The Cyclical Behaviour of Equilibrium Unemployment and Vacancies: Evidence from Italy

Gabriele Cardullo[*]and Marco Guerrazzi[†]

Department of Economics

University of Genoa

Abstract

In this paper, we analyse the fluctuations of unemployment, vacancies, tightness and productivity in the Italian labour market over the last twenty years. For reasons of data availability on job vacancies, this period is split in two parts. The first (1993-2003) is covered by the ISFOL HWTS, while the second (2004-2012) is analyzed by means of the harmonized ISTAT vacancy rate. In both periods, in line with previous findings on the unemployment volatility puzzle, we find that that the labour market tightness indicator is much volatile than labour productivity. In addition, we show that a matching model with segmented labour markets and on-the-job search has the potential to provide a rationale for this pattern.

[*]Assistant Professor, Department of Economics, University of Genoa, via F. Vivaldi n. 5, 16126 Genoa (Italy), Phone (+39) 010 20952210, Fax (+39) 010 2095269, E-mail cardullo@economia.unige.it

[†]Assistant Professor, Department of Economics, University of Genoa, via F. Vivaldi n. 5, 16126 Genoa (Italy), Phone (+39) 010 2095225, Fax (+39) 010 2095269, E-mail guerrazzi@economia.unige.it

1 Introduction

After the publication of two influential articles by Shimer (2004, 2005) - in which he shows that the standard Mortensen-Diamond-Pissarides (MDP) model is unable to replicate observed fluctuations in unemployment and job vacancies in response to productivity shocks of plausible magnitude - the empirical appraisal of the cyclical behaviour of equilibrium unemployment and vacancies has regained a lot of interest in the macroeconomic debate (e.g. Cardullo 2010 and Guerrazzi 2011).

Retrieving US quarterly data over a fifty-year time horizon, Shimer (2004, 2005) measures, inter-alia, the autocorrelation and the volatility of unemployment, vacancies and labour productivity. One of the most striking finding of his empirical explorations is that the standard deviation of the vacancy-unemployment ratio, i.e., the labour market tightness indicator, is almost twenty times as large as the standard deviation of labour productivity over the period under examination. The so-called 'Shimer puzzle' (or 'unemployment volatility puzzle') comes from the fact that the MDP model predicts that those two variables should have nearly the same volatility.[1]

A number of contributors tried to address this theoretical inconsistency within the US economy. For instance, Shimer (2004) and Hall (2005a,b) rely on real wage stickiness questioning the Nash bargaining hypothesis. Furthermore, Hagedorn and Manovskii (2008) show that calibrating the value of non-market activities close to labour productivity amplifies the volatility of labour market tightness. Moreover, Pissarides (2009) suggests that taking into account that only the wages of newly hired workers respond to productivity shocks brings the model close to available evidence. In addition, Silva and Toledo (2009) and Petrosky-Nadeau and Wasmer (2013) argue, respectively, that labour turnover costs and financial frictions may generate the required amplification mechanism.

Spurred on by such a stream of theoretical efforts aimed at reconciling Shimer's (2004, 2005) empirical findings with the well-established theory of equilibrium unemployment, a number of scholars become active in testing the soundness of the unemployment volatility puzzle even outside the US economy. First, Zhang (2008) explores the cyclical behaviour of unemployment and vacancies in Canada. Moreover, Miyamoto (2011) replicates the experiment with Japan data. Furthermore, Gartner et al. (2012) do the same in Germany. Even if they provides different theoretical rationales for such a puzzle, all those authors find that in all the countries taken into consideration the volatility of labour market

---

[1]The intuition for those results is that wages bargained according to the Nash rule should absorb a great deal of productivity shocks. Therefore, vacancies and unemployment should be only partially affected by erratic disturbances affecting the value of output.

tightness is much higher than the one attached to productivity.[2]

More recently, Justiniano and Michelacci (2011) as well as Amaral and Tasci (2012) test Shimer's (2004, 2005) empirical results over a set of OECD countries in which a number of EU members is included. However, until now, nothing has been said about the cyclical behaviour of unemployment and vacancies in the Italian context. As a consequence, in this paper, we aim at filling this gap by addressing the macroeconomic fluctuations of unemployment, job vacancies, tightness and productivity in the Italian labour market over the last twenty years.

All over this period, the Italian labour force survey together with official national accounts are able to provide homogeneous quarterly figures for both unemployment and labour productivity measured, respectively, as the fraction of the labour force actively searching for work and added value per worker evaluated at constant prices. Unfortunately, the same does not hold for job vacancies and the corresponding vacancy rate. As a consequence, according to data availability on unfilled job openings, our empirical analysis is split in two sub-periods. Specifically, the first (1993-2003) is covered by the help-wanted time series (HWTS) provided by the Italian Institute for the Development of Vocational Training (ISFOL), while the second (2004-2012) is analyzed by means of the harmonized vacancy rate worked out by the Italian National Institute of Statistics (ISTAT). Retrieving data from both series, we explore the macroeconomic fluctuations of the Italian labour market by following the estimative approach adopted throughout the literature mentioned above.

The main results achieved in this paper are the following. On the empirical ground, confirming previous findings on the unemployment volatility puzzle, we find that, in both periods, despite some heterogeneities in the correlation matrix of the involved series, the labour market tightness indicator is much more volatile than labour productivity. Specifically, in the time-span covered by the ISFOL HWTS (ISTAT harmonized vacancy rate) the ratio between the volatility of labour market tightness and the corresponding index for productivity is around five (thirty-three).

In addition, from a theoretical point of view, we show that a baseline MDP model is fairly unable to replicate available Italian data. By contrast, we show that a matching model with segmented labour markets with on-the-job search build along the lines of Krause and Lubik (2006) has the potential to provide the required amplification mechanism. In details, we show that when intermediate goods produced in the primary and

---

[2]Specifically, the reported ratio between those indexes is about eighteen for Canada, thirteen in Japan and fifty in Germany.

in the secondary sector of the economy becomes sufficiently substitutable, the theoretical model with segmented labour market and on-the-job search amplifies productivity shocks by an order of magnitude consistent with available official empirical observations.

The paper is arranged as follows. Section 2 analyses the patterns of unemployment, vacancies, tightness and productivity in the Italian labour market over the last twenty years by exploiting available unfilled job openings series. Section 3 develops a matching model with segmented labour markets and on-the-job search by exploring its amplification mechanism. Finally, section 4 concludes.

2   Italian labour market facts

In this section we provide an empirical analysis of the Italian labour market dynamics over the period 1993-2012. However, as we stated above, this time-horizon is not covered by a unique time series for unfilled job openings. To the best of our knowledge, the longest and more reliable Italian vacancy series available the period under examination are essentially two, i.e., the ISFOL HWTS that covers about the first decade and the harmonized ISTAT vacancy rate that holds all over the second one.

