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Abstract

The Great Recession in Continental Europe sparked a Great Unemployment Divergence, led by a catastrophic job destruction in some countries. To study such phenomena, I resort to a Bewley-Hugget-Aiyagari incompletemarkets model with an indirect search frictional labour market as in Krusell et al. (2010). I complement it with: (a) financial constraints à la Kiyotaki and Moore (1997), (b) labour market dualism in employment protection, and (c) downward real wage rigidity, and I show that a catastrophic job destruction phenomena can take place in the presence of medium to severe aggregate negative and temporary productivity shocks, leading to a costly process of destruction of inter-temporally viable existing jobs - *a last dance phenomena*. For that outcome, the initial leverage positions of firms and the management of liquidity along the downturn are critical, and can justify divergent paths in unemployment. In a calibration to portray the Portuguese economy, no intervention would imply an elasticity between the initial downturn and the unemployment rate of close to 1 for initial downturns of more than 5-6 percent. In such context, several balanced-budget policies targeting the liquidity in permanent contracts are considered, and they prove to be capable to alleviate sizably the job destruction mechanism.

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1. Introduction

The Great Recession in the European Union was defined by its *Great Unemployment Divergence*, particularly between neighbouring countries as France, Portugal and Spain, or Italy and Germany (see Boeri and Jimeno (2015) for an overview). Despite similar starting points, and often identical institutional designs (see Botero et al. (2004)), some sailed through without significant unemployment spikes, while others recorded *catastrophic job destruction spells*, defined by a strong spike in job destruction that led to a significant rise in unemployment, as presented by Carneiro et al. (2014). What could justify such divergent paths?

A standard agency problem between borrowers and lenders imply the existence of credit constraints, which weaken the liquidity of firms in the presence of a negative MIT type shock (see Gertler and Gilchrist, 2018 for a broad overview). Even from an aggregate perspective, Holmstrom and Tirole (2011) highlights that the constant need of firms to manage and forecast liquidity and the *limited pledgeability* of the entrepreneur, as only part of their income or wealth can be borrowed upon or used as collateral, provides a conceptual framework where liquidity provision would be insufficient in the presence of aggregate liquidity shocks. Empirically, among a plethora of studies in the context of the Great Recession, Benmelech et al. (2011), Bentolila et al. (2013), Berg (2016) and Chodorow-Reich (2014) shows a significant quantitative importance of financial constraints in the employment decisions of firms, and Boeri et al. (2012) reports a clear link between firm leverage and job destruction. In the latter, more leveraged firms may see their liquidity being suddenly called back by the lender, thus affecting their ability to run and manage existing jobs, forcing them to destroy some. In toto, if the external financing channel shuts, and the shareholders do not inject further equity, firms can only turn to the adjustment of input utilization, and their prices, to correct liquidity shortages, and alleviate the risk of failure.

To address the ability of firms to resort to this *internal financing channel*, one needs to define the institutional framework of the labour market and the financing of the economy, as those shape the magnitude and the intensive-extensive margin calibration of labour and capital adjustments. Regarding the labour market, I define the continental labour market institution, where there is downward real wage rigidity, and following the insights of Bentolila et al. (2013) and Boeri and Jimeno (2015), contract duality, with permanent jobs enjoying higher employment protection, expressed by the existence of firing taxes and higher predicted job tenures. Regarding the financing of the economy, while studying the shape of policies devised to alter it, such as monetary policy, is worth pursuing, in this study I focus on the role of labour market policies. Accordingly, a stable and comparable environment of external financing provides an orderly benchmark, and thus, I keep the external financing of firms constant as percentage of assets.

Altogether, I propose a Bewley-Huggett-Aiyagari incomplete-markets setting with an indirect search frictional labour market type model, in the spirit of Krusell et al. (2010).¹ Then, I extend it to display: (a)

¹See de Almeida Vilares and Reis (2021) for an empirical analysis of the suitability of a search and matching structure of the wage setting for Portugal.

increasing productivity with wealth, as proposed by Eeckhout and Sepahsalari (2020);² (b) financial constraints on the firm side in the spirit of Kiyotaki and Moore (1997);³ (c) the referred permanent-temporary contract duality; and (d) the downward real wage rigidity, in the spirit of Schmitt-Grohé and Uribe (2016). The heterogeneity of the model, coupled with its realistic wage setting, the considered labour market institutions, and the analysis of the workers' savings problem allows for a just sufficiently rich environment to analyse what conditions spark a catastrophic burst of employment in the presence of stable financing of the economy, and which are the most targeted workers.

The first contribution of this paper is to clearly show that a model of this class is capable to display catastrophic job destruction spells, for a model reasonably calibrated to match the Portuguese economy. Indeed, for medium to severe aggregate temporary downturns in productivity, above 5-6 percent f the stationary equilibrium levels, the unemployment rate responds on a 1-to-1 basis to the initial productivity shock. This process is led by job destruction of existing jobs, leading to a reallocation process that is more costly than ensuring the continuity of most of the job assignments, in what I coin as a *last dance phenomena*, as most of those jobs are inter-temporally viable.⁴

The second contribution of this paper is to clearly establish a comparison between the downward real wage rigidity environment and a benchmark wage flexibility environment, while preserving the other characteristics of the model. This provides a useful comparison, where one can decouple the job creation weakening in the outcome of the productivity downturn, and the job destruction mechanism. From this comparison, this paper shows that the job destruction mechanism becomes particularly acute for medium to severe downturns and coincides with the sizable spike in unemployment rate levels the model prescribes. Consequently, it highlights the initial levels of leverage and its the management along the adjustment path as decisive in determining the path of the unemployment rate, and potentially in explaining the diverging paths undertaken by apparently similar labour markets.

The third contribution of the paper is to explore a set of budget balanced labour market policies focused on permanent contracts, and their ability to alleviate the liquidity constraint, and thus the job destruction phenomena. Concretely, I consider: (a) a cash transfer to firms balanced with the saved unemployment benefits' proceeds; (b) a mandated reduction of wages; (c) a cash transfer to firms balanced with firms' future payments; and (d) temporary wage flexibility. Implemented for a year, with a quarter of implementation lag, I found that those policies can alleviate job destruction in the presence of aggregate productivity shocks, thus reducing the unemployment peak in the trough. Moreover, those improve worker's welfare along the worker's wealth

 $^{^{2}}$ See Bernstein et al. (2021) for an empirical analysis of the effect of negative wealth shocks on the productivity of U.S. innovative workers. It confirms the wealth-productivity relationship proposed.

 $^{^{3}}$ Holmstrom and Tirole (2011) focuses in the ability of the *private sector* to generate enough liquidity to fund aggregate liquidity needs to implement a second best production plan. Here we follow Kiyotaki and Moore (1997) in defining financial constraints at firm level.

 $^{^{4}}$ The Last Dance corresponds to a documentary describing the decision of the Chicago Bulls to dismantle its current winning team and undergo into a deep restructure. It may find parallel in the phenomena to dissolve inter-temporally viable firms due to instantaneous leverage concerns.

distribution and preserve higher levels of capital stocks, while prescribing substantially different wage profiles along the adjustment paths. Consequently, the paper puts forward a wide set of policies that could be adopted in the presence of substantial aggregate downturns.

The paper is organized as follows. Section 2 presents an analysis of the Continental European Labour market framework and some stylized facts on the divergent paths among the economies sharing such taxonomy. Section 3 takes a closer look on the stylized facts of the Portuguese labour market adjustment. Section 4 presents the model. Section 5 explains the calibration method and the most relevant stationary equilibrium results. Section 6 presents the transitional dynamics of the model following a aggregate productivity downturn. Section 7 relaxes the wage rigidity component and compares the transitional dynamics with the benchmark economy. Section 8 prescribes several labour market policies toward permanent contracts and their ability to alleviate job destruction. Section 9 concludes.

2. Continental European labour market institutions and the Great Recession

The cross country differences in labour market institutions have been at the core of the discussion of economic performance for decades. Traditionally, the Continental European countries have significant higher levels of labour market regulations and more generous labour policies than the U.S. or the United Kingdom, ranging from higher levels of employment protection and more centralization of collective bargaining to more generous unemployment benefit systems and a more frequent use of active labour market policies.

Even within Continental Europe, there is a high institutional heterogeneity. Boeri (2011) singles out the *Scandinavian*, the *Continental* and the *Mediterranean* clusters as the sensible partition. The first is characterized by a strong emphasis on fiscal policy rather than employment protection, thus resorting to a generous unemployment benefit system, substantial use of active labour market policies and significant tax wedges instead of preventing unemployment spells. The continental cluster, formed by France, Germany, Austria and Belgium, presents a more balanced use of both types of instruments, resorting jointly to high levels of employment protection and a substantial system of unemployment benefits and active labour market policies. Finally, the Mediterranean concept of Portugal, Spain, Italy and Greece relies fundamentally on very substantial employment protection laws and wage rigidity, while the unemployment insurance is relatively low.

Despite different policy mixes, a notable common feature of every Continental European labour market is duality (see Boeri and van Ours (2013) and Boeri (2011) for recent overviews on the topic). In every geography coexists permanent and temporary contracts, where the former are highly protected from dismissal, have a clearly defined career path, and enjoy a sizable wage premium. The latter have hardly any protection, and are designed to fill temporary necessities of firms. Table 1 presents a snapshot of duality indicators, before and after the Great Recession. Beyond its across board prevalence, duality seems more intense among the Southern European economies, where employment protection of permanent contracts is particularly stringent, the use of temporary contracts is frequent and the wage premia is sizable.

Countries:	Str: Dismissa	ictness of EPL ls of Perm. Workers	Strict Hiring 7	ness of EPL Temp. Workers	Temporary ers employees (%)		Wage Premia Perm. Contracts (%)
	2007	2015	2007	2015	2007	2015	2010
Portugal	4.42	3.14	2.56	1.81	17.1	18.3	13.8
Spain	2.36	1.96	3.00	2.47	25.2	20.7	10.5
Greece	3.13	2.45	2.75	2.25	7.1	7.8	-
Italy	3.02	2.93	2.00	1.63	9.5	10.6	11.4
France	2.71	2.50	3.13	3.13	10.8	13.3	7.5
Germany	2.60	2.60	1.00	1.13	10.6	10.1	17.3
Austria	2.29	2.29	1.31	1.31	4.5	5.4	10.1
Belgium	1.69	2.07	2.25	2.06	6.8	7.3	8.3
Netherlands	3.30	3.24	0.94	0.94	12.9	14.0	16.5
Finland	2.08	2.08	1.56	1.56	12.7	12.1	9.4
Sweden	2.45	2.45	1.44	0.81	13.5	13.5	9.8
Denmark	1.47	1.53	1.38	1.63	7.4	6.9	3.4
U.S.	0.09	0.09	0.25	0.25	-	-	-
U.K.	1.51	1.35	0.38	0.38	4.4	4.6	12.1

Table 1: Labour market dualism in European Continental labour markets

Notes: On columns (1) and (2), the higher the index indicator the more stringent it is to dismiss permanent workers or resort to hiring temporary workers respectively. The wage premia for temporary contracts is the unexplained component of an Oaxaca-Blindler decomposition using data of 2010 European Structure of Earnings Survey, as estimated by Silva and Turrini (2015). *Sources: OECD* for employment protection legislation indexes (EPL). *Eurostat* for percentage of temporary workers as percentage of all employees. Silva and Turrini (2015) for the wage premia of temporary contracts.

The institutional framework of the market is particularly scrutinized during downturns. Among those, the Great Recession triggered by the financial turnoil in 2007-2008 sparked a severe recession across the board. As showed in figure 1, the initial shock was broadly symmetric, while the aftermath was influenced by an asymmetric sovereign debt crisis, which particularly affected the Southern European countries. Altogether, one of the most distinctive features of the Great Recession in Continental Europe was its *unbearable unemployment divergence* which cannot be explained by the size or the specific nature of the shock, or even by region-sector idiosyncrasies, as noted by Boeri and Jimeno (2015). Bentolila et al. (2012)) in a comparison between the Spanish and the French labour market highlights the role of labour market duality in mounting two remarkably different unemployment realities across a single border, reinforcing the evidence of Boeri and Garibaldi (2007), Costain et al. (2010) and Sala et al. (2012) that dual labour markets are more prone to employment volatility.



Figure 1: Real GDP growth and Unemployment across Continental Europe.

 $Sources:\ Eurostat.$

In a complementary view, the pre-existing cross country financial asymmetries both on the financial standing of firms and on the financial system resilience also played significant roles. Resorting to the U.S., Duygan-Bump et al. (2015) established the link between higher financing needs and unemployment particularly for small firms. For Italy, Barone et al. (2016) negatively links outstanding loans and credit supply and find a significant effect of a credit crunch on productivity and employment, particularly for small firms and in areas dependent of external financing. Bentolila et al. (2013) refers to the Spanish credit crunch and confirm the existence of significant job losses on firms that were particularly linked to frailer banks. Identical findings are found in Chodorow-Reich (2014) for the US. For Portugal, Blattner et al. (2017) extends this evidence by presenting the role of weaker financial institutions in misallocating resources towards stressed firms, rather than recognizing further losses.

In this context, figure 2 presents key stylized facts on the financial standing of firms in several Continental European economies. First, there is a high asymmetry in the leverage of firms at the start of the Great Recession, with southern European economies presenting significant higher leverage levels, notably Portugal, Italy, Greece or even Spain. Second, beyond preexisting spreads across countries, some didn't witness the same sizable reduction in the cost of borrowing for non financial corporations as the majority of the Eurozone. Thus, particularly for those countries that witnessed a severe repercussion of the sovereign debt crisis component of the Great Recession, namely Portugal, Greece or Ireland, the costs of borrowing didn't witness a significant and durable reduction, thus allegedly muting the effect of a potential financing ease along the downturn on the liquidity of firms. Altogether, for an analysis of the Portuguese case, such stylized facts are compatible with a stable financing environment as defined in our assumption framework.⁵

 $^{^{5}}$ See Carneiro et al. (2014) for an empirical analysis of the influence of the credit channel in the job destruction process.





Panel A: Debt to financial assets ratio of non financial corporations

Panel B: Annual non financial corporations cost of borrowing for new business



Sources: OECD and ECB.

3. Stylized facts of the labour market adjustment in Portugal throughout the Great Recession

The analysis of this paper will adopt a calibration that is focused in the Portuguese case. Beyond the accessibility to administrative datasets to inform the calibration, another relevant advantage of calibrating the model to Portugal resides in the severe magnitude of the Great Recession shock to the economy and the labour market (see Blanchard (2007) and Blanchard and Portugal (2017) for context macroeconomic analysis of the Portuguese economy). In detail, the Portuguese economy was severely hit by two germane crisis, namely the

2008 Financial crisis, and the subsequent European debt crisis. By 2013 the Portuguese economy was in such a frail standing that a significant loan from the European institutions and IMF, coupled with an economic conditionality program was needed. From 2008 to just before the start of the *Financial Assistance Program*, the Portuguese economy recorded a 7 percent fall in real gdp.



Figure 3: The real GDP growth and the unemployment rate in Portugal.

Sources: INE.

From a labour market perspective, as displayed in figure 3, the most notable consequence was the *catastrophic job destruction*, as coined by Carneiro et al. (2014), that saw the unemployment rate jump from around 8 percent in 2008 to 16 percent in 2013, displaying an unemployment rate growth that surpassed the real gdp fall in the same period. In inspecting closely the drivers of such dramatic unemployment surge, the market was severely affected in its flows. Leading the effects, figure 4 presents the displacement caused by the closing of firms, many of which bankrupted over the period, or simply closed due to striking financial constraints. Simultaneously, the creation of new firms more than halved in the trough in 2012, confirming a significant adverse conditions on firm survival and creation. For the staying firms over the period, the job flows are significantly reduced, with the reduction on job displacement not being enough to overturn the supra-cited adverse dynamics.

Beyond flows, if one assesses the behaviour of wages, the existence of some degree of wage rigidity is clear, as seen in figure 5. In the acute downturn moments, the base wage freezes significantly spiked to record highs of around 70 percent of the existing labour contracts in 2012 and 2013, the total wage freezes increased to levels close to 20 percent, and the number of minimum wage recipients continuously increased.⁶ If one measures the fall in real wages, it is around 7.5 percent for median wages, having a monotonic increasing relationship with the percentile of the 2007 initial wage distribution.

 $^{^{6}}$ The difference between total and base wages regards wage supplements, which typically may include regular bonuses, meals subsidies, overtime compensation, and other fringe benefits.

Figure 4: The evolution of job flows in Portugal.



Sources: Quadros de Pessoal.

Figure 5: The evolution of the wage distribution in Portugal.



Notes: The percentiles of wages consider real wages. The year-on-year wage freeze calculations consider nominal wages. *Sources: Quadros de Pessoal.*

In a general sketch of the Portuguese adjustment from 2008 too 2013, the Great Recession meant a downturn that reached around 7 percent, leading to a significant burst of employment, with the unemployment rate doubling to around 16 percent, in a process significantly led by the reduction of flows in surviving firms, the reduction in the creation of firms and a significant firm destruction. In the process wages displayed some degree of wage rigidity, even though real wages sizably adjusted, with a fall of 7,5 percent of median wages. Consequently, compositional effects played a significant role. A model produced to describe the adjustment should be capable to mimic these findings.

4. The model

Population and technology. Time is continuous. There is a continuum of consumers in the economy, with measure 1, whom have standard time-additive preferences with discount factor ρ . The consumers do not value leisure and they may be either unemployed or employed. When employed, their contract may be either permanent (P) or temporary (T).

Production is decentralized. There is a large mass of potential single job firms, which operate in a perfectly competitive environment. Each firm produces z(a, j, t)F[k(a, j, t)], with $j \in \{P, T\}$ corresponding to the type of contract. k(a, j, t) is the capital stock used by the worker and z(a, j, t) represents the productivity level of the match, which is assumed to depend positively on the level of the worker's assets - a, and it is heterogeneous by types of contracts.⁷ $F(\cdot)$ is increasing and strictly concave. Capital depreciates geometrically at rate δ . The output is either consumed, invested in further capital or in vacancy creation, as I will subsequently detail.

The frictional financial market. Workers are at risk of unemployment, which is assumed to be uninsurable. However, workers can save to smooth out the impact of such adverse shock. For that purpose, workers hold an heterogeneous amount of assets a, which they lend to a banking institution, and obtain a deposit rate r(t).

