Age Composition of the Demand and the Secular Stagnation *

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Abstract

In this paper, we analyze the effect of demographic changes on labor productivity through the demand channel. We document the relevance of this channel using the change in the age composition of the demand coming from abroad. We find that aging does not have a monotonic effect on the economy as middle-aged consumers drive up prices, and drive down production and productivity growth. We show that middle-aged consumers face higher searching cost relative to young and older consumers as they have a higher opportunity cost of time relative to the elderly, and are less efficient in searching for goods relative to the young consumers. We rationalize our findings using a sequential search model in which middle-aged consumers have relatively higher search costs. An increase in the share of middle-aged consumers increases firms' market power and reduces the incentive to invest in technologies with depressing effects on productivity. We show that the model is consistent with our empirical findings, and show that the change in the age composition of the demand has been a major driver of the slowing down of the labor productivity growth in aging economies between 1995 and 2007.

JEL codes: J11, E21, E22, O47

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1 Introduction

As countries advance along the path of demographic transition first middle aged and then older people represent an increasing share of the population. As we know from earlier works, consumption behaviors vary greatly with age: younger people substitute more easily across brands (Bornstein, 2019) while old people use more of their time to shop (Aguiar and Hurst, 2007). In this paper we provide empirical evidence that, at the sector level, young and old people, relatively to middle aged, accentuate competition across firms. This materializes into a higher average price, an increase in the quantity sold and higher productivity growth. In a stylized model, we show that demographic transition can explain the underlying trends in productivity growth of the past decades, the secular stagnation, and provide projections for the future.

Since the mid 90s the share of young adults reduced sharply while the share of middle aged and old people increased (Figure 1). At world level, aging of the population has been mostly driven by the increase of middle aged people while the share of older people remained almost constant and is expected to increase in the future.

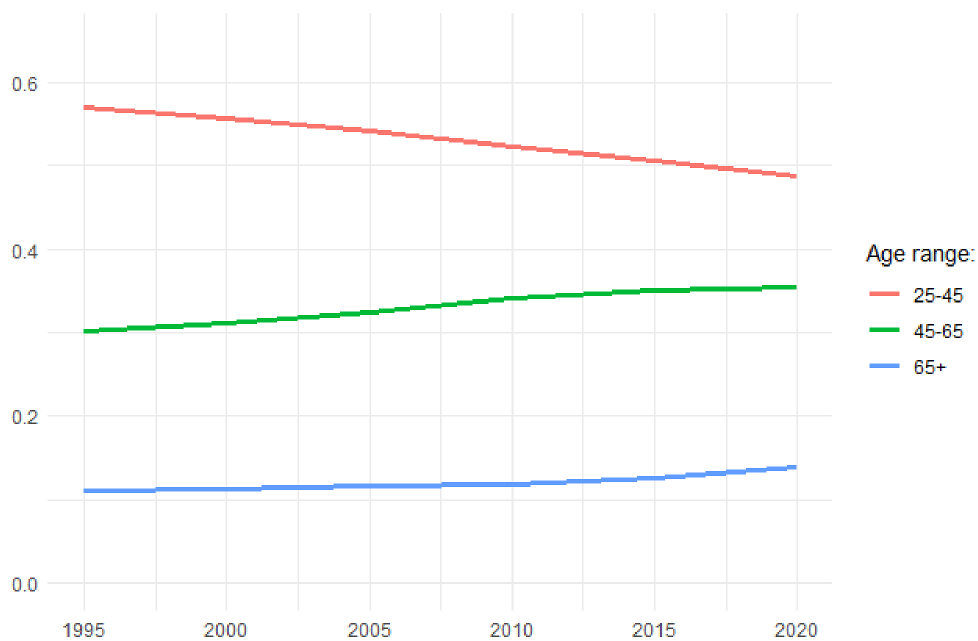


Figure (1) Share of the population in different age categories at world level. Source: United Nation World Population Prospects 2019.

Aging represents an increasing concern all around the world since it affects the economy through several channels with effects on key determinants of growth such as productivity. Gordon and Murray (2016) recognizes the structural demographic change undergoing at world level as one the headwinds that contributed to the secular stagnation suggesting that countries facing a faster aging of the population would suffer from lower productivity and economic growth. Acemoglu and

Restrepo (2017), however, shows that there is no clear evidence of a negative relation between aging and economic growth. The authors argue that the shortage of prime-aged workers could have triggered the adoption of new and automated technologies leading to higher productivity and economic growth. Acemoglu and Restrepo (2018) analyzes this hypotheses showing that, as population ages, productivity increases in industries that are most amenable to automation. However, the evidence of the relevance of the labor market channel is limited as the effect is not disentangled from the other possible channels. Indeed, they find no general evidence that aging boosts productivity implying that the demographic change might affects the economy through different channels offsetting each other.

This paper analyzes a novel channel through which the demographic process can affect the economy: the aging demand channel. It relies on two strands of the literature. First, consumption behaviors vary greatly over the life-cycle. Bornstein (2019) shows that consumers update the composition of their good basket less frequently already from middle age, while Aguiar and Hurst (2005, 2007, 2013) provide evidence of the decreasing opportunity cost of time with age. Second, structural changes of the demand can affect productivity growth at the business cycle frequency.[1] It is therefore relevant to ask: which structural changes fosters an aging demand in terms of market structure and long term trends? How did the aging demand channel contribute to the relative decrease of labour productivity growth in developed aging economies?

First empirically we provide evidence that an aging demand shock reduces labour productivity growth and materializes into a typical decrease of competition. We identify the aging demand channel through changes in the age composition of the foreign demand. Indeed, foreign demographic changes affect firms through their exports, both directly and through the value chains, while keeping the labour supply and other potential channels constant. We show that changes in the age structure of the demand affects the labour productivity growth, production and prices. In particular, an increase in the share of young adults (25-44) is associated with higher productivity and production, and lower prices, while an increase in the share of middle aged consumers (45-64) has the opposite effect. Interestingly, an increase in the share of older consumers (65+) has similar effects as an increase in the share of young adults meaning that the effect of aging on productivity through the demand channel is non-linear and non-monotonic. We argue that young and old consumers have similar effects on prices because they both have lower search cost with respect to middle aged consumers. Young consumers have larger social networks East et al. (2014), higher cognitive abilities Gutchess (2011); Peters (2011), are more familiar with technologies and use them to make online purchasing, and, therefore, they are able to compare prices and quality at a lower cost resulting in more informed consumption decision choices. Lambert-Pandraud et al. (2005) shows, indeed, that young consumers compare more brands and more sellers before purchasing. Old consumers, instead,

[1] Swanson (2006); Bai et al. (2012); Jaravel and Olivi (2019) show that demand shocks such as government expenditures, preference shocks, and changes in the tax schedule can affect productivity.

have a low search cost because the opportunity cost of time drops with retirement allowing older people to devote more time in searching as shown by Aguiar and Hurst (2007). To our knowledge, this paper is the first one to provide evidence of the non-monotic anti-competitive effect of an aging demand on the market equilibrium and its consequences for the labour productivity growth.

To disentangle the mechanisms at play we build a general equilibrium model with oligopolistic competition and sequential search with heterogeneous agents as in Stahl (1989). The economy is populated by two types of consumers: consumers with low search costs (young and old), and consumers with high search costs (middle aged).[2] On the supply side, ex-ante identical firms set prices and invest in labor-saving technologies to enhance their labor productivity. Within each sector, firms are ex-ante identical and produce an homogenous good. Finally, consumers get utility by consuming a good composed of goods from each sector. We characterize the unique mixed strategies equilibrium: a distribution of ex-post heterogeneous firms with respect to their price, quantity sold and productivity. Low-price firms attract more buyers and concentrate the vast majority of the total investments.

In line with the empirical analysis, we find that as population ages the advantage of charging a low price reduces and firms increasingly favor high-price (and low-investment) strategies with a negative effect on aggregate production and productivity growth. We define aging as an increase in the share of high search costs buyers. The effect is though not homogeneous across firms. As consumers search less on average low-price (and high-investment) firms attract a smaller share of the demand. This leads to a reduction in investment by low-price firms as they now spread their investment costs over a smaller production. On the contrary, high-price firms attract more buyers, allowing them to spread their investment costs over a larger production. Hence, the aging of the population reallocates buyers from low-price to high-price firms and the net effect on investment is negative because the small increase in small firms' investment only partially compensate the significant drop of investments made by the largest firms. The heterogenous effect of demographic change on firms' productivity is in line with findings in Acemoglu and Restrepo (2018) which shows that an aging population increases productivity in firms that are most amenable of automation, and it reduces it in firms with a low opportunity to replace human labor with machines.

Our paper relates to several different literatures. In particular, it relates to the secular stagnation literature identifying the aging of the population as one of the causes of the slowing down in productivity in recent decades. Feyrer (2007) shows that the age structure of the population is, indeed, a strong predictor of productivity as workers' productivity and ability to innovate systematically change over the life-cycle. In more recent papers, Aiyar and Ebeke (2017) and Aksoy et al. (2019)

[2]We make no distinction between young and old consumers in terms of their search costs. However, in an extension of the model we will distinguish young from old people by assuming that that old consumers do not work. This would have implications in terms of their budget constraint, and in terms of the aggregate labor supply.

show that the projected workforce aging could significantly reduce production and productivity growth over the next decades. Acemoglu and Restrepo (2018) and Abeliansky and Prettner (2017), instead, argue that the change in the age composition of the workforce and the reduction of the labor supply driven by an aging population could trigger an increase in the adoption of new and automated technologies with positive effects on productivity. Differently from this literature that only focuses on the workforce aging, we consider the change in the age composition of the demand as Swanson (2006); Bai et al. (2012); Jaravel and Olivi (2019) have shown that demand shocks such as government expenditures, preference shocks, and changes in the tax schedule can affect productivity.

Our paper relates to the literature that studies the difference in consumption behavior across age categories. Aguiar and Hurst (2007) shows that the opportunity cost of time is higher for young adults implying that they devote less time to search and face the highest price. Our results are in contrast with this finding as we show that young consumers drive down prices. Differently from their paper that considers only grocery shopping in physical stores for which the opportunity cost of time is probably the key determinant of the price faced, in our analysis we consider the entire production for which other factors such as the ability to compare prices, quality, and to purchase online play a key role in determining the price at which the good is sold. Similar to Aguiar and Hurst (2007) that argue that retired people enjoy a large amount of free time and, therefore, a low opportunity costs of time allowing them to search for cheaper prices, we find that old consumers tend to drive down prices. Consumers of different age categories also differ in terms of the goods they consume. Cravino et al. (2019) shows that older consumers, indeed, consume a different basket of goods with respects to younger consumers, and analyzes the macroeconomic effects of aging through this channel. In our analysis, we abstract from differences in the consumption baskets as we mostly focus on middle aged and young adults which have relatively similar consumption baskets. The closest paper to our analysis is Bornstein (2019) which provides empirical evidence of the increased inertia in consumers' basket from middle age. An aging demand affects firms' entry and markup through a decrease in the elasticity of substitution of the representative consumer. Our paper adds to this work by providing a micro-founded model with heterogeneous agents and by studying the effect of an aging demand on investment and productivity growth.

The remainder of the paper is structured as follows: Section 2 presents the empirical evidence showing the non-monotonic effect of aging on prices, production and productivity growth, and the contribution of the aging demand to the slowing down in the productivity growth; Section 3 presents the model which provides the mechanism relating aging of the demand and prices, investment, production, and productivity in a general equilibrium framework; in Section 4 we present our prospective exercise for different countries at different stages of the demographic process; finally, Section 5 concludes.

2 Empirical Evidence

The median age in developed countries is significantly higher than in the rest of world. This observation, coupled with the evidence that productivity and the ability to innovate systematically change over the life-cycle, lead to the hypothesis that the aging of the population is one of the causes contributing to the secular stagnation, i.e. the slowing down of the productivity growth in developed countries. However, we find no clear evidence of such a relation (see Figure 7 in Appendix A.1). The correlation between aging and productivity growth is, indeed, slightly positive , but not significant. Our findings do not change when controlling for a measure of the initial age, and for regional fixed effects (see Table 6 in Appendix A.1). This result is in line with Acemoglu and Restrepo (2017) showing that the raw correlation between GDP per capita and aging is slightly positive, but not significant.

The zero correlation results between aging and productivity is very unlikely evidence of the neutrality of aging on productivity. Most probably aging affects productivity through several channels offsetting each other. Acemoglu and Restrepo (2018) shows, indeed, the relevance of the labor market channel through which aging affects the adoption of new and technology with positive effects on productivity. Aging, however, does not only affect the labor marker, but also the demand side of the economy by changing the age composition of the consumers. Failing to identify these two different channels leads to confounding results.

In this section, we empirically document the relevance of the demand channel in affecting productivity as populations undergo through demographic changes. To isolate the effect of the demographic shock in the demand, we consider demographic changes coming from foreign markets affecting the domestic economy through the exports. Indeed, since the labor supply and the age composition of the labor force are not affected by demographic changes coming from foreign markets, by considering only the demand coming from exports, we are able to disentangle the effect of changes in the age composition of the demand from the effect of demographic changes in the labor market. We use a shift-share design estimation procedure assuming exogenous shifts meaning that foreign demographic changes do not depend on the domestic variables of interest (productivity, production and prices). We compute the shares of each sector in each country as a measure of the exposure to the final demand in each country using the world input-output matrix. The world input-output matrix records the production of each sector in each country, and the ways industries trade with one another at the world level. Using the world input-output matrix, we are able to compute the actual exposure of each country-sector to each other country's final demand by tracking down the intermediate input trade patterns. Indeed, by considering only the direct flows, we might ignore a relevant share of the final demand exposure. We, therefore, estimate a measure of the foreign demographic change demand by weighting each foreign country's demographic change with the estimated exposure to foreign demand faced by each country-sector. Using this measure of demographic demand shock, we analyze the effect of a change in the age composition of the demand coming from foreign markets on productivity, production, and prices.

In the next paragraphs, we describe the data, and present our empirical methodology. We show how the different age categories affect key economic variables such as productivity, production and prices differently. Finally, we estimate the effect of the demographic change on productivity, and discuss the bias that arising in analysis missing the demand channel.

2.1 Data

To perform our empirical analysis, we use the World Input-Output Database. The database covers the 27 EU countries, 13 other major countries in the world, and an estimate for the rest of the world for the period from 1995 to 2009.The structure of the WIOTs is represented in Figure 2.