As a consequence, in what follows, we provide distinct empirical analyses for the two relevant sub-periods.[3] Specifically, for each selected time-span, we take into consideration the level and trend paths of unemployment, vacancy, labour market tightness and productivity. In addition, a special attention is paid to the Beveridge curve, i.e., the well-known relationship between unemployment and vacancies conveyed both in levels and trend log-deviations. Moreover, for each series, taking into account trend log-deviations only, we report detailed statistics on volatility, persistence and correlation.

2.1   The period covered by the ISFOL HWTS (1993.1-2003.4)

The ISFOL HWTS is grounded on help-wanted job advertisements collected by the Centre for Business Statistics (Centro di Statistica Aziendale) headquartered in Florence. Data collection started about 30 years ago. However, essentially for reasons of attrition, homogeneous data are available only for the period 1993-2003 (e.g. Mandrone 2012). All

---

[3]The two series for unfilled job openings do not overlap and this does not allow to implement consistent merging procedures. Moreover, as suggested by Valletta (2005), the use of an HWTS to retrieve vacancies beyond the 90s will be rather misleading for the growing reliance of alternative sources such as internet posting.

over this time-horizon, the ISFOL HWTS retrieves information about job advertisements on the main thirty-two Italian newspapers.

The basic unit of survey of the ISFOL HWTS are job advertisements. However, the information collected in each newspaper is processed in order to derive an estimation for the corresponding number of job vacancies posted by firms on a quarterly basis. Thereafter, following the official definition also implemented by ISTAT, the corresponding vacancy rate is obtained by dividing the total number of available jobs, i.e., vacancies plus the number of employed people, by total employment.

The level and trend paths of unemployment (u), vacancies (v), labour market tightness (v/u) and labour productivity (a) all over the time-span taken into consideration by the ISFOL HWTS are illustrated in the four panels of figure 1 (all the series are seasonally adjusted; HP trends obtained by setting the smoothing parameter at $1,600$).
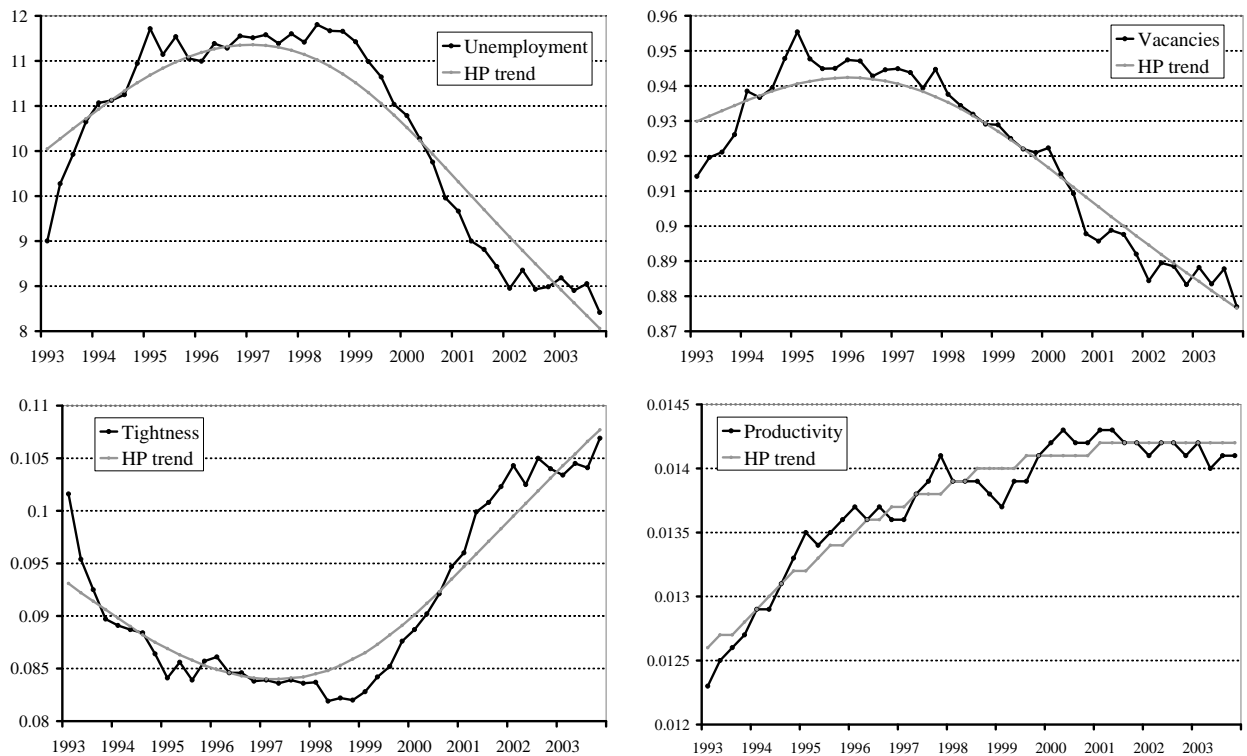

Figure 1: Italian labour market facts (1993.1-2003.4)

The four diagrams show that the period covered by the ISFOL HWTS is characterized by a recover of labour market tightness as well as a mildly upward trend in labour productivity. It is quite likely that those encouraging patterns are the joint outcome of

labour market reforms aimed at increasing employment flexibility (cfr. Treu Reform) and the end of austerity policies carried out after the European monetary crisis bursted at the beginning of the 90s.

The level and trend log-deviation relationships between unemployment and vacancies are illustrated in the two panels of figure 2 (both diagrams include the results of a linear OLS regression among the involved variables).
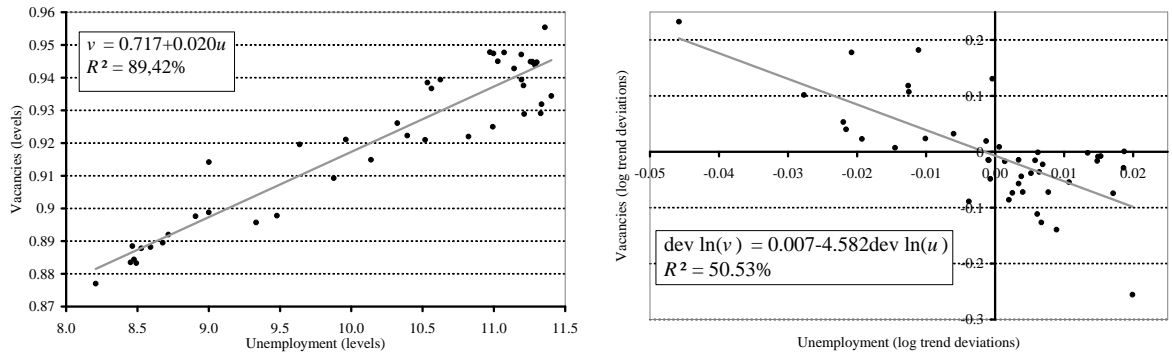
Figure 2: Unemployment versus vacancies (1993.1-2003.4)

The two diagrams reveals that the series of vacancies retrieved from the ISFOL HWTS is consistent with textbook negative shape of the Beveridge curve only when trend log-deviations are concerned. By contrast, the level relationship between unemployment appears definitely upward-sloped with a strong degree of statistical significance.[4]

A set of summary statistics concerning the trend log-deviation dynamics of the series depicted in figure 1 can be found in table 1.

