Human Capital, Growth and Financial Imperfections

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Abstract

This paper presents a model of endogenous growth featuring two engines of growth and two frictions in financial markets. The two engines are industrial innovation and human capital accumulation. The two frictions are overpriced annuities and moral hazard in lending. The main result of the paper is that human capital accumulation is harmed by moral hazard but is favoured by overpriced annuities.

1 Introduction

In the last two decades the nexus between finance and growth has been the subject of a large number of studies. Most of these studies share the view that growth is caused by finance and focus on the impact of those imperfections that characterise trade in financial markets. In the prototypical study of this literature, an informational asymmetry plagues the borrower-lender relationship and reduces resources devoted to productive or innovative investments. A lower rate of growth follows quite logically.

Imperfections in financial markets, however, are not only confined within the borrower-lender relationship. These markets, in fact, also provide insurance and pension services to individuals. In the US and UK, in particular, the largest institutional holders of assets are pension funds and insurance corporations rather than standard investment funds. In the US these funds manage 59% of total financial wealth while this figure increases to 80% for the UK (source: Oecd). Thus, in the light of the pervasiveness of asymmetric information in insurance relationships, financial markets are bound to be imperfect even before the borrower-lender relationship enters into the picture.

One of the most striking consequences of these imperfections is the insufficient supply of annuities. This is by all means a relevant deficiency of financial markets in the ligth of the welfare enhancing function of annuities in a world with uncertain lifespans and risk aversion (Cannon and Tonks, 2008).

The objective of this paper is to compare imperfections in annuity markets and in standard lending relationships with respect to their impact on growth. For this purpose, we build an endogenous growth model that contain both types of imperfections. In the model, the lending relationship is plagued by moral hazard while annuities are overpriced due to some underlying informational asymmetry between traders. A second feature of the model is the fact that it allows either industrial innovation and human capital accumulation as engines of growth.

With the groundings offered by the model, we find that the two imperfections operate in similar ways with respect to industrial innovation. Both imperfections, in fact, insert a wedge between the return on savings and the return on investments so that their distinction appears to be immaterial on theoretical grounds. By constrast, the two imperfections operate in opposite directions with respect to the accumulation of human capital. While moral hazard in lending decreases accumulation because future labour income is discounted at a larger rate, imperfect annuities operate as a stimulus. The key difference between human and financial wealth is that the first can not buy annuities. Thus, a less efficient annuity market makes financial wealth relatively less appealing with respect to human wealth. It follows that, for those that are faced with schooling decisions, inefficient annuities increase the incentive to invest in education.

While the link between growth and asymmetric information in lending relationships has been the subject of a vast literature[1], the connection between imperfect annuities and growth has been largely overlooked. A notable exception is Heijdra and Mieirau (2012), which studies the impact of overpriced annuities on growth and retirement decisions within an AK model and with a sophisticated demographic dynamics. With respect to this contribution, the present paper adopts the simple demographics of Blanchard-Yari but adds human capital investments and moral hazard in lending. Hu (1999) studies an economy that combines human capital accumulation with Blanchard-Yari demographics, the market for annuities is either supposed to be perfect or completely absent. With respect to this contribution, the present paper adds industrial innovation and moral hazard in lending and allows for a continuous degree of imperfections in the annuity market.

The paper is composed as follows. In section 2 we set up and solve the model without imperfections. In section 3 we introduce imperfections and study the comparative statics for the rate of growth. Section 4 contains some concluding remarks. Finally, mathematical details are provided in the appendix.

2 The Economy

Individuals

[1] A largely non-exaustive list includes Bencivegna and Smith (1991), Pagano (1993), Aghion *et al.* (2005, 2010). Levine (2005) provides an excellent survey of the theoretical and empirical literature.

The population consists of an infinite number of overlapping generations with constant death rate and constant birth rate (Blanchard, 1985). The death and the birth rates are also equal so that the size of the population, normalised to one, is constant. Individuals are born with zero financial wealth and with a skill endowment which is uniform for all members of the same generation. In addition, individuals are endowed with a unit flow of time which can be allocated to labour and to educational activities. Labour earns wages while education increases the skill endowment. Individuals have access to a risk free financial activity and to a mutual fund that remunerates current wealth at a financially fair rate in exchange of total wealth transfer at death.