The financial system is composed by an investment institution and a banking institution. The former manages a portfolio composed by the equity of the mass of firms (Q(t)), whose flow dividends (D(t)) accrue from the different between the firms' flow profits and the costs with labour flows, namely the investment in job creation and the taxes of firing workers. The latter lends capital to firms, obtaining a lending rate $r_l(t)$, so that $r_l(t) - \delta = r(t)$. For parsimony purposes, we assume there is no seniority between loans and capital in case of firm insolvency. Therefore, the arbitrage condition in the allocation of the resources available to the financial system entails:

$$r_l(t) - \delta = \frac{Q(t) + D(t)}{Q(t)}.$$
(1)

The lending market is frictional. The investment institution may withdraw activity or declare bankruptcy of specific firms. Aware of such risk, the lender, in an attempt to insure itself, may establish lending limits or tighten the surveillance of firm's activity when the EBITDA flows of each firm, or prospective firms, are sufficiently low. Such enhanced surveillance affects the firms' decisions regarding hiring, promotion and/or dismissal of workers, as we will see. The EBITDA is defined by:

$$\xi(a, j, t) = z(a, j, t)F[k(a, j, t)] - w(a, j, t), \quad j \in \{T, P\},$$
(2)

where w(a, i, t) corresponds to the wage of a given worker with contract type *i*, which, as will be evident in this section, depends on the asset level of the worker.

On top of the resources obtained from the wealth of domestic consumers, the banking institution may may

 $^{^{7}}$ Intuitively, we imply that a wealthier worker is inherently more productive. In the context of direct search models, Eeckhout and Sepahsalari (2020) rationalizes an identical relationship.

obtain resources from abroad, under the same conditions as the domestic resources. Accordingly, ϑ corresponds to the ratio of domestically owned assets to total assets in the economy, and the stock of assets owned by domestic consumers is given by:

$$A = \vartheta [e_T(t)K_T(t) + e_P(t)K_P(t) + Q(t)], \tag{3}$$

where: (a) $e_T(t)$ and $e_P(t)$ are respectively the stock of temporary and permanent contract employed workers; (b) $K_T(t)$ corresponds to the aggregate capital in temporary contracts; and (c) $K_P(t)$ corresponds to the aggregate capital in permanent contracts. The aggregate capital by type of contract will be defined later. Further, we assume that external funding is stable, thus ϑ is constant.

The frictional labour market. The labour market is assumed to be frictional. Firms are required to post vacancies to hire workers, and pay a cost γ per vacancy posted. The flow of worker-firm meets is determined by a typical constant returns to scale aggregate matching function, M(u(t), v(t)), where u(t) is the measure of unemployed workers searching for a job, and v(t) is the measure of open vacancies. Correspondingly, $\theta(t) =$ v(t)/u(t) is the market tightness; $\theta(t)q[\theta(t)] = M(u(t), v(t))/v(t)$ represents the Poisson rate at which an unemployed worker meets a firm; $q[\theta(t)]$ is the Poisson rate at which a firm meets a candidate, per vacancy posted; and $\gamma v(t)$ corresponds to the vacancy costs of the investment institution.

At the hiring moment, any worker is hired on a temporary contract (T). Once the firm and the worker meet, the probability of a successful completion of the match, $h[\xi(a,T,t)]$, follows a logistic decreasing function $h(\cdot)$, which is dependent of the EBITDA of the newly formed match, $\xi(a,T,t)$. While on a temporary contract, the match may be dissolved with probability $\sigma_T[\xi(a,T,t)]$, where $\sigma_T(\cdot)$ corresponds to a logistic decreasing function. In the event of a dissolution it occurs costlessly, i.e. without any firing tax involved ($S_T = 0$).

Simultaneously, the temporary worker can be promoted into a permanent contract (P). Such event occurs with probability $\varphi[\xi(a, P, t)]$, where $\varphi(\cdot)$ is an increasing logistic function, and $\xi(a, P, t)$ corresponds to the EBITDA of the firm after the potential promotion of the worker. Once on a permanent contract, the match may be dissolved with probability $\sigma_P[\xi(a, P, t)]$, where $\sigma_P(\cdot)$ is a decreasing logistic function. However, contrary to a temporary contract, the dissolution entangles a positive firing tax - S_P - paid by the firm.

Notation note. For notation simplicity, in the remainder of this section, consider $j = \{P, T\}$ and: (a) $\ddot{z} = z(a, j, t)$; (b) $\ddot{k} = k(a, j, t)$; (c) $\ddot{k} = k(a, j, t)$; (d) $\ddot{\xi}_P = \xi(a, P, t)$ and (e) $\ddot{\xi}_T = \xi(a, T, t)$.

Consumers. Following the insight of Kaplan et al. (2018), beyond the wage, the worker is also entitled to a percentage of the flow profit of the firm, ψ , in a form of a *bonus*.⁸ Consequently:

$$\eta(a, j, t) = \psi \left[\ddot{z} F[\ddot{k}] - r_l(t) \ddot{k} - w(a, j, t) \right], \quad j \in \{T, P\}.$$
(4)

⁸Intuitively, in this model the term ψ controls the percentage of the firms' profits that are distributed as a lump sum to the population, versus the share that is distributed directly to the involved workers.

In a recursive form, the employed workers in temporary contracts choose their consumption by resorting to the following HJB equation and state-constraint boundary condition:

$$\rho W(a,T,t) = \max_{c} \left\{ u(\ddot{c}) + \partial_{a} W(a,T,t) \left[w(a,T,t) + \eta(a,T,t) + (r_{l}(t) - \delta)a - \ddot{c} \right] + \sigma_{T}(\ddot{\xi}_{T}) \left(W(a,U,t) - W(a,T,t) \right) + \varphi(\ddot{\xi}_{P}) \left(W(a,P,t) - W(a,T,t) \right) + \partial_{t} W(a,T,t) \right\}$$
(5)

subject to:

$$\partial_{a}W(\underline{\mathbf{a}}, P, t) \ge u' \bigg(w(\underline{\mathbf{a}}, P, t) + \eta(\underline{\mathbf{a}}, T, t) + (r_{l}(t) - \delta)\underline{\mathbf{a}} \bigg),$$

where: (a) the utility function, $u(\bullet)$, is assumed to be increasing and concave; (b) the W(a, U, t) corresponds to the value function of the unemployed worker; (c) W(a, T, t) represents the value function of an employed worker in a temporary contract; (d) W(a, P, t) corresponds to the value function of a worker in a permanent contract; and (e) <u>a</u> represents the lowest admissible level of wealth.

Beyond the proper structure of the career, which predicts no possible promotion for permanent workers, the fundamental difference between a temporary contract and a permanent contract is the existence of the positive firing tax S_P in case of a termination of the match of the latter type. Consequently, the recursive problem of permanent workers is given by:

$$\rho W(a, P, t) = \max_{c} \left\{ u(\ddot{c}) + \partial_{a} W(a, P, t) \left[w(a, P, t) + \eta(a, P, t) + (r_{l}(t) - \delta)a - \ddot{c} \right] + \sigma_{P}(\ddot{\xi}_{P}) \left[W(a, U, t) - W(a, P, t) \right] + \partial_{t} W(a, P, t) \right] \right\}$$

$$(6)$$

subject to:

$$\partial_a W(\underline{\mathbf{a}}, P, t) \ge u' \left(w(\underline{\mathbf{a}}, P, t) + \eta(\underline{\mathbf{a}}, P, t) + (r_l(t) - \delta)\underline{\mathbf{a}} \right)$$

Consistently with Krusell et al. (2010), the unemployed workers earn some exogenous income of level b. Their recursive problem is given by:

$$\rho W(a,U,t) = \max_{c} \left\{ u(\ddot{c}) + \partial_{a} W(a,U,t) \left[b + r(t)a - \ddot{c} \right] + \theta(t)q[\theta(t)]h(\ddot{\xi}_{T}) \left[W(a,T,t) - W(a,U,t) \right] + \partial_{t} W(a,U,t) \right] \right\}$$

subject to:

$$\partial_{a}W(\underline{\mathbf{a}},U,t) \ge u' \bigg(b + (r_{l}(t) - \delta)\underline{\mathbf{a}} \bigg).$$
(7)

The first order conditions of the described recursive problems corresponds to the respective policy functions

of consumption and savings, which due to symmetry can be represented as:

$$\ddot{c} = u'^{-1} \Big(\partial_a W(a, i, t) \Big),$$

$$\dot{a}(a, i, t) = y(a, i, t) + (r_l - \delta)a - \ddot{c}, \quad \text{where } y(a, i, t) = \begin{cases} w(a, P, t) + \eta(a, P, t) & \text{if } i = P \\ w(a, T, t) + \eta(a, T, t) & \text{if } i = T \\ b & \text{if } i = U \end{cases}$$

$$\tag{8}$$

The savings policy functions are then considered in Kolmogorov forward equations defined as:

$$\partial_t g(a, P, t) = -\partial_a [\dot{a}(a, P, t)g(a, P, t)] + \varphi(\ddot{\xi}_P)g(a, T, t) - \sigma_P(\ddot{\xi}_P)g(a, P, t),$$

$$\partial_t g(a, T, t) = -\partial_a [\dot{a}(a, T, t)g(a, T, t)] + h(\ddot{\xi}_T)\theta(t)q[\theta(t)]g(a, U, t) - [\sigma_T(\ddot{\xi}_T) + \varphi(\ddot{\xi}_P)]g(a, T, t),$$

$$\partial_t g(a, U, t) = -\partial_a [\dot{a}(a, U, t)g(a, U, t)] + \sigma_T(\ddot{\xi}_T)g(a, T, t) + \sigma_P(\ddot{\xi}_P)g(a, P, t) - h(\ddot{\xi}_T)\theta(t)q[\theta(t)]g(a, U, t).$$
(9)

Equation (9) describes the dynamics of the distribution of workers over states, g(a,i,t), where $i \in \{P,T,U\}$. Consequently, given the presented structure, the labour market transitions are described by the following system:

$$\begin{split} \dot{u} &= \int_{\underline{\mathbf{a}}}^{\infty} \sigma_{T}(\ddot{\xi}_{T})g(a,T,t)da + \int_{\underline{\mathbf{a}}}^{\infty} \sigma_{P}(\ddot{\xi}_{P})g(a,P,t)da - \int_{\underline{\mathbf{a}}}^{\infty} h(\ddot{\xi}_{T})\theta(t)q[\theta(t)]g(a,U,t)da \\ \dot{e}_{T} &= -\int_{\underline{\mathbf{a}}}^{\infty} [\sigma_{T}(\ddot{\xi}_{T}) + \varphi(\ddot{\xi}_{P})]g(a,T,t)da + \int_{\underline{\mathbf{a}}}^{\infty} h(\ddot{\xi}_{T})\theta(t)q[\theta(t)]g(a,U,t)da \\ \dot{e}_{P} &= -\int_{\underline{\mathbf{a}}}^{\infty} \sigma_{P}(\ddot{\xi}_{P})g(a,P,t)da + \int_{\underline{\mathbf{a}}}^{\infty} \varphi(\ddot{\xi}_{P})g(a,T,t)da \\ 1 &= \underbrace{\int_{\underline{\mathbf{a}}}^{\infty} g(a,T,t)da}_{\equiv e_{T}(t)} + \underbrace{\int_{\underline{\mathbf{a}}}^{\infty} g(a,P,t)da}_{\equiv e_{P}(t)} + \underbrace{\int_{\underline{\mathbf{a}}}^{\infty} g(a,U,t)da}_{\equiv u(t)} \end{split}$$
(10)

Firms. The firms are owned by the investment institution, rent capital from the bank, create jobs and produce. The firms maximize the present value of the profits, or identically, the value of equity, which is not zero due to the existence of labour market frictions. Further, to create jobs, firms are required to post vacancies and search for workers. The time-dependent HJB equation for the value of posting a vacancy, V(t), is given by:

$$(r_l(t) - \delta)V(t) = -\gamma + q(\theta) \int_{\underline{a}}^{\infty} J(a, T, t) \frac{g(a, U, t)}{u(t)} da,$$
(11)

where J(a, T, t) corresponds to the value of a temporary contract job performed by a worker with asset level a; and g(a, U, t)/u(t) represents the density function of the unemployed workers. Noteworthy, due to labour market frictions, the firm posting a vacancy is unable to *target* potential workers based on their level of assets.

The free-entry condition in the market implies that firms will post vacancies until $V(t) \equiv 0$. Consequently, the number of vacancies in equilibrium is implicitly given by:⁹

$$\gamma = q[\theta(t)] \int_{\underline{\mathbf{a}}}^{\infty} J(a, T, t) \frac{g(a, U, t)}{u(t)} da.$$
(12)

Once a vacancy is filled, the value of the corresponding filled temporary contract job, given the assets owned

 $^{^{9}}$ Notice that new firms enter the market based on the comparison between the expected inter-temporal value of the job and the cost of the vacancy. It is possible that the firm would initially be formed with very high leverage. In this situation, the job is not created, even though the firm undertakes the cost of the vacancy creation.

by the worker, results from the following recursive problem:

$$(r_{l}(t) - \delta)J(a, T, t) = \max_{k} \left\{ \ddot{z}F(\ddot{k}) - r_{l}(t)\ddot{k} - w(a, T, t) - \eta(a, T, t) + \partial_{a}J(a, T, t)\dot{a}(a, T, t) + [\sigma_{T}(\ddot{\xi}_{T}) + \varphi(\ddot{\xi}_{P})][V(t) - J(a, T, t)] + \partial_{t}J(a, T, t) \right\}.$$
(13)

Notice I consider that a job that gets promoted means the current temporary job is dissolved. For a permanent contract job, its value corresponds to the solution of the respective problem given by:

$$(r_{l}(t) - \delta)J(a, P, t) = \max_{k} \left\{ \ddot{z}F(\ddot{k}) - r_{l}(t)\ddot{k} - w(a, P, t) - \eta(a, P, t) + \partial_{a}J(a, P, t)\dot{a}(a, P, t) + \sigma_{P}(\ddot{\xi}_{P})[V(t) - S_{P} - J(a, P, t)] + \partial_{t}J(a, P, t) \right\}.$$
(14)

Notice that independently of the type of contract, the investment institution will borrow capital optimally, and therefore we have that:

$$r_l(t) = \ddot{z} F_k'(\ddot{k}),\tag{15}$$

which implicitly defines the capital stock available to each match.¹⁰ The dividends collected by the investment institution are given by:

$$d = (1 - \psi) \int_{\underline{a}}^{\infty} \left[\ddot{z}F(\ddot{k}) - r_l(t)\ddot{k} - w(a, T, t) \right] g(a, T, t) da + (1 - \psi) \int_{\underline{a}}^{\infty} \left[\ddot{z}F(\ddot{k}) - r_l(t)\ddot{k} - w(a, P, t) \right] g(a, P, t) da - \gamma v,$$
(16)

and the aggregate capital stock by each type of contract is:

$$K_j(t) = \int_{\underline{\mathbf{a}}}^{\infty} \ddot{k}g(a, j, t) da, \quad j \in \{T, P\}.$$
(17)

Wage Setting Mechanism and Worker's Bonus. The wage is determined considering: (a) a rent-sharing rule, subjected to a downward wage rigidity constraint;¹¹ and (b) the parties set wages disregarding the subsequent attribution of bonuses, that are only defined once profits are determined. Therefore, for wage setting purposes, the parties consider $\eta(a, i, t) = 0.^{12}$ Accordingly, the rent sharing rule is given by:

$$\beta \left[J(a,j,t) + S_j - V(t) \right] = (1 - \beta) \left[W(a,j,t) - W(a,U,t) \right], \text{ s. to } \eta(a,i,t) = 0, \ j \in \{T,P\},$$
(18)

where β corresponds to the worker's bargaining power, J(a, j, t) value function of the job, W(a, j, t) the value function of the employed worker, and W(a, U, t) the value function of the unemployed. The path of wages is subjected to downward wage rigidity, in the spirit of Schmitt-Grohé and Uribe (2016), so that:

$$w(a, j, t) \ge \lambda w(a, j, t - \epsilon), \quad j \in \{T, P\},$$
(19)

where λ corresponds to the maximum quarterly downward adjustment of real wages.

 $^{^{10}}$ Noteworthy, independently of a firm's EBITDA, I assume each firm will acquire the optimal level of capital. Consequently, in the proposed model the potential financial constraints affect the probability of dissolution of the firm, and not the capital available to it.

¹¹An alternative approach is to consider a (Generalized) Nash Bargain as in Krusell et al. (2010). However, l'Haridon et al. (2013) shows that the difference is negligible in models without precautionary savings, and the derivation of wages in a Nash Bargain arrangement in continuous time raises computational issues regarding the marginal propensity to consume.

 $^{^{12}}$ Intuitively, the wage corresponds to the fixed component of worker's compensation. Then, once the parties determine the profit of the firm and split the profit according to the profit-sharing rule, the variable compensation component is determined.