			Input use & value added							**Final use**			**Total use**
			Country 1			$\cdots$	Country J			Country 1	$\cdots$	Country J	
			Industry 1	$\cdots$	Industry S	$\cdots$	Industry 1	$\cdots$	Industry S				
Intermediate	Country 1	Industry 1	Z_{11}^{11}	$\cdots$	Z_{11}^{1S}	$\cdots$	Z_{1J}^{11}	$\cdots$	Z_{1J}^{1S}	F_{11}^{1}	$\cdots$	F_{1J}^{1}	Y_1^1
		$\cdots$	$\cdots$	Z_{11}^{rs}	$\cdots$	$\cdots$	$\cdots$	Z_{1J}^{rs}	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
		Industry S	Z_{11}^{S1}	$\cdots$	Z_{11}^{SS}	$\cdots$	Z_{1J}^{S1}	$\cdots$	Z_{1J}^{SS}	F_{11}^{S}	$\cdots$	F_{1J}^{S}	Y_1^S
inputs	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	Z_{ij}^{rs}	$\cdots$	$\cdots$	$\cdots$	$\cdots$	F_{ij}^{r}	$\cdots$	Y_i^r
	Country J	Industry 1	Z_{J1}^{11}	$\cdots$	Z_{J1}^{1S}	$\cdots$	Z_{JJ}^{11}	$\cdots$	Z_{JJ}^{1S}	F_{J1}^{1}	$\cdots$	F_{JJ}^{1}	Y_J^1
supplied		$\cdots$	$\cdots$	Z_{J1}^{rs}	$\cdots$	$\cdots$	$\cdots$	Z_{JJ}^{rs}	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
		Industry S	Z_{J1}^{S1}	$\cdots$	Z_{J1}^{SS}	$\cdots$	Z_{JJ}^{S1}	$\cdots$	Z_{JJ}^{SS}	F_{J1}^{S}	$\cdots$	F_{JJ}^{S}	Y_J^S
Value added			VA_1^1	$\cdots$	VA_1^S	VA_j^s	VA_J^1	$\cdots$	VA_J^S				
Gross output			Y_1^1	$\cdots$	Y_1^S	Y_j^s	Y_J^1	$\cdots$	Y_J^S	$\cdots$			

Figure (2) World Input Output Table

Using the WIOD tables allows us to capture the interdependence coming from the integrated production structure of the world's economies. Indeed, focusing directly only on export measures would have not allowed us to capture the intermediate goods exchanges which constitute a large part of the trade between sectors and countries. From the WIOD, we also get socio-economic data such as value added, hours worked by employees, and price deflator measures. The demographic data are taken from the UN World Population Prospects 2019 (WPP) dataset.

2.2 Empirical Strategy

Our analysis follows the shift-share design as described in Adão et al. (2019) by assuming exogeneity from the "shift" variables, i.e. the change in the age structure. Indeed, since firms endogenously choose the exposure to the different markets (the "shares"), we treat the shares as endogenous, and the shift as exogenous meaning that our identification approach relies on the assumption that foreign demographic changes do not depend on the dependent variables in our model (domestic productivity, production and prices). In the likely case in which the information related to future changes in the foreign demographic structure is already part of the firms' information set, firms would take that into account when choosing the markets to link to. To avoid this concern over the contemporaneous changes in the shares, we use lagged shares of exposure to those markets as in Huneeus (2018). Moreover, to remove the effect of relevant determinants of the variables of interest that are fixed over time, we employ a long difference specification. We, therefore, consider the following empirical model:

$$\Delta logY_i^s = \beta_0 + \beta_1 \Delta mid_i^s + \beta_2 \Delta old_i^s + \Gamma X_i^s + \epsilon_i^s$$

where $\Delta logY_i^s$ is defined as the change in log productivity (real value added per hour worked by employees) in country i, sector s between 1995 and 2007; Δmid_i^s (Δold_i^s) is the estimated demographic demand shock for middle aged (old) consumers. We, therefore, consider three age categories: young adults (25-44), middle aged (45-64), and old (65-84) using young adults as the reference age category. We exclude younger and older consumers (<25 and 85+) as we focus on independent consumers.[3] The demographic demand shocks are defined as:

$$\Delta mid_i^s = \sum_{j \neq i} \bar{\xi}_{i,j}^s(t-1)\Delta mid_j(t)$$

$$\Delta old_i^s = \sum_{j \neq i} \bar{\xi}_{i,j}^s(t-1)\Delta old_j(t)$$

where $\bar{\xi}_{i,j}^s(t-1)$, the share, is the exposure of country i, sector s to the final demand of country j in period $t-1$ (see the estimation procedure in Appendix A.2). $\Delta mid_j(t)$ ($\Delta old_j(t)$), the shift, is the change in the fraction of middle aged (old) in country j in period t. We use lagged values of the shares to avoid contemporaneous effects that changes in the age composition of the foreign countries could have on the shares. We consider the demographic change between 1995 and 2020 fixing the shares at the 1995 levels. Our demographic demand shock is, therefore, defined as the sum of the change in the fraction of middle aged (and old) in foreign countries weighted by the initial domestic country-sector exposure to the foreign final demand. We consider the change between 1995 and 2007 in productivity to avoid the years of the crisis. We instead consider the demographic change between 1995 and 2020 since firms are forward looking and take into account the demographic trends predictions when making strategical choices as also argued in Acemoglu and Restrepo (2018).

Finally, X_i^s is a vector of control variables. We consider the initial age of the demand, the initial exposure to exports, and domestic demographic variables such as the change in the shares of the different age categories. The effect of aging on labor productivity might, indeed, not be linear. Controlling for the initial age of the demand controls for such non-linearity. Moreover, since foreign aging demand shocks might have different effects on countries depending on their

[3] Younger consumers (<25) might still have no income and depend on their parents regarding their consumption decisions; older consumer (85+) might delegate their consumption decision choices to someone else (think, for instance, of elderly in nursing homes).

initial exposure to foreign markets and countries exporting more might also be more productive, we include a measure of initial export exposure as a control. We include domestic demographic variables since the change of the domestic age structure might be correlated with the change of the age structure abroad; including such variables, we control for the effect of demographic change on productivity coming from the domestic labor market and the domestic demand.

Using the same empirical strategy, we analyze the effect of the change in the demographic composition of the demand on production measured as real value added, and prices measured as the value added price deflator.

2.3 Results

In this section, we analyze the results of our empirical analysis. Table 1 shows the effect of a change in the age composition of the foreign demand on productivity highlighting the relevance of the demand channel in affecting productivity as population undergo a structural demographic change. We use the young adults as the baseline age category. We find that an increase in the foreign demand coming from middle aged consumers ("Foreign ΔMid-age Demand") has a negative and significant effect on domestic productivity growth with respect to the young adults category while an increase in the foreign demand coming from old consumers ("Foreign ΔOld Demand") has no significantly different effect on productivity with respect to young adults consumers. These results are robust to the inclusion of regional and sectoral dummies, domestic changes in the age composition of the population ("Domestic ΔMid-age" and "Domestic ΔOld"), initial demographic structure ("Initial Mid-age Demand" and "Initial Old Demand"), and initial exposure to final foreign demand ("Initial Export Exposure").

We also find that while the increase in the share of foreign middle aged demand has a negative effect on productivity growth, the increase in the share of domestic middle aged has no significant effect. This is consistent with the hypothesis that structural demographic changes affect the economy through several channels offsetting each other. Moreover, while the increase in the share of foreign old demand has a no significant effect on productivity growth, the increase in the share of domestic old has a positive effect on productivity growth. This is consistent with the idea that an increase in the domestic elderly reduces the labor supply triggering the adoption of automated technologies as shown by Abeliansky and Prettner (2017) with possible positive effects on productivity.

Controlling for initial conditions, we find that while productivity growth is independent on the initial share of middle-aged, it is negatively influenced by the initial share of old people. This implies that the effect of a structural demographic change through the demand channel is not linear depending on the initial demographic conditions in the country. Moreover, we find that the initial exposure to exports is positively related to productivity growth. This is consistent with the idea that more integrated markets exhibit higher productivity as argued in Melitz and Ottaviano (2008).

Table (1) Change in the share of middle-aged (45-64) and old (65-84) foreign demand on productivity (real value added per hour worked). Baseline age category: young adults (25-44).

	Dependent variable:			
	Δlog Productivity			
	(1)	(2)	(3)	(4)
Foreign ΔMid-age Demand	10.683	8.022	8.095	6.614
	(1.781)	(1.829)	(1.868)	(1.872)
Foreign ΔOld Demand	1.024	1.329	0.939	1.669
	(1.630)	(1.810)	(1.917)	(1.934)
Domestic ΔMid-age			0.678	0.981
			(0.497)	(0.674)
Domestic ΔOld			1.197	2.460
			(0.648)	(0.813)
Initial Mid-age Demand				4.787
				(2.083)
Initial Old Demand				1.589
				(1.180)
Initial Export Exposure				0.120
				(0.083)
Region Fixed Effects		X	X	X
Sector Fixed Effects		X	X	X
Observations	1,347	1,347	1,347	1,347
R^2	0.026	0.202	0.205	0.227
Adjusted R^2	0.025	0.178	0.179	0.201
Note:		p 0.1;	p 0.05;	p 0.01

We employ the same identification approach to analyze the effect of a structural demographic change on production and prices. Table 2 summarizes our results showing that the change in the share of middle-age foreign demand has a negative effect on production growth, and a negative effect on prices with respect to the reference age category while elderly people do not significantly differ from young adults. Tables 7 and 8 in Appendix A.3 show that results for production and prices are robust to different specifications.[4] In Appendix A.4, we also show that these results are robust when we consider a subset of sectors, namely manufacturing sectors (Table 12). Since in our empirical identification approach we use demographic changes in the demand coming from abroad through exports, as a robustness check, we focus on manufacturing sectors since those sectors are the ones that mostly produce goods that are suitable to be traded in international markets. We find, indeed, that the estimated coefficients for productivity, production, and prices are significantly larger in magnitude.

As we find that a change in the share of foreign young and old have a not significantly different effect on growth levels of productivity, production and prices through the demand channel, we collapse the age variables to a single one to simplify the interpretation. We define the change in foreign age structure of demand ("Foreign ΔAge-composition Demand") as the change of middle-aged with respect to young adults and old foreign demand.[5] Column 1 in Table 3 shows the effect of a relative increase in middle-aged foreign demand on productivity. A 10 percentage points increase in the share of middle aged consumers relatively to young adults and old consumers (which is close to the average increase occurred in developed economies between 1995 and 2020) is associated with an average 20% decline in value added per hour worked through the demand channel corresponding to an average of a 1.52% loss in productivity per year through the demand channel between 1995 and 2007. Although such a result is quite large, it is similar in the order of magnitude to the findings in Acemoglu and Restrepo (2018) and Aksoy et al. (2019) showing that aging accounts for 1.1% and 0.64% in productivity loss per year respectively. However, our coefficient are significantly larger than in previous studies as they do not attempt to disentangle the different channels meaning that their findings entail different effects possibly offsetting each other. Moreover, our estimation of the loss in productivity through the demand channel has been computed as a counterfactual exercise under the assumption that in the period 1995-2020 no demographic change occurred at global level. Since in this period the demographic change has been huge and involved most of the countries around the world, the estimated effects are unsurprisingly large.

[4] While for the effects on production results are statistically significant for all the specifications considered (Table 7), for the effects on prices results become statistically significant once regional and sectoral dummies, and domestic demographic statistics are included (Table 8).

[5] For clarity, we define Foreign ΔAge-composition Demand as:

$$\Delta age_i^s \quad {}^{\circ}{}_{j} \; {}_{i} \; \bar{}^{s}_{i,j} \Delta age_j \text{ where } \Delta age_j \quad \Delta \frac{\#\,\text{Mid-age}}{\#\,\text{Young} \quad \#\,\text{Old}} \mathrm{p} j \mathrm{q}.$$

Table (2) Change in the share of middle-aged (45-64) and old (65-84) foreign demand on productivity, production, and prices. Baseline age category: young adults (25-44).

	Dependent variable:		
	Δlog Productivity	Δlog Production	Δlog Price
	(1)	(2)	(3)
Foreign ΔMid-age Demand	6.614	5.843	14.548
	(1.872)	(1.953)	(2.807)
Foreign ΔOld Demand	1.669	1.063	3.710
	(1.934)	(2.017)	(2.889)
Domestic ΔMid-age	0.981	1.390	1.085
	(0.674)	(0.703)	(1.011)
Domestic ΔOld	2.460	1.632	10.331
	(0.813)	(0.848)	(1.219)
Initial Mid-age Demand	4.787	2.283	12.541
	(2.083)	(2.173)	(3.124)
Initial Old Demand	1.589	5.222	21.868
	(1.180)	(1.231)	(1.770)
Initial Export Exposure	0.120	0.219	0.616
	(0.083)	(0.087)	(0.125)
Region Fixed Effects	X	X	X
Sector Fixed Effects	X	X	X
Observations	1,347	1,347	1,348
R^2	0.227	0.242	0.451
Adjusted R^2	0.201	0.216	0.432
Note:			p 0.1; p 0.05; p 0.01

It is important to stress that according to our analysis the effect of aging on productivity through the demand channel is not monotonic, and that countries have a very different age structure meaning that the demographic change could have a very different effect on productivity through this channel in different countries. Indeed, while relatively old countries such as Italy and Germany have experienced a modest increase in the share of middle aged people, this increase is higher in relatively younger countries such as the United Stated meaning that the aging of the population had a stronger negative effect through the demand channel in the United States. Moreover, a very old country such as Japan experienced a reduction in the share of middle aged people as population aged and the share of elderly rapidly increase meaning a positive effect of aging on productivity through the demand channel.[6]

Table (3) Change of middle-aged (45-64) with respect to young adults (25-44) and old (65-84) foreign demand on productivity, production, and prices.

	Dependent variable:		
	Δlog Productivity	Δlog Production	Δlog Price
	(1)	(2)	(3)
Foreign ΔAge-composition Demand	2.015	1.799	3.362
	(0.641)	(0.672)	(1.020)
Domestic ΔMid-age	1.060	0.238	3.363
	(0.573)	(0.600)	(0.910)
Domestic ΔOld	2.721	0.083	3.992
	(0.682)	(0.715)	(1.086)
Initial Age-composition Demand	2.203	1.790	5.351
	(0.395)	(0.414)	(0.628)
Initial Export Exposure	0.110	0.169	0.842
	(0.081)	(0.085)	(0.129)
Region Fixed Effects	X	X	X
Sector Fixed Effects	X	X	X
Observations	1,347	1,347	1,348
R^2	0.224	0.232	0.379
Adjusted R^2	0.198	0.207	0.359
Note:		p 0.1; p 0.05;	p 0.01

Column 2 and 3 in Table 3 also show that a relative increase of middle aged leads to a reduction in production, and an increase in prices. These results mean that structural demographic change leads to a non-standard demand shock. Indeed, while standard demand shocks are characterized by a positive co-movement between prices and quantities, our demographic demand shock leads

[6] More on this in Appendix A.5.

to a negative co-movement between these variables meaning that the change in the structure in the economy triggers a supply response which is consistent with the negative co-movement between prices and quantities. There is, indeed, evidence that the age structure of the population affects the market structure (Bornstein, 2019), and that firms respond to demand shocks by updating their beliefs and strategies (Berman et al., 2019). In the model section, we analyze how a structural demographic change can affect firms' strategies leading to the negative co-movement between prices and quantities, and the negative effect on productivity that we see in the data.