		$\ln(u)$	$\ln(v)$	$\ln(v/u)$	$\ln(a)$
Standard deviation		0.013	0.089	0.011	0.002
Quarterly autocorrelation		0.706	0.665	0.665	0.646
Correlation matrix	$\ln(u)$	1	-0.710	0.987	-0.124
	$\ln(v)$	$-$	1	-0.599	0.550
	$\ln(v/u)$	$-$	$-$	1	-0.038
	$\ln(a)$	$-$	$-$	$-$	1

Table 1: Summary statistics, quarterly Italian data (1993.1-2003.4)

[4]Using the same data source, a similar result was derived by Sestito (1988) over the period 1980-1985.

The figures in table 1 suggest some interesting but rather preliminary conclusions. First, as we already said in commenting figure 2, there is a negative relationship between the trend log-deviations of unemployment and vacancies confirming the conventional wisdom about the shape the Beveridge curve. However, data also reveal an unconventional counter-cyclicality of the labour market tightness indicator. Second, the series of vacancies is more volatile than unemployment and productivity corroborating the results obtained over the other OECD countries (see Amaral and Tasci 2012), Moreover, somehow in line with Shimer's (2004-2005) findings, the volatility of labour productivity is far below the volatility of labour tightness; indeed, the ratio between the latter and the former is around five, a value higher than the one predicted by the conventional theory but far below the ones found in other countries.

Despite the short length of the time-horizon, the latter result is a first signal that even in Italy the standard MDP model might not good enough to replicate observed data. Obviously, all the issues related to the derivation of a vacancy index from newspaper advertisements without any reference to shared official criteria call for an additional examination.[5]

2.2 The period covered by the ISTAT vacancy rate (2004.1-2012.4)

In the third quarter of 2003, ISTAT started a new survey on job vacancies and worked hours. In compliance with the official Eurostat guidelines on job vacancies, this survey aims at measuring vacancies, job flows and worked hours in firms that employ more than ten employees.[6] As a consequence, this survey provides a harmonized vacancy rate that directly measures the extent of search externalities experienced by Italian firms in their recruiting processes.

Up to present, ISTAT discloses the official harmonized vacancy rate without any adjustment. Therefore, in order to remove seasonal patterns that may artificially affect the volatility of data, we filtered the series by averaging, on a quarterly basis, deviations from

---

[5]For instance, Abrahm (1987) argue that occupation composition of employment, equal employment opportunity pressures as well as consolidation in newspaper markets may lead to substantial distortions in the estimation of vacancies from job advertisements. Furthermore, in the Italian labour market additional inconsistencies may derive from the fact that usually recruitments occur through informal links (e.g. Pistaferri 1999).

[6]The sample size is about 15,000 firms selected by drawing on the National Statistical Register of Active Firms (ASIA).

a linear decreasing trend.[7] After the ISFOL HWTS analyzed above, this series is intended to become the longest data source on Italian unfilled job openings.

The paths of unemployment (u), vacancies (v), labour market tightness (v/u) and labour productivity (a) all over the period covered by the ISTAT vacancy rate are illustrated in the four panels of figure 3 (all the series are seasonally adjusted; HP trends obtained by setting the smoothing parameter at $1,600$).
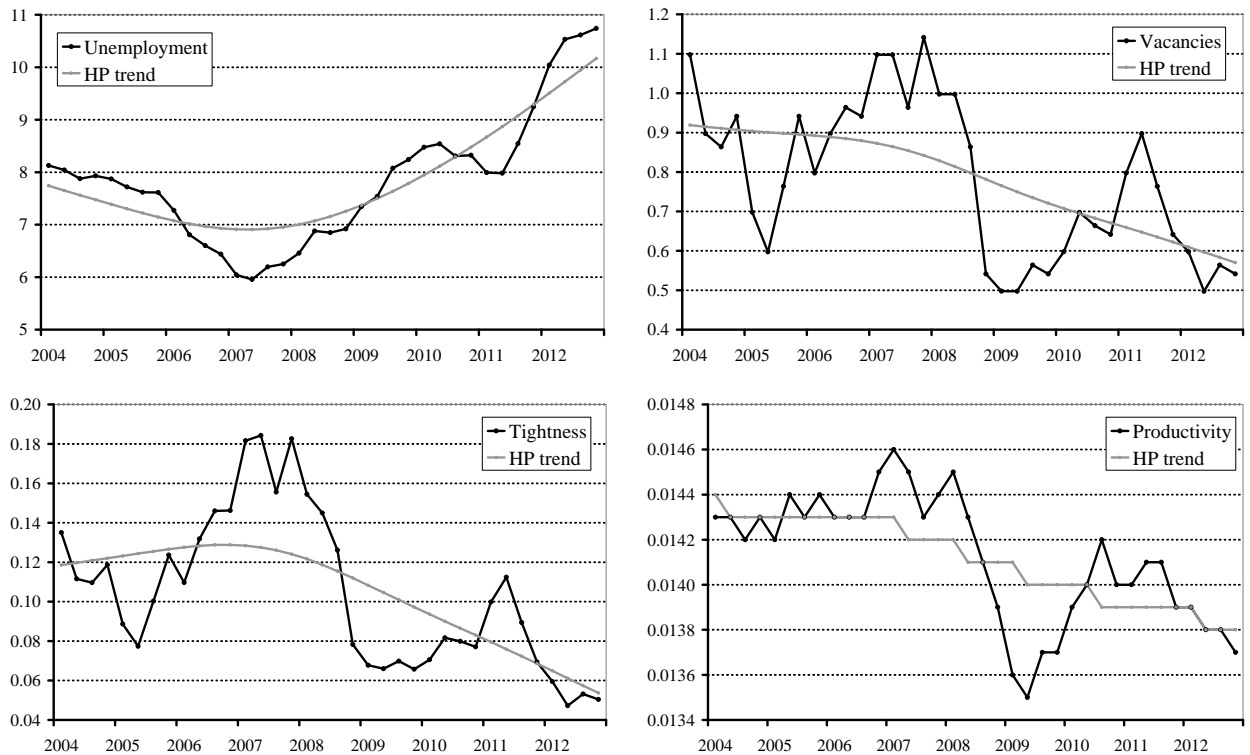
**Figure 3:** Italian labour market facts (2004.1-2012.4)

The time horizon described by the ISTAT vacancy rate is quite different from the one illustrated in figure 1. As shown in the four panels of figure 3, this period is characterized by quite adverse labour market conditions; indeed, after the satisfying performance of the beginning, unemployment begun to grow at increasing rates while labour market tightness displays a quite clean decreasing trend. In addition, the path of labour productivity appears seriously hit by the strong recession of 2008-2009.