At any time, individuals decide how much to consume and to save and how to split their time endowment between labour and education. Below we present the problem faced at t by an individual born at τ:

$$\max_{c_{\tau,t},s_{\tau,t}} \int_t^\infty e^{-(\delta+\rho)(z-t)}\log(c_{\tau,z})dz \tag{1}$$

$$\begin{aligned}
\dot{a}_{\tau,t} &= (r+\rho)a_{\tau,t} + (1-s_t)q_{\tau,t}w_t - c_{\tau,t} &(2)\\
\dot{q}_{\tau,t} &= hs^{\alpha}_{\tau,t}q_{\tau,t} \qquad \alpha<1 &(3)\\
\dot{w}_t &= g_w w_t &(4)\\
0 &\le s_{\tau,t}\le 1 \quad q_{\tau,\tau},\ w_{\tau,\tau}\text{: given} \quad a_{\tau,\tau}=0
\end{aligned}$$

Future consumption utility is discounted at rate $\delta+\rho$, the first parameter is the rate of time preference while the second represents the instantaneous death probability. Equation 2 represents the dynamic budget constraint. Financial wealth $a_{\tau,t}$ is remunerated at rate $r+\rho$ as r represents the interest rate on the riskless activity and ρ the rate of return from the mutual fund. Current labour income is given by $(1-s_t)q_{\tau,t}w_t$ as $(1-s_{\tau,t})$ represents the fraction of time devoted to the labour market, $q_{\tau,t}$ the current skill endowment and w_t the wage paid to a unit of skills. Equation 3 implies that time devoted to education exhibits decreasing marginal returns while the efficiency of the educational system is conveyed by h. Finally, equation 4 posits that wages increase at a constant rate g_w. The interest rate and the rate of growth of wages are exogenous from the point of view of single agents.

We assume that the following restriction holds:

$$\rho+\delta\ge h \tag{5}$$

In addition, when solving the problem 1-4 we conjecture that

$$r+\rho-g_w-h>0 \tag{6}$$

This conjecture will be proved to be true in equilibrium.

In the appendix, we formally solve the problem 1-4. We show that the optimal dynamics of s exhibits a saddle-path property and provide arguments that rule out paths leading to the boundaries of the interval $[0,1]$. We also show that the unique rational expectation solution for s is time-independent and cohort-independent. This solution turns out to be implicitly defined by the condition

$$\alpha h s^{\alpha-1}(1-s) = r + \rho - g_w - h s^{\alpha} \tag{7}$$

The cost of shifting a marginal amount of time ds from labour to education is given by $q_{\tau,t} w_t ds$. By contrast, the return from the shift is given by the increase in the income flow guaranteed by the larger skill endowments from the next period onwards. In fact, following the shift, the endowment path moves upwards. Immediately after the shift, labour income increases by $\alpha h s^{\alpha-1}(1-s)\, q_{\tau,t} w_t ds$ but in the future this variation enlarges at rate $g_w + h s^{\alpha}$. In turn, since individuals face a total rate of return on financial wealth given by $r+\rho$, the present discounted value of this additional flow of income is given by $q_{\tau,t} w_t (1-s) ds/(r+\rho-g_w-hs^{\alpha})$. Thus, the interpretation of equation 7 is that it imposes equality between the marginal cost and the marginal return from time devoted to education.

The rate of growth of individual consumption is time and cohort invariant:

$$g_c = r - \delta \tag{8}$$

As with infinite lives, the rate of growth of individual consumption is given by the difference between the riskless interest rate and the rate of time preference. In fact, individuals face a positive death rate, which represents an incentive to anticipate consumption. However, this incentive is completely offset by the incentive to postpone consumption due to the remuneration of wealth from the mutual fund.

Manufacturing - Commodity Producers

The manufacturing sector is made of two segments, one produces a general purpose commodity while the other produces intermediate goods. The commodity represents the numeraire of the economy and can be used as a consumption good, as an investment good and as an input in the intermediate segment. By constrast, intermediate goods can be used only as inputs in the commodity segment.

Commodity producers are perfectly competitive and operate with a constant returns to scale technology:

$$Y_t = L_t^{1-\theta} N_t^{\sigma(1-\theta)-\theta\left(\frac{1}{\pi}-1\right)} \left[\int_0^{N_t} x_{i,t}^{\pi} di\right]^{\frac{\theta}{\pi}} \qquad 0<\pi,\theta<1 \tag{9}$$

In this expression, Y_t represents the physical output, L_t represents the skill input and $x_{i,t}$ the physical amount of the $i-th$ intermediate input. The number of intermediate inputs produced at t is given by N_t. The marginal productivity of inputs is controlled by θ while the substitutability between any couple of intermediate goods is controlled by π. The return from variety is controlled by σ (Benassy, 1998).