2.4 Discussion: Bias from missing the Demand channel

Before presenting the theoretical model, we compare our results with those in Acemoglu and Restrepo (2018) to highlight the bias arising in analysis missing the demand channel, and to attempt to estimate the aging effect on productivity through the labor market channel. We consider a comparable subset of countries and define the variable (domestic) "aging" accordingly considering the change in 55-64 relative to 20-54.[7;8] We proceed in two ways to estimate the aging effect on productivity through the labor market channel. In the first approach we estimate the bias arising from not considering the demand channel, and we indirectly estimate the effect through the labor market; in the second approach, we attempt to directly estimate the effect controlling for the demand channel in the empirical specification.

Indirect estimation We estimate the aging effect on productivity growth through the labor market channel as the unexplained effect of aging on productivity when subtracting the demand channel effect. We estimate the demand channel effect using the same identification approach in the previous analysis (Table 15 in Appendix A.6). Using Acemoglu and Restrepo (2018) estimates, we find a positive effect of aging on productivity of 1.62% per year through the labor market channel in the period between 1995 and 2007.[9] However, as we only have a reliable strategy only to identify the demand channel and we compute the labor market effect as a residual, this approach relies on the assumption that aging affects productivity only through the labor market and the demand channel.

[7] In their analysis, Acemoglu and Restrepo (2018) use EUKLEMS and OECD data, while in our analysis we rely on WIOD data to estimate the demand shock. When comparing the results we only consider European countries as those are the ones included in every analysis.

[8] We consider "old" those between 55 and 65 as in their analysis they consider old workers above 55.

[9] Table 15 shows that an increase in 10 percentage points in the relative share of 55-64 would negatively affect productivity by 38% meaning an average of 2.72% per year between 1995 and 2007 through the demand channel. Acemoglu and Restrepo (2018) finds that a 10 percentage points increase in the workers above 55 negatively affects productivity by 14% (1.1% per year) in the same period

Direct estimation Another approach to attempt to estimate the labor market effect of aging on productivity is to directly estimate it including proxies for domestic and foreign aging demand as controls. To control for the aging demand coming from abroad, we include the estimates of the foreign aging demand as in the previous analysis. To control for domestic aging demand, we weight the domestic aging by the domestic exposure in terms of final consumption of each country-sector. This implies that the domestic aging demand variable is at sector level instead of country level. This approach obviously does not allow us to identify the labor market channel, but still offers an attempt to control for the demand channel.

Table (4) Effect of domestic aging (change in the share of 55-64 relative to 20-54) on productivity (direct estimation approach).

	Dependent variable:			
	Δlog Productivity			
	(1)	(2)	(3)	(4)
Domestic Aging	0.200	4.245	2.077	1.994
	(0.718)	(0.860)	(0.971)	(1.029)
Domestic Aging Demand		6.818	3.080	3.213
		(0.853)	(1.210)	(1.233)
Foreign Aging Demand		1.983	2.434	2.139
		(1.574)	(1.951)	(1.953)
Initial Domestic Age				2.551
				(1.101)
Initial Age Demand				1.196
				(1.025)
Sector Fixed Effects			X	X
Observations	909	909	909	909
R^2	0.0001	0.068	0.177	0.182
Adjusted R^2	0.001	0.065	0.143	0.146
Note:		p 0.1;	p 0.05;	p 0.01

Table 4 shows that there is a positive, not significant raw correlation between aging and productivity growth which becomes significant and positive as the demand channel controls are included. Assuming that the domestic and foreign aging demand variables remove the effect of the demand channel on the domestic aging variable (as expected their coefficients are both consistently negative, even if only the domestic aging demand one is significant), such variable represents the effect of aging on productivity from the labor market side which is positive. As in the previous estimation, this interpretation holds only if aging affects productivity mainly by the labor market and the demand channel. Our estimates mean that domestic aging increased productivity by 1.53% per year through the labor market channel, while it reduced productivity by 2.34% per year through the demand channel.

As we can see from Table 14, the results coming from the different estimation procedures are very similar. Our findings, therefore, suggest the relevance of the labor market and the demand channel in affecting productivity through aging in the period 1995-2007.

Table (5) Labor market and demand channel effect of aging on productivity.

	Labor market	Demand channel	Net effect
Indirect estimation	1.62%	-2.72%	-1.1%
Direct estimation	1.53%	-2.34%	-0.81%

3 A Model of Oligopolistic Competition with Technology Investment

In this section, we study the effects of a change in the age composition of the demand on firm strategies and productivity in a given sector using a sequential search model based on Stahl (1989) with two types of consumers: consumers with low search costs (young and old), and consumers with high search cost (middle aged). Although young and old consumers greatly differ in the way the reach information over prices and goods' quality, they both have lower search costs relatively to middle aged consumers. The idea is that the search cost is a function of both the time that the consumer is willing to devote in searching, and the search efficiency. Young consumers are very efficient in searching (because of internet, social networks, and their higher cognitive abilities), but have little time because they work (and have a high opportunity cost of time). Middle aged consumers also have little time as they still need to work, but they are less efficient than young consumers in searching as they are less familiar with technologies, have smaller networks, and lower cognitive abilities. This implies that middle aged consumers have an implicit higher search cost relative to young consumers. Old consumers, instead, are able to devote more time in searching as they have a lower opportunity cost of time (old people are usually retired) and are able to compensate for the lower efficiency in searching resulting in relatively low search costs.[10]

On the other side, firms set strategies in terms of investment in labor saving technologies and prices. We model the demographic change as an increase in the share of middle aged consumers with higher search costs. As the aging process at world level has mostly resulted in an increase in the share of the middle aged, we will generally refer at this increase simply as "aging" keeping in mind that in older countries, aging actually mostly resulted in an increase in the elderly rather

[10]In Appendix B.1, we show that in a simple calibrated model in which agents have age-specific opportunity cost of time and search efficiency, young and old face lower search costs relative to the middle aged consumers.

than the middle aged.[11] In an extension of the paper, we will perform a prospective exercise in which we analyze the heterogenous effect of aging in countries with different age distributions.

3.1 Theoretical Framework

3.1.1 Optimal Consumer Search

The economy is populated by a measure 1 of consumers, of which a fraction λ is middle aged and $(1-\lambda)$ young or old. All consumers observe the first price for free. Middle aged comsumers pay a search cost $s > 0$, while young and old consumers pay no search costs.

Given the lowest previously observed price z, we define the utility surplus of an older buyer $j \in [0, \lambda]$ of finding a price $p < z$ as:

$$CS^j(p;\ z) = \int_p^z D^j(x)\, dx. \tag{1}$$

with $D^j(p)$, such that $D^{j\prime}(p) \leq 0$, her demand function. The expected benefit for an old buyer of randomly observing another price is then simply the integral of $CS^j(p;\ z)$ over the price distribution $F(p)$:

$$ECS^j(z) = \int_b^z CS^j(p;\ z)\, dF(p), \tag{2}$$

where b is the minimum price in the distribution.

Definition 3.1 (Reservation price). *We define the reservation price r_F such that $ECS^j(r_F) = s$ if $ECS^j(p)$ has a root, $r_F = \infty$ otherwise. It implies the following search rule:*

- *if $p \leq r_F$, she stop its search and purchase at price z*
- *if $p > r_F$, she continues to search*
- *if $p > r_F$ for all firms, she picks the lowest observed price*

The optimal search rule for old consumers simply states that she continues searching if the lowest observed price, z, is greater than the reservation price r_F, and purchase if $z = p$. If all firms have prices exceeding r_F, then the consumer picks the lowest price observed.[12]

Young and old consumers pay a zero search cost, so they do not stop searching until they observe $p = b$.[13] However, since the event $p = b$ has zero probability in equilibrium as we show in

[11] See Appendix A.5.

[12] As it will become clear below, the definition of the indifference case $p = r_F$ is not important since, as in equilibrium, the probability of a firm setting price $p = r_F$ is null.

[13] In order to rule out the monopolistic price equilibrium in which all firms set the price equal to the monopolistic price, we assume that young and old consumers stop searching only once they observe twice the lowest price b. Otherwise, firms could all charge the monopolistic price, and the young and old consumers would stop at the first observation since $b = \hat{p}$ (i.e. the lowest price is the monopolistic price). Since the young and old consumers stop searching only once she faces the lowest price twice, the monopolist price equilibrium can not be sustained since firms have a profitable deviation of lowering their price marginally and attract all the young buyers.

the next section, young and old consumers will sample the prices of all N stores and purchase at the lowest price.

3.1.2 Production

A finite number N of ex-ante identical firms produce a homogeneous good and compete through prices. Each firm produces goods using solely labor ℓ, paid at wage w, with a constant return to scale technology:

$$y^s = \alpha\ell$$

with α the productivity of labor. Firms can invest in labor saving technologies to reduce the amount of labor required to produce each good by a at a quadratic investment costs $z(a) = \frac{a^2}{z}$. The marginal cost of production is then:

$$c(a) = \frac{w}{\alpha} = w(\bar{a} - a)$$

where $\bar{a}$ is the quantity of labour required to produce one unit of good in the case a firm does not invest in labour saving technologies.

3.1.3 Firm Strategies

A firm strategy is a 2-tuple $(p, a) \in \mathbb{R}_+ \times [0, \bar{a}]$, i.e. the price set by a firm and its investment in labour-saving technologies.[14] We assume that the choice of the price, p, and the level of investment, a, are simultaneous. In particular, we can show that the choice of the price (investment) determines the other. It is, however, relevant to stress that the choice of the strategy is made ex-ante to the realization of the demand.

We use symmetric NE as equilibrium definition (from now on, we will refer to symmetric NE simply as NE). We assume an exogenous consumer reservation price r (we will characterize a consistent r later on) to define $F(p; r)$,the equilibrium cumulative probability distribution of prices, and its support $[b(r), r]$, with $b(r)$ the minimum price in the economy.[15]

Defying $\hat{p}$ as the monopolistic price we get:

Lemma 3.2. *Given $\lambda \in (0, 1)$, the NE-distribution of prices $F(p; r)$ is atomless.*

Lemma 3.3. *Given the NE-distribution of prices $F(p; r)$, then $P_r = min\{r, \hat{p}\}$.*

Lemma (3.2) tells us that the NE-distribution is continuous and that all possible NE equilibrium of this model are in mixed-strategy. Lemma (3.3) sets the limit of the upper bound of the NE-distribution of prices which can not be greater than the monopolistic price (since a price higher

[14] Firms cannot price discriminate consumers according to their age.

[15] As shown in Stahl (1989), the distribution of prices $F(p; r)$ will produce a reservation price r_F; a consistent reservation price r^* producing a NE price distribution $F(p; r^*)$, is a reservation price such that the price distribution $F(p; r^*)$ produces $r_F = r^*$.

than the monopolistic price can never be optimal), and it needs to be equal to the reservation price if $r \leq \hat{p}$ since by setting p_j such that $r < p_j \leq P_r - b(r)$, then P_r lead to less revenues than setting $p_j = r$. Since $p_j \leq r$, consumers with positive search cost will always stop with probability one at the first price observed, while informed consumers will purchase at the lowest price. We can define the expected number of buyers a firm attract, $m(p)$ as:

$$m(p) = \frac{\lambda}{N} + (1-\lambda)[1-F(p;r)]^{N-1} \tag{3}$$

The first expression is the equal share of middle aged buyers allocated to each firm. The second expression is the number of young and old buyers multiplied by the probability for a firm with price p to be the lowest pricing firm.

Given Lemmas (3.2) and (3.3), for a given price $p_j \leq \hat{p}$, and assuming $y^s(p_j) \geq y^d(p_j)$, each producer maximizes the following expected profit:

$$E[\pi_j(p, F)] = \max_{\{a\}} E[y^d(p)](p - c(a)) - z(a) \tag{4}$$

where $E[y^d(p)] = \frac{\lambda}{N} D^M(p) + (1-\lambda)[1-F(p;r)]^{N-1} D^{Y,O}(p)$ is the expected aggregate demand faced by a firm setting price p_j. $D^M(p)$ is the demand of a middle aged buyer at price p, and $D^{Y,O}(p)$ is the demand of a young or old buyer at price p_j (the demand of a young and an old consumer do not differ as they are identical in terms of their search costs). The demand is composed of two elements. Middle aged buyers randomly observed a single price, so they are equally assigned across firms and each firm faces the demand of exactly λ/N middle aged buyers. Young and old buyers observe all the prices, so they purchase at the lowest pricing firm, and $[1-F(p;r)]^{N-1}$ is the probability of the firm setting price p of being the lowest pricing firm.

As in Stahl (1989), we consider the following definition of symmetric NE:

Definition 3.4. *A distribution $F(p;r)$ is a conditional NE iff $E\pi(p,F)$ is equal to a constant (say π) for all p in the support of F and not greater than π for any p. $F(p;r)$ is a consistent NE iff F is a conditional NE and r is a consistent reservation price.*

The equilibrium is, therefore, defined through an iteration process: one guesses first a reservation price and obtain an equilibrium cumulative distribution function. Then, one checks if this equilibrium implies a reservation price consistent with the original guess.

Proposition 3.5. *There is a unique solution of the equilibrium profit π, and given a price strategy, p, there is a unique optimal investment in labor saving technology $a(p;\pi)$:*

$$a(p;\pi) = \left[(\frac{p}{w} - \bar{a})^2 - \bar{z}\pi\right]^{\frac{1}{2}} + (\frac{p}{w} - \bar{a}) \tag{5}$$

It is negatively correlated with price, $a'(p) < 0$, and it is decreasing in the reservation price, $da/dr < 0$.

Proof. *in appendix B.2* ☐

This proposition allows a first characterization of firms. The model predicts a negative relation between price and productivity, which implies a positive relation between size (in terms of buyer, total sales or sales per buyers) and productivity and finally, a positive relation between the share of young and old buyers in a firms total demand and productivity.