---

[7]In this way, the adjusted series results quite smoother than the original one. More technical details on seasonal adjustments are given by Gómez and Maraval (1996).

The level and trend log-deviation relationships between unemployment and vacancies are illustrated in the two panels of figure 4 (both diagrams include the results of a linear OLS regression among the involved variables).
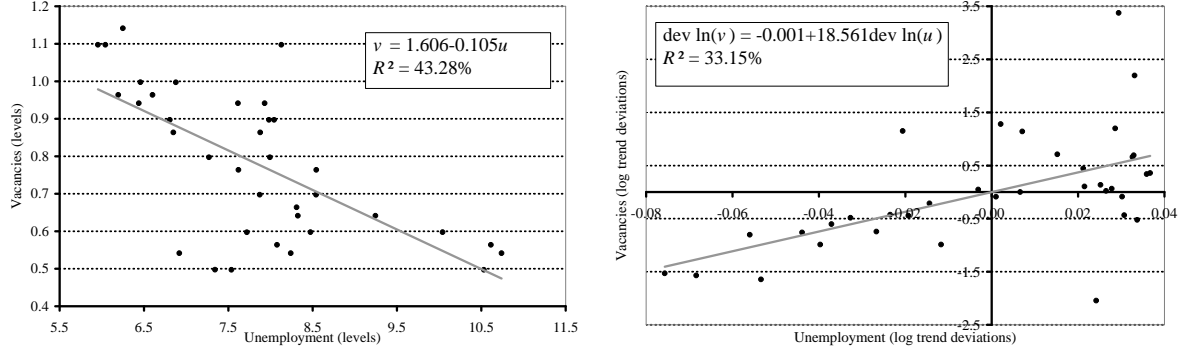
Figure 4: Unemployment versus vacancies (2004.1-2012.4)

Even the picture of the relationships between unemployment and vacancies obtained by means of the ISTAT harmonized vacancy rate are completely at odds with the ones retrieved in figure 3; indeed, the textbook negative shape of the Beveridge curve is obtained only by considering the levels of the involved variables but it fails as far as trend log-deviations are concerned.[8] However, the fit of the upward-sloped regression line appears quite low.

A set of summary statistics concerning the trend log-deviation dynamics of the series depicted in figure 3 can be found in table 2.

		$\ln(u)$	$\ln(v)$	$\ln(v/u)$	$\ln(a)$
Standard deviation		0.033	1.079	0.112	0.003
Quarterly autocorrelation		0.884	0.567	0.782	0.753
Correlation matrix	$\ln(u)$	1	0.575	0.787	0.575
	$\ln(v)$	–	1	0.894	0.556
	$\ln(v/u)$	–	–	1	0.769
	$\ln(a)$	–	–	–	1

Table 2: Summary statistics, quarterly Italian data (2004.1-2012.4)

[8]This result was already stressed by Baldi and Sorrentino (2009) on a pioneering study on the properties of the harmonized Italian vacancy rate series.

¿From the point of view of co-movements, the figures in table 2 are somehow more unconventional than the ones in table 1; indeed, all over the concerned period, vacancies, tightness and productivity are all counter-cyclical. In addition to measurement issues due to the not too large number of observations, this pattern suggests that supply shocks may be not the unique driver of business cycles (see Justiniano and Michelacci 2011).[9] However, in line with previous findings on the unemployment volatility puzzle reviewed in the introduction, the figures in table 2 strengthen some results shown in table 1 by conveying that the volatility of the official labour market tightness indicator exceeds the one on productivity by an order of magnitude; indeed, the ratio between the latter and the former is about thirty-three, a figure that seems to approach the biggest value of about fifty obtained by Gartner et al. (2012) with German data covering the period 1980-2004.[10]

In the next section, we present a theoretical search model with segmented labour markets and on-the-job search. Thereafter, adopting the same calibration procedure implemented by Shimer (2005), we show that it fits Italian business cycle data presented in tables 1 and 2 better than a baseline MDP framework .

3   Towards a theoretical explanation

Data analyzed in section 2 do not provide robust cyclical regularities, especially in terms of co-movements among the involved series. The only established finding is that even in the Italian context the volatility of the labour market tightness indicator is much higher than the volatility of productivity. As a consequence, in order to explain this dynamic pattern, we develop a theoretical framework able to generate the suitable amplification mechanism. In this regard, we present an equilibrium version of the matching framework put forward by Krause and Lubik (2006) and we show that labour market segmentation and the possibility of on-the-job search allow to outperform the baseline MDP model magnifying vacancy and unemployment fluctuations.

Krause and Lubik (2006) develop a matching model in which consumption goods are produced in two intermediate sectors which are assumed to differ in terms of the costs of vacancy posting. Specifically, one of the two intermediate sectors is assumed to be characterized by higher costs of vacancy posting with respect to the other. Therefore,

---

[9]Among OECD countries, Amaral and Tasci (2012) find that labour productivity is counter-cyclical in Australia, Poland and Spain.

[10]Focusing on a longer time-horizon, Amaral and Tasci (2012) report a figure of thirty-three for Germany and Spain.

as far as labour market conditions are concerned, the former will be less tight than the latter. Moreover, for a decreasing return argument, the former will be characterized by higher productivity and wages with respect to the latter mirroring traditional segmentation processes undergoing in actual labour markets usually labelled with the distinction among primary and secondary sector.[11] In turn, this segmentation of labour market conditions will lead workers employed in the low-wage, high-tightness sector to make some efforts to find a job in the high-wage, low-tightness sector. Calibrating and simulating the model, Krause and Lubik (2006) show that their framework is able to fairly reproduce the observed path of US vacancy and unemployment fluctuations in response to productivity shocks of plausible magnitude.

