}^{soc,k,dm} = r_{t+1}^{priv,k,dm} + [w^{dm}(k_{t+1}, A_{t+1}, Z_{t+1}) - \mathbb{E}] \frac{\partial l^{dm}(Z_{t+1}, A_{t+1}, k_{t+1})}{\partial k} \tag{74}$$

$$r_{t+1}^{soc,A,dm} = r_{t+1}^{priv,A,dm} + [w^{dm}(k_{t+1}, A_{t+1}, Z_{t+1}) - \mathbb{E}] \frac{\partial l^{dm}(Z_{t+1}, A_{t+1}, k_{t+1})}{\partial A} \tag{75}$$

$$r_{t+1}^{soc,Z,dm} = r_{t+1}^{priv,Z,dm} + [w^{dm}(k_{t+1}, A_{t+1}, Z_{t+1}) - \mathbb{E}] \frac{\partial l^{dm}(Z_{t+1}, A_{t+1}, k_{t+1})}{\partial Z} \tag{76}$$

where $w^{dm}(k_t, A_t, Z_t)$ is the marginal productivity of labor:

$$w^{dm}(k_t, A_t, Z_t) = \frac{sn^{1-r} A_t^r (k_t)^r \left(\ln \left(\frac{l^{dm}(A_t, k_t, Z_t)}{n} \right) + Z_t \right)^s}{\left(\ln \left(\frac{l^{dm}(k_t, A_t, Z_t)}{n} \right) + Z_t \right) l^{dm}(k_t, A_t, Z_t)}$$

It follows from the bargaining process between entrepreneurs and workers (see equation 65) that in the high wage sector that the wage in such sector should be always be larger than in the low wage sector (this is proven in the appendix):

$$w^{dm}(k_t, A_t, Z_t) > \mathbb{E} \tag{77}$$

Thus, it follows from the above equations that the social return to invest in labor saving technological change is lower than the private, while in the case of investment in capital and neutral technological change the opposite happens:

$$r_{t+1}^{soc,Z,dm} < r_{t+1}^{priv,Z,dm}; \quad r_{t+1}^{soc,k,dm} > r_{t+1}^{priv,k,dm}; \quad r_{t+1}^{soc,A,dm} > r_{t+1}^{priv,A,dm} \quad (78)$$

The reason for this divergence between the private and the social return of investment is clear: Labor saving technological change generate a “negative external effect” on the employment of high wage sector, which has a higher marginal productivity of labor than in the low wage sector (see proposition 2). This “negative external effect” makes the social return of labor saving technological change to be lower than the private return. On the opposite, neutral technological change and capital generates a “positive external effect” on the employment in the high wage sector. This positive “external effect” of the investment on neutral technological change and capital is neither taken into account by respectively the R&D firm nor the production firms. Thus, when the R&D firm makes decisions about the investment in labor saving and neutral technological change, does not take account of either the negative effect that labor saving technological change generates in the employment in the high wage sector or the positive effect of neutral technological change. As a consequence, there is too much investment in labor saving technology at equilibrium when compare with the second-best allocation. This result is formalized in the next proposition (see the proof in the appendix) and is displayed in figure 4.

Proposition 3: Let's define $k^{me.dm}(b_t)$, $A^{me.dm}(b_t)$, $Z^{me.dm}(b_t)$ and $k^{sb.dm}(b_t)$, $A^{sb.dm}(b_t)$, $Z^{sb.dm}(b_t)$ the amount of capital, neutral technological change and labor saving technological change for an amount of total investment in respectively the market allocation and the second-best allocation (in the unemployment model “um”). If $Z^{me.dm}(b_t) > 0$, then: $k^{me.dm}(b_t) < k^{sb.dm}(b_t)$, $A^{me.dm}(b_t) < A^{sb.dm}(b_t)$, $Z^{me.dm}(b_t) > Z^{sb.dm}(b_t)$.

3. CONCLUSION

This paper arises two important questions ¿Does labor saving technological change generate unemployment? ¿Does labor saving technological change is efficient? To answer these questions this paper has developed a growth model in which technological change adopt two forms: i) neutral technological change that increases the productivity of both capital and labor and, ii) labor-saving technological change that increases the productivity of capital but reduces the marginal product of labor, and therefore reduces the demand for labor and the share of labor in income. We compared two different frameworks: in first one, the benchmark model, labor markets are competitive and therefore unemployment does not arise; in the second one a bargaining process between a union of insider workers and the firm takes place, as a consequence unemployment arises. The benchmark model is designed to make market equilibrium efficient. This is not very common in the R&D literature, but the model delivers the result for which the model was designed for: market equilibrium is efficient. This result allows us to identify in the modified versions of the model, in which frictions in the labor market are introduced, inefficiency sources that are not related with the conventional problems in the R&D literature, but related exclusively with distortions associated to frictions in the labor market. Analyzing the second model, it turns out that unemployment decreases with the share of labor in income. Since labor saving technological change reduces the labor share, it generates unemployment. The technological change is efficient in the first framework; in which unemployment does not arise. In the second framework, labor saving technological change generates unemployment and as a consequence technological change is inefficient: an inefficiently high amount of resources is devoted to labor saving technological change at the expense of an inefficiently low amount of resources devoted to neutral technological change.

Proposition 3: Let's define $k^{me,dm}(b_t)$, $A^{me,dm}(b_t)$, $Z^{me,dm}(b_t)$ and $k^{sb,dm}(b_t)$, $A^{sb,dm}(b_t)$, $Z^{sb,dm}(b_t)$ the amount of capital, neutral technological change and labor saving technological change for an amount of total investment in respectively the market allocation and the second-best allocation (in the dual market model “ dm ”). Then: $k^{me,dm}(b_t) < k^{sb,dm}(b_t)$, $A^{me,dm}(b_t) < A^{sb,dm}(b_t)$, $Z^{me,dm}(b_t) > Z^{sb,dm}(b_t)$.

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