Proposition 3.6 (Uniqueness of the Equilibrium)**.**
With a constant demand function, the expected continuation profit $ECS(r)$ *is strictly increasing in* r*. If an equilibrium exists, it is unique and defined as:* $ECS(r^*) - s = 0$

Proof. *in appendix B.3* ☐

We show that a sufficient condition for $F(p;r)$ to be fully characterized is that the demand is constant. Although this condition is restrictive, we numerically show that r^* exist and is unique, and that $F(p;r^*)$ is fully characterized also for a wider range of demand function including the ones of the form $D^j(p) = A^j p^{-\sigma}$, with $\sigma \geq 0$, as obtained from a CES aggregator.

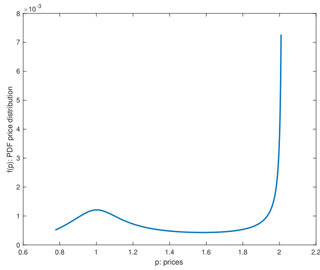
3.1.4 Calibration and numerical results

Young and old buyers have identical demand functions of the form $D^j(p) = A^j p^{-\sigma}$, with $A^M = A^{Y,O}$. We make this agnostic choice to limit, for now, our analysis to the mechanism. We set $\sigma = 1.2$, as it is usually assumed in model of oligopolistic competition with constant elasticity of substitution [16]. $\bar{a} = 1$ so that the labour productivity α is defined between 1, the unit labour costs, and ∞. [17] The solution algorithm is described in the appendix B.6.

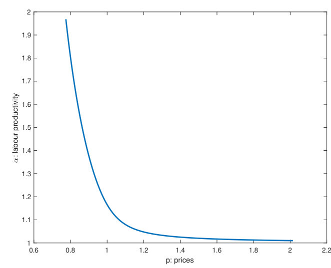
We see that in equilibrium there are two large density areas of strategies. The first one, an almost mass point at the reservation price, includes all the strategies focused on middle aged buyers and low labor productivity; the so called "mam and pop stores". The second one, a bell-shaped distribution above the minimum price, includes all the strategies focused on buyers with no search costs (young and old), high labor productivity and large production. This is in stark contrast with Stahl (1989), which obtains an almost symmetric PDF, with an almost mass point at the minimum price. Interestingly, little probability is allocated to the strategies around the minimum price though they imply a higher markup (defined as the price over the marginal cost). This result arises from the convexity of the cost of investment in labor saving technologies.

[16] see among others Bilbiie et al. (2007); Devereux et al. (1996); Etro and Colciago (2010); Faia (2012); Jaimovich and Floetotto (2008)

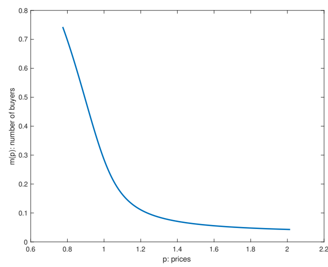
[17] Other parameters are set agnostically, we show in appendix B.7 that they do not affect qualitatively the results. Cost of search $s = 0.15$, the investment costs $z = 2$ and the number of firms $N = 7$



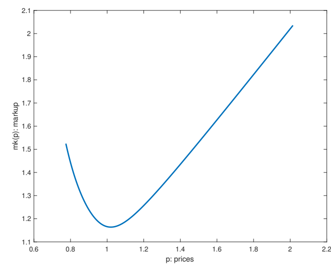
(a) PDF of Strategies



(b) Productivity of Labour



(c) Number of Buyers



(d) Markups

Figure (3) Graphs from the numerical solution of the model calibrated as described below and the share of middle aged consumers set to $\lambda = 0.3$.

3.2 Aging of the population and labor productivity

In this section, we analyze the effect of an increase in the share of middle aged consumers with positive search costs on individual firm strategies and aggregate outcomes, especially focusing on the firm level investment in the labor saving technology and aggregate long-term productivity.

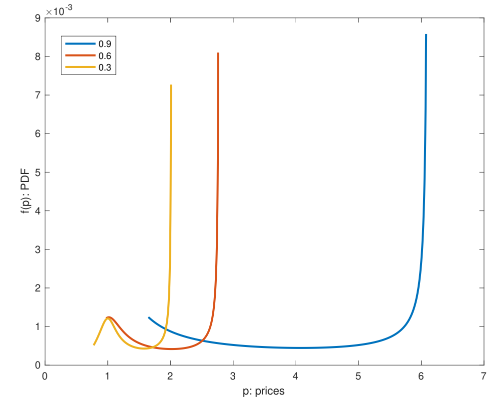
We define the policy functions as functions of the deep parameters of the model: (λ, N, s): $b(\rho(\lambda, N, s))$, $r(\lambda, N, s) = \min\{\rho(\lambda, N, s), \hat{p}\}$, $F(p; \rho(\lambda, N, s), \lambda, N)$, $\pi(\rho(\lambda, N, s), \lambda, N)$. We assume no free entry (N constant and π endogenous). Results in the free entry case are exactly the same with the only difference that profits (which are pinned down by entry costs, i.e. $\pi = \kappa$) determine the equilibrium amount of of firms in the economy.

Proposition 3.7 (Relative price effect). *For a given price strategy, an increase in the share of middle aged consumers increases the investment in the labour saving technology:*

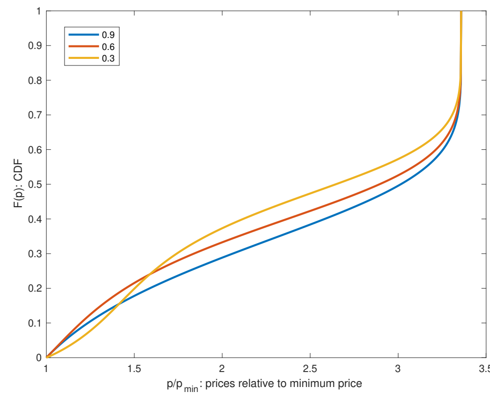
$$\frac{da(p; \pi(r(\lambda, N, s), \lambda, N))}{d\lambda} = \frac{\partial a}{\partial \pi}\left(\frac{\partial \pi}{\partial \lambda} + \frac{\partial \pi}{\partial r}\frac{\partial r}{\partial \lambda}\right) < 0 \tag{6}$$

Proof. *in appendix B.4* □

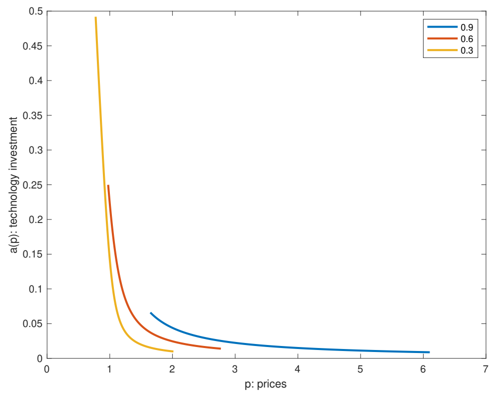
The intuition behind this result is the following. As the population ages, and the relative share of middle aged people increases, the reservation price for middle aged people increases, so a given price p is now relatively cheaper than before. Firms with such prices are competing to attract buyers



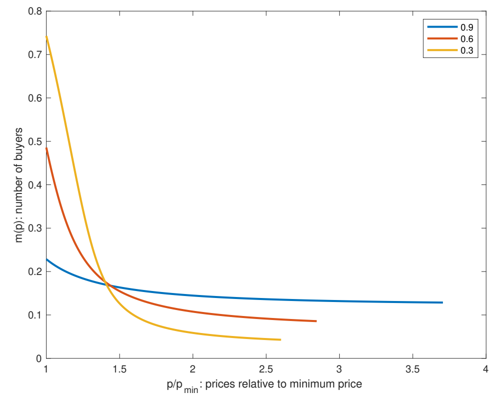
(a) PDF of Strategies; $[b(r), r]$



(b) CDF of Strategies; $[1, r/b(r)]$



(c) Investment; $[b(r), r]$



(d) Number of buyers; $[1, r/b(r)]$

Figure (4) Heterogeneous effect of an increase in the share of middle aged consumers along the strategy distribution. Firms strategies for different levels of middle aged population: $\lambda = 0.3$, yellow line; $\lambda = 0.6$, red line; ($\lambda = 0.9$, blue line. In figure (b) and (d), we scale the price range by the minimum price $b(r)$.

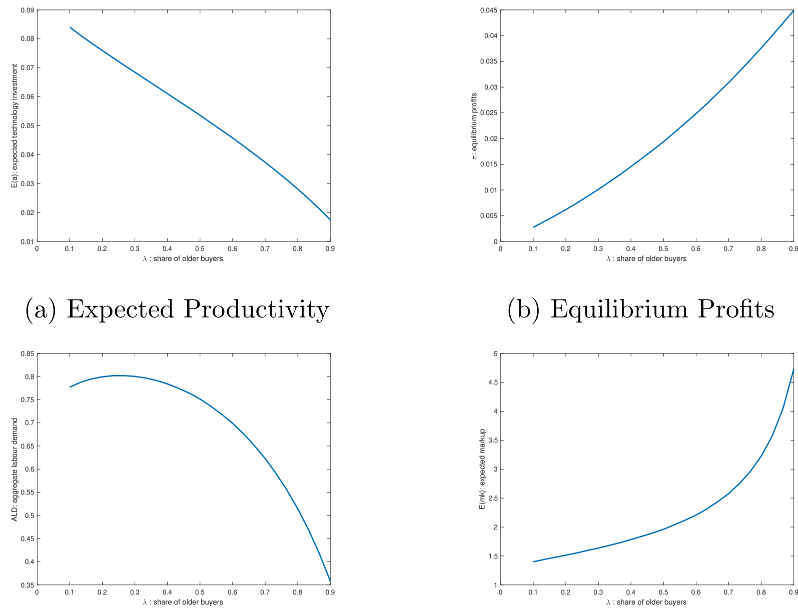
with no search costs more. In a reverse perspective, the result can intuitively be interpreted as for a given level of productivity, a firm increases its price when the share of people with positive search costs increases. This *relative price effect* is visible on figure 4c which represents the distribution of investment strategies along the equilibrium price ranges, for example at $p = 2$ a firm would be the highest pricing in a $\lambda = 0.3$ economy and the lowest pricing in a $\lambda = 0.9$ economy, leading to an investment approximately six times as high.

Proposition 3.8 (Aggregate effect)**.** *The effect of aging on the long term investment in the labour saving technology, defined as* $\mathrm{E}_p(a) = \int_{b(r)}^{r} a(p)dF(p)$*, can be decomposed into a positive* ***relative price effect*** *and a negative* ***distribution effect****.*

Proof. *in appendix B.5* □

The *distribution effect* corresponds to the change in the incentive to adopt different strategies as the share of population with search costs increases. The strategy distribution changes in two ways.. First, the price range increases with both the minimum price $b(r)$ and the reservation price r increasing (figure 4a). As we have seen in Proposition 3.5, this leads to an aggregate decrease in investment as prices increase in absolute terms. Second, with higher probabilities firms choose high-price low-investment strategies. This effect is visible in figure 4b picturing the cumulative distribution function of strategies in an economy with a different age structure and in which we scaled the price range by the minimum price $b(r)$. Indeed, as the share of middle aged increases, there are fewer buyers with no search costs to compete for, and therefore, more probability is put on strategies focusing on buyers with positive search costs.

These effects have heterogeneous on firms along the strategy distribution. The increase in the share of agents with positive search costs leads to a redistribution of buyers from low price-high productivity firms to the other ones. This phenomenon is visible in figure 4d in which we plot the expected number of buyers a firm attracts, scaling the price range by the minimum price $b(r)$. Relatively high price-low productivity firms spread the investment costs over the production of more goods, and, therefore, increase their investment (marginally). While relatively low price-high productivity firms experienced a large drop in demand, leading to a large drop in production and, therefore, investment. As shown in Figure 4c, the most productive firm in a $\lambda = 0.3$ economy invests approximately ten times as much as in a $\lambda = 0.9$ economy.



(a) Expected Productivity (b) Equilibrium Profits

(c) Aggegate Labour Demand (d) Expected Markups

Figure (5) Aggegate effects of aging. Main economic aggregates along the age range, $\lambda \in [0.1, 0.9]$. Expected productivity and markups are computed taking the expectation of firm strategies over the strategy PDF.

The net effect on the average productivity in the economy, as captured by the expected investment, is ambiguous theoretically but consistently negative numerically. We see indeed that the expected investment in the labor saving technology, a measure of average labor productivity in the economy, monotonically decreases as the share of middle aged people increase (see figure 5a). [18] At the same time, equilibrium profit and expected markups increase, showing the drop of competition in the economy as the middle aged increase (see figure 5b and 5d). Finally, the aggregate labor demand takes the form of an inverted U-shaped function. From around $\lambda \approx 0.3$ the demand for labor decreases because the aggregate demand drop (the price increase decreases the demand per buyer which is exacerbated by the fact that middle aged people observe higher prices on average) is not fully compensated by the drop in productivity.

4 A General Equilibrium Perspective [PRELIMINARY]

Until now we focused on a sector analysis in which the demand functions are defined exogenously and the labour market does not clear. We develop a general equilibrium analysis with an economy made of a continuum of sectors of measure 1 and in which the real wage adjusts to clear the labour

[18]This result is robust to large deviations from the baseline calibration. See appendix B.7 for these robustness checks.

market.