Considering the solutions of the presented recursive problems and the downward wage rigidity constraint, the wage functions are defined as:

$$w(a,j,t) = \begin{cases} \tilde{w}(a,j,t) & \text{if } \tilde{w}(a,j,t) \ge \lambda w(a,j,t-\epsilon) \\ w(a,j,t-\epsilon) & \text{if } \tilde{w}(a,j,t) < \lambda w(a,j,t-\epsilon) \end{cases}, \quad j \in \{T,P\},$$
(20)

where $\tilde{w}(a, T, t)$ corresponds to the unconstrained wages, resulting from: (a) the rent-sharing rule; and (b) the lack of firm commitment, so that those unconstrained wages are set on a period by period basis. However, when the resulting unconstrained wage is lower than the existing wage in moment $t - \epsilon$, the wage is not lowered. Accordingly, the unconstrained wages are given by:¹³

$$\tilde{w}(a,T,t) = \frac{\beta \left[\rho + \varphi(\ddot{\xi}_{P}) + \sigma_{T}(\ddot{\xi}_{T}) \right] \left\{ \ddot{z}F(\ddot{k}) - r_{l}(t)\ddot{k} + \partial_{a}J(a,T,t) \left(r(t)a - \ddot{c} \right) + \partial_{t}J(a,T,t) \right\}}{\left[1 - \beta \right] [r(t) + \varphi(\ddot{\xi}_{P}) + \sigma_{T}(\ddot{\xi}_{T})] \partial_{a}W(a,P,t) + \beta [\rho + \varphi(\ddot{\xi}_{P}) + \sigma_{T}(\ddot{\xi}_{T})] [1 - \partial_{a}J(a,T,t)]} - \frac{(1 - \beta) \left[r(t) + \varphi(\ddot{\xi}_{P}) + \sigma_{T}(\ddot{\xi}_{T}) \right] \left\{ u(\ddot{c}) + \partial_{a}W(a,P,t) \left(r(t)a - \ddot{c} \right) + \varphi(\ddot{\xi}_{P})W(a,P,t) - [\rho + \varphi(\ddot{\xi}_{P})]W(a,U,t) + \partial_{t}W(a,P,t) \right\}}{\left[1 - \beta \right] [r(t) + \varphi(\ddot{\xi}_{P}) + \sigma_{T}(\ddot{\xi}_{T})] \partial_{a}W(a,P,t) + \beta [\rho + \varphi(\ddot{\xi}_{P}) + \sigma_{T}(\ddot{\xi}_{T})] [1 - \partial_{a}J(a,T,t)]};$$

$$\tilde{w}(a,P,t) = \frac{\beta \left[\rho + \sigma_{P}(\ddot{\xi}_{P}) \right] \left\{ \ddot{z}F(\ddot{k}) - r_{l}(t)\ddot{k} + \partial_{a}J(a,P,t) \left(r(t)a - \ddot{c} \right) + r(t)S_{P} + \partial_{t}J(a,P,t) \right\}}{\left[1 - \beta [r(t) + \varphi(\ddot{\xi}_{P}) + \sigma_{T}(\ddot{\xi}_{T})] \partial_{a}W(a,P,t) - \beta [\rho + \varphi(\ddot{\xi}_{P}) + \sigma_{T}(\ddot{\xi}_{T})] - \partial_{a}J(a,T,t)]} - \frac{\beta \left[\rho + \sigma_{P}(\ddot{\xi}_{P}) \right] \left\{ \ddot{z}F(\ddot{k}) - r_{l}(t)\ddot{k} + \partial_{a}J(a,P,t) \left(r(t)a - \ddot{c} \right) + r(t)S_{P} + \partial_{t}J(a,P,t) \right\}}{\left[1 - \beta [r(t) + \varphi(\ddot{\xi}_{P}) + \sigma_{T}(\ddot{\xi}_{T})] \partial_{a}W(a,P,t) - \beta [\rho + \varphi(z,P,t)]} - \frac{\beta \left[\rho + \sigma_{P}(\ddot{\xi}_{P}) \right] \left\{ \ddot{z}F(\ddot{k}) - r_{l}(t)\ddot{k} + \partial_{a}J(a,P,t) \left(r(t)a - \ddot{c} \right) + r(t)S_{P} - \partial_{t}J(a,P,t) \right\}}{\left[1 - \beta [r(t) + \varphi(z,P,t) + \beta [\rho + \varphi(z,P,t)] - \beta [\rho + \varphi(z,P,t)] - \beta [\rho + \varphi(z,P,t)]} - \frac{\beta \left[\rho + \sigma_{P}(\ddot{\xi}_{P}) \right] \left\{ \ddot{z}F(\ddot{k}) - r_{l}(t)\ddot{k} + \partial_{a}J(a,P,t) \left(r(t)a - \ddot{c} \right) + r(t)S_{P} - \partial_{t}J(a,P,t) \right\}}{\left[1 - \beta [r(t) + \varphi(z,P,t) + \beta [\rho + \varphi(z,P,t)] - \beta [\rho$$

$$f(a, P, t) = \frac{[1 - \beta][r(t) + \sigma_P(\ddot{\xi}_P)]\partial_a W(a, P, t) + \beta[\rho + \sigma_P(\ddot{\xi}_P)][1 - \partial_a J(a, P, t)]}{[1 - \beta][r(t) + \sigma_P(\ddot{\xi}_P)]\left\{u(\ddot{c}) + \partial_a W(a, P, t)\left(r(t)a - \ddot{c}\right) - \rho W(a, U, t) + \partial_t W(a, P, t)\right\}}{[1 - \beta][r(t) + \sigma_P(\ddot{\xi}_P)]\partial_a W(a, P, t) + \beta[\rho + \sigma_P(\ddot{\xi}_P)][1 - \partial_a J(a, P, t)]}.$$

Notice that by the proper definition of a stationary recursive equilibrium, the downward wage rigidity constraint would not bind in computing such equilibrium.

5. Calibration, computation and stationary equilibrium results

In this section, the objective is to analyze the ability of the stationary equilibrium of the model, and its proposed calibration strategy, to mimic a steady state for the Portuguese economy. Once performed, such calibration strategy will serve as the basis for the transitional dynamics analysis of the subsequent sections, and the impact of the labour market policy alternatives considered.

Broadly, the three novel features of the model are the integration of a detailed description of the labour market institutional setting, the prevalence of endogenous job destruction associated with the level of leverage of firms, and the link between productivity and wealth. Those will be key in the assessment of the adjustment paths of the economy during downturns, and thus particular emphasis will be placed in comparing key untargeted moments of the calibrated model and their empirical counterparts.

Computation. I formally define the recursive stationary equilibrium of this economy in appendix B. The proposed model corresponds to a Krusell et al. (2010) type model in a continuous time setting, solved through

 $^{^{13}\}mathrm{See}$ appendix A for further details in deriving the unconstrained wage equations.

the finite-difference method of Achdou et al. (2017). Differently from the traditional one-dimensional fixed-point problem of the Bewley-Huggett-Aiyagari standard paradigm, this model corresponds to a functional fixed-point problem, given the need to find the endogenous wage schedule.

Compared to the standard Krusell et al. (2010) model in continuous time, as presented in Bardóczy (2017), our model presents two additional layers of complexity: (a) the existence of two categories of workers, translating into two sets of HJB equations for employed workers and firms, and a transition matrix defined for employed workers for both types of contracts and the unemployed; and (b) the existence of endogenous job destruction functions.

Despite the additional computational intensity, the algorithm I use is standard and works as follows: (1) set up a grid for assets; (2) guess the market tightness, θ , the interest rate, r(t), and define the wage schedule for temporary and permanent workers, w(a, T, t) and w(a, P, t); (3) compute the leverage of each type of worker along the grid and compute the transition matrix; (4) solve the contraction mapping of the 3 types of HJB equations for the workers; (5) compute the stationary distribution of workers; (6) given the distribution solve the contraction mapping of the 2 types of HJB equations for the firms; (7) evaluate the free entry condition and the asset market clearing condition; (8) perform the rent-sharing rule and update the wage function; and (9) update r_l and θ until there is convergence from step 2 onwards in (w, r, θ) . Appendix C presents further details on the algorithm used.

Benchmark calibration. The calibration follows two main objectives. First, the calibration should present a consistent mapping between the model and several labour market measures. Second, it should present a realistic representation of the financial standing of firms and of households. For this purpose, I make the mapping between the model parameters and quarterly data from various sources, and take the years of 2016-2017 as the benchmark steady state of the economy.

Regarding functional form assumptions, the utility function is $u(\ddot{c}) = ln(\ddot{c})$. The production function $\ddot{z}F(\ddot{k})$ corresponds to $\ddot{z}F(\ddot{k}) = \ddot{z}\ddot{k}^{\alpha}$, where $\ddot{z} = \tilde{z}_t z(a, j)$, where $j\{P, T\}$. In the stationary equilibrium $\tilde{z}_t = 1$, and z(a, j) is set as a logistic type function:

$$z(a,j) = \frac{\overline{z}}{1 + e^{-\kappa_z [a-z^{inv}]}} + \tilde{z}_j, \quad j \in \{T, P\}.$$
(22)

The matching function M(u, v) is set as $M[u(t), v(t)] = \chi u(t)^{\eta} v(t)^{1-\eta}$. The labour flows also follow logistic functions of the interest payment to EBITDA $(\frac{r_t \ddot{k}}{\ddot{\xi}})$, so that the displacement, promotion and hiring of workers is given by:

$$\sigma_{j}(\ddot{\xi}_{j}) = \frac{\overline{\sigma_{j}}}{1 + e^{\kappa_{\sigma}\left[\frac{r_{l}(t)\ddot{k}}{\ddot{\xi}} - \sigma_{j}^{inv}\right]}} + \tilde{\sigma}_{j}, \qquad \varphi(\ddot{\xi}_{P}) = \frac{\overline{\varphi}}{1 + e^{\kappa_{\varphi}\left[\varphi^{inv} - \frac{r_{l}(t)\ddot{k}}{\ddot{\xi}_{P}}\right]}}, \qquad h(\ddot{\xi}_{T}) = \frac{1}{1 + e^{\kappa_{h}\left[h^{inv} - \frac{r_{l}(t)\ddot{k}}{\ddot{\xi}_{T}}\right]}}, \qquad j \in \{T, P\}.$$

$$(23)$$

The benchmark parameter calibration is presented in table 2, and the mapping between the empirical counterparts and several endogenous outcomes of the model is presented in table 3. As in Shimer (2005), home

production is commensurate with the average level of unemployment insurance. Accordingly, the flow of home production is set at b = 0.85, which corresponds to 47% of the average wage of the economy, matching the 5-year average of the OECD net replacement ratio indicator. It is significantly higher than the 40% in Shimer's work, thus representing the more beneficial unemployment insurance protection prevalent in Southern European economies, and particularly in Portugal.

Table	2:	Proposed	Calibration.
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Parameter				
		Expression	. varao	
	Discount factor	ρ	0.005	
Consumers	Home Production	b	0.85	
	Minimum asset level	<u>a</u>	0	
	Matching Efficiency	χ	0.65	
	Worker Bargaining Power	β	0.4	
	Share of EBIT Paid as Variable Component	$\eta(a,j,t)$	0.3	
	Matching Elasticity of Unemployed	η	0.4	
	Firing Tax	S_P	0.4	
Labour	Vacancy Cost	γ	0.772	
Market	Downward rigidity parameter	λ	0.995	
	Steepness of flow logistic functions	$\kappa_{\sigma} = \kappa_{\varphi} = \kappa_h$	200	
	Inversion of logistic functions	$\sigma_i^{inv} = h^{inv}$	0.95	
	Non-stressed promotion rate	$\overline{\varphi}$	0.11	
	Non-stressed displacement rate of temp. workers	$ ilde{\sigma_T}$	0.03	
	Non-stressed displacement rate of perm. workers	$\tilde{\sigma_P}$	0.12	
	Fully-stressed displacement rate of temp. workers	$\overline{\sigma_T}$	0.4	
	Fully-stressed displacement rate of perm. workers	$\overline{\sigma_P}$	0.15	
Financial	Ratio of domestically owned assets	ϑ	0.625	
Financiai	Share of Capital	α	0.225	
Sector	Depreciation rate	δ	0.018	
	Steepness Parameter	κ_z	0.03	
Productivity	Inversion Parameter	z^{inv}	12	
Function	Location Parameter (Permanent Contracts)	$ ilde{z}_P$	1.03	
1 unction	Location Parameter (Temporary Contracts)	$ ilde{z}_T$	0.97	
	Maximum Spread of Productivity	\overline{z}	0.20	

Table 3: Endogenous Outcomes.

	Moment		Value	Target	Period	Source
		Expression		8		
Labour Market	Labour Market Tightness	θ	1	1	-	Internally calibrated
	Unemployment Rate	100 imes u%	7.59%	8%	-	$Eurostat^{(a)}$
	Net Replacement Ratio	$100 \times \frac{h}{E[w]}\%$	46.55%	48%	2016 - 2017	$OECD^{(b)}$
	Proportion of Temporary contracts	$100 \times \frac{e_T}{(e_P + e_T)} \%$	21.4%	17.85%	2016-2017	Eurostat
	Permanent contract wage gap	$100 \times E[\frac{w(a, P, t) - w(a, T, t)}{w(a, T, t)}]\%$	18%	13.8%	-	Silva and Turrini (2015)
	Minimum to average wage ratio	$100 \times \left[\frac{E[w]}{W}\right]\%$	122.9%	136.3%	2016-2017	Quadros de Pessoal ^(c)
	Non-distressed average displacement rate	$100 \times \frac{e_T \sigma_T(\xi_T) + e_P \sigma_P(\xi_P)}{(e_P + e_T)} \%$	4.93%	5.16%	2016-2017	$Eurostat^{(d)}$
	Annual maximum real wage decline	$100 \times (1 - e^{-4\lambda})$	1.98%	2%	-	Inflation Benchmark
	Coefficient of variation of wages	$\frac{\sigma_w}{E[w]}$	0.81	1.51	2016	Quadros de Pessoal
	Average tenure of temporary contracts	$\frac{1}{4} \times \frac{1}{\tilde{\sigma}_T}$	2.08	1	2016	Quadros de Pessoal
	Average tenure on permanent contract	$\frac{1}{4} \times \frac{1}{\tilde{\sigma}_P} - \frac{1}{4} \times \frac{1}{\tilde{\sigma}_T}$	8.33	9	2016	Quadros de Pessoal
Financial Sector	Quarterly gross interest rate on capital	$100 \times [1 - e^{-4(r_l)}]$	1.99%	2.64%	-	Card et al. (2014) (Italy)
	Investment-output ratio	$100 \times \delta k^{1-\alpha}$	21.29%	17.15%	2016 - 2017	Eurostat
	Average Leverage	$100 \times E\left[\frac{r_l k}{\epsilon}\right]\%$	83.7%	90.23%	2016-2017	SCIE
	Annual deposit rate	$100 \times [1 - e^{-4(r_l - \delta)}]$	0.72%	0.54%	2016 - 2017	Banco de $Portugal^{(e)}$
	Average savings' rate	$100 \times E[\dot{a}]$	-7.6%	-1.775%	2016-2017	OECD
	Net External Debt to output	$100 \times \frac{(1-\vartheta)K}{z+k(e_T+e_P)}$	111.92%	94.6%	2016 - 2017	Bank of Portugal
	Consumers net worth as perc. of output	$100 \times \frac{A}{z_t k (e_T + e_P)}$	186.54%	183.63%	2016-2017	$OECD^{(f)}$

Figure 6: Calibrated Functions

Panel B: Productivity function



Notes: (a) the indicator corresponds to unemployment as percentage of total population for individuals between 20-64 years; (b) Indicator in 2009 OECD Emplooyment Outlook; (c) the indicator considers the average net average wage and net minimum wage; (d) the indicator resorts to the labour market transition database and include transitions from job-to-unemployment and job-to-inactivity; (e) the indicator corresponds to the average annual deposit rate of term deposits; (f) the indicator corresponds to the product of the household net worth and the household proportion of financial assets. Implicitly, as a simplifying assumption I assume the liquidity of assets as a proxy of the liquidity of the net worth. *Sources: Eurostat, OECD, Banco de Portugal, SCIE 2016-2017, Quadros de Pessoal 2016*, Silva and Turrini (2015), Card et al. (2014).

The firing tax of permanent workers is set at $S_P = 0.4$, it represents 22.5% of the average wage, and it is consistent with survey data for Oslington and Freyens (2005). Notice that following Boeri (2011), a firing tax should be distinguished from a severance payment, which can be fully offset with a compensating wage adjustment in this framework (Lazear, 1990). Thus, the firing tax corresponds to a pure deadweight loss, accruing from the cost on the resources required to obtain legal advice, make the decision, gather the case for the dismissal decision, meet with the worker, and potentially go to court. On the other side, the cost of post a vacancy is set so that the labour market tightness is calibrated at $\theta = 1$. It corresponds to 51.6% of the average wage which is similar to the calibrations of Krusell et al. (2010) and Hagedorn and Manovskii (2008) whom set the vacancy cost at 60% and 58.4%, respectively.

The non-stressed separation rates of unemployment, $\tilde{\sigma}_P = 0.03$ and $\tilde{\sigma}_T = 0.12$, are set considering the average tenure for each type of contract, the model's postulated career path, and the fact that job-to-job transitions are particularly noticeable in temporary jobs, which increases the effective spell of employment of workers vis-á-vis firm tenure. The promotion rate, $\bar{\varphi} = 0.1$, and the matching efficiency parameter, $\chi = 0.65$, are set so that conditional on separation rates the calibration approximates: (a) the empirical counterpart of the percentage of temporary workers in the economy; (b) the unemployment rate; and (c) the duration of a temporary contract until promotion, $\frac{1}{4}\frac{1}{\varphi}$, which is set at 2.5 years.¹⁴

Then, as the interest payment to EBITDA ratio approaches 95 percent, the displacement probabilities increase logistically, and the hiring and promotion probability decrease, also logistically, as presented in figure

 $^{^{14}}$ The maximum allowed number of successive temporary contracts in a firm is 3. Given the typical temporary contract is of 1 year, 2.5 years of average length until promotion, conditional on not being dismissed before, is deemed adequate.

6. The intuition behind the calibration is that a very indebted firm will neither hire nor promote a worker. For a temporary worker in a stressful firm, the hazard of being dismissed becomes 52 percent, translating into an additional expected job duration of 5.8 months, while for a permanent worker in an equally stressed firm, the additional expected job duration is shortened to around 1,4 years.

Regarding the productivity function, two features are noteworthy. First, I assume the insight of Eeckhout and Sepahsalari (2020). The productivity of the match is an increasing function of the wealth of the worker, as presented in figure 6.¹⁵ Second, the permanent jobs are more productive, following a standard reasoning of positive productivity effects of tenure, which is fully consistent with the structure of the adopted model.

The discount rate and the deposit rate influences savings decisions of consumers. The discount rate is set at $\rho = 0.005$, which corresponds to a 2% annual discounting. The annual deposit rate of assets is endogenously determined at 0,7%, leading to an average savings rate of -7.6%, which is lower than the *OECD* estimate of the savings of households. Further, the ratio of domestically owned assets, $\vartheta = 0.625$, pin-points the net external debt, endogenously defined as 112% and the consumers net financial worth of around 187%. Both are not substantially different from the empirical counterparts from *Bank of Portugal* and *OECD*.¹⁶

The deposit rate jointly with the depreciation rate, leads to a gross annual cost of capital of 7.59%, which is consistent with Card et al. (2014) for Italy. Then supplemented with the capital share leads to an investment share of 21.29%, which is not at the odds with the *Eurostat* data. Noteworthy in the financial side, particularly given the mechanism of job destruction proposed, the average leverage of firms, of 83.7% is broadly in line, or being slightly more conservative, when compared with the information of the administrative panel data of firms (*SCIE*).

An important feature of the model is its ability to generate wage and wealth dispersion. When compared with the benchmark, the model generates around 54% of the empirical standardized variability of wages, as directly measured in *Quadros de Pessoal* for 2016. This lack of dispersion is consequence of the simplified nature of the productivity functions. While it moderately depends on the wealth of the worker and its type of contract, it does not vary exogeneously, as I do not assume the existence of inherently different *firm types*.

In a more detailed analysis, if one focuses in the wage variation associated with worker's components of a typical AKM wage decomposition, the benchmark empirical coefficient of variation of wages becomes around 0.92, which implies that the model generates 88% of such empirical within variability of wages.¹⁷ Altogether, I abstain from undertaking adjustments to the model in order to allow it to generate wage variability compatible with its empirical counterpart. That would require the adoption of a distribution of *firm types* that would

 $^{^{15}}$ The model presented by Eeckhout and Sepahsalari (2020) is a direct search model, where the wealthier worker is more selective when searching for a job, thus landing on average on a more productive firm. Here we take such relationship as exogenous.

 $^{^{16}}$ The OECD indicator used for the net financial worth of the agents corresponds to the product of the household net worth and the household proportion of financial assets. Implicitly, as a simplifying assumption I assume the liquidity of assets as a proxy of the liquidity of the net worth.