In this paper we follow a different approach. Specifically, we analytically solve a steady-state version of Krause and Lubik (2006) model and then we assess the elasticities of labour market tightness with respect to productivity under different scenarios. As shown by Shimer (2005), such elasticities are useful approximations to the volatilities of the corresponding variables in the dynamic stochastic set-up.[12]

3.1   The model

We assume that the economy is populated by a measure one of risk-neutral workers. Time is continuous and the discount rate is denoted by r. There are two intermediate good sectors in the economy, denoted by g (good) and b (bad). The two intermediate goods are combined together via a CES aggregator to produce the unique final consumption good. Therefore,

$$Y \; = \; a \left(Q_g^{\frac{\rho-1}{\rho}} + Q_b^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}} \qquad \rho > 1 \qquad (1)$$

where Y is the consumption good, a is a productivity parameter, Q_i is output in sector i, with $i = \{g, b\}$, and ρ is the elasticity of substitution between the two intermediate goods.

Product markets are perfectly competitive. As a consequence, taking the consumption good as numeraire, cost minimization leads to the following inverse demand functions:

---

[11]Evidence of labour market segmentation in the Italian context is given by Cipollone (2001) and, more recently, by Battisti (2013).

[12]Indeed, Mortensen and Nagypal (2007) show that the two outcomes coincide in the limit when the arrival rate of shocks is close to zero or changes in productivity are small.

$$p_i(Q_i) = a\left(\frac{Q_i}{Y}\right)^{-\frac{1}{\rho}} \qquad i = \{g, b\} \tag{2}$$

In each intermediate sector, each firm can hire one worker at most, that in turn produces one unit of the intermediate good. Therefore, if E_i is the level of employment in sector i, then $E_i = Q_i$, with $i = \{g, b\}$.

The only difference between the two sectors is in the flow cost paid to keep a vacancy open, namely, we assume that $c_g > c_b$. As it will be clear later on, the sector with higher vacancy costs will exhibit lower employment and, for decreasing returns in the production function of consumption goods, higher productivity and wages. So, employed workers in sector b exert some effort in searching for a job in sector g.

The flow of new matches in sector b is denoted by $m(v_b, u_b) = \mu v_b^{1-\eta} u_b^{\eta}$, with $\mu > 0$ and $0 < \eta < 1$. Denoting $\theta_b \equiv v_b/u_b$, the job finding rate is given by $f(\theta_b) \equiv m(v_b, u_b)/u_b$ and the rate at which vacancies are filled is $q(\theta_b) \equiv m(v_b, u_b)/v_b$, which is positive, decreasing and a convex function of θ_b. It is worth noting that η is the elasticity of the expected duration of filling a vacancy $q(\theta)^{-1}$ with respect to tightness. In sector g, the matching function is $m(v_g, u_g + se_b) = \mu v_g^{1-\eta}(u_g + se_b)^{\eta}$, in which s is the amount of search effort spent by employees in sector b to look for a better-paid position in the other sector. If we define $\theta_g \equiv v_g/(u_g + se_b)$, then we can again convey the job finding rate and the job filling rate as functions of tightness only, i.e., $f(\theta_g) \equiv m(v_g, u_g + se_b)/(u_g + se_b)$ and $q(\theta_g) \equiv m(v_g, u_g + se_b)/v_g$. In both sectors, at an exogenous rate δ, a firm-worker pair is destroyed.

Denoting by ϕ the share of employees working in sector g, the prices of the intermediate goods can be written as:

$$p_g = a\left(1 + \left(\frac{1-\phi}{\phi}\right)^{\frac{\rho-1}{\rho}}\right)^{\frac{1}{\rho-1}} \tag{3}$$

$$p_b = a\left(1 + \left(\frac{\phi}{1-\phi}\right)^{\frac{\rho-1}{\rho}}\right)^{\frac{1}{\rho-1}} \tag{4}$$

In steady-state, the flows in and out of employment status must be equal. Therefore,

$$\delta\phi E = sf(\theta_g)(1-\phi)E + f(\theta_g)U_g \tag{5}$$

$$\delta(1-\phi)E = f(\theta_b)U_b - sf(\theta_g)(1-\phi)E \tag{6}$$

The on-the-job mechanism implies there is a share of workers $sf(\theta_g)(1 - \phi)E$ that quit their job in sector b to become employed in sector g. The discounted present value in employment in sector g, i.e., $V_{E,g}$, verifies the following Bellman equation:

$$rV_{E,g} = w_g + \delta(V_{U,g} - V_{E,g}) \tag{7}$$

where w_g is the real wage in intermediate sector g and $V_{U,g}$ represents the discounted expected lifetime income of an unemployed worker in the same sector.

Similarly, the discounted present value in employment in sector b, i.e., $V_{E,b}$, is assumed to verify

$$rV_{E,b} = \max_s w_b - k(s) + \delta(V_{U,g} - V_{E,g}) + sf(\theta_g)(V_{E,g} - V_{E,b}) \tag{8}$$

Workers employed in sector b may exert some effort in searching for a job in sector g. The disutility of on-the-job search is captured by a convex function $k(\cdot) = \kappa s^\sigma$, with $\sigma > 1$. The first-order condition (FOC) for the problem in (8) is given by

$$k'(s) = f(\theta_g)(V_{E,g} - V_{E,b}) \tag{9}$$

Denoting the value for leisure by z, $V_{U,i}$ verifies the following Bellman equation:

$$rV_{U,i} = z + f(\theta_i)(V_{E,i} - V_{U,i}) \qquad i = \{g, b\} \tag{10}$$

Unemployed workers are free to direct their search towards either sector. However, a non-arbitrage condition ensures that the value of being unemployed must be equal across sectors. Hence,

$$rV_{U,g} = rV_{U,b} = rV_U \tag{11}$$

On the firms' side, the expected discounted profits of a vacancy takes the following form:

$$rJ_{V,i} = -c_i + q(\theta_i)(J_{E,i} - J_{V,i}) \qquad i = \{g, b\} \tag{12}$$

Moreover, the expected utilities of a filled vacancy in the two sectors are equal to

$$J_{E,g} = p_g - w_g + \delta(J_{V,g} - J_{E,g}) \tag{13}$$

$$J_{E,b} = p_b - w_b + \delta(J_{V,b} - J_{E,b}) + sf(\theta_g)(J_{V,b} - J_{E,b}) \tag{14}$$

13

A firm-worker pair in sector b breaks down not only at the exogenous destruction rate δ but also whenever the employee finds a job in sector g. In each sector, firms post vacancies as long as the discounted expected profits are nonnegative, so that $J_{V,i} = 0$, for $i = \{g, b\}$. Using (12), (13), and (14) we derive

$$\frac{p_b - w_b}{r + \delta + sf(\theta_g)} = \frac{c_b}{q(\theta_b)} \tag{15}$$

$$\frac{p_g - w_g}{r + \delta} = \frac{c_g}{q(\theta_g)} \tag{16}$$

The free-entry zero profit condition in sector g, i.e., eq. (16), is qualitatively identical to the one retrieved in the standard MDP model and it equates the expected cost of filling a vacancy (the RHS) with the expected revenues (the LHS). By contrast, in sector b, expected revenues are discounted by a further term, i.e., $sf(\theta_g)$, that takes into account the probability that workers may leave their job in order to switch in the other sector.