4.1 Household demand

Each household j provides a unit of labour and gets utility from consuming a composite good, $U(C^j) = C^j$. The composite good aggregates consumption c_s^j of goods from each sector $s \in \mathcal{S}$ such that: $C^j = \left(\int_s (c_s^j)^{\frac{\sigma-1}{\sigma}} ds\right)^{\frac{\sigma}{\sigma-1}}$. We define a price aggregator for each individual: $\widetilde{P}^j = \left(\int_s (p_s^j)^{1-\sigma} ds\right)^{\frac{1}{1-\sigma}}$. We are interested in real variables, therefore, following Ghironi and Melitz (2005) we normalize prices and assume that the aggregate price in the economy stay constant, i.e. $P_{eq} = 1$. To ensure this condition, we normalize the price aggregator of each type of agent by the average price in the economy:

$$\underbrace{\frac{\lambda \widetilde{P}^M}{\lambda \widetilde{P}^M + (1-\lambda)\widetilde{P}^{Y,O}}}_{P^O} + \underbrace{\frac{(1-\lambda)\widetilde{P}^{Y,O}}{\lambda \widetilde{P}^M + (1-\lambda)\widetilde{P}^{Y,O}}}_{P^Y} = 1 = P_{eq}$$

Following Melitz (2003) and using the condition $R^j = P^j Q^j$, we then obtain the sector specific demand for each type of agent. [19]

$$c_s^j = D(p_s^j) = C^j \frac{p_s^j}{P^j}$$

In this baseline specification of the model, we assume that agents own equally shares in each firms, and that all types of agents work meaning that the budget constraints do not differ across the different age categories. We stay agnostic to ensure that our results are not driven by a specific revenue sharing assumption but only by the demand channel. More refine modelling assumptions about the labour supply and the revenues heterogeneity would be interesting in a more global analysis of aging, though we here favor an agnostic assumption to highlight the precise channel we are interested in.[20] Aggregate profits in the economy amount to: $N\pi$, which are equally divided among the consumers, their revenues are then identical: $R = w + N\pi$. Informed buyers (young and old) buy in each market at the lowest price, which means that out of N independent draws from $F(p)$, they pick the lowest one.

$$F^{Y,O}(p) = 1 - (1 - F(p))^N \; ; \quad f^{Y,O}(p) = \frac{\partial F^{Y,O}(p)}{\partial p} = N(1 - F(p))^{N-1} f(p) > 0$$

4.2 Aggregate measures

All the sectors are identical, and therefore agents face the same price distribution in each sector. By the law of large numbers, we obtain an equality between the expectation of price over sector

[19] See appendix C.1 for the detailed derivations

[20] In an extension to this model, we will assume that young and middle aged only work, and old only receive profits.

and the expected price. [21]

The aggregate production in the economy Y is simply the aggregation of the sector level demand Y_s, $Y = \int_s Y ds$. The production in each sector fulfills the market clearing condition:

$$Y = \int_s \int_j (c_s^j) dj ds = \int_j \frac{C^j}{P^{j\ \sigma}} \int_s (p_s^j) \; ds \; dj$$

And buyers with the same information costs have the same price aggregator and, therefore, their consumption of the composite good is identical. We can then rewrite the aggregate production simply as a function of the share of older buyers, the consumption and price aggregators and the price distributions.

$$Y = (1 - \lambda)\frac{C^{Y;O}}{P^{Y\ \sigma}} \mathrm{E}^{I}[p] + \lambda \frac{C^{M}}{P^{O\ \sigma}} \mathrm{E}^{O}[p] \tag{7}$$

In the labour market, each agent provides a unit of labour. On the demand side, using the market clearing conditions at the sector level, we have that the aggregate labour demand is the sum of the products between the number of goods sold at each price level with the quantity of labour required to produce each of these goods in firms with this price strategy.

$$ALD = \int_s \int_j \frac{c_s^j}{\alpha_s^j} \, dj ds = \int_j \frac{C^j}{P^{j\ \sigma}} \int_s (p_s^j) (\bar{a} - a(p_s^j)) \, ds \; dj$$

Following the argument presented above, we obtain as well a simplified version of the aggregate labour demand. [22]

$$ALD = (1 - \lambda) \frac{C^{Y;O}}{P^{Y\ \sigma}} \mathrm{E}^{Y}[p(\bar{a} - a(p))] + \lambda \frac{C^{M}}{P^{O\ \sigma}} \mathrm{E}^{O}[p(\bar{a} - a(p))] \tag{8}$$

In this case, the equilibrium definition is identical from above with the extra condition that the real wage w should adjust so that the labour market clears. [23]

4.3 Results

In economies with a larger share of middle aged consumers with positive search costs, higher prices decrease individual demand and the higher share of less informed buyers decreases aggregate

[21] More precisely, let's define $g(p_s^j)$ a function of the random variable $p_s^j \sim F^j(p)$. Since all the sectors are identical, by the law of large number we have that: $\int_{s \in S} g(p_s^j) ds = S \frac{\int_{s\in S} g(p_s^j) ds}{S} = S\, \mathrm{E}[g(p_s^j)] = S \int_{b(r)}^{r} g(p_s^j) dF^j(p)$.

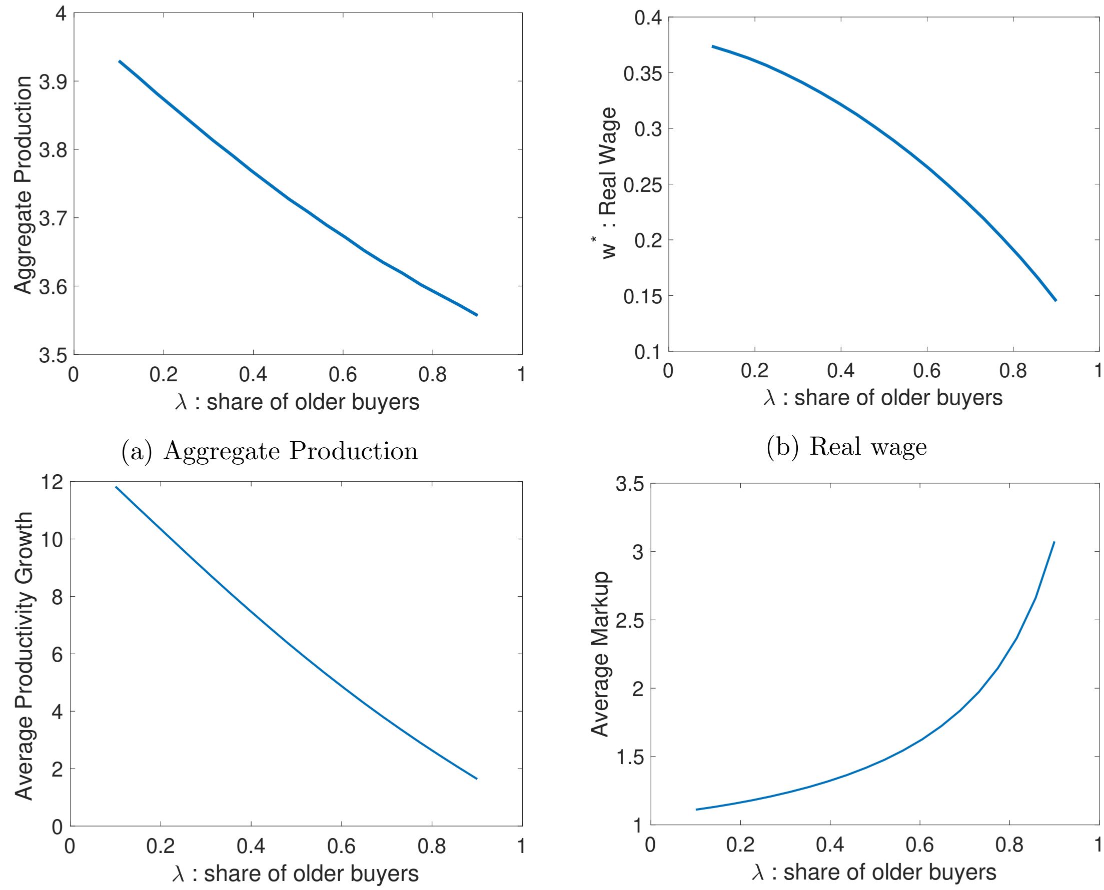
[22] An equivalent, but more complex, computation of the aggregate labour demand would sum the firm level labour demand.

$$ALD = N \int_{b(r)}^{r} l(p) f(p) dp$$

with:

$$l(p) = (\bar{a} - a(p)) y(p) = (\bar{a} - a(p)) \frac{1}{N} D^{M}(p) (1 - \ldots)(1 - F(p))^{N-1} D^{Y,O}(p)$$

[23] See appendix for details about the algorithm.



(a) Aggregate Production

(b) Real wage

(c) Average Productivity Growth

(d) Average Markups

Figure (6) Macroeconomic aggregates in economies along the age gradient.

P r0:1; 0:9s. Average productivities are computed weighing each firm by its labour. Average markups are computed weighing each firm by its total sales.

demand, leading to an overall drop in aggregate production (figure 6a). The lower demand for labour is compensated by a lower real wage and a drop in investment, and therefore, a lower labour productivity growth.

These results summarize well the evolution in most of the developed economies over the last decades in which aging mostly resulted in an increase in the share of middle aged consumers including the relative decrease in the labour productivity growth and the increased market power of firms (as summarized by the increased average markups).

5 Conclusion

We presented evidence of a novel channel through which aging affects the economy: the *demand channel*. We empirically show the relevance of such a channel in terms of its effects on prices, and production and productivity growth. We have also shown that aging does not have a monotonic effect on productivity since young and old consumers tend to drive down prices, increase production and productivity growth (opposite to middle aged consumers). We have shown the bias rising from analysis overlooking such a channel, and tried to disentangle the different channels through which a demographic change affects the economy. In this way we have estimated the effect of the demographic change on productivity through the labor market channel, and showed that it is positive as suggested by Acemoglu and Restrepo (2017, 2018); Abeliansky and Prettner (2017); and the effect through the demand side which, given the demographic trends in the period 1995-2020, is negative contributing to the slowing down in productivity growth in the period 1995-2007.

In order to rationalize such results, we have built a search model in which young and old agents have low search costs, and middle aged agents have high search costs as suggested by our empirical evidence and our calibration exercise in Appendix B.1. We show that as the share of middle aged people increases, the demand is more equally allocated across firms, leading to an aggregate reduction in investment in productivity enhancing technologies and eventually translating into a drop of the labour productivity growth. A lower aggregate labour demand translates, in general equilibrium, to a drop in the real wage and aggregate production.

In an extension of the paper, we will provide a model in which aging results in both an increase in the middle aged (leading to a reduction in productivity through the demand channel), and an increase in the share of old people (leading to a reduction in the labor supply, an increase in investment in labor saving technologies, and an increase in productivity through the supply channel).

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A Empirical Evidence

A.1 Statistics

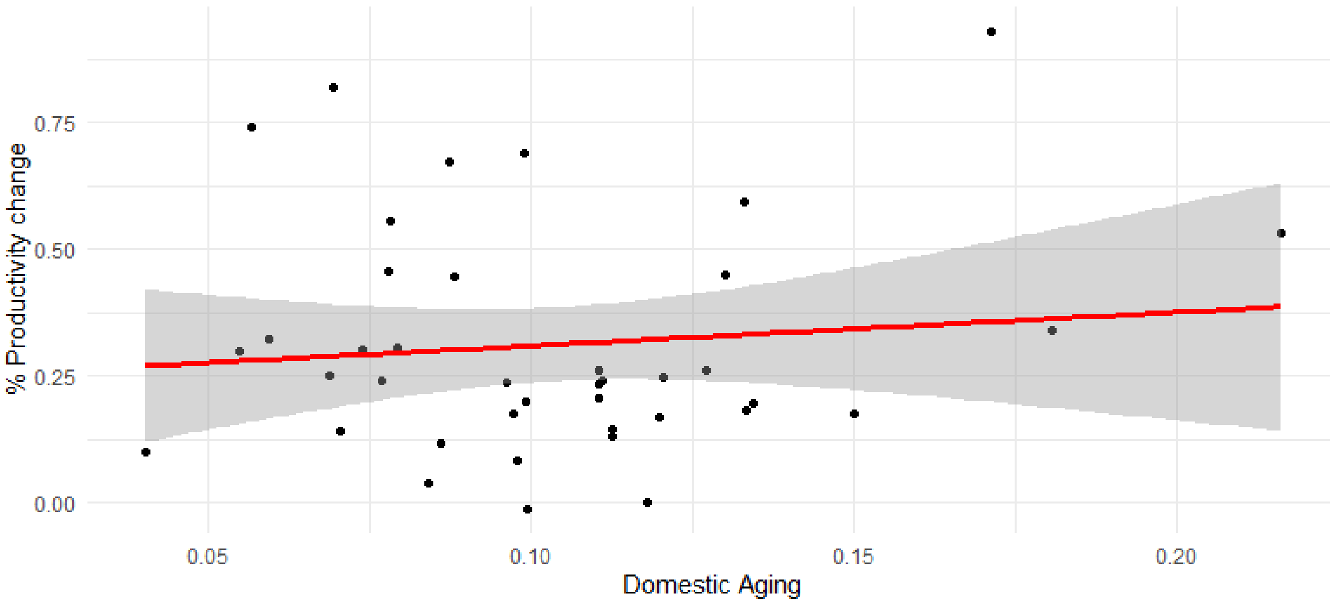


Figure (7) Raw correlation between Domestic Aging (change in the share of 50+ relative to 20+) and productivity growth (% change in real value added per hour worked).

Table (6) Correlation between Domestic Aging (change in the share of 50+ relative to 20+) and productivity growth (% change in real value added per hour worked).

	Dependent variable:		
	Δlog Productivity		
	(1)	(2)	(3)
Domestic Aging	0.660	0.436	0.310
	(1.025)	(1.029)	(0.893)
Initial Domestic Age			3.778
			(1.096)
Region Fixed Effects		X	X
Observations	40	40	40
R^2	0.011	0.303	0.491
Adjusted R^2	0.015	0.176	0.380
Note:	p 0.1;	p 0.05;	p 0.01

A.2 Estimation of the Aging Demand Shocks

In order estimate our foreign aging demand shock, we compute the exposure of each sector s in country i to country j making use of the Leontief input-output model:

$$\xi^s_{i,j} = (I - A)^{-1} f^s_{i,j}$$

where $\{\xi^s_{i,j}\}$ is the share matrix with dimension $(S \times J)$ by J showing the share of production for each sector s in each country i consumed in country j; $(I - A)^{-1}$ is the Leontief inverse matrix with dimension $(S \times J)$ by $(S \times J)$ showing the gross output values that are generated in all stages of the production process of one unit of consumption; A is the input-output matrix with dimension $(S \times J)$ by $(S \times J)$ showing the input amount required to produce 1 unit of output; and $f^s_{i,j}$ is the final demand matrix with dimension $(S \times J)$ by J showing the amount of output produced by sector s in country i consumed or invested in country j as a share of the total output produced by sector s in country i.[24]

To obtain aging demand shocks orthogonal to the supply aging shock (i.e. aging of the domestic labor force), we exclude aging coming from the domestic economy, and we normalize the shares as follows:

$$\bar{\xi}^s_{i,j} = \frac{\xi^s_{i,j}}{\sum_{k \neq i} \xi^s_{i,k}}, \text{for } j \neq i$$

so that the sum of the shares of each sector in each country with respect to each other country sums up to one (i.e. $\sum_{j \neq i} \bar{\xi}^s_{i,j} = 1$).