 $^{^{17}}$ See Torres et al. (2018) for an empirical analysis of the sources of wage variability in Portugal, resorting to the same dataset. I use their findings in calculate the within variability of wages, as the sum of the variability associated with the worker fixed effect and time-varying covariates, in their two high dimensional fixed effect model.

significantly increase computational burden. Given the match of the model's variability with the within empirical results, one should interpret the results as representing the adjustment of a representative firm. The implied stationary wealth distribution is given in figure 7.



Figure 7: Steady state worker's wealth distribution, savings and firm leverage

One of the key model mechanisms of job destruction is associated with the leverage of firms, namely the interest payment to EBITDA ratio. In figure 7, it is clear that in steady state the firms are not financially stressed, and the permanent contract jobs are around 20 percent more leveraged than the temporary jobs. In what concerns savings, the shape of the savings functions are consistent with the results of the standard Krusell et al. (2010) model, and establishes a reasonable ranking of types of workers, where the permanent contract workers are the ones saving the most.

Productivity shock. With the presented calibration, the core of the exercise of the paper is to study the adjustment paths of the economy subjected to a negative, aggregate, unexpected and one time productivity shock, with a subsequent productivity path until the steady state values. In this process, instead of a defined magnitude of the shock, I build a grid of different magnitudes of an initial shock, z_0 , from zero to 10% of the steady state productivity levels. The function of the shock is given by:

$$z_t = 1 + (z_0 - 1)e^{-0.3t}.$$
(24)

Thus, the shape of the calibration of those shocks is presented in figure 8. As we will inspect, the adoption of an interval of shocks allows to further highlight the mechanisms generating labour market adjustments, particularly

under different wage rigidity regimes and policy options. The algorithm to analyse the transitional dynamics of the proposed economy throughout the downturns implied by the grid of productivity shocks is presented in appendix D.



Figure 8: Productivity shock

6. Transitional dynamics with downward real wage rigidity

In the standard description of the modelled labour market, wages are rigid in real terms, there persist a significant contract duality, where 21 percent of the workers have their labour relationship governed by a more precarious temporary contract, and firms are at hazard of financial distress if their leverage increases substantially, thus increasing the odds of an episode of match dissolution or insolvency. Such dissolution episodes, when affecting a permanent contract firm consumes a share of the capital available to the insolvent firm in the form of a firing tax.

Under these cornerstone conditions, the model generates substantial unemployment spikes in the outcome of sufficiently negative productivity shocks, particularly at the higher end of the shock magnitude, in absolute terms. As presented in figure 9, the unemployment growth in the first quarters of the downturn, consistently exceeds the initial size of the downturn for initial shocks of 3.5% or higher, reaching a job destruction of 11.8% for the maximum considered downturn of 10%. By definition, such unemployment behaviour corresponds to the aggregate of job creation and destruction dynamics. In our model it intimately relates with the leverage standing of firms. As presented in figure 10, a sizable productivity shock creates significant liquidity constraints at firm level. Those substantially increases the odds of firm dissolution.

The results presented partially follows Mortensen and Pissarides (1994), where endogenous job destruction generates increased volatility of employment when compared to the standard textbook search and matching model. However, in our model, the destruction mechanism is not directly determined by an idiosyncratic productivity shock coupled with a general exogenous threshold, but rather due to an increased hazard rate of bankruptcy of highly indebted firms, which increases significantly the odds of a terminal financial disruption.



Figure 9: Unemployment and job finding rate transitional dynamics

Figure 10: Firm level mechanisms of adjustment



Additionally, differently from the standard mechanism, our model establishes a firm liquidity channel to job creation. As presented in figure 6, in this labour market, no highly leveraged firm is ever created, as firms cease to hire temporary workers, and promotions into highly indebted firms is also ceased, thus stopping the steady state dynamic of contract graduation. Consequently, the leverage also refrains the job creation dynamics, over and above the standard receding caused by the fall of the textbook inter-temporal job value *vis-à-vis* the cost of posting a vacancy.

The increased leverage of firms with impact on the firm survival rate also impacts capital, over the standard prescribed mechanism. Traditionally, the fall in productivity reduces the capital utilization of firms, reducing the annual cost of capital rate and the deposit rate, thus creating a pull of investment into consumption. In this model, on top of the traditional mechanism, when firms become highly leveraged, some of then bankrupt, which leads to a capital loss due to increased firing taxes, which further weightens the decline in capital, while on this part not transforming it into consumption.

Then as the productivity starts to recover, leverage starts to fall, firms start employing more capital, the distress in the labour market flows recedes, with direct impacts on job creation and job destruction, and the unemployment starts to converge to the steady state values. Altogether, the aggregate dynamics highlight a strong propagation mechanism of productivity downturns into unemployment, which can be described as a *catastrophic job destruction*, as the empirical study of Carneiro et al. (2014) as coined.

To further understand the mounting of the described adverse mechanisms, one should assess the behaviour of wages and contract types, which are displayed in figure 11. Generally, in search and matching models, the instantaneous bargaining of wages can constitute an internal financing of the firm in the presence of downturns, as wages are automatically reduced in the response to the downturn. However, in this setting the fall in wages in anchored to a maximum rate of allowed decline, which is compatible with a calibrated year inflation rate of 2%. In these regards, it is assumed that any potential new hires are also subjected to the same degree of rigidity of their wages in relation to steady state wages. Beyond computational concerns, such assumption seems a reasonable assumption, as the hiring during the acute phase of downturn is particularly hampered thus not substantially altering the implied wage rigidity, there are tabled wages and compensation policies sizably bargained at sector bargaining level, and there are equal pay equal job concerns. Consequently, with the wage financing channel impaired, firms witness a stronger leverage increase that sparks the described labour market distress.



Figure 11: Segmented wage and employment transitional dynamics

When one partials out the employment dynamics by type of contract, the initial response is lead by a fall in both temporary and permanent employment. Then, as the leverage of actual and potential temporary contract firms is reduced, from an significantly lower level when compared with permanent contract firms, both the current temporary workers see the survival rate of their jobs increase substantially, and the job creation recovers substantially, even though still adversely affected by free entry condition fundamentals. Thus, one starts witnessing an increase in temporary contract employment soon after the initial shock. However, the possibility of graduation into permanent contracts is still impaired while the potential permanent contracts would result into highly leveraged firms. Consequently, the adjustment is marked by a cleansing of permanent contract firms which are dissolved, and the mounting of temporary contract firms which do not graduate during the first phase of the transition. All in all, such adjustment in flows leads to compositional effects that imply a more sizable economy wide average wage adjustment, than the ones witness at contract level, which may potentially mask at aggregate level the degree of wage rigidity.

7. Flexible wages paradigm

In this section I start from the same benchmark economy description, but the wages are assumed to be flexible, so that they are continuously determined through the rent-sharing rule, disregarding the wage rigidity rule.¹⁸ Beyond the usefulness of analysing transitional dynamics under this policy change on the standard

 $^{^{18}}$ See appendix E for further details on implementing the flexible wage transitional dynamics algorithm.

environment, this analysis allows to further highlight the consequences of the shutdown of the wage financing channel present in the previous section.



Figure 12: Firm level mechanisms of adjustment under a flexible wage regime

As presented in figure 12, the first stark difference between the considered environments concerns leverage. As wages immediately adjust in response to a fall in productivity, and thus of the match surplus, the leverage of firms increase by a substantially less degree when compared to the rigid-wage scenario. Consequently, firms are less financially distressed, with temporary job firms not even trespassing to a region of substantial increase in the dissolution hazard rate. Thus, the cleansing effect on permanent contract firms is more short-lived and less sizable. The reduction of hiring dynamics is fundamentally driven by the traditional free entry condition dynamics, rather than by the influence of leverage on hiring, and the promotion hold-up lasts fewer periods. Altogether, it implies a lower reduction of the capital stock, due to less firing taxes paid, and less adverse job dynamics. The annual cost of capital thus fall by less.

The underlying mechanism that hampers the rise in leverage is displayed in the behaviour of wages in figure 13. When compared with the rigid-wage case, wages fall substantially more, for example by more than the triple in a 10% initial negative shock in productivity. Such adjustment immediately provides the referred internal financing mechanism, which translates in significantly less employment adjustments across types of contracts, thus leading to less powerful compositional between temporary and permanent contracts. Consequently, the fall in wages in both contract types and economy wide are identical. Noteworthy, while the fall in wages is more pronounced the recovery path is also stronger, implying a return of wages to the steady state levels at a much

faster pace.



Figure 13: Segmented wage and employment transitional dynamics under a flexible wage regime

The less impactful financial distress also affects the job creation dynamics, as seen in figure 14, which are now solely driven by the dynamics of the inter-temporal free entry condition. Altogether, the difference in unemployment is very sizable. With the presence of flexible wages, the unemployment rate increase is fairly moderate and never surpasses, or comes close, to the amount of the initial productivity downturn. For example for a 10% downturn the unemployment rate increase reaches a maximum of 3%.

Notice that the *defusal* of a catastrophic job destruction is not operated by assuming a textbook drop of an endogenous job destruction mechanism, thus assuming a standard search and matching model unable to generate significant unemployment spikes, or in a more broad discussion a significantly high elasticity of productivity to labour market tightness (see Shimer (2005) for further details). In this environment, I keep the endogenous job destruction mechanism untouched, in the sense that equally high leverage still causes financial distress with the previously described consequences. However, the simple relaxation of wage rigidity, with the consequent opening of the wage bill internal financing channel of firms, substantially mutes the adverse effects witnessed in the downward rigid wage environment.

From a comparative perspective, the adoption of downward real rigid wages does not have a substantial impact on unemployment behaviour for minor or moderate productivity downturns, namely in the range of 0% to 2%. For more sizable downturns the labour market disruption is increasingly more noticeable in downward

real rigid wages. In a nutshell, this comparison highlights the potential nefarious consequence of the job destruction leverage based mechanism coupled with downward real rigid wages. Indeed, such mechanism is capable to generate a costly process of destruction of inter-temporally viable jobs, particularly permanent jobs, due to the bite of the instantaneous leverage effect on firm survival. This constitutes what I coin as *a last dance phenomena*. The subsequent policy section of the paper will discuss different alternatives of aggregate policy designs in trying to defuse this catastrophic job destruction.





In a welfare based perspective, we adopt an adapted continuous-time version of the welfare measure proposed by Krusell et al. (2010). I follow the percentiles of the distribution of workers' asset holdings for each relevant situation of the workers, under the relevant transition path, and record his consumption path. Then, I compare the consumption path of the worker's percentile with the consumption path of the equivalent worker in the equivalent percentile of asset holdings under the stationary distribution. Denote this welfare measure as π . The welfare measure is then calculated holding the following condition:

$$E_0 \left[\int_0^\infty e^{-\rho t} log[(1+\pi)c(a,j,t)] dt \right] = E_0 \left[\int_0^\infty e^{-\rho t} log[c^s(a,j,t)] dt \right], \quad j \in \{P,T\}$$
(25)

where $c^{s}(a, j, t)$ represents the consumption level in the steady state.¹⁹ Accordingly, I will focus in assessing π

$$\pi = e^{-\rho \left\{ E_0 \left[\int_0^\infty e^{-\rho t} log[c^s(a,j,t)] dt \right] - E_0 \left[\int_0^\infty e^{-\rho t} log[c(a,j,t)] dt \right] \right\}} - 1.$$
(26)

 $^{^{19}}$ Notice that the relevant welfare measure, $\pi,$ can be calculated as:

to measure welfare losses or gains.

Figure 15 resumes the findings for the downward real wage rigidity and wage flexible regimes, coupled with an analysis of the path of average savings in both environments. Regarding the welfare measure, the results broadly confirm the insight of a direct unemployment analysis. Until 2%-3%, the welfare loss associated with either regime is broadly identical, whereas for stronger downturns, the downward real wage rigidity regime significantly penalizes the workers, in an across board effect, namely targeting the considered percentiles for every condition the worker finds himself into.



Figure 15: Welfare and savings comparison across different wage regimes

A notable characteristic is the strong consistency of the results for the different labour market groups, and broadly for percentiles. This implies that in the presented model, with either wage adjustment regime, an aggregate productivity shock affects the different groups of workers in a broadly identical percentage of the stationary equilibrium initial welfare state. In terms of percentiles, while in the flexible wage regime there is a substantial homogeneity in the percentage effect, in the case of downward real wage rigidity there is a slightly lower impact for the lower percentiles of temporary and unemployed workers. For example, in such regime in the presence of a 10 percent downfall, the ratio of the fall in the welfare measure between the 25^{th} and the 50^{th} percentiles of those workers is on average 1.06. Noteworthy, an identical fall in consumption paths, signalled by π , imply a loss in the utility levels that is decreasing in percentage terms as the initial consumption level increases. Consequently, the utility loss is greater for lower levels of assets, which predominantly populates the unemployed and temporary worker groups.

Regarding the average savings, those are fundamentally influenced by the earnings, and the relationship between the discount rate and the deposit rate, with the latter being indexed to the annual cost of capital. The differences in the trough of the savings adjustment path, across regimes are not substantial. In the downward real rigid wage case, while the wage rigidity binds, and the job destruction does not sufficiently kicks in, the adjustment in savings happens at a slower pace, and the trough is attained later, when compared with the flexible wage regime. There, the immediate fall in earnings entices the consumers to immediately resort to savings in order to smooth out consumption. Overall, the rigid wage regime delays slightly the path of adjustment, but do not substantially change it, neither in terms of magnitude, nor in terms of shape. Consequently, it adds additional evidence that the potential intent of downward rigid wages to secure income of the consumers is *trampled* by the job destruction dynamics.

8. Labour market policies on permanent contracts

While the previous section provides a useful benchmark to assess the mechanisms underlying the adjustment of a more typical Continental European labour market as modelled, it is often the case that wage flexibility is not a feasible policy option, even on dire downturns. By standard, these labour markets have been defined around the notion of some degree of either nominal or real wage security, which is often enshrined in the law. Its relaxation frequently requires either clear popular support, or a broad collective bargaining consensus. Those have not been reached in an aggregate, or even large sector sense in past downturns as the Great Recession. Consequently, in this section I will explore other policy alternatives, focusing in defusing the mounting of very high leverage levels in permanent contract firms.²⁰

Simultaneously, as the stance of the fiscal policy in the response to a downturn, and particularly a sizable downturn is often uncertain, and frequently constrained, I will abstain from assuming any degree of fiscal expansion, or any acceptable level of public debt. Thus, the policies considered will be budget balanced, either inter-temporally or instantaneously. Moreover, I will assume that a policy response takes time. To ensure a comparable basis, every policy entangles one quarter of policy lag implementation. The policy objective will be labour market stabilization, with the policy maker willing to fasten the closure of the gaps vis-à-vis the stationary equilibrium levels, in terms of employment in both types of contracts and of unemployment.²¹ Consequently, when selecting the optimal calibration of the policy, the policy maker will minimize:

$$\int_{0}^{\infty} e^{-\rho t} \left\{ \left| e_{T}(t) + e_{P}(t) - e_{T}^{s} - e_{P}^{s} \right| + \left| u(t) - u^{s} \right| \right\} dt.$$
(27)

 $^{^{20}}$ See appendix E for the details of the algorithms to estimate the transitional dynamics implied by the productivity shocks, while accounting for each of the proposed policies.

 $^{^{21}}$ Alternative, one could consider a policy maker that intents to maximize a welfare function. Here, I take the approach to assume the policy maker intents to act over a visible indirect measure of welfare and functioning of the labour market, and by the end of the section we will make a detailed analysis of the welfare implications of such approach.

Finally, also to ensure comparability, and given the nature of the policies to be considered, I will assume their liquidity provision phase will last for 1 year in the calibration proposed. Altogether, the core of this section is to explore a set of policies that are comparable at face value.

Policy #1: Unemployment benefit transfer. The first policy considered starts with a slight change in the assumption framework. If one assumes h is home production that is provided by an unemployment benefit transfer, a natural policy is to increase the value of such transfer by *front-loading* the resources that latter, throughout the adjustment path, would be spent if the front-loading does not take place. Consequently, the policy maker pools such resources and provides a transfer to firms in form of a *ad-valorem* subsidy on the wage of the worker - τ^{ub} . Accordingly, from the end of the 1st quarter and until the 5th (i.e. $\Upsilon = [v; \bar{v}]$), while the wage of the worker is not altered, the profit of the firm, before the payment of bonuses, becomes:

$$\tilde{\eta}(a,P,t) = \left[\ddot{z}F(\ddot{k}) - r_l(t)\ddot{k} - (1 - \tau^{ub})w(a,P,t)\right], \text{ if } t \in \Upsilon.$$
(28)

The referred budget balance condition is given:

$$\int_{\underline{v}}^{\infty} h \times [u^{NP}(t) - u(t)] dt = \int_{t \in \Upsilon} [\tau^{ub} w(a, P, t)] dt,$$
⁽²⁹⁾

where u_t^{NP} corresponds to no-policy scenario unemployment paths, which are naturally the ones recorded in the downward real rigid wage environment. Given conditions (28) and (29), a calibration of such transfer is attainable, and it will be performed numerically.

Policy #2: Income Tax and firm subsidy transfer.²² The second policy represents a direct attempt to unmute the wage internal financing channel of firms, through an *ad-valorem* tax on wages and the equivalent subsidy to firms - τ^{ts} . From the side of permanent workers, their period income becomes:

$$y(a, P, t) = (1 - \tau^{ts})w(a, P, t) + \eta(a, P, t), \text{ if } t \in \Upsilon.$$
(30)

From the side of the firm, the flow profit before bonuses becomes:

$$\tilde{\eta}(a,P,t) = \left[\ddot{z}F(\ddot{k}) - r_l(t)\ddot{k} - (1 - \tau^{ts})w(a,P,t)\right], \quad \text{if } t \in \Upsilon.$$
(31)

Differently from policy #1, this policy requires an objective function, as it is instantaneously balanced by nature. Thus the policy maker will select τ^{ts} , holding conditions (30) and (31), that minimizes condition (27).

Policy #3: Moratorium in cost of capital transfer. This policy entails a temporary administrative reduction in the cost of capital, which is subsequently balanced by a future increase in the cost of capital. After the initial policy lag in the period [0, v], the reduction covers the period Υ , which is calibrated to cover 1 year. Then the repayment period is calibrated as 5Υ , in the period $[\Upsilon, 5\Upsilon]$.

 $^{^{22}}$ An identical policy based on a reduction of the social security contribution paid by the firms, and an increase of the social contribution paid by workers was proposed in Portugal, with a calibration of 7 percent, during the Great Recession in 2012. Due to strong public resistance it was abandoned.

The present policy does not consist in a pure loan. Rather, it is drafted as a loan with an adjusted interest rate which accounts for the entry and exit of firms along the policy implementation and repayment periods, and assumes that every firm is subjected to the policy at any given moment, independently of its history. Consequently, for instance firms that enter the market in the repayment period are always subjected to the increased cost of capital, whereas a firm that bankrupts in the middle of the policy implementation period does not have a repayment requirement. The interest rate considered in the policy will be calibrated to ensure the policy is balanced in aggregate flows.