When a worker and an employer form a match, the surplus $V_{E,i} - V_U + J_{E,i}$ with $i = \{g, b\}$, is shared through Nash bargaining. If β denotes the bargaining power of the workers, then the splitting-the-surplus condition can be written as

$$(1 - \beta)\,(V_{E,i} - V_U) = \beta J_{E,i} \quad i = \{g, b\}, \quad 0 < \beta < 1 \tag{17}$$

Consistently with Krause and Lubik (2006), we do not consider the option of recall. As a consequence, this means that wages in previous jobs are not part of the outside options of workers.[13]

Using eq.s $(10) - (12)$ and (17) it is possible to derive that

$$\theta_b\, c_b = \theta_g\, c_g \tag{18}$$

Since $c_g > c_b$, then $\theta_g < \theta_b$. Because of the higher vacancy costs, less firms enter sector g leading to a lower tightness, lower employment and a higher sector productivity implied by the decreasing marginal returns characterizing the production function of final goods.

Exploiting eq.s (10), (12), and (17), the FOC for the search effort in eq. (9) can be written as

$$s = \left(\frac{c_b\, \theta_b \beta}{\kappa \sigma\,(1 - \beta)} \left(1 - \left(\frac{c_b}{c_g} \right)^{1-\eta} \right) \right)^{\frac{1}{\sigma - 1}} \tag{19}$$

---

[13]Krause and Lubik (2006) extensively discuss the consequences of imposing a recall in the Nash bargaining game.

Furthermore, using the eq.s (7), (8), (10), (13), and (14), the Nash bargaining solution in eq. (17) allows to write the wage equation in both sectors. Specifically,

$$w_g = \beta\,(\,p_g\,+\,c_g\theta_g\,)\,+\,(1-\beta)z \tag{20}$$

$$w_b = \beta\,(p_b\,+\,(1-s)c_b\theta_b\,)\,+\,(1-\beta)\,(\,z\,+\,k(s)) \tag{21}$$

It is worth noting that eq. (19) implies that $d\,w_b/d\,s = (1-\beta)k'(s) - \beta c_b\theta_b < 0$. This inequality follows because the stronger the effort devoted to search for a job in sector g, the lower the expected surplus of a match in sector b, as it breaks up more easily. Therefore, this translates into a lower wage w_b.

Definition 1 *A steady-state equilibrium is defined by a vector* $(\theta_i, w_i, p_i\,(Q_i))$, *with* $i = \{g, b\}$, *a value of search effort s, a share of employment in the g sector ϕ, and a value of consumption output Y satisfying:*

- *The wage equations (20) and (21);*

- *The non-arbitrage condition in eq. (18);*

- *The FOC for the optimal search effort in eq. (19);*

- *The zero-profit conditions in eq.s (15) and (16);*

- *The FOCs in the final good sector in eq. (2) and eq. (1) for Y.*

To find the steady-state equilibrium, we insert the expressions for the real wages in eq.s (20) and (21) in the zero-profit conditions conveyed by eq.s (15) and (16) and then we substitute θ_g for θ_b via the non-arbitrage condition in eq. (18). In this way, we derive

$$p_b - z - \beta c_b\theta_b - \frac{c_b\,(r+\delta)}{q(\theta_b)} - (1-\beta)\kappa s^\sigma + c_b\theta_b s\left(\beta - \left(\frac{c_b}{c_g}\right)^{1-\eta}\right) = 0 \tag{22}$$

$$p_g - z - \beta c_b\theta_b - \frac{c_g\,(r+\delta)}{q(\theta_b)}\left(\frac{c_b}{c_g}\right)^\eta = 0 \tag{23}$$

Eq. (19) recalls that search effort s is a monotonically increasing function of θ_b. As a consequence, the system in eq.s (22) and (23) is composed by two equations in two unknowns, i.e., θ_b and ϕ.[14] If a (unique) solution of the system exists, then all the other remaining variables of the model can be easily obtained by using the eq.s cited in definition 1. The following proposition summarizes the results:

---

[14]Recall that, according to eq.s (3) and (4), p_g and p_b depend on ϕ only.

Proposition 1 *If $z < 1$, then there is a unique solution for the system in eq.s (22) and (23). Therefore, the steady-state equilibrium of the model exists and it is unique.*

The intuition for the statement in proposition 1 can be given as follows. For the Inada conditions in the matching function and in the CES production function and the decreasing marginal productivity in the production function, it is easy to show that eq. (22) is an increasing function in (ϕ, θ_b) space, that intersects the horizontal axis in a point greater than zero, and that goes to $+\infty$ as $\phi = 1$. Moreover, eq. (23) is a decreasing function in (ϕ, θ_b) space, that intersects the vertical axis in a point between zero and one, and that goes to $+\infty$ as $\phi = 0$. As a consequence, a solution for system in eq.s (22) and (23) exists and it is unique.[15]

3.2   Elasticities

We now compute the elasticity of tightness with respect to the productivity parameter a. Taking into account the non-arbitrage condition in eq. (18), the elasticity of tightness in sector g has the same magnitude of the one in sector b. In other words,

$$\frac{d\theta_b}{da}\frac{a}{\theta_b} = \frac{d\theta_g}{da}\frac{a}{\theta_g} \equiv \epsilon_{\text{KL}} \tag{24}$$

Totally differentiating the system in eq.s (22) and (23), allows to derive the following expression:

$$\epsilon_{\text{KL}} = \frac{p_b}{\Phi_0 + \frac{c_b(\eta(r+\delta)+\beta f(\theta_b)+\Phi_1)}{(1-\beta)q(\theta_b)}} \tag{25}$$

where

$$\Phi_0 \equiv -\frac{\partial p_b}{\partial \phi}\frac{\partial \phi}{\partial \theta_b}\theta_b = \frac{\phi c_b\left(\beta f(\theta_b) + \eta(r+\delta)\left(\frac{c_b}{c_a}\right)^{\eta-1}\right)}{(1-\phi)(1-\beta)q(\theta_b)} \tag{26}$$

$$\Phi_1 \equiv sf(\theta_b)\left(\frac{c_b}{c_g}\right)^{1-\eta}\left(1+\frac{1-\beta}{1-\sigma}\right) - \beta \tag{27}$$

Two terms distinguish ϵ_{KL} from the corresponding expression of the elasticity in a standard MDP model. The first term is the derivative in eq. (26), i.e., the term stemming from the assumption that there is a decreasing demand for the intermediate goods

---

[15]Details of the proof are available from the authors on request.

produced by firms. In the textbook MDP model, it is implicitly assumed that firms face an infinitely elastic demand so that output price does not change. In addition, the second term is given by the expression in eq. (27), i.e., the term that conveys the presence of on-the-job search in sector b.