We define the demographic shock for sector s in country i at time t as the sum of the change in the age variable in each other country weighted by the standardized shares:

$$\Delta age^s_i = \sum_{j \neq i} \bar{\xi}^s_{i,j}(t-1)\Delta age_j(t)$$

where the shares are taken in the previous period $t-1$ to avoid the possible contemporaneous effects in the shares. Indeed, since shares are endogenous, they could be affected by the aging of the population; by fixing the shares to the previous period, we mitigate for this endogeneity problem.

[24] Borrowing from the notation in Table 2, S (J) represents the number of sectors (countries).

A.3 Tables

Table (7) Change in the share of middle-aged (45-64) and old (65-84) foreign demand on production (real value added). Baseline age category: young adults (25-44).

	Dependent variable:			
	Δlog Production			
	(1)	(2)	(3)	(4)
Foreign ΔMid-age Demand	8.393	8.992	7.341	5.843
	(1.887)	(1.919)	(1.950)	(1.953)
Foreign ΔOld Demand	0.788	3.094	0.476	1.063
	(1.727)	(1.898)	(2.001)	(2.017)
Domestic ΔMid-age			1.603	1.390
			(0.519)	(0.703)
Domestic ΔOld			1.202	1.632
			(0.676)	(0.848)
Initial Mid-age Demand				2.283
				(2.173)
Initial Old Demand				5.222
				(1.231)
Initial Export Exposure				0.219
				(0.087)
Region Fixed Effects		X	X	X
Sector Fixed Effects		X	X	X
Observations	1,347	1,347	1,347	1,347
R^2	0.015	0.209	0.219	0.242
Adjusted R^2	0.013	0.185	0.194	0.216
Note:		p 0.1;	p 0.05;	p 0.01

Table (8) Change in the share of middle-aged (45-64) and old (65-84) foreign demand on prices (value added price deflator). Baseline age category: young adults (25-44).

	Dependent variable:			
	Δlog Price			
	(1)	(2)	(3)	(4)
Foreign ΔMid-age Demand	1.735	2.471	6.843	14.548
	(3.207)	(3.054)	(3.040)	(2.807)
Foreign ΔOld Demand	3.040	3.980	4.059	3.710
	(2.928)	(3.014)	(3.111)	(2.889)
Domestic ΔMid-age			0.862	1.085
			(0.809)	(1.011)
Domestic ΔOld			8.275	10.331
			(1.055)	(1.219)
Initial Mid-age Demand				12.541
				(3.124)
Initial Old Demand				21.868
				(1.770)
Initial Export Exposure				0.616
				(0.125)
Region Fixed Effects		X	X	X
Sector Fixed Effects		X	X	X
Observations	1,348	1,348	1,348	1,348
R^2	0.001	0.297	0.334	0.451
Adjusted R^2	0.0004	0.275	0.312	0.432
Note:		p 0.1;	p 0.05;	p 0.01

Table (9) Change of middle-aged (45-64) with respect to young adults (25-44) and old (65-84) foreign demand on productivity.

	Dependent variable:			
	Δlog Productivity			
	(1)	(2)	(3)	(4)
Foreign ΔAge-composition Demand	3.933	2.948	2.848	2.420
	(0.703)	(0.722)	(0.736)	(0.745)
Domestic ΔAge-composition			0.127	0.481
			(0.184)	(0.237)
Initial Age-composition Demand				1.814
				(0.466)
Initial Export Exposure				0.128
				(0.081)
Region Fixed Effects		X	X	X
Sector Fixed Effects		X	X	X
Observations	1,347	1,347	1,347	1,347
R^2	0.023	0.201	0.201	0.213
Adjusted R^2	0.022	0.177	0.177	0.187
Note:		p 0.1;	p 0.05;	p 0.01

Table (10) Change of middle-aged (45-64) with respect to young adults (25-44) and old (65-84) foreign demand on production.

	Dependent variable:			
	Δlog Production			
	(1)	(2)	(3)	(4)
Foreign ΔAge-composition Demand	3.219	3.373	2.775	2.271
	(0.744)	(0.759)	(0.769)	(0.777)
Domestic ΔAge-composition			0.764	0.029
			(0.193)	(0.247)
Initial Age-composition Demand				2.180
				(0.486)
Initial Export Exposure				0.169
				(0.085)
Region Fixed Effects		X	X	X
Sector Fixed Effects		X	X	X
Observations	1,347	1,347	1,347	1,347
R^2	0.014	0.205	0.214	0.230
Adjusted R^2	0.013	0.181	0.190	0.205
Note:		p 0.1;	p 0.05;	p 0.01

Table (11) Change of middle-aged (45-64) with respect to young adults (25-44) and old (65-84) foreign demand on prices.

	Dependent variable:			
	Δlog Price			
	(1)	(2)	(3)	(4)
Foreign ΔAge-composition Demand	1.282	0.674	1.289	4.218
	(1.265)	(1.205)	(1.226)	(1.178)
Domestic ΔAge-composition			0.788	1.903
			(0.307)	(0.374)
Initial Age-composition Demand				8.703
				(0.736)
Initial Export Exposure				0.857
				(0.128)
Region Fixed Effects		X	X	X
Sector Fixed Effects		X	X	X
Observations	1,348	1,348	1,348	1,348
R^2	0.001	0.296	0.299	0.379
Adjusted R^2	0.00002	0.275	0.278	0.359
Note:		p 0.1;	p 0.05;	p 0.01

A.4 Robustness Checks

Table (12) Change of middle-aged (45-64) and old (65-84) foreign demand on productivity, production, and prices in **manufacturing** sectors. Baseline age category: young adults (25-44).

	Dependent variable:		
	Δlog Productivity	Δlog Production	Δlog Price
	(1)	(2)	(3)
Foreign ΔMid-age Demand	10.771	10.027	21.622
	(3.383)	(3.804)	(4.881)
Foreign ΔOld Demand	2.648	0.176	3.864
	(3.398)	(3.822)	(4.865)
Domestic ΔMid-age	0.523	2.295	0.676
	(1.131)	(1.272)	(1.634)
Domestic ΔOld	1.658	1.246	8.666
	(1.312)	(1.475)	(1.894)
Initial Mid-age Demand	1.658	3.499	9.761
	(3.995)	(4.493)	(5.770)
Initial Old Demand	2.473	5.017	22.892
	(2.281)	(2.565)	(3.293)
Initial Export Exposure	0.077	0.123	0.385
	(0.128)	(0.144)	(0.185)
Region Fixed Effects	X	X	X
Sector Fixed Effects	X	X	X
Observations	556	556	557
R^2	0.181	0.271	0.435
Adjusted R^2	0.142	0.236	0.408
Note:			p 0.1; p 0.05; p 0.01

Table (13) Change of middle-aged (45-64) with respect to young adults (25-44) and old (65-84) foreign demand on productivity, production, and prices in **manufacturing** sectors.

	Dependent variable:		
	Δlog Productivity	Δlog Production	Δlog Price
	(1)	(2)	(3)
Foreign ΔAge-composition Demand	3.710	3.527	7.750
	(1.304)	(1.465)	(1.968)
Domestic ΔAge-composition	0.0001	0.456	1.871
	(0.393)	(0.441)	(0.591)
Initial Age-composition Demand	1.279	1.668	10.198
	(0.902)	(1.013)	(1.358)
Initial Export Exposure	0.036	0.063	0.744
	(0.120)	(0.134)	(0.180)
Region Fixed Effects	X	X	X
Sector Fixed Effects	X	X	X
Observations	556	556	557
R^2	0.169	0.262	0.374
Adjusted R^2	0.135	0.232	0.348
Note:		p 0.1; p 0.05;	p 0.01

A.5 Different stages in the Demographic process

In this exercise, we analyze the contribution of the structural demographic change on the slowing down in productivity in the period 1995-2007 in different countries. We base our analysis on the results presented in the previous section showing that an increase in the middle aged relatively to young and old consumers drives down productivity. However, it is extremely relevant to recognise that countries are at very different stages in their demographic process. Indeed, we have shown the aging of the population has not a monotonic effect on productivity through the demand channel as old consumers are not significantly different from young consumers in terms of their effects on the economy relatively to the middle aged. This implies that potentially, very old countries might have experienced a positive effect on productivity of aging through the demand channel. This is the case of Japan in which the relative share of middle aged population reduced driven by the increase in the share of elderly. Table 14 shows the estimated change in productivity due to the change in the age composition of the demand. As we can see, the United States present a higher productivity loss relatively to old countries such as Germany and Italy due to the higher increase in middle-aged and the relative low increase in the old population. Differently from the European countries, Japan showed a drastic reduction in the relative share of middle-age population as the very old sharply increased meaning that Japan has most likely gained because of the aging of the consumers. It is, however, relevant to stress that this analysis of the effect of aging on productivity is extremely partial as we are only considering the demand channel, and we consider domestic aging abstracting from from foreign demographic changes that influence the economy through exports. Countries have, indeed, different exposure to foreign markets meaning that they are affected differently by foreign aging countries.[25]

Table (14) Change in productivity in different countries due to the domestic demographic change through the demand channel.

Countries	$\Delta \frac{\#Mid\ \ age}{\#Young\ \ \#Old}$ 100 (domestic)	Annual %Δ Productivity (demand channel)
United States of America	14	-2
Germany	11	-1.67
Italy	11	-1.67
Japan	-15	2.21

[25] We used exports to estimate the effect of demographic change through the demand channel in order to disentangle it from the labor market channel. Now, using this estimate, we compute the demand channel effect of demographic change for different countries considering their domestic demographic process.

A.6 Bias from missing the Demand channel

Table (15) Effect of an increase in the share of 55-64 relative to 20-54 on productivity in European countries trough the demand channel.

	Dependent variable:			
	ch_productivity.x			
	(1)	(2)	(3)	(4)
foreign_aging_demand	2.072	1.106	1.088	3.880
	(1.627)	(1.882)	(1.883)	(1.975)
domestic_aging.x			0.275	0.356
			(0.667)	(0.684)
initial_age_demand				1.176
				(0.999)
initial_export_exposure				0.464
				(0.107)
Sector Fixed Effects		X	X	X
Observations	909	909	909	909
R^2	0.002	0.171	0.171	0.188
Adjusted R^2	0.001	0.138	0.137	0.154
Note:		p 0.1;	p 0.05;	p 0.01

B Micro-model

B.1 Search cost Microfoundation

In this section, we microfound the search costs as an evidence of our theoretical assumption that young and old have similar search costs even if they differ in terms of opportunity cost of time and efficiency in searching, and have lower search costs relatively to middle age consumers. We proceed by solving the utility maximization problem of young, middle aged, and old consumers choosing the amount of time to devote searching given the age-specific opportunity cost of time, and search efficiency. We then calibrate the model using Aguiar and Hurst (2007) results on opportunity cost of time to show that young and old have lower search cost relatively to middle aged consumers.

Consumer $i \in \{y, m, o\}$ (young, middle aged, old) solves the following utility maximization problem by choosing the amount of time to devote to shopping given that spending time in searching is costly and depends on the age category:

$$max_{\{t\}} \quad C(t, e_i) \quad \mu_i \quad t \tag{9}$$

where μ_i is the age-specific opportunity cost of time. Consumption depends on the time devoted to search (t), on the age-specific search efficiency (e_i). For simplicity, we define:

$$C(t, e_i) \quad c \quad e_i \, t^{1} \quad , \tag{10}$$

i.e. consumption depends on search efficiency and time devoted to search which are imperfect substitutes. From the FOC, we get:

$$t_i \quad e_i \quad \frac{c \quad (1 \quad \alpha)}{\mu_i}^{\frac{1}{\alpha}} \tag{11}$$

We define the search cost (s) such that:

$$1 \quad s_i \quad e_i \, t^{1} \quad . \tag{12}$$

From which we get:

$$s_i \quad 1 \quad e_i \quad \frac{c \quad (1 \quad \alpha)}{\mu_i}^{\frac{1 \quad \alpha}{\alpha}} \tag{13}$$

Calibration We calibrate the model using the results in Aguiar and Hurst (2007) for the age-specific opportunity cost of time. We consider the simple mean across the different age categories in order to recover the opportunity cost of time of our three aggregate age categories (young, middle aged, old) consumers. We normalize our results such that the opportunity cost of time for young consumers is equal to 1. Since the results in Aguiar and Hurst (2007) are in terms of log deviations from a baseline age category, our results are in terms of a parameter representing the opportunity cost of time of the baseline category (χ). We recover a proxy for the search efficiency using Eurostat data ("isoc_ec_ibuy") on Internet purchases by individuals in Germany in 2007 by averaging the percentage of online shoppers (in the last 3 months) across the different age categories to recover our three aggregate age categories, and normalizing such that young consumers had an opportunity cost of time equal to 1. Table 16 shows a summary of our calibration.

Table (16) Calibration

mu_y	1	Aguiar and Hurst (2007)
mu_m	0.6135	Aguiar and Hurst (2007)
mu_o	0.0258	Aguiar and Hurst (2007)
e_y	1	Eurostat ("isoc_ec_ibuy")
e_m	0.525	Eurostat ("isoc_ec_ibuy")
e_o	0.15	Eurostat ("isoc_ec_ibuy")
α	0.66	target: $\frac{t_m}{t_y}$
c	3	scaling parameter
χ	0.278	-

To recover α, we targeted the time spent shopping by middle aged relative to young consumers using Eurostat data ("tus_00age") from on Time spent in shopping and services in Germany and other European countries in 2010. c is a scaling parameter; we have chose it in order to obtain an estimation of the searching costs which is consistent with the theoretical model assumption to be able to calibrate the model, i.e. $s_y \quad s_o \quad 0$ and $s_m < 0$. With the calibration above, we, indeed obtain $s_y \quad s_o \quad 0$ and $s_m \quad 0.3178$ meaning that even if young and old consumers are very different in the way they search in terms of search efficiency and time spent searching, they face low search cost relative to middle aged consumers which is consistent with the aggregate empirical evidence we have shown in the empirical section of the paper.

B.2 Proof of proposition 3.5

Proof. Using the FOC from problem (4), and definition (3.4), we obtain a polynomial of order 2:

$$\mathsf{E}\pi \quad a^2 \quad 2a\left(\frac{p}{w} \quad \bar{a}\right) \quad \bar{z}\pi \quad 0$$

It has two solutions:

$$a_1 \quad \left(\left(\frac{p}{w} \quad \bar{a}\right)^2 \quad \bar{z}\pi\right)^{1/2} \quad \left(\frac{p}{w} \quad \bar{a}\right)$$

$$a_2 \quad \quad \left(\left(\frac{p}{w} \quad \bar{a}\right)^2 \quad \bar{z}\pi\right)^{1/2} \quad \left(\frac{p}{w} \quad \bar{a}\right)$$

$a_2 \quad 0$ therefore, there is a unique solution for the investment: $a(p; r, \pi) \quad a_1$.