Given this structure, defining LC(a, P, t) as the function that perturbs the standard cost of capital, the flow profit before bonuses becomes:

$$\tilde{\eta}(a,P,t) = \ddot{z}F(\ddot{k}) + LC(a,P,t) - r_l(t)\ddot{k} - w(a,P,t),$$
(32)

with LC(a, P, t) being positive in the implementation period, and negative in the repayment period. The relevant interest rate applied to the profile of the moratorium, r_{loan} , is adjusted so that on aggregate, the policy is budget balanced. Therefore, it holds:

$$\int_{t\in\Upsilon} \int_{a} [LC(a,P,t)g(a,P,t)e^{-r_l(t)t}]dadt = 0.$$
(33)

As in the case of a tax-subsidy policy 2, the pin-point of the calibration requires the policy maker to minimize the problem in equation (27), as any design policy entangling equations (32) and (33) is by definition intertemporally balanced on aggregate terms.

Policy #4: Joint transfer policy. This policy combines the policy instruments of policies 1, 2 and 3. The policy has a balanced unemployment benefit transfer component, a income tax - firm transfer component and a cost of capital moratorium transfer component. Accordingly, the flow profit of the firm, before the payment of bonuses is given by:

$$\tilde{\eta}(a, P, t) = \begin{cases} \ddot{z}F(\ddot{k}) + LC(a, P, t) - r_l(t)\ddot{k} - (1 - \tau^{ub, j} - \tau^{ts, j})w(a, P, t), & \text{if } t \in \Upsilon. \\ \\ \ddot{z}F(\ddot{k}) + LC(a, P, t) - r_l(t)\ddot{k} - w(a, P, t), & \text{if } t \in [\Upsilon, 5\Upsilon]. \end{cases}$$
(34)

The balancing of the policy entangles the moratorium and income tax and firm subsidy transfer components. Accordingly, the following conditions hold:

$$\int_{t \in \Upsilon} \int_{a} [LC(a, P, t)g(a, P, t)e^{-r_{l}(t)t}] dadt = 0;$$

$$\int_{\underline{v}}^{\infty} h \times [u^{NP}(t) - u(t)] dt = \int_{t \in \Upsilon} [\tau^{ub, j}w(a, P, t)] dt.$$
(35)

Finally, the calibration of the policy is achieved by the minimization of the objective function in equation (27).

Policy #5: Temporary wage flexibility transfer. Differently from the previous policies, this policy will not be based on transfers and taxes. Rather, it will impose a temporary wage flexibility of wages, which could

be implemented by forcing a renegotiation of the collective agreements in the market to define temporary paying clauses that will bind in the period Υ . So while the wages will follow a flexible wage path in the period Υ , they will recover the value they would have under the downward real rigid wage regime afterwards. Quantitatively, instead of equation (20), we have:

$$w(a, P, t) = \begin{cases} \tilde{w}(a, P, t) & \text{if } t \in [0, \underline{v}] \cup (\bar{v}, \infty), \ \tilde{w}(a, T, t) \ge \lambda w(a, P, t - \epsilon) \\ w(a, P, t - \epsilon) & \text{if } t \in [0, \underline{v}] \cup (\bar{v}, \infty), \ \tilde{w}(a, P, t) < \lambda w(a, P, t - \epsilon) \\ \tilde{w}(a, P, t) & \text{if } t \in \Upsilon, \\ \tilde{w}(a, P, t) & \text{if } t = \bar{v}, \ e^{-\lambda(t - t_0)}w(a, P, t_0) < \tilde{w}(a, P, t) \\ e^{-\lambda(t - t_0)}w(a, P, t_0) & \text{if } t = \bar{v}, \ e^{-\lambda(t - t_0)}w(a, P, t_0) \ge \tilde{w}(a, P, t) \end{cases}$$
(36)

where t_0 is the moment of the downturn shock. This policy will be different from the previously considered flexible wage regime in two fundamental characteristics. First, I assume that the market is initially functioning as a downward real rigid wage regime. When faced with the downturn, the policy maker assesses the situation and during the policy lag implementation forces the collective bargaining sides to bargain temporary wage relief clauses. Second, the policy is temporary. The reintroduction of the wage rigidity environment takes place at moment \bar{v} . In that moment, the wage rigidity constraint for that period is compatible with the wage rigidity that would have bound if the downward real rigidity constraint was not temporarily relaxed.

Calibration and policy effects. The calibration of the policies is presented in figure 16. For policies #1-#3, the calibration prescribes an increasing path of intervention, particularly for downturns of higher magnitude, in which the growth of the rates assumes an exponential path. There, the policies that are balanced through transfers from the market, namely the income tax firm subsidy, has higher rates, when compared with a policy that is back-up by future savings in unemployment benefits. Also notice that the rate of the moratorium policy is not directly comparable with the other two, given that the rate focuses in the cost of capital flow instead of wages.

Regarding the joint policy approach #4, the broad trend of rates follows the findings of the isolated policies #1-#3, but there is some degree of substitution of the magnitudes of each component of the policy. This interaction is expected, as in theory there may exist trade offs among the components of the policy. For example, the opening of the firm's wage bill financing channel through the income tax firm subsidy component could reduce the front-loading of future savings in the unemployment benefits component.

Figure 16: Optimal policy calibration



Figure 17: Unemployment effects



The main policy maker priority is to contribute to the stabilization of labour market flows. In figure 17, the comparison of the unemployment rate dynamics of the market under a no policy approach *vis-à-vis* each considered policy is presented. The first notable finding is that, in presence of a small to moderate downturns, namely below 6 percent, every policy is largely ineffective in enhancing a faster convergence to the stationary

equilibrium flow values. For those downturn levels, the leading mechanism for the dynamics of job creation and destruction is rather inter-temporal, focused in the free entry condition and non-distressed hazard rates, as the period financial distress due to very high leverage levels is not triggered. Consequently, the temporary short-lived policies are fundamentally transfers into the profits before bonuses of the firms and wages, rather than having a sizable effect on job creation and job destruction. In a nutshell, given the budget balance nature of the policies, they are simply not sufficiently powerful to translate into significant gains in the dynamics of job creation, and thus their effects are very moderate.

However, in the presence of significant downturns, namely with an initial shock above 6 percent, the policies become significantly effective, reaching a reduction in unemployment in the peak of around 20 percent for a 10 percent downturn. In levels that would translate in a reduction of around 4 percentage points in unemployment rate, which is notable given that every policy does not entail a change in the stance of the fiscal policy. Moreover, the bucket of considered policies present an identical unemployment effect along the considered paths of productivity shocks. These findings highlight the ability of the policies to reduce the sharp increase of the leverage of firms, thus stimulating a quicker reduction in its levels from the disruptive region that, as presented before, constitutes a powerful mechanism of job destruction and weakening of job creation. Thus, by attenuating the adverse contribution of the leverage mechanism, the policy maker effectively attenuates the catastrophic job destruction spell.



Figure 18: Capital effects

The referred attenuation translates into visible capital stock effects, as presented in figure 18. As the job

destruction is reduced, the loss in the capital stock associated with firing taxes is also attenuated. Here, while the effects are broadly shape identical across policies, it is visible the moderate adverse effects of the moratorium policy in the repayment period, where the capital gains are attenuated when compared to the other policy alternatives, and inclusively when compared with the joint policy alternative that resorts to a less sizable moratorium component.

Regarding the effects of the policies in the income profiles along the adjustment path, figure 19 presents the path of the average income. Here, significantly different paths co-exist. On one hand, the policies that focus in reposing some degree of wage flexibility, namely the income tax firm subsidy and the temporary wage flexibility, imposes an initial fall in real terms on the income of workers, when compared with the no policy path. Further, those declines are prescribed even for moderate and small downturn magnitudes. Consequently, these policies confirm the ability to be imperfect substitutes to a wage flexible regime. Differently, the policies that do not directly affect the wages of workers, namely the unemployment benefit subsidy and the moratorium policies, do not entangle an initial real income decline, prescribing immediate sizable income gains.



Figure 19: Average income effects

These different income profiles offers are a very relevant finding in this discussion. Those constitute alternatives that enable the policy maker to select a preferred shape of income profiles along the adjustment path, while attaining identical unemployment and capital effects. For instance, an initial decline of real income may encounter strong political and public opposition, as for instance was the case in several continental European countries during the Great Recession. The final point of analysis of this section focuses in the welfare, along the percentiles of the wealth distribution for the different types of workers. The approach is identical to the welfare discussion undertaken before, but the benchmark policy to compare with is the standard model with downward real rigid wages without any policy intervention. Consequently, the welfare measure is now given by:

$$E_0 \left[\int_0^\infty e^{-\rho t} log[(1+\pi)c(a,j,t)] dt \right] = E_0 \left[\int_0^\infty e^{-\rho t} log[c^{NP}(a,j,t)] dt \right], \quad j \in \{T,P\}$$
(37)

where $c^{NP}(a, j, t)$ are the consumption paths under a no policy intervention.

The results are summarized in figure 20, and supplemented with a comparison between the pure wage flexible regime and the downward real wage rigidity regime. Noteworthy, in the presence of aggregate productivity shocks, the welfare effects are quite identical in shape and magnitude across workers in different positions in the labour market, concretely permanent and temporary contracts and unemployed, and among the considered wealth percentiles.

A fully flexible wage regime has sizable welfare gains for any moderate to strong downturn when compared to any of the adopted policy interventions in the downward real wage rigidity environment. In fact, confirming previous findings, those policies only have visible effects for strong downturns. In a ranking, the consumers would prefer a policy based on a subsidy backed by future savings in the unemployment benefit system, followed by the joint policy alternative, and then the remainder policies, concretely the temporary wage flexibility, the moratorium and the income tax firm subsidy alternatives, which attain identical welfare results.





Notes: The presented welfare measure is the ratio of the standard welfare measure under the policy intervention to the standard welfare measure under the no policy intervention regime.

Altogether, this section highlights several key findings. First, when facing severe aggregate productivity downturns in an environment where there are downward real rigid wages and instantaneous financial distress in the presence of high firm leverage, there are budget-balanced policy interventions that can significantly attenuate the spike in unemployment, reduce the loss in capital stock arising from labour market disruption, and provide across board welfare gains to consumers. Those are particularly ineffective for lower magnitude shocks. Second, the bucket of policy alternatives is sufficiently rich to provide an ample set of alternatives to the policy maker concerned in stabilizing labour market flows. Third, those alternatives are visibly different in the wage profiles they imply, with some potentially collecting more public and political support than others. Fourth, either current financing constraints of the fiscal policy, or time-consistency constraints in the implementation of some of the prescribed policy alternatives can be surpassed by alternatives that, while not entailing the same type of policy structure, produces similar unemployment and capital effects. All in all, in our environment, when facing a severe downturn there is a high cost of not intervening.

9. Conclusion

I have analyzed the effect of negative aggregate productivity shocks varying from 0.5 to 10 percent of the benchmark productivity levels, and the subsequent labour market adjustments, in the context of a Bewley-Huggett-Aiyagari incomplete markets model with search frictions, financial constraints, and a labour market of the Continental European type, namely one that combines contract duality and downward real wage rigidity. With a benchmark calibration to portray the Portuguese case, I analyze the adjustment paths of the main labour market indicators.

Along the grid of negative aggregate productivity shocks, job creation is increasingly negatively affected, due to the fall of the inter-temporal value of a job, which restricts the creation of new vacancies. However, this mechanism alone is not sufficiently powerful to cause a burst in employment, as the decline in the inter-temporal value of a job facing a short-lived productivity downturn is not sizable enough to generate the required and sufficiently permanent fall in job creation. Even if it was, that would not mimic the stylized empirical facts of the market in the Great Recession, where a job destruction spike carried a sizable role.

Moreover, under standard wage flexibility, the model does not predict any other sizable adverse mechanism to employment. When one allows wages to instantaneously adjust to the value of the worker-firm surplus, firms facing productivity downturns resort to internal financing to face liquidity shortages, as they can reduce the wage bill. Intuitively, they share with the workers, as per the magnitude of the relative bargaining powers plus the fall in bonuses, the costs of the adjustment and largely ensure their survival, even in the presence of severe downturns. However, this mechanism contradicts the stylized facts of wage adjustments, where one sees a significant rise of wage freezes, and few to none wage reductions. Thus, I propose a mechanism that hampers the internal financing channel of firms and places them at greater risk of failure.

The mechanism consists in combining downward real wage rigidity, which largely disables real downward

wage adjustments, with instantaneous liquidity constraints, where firms with very high leverage levels, measured as the comparison of the period cost of capital versus the EBITDA of the firm, significantly increases the firm's failure rate when it reaches very sizable levels, of above 1. Indeed, as the magnitude of the shock becomes severe, namely above 5 to 6 percent of the stationary equilibrium levels, the model with this mechanism predicts a sizable unemployment surge, with an around 1-to-1 elasticity between the shock magnitude and the unemployment surge, lead by the failure of firms. This finding is compatible with a *catastrophic job destruction phenomena* described in the empirical literature during the Great Recession for Portugal.

Noteworthy, even in the presence of a temporary productivity shock, this mechanism predicts a sizable phenomenon of firms' failure even if they are solvent, as the critical binding constraint becomes liquidity. Thus, the market enters in turmoil with a significant number of workers, particularly permanent workers, facing unemployment with their reemployment dynamics being led by the job creation dynamics. This process is more costly than ensuring the continuity of their job assignments, and can be coined as a *last dance effect* of terminating inter-temporally viable jobs.

The *last dance effect* is critically determined by the leverage level at the dawn of the productivity shock, and the liquidity of firms along the adjustment path. While the managing of the former is fundamentally to be performed prior to the shock, the managing of the latter can be made once the shock has materialized. Accordingly, I devise several balanced-budget policies to enhance the internal financing channel in the trough of the downturn, in order to alleviate the last dance effect. Options entangle instantaneously transferring resources from permanent workers to firms, inter-temporally changing the profiles firm's interest rate payments when they have permanent workers, relaxing them in the trough of the downturn, or even by resorting to future savings in unemployment benefits to subsidize permanent jobs. For severe downturns above 7 percent, every policy, or combination of policies, proved to be effective in reducing by almost 20 percent the maximum recorded unemployment rate under a no policy scenario. Those also translates into less capital destruction, and they have positive welfare effects across the board. Interestingly, they prescribe different wage profiles along the adjustment, opening the possibility of the policy maker to select the most acceptable path, also considering potential public financing constraints.

All in all, in the presence of a market where wage flexibility is impaired and there are instantaneous financing constraints, the labour market adjustment would benefit from policies devised to partially open the internal financing channel of firms, when the downfall is predicted to be strong or severe. The lack of action can spark catastrophic job destruction spells, with significant costs in terms of unemployment and welfare. In this paper, I addressed the question resorting to aggregate shocks and homogeneous policies targeting permanent jobs. While this proves to be effective under the conditions of the proposed model, it also opens the question of how much more powerful could be targeted policies that takes into account the much richer information about each firm's financial standing that policy makers have access to. Altogether, it reinforces the merit of relaxing institutionally defined constraints in extraordinary periods, too minimize costly *last dance effects*.

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Appendix A: Derivation of Unconstrained Wages

As referred in the text, the wage setting follows a rent-sharing rule assuming: (a) the potential promotion of a temporary job into a permanent job entails the extinguishing of the temporary job position, requiring the firm to potentially post a new vacancy; and (b) the wage setting do not consider the subsequent attribution of bonuses, that is only defined once profits are determined. Hence for wage setting purposes $\eta(a, j, t) = 0$ with $j \in \{T, P\}$.

Permanent contract wages. Given the assumption framework, the wages are defined considering the value functions and the rent-splitting rule. Accordingly, consider the set of equations:

$$\beta \bigg[J(a, P, t) + S_P \bigg] = (1 - \beta) \bigg[W(a, P, t) - W(a, U, t) \bigg];$$

$$r(t) J(a, P, t) = \ddot{z}F(\ddot{k}) - r_l(t)\ddot{k} - \tilde{w}(a, P, t) + \partial_a J(a, P, t)\dot{a}(a, P, t) + \sigma_P(\ddot{\xi}_P)[-S_P - J(a, P, t)] + \partial_t J(a, P, t);$$

$$\rho W(a, P, t) = u(\ddot{c}) + \partial_a W(a, P, t)\dot{a}(a, P, t) + \sigma_P(\ddot{\xi}_P) \bigg[W(a, U, t) - W(a, P, t) \bigg] + \partial_t W(a, P, t)$$
(1)

 $\dot{a}(a,P,t) = \tilde{w}(a,P,t) + (r_l - \delta)a - \ddot{c}$

Solving this set of equations, I have:

$$\tilde{w}(a,P,t) = \frac{\beta \left[\rho + \sigma_P(\ddot{\xi}_P) \right] \left\{ \ddot{z}F(\ddot{k}) - r_l(t)\ddot{k} + \partial_a J(a,P,t) \left(r(t)a - \ddot{c} \right) + r(t)S_P + \partial_t J(a,P,t) \right\} }{[1 - \beta][r(t) + \sigma_P(\ddot{\xi}_P)]\partial_a W(a,P,t) + \beta[\rho + \sigma_P(\ddot{\xi}_P)][1 - \partial_a J(a,P,t)]} - \frac{(1 - \beta) \left[r(t) + \sigma_P(\ddot{\xi}_P) \right] \left\{ u(\ddot{c}) + \partial_a W(a,P,t) \left(r(t)a - \ddot{c} \right) - \rho W(a,U,t) + \partial_t W(a,P,t) \right\} }{[1 - \beta][r(t) + \sigma_P(\ddot{\xi}_P)]\partial_a W(a,P,t) + \beta[\rho + \sigma_P(\ddot{\xi}_P)][1 - \partial_a J(a,P,t)]}$$

$$(2)$$

Then notice that in the case of a stationary equilibrium, by definition $\partial_t W(a, P, t) = \partial_t J(a, P, t) = 0$.