If both terms are equal to zero, we derive an expression for the elasticity equivalent to the one considered by Shimer (2005). Specifically,

$$\epsilon_{\mathrm{S}} = \frac{p_b}{\frac{c_b(\eta(r+\delta)+\beta f(\theta_b))}{(1-\beta)q(\theta_b)}} \tag{28}$$

The amplification potential of eq. (25) is conveyed by the following proposition:

Proposition 2   *If* $\left(\frac{c_b}{c_g}\right)^{1-\eta} < \frac{\beta(\sigma-1)}{\sigma-\beta}$, *then* ϵ_{KL} *is increasing in* s.

Proposition 2 claims that the introduction of on-the-job search increases the elasticity of tightness in response to an increase in productivity. Under plausible assumptions about the arrival rate of shocks in a and the magnitude of its change, this implies that the introduction of on-the-job search amplifies the volatility of tightness in response to shocks on productivity.

The rationale of this result goes as follows. When a positive productivity shock hits intermediate sectors of the economy, firms post more vacancies both in the high-paid and in the low-paid sector. Higher values of v_g raises search effort s. Thereafter, more job-seekers in sector g imply a lower expected duration of vacancies $q(\theta_g)^{-1}$, so even more vacancies are posted in sector g. Vacancy creation and employees' search effort are strategic complements, in other words, an increase in v_g triggers search effort that, in turn, raises v_g even more. The process ends because of the convexity assumption about the search cost. The final result is a larger amplification in vacancy posting in response to a productivity shock. A complementarity between sectors also arises. If search effort of low-paid employees goes up, then congestion effect in the matching technology will make more difficult for unemployed workers to find a high-paid job. As a consequence, they will direct their search toward the low-paid sector. This in turn will boost vacancy creation in that sector.

3.3   A calibration experiment

The model developed above is quite non-linear. Therefore, in order to provide a quantitative assessment of the theoretical results underlying proposition 2, we rely on a com-

putational experiment. Specifically, we calibrate the model and we evaluate how ϵ_{KL} outperforms its baseline reference ϵ_{S} in terms of amplification of productivity shocks by taking into account different degrees of substitutability among the two intermediated goods produced in sectors b and g.[16] Moreover, for reasons of comparability, we take as reference period of calibration the last decade only, i.e., the period covered by the ISTAT harmonized vacancy rate.

| Parameter	Description	Value
μ	*Scale parameter of the matching function*	1.548
η	*Elasticity of the matching function*	0.519
δ	*Job separation rate*	0.090
β	*Workers' bargaining power*	0.519
a	*Average productivity*	0.014
z	*Value of leisure*	0.005
r	*Interest rate*	0.012
c_b	*Bad job creation cost*	0.200
c_g	*Good job creation cost*	0.800
σ	*Search elasticity parameter*	1.100
κ	*Search cost function parameter*	0.040

Table 3: Calibration

The model is calibrated as follows. First, consistently with Shimer (2005), the parameters of the matching function and the job destruction rate, respectively, μ, η and δ, are retrieved by computing job finding and separation rates exploiting OECD data on long-term unemployment.[17] Thereafter, in order to consider efficient fluctuations, we set $\beta = \eta$ (e.g. Hosios 1990). The value for productivity, i.e., a, is derived by averaging data in the forth panel of figure 3. Similarly, the value of leisure, i.e., z, is obtained by averaging labour productivity all over the period and taking into account OECD labour shares and replacement rates (e.g. Martin 1996). The figure of the interest rate is consistent with an annual real interest rate slightly above 4%. In addition, the remaining model parameters are calibrated following the contribution by Krause and Lubik (2006). Specifically, bad and good job creation costs, namely, c_b and c_g, are set in order to mimic a 1 to 4 ratio. Moreover, the search elasticity and the cost function parameters, i.e., σ and κ, are

---

[16]MATLAB[TM] codes are available from the authors.

[17]Identification issues are addressed by instrumenting with trend values of the involved variables.

fixed at the same theoretical values suggested by Krause and Lubik (2006). Furthermore, the model economy is simulated by using different values of the elasticity of substitution among the two intermediate goods necessary to produce the final consumption good, i.e., ρ.

The whole set of parameter values is collected in table 3 while simulation results are illustrated in figure 5.
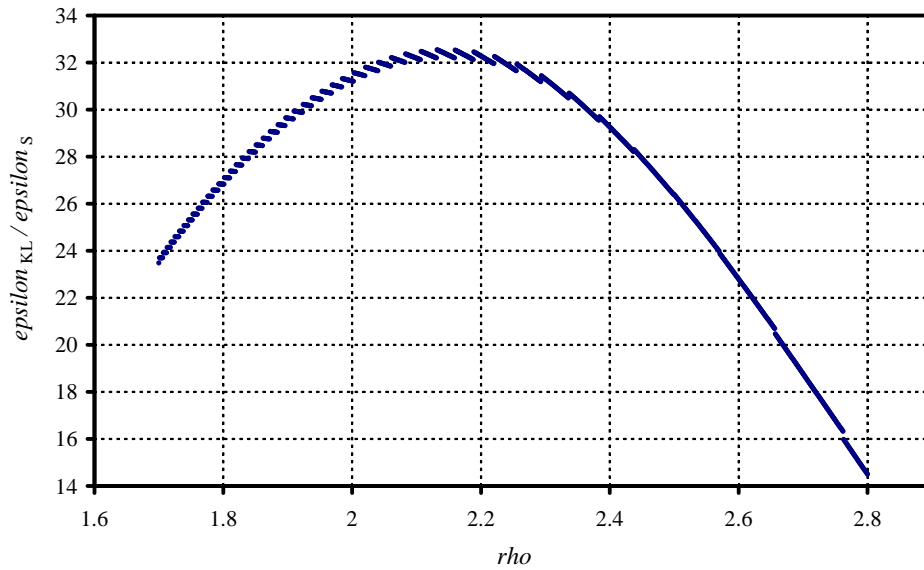
Figure 5: Simulation results

The reverse-u parabola in figure 5 shows that there are some critical values of ρ such that the model with elastic demand and on-the-job search is able to produce an amplification of productivity shocks much higher then the one generated when those factors are neglected. Specifically, when the elasticity of substitution among the two intermediate goods approaches 2.15, the ratio between ϵ_{KL} and ϵ_S becomes quite close to the magnitude implied by the figures in table 2.