From lemma (B.2), we can obtain a closed form solution for $a(p; r, \pi)$, and:

$$a'(p) \quad \frac{\partial a}{\partial p} \quad \frac{1}{w} \quad \frac{\frac{p}{w} \quad \bar{a}}{\sqrt{\left(\frac{p}{w} \quad \bar{a}\right)^2 \quad \bar{z}\pi}} \quad 1 \quad 0$$

□

Lemma B.1. *The NE equilibrium expected profit π is unique and is increasing in the reservation price.*

Proof. From definition (3.4), we know that all prices in the support of F give the same expected profit. At price P_r, $F(P_r, r) \quad 1$ and a firm is for certain the highest pricing firm, which means that it doesn't attract any young buyers. We then obtain a simple expression for π the conditional NE-expected profits:

$$\pi \quad \mathbb{E}\,\pi(P_r, F) \quad \frac{\lambda}{N} \quad R^O(P_r) \quad z(a(P_r)) \tag{14}$$

where $R^j(P_r) \quad D^j(P_r) \; (P_r \quad c(a(P_r)))$.

By expanding 14, we obtain the following polynomial of π:

$$0 \quad \pi \quad \frac{\lambda}{N} D^M(r)w \left[\left(\left(\frac{r}{w} \quad \bar{a}\right)^2 \quad \bar{z}\pi \right)^{1/2} \quad \frac{1}{\bar{z}} \left(\left(\frac{r}{w} \quad \bar{a}\right)^2 \quad \bar{z}\pi \right)^{1/2} \quad \frac{r}{w} \quad \bar{a} \right]^2$$

This polynomial has three solutions:

$$\pi_1 \quad \frac{(r \quad \bar{a}w)^2}{w^2 \bar{z}} \quad 0$$

$$\pi_2 \quad \frac{D^M(r)\lambda\,(4N\,(r \quad \bar{a}w) \quad \lambda D^M(r) w^2 \bar{z})}{4N^2}$$

$$\pi_3 \quad \frac{D^M(r)\lambda\,(4N\,(r \quad \bar{a}w) \quad \lambda D^M(r) w^2 \bar{z})}{4N^2}$$

Under the condition that $r \quad \bar{a}w \geq 0$, $\pi_3 \quad 0$. This condition simply states that the reservation price is larger than the marginal costs of a firm which does not invest in the labour saving technology. Let's assume that $r \quad \bar{a}w \quad 0$, then a firm setting its price at r would make negative profit, which would imply that r is not in the equilibrium support of F in obvious contradiction with the definition of r. We have then $\pi_3 \quad 0$ and a unique positive solution $\pi \quad \pi_2$.

From the proposition 3.5, it also implies that $a(p) > a(r) > 0$, which means that for all strategies firms have strictly positive investment.

We then know that: $\partial\pi/\partial r > 0$ since $dD^M(r)/dr \quad 0$. □

B.3 Characterization and proof of proposition 3.6

B.3.1 Characterization of the NE price strategy distribution:

Using equation (14), and solving $E\{\pi[p, F(p; r)]\} = \pi$ for $F(p; r)$, we get the NE-distribution of prices:

$$F(p;\, r) = 1 - \left[\frac{1}{N}\frac{\lambda}{1-\lambda}\frac{R^O(P_r) - R^O(p)}{R^Y(p)} - \frac{z(a(P_r)) - z(a(p))}{(1-\lambda)R^Y(p)}\right]^{\frac{1}{N-1}} \tag{15}$$

Where $R^Y(p) = D^{Y,O}(p)(p - c(a))$ To complete the characterization of $F(p; r)$, we need to derive the bounds of the support, $b(r)$ and r. We pin down $b(r)$ using the fact that $F(b(r); r) = 0$ which leads to:

$$\frac{\lambda}{N}R^O(b(r)) + (1-\lambda)R^Y(b(r)) - z(b(r)) = \frac{\lambda}{N}R^O(P_r) - z(a(P_r)) \tag{16}$$

In order to show that $b(r)$ is the unique solution of equation (16), it is sufficient to show that $\frac{\lambda}{N}R^O(p) + (1-\lambda)R^Y(p) - z(p)$ is increasing in p since the right-hand-side of equation (16) is constant in p. A sufficient condition for this to hold is that demand is constant in p.[26]

B.3.2 Characterization of r^*:

As a last step, we need to show that there exists at least one $r = r^* \le \hat{p}$ which is consistent with $F(p; r)$ as defined by equations (15) and (16). Since r_F is defined as the unique solution to $ECS(z) = s$, if one exists, or ∞ otherwise, a consistent reservation price (r^*) must satisfy:

$$H(r^*;\, \lambda, N, \bar{a}, s) = \int_{b(r^*)}^{r^*} D^M(p)\, F(p;\, r^*, \lambda, N, \bar{a})\, dp - s = 0 \tag{18}$$

Notice that if $\int_{b(r^*)}^{r^*} D^M(p)\, F(p;\, r^*, \lambda, N, \bar{a})\, dp$ is increasing in p then, if it exists, the root is unique. Assuming a constant demand function, we can show that H is increasing in p and, that, if it exists, the root is unique.[27]

We can now formally define the consistent reservation price as follows:

$$\rho(\lambda, N, \bar{a}, s) = \begin{cases} r^*, & \text{if } H(r^*;\, \lambda, N, \bar{a}, s) = 0 \text{ and } r^* \in [0, \hat{p}] \\ \infty, & \text{otherwise} \end{cases} \tag{19}$$

[26] Assuming that demand is constant in p means that as price increases, revenues $R(p)$ and $R^Y(p)$ necessarily increase. Moreover since:

$$z'(a(p)) = \frac{dz}{dp} = \frac{2a}{z}a' \geq 0 \tag{17}$$

, then $\frac{\lambda}{N(1-\lambda)}R^O(b(r)) + (1-\lambda)R^Y(b(r)) - z(b(r))$ is increasing in p, since $a'(p) \geq 0$

[27] Proof of Lemma 3 in ? showing that $\frac{\partial H(r, s)}{\partial r} > 0$ for all $r \le \hat{p}$.

B.3.3 Uniqueness of equilibrium

Proof. A sufficient condition for the uniqueness of the solution, r^{*}, is that H is strictly monotonic in r, in this case increasing; or equivalently $\partial H/\partial r > 0$. Using the Leibniz rule we obtain:

$$\frac{\partial H}{\partial r} = D(r) - \int_{b(r^{*})}^{r} D(p)\frac{\partial F(p)}{\partial r}dp$$

If $D(p)\frac{\partial F(p)}{\partial r} \geq -D(r)\frac{\partial F(p)}{\partial p}$:

$$\frac{\partial H}{\partial r} = D(r) - \int_{b(r^{*})}^{r} D(p)\frac{\partial F(p)}{\partial r}dp \geq D(r) + \int_{b(r^{*})}^{r} D(r)\frac{\partial F(p)}{\partial p}dp = D(r)\left(1 - \int_{b(r^{*})}^{r} f(p)dp\right) = 0$$

We will now check if this sufficient condition is fulfilled.

$$f(p;r) = \frac{\partial F}{\partial p} = \frac{1}{N-1}\left[\frac{\lambda}{N}\frac{R(r)-R(p)}{(1-\lambda)R^{Y}(p)} + \frac{z(r)-z(p)}{(1-\lambda)R^{Y}(p)}\right]^{\frac{2-N}{N-1}}\left[-\frac{\lambda}{N}\frac{(R(r)-R(p))R^{1I}(p)+R^{1}(p)R^{Y}(p)}{((1-\lambda)R^{Y}(p))^{2}} - \frac{(z(r)-z(p))R^{1I}(p)+z^{1}(p)R^{Y}(p)}{((1-\lambda)R^{Y}(p))^{2}}\right]$$

$$\frac{\partial F}{\partial r} = \frac{1}{N-1}\left[\frac{\lambda}{N}\frac{R(r)-R(p)}{(1-\lambda)R^{Y}(p)} + \frac{z(r)-z(p)}{(1-\lambda)R^{Y}(p)}\right]^{\frac{2-N}{N-1}}\left[\frac{\lambda}{N}\frac{R^{1}(r)}{(1-\lambda)R^{Y}(p)} + \frac{z^{1}(r)}{(1-\lambda)R^{Y}(p)}\right]$$

We combine the partial derivatives of the CDF of prices with respect to p and r, and obtain:

$$\frac{\frac{\partial F}{\partial r}D(p)}{-\frac{\partial F}{\partial p}D(r)} = \frac{\frac{1-(1-\,)}{N}R^{1}(r) + z^{1}(r)}{\left(\frac{\,}{N}(R(r)-R(p)) + (z(r)-z(p))\right)\frac{R^{1I}(p)}{R^{Y}(p)} + \frac{\,}{N}R^{1}(p) + z^{1}(p)}\frac{D(p)}{D(r)}$$

Since: $\pi = \frac{\,}{N}R(r) + z(r)$, we can rewrite the denominator:

$$\frac{\frac{\partial F}{\partial r}D(p)}{-\frac{\partial F}{\partial p}D(r)} = \frac{\frac{\,}{N}R^{1}(r) + z^{1}(r)}{\left(\pi - \frac{\,}{N}R(p) - z(p)\right)\frac{R^{1I}(p)}{R^{Y}(p)} + \frac{\,}{N}R^{1}(p) + z^{1}(p)}\frac{D(p)}{D(r)}$$

Since: $\pi = \frac{\,}{N}R(p) + (1-\lambda)(1-F(p))^{N-1}R^{Y}(p) + z(p)$, we can rewrite the denominator:

$$\frac{\frac{\partial F}{\partial r}D(p)}{-\frac{\partial F}{\partial p}D(r)} = \frac{\frac{\,}{N}R^{1}(r) + z^{1}(r)}{[(1-\lambda)(1-F(p))^{N-1}R^{Y}(p)]\frac{R^{1I}(p)}{R^{Y}(p)} + \frac{\,}{N}R^{1}(p) + z^{1}(p)}\frac{D(p)}{D(r)}$$

$$= \frac{\frac{\,}{N}R^{1}(r) + z^{1}(r)}{(1-\lambda)(1-F(p))^{N-1}R^{1I}(p) + \frac{\,}{N}R^{1}(p) + z^{1}(p)}\frac{D(p)}{D(r)}$$

From the equilibrium condition: $\partial\pi/\partial p = \lambda/N R^1(p) - z^1(p) - (1-\lambda)(1-F(p))^{N-1} R^{1\prime}(p) - (1-\lambda)(N-1) f(p)(1-F(p))^{N-2} R^Y(p)$, as well as: $\partial\pi/\partial p|_{p=r} = \lambda/N R^1(r) - z^1(r)$, which implies:

$$\frac{\frac{\partial F}{\partial r} D(p)}{\frac{\partial F}{\partial p} D(r)} = \frac{\partial\pi/\partial p|_{p=r}}{\partial\pi/\partial p + \underbrace{(1-\lambda)(N-1)f(p)(1-F(p))^{N-2}R^Y(p)}_{>0}} \frac{D(p)}{D(r)}$$

If the demand for uninformed is constant, then we obtain the required inequality:

$$\frac{\frac{\partial F}{\partial r} D(p)}{\frac{\partial F}{\partial p} D(r)} = \frac{\partial\pi/\partial p|_{p=r}}{\partial\pi/\partial p + \underbrace{(1-\lambda)(N-1)f(p)(1-F(p))^{N-2}R^Y(p)}_{>0}} > 1$$

□

B.4 Proof of proposition 3.7

Proof. We decompose the derivative of the investment strategy with respect to the share of older consumers the following way:

$$\frac{\partial a}{\partial \pi}\left(\frac{\partial \pi}{\partial \lambda} + \frac{\partial \pi}{\partial r}\frac{\partial r}{\partial \lambda}\right) = \frac{\partial a}{\partial \pi}\left(\frac{\partial \pi}{\partial \lambda}\Big|_{a} + \frac{\partial \pi}{\partial r}\Big|_{a}\frac{\partial r}{\partial \lambda}\Big|_{a;}\right) \tag{20}$$

- $\frac{\partial a}{\partial} = \frac{z}{2}\left(\left(\frac{p}{w} - \bar{a}\right)^2 - \bar{z}\pi\right)^{-\frac{1}{2}} > 0$
- $\frac{\partial}{\partial}\Big|_a = \frac{R^O(r)}{N} > 0$
- $\frac{\partial}{\partial r}\Big|_a = \frac{}{N}\frac{\partial R^O(r)}{\partial r}\Big|_a - \frac{}{N} D^M > 0$
- $\frac{\partial r}{\partial}\Big|_{a;} = -\frac{\frac{\partial H}{\partial \lambda}}{\frac{\partial H}{\partial r}} > 0$ since

  $\frac{\partial H}{\partial}\Big|_{a;\,;r} = \int_{b(r)}^{r} D^M \frac{\partial F}{\partial}\Big|_{a;\,;r} dp < 0$ where

  $\frac{\partial F}{\partial}\Big|_{a;\,;r} = \frac{1}{N-1}\left(\right)^{\frac{1}{N-1}-1}\left(\frac{pR^O(r) - R^O(p)(N R^Y(p))}{(p)^2} - \frac{z(r) - z(p)}{R^Y(1 \;)^2}\right) < 0$ and

  $\frac{\partial H}{\partial r} > 0$ from the proof of uniqueness.

which implies that the equation 6 is positive. □

B.5 Proof if proposition 3.8

Proof. The expected research intensity is given by $E(a) = \int_{b(r)}^{r} a(p)dF(p)$. To simplify notations, we simply write the policy function without their arguments. Integrating by parts, we get:

$$E(a) = a(r) - \int_{b(r)}^{r} a_p(p)F(p)\, dp \tag{21}$$

$$E(a) = a(r,\pi) - \int_{b(r)}^{r} a'(p;\pi)\, F(p;\lambda,r)\,dp \tag{22}$$

where $a'(p) = \partial a / \partial p$. We use under scripts to indicate the parameter or the argument we are deriving with respect to. Taking the derivative with respect to λ, and using the Leibniz rule, we get:

$$\frac{d\,E(a)}{d\lambda} = \underbrace{\frac{\partial a}{\partial r}\frac{\partial r}{\partial \lambda}}_{a} + \underbrace{\frac{\partial a}{\partial \pi}\left(\frac{\partial \pi}{\partial \lambda} + \frac{\partial \pi}{\partial r}\frac{\partial r}{\partial \lambda}\right)}_{b} - \underbrace{\frac{dr}{d\lambda}\, a_p|_{p=r}\, F(p)|_{p=r}}_{c} + \underbrace{\frac{db(r)}{d\lambda}\, a_p|_{p=b(r)}\, F(p)_{p=b(r)}}_{d}$$

$$- \int_{b(r)}^{r} \underbrace{F(p)\frac{\partial a_p}{\partial \pi}\left(\frac{\partial \pi}{\partial \lambda} + \frac{\partial \pi}{\partial r}\frac{\partial r}{\partial \lambda}\right)}_{e} + \underbrace{a_p\left(\frac{\partial F}{\partial r}\frac{\partial r}{\partial \lambda} + \frac{\partial F}{\partial \lambda}\right)}_{f} \tag{23}$$

- (a) represents the price effect for $p = r$;
- (b) represents the profit effect for $p = r$;
- (c) and (d) take into account the change in the boundaries of the price distribution;
- (e) represents the profit effect (the second term in the brackets) given the price distribution $F(p)$;
- (f) represents the change in the expected research intensity coming from changes in the shape of the price distribution.