Temporary contract wages. The wages for the temporary contracts solve the following set of equations:

$$\beta \left[J(a,T,t) \right] = (1-\beta) \left[W(a,T,t) - W(a,U,t) \right];$$

$$r(t)J(a,T,t) = \ddot{z}F(\ddot{k}) - r_l(t)\ddot{k} - \tilde{w}(a,T,t) + \partial_a J(a,T,t)\dot{a}(a,T,t) + [\varphi(\ddot{\xi}_P) + \sigma_T(\ddot{\xi}_T)][-J(a,T,t)] + \partial_t J(a,T,t);$$

$$\rho W(a,T,t) = u(\ddot{c}) + \partial_a W(a,T,t)\dot{a}(a,T,t) + \sigma_T(\ddot{\xi}_T) \left[W(a,U,t) - W(a,T,t) \right] + \varphi(\ddot{\xi}_P) \left[W(a,P,t) - W(a,T,t) \right] + \partial_t W(a,T,t);$$

$$\dot{a}(a,T,t) = \tilde{w}(a,T,t) + (r_l - \delta)a - \ddot{c};$$
(3)

Solving this set of equations, I have:

$$\tilde{w}(a,T,t) = \frac{\beta \left[\rho + \varphi(\ddot{\xi}_{P}) + \sigma_{T}(\ddot{\xi}_{T}) \right] \left\{ \ddot{z}F(\ddot{k}) - r_{l}(t)\ddot{k} + \partial_{a}J(a,T,t) \left(r(t)a - \ddot{c} \right) + \partial_{t}J(a,T,t) \right\}}{[1 - \beta][r(t) + \varphi(\ddot{\xi}_{P}) + \sigma_{T}(\ddot{\xi}_{T})]\partial_{a}W(a,P,t) + \beta[\rho + \varphi(\ddot{\xi}_{P}) + \sigma_{T}(\ddot{\xi}_{T})][1 - \partial_{a}J(a,T,t)]} - \frac{(1 - \beta) \left[r(t) + \varphi(\ddot{\xi}_{P}) + \sigma_{T}(\ddot{\xi}_{T}) \right] \left\{ u(\ddot{c}) + \partial_{a}W(a,P,t) \left(r(t)a - \ddot{c} \right) + \varphi(\ddot{\xi}_{P})W(a,P,t) - [\rho + \varphi(\ddot{\xi}_{P})]W(a,U,t) + \partial_{t}W(a,P,t) \right\}}{[1 - \beta][r(t) + \varphi(\ddot{\xi}_{P}) + \sigma_{T}(\ddot{\xi}_{T})]\partial_{a}W(a,P,t) + \beta[\rho + \varphi(\ddot{\xi}_{P}) + \sigma_{T}(\ddot{\xi}_{T})][1 - \partial_{a}J(a,T,t)]}}$$

$$(4)$$

Then notice that in the case of a stationary equilibrium, by definition $\partial_t W(a, T, t) = \partial_t J(a, T, t) = 0$.

Appendix B: The Definition of the Stationary Equilibrium

In this section we provide a calibration for the model and present its consequent stationary equilibrium. Formally, the stationary recursive equilibrium consists of (a) set of value functions $\{J(a, P, t), J(a, T, t), W(a, P, t), W(a, T, t), V(t), W(a, U, t)\}$; (b) a set of policy functions $\{c(a, P, t), c(a, T, t), c(a, U, t), \dot{a}(a, P, t), \dot{a}(a, T, t), \dot{a}(a, U, t)\}$; (c) a distribution over assets and employment $\{g(a, P, t), g(a, T, t), g(a, U, t)\}$; (d) a capital stock per firm k(a, T, t) and k(a, P, t); (e) a labour market tightness $\theta(t)$; (f) the firm's dividents D(t) and (g) a set of prices $\{r(t), r_l(t), Q(t)\}$ such that:

- the workers solve the problems in equations (5), (6) and (7) given prices, the capital stock and labour market tightness;
- the firms defines the capital stock by solving the problems in equations (13) and (14), given prices and the labour market tightness;
- the job finding and filling probabilities are as explained in the frictional labour market description part;
- free entry condition holds as presented in equation (12);
- the stationary distributions over assets and employment satisfy the standard Kolmogorov forward equations;
- the wage setting satisfies the rent-sharing rule;
- the asset market clears according to equation (3), the price of equity corresponds to the present value of the dividend string as in equation (16), and the arbitrage condition as in equation (1) holds;
- The final good is produced in firms and at home and it is spent in consumption and investment in physical capital, vacancy creation and firing taxes:

$$\int_{a}^{\infty} \sum_{i=\{P,T,U\}} c(a,i,t)g(a,i,t)da + \int_{a}^{\infty} \sum_{i=\{P,T,U\}} [\dot{k}(a,i,t) - \delta k(a,i,t)]g(a,i,t)da + \xi v = \int_{a}^{\infty} \sum_{i=\{P,T\}} [z_{t}(a,i)F(k(a,i,t)]g(a,i,t)da + u_{t}h]$$
(1)

Appendix C: Algorithm of Stationary Equilibrium

The stationary equilibrium results from a fixed-point algorithm over $\{\theta, r(t)\}$. The HJB equations are solved via the finite difference method as described in Achdou et al. (2017). This section follows closely the work of Bardóczy (2017), with the required adjustments to the concrete model implemented. Accordingly, the algorithm unfolds through the following steps:

Step 1. Set up the grid for assets. In this computations, I have $a \in [0, 150]$, with 501 points. The choice reflects: (a) the resulting stationary equilibrium estimate, which is well covered in the grid; (b) the idea that the upper bound is quite irrelevant for the agent's decisions in the relevant range; and (c) the computational requirements to efficiently run the algorithm.

Step 2. Guess the tightness θ in the market. The calibration implemented seeks a $\theta = 1$ in equilibrium. Accordingly, the cost of posting a vacancy, γ , is adjusted to attain such objective. Then, given tightness, one obtains:

$$\theta q(\theta) = \chi \theta^{1-\eta}; \quad q(\theta) = \chi \theta^{-\eta}; \tag{1}$$

Step 3. Guess the capital per worker k for each type of contract, and the cost of capital $r_1(t)$. So we use as initial guess:

$$k(a,T,t) = \left[\frac{\alpha z(a,T,t)}{r_l(t)}\right]^{\frac{1}{1-\alpha}}; \quad k(a,P,t) = \left[\frac{\alpha z(a,P,t)}{r_l(t)}\right]^{\frac{1}{1-\alpha}}; \quad r_l(t) = r(t) + \delta;$$
(2)

where r(t) is initially set at 0.004.

Step 4. Define the wage schedules w(a, T, t) and w(a, P, t). I start with an initial guess where we have:

$$w(a, j, t) = \beta \left[z(a, j, t) k(a, j, t)^{\alpha} - r_l(t) k(a, j, t) \right], \quad j \in \{T, P\}.$$
(3)

Step 5. Define the leverage of firms $\xi(\mathbf{a}, \mathbf{T}, \mathbf{t})$ and $\xi(\mathbf{a}, \mathbf{P}, \mathbf{t})$, and define the flow probabilities. Accordingly, the leverage of the firms is defined as:

$$\xi(a,j,t) = \frac{r_l(t)k(a,j,t)}{z(a,j,t)k(a,j,t)^{\alpha} - w(a,j,t)}, \quad j \in \{T,P\}.$$
(4)

Then, define the flow probabilities $\sigma_j[\xi(a, j, t)]$ with $j \in \{T, P\}$, $h[\xi(a, T, t)]$ and $\varphi[\xi(a, P, t)]$.

Step 6. Solve the worker's problem. The discretized HJB equations become:

$$\rho W(a,i,t) = u[c(a,i,t)] + \underbrace{\frac{W(a+\epsilon,i,t) - W(a,i,t)}{\Delta a}}_{\partial_a W^F(a,i,t)} \dot{a}(a,j,t)^+ + \underbrace{\frac{W(a,i,t) - W(a-\epsilon,i,t)}{\Delta a}}_{\partial_a W^B(a,i,t)} \dot{a}(a,j,t)^- + \lambda_{i,i'}[W(a,i',t) - W(a,i,t)]$$

$$(5)$$

with $i \in \{T, P, U\}$. The consumption associated with increasing, decreasing or unchanging assets is defined as:

$$c^{F}(a,i,t) = (\partial_{a}W^{F}(a,i,t))^{-1}, \quad c^{B}(a,i,t) = (\partial_{a}W^{B}(a,i,t))^{-1}, \quad c^{0}(a,i,t) = y(a,i,t) + r(t)a.$$
(6)

Accordingly, the savings decisions become:

$$\dot{a}^{F}(a,i,t) = y(a,i,t) - r(t)a - c^{F}(a,i,t), \quad \dot{a}^{B}(a,i,t) = y(a,i,t) - r(t)a - c^{B}(a,i,t).$$
(7)

By concavity of the value function,

$$\partial_a W^F(a,i,t) < \partial_a W^B(a,i,t) \to \dot{a}^F(a,i,t) < \dot{a}^B(a,i,t).$$
(8)

As standard, one defines 3 cases:

$$I^{B} = \{(a, i, t) : \dot{a}^{B} < 0\}; \quad I^{F} = \{(a, i, t) : 0 < \dot{a}^{F}\}; \qquad I^{0} = \{(a, i, t) : \dot{a}^{F} < 0 < \dot{a}^{B}\}, \tag{9}$$

representing a certain decreasing in assets, a certain increasing in assets of an ambiguous situation. Then, I implement the upwinding scheme, with the final policies as:

$$c(a,i,t) = \mathbb{1}_{\{I^F\}} c^F(a,i,t) + \mathbb{1}_{\{I^B\}} c^B(a,i,t) + \mathbb{1}_{\{I^0\}} c^0(a,i,t),$$

$$\dot{a}(a,i,t) = y(a,i,t) - r(t)a - c(a,i,t).$$
(10)

Let l index to represent the iterations of the value function. In matrix form, the HJB equation can be written as:

$$\frac{W^{l+1} - W^l}{\Delta} = u^l + (\mathbf{A}^l + \mathbf{\Lambda}^l)W^{l+1} - \rho W^l$$
(11)

where \mathbf{A}^l represents the transition matrix along the asset dimension. It is defined as:

$$\mathbf{A}^{l} = \begin{bmatrix} \mathbf{A}_{\mathbf{P}} & 0 & 0 \\ 0 & \mathbf{A}_{\mathbf{T}} & 0 \\ 0 & 0 & \mathbf{A}_{\mathbf{U}} \end{bmatrix}, \quad \mathbf{A}_{\mathbf{i}} = \begin{bmatrix} y(1,i,t) & z(1,i,t) & 0 & 0 & \dots & 0 \\ x(2,i,t) & y(2,i,t) & z(2,i,t) & 0 & \dots & 0 \\ 0 & x(3,i,t) & y(3,i,t) & z(3,i,t) & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & x(150,i,t) & y(150,i,t) \end{bmatrix}, \quad \begin{cases} x(a,i,t) = \frac{-[(a)(a,i,t)^{-}]}{\Delta a} \\ z(a,i,t) = \frac{-[(a)(a,i,t)^{+}]}{\Delta a} \\ y(a,i,t) = -x(a,i) - z(a,i,t). \end{cases}$$
(12)

The matrix $\mathbf{\Lambda}^l$ represents the transition matrix on the employment dimmension, and it is defined as:

$$\Lambda^{l} = \begin{bmatrix} \Lambda_{\mathbf{P},\mathbf{P}} & \mathbf{0} & \Lambda_{\mathbf{P},\mathbf{U}} \\ \Lambda_{\mathbf{T},\mathbf{P}} & \Lambda_{\mathbf{T},\mathbf{T}} & \Lambda_{\mathbf{T},\mathbf{U}} \\ \mathbf{0} & \Lambda_{\mathbf{U},\mathbf{T}} & \Lambda_{\mathbf{U},\mathbf{U}} \end{bmatrix}$$
(13)

where $\Lambda_{i,i'}$ is a diagonal matrix with the flow probability of transitioning from *i* to *i'* along the diagonal, given the level of leverage of the particular assignment. Accordingly, the set of flow probabilities are given by the functions:

$$\begin{cases} \mathbf{\Lambda}_{\mathbf{P},\mathbf{P}}(m,m) = -\sigma_{P}[\xi(m,P,t)] \\ \mathbf{\Lambda}_{\mathbf{T},\mathbf{T}}(m,m) = \sigma_{P}[\xi(m,P,t)] \end{cases}, \begin{cases} \mathbf{\Lambda}_{\mathbf{T},\mathbf{P}}(m,m) = \varphi[\xi(m,P,t)] \\ \mathbf{\Lambda}_{\mathbf{T},\mathbf{T}}(m,m) = -\varphi[\xi(m,P,t)] - \sigma_{T}[\xi(m,T,t)] \end{cases}, \begin{cases} \mathbf{\Lambda}_{\mathbf{U},\mathbf{T}}(m,m) = \theta q(\theta) h[\xi(m,T,t)] \\ \mathbf{\Lambda}_{\mathbf{U},\mathbf{U}}(m,m) = -\theta q(\theta) h[\xi(m,T,t)] \end{cases} \end{cases}$$

$$(14)$$

with m representing the positioning in the grid of assets.

Step 7. Stationary distribution of workers. I solve the discretized version of the Kolmogorov forward equations as:

$$(\mathbf{A}^l + \mathbf{\Lambda}^l)' g = 0. \tag{15}$$

Given this equation has infinitely many solutions, the solution is pinned down by the fact that g is a density and thus integrate to 1. So as standard, I fix the first row of $(\mathbf{A}^{l} + \mathbf{\Lambda}^{l})'$ to be [1, 0, ..., 0] and then rescale the density g to integrate to 1.

Step 8. Solve the firm's problem. The discretized HJB equation for the firm is solved through the system given by:

$$\frac{J^{l+1} - J^{l}}{\Delta} = \ddot{z}F(\ddot{k}) - r_{l}(t)\ddot{k} - w(a,j,t) - \sigma_{j}(\xi(m,j,t))S_{j} + \mathbf{A}_{j}^{l} - (r(t) + \tilde{\sigma}[\xi(m,j,t)]J^{l+1}, \quad j \in \{T, P\},
\tilde{\sigma}[\xi(m,j,t)] = \begin{cases} \tilde{\sigma}[\xi(m,P,t)] = \sigma_{P}[\xi(m,P,t)]; \\ \tilde{\sigma}[\xi(m,T,t)] = \sigma_{T}[\xi(m,T,t)] + \varphi[\xi(m,P,t)]. \end{cases}$$
(16)

Step 9. Evaluate the Free Entry condition:

$$FE = -\gamma + q[\theta(t)] \sum_{a} J(a, T, t) \frac{g(a, U, t)}{u(t)} \Delta a.$$
(17)

If FE is positive, more firms enter the market of temporary jobs and thus θ will have to increase. If negative, more firms leave or not enter the market and thus θ will be decreased. Accordingly, the function that adjusts the guess is given by:

$$\theta^{l+1} = \theta^l + \Delta_\theta F E^l, \tag{18}$$

where Δ_{θ} regulates the adjustment of the guess.

Step 10. Evaluate the asset clearing condition. The asset clearing condition is given by:

$$AD = \sum_{a} \sum_{i \in \{T,P,U\}} ag(a,i,t)\Delta a - \vartheta \left[\sum_{a} \sum_{i \in \{T,P,U\}} k(a,i,t)g(a,i,t)\Delta a + \frac{\sum_{a} \sum_{j \in \{T,P\}} [\ddot{z}F(\ddot{k}) - r_l(t)\ddot{k} - w(a,j,t)]g(a,i,t)\Delta a - \gamma v(t)}{r(t)} \right],$$

$$(19)$$

where v(t) is calculated given $\theta(t) = \frac{v(t)}{u(t)}$, and:

$$u(t) = \sum_{a} [g(a, T, t) + g(a, P, t)] \Delta a.$$
(20)

If AD^{l} is positive, then the guess on r(t) is too low and r(t) should be increased. If AD^{l} is negative then the guess on r(t) is too high. Accordingly, I adjust it following:

$$r(t)^{l+1} = r(t)^{l} + \Delta r.AD^{l}, \tag{21}$$

with Δr regulating the speed of adjustment of the guess.

Step 11. Update the wages. Update wages following the results of appendix A.

Step 12. Update θ and r(t). Update θ and r(t) until there is convergence from step 6 onward.

Appendix D: Algorithm of Transitional Dynamics

The algorithm to describe the transitional dynamics given the aggregate productivity shock is standard, as described in Bardóczy (2017). The economy starts from the stationary equilibrium and returns to it. The algorithm iterates backwards the forward looking variables as prices and value functions, and iterates states forwards. The fixed point theorem algorithm is implemented over p_0 and the sequences of $\{r(t), \theta(t)\}_t$. The algorithm needs to guess the equity price on impact, p_0 as it implies a revaluation of the assets of the agents in the economy. In detail, the algorithm entails solving the following steps:

Step 1. Solve the stationary equilibrium. The stationary equilibrium described in appendix C will provide the initial and terminal description of the economy.

Step 2. Set up a grid for time steps. The algorithm defines N - 1 time steps from N time points where the economy will be evaluated. Accordingly, we have:

$$t_n = \begin{cases} 0, & n = 1 \\ \sum_{m=1}^{N-1} \Delta t_m & n = 2, \dots, N \end{cases}$$
(1)

The grid is defined as a non-uniform grid, that gets wider as n increases. As standard, the objective is to have a dense map of points right after the initial shock, which then gets sparser as the time elapses and consequently the transition becomes smoother towards the stationary equilibrium.