4   Concluding remarks

In this paper, we explore the macroeconomic fluctuations of unemployment, job vacancies, tightness and productivity in the Italian labour market over the last twenty years. Since the selected time-horizon is not covered by a unique time series for unfilled job openings,

the empirical analysis is divided in two parts. The first (1993-2003) is covered by the ISFOL HWTS, while the second (2004-2012) is analyzed by means of the harmonized ISTAT vacancy rate.

The main results achieved in this paper can be summarized as follows. First, on the empirical ground, we find that in two periods under examination, despite some significant differences in the co-movements of involved series, the volatility of the labour market tightness indicator is much higher than the volatility of labour productivity. This finding appears quite robust and extends to Italy previous results on the unemployment volatility puzzle derived in other countries (e.g. Zhang 2008, Miyamoto 2012 and Gertner et al. 2012).

Moreover, from a theoretical perspective, we show that a matching model with segmented labour markets and on-the-job search build along the lines of Krause and Lubik (2006) has the potential to provide the required amplification mechanism. Specifically, we show that a reasonably calibrated version of that model is fairly able to reproduce an amplification of productivity shocks consistent with available official empirical evidence.

References

[1] Abrahm K.G. (1987), Help-Wanted Advertising, Job Vacancies, and Unemployment, *Brookings Papers on Economic Activity*, Vol. 18, No. 1, pp. 207-248.

[2] Amaral P.S. and M. Tasci (2012), The Cyclical Behavior of Equilibrium Unemployment and Vacancies across OECD Countries, *Working Paper of the Federal Researve Bank of Cleveland*, No. 36.

[3] Baldi C. and M. Sorrentino (2009), I Posti Vacanti in Italia e in Europa. Le Nuove Statistiche Trimestrali Armonizzate: Prime Analisi delle Serie Storiche, *Presented during the XXIV National Conference of Labour Economics*, Sassari.

[4] Battisti M. (2013), Reassessing Segmentation in the Labour Market: An Application for Italy 1994-2004, *Bulletin of Economic Research*, Vol. 65, No. s1, pp. s38-s55.

[5] Cardullo G. (2010), Matching Model Under Scrutiny: An Appraisal of the Shimer Puzzle, *Journal of Economic Survey*, Vol. 24, No. 9., pp. 622-656.

[6] Cipollone P. (2001), Is the Italian Labour Market Segmented?, *Temi di Discussione della Banca d'Italia*, No. 400.

[7] Gartner H., Merkl C. and T. Rothe (2012), Scerosis and Large Volatilities: Two Sides of the Same Coin, *Economic Letters*, Vol. 117, No. 1, pp. 106-109.

[8] Gómez V. and A. Maraval (1996), *Programs TRAMO and SEATS, Instruction for User*, Electronic Dissemination of Information Unit, Research Department, Banco de España.

[9] Guerrazzi M. (2011), Search and Stochastic Dynamics in the Old Keynesian Economics: A Rationale for the Shimer Puzzle, *Metroeconomica*, Vol. 62, No. 4, pp. 561-586.

[10] Hagedorn M. and I. Manovskii (2008), The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited, *American Economic Review*, Vol. 98, No. 4, pp. 1692-1706.

[11] Hall R.E. (2005a), Employment Fluctuations with Equilibrium Wage Stickiness, *American Economic Review*, Vol. 95, No. 1, pp. 50-65.

[12] Hall R.E. (2005b), Employment Efficiency and Sticky Wages: Evidence from Flows in the Labor Market, *Review of Economic and Statistics*, Vol. 87, No. 3, pp. 397-407.

[13] Hosios A.J. (1990), On the Efficiency of Matching and Related Models of Search and Unemployment, *Review of Economic Studies*, Vol. 57, No. 2, pp. 279-298.

[14] Justiniano A. and C. Michelacci (2011), The Cyclical Behavior of Equilibrium Unemployment and Vacancies in the US and Europe, *NBER Working Paper Series*, No. 17429.

[15] Krause M.U. and T.A. Lubik (2006), The Cyclical Upgrading of Labor and On-the-Job Search, *Labour Economics*, Vol. 13, No. 4, pp. 459-477.

[16] Martin J.P. (1996), Measure of Replacement Rates for the Purpose of International Comparison: A Note, *OECD Economic Studies*, No. 26.

[17] Miyamoto H. (2011), Cyclical Behaviour of Unemployment and Job Vacancies in Japan, *Japan and the World Economy*, Vol. 23, No. 3, pp. 214-225.

[18] Mandrone E. (2012), *Job Vacancies in the Italian Labour Market: The New ISFOL Help Wanted Time Series*, ISFOL Temi & Ricerche, No. 2.

[19] Mortensen D.T. and E. Nagypàl (2007), More on Unemployment and Vacancy Fluctuations, *Review of Economics*, Vol. 10, No. 3, pp. 327-347.

[20] Petrosky-Nadeau N. and E. Wasmer (2013), The Cyclical Volatily of Labor Markets under Frictional Financial Markets, *American Economic Journal: Macroeconomics*, Vol. 5, No. 1, pp. 193-221.

[21] Pissarides C.A. (2000), *Equilibrium Unemployment Theory*, 2nd Edition, MIT Press, Cambridge, Mass.

[22] Pissarides C.A. (2009), The Unemployment Volatility Puzzle: Is Wage Stickiness the Answer?, *Econometrica*, Vol. 77, No. 5, pp. 1339-1369.

[23] Pistaferri L. (1999), Informal Networks in the Italian Labour Market, *Giornale degli Economisti e Annali di Economia*, Vol. 58, No. 3-4, pp. 355-375.

[24] Sestito P. (1988), Esiste una Curva di Beveridge per l'Italia?, *Temi di Discussione del Servizio Studi della Banca d'Italia*, No. 101.

[25] Shimer R. (2005), The Cyclical Behavior of Equilibrium Unemployment and Vacancies, *American Economic Review*, Vol. 95, No. 1, pp. 25-48.

[26] Shimer R. (2005), The Consequences of Rigid Wages in Search Models, *Journal of the European Economic Association*, Vol. 2, No. 2-3, pp. 469-479.

[27] Silva J. and M. Toledo (2009), Labor Turnover Costs and the Cyclical Behavior of Vacancies and Unemployment, *Macroeconomic Dynamics*, Vol. 13, No. 1, pp. 76-96.

[28] Valletta R. (2005), Help-Wahted Advertising and Job Vacancies, *FRBSF Economic Letter*, No. 2005-02.

[29] Zhang M. (2008), Cyclical Behavior of Unemployment and Job Vacancies: A Comparison between Canada and United States, *B.E. Journal of Macroeconomics*, Vol. 8, No. 1, pp. 1-27.