We notice that:

- (a) simplifies with (c) since $F(p)|_{p=r} = 1$;
- (d) is equal to 0 since $F(p)|_{p=b(r)} = 0$

We can, therefore, rewrite:

$$\frac{d\,E(a)}{d\lambda} = \underbrace{\frac{\partial a}{\partial \pi}\left(\frac{\partial \pi}{\partial \lambda} + \frac{\partial \pi}{\partial r}\frac{\partial r}{\partial \lambda}\right)_{p=r} - \int_{b(r)}^{r} F(p)\frac{\partial a_p}{\partial \pi}\left(\frac{\partial \pi}{\partial \lambda} + \frac{\partial \pi}{\partial r}\frac{\partial r}{\partial \lambda}\right) dp}_{\text{aggregate profit effect}}$$

$$- \underbrace{\int_{b(r)}^{r} a_p\left(\frac{\partial F}{\partial r}\frac{\partial r}{\partial \lambda} + \frac{\partial F}{\partial \lambda}\right) dp}_{\text{distribution effect}} \tag{24}$$

where the *profit effect* is unambiguously positive since the first term is positive as shown by equation (6), and $\frac{\partial a_p}{\partial} \quad 0$.[28] The *distribution effect* is instead positive since $a_p \quad 0$, $\frac{\partial F}{\partial r} \quad 0$, $\frac{\partial r}{\partial} < 0$, and $\frac{\partial F}{\partial} \quad 0$. □

B.6 Numerical Solution

We solve the theoretical model numerically using the following steps:

1. Guess $r \in [p^{comp}, p^{mon}]$, with p^{comp} the equilibrium price if the market is competitive and p^{mon} the equilibrium price if the market is monopolistic.

2. Compute π the equilibrium profits

3. Compute $b(r)$ the lower bound of the CDF $F(p)$

4. Compute the CDF on the interval $[b(r), r]$

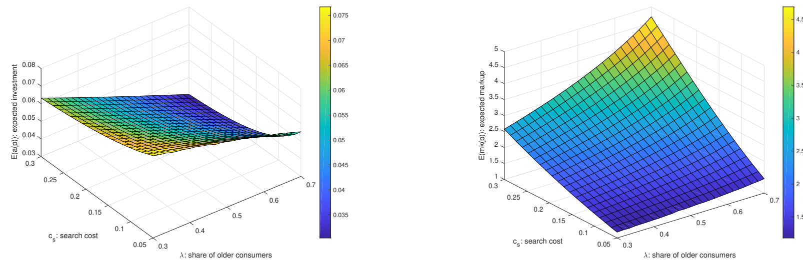
5. Find r s.t. $H \quad 0$

B.7 Calibration robustness checks

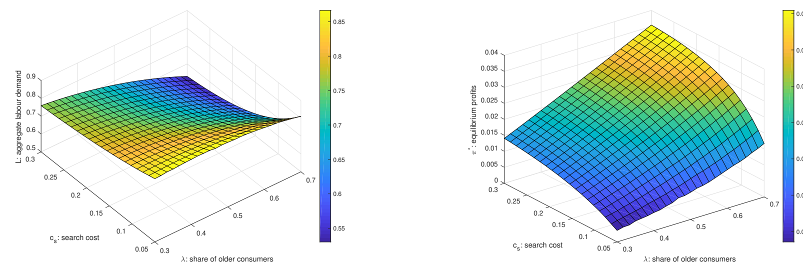
We plot the main aggregates from the theoretical model and check how the effect of aging is consistent across a wide range of calibration of the model.

---

[28]
$$a_{p\pi} \quad \frac{\partial a_p}{\partial} \quad \frac{\bar{z}}{2 \; w}\left(\frac{p}{w} \quad \bar{a}\right) \left(\frac{p}{w} \quad \bar{a}\right)^2 \quad \bar{z} \quad ^{\frac{3}{2}} \quad 0 \qquad (25)$$

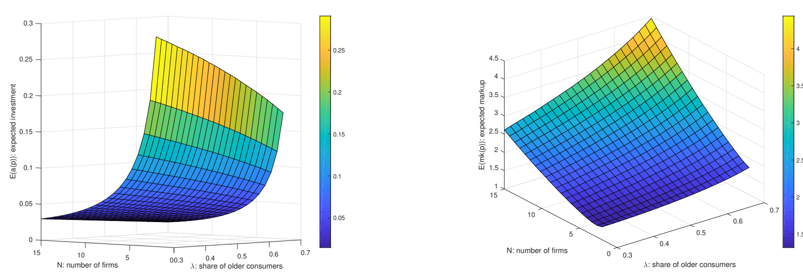


(a) Expected Investment (b) Expected Markup

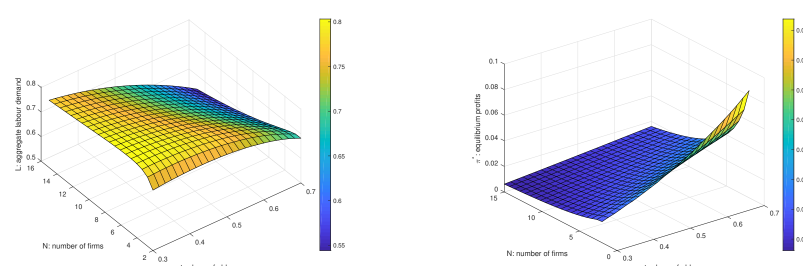


(c) Aggegate Labour Demand (d) Profit

Figure (8) Robustness check on the search cost for older buyers s. Aggregates from the numerical solution of the model calibrated as described in the main text.

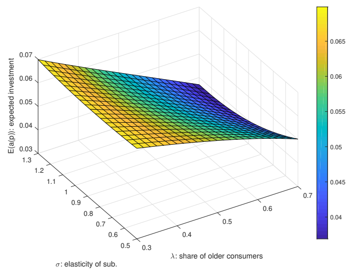


(a) Expected Investment (b) Expected Markup

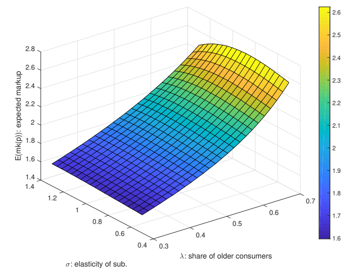


(c) Aggegate Labour Demand (d) Profit

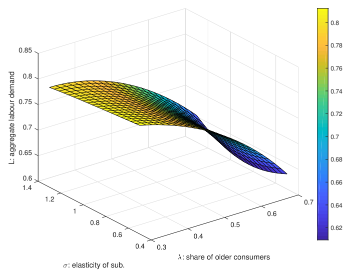
Figure (9) Robustness check on the number of firms N. Aggregates from the numerical solution of the model calibrated as described in the main text.



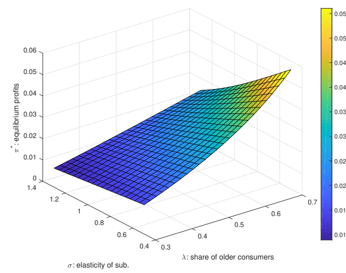
(a) Expected Investment



(b) Expected Markup

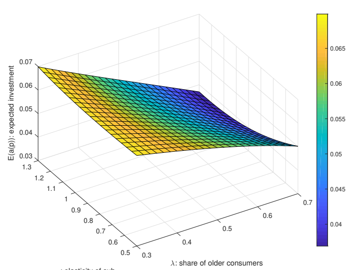


(c) Aggegate Labour Demand

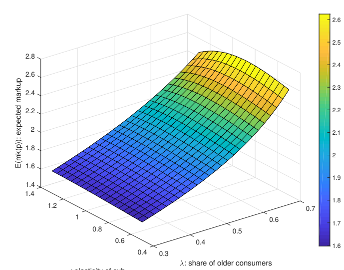


(d) Profit

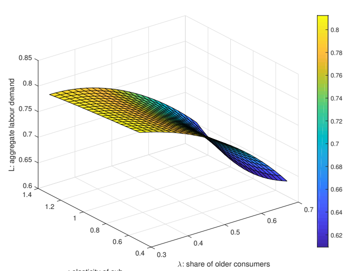
Figure (10) Robustness check on the demand function elasticity σ. Aggregates from the numerical solution of the model calibrated as described in the main text.



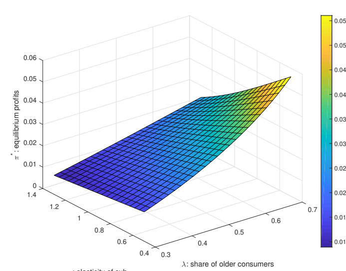
(a) Expected Investment



(b) Expected Markup

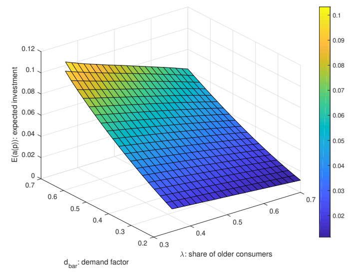


(c) Aggegate Labour Demand

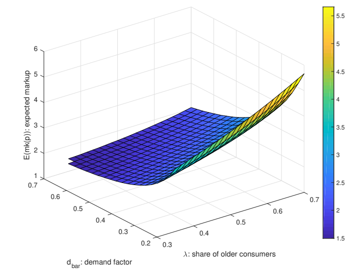


(d) Profit

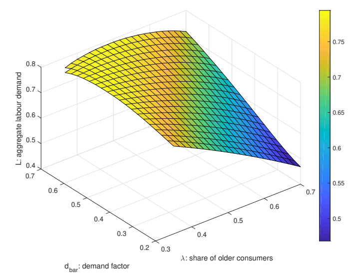
Figure (11) Robustness check on the demand function elasticity σ. Aggregates from the numerical solution of the model calibrated as described in the main text.



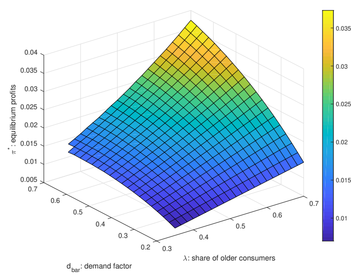
(a) Expected Investment



(b) Expected Markup

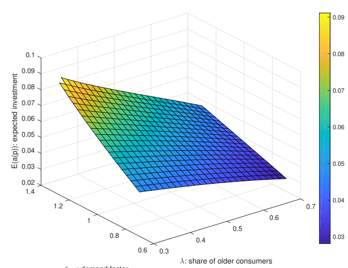


(c) Aggegate Labour Demand

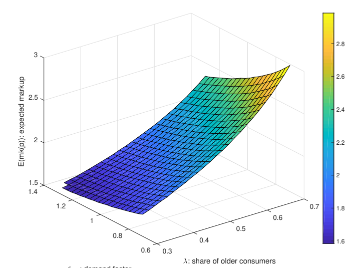


(d) Profit

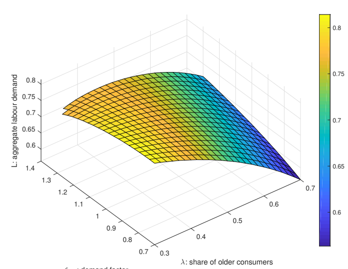
Figure (12) Robustness check on the demand function parameter $\bar{d}$. Aggregates from the numerical solution of the model calibrated as described in the main text.



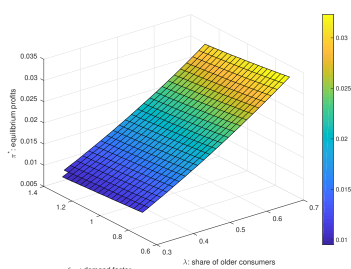
(a) Expected Investment



(b) Expected Markup



(c) Aggegate Labour Demand



(d) Profit

Figure (13) Robustness check on the demand function heterogeneity parameter ξ. Aggregates from the numerical solution of the model calibrated as described in the main text.

C Macro Model

C.1 Derivation of the sector-specific demand

The consumer's problem is to maximize utility subject to the budget constraint:

$$\max_{c_s^j} \left(\int_{s \in \mathcal{S}} c_s^{j\,\frac{\sigma-1}{\sigma}} ds \right)^{\frac{\sigma}{\sigma-1}} \quad \text{s.t.} \quad \int_{s \in \mathcal{S}} p^j(s)\, c_s^j ds \leq R$$

The first order conditions for a good j:

$$\left(\int_{s \in \mathcal{S}} c_s^{j\,\frac{\sigma-1}{\sigma}} ds \right)^{\frac{1}{\sigma-1}} c_s^{j\,-\frac{1}{\sigma}} - \lambda p^j(s) = 0$$

We combine the FOCs for two distinct products s and t:

$$c_s^j = \frac{p^j(s)}{p^j(t)} c_t^j$$

Replacing c_s^j in the budget constraint, we obtain the expenditure in a given market t:

$$p^j(t)\, c_t^j = R^j \left(\frac{p(t)}{P^j} \right)^{1}$$

Using the condition $R^j = P^j Q^j$, we then obtain the demand from each product:

$$c_t^j = D(p^j(t)) = C^j \frac{p^j(t)}{P^j}$$

C.2 Numerical solution

We solve the general equilibrium model using the following algorithm:

1. Outer-loop: guess w
2. Inner-loop: guess the aggregators
3. Solve model following the steps presented in appendix B.6 and verify that the aggegators are consistent; otherwise, update the guesses for aggregators until consistent
4. Outer-loop: compute the aggegate labour demand and verify if consistent with the aggegate labour supply ($ALS = 1$); otherwise, update guess for w until consistent.