Step 3. Define the aggregate shock process. The aggregate shock and the subsequent transition path of productivity towards stationary equilibrium levels is given by:

$$z_t = 1 + (z_0 - 1)e^{-0.3t}.$$
(2)

Step 4. Guess the path for tightness $\theta(t)$ and the interest rate r(t). The guess for theta allows for the definition of:

$$\theta(t)q[\theta(t)] = \chi\theta(t)^{1-\eta}; \quad q[\theta(t)] = \chi\theta(t)^{-\eta}.$$
(3)

With the interest rate guess, the capital variables are also defined as:

$$k(a,T,t) = \left[\frac{\alpha z(a,T,t)}{r_l(t)}\right]^{\frac{1}{1-\alpha}}; \quad k(a,P,t) = \left[\frac{\alpha z(a,P,t)}{r_l(t)}\right]^{\frac{1}{1-\alpha}}.$$
(4)

Step 5. Iterate the HJB equations. The idea is to iterate the HJB equations backwards. So start from the terminal conditions, $W^N = W$ and $J^N = J$, where J and W corresponds to the HJB of the stationary equilibrium. Then, the method is analogous to the stationary equilibrium algorithm and solves the following

system:

$$\frac{W^{n} - W^{n+1}}{\Delta t_{n}} = u^{n+1} + (\mathbf{A}^{n+1} + \mathbf{\Lambda}^{l})W^{n} - \rho W^{n+1}
\frac{J^{n} - J^{n+1}}{\Delta} = [\ddot{z}F(\ddot{k}) - r_{l}(t)\ddot{k} - w(a, j, t) - \sigma_{j}(\ddot{\xi}_{j})S_{j}]^{n+1} + \mathbf{A}_{j}^{n+1} - [(r(t) + \tilde{\sigma}(\ddot{\xi}_{j})]^{n+1}J^{n}, \quad j \in \{T, P\},
\tilde{\sigma}[\ddot{\xi}_{j}]^{n+1} = \begin{cases} [\tilde{\sigma}(\ddot{\xi}_{P})]^{n+1} = [\sigma_{P}(\ddot{\xi}_{P})]^{n+1}; \\ [\tilde{\sigma}(\ddot{\xi}_{T})]^{n+1} = [\sigma_{T}(\ddot{\xi}_{T}) + \varphi(\ddot{\xi}_{P})]^{n+1}. \end{cases}$$
(5)

Step 6. Adjust wages. In the standard transitional dynamics algorithm, wages are subjected to downward real wage rigidity. Accordingly, we have that wages are adjusted given:²³

$$\begin{split} \tilde{w}(a,T,n+1) &= \frac{\beta \left[\rho + \varphi(\tilde{\xi}_{P}) + \sigma_{T}(\tilde{\xi}_{T}) \right] \left\{ \tilde{z}F(\tilde{k}) - r_{l}(n+1)\tilde{k} + \partial_{a}J(a,T,n+1) \left(r(n+1)a - \tilde{c} \right) + \partial_{t}J(a,T,n+1) \right\} \\ &= \frac{\beta \left[\rho + \varphi(\tilde{\xi}_{P}) + \sigma_{T}(\tilde{\xi}_{T}) \right] \partial_{a}W(a,P,n+1) + \beta \left[\rho + \varphi(\tilde{\xi}_{P}) + \sigma_{T}(\tilde{\xi}_{T}) \right] \left[1 - \partial_{a}J(a,T,n+1) \right]}{\left[1 - \beta \right] \left[r(n+1) + \varphi(\tilde{\xi}_{P}) + \sigma_{T}(\tilde{\xi}_{T}) \right] \left\{ u(\tilde{c}) + \partial_{a}W(a,P,n+1) \left(r(n+1)a - \tilde{c} \right) + \varphi(\tilde{\xi}_{P})W(a,P,n+1) - \left[\rho + \varphi(\tilde{\xi}_{P}) \right] W(a,U,n+1) + \partial_{t}W(a,P,n+1) \right\}}{\left[1 - \beta \right] \left[r(n+1) + \varphi(\tilde{\xi}_{P}) + \sigma_{T}(\tilde{\xi}_{T}) \right] \partial_{a}W(a,P,n+1) + \beta \left[\rho + \varphi(\tilde{\xi}_{P}) + \sigma_{T}(\tilde{\xi}_{T}) \right] \left[1 - \partial_{a}J(a,T,n+1) \right]} \end{split}$$
(6)
$$\tilde{w}(a,P,n+1) &= \frac{\beta \left[\rho + \sigma_{P}(\tilde{\xi}_{P}) \right] \left\{ \tilde{z}F(\tilde{k}) - r_{l}(n+1)\tilde{k} + \partial_{a}J(a,P,n+1) \left(r(t)a - \tilde{c} \right) + r(n+1)S_{P} + \partial_{t}J(a,P,n+1) \right\} \right.}{\left[1 - \beta \right] \left[r(n+1) + \sigma_{P}(\tilde{\xi}_{P}) \right] \partial_{a}W(a,P,n+1) + \beta \left[\rho + \sigma_{P}(\tilde{\xi}_{P}) \right] \left[1 - \partial_{a}J(a,P,n+1) \right]} - \frac{\left(1 - \beta \left[r(n+1) + \sigma_{P}(\tilde{\xi}_{P}) \right] \partial_{a}W(a,P,n+1) + \beta \left[\rho + \sigma_{P}(\tilde{\xi}_{P}) \right] \left[1 - \partial_{a}J(a,P,n+1) \right]}{\left[1 - \beta \right] \left[r(n+1) + \sigma_{P}(\tilde{\xi}_{P}) \right] \partial_{a}W(a,P,n+1) + \beta \left[\rho + \sigma_{P}(\tilde{\xi}_{P}) \right] \left[1 - \partial_{a}J(a,P,n+1) \right]} \right]}{\left[1 - \beta \left[r(n+1) + \sigma_{P}(\tilde{\xi}_{P}) \right] \partial_{a}W(a,P,n+1) + \beta \left[\rho + \sigma_{P}(\tilde{\xi}_{P}) \right] \left[1 - \partial_{a}J(a,P,n+1) \right]} \right]}.$$

$$w(a,j,n+1) = \begin{cases} \tilde{w}(a,j,n+1) & \text{if } \tilde{w}(a,j,n+1) \ge \lambda w(a,j,n) \\ w(a,j,n) & \text{if } \tilde{w}(a,j,n+1) < \lambda w(a,j,n) \end{cases}, \quad j \in \{T,P\},$$

$$(7)$$

If in a particular exercise wages are nt subjected to downward real wage rigidity then it is simply a matter to relax the constraint imposed in equation (7).

Step 7. Revalue assets in the after the initial shock. Let p represent the price of equity in the stationary equilibrium and p_0 the price on impact. An agent that owns s_i share of the total assets in the economy, $a_i = s_i(K + p)$, will have its assets updated on impact as:

$$a'_{i} = s_{i}(K + p_{0}) = a_{i} \left[1 + \frac{p_{0} - p}{K + p} \right].$$
(8)

Further, assume that agents can't do anything about the revaluation, and thus $g_0(a',i) = g(a,i)$, where $i = \{T, P, U\}$. The g_0 density at old grid points $\{a_i\}$ is found be interpolation. I follow the insight of Bardóczy (2017) and interpolate directly g by resulting to MATLAB's *pchip* method, and then rescale it.

Step 8. Iterate the Kolmogorov forward equations. Given the revalued stationary distribution, after the reassessment of prices, I iterate the Kolmogorov forward equations. Accordingly, using the implicit method, it

²³As in step 5 the $\ddot{\bullet}$ variables refer to period n+1. For notation ease I don't make it explicit in the formulae as it was done for step 5 with $[\bullet]^{n+1}$.

consists in solving:

$$\frac{g_{n+1} - g_n}{\Delta t_n} = \left[\mathbf{A}^n + \mathbf{\Lambda}^n\right]' g_{n+1}.$$
(9)

Step 9. Calculate unemployment and vacancies. So given the iterated distributions, we have:

$$u(n) = \sum_{a} g(a, U, n) \Delta a, \quad v(n) = \theta(n)u(n).$$
(10)

Step 10. Evaluate the free entry condition. The set of free entry conditions along the transitional dynamics is given by:

$$FE_n = -\gamma + q[\theta(t)]_n \sum_a J(a, T, n) \frac{g(a, U, n)}{u(n)} \Delta a.$$
(11)

Step 11. Evaluate the asset market clearing condition. The arbitrage condition implies that one can solve the price of equity backwards. Thus one obtains:

$$\frac{p_{n+1} - p_n}{\Delta t_n} = p_n(r(n) - d(n)), \tag{12}$$

where d(n) corresponds to the dividends, given by:

$$d(n) = (1-\psi) \sum_{a} \left[\ddot{z}F(\ddot{k}) - r_l(n)\ddot{k} - w(a,T,n) \right] g(a,T,n) \Delta a + (1-\psi) \sum_{a} \left[\ddot{z}F(\ddot{k}) - r_l(n)\ddot{k} - w(a,P,n) \right] g(a,P,n) \Delta a - \gamma v.$$
(13)

Given the path of prices, the capital in each firm and the distribution of assets, we have:

$$AD_{n} = \sum_{a} \sum_{i \in \{T, P, U\}} ag(a, i, n) \Delta a - \vartheta \left[\sum_{a} \sum_{i \in \{T, P, U\}} k(a, i, n) g(a, i, n) \Delta a + \frac{\sum_{a} \sum_{j \in \{T, P\}} [\ddot{z}F(\ddot{k}) - r_{l}(t)\ddot{k} - w(a, j, n)]g(a, i, n) \Delta a - \gamma v(n)}{r(n)} \right].$$

$$(14)$$

Step 12. Adjust \mathbf{p}_0 and $\{\mathbf{r}(\mathbf{n}), \theta(\mathbf{n})\}_{\mathbf{n}}^{\mathbf{N}-1}$. Accordingly, adjust: (a) the price on impact based on the no-arbitrage condition; (b) the labour market tightness based on the free entry condition; and (c) the interest rate based on the excess demand for assets condition. Consequently, consider:

$$p_0^{l+1} = \Delta_p p_0 + (1 - \Delta_p) p_0^l$$

$$r(n)^{l+1} = r(n)^l + \Delta_r A D_n \qquad (15)$$

$$\theta(n)^{l+1} = \theta(n)^l + \Delta_{FE} F E_n.$$

The iterate from step 5 onward, until reaching convergence.

Appendix E: Details on the algorithms for policy analysis.

Algorithm for transitional dynamics under flexible wages. The algorithm to estimate the transitional dynamics under flexible wages is identical to the algorithm in appendix D, with the sole exception residing in relaxing equation (7), which has no applicability.

Altogether, the transitional dynamics are therefore estimated in a fully comparable way with the downward real rigid wages. Paramount to this approach is the fact that the stationary equilibrium under both scenarios is identical, as the downward real wage rigidity is not binding in the equilibrium.

Algorithm for transitional dynamics under the unemployment benefit transfer policy. This algorithm differs from the one presented in appendix D in two major ways. First, during the implementation period of the subsidy, wages, leverage and profits are adjusted to include the subsidy. Secondly, the algorithm needs to ensure the budget balancing of the policy.

Notice that the discretization of the algorithm implies that the time steps are matched to quarters. Accordingly, the subsidy is provided in the periods $t \in [1; 5]$, corresponding to 4 quarters of implementation and 1 quarter of implementation lag.

The profits become:

$$\tilde{\eta}(a, P, t) = \left[\ddot{z}F(\ddot{k}) - r_l(t)\ddot{k} - (1 - \tau^{ub})w(a, P, t)\right], \text{ if } t \in [1, 5],$$
(16)

and the leverage of the firm becomes:

$$\xi(a, P, t) = \frac{r_l(t)}{z(a, P, t)F[k(a, P, t)] - (1 - \tau^{ub})w(a, P, t)}, \quad \text{if } t \in [1, 5].$$
(17)

Accordingly, the step 5 of the standard algorithm is duly adjusted, namely the HJB equations for the value of the job. Noteworthy, the HJB equations for the workers do not change.

The set of equations defining the permanent contract wages during the implementation period becomes:

$$\beta \left[J(a, P, t) + S_P \right] = (1 - \beta) \left[W(a, P, t) - W(a, U, t) \right];$$

$$r(t)J(a, P, t) = \ddot{z}F(\ddot{k}) - r_l(t)\ddot{k} - (1 - \tau^{ub})\tilde{w}(a, P, t) + \partial_a J(a, P, t)\dot{a}(a, P, t) + \sigma_P(\ddot{\xi}_P)[-S_P - J(a, P, t)] + \partial_t J(a, P, t) + \rho_W(a, P, t) = u(\ddot{c}) + \partial_a W(a, P, t)\dot{a}(a, P, t) + \sigma_P(\ddot{\xi}_P) \left[W(a, U, t) - W(a, P, t) \right] + \partial_t W(a, P, t)$$

$$\dot{a}(a, P, t) = \tilde{w}(a, P, t) + (r_l - \delta)a - \ddot{c}$$
(18)

with the resulting permanent contract wages, relevant for step 6 defined as:

$$\tilde{w}(a,P,t) = \frac{\beta \left[\rho + \sigma_P(\ddot{\xi}_P) \right] \left\{ \ddot{z}F(\ddot{k}) - r_l(t)\ddot{k} + \partial_a J(a,P,t) \left(r(t)a - \ddot{c} \right) + r(t)S_P + \partial_t J(a,P,t) \right\}}{[1 - \beta][r(t) + \sigma_P(\ddot{\xi}_P)]\partial_a W(a,P,t) + \beta[\rho + \sigma_P(\ddot{\xi}_P)][(1 - \tau^{ub}) - \partial_a J(a,P,t)]} - \frac{(1 - \beta) \left[r(t) + \sigma_P(\ddot{\xi}_P) \right] \left\{ u(\ddot{c}) + \partial_a W(a,P,t) \left(r(t)a - \ddot{c} \right) - \rho W(a,U,t) + \partial_t W(a,P,t) \right\}}{[1 - \beta][r(t) + \sigma_P(\ddot{\xi}_P)]\partial_a W(a,P,t) + \beta[\rho + \sigma_P(\ddot{\xi}_P)][(1 - \tau^{ub}) - \partial_a J(a,P,t)]}, \quad \text{if } t \in [1,5].$$

$$(19)$$

The temporary contract wages are unchanged.

The remainder of the steps from step 7 to step 12 are unchanged, other than for the adjustment of dividends during the implementation period, where it becomes:

$$d(n) = (1 - \psi) \sum_{a} \left[\ddot{z}F(\ddot{k}) - r_{l}(n)\ddot{k} - w(a, T, n) \right] g(a, T, n) \Delta a + + (1 - \psi) \sum_{a} \left[\ddot{z}F(\ddot{k}) - r_{l}(n)\ddot{k} - (1 - \tau^{ub})w(a, P, n) \right] g(a, P, n) \Delta a - \gamma v, \text{ if } t \in [1, 5].$$
(20)

In step 12, the convergence needs to be supplemented with the balanced budget criteria. The savings and costs from the policy are given by:

$$ub^{sav} = \sum_{n=1}^{N} \left(u(n) - u(n)^{DRWR} \right) b;$$

$$ub^{exp} = \sum_{n \text{ if } t \in [1,5]} \sum_{a} \tau^{ub} \left(w(a, P, n)g(a, P, n) \right) \Delta a.$$
(21)

The convergence of the fixed point algorithm is supplemented with the balanced budget condition as:

$$B^{ub,l} = ub^{sav,l} - ub^{exp,l};$$

$$\tau^{ub,l+1} = \tau^{ub,l} + \Delta_{ub}B^{ub,l}.$$
(22)

Intuitively, notice that if the policy is generating savings, then the *ad-valorem* rate of the subsidy can be increased, whereas if the policy is generating deficits the rate should be reduced.

The algorithm iterates from step 5, with the referred changes until a convergence is reached.

Algorithm for transitional dynamics under the income tax and firm subsidy transfer. In this policy the budget is permanently balanced, as the policy consists in an instantaneous transfer between workers and firms. However, given this property, just ensuring budget balance do not pin point a single calibration of the policy. For that purpose, as presented in the text, the policy maker attempts to select the *ad-valorem* rate of transfer that minimizes the disturbance in employment and unemployment flows.

Consequently, the algorithm to run this policy consists in an inner loop algorithm, which consists in an adjusted version of the one proposed in appendix D, whereas the outer loop consists in a minimization algorithm of the policy objective, by selecting the *ad-valorem* rate of transfer.

As in every policy algorithm, the discretization of the algorithm implies that the time steps are matched to quarters. Accordingly, the subsidy is provided in the periods $t \in [1;5]$, corresponding to 4 quarters of implementation and 1 quarter of implementation lag. Thus, by comparison with the algorithm in appendix D, the inner algorithm is adjusted to incorporate the policy. The profits become:

$$\tilde{\eta}(a, P, t) = \left[\ddot{z}F(\ddot{k}) - r_l(t)\ddot{k} - (1 - \tau^{ts})w(a, P, t)\right], \text{ if } t \in [1, 5],$$
(23)

and the leverage of the firm becomes:

$$\xi(a, P, t) = \frac{r_l(t)k(a, P, t)}{z(a, P, t)F[k(a, P, t)] - (1 - \tau^{ts})w(a, P, t)}, \quad \text{if } t \in [1, 5].$$
(24)

Accordingly, the step 5 of the standard algorithm is duly adjusted, namely the HJB equations for the value of the job. Noteworthy, the HJB equations for the workers do not change, even though the income of the individuals is adjusted so that:

$$y(a, i, t) = \begin{cases} (1 - \tau^{ts})w(a, P, t) + \eta(a, P, t) & \text{if } i = P \\ w(a, T, t) + \eta(a, T, t) & \text{if } i = T \\ b & \text{if } i = U \end{cases}$$
(25)

The set of equations defining the permanent contract wages during the implementation period becomes:

$$\beta \left[J(a, P, t) + S_P \right] = (1 - \beta) \left[W(a, P, t) - W(a, U, t) \right];$$

$$r(t)J(a, P, t) = \ddot{z}F(\ddot{k}) - r_l(t)\ddot{k} - (1 - \tau^{ts})\tilde{w}(a, P, t) + \partial_a J(a, P, t)\dot{a}(a, P, t) + \sigma_P(\ddot{\xi}_P)[-S_P - J(a, P, t)] + \partial_t J(a, P, t);$$

$$\rho W(a, P, t) = u(\ddot{c}) + \partial_a W(a, P, t)\dot{a}(a, P, t) + \sigma_P(\ddot{\xi}_P) \left[W(a, U, t) - W(a, P, t) \right] + \partial_t W(a, P, t)$$

$$\dot{a}(a, P, t) = (1 - \tau^{ts})\tilde{w}(a, P, t) + (r_l - \delta)a - \ddot{c}$$
(26)

with the resulting permanent contract wages, relevant for step 6 defined as:

$$\tilde{w}(a,P,t) = \frac{\beta \left[\rho + \sigma_P(\ddot{\xi}_P) \right] \left\{ \ddot{z}F(\ddot{k}) - r_l(t)\ddot{k} + \partial_a J(a,P,t) \left(r(t)a - \ddot{c} \right) + r(t)S_P + \partial_t J(a,P,t) \right\}}{[1 - \beta][r(t) + \sigma_P(\ddot{\xi}_P)](1 - \tau^{ts})\partial_a W(a,P,t) + \beta[\rho + \sigma_P(\ddot{\xi}_P)](1 - \tau^{ts})[1 - \partial_a J(a,P,t)]} - \frac{(1 - \beta) \left[r(t) + \sigma_P(\ddot{\xi}_P) \right] \left\{ u(\ddot{c}) + \partial_a W(a,P,t) \left(r(t)a - \ddot{c} \right) - \rho W(a,U,t) + \partial_t W(a,P,t) \right\}}{[1 - \beta][r(t) + \sigma_P(\ddot{\xi}_P)](1 - \tau^{ts})\partial_a W(a,P,t) + \beta[\rho + \sigma_P(\ddot{\xi}_P)](1 - \tau^{ts})[1 - \partial_a J(a,P,t)]}, \quad \text{if } t \in [1,5].$$

The temporary contract wages are unchanged.

The remainder of the steps from step 7 to step 12 are unchanged, other than for the adjustment of dividends during the implementation period, where it becomes:

$$d(n) = (1 - \psi) \sum_{a} \left[\ddot{z}F(\ddot{k}) - r_{l}(n)\ddot{k} - w(a, T, n) \right] g(a, T, n) \Delta a + + (1 - \psi) \sum_{a} \left[\ddot{z}F(\ddot{k}) - r_{l}(n)\ddot{k} - (1 - \tau^{ts})w(a, P, n) \right] g(a, P, n) \Delta a - \gamma v, \text{ if } t \in [1, 5].$$
(28)

Given these adjustments to the inner algorithm, one obtains the employment and unemployment sequences for each level of τ^{ts} . Then, resorting to the *Fminsearch* procedure of Matlab, I pinpoint the rate that minimizes the objective function of the policy maker, given by:

$$\underset{\tau^{ts} \in [0,1]}{\arg\min} \sum_{n-1}^{N} e^{-\rho n} \left\{ \left| e_T(n) + e_P(n) - e_T^s - e_P^s \right| + \left| u(n) - u^s \right| \right\} dn.$$
(29)

where e_P^s , e_T^s and u^s correspond to the stationary equilibrium levels of employment and unemployment.

Altogether, this last step solves the outer loop ensuring a policy that is budget balanced by definition, and implemented with the objective to minimize employment fluctuations over the adjustment. Intuitively, it consists in computing the inner algorithm sufficient times to search over the grid of admissible *ad-valorem* rates the one that most fulfills the objective.

Algorithm for temporary wage flexibility transfer. This policy consists in relaxing the downward real wage rigidity for the periods $t \in [1, 5]$. Accordingly, the algorithm is analogous to the one in appendix D with the replacement of equation (7) of the appendix D with equation (36) of the text.

Algorithm for Moratorium in cost of capital transfer. This policy consists in an adjustment of the cost of capital. Critically, during part of the downturn, the policy will temporarily reduce the loan size to the banking institution, providing relief in the leverage ratio perceived by such institution. Then, in the repayment period, the policy will increase the cost of capital to ensure the repayment of the subsidy provided. Given the discretization implemented, the policy is implemented with a quarter of implementation lag, and it is calibrated to consist in a 1-year subsidy, with a 5-year repayment. The interest rate of the loan is set to ensure the budget-balance of the policy.

As the policy is budget-balanced by construction, it requires the calibration of the size of the initial of the subsidy to be selected given the objective of the policy maker. As in the income tax firm subsidy transfer case, I will consider the policy maker intends to minimize the employment fluctuations over the adjustment.

Accordingly, the algorithm of this policy is as defined in appendix D with the required changes to ensure the incorporation of the subsidy, the definition of the interest rate associated with the repayment, and the outer loop to define the optimal size of the credit line to ensure the minimization of employment fluctuations. The step 4 of the algorithm is enlarged to calculate the specificity of the subsidy and repayment profits. For the implementation period, the firms receive a flow as:

$$LC(a, P, t) = \tau^{m} r_{l}^{s} K(a, P)^{s}, \quad \text{if} \quad t \in [1, 5],$$
(30)

where $r_l^s K(a, P)^s$ corresponds to the cost of capital the firm was incurring in the stationary equilibrium. By the end of the implementation period, the total amount provided for the firm corresponds to:

$$TL(a,P) = \tau^{m} r_{l}^{s} K(a,P)^{s} \left[\frac{1 - e^{-4r_{loan}}}{r_{loan}} \right] e^{4r_{loan}},$$
(31)

with r_{loan} corresponding to the quarterly interest rate considered to ensure the budget balance of the policy, and the factor 4 maps into the idea that the cost of capital ease takes place for 4 quarters. Then the loan is paid back in installments for 5 years. Accordingly, the installments become:

$$LC(a, P, t) = -\left[\tau^{m} r_{l}^{s} K(a, P)^{s} \left[\frac{1 - e^{-4r_{loan}}}{r_{loan}}\right] e^{4r_{loan}} \right] \left[\frac{1 - e^{-20r_{loan}}}{r_{loan}}\right]^{-1} \quad \text{if} \quad t \in (5, 25].$$
(32)

Notice that for t > 25 then LC(a, P, t) = 0. Given the policy details as defined, the capital variables are unchanged, as it results from first order condition of the capital. The profit becomes:

$$\tilde{\eta}(a,P,t) = \left[\ddot{z}F(\ddot{k}) - r_l(t)\ddot{k} + LC(a,P,t) - w(a,P,t)\right],\tag{33}$$

and the leverage of the firm becomes:

$$\xi(a, P, t) = \frac{r_l(t)k(a, P, t) - LC(a, P, t)}{z(a, P, t)F[k(a, P, t)] - w(a, P, t)}.$$
(34)

The guess of tightness and interest rate are as defined in step 4. Further, step 5 is unchanged, other than for the adjustment of profits in the HJB equation of the value of the job, and the changes in the leverage of the permanent contract jobs.

Regarding wages, the wages of the permanent contracts are adjusted to consider the existence of the credit

line. Accordingly one have the relevant set of equations as:

$$\beta \bigg[J(a, P, t) + S_P \bigg] = (1 - \beta) \bigg[W(a, P, t) - W(a, U, t) \bigg];$$

$$r(t) J(a, P, t) = \ddot{z}F(\ddot{k}) - r_l(t)\ddot{k} + LC(a, P, t) - \tilde{w}(a, P, t) + \partial_a J(a, P, t)\dot{a}(a, P, t) + \sigma_P(\ddot{\xi}_P)[-S_P - J(a, P, t)] + \partial_t J(a, P, t);$$

$$\rho W(a, P, t) = u(\ddot{c}) + \partial_a W(a, P, t)\dot{a}(a, P, t) + \sigma_P(\ddot{\xi}_P) \bigg[W(a, U, t) - W(a, P, t) \bigg] + \partial_t W(a, P, t)$$

$$\dot{a}(a, P, t) = \tilde{w}(a, P, t) + (r_l - \delta)a - \ddot{c};$$
(35)

which results into permanent contract wages as:

$$\tilde{w}(a,P,t) = \frac{\beta \left[\rho + \sigma_P(\ddot{\xi}_P) \right] \left\{ \ddot{z}F(\ddot{k}) - r_l(t)\ddot{k} + LC(a,P,t) + \partial_a J(a,P,t) \left(r(t)a - \ddot{c} \right) + r(t)S_P + \partial_t J(a,P,t) \right\} - \left[1 - \beta \right] [r(t) + \sigma_P(\ddot{\xi}_P)] \partial_a W(a,P,t) + \beta [\rho + \sigma_P(\ddot{\xi}_P)] [1 - \partial_a J(a,P,t)] - \left[1 - \beta \right] [r(t) + \sigma_P(\ddot{\xi}_P)] \left\{ u(\ddot{c}) + \partial_a W(a,P,t) \left(r(t)a - \ddot{c} \right) - \rho W(a,U,t) + \partial_t W(a,P,t) \right\} - \left[1 - \beta \right] [r(t) + \sigma_P(\ddot{\xi}_P)] \partial_a W(a,P,t) + \beta [\rho + \sigma_P(\ddot{\xi}_P)] [1 - \partial_a J(a,P,t)] \right].$$
(36)

From steps 7 to step 11 the algorithm remains analogous to the one presented in appendix D, other than for the adjustment of dividends during the implementation period, where it becomes:

$$d(n) = (1 - \psi) \sum_{a} \left[\ddot{z}F(\ddot{k}) - r_{l}(n)\ddot{k} - w(a, T, n) \right] g(a, T, n)\Delta a + (1 - \psi) \sum_{a} \left[\ddot{z}F(\ddot{k}) - r_{l}(n)\ddot{k} + LC(a, P, n) - w(a, P, n) \right] g(a, P, n)\Delta a - \gamma v, \text{ if } t \in [1, 5].$$
(37)

Regarding step 12, one needs to enlarge the convergence adjustment conditions to include the definition of the interest rate of the policy. Accordingly, consider that:

$$ADT(r_{loan}) = \sum_{n \text{ if } t \in [1,5]} \sum_{a} \tau^m r_l^s K(a, P)^s e^{-r_{loan}t} g(a, P, t) \Delta a,$$
(38)

corresponds to the aggregate level of the subsidy provided to firms, where as:

$$ADR(r_{loan}) = \sum_{n \text{ if } t \in (5,25]} \sum_{a} \left[\tau^m r_l^s K(a, P)^s \left[\frac{1 - e^{-4r_{loan}}}{r_{loan}} \right] e^{4r_{loan}} \right] \left[\frac{1 - e^{-20r_{loan}}}{r_{loan}} \right]^{-1} e^{-r_{loan}t} g(a, P, t) \Delta a, \quad (39)$$

coorresponds to the aggregate level of repayments. Accordingly, the policy balance corresponds to:

$$BP(r_{loan}) = ADR(r_{loan}) - ADT(r_{loan}).$$

$$\tag{40}$$

Given this balance, the r_{loan} is adjusted according to:

$$r_{loan}^{l+1} = r_{loan}^{l} - \Delta_r B P^l.$$

$$\tag{41}$$

Notice that when the policy results in savings, the interest rate of the policy can be reduced, whereas when it results in expenditure, the interest rate of the policy needs to be increased. The resulting interest rate will therefore include the impact of the employment and unemployment flows in the revenues and expenditure of the policy.

Given these adjustments to the inner algorithm, one obtains the employment and unemployment sequences for each level of τ^m - a measure of the magnitude of the flow subsidy in the level of loans of the permanent job firms. Then, resorting to the *Fminsearch* procedure of Matlab, I pinpoint the rate that minimizes the objective function of the policy maker, given by:

$$\underset{\tau^m \in [0,1]}{\arg\min} \sum_{n=1}^{N} e^{-\rho n} \left\{ \left| e_T(n) + e_P(n) - e_T^s - e_P^s \right| + \left| u(n) - u^s \right| \right\} dn.$$
(42)

where e_P^s , e_T^s and u^s correspond to the stationary equilibrium levels of employment and unemployment.

Altogether, as in the case of the income tax firm subsidy transfer, this last step solves the outer loop ensuring a policy that is budget balanced by definition, and implemented with the objective to minimize employment fluctuations over the adjustment.

Algorithm for joint transfer policy. This policy combines the moratorium policy, the income tax and firm subsidy transfer policy and the unemployment benefit transfer policy. Accordingly, the algorithm combines the features described in the previous algorithms for the relevant policies.

Accordingly, for the implementation period of the policy, the moratorium policy implies the firms receive a flow as:

$$LC(a, P, t) = \tau^m r_l^s K(a, P)^s, \text{ if } t \in [1, 5],$$
(43)

where $r_l^s K(a, P)^s$ corresponds to the cost of capital the firm was incurring in the stationary equilibrium. By the end of the implementation period, the total amount of the loan provided for the firm corresponds to:

$$TL(a,P) = \tau^{m} r_{l}^{s} K(a,P)^{s} \left[\frac{1 - e^{-4r_{loan}}}{r_{loan}} \right] e^{4r_{loan}},$$
(44)

with r_{loan} corresponding to the quarterly interest rate considered to ensure the budget balance of the policy, and the factor 4 maps into the idea that the cost of capital ease takes place for 4 quarters. Then the subsidy is paid back in installments for 5 years. Accordingly, the installments become:

$$LC(a, P, t) = -\left[\tau^m r_l^s K(a, P)^s \left[\frac{1 - e^{-4r_{loan}}}{r_{loan}}\right] e^{4r_{loan}} \right] \left[\frac{1 - e^{-20r_{loan}}}{r_{loan}}\right]^{-1} \quad \text{if} \quad t \in (5, 25].$$
(45)

Notice that for t > 25 then LC(a, P, t) = 0. Given the policy details as defined, the capital variables are unchanged, as it results from first order condition of the capital.

Given the moratorium policy block, the profits also incorporate the other two policies. Thus, the profit becomes:

$$\tilde{\eta}(a, P, t) = \left[\ddot{z}F(\ddot{k}) - r_l(t)\ddot{k} + LC(a, P, t) - (1 - \tau^{ub} - \tau^{ts})w(a, P, t) \right],\tag{46}$$

and the leverage of the firm becomes:

$$\xi(a, P, t) = \frac{r_l(t)k(a, P, t) - LC(a, P, t)}{z(a, P, t)F[k(a, P, t)] - (1 - \tau^{ub} - \tau^{ts})w(a, P, t)}.$$
(47)

Regarding wages, the wages of the permanent contracts are adjusted to consider the existence of the joint

policy. Accordingly one have the relevant set of equations as:

$$\beta \bigg[J(a, P, t) + S_P \bigg] = (1 - \beta) \bigg[W(a, P, t) - W(a, U, t) \bigg];$$

$$r(t) J(a, P, t) = \ddot{z}F(\ddot{k}) - r_l(t)\ddot{k} + LC(a, P, t) - (1 - \tau^{ub} - \tau^{ts})\tilde{w}(a, P, t) + \partial_a J(a, P, t)\dot{a}(a, P, t) + \sigma_P(\ddot{\xi}_P)[-S_P - J(a, P, t)] + \partial_t J(a, P, t);$$

$$\rho W(a, P, t) = u(\ddot{c}) + \partial_a W(a, P, t)\dot{a}(a, P, t) + \sigma_P(\ddot{\xi}_P) \bigg[W(a, U, t) - W(a, P, t) \bigg] + \partial_t W(a, P, t)$$

$$\dot{a}(a, P, t) = (1 - \tau^{ts})\tilde{w}(a, P, t) + (r_l - \delta)a - \ddot{c};$$
(48)

which results into permanent contract wages as:

$$\tilde{w}(a,P,t) = \frac{\beta \left[\rho + \sigma_P(\ddot{\xi}_P)\right] \left\{ \ddot{z}F(\ddot{k}) - r_l(t)\ddot{k} + LC(a,P,t) + \partial_a J(a,P,t) \left(r(t)a - \ddot{c}\right) + r(t)S_P + \partial_t J(a,P,t) \right\}}{[1 - \beta][r(t) + \sigma_P(\ddot{\xi}_P)](1 - \tau^{ts})\partial_a W(a,P,t) + \beta[\rho + \sigma_P(\ddot{\xi}_P)][(1 - \tau^{ts} - \tau^{ub}) - \partial_a J(a,P,t)(1 - \tau^{ts})]} - \frac{(1 - \beta)\left[r(t) + \sigma_P(\ddot{\xi}_P)\right] \left\{u(\ddot{c}) + \partial_a W(a,P,t) \left(r(t)a - \ddot{c}\right) - \rho W(a,U,t) + \partial_t W(a,P,t)\right\}}{[1 - \beta][r(t) + \sigma_P(\ddot{\xi}_P)](1 - \tau^{ts})\partial_a W(a,P,t) + \beta[\rho + \sigma_P(\ddot{\xi}_P)][(1 - \tau^{ts} - \tau^{ub}) - \partial_a J(a,P,t)(1 - \tau^{ts})]} \right\}}{(49)}$$

From steps 7 to step 11 the algorithm remains analogous to the one presented in appendix D, other than for the adjustment of dividends during the implementation period, where it becomes:

$$d(n) = (1 - \psi) \sum_{a} \left[\ddot{z}F(\ddot{k}) - r_{l}(n)\ddot{k} - w(a, T, n) \right] g(a, T, n)\Delta a + (1 - \psi) \sum_{a} \left[\ddot{z}F(\ddot{k}) - r_{l}(n)\ddot{k} + LC(a, P, n) - w(a, P, n) \right] g(a, P, n)\Delta a - \gamma v, \text{ if } t \in [1, 5].$$
(50)

Regarding step 12, one needs to enlarge the convergence adjustment conditions to include the definition of the interest rate of the moratorum and the definition of the τ^{ub} parameter. Accordingly, consider that:

$$ADT(r_{loan}) = \sum_{n \text{ if } t \in [1,5]} \sum_{a} \tau^m r_l^s K(a, P)^s e^{-r_{loan}t} g(a, P, t) \Delta a,$$
(51)

corresponds to the aggregate level of loans provided to firms, where as:

$$ADR(r_{loan}) = \sum_{n \text{ if } t \in (5,25]} \sum_{a} \left[\tau^m r_l^s K(a,P)^s \left[\frac{1 - e^{-4r_{loan}}}{r_{loan}} \right] e^{4r_{loan}} \right] \left[\frac{1 - e^{-20r_{loan}}}{r_{loan}} \right]^{-1} e^{-r_{loan}t} g(a,P,t) \Delta a.$$
(52)

Accordingly, the policy balance corresponds to:

$$BP(r_{loan}) = ADR(r_{loan}) - ADT(r_{loan}).$$
(53)

Given this balance, the r_{loan} is adjusted according to:

$$r_{loan}^{l+1} = r_{loan}^{l} - \Delta_r B P^l.$$
(54)

Notice that when the policy results in savings, the interest of the loan can be reduced, whereas when it results in expenditure, the interest of the loan needs to be increased. The resulting interest rate of the loan will therefore include the impact of the employment and unemployment flows in the revenues and expenditure of the credit line. Regarding the savings and costs from the unemployment benefit transfer policy are given by:

$$ub^{sav} = \sum_{n=1}^{N} \left(u(n) - u(n)^{DRWR} \right) b;$$

$$ub^{exp} = \sum_{n \text{ if } t \in [1,5]} \sum_{a} \tau^{ub} \left(w(a, P, n)g(a, P, n) \right) \Delta a.$$
(55)

The convergence of the fixed point algorithm is supplemented with the balanced budget condition as:

$$B^{ub,l} = ub^{sav,l} - ub^{exp,l}; (56)$$

$$\tau^{ub,l+1} = \tau^{ub,l} + \Delta_{ub} B^{ub,l}.$$

Intuitively, notice that if the policy is generating savings, then the *ad-valorem* rate of the subsidy can be increased, whereas if the policy is generating deficits the rate should be reduced.

Altogether, the convergence criteria entails the adjustment of: (a) the price on impact based on the noarbitrage condition; (b) the labour market tightness based on the free entry condition; (c) the interest rate based on the excess demand for assets condition; (d) the unemployment benefit subsididy transfer τ^{ub} ; and (e) the interest rate of the moratorium, τ^m . Consequently, consider:

$$p_{0}^{l+1} = \Delta_{p} p_{0} + (1 - \Delta_{p}) p_{0}^{l},$$

$$r(n)^{l+1} = r(n)^{l} + \Delta_{r} A D_{n},$$

$$\theta(n)^{l+1} = \theta(n)^{l} + \Delta_{FE} F E_{n},$$

$$\tau^{ub,l+1} = \tau^{ub,l} + \Delta_{ub} B^{ub,l}, r_{loan}^{l+1} = -r_{loan}^{l} - \Delta_{r} B P^{l}.$$
(57)

The iteration of the inner loop of appendix D, from step 5 onward, with the prescribed adjustments in this appendix, allows for the convergence of the inner loop.

The given the convergence of the inner loop algorithm, one obtains the employment and unemployment sequences for each level of $\{\tau^m, \tau^{ts}\}$. Then, resorting to the *Fminsearch* procedure of Matlab, I pinpoint the rate that minimizes the objective function of the policy maker, given by:

$$\arg\min_{t^{m} \in [0,1], \tau^{ts} \in [0,1]} \sum_{n=1}^{N} e^{-\rho n} \left\{ \left| e_{T}(n) + e_{P}(n) - e_{T}^{s} - e_{P}^{s} \right| + \left| u(n) - u^{s} \right| \right\} dn.$$
(58)

where e_P^s , e_T^s and u^s correspond to the stationary equilibrium levels of employment and unemployment.

Altogether, as in the relevant isolated policies, this last step solves the outer loop ensuring a policy that is budget balanced by definition, and implemented with the objective to minimize employment fluctuations over the adjustment.