Commodity producers demand labour so as to equate the marginal productivity to the wage:

$$w_t = (1-\theta)\,\frac{Y_t}{L_t} \tag{10}$$

Analogously, these firms demand the $i-th$ intermediate input so as to achieve equality between the price $p_{i,t}$ and the marginal productivity. This boils down to the following aggregate demand for $x_{i,t}$ and the subsequent aggregate expenditure for intermediate goods:

$$p_{i,t} = \theta L_t^{1-\theta} N_t^b \left[\int_0^{N_t} x_{i,t}^{\pi} di\right]^{\frac{\theta}{\pi}-1} x_{i,t}^{\pi-1} \tag{11}$$

$$\int_0^{N_t} p_{i,t} x_{i,t} di = \theta Y_t \tag{12}$$

Equations 10 and 12 make clear that commodity producers make no profit since all their revenues are distributed to workers and to intermediate producers.

Manufacturing - Intermediate Producers

Any intermediate good is produced by a monopolist by means of a technology that requires only one unit of the commodity to produce one unit of output. Equation 11 implies that the absolute value of the elasticity of demand is given by $\frac{1}{1-\pi}$. Thus, monopolists price uniformly their goods by imposing a common mark-up over their common marginal cost: $p_{i,t} = 1/\pi$.

A consequence of uniform pricing is that commodity producers demand equal amounts of all intermediate goods. Let H_t represents the aggregate amount of Y_t that is devoted to the production of intermediate goods, uniform expenditure over these goods gives

$$x_{i,t} = x_t = \frac{H_t}{N_t} \tag{13}$$

Thus, the profit flow accruing to monopolists is given by

$$\Pi_{i,t} = (p_{i,t} - 1)x_{i,t} = \frac{1-\pi}{\pi}\frac{H_t}{N_t} \tag{14}$$

Research and Development

Monopoly for any intermediate good is due to endless patent protection guaranteed to the R&D firm that firstly designed the good. We assume that entering the R&D sector is free but setting up a new R&D firm requires to invest an amount kY_t of the commodity. The duration of R&D firms is very short. Once a new firm is started, it immediately files λN_t new patents, sell these patents to monopolists and exit from the market. Thus we assume that the range of applications that spur from any given R&D project enlarges with respect to the number of existing goods. This is the usual knowledge externality that drives growth in models featuring expanding variety (Romer, 1990).

Let E_t represents the aggregate amount of the general purpose commodity invested in the creation of R&D firms. The mechanics of innovation boil down to the following rate of growth for N_t:

$$g_N = \frac{\lambda}{k}\frac{E_t}{Y_t} \tag{15}$$

Due to free entry in the production of intermediate goods, the price of patents is bid to the point it equals the discounted flow of monopolistic profits J_t:

$$J_t = \frac{1-\pi}{\pi}\int_t^{\infty} e^{-r(\mu-t)}\frac{H_\mu}{N_\mu}d\mu \tag{16}$$

In turn, free entry in the R&D sector implies equality between entry costs and patent revenue:

$$kY_t = \lambda N_t J_t \tag{17}$$

3 Aggregation and Equilibrium

In this section we solve the model along a stable path. All non-stationary variables increase at a constant rate while all stationary variables are constant.

Equilibrium in the labour market

The aggregate demand of skills is given by equation 10. The supply of skills is given by the aggregation of individual skill endowments. Equation 3 implies that individual endowments increase at rate hs^α. Thus, to perform the aggregation we need an assumption as for the skill endowment at birth. Let us assume that the initial endowment increases across generations at rate g_q:

$$q_{t,t} = q_0 e^{g_q t}$$

Based on this assumption, aggregate skill supply reads as follows

$$L_t = \int_{-\infty}^{t} \rho e^{-\rho(t-\tau)}(1-s)q_{\tau,\tau}e^{hs^\alpha(t-\tau)}d\tau = q_{t,t}\frac{\rho(1-s)}{\rho+g_q-hs^\alpha} \tag{18}$$

The term $\rho e^{-\rho(t-\tau)}$ represents the number of individuals born at τ that are still alive at t while the term $q_{\tau,\tau}e^{hs^\alpha(t-\tau)}$ represents their individual skill endowment. Equation 18 implies that aggregate skills increase at the rate g_q.

The equilibrium wage rate w_t is solved by equating the demand and the supply of skills given by equations 10 and 18. It is straightforward to observe that wages increase at a rate given by the difference between g_Y - i.e. the rate of growth of Y_t - and g_q:

$$g_w = g_Y - g_q \tag{19}$$

Equilibrium in financial markets

The aggregate supply of financial wealth corresponds to $N_t J_t$, which is the total value of claims over monopolistic profits. By constrast, the total demand for financial wealth comes from aggregating individual portfolio holdings:

$$A_t = \int_{-\infty}^{t} \rho e^{-\rho(t-\tau)} a_{\tau,t} d\tau \tag{20}$$

Since the free entry condition 17 dictates $N_t J_t = kY_t/\lambda$, equilibrium in financial markets requires

$$A_t = \frac{k}{\lambda} Y_t \tag{21}$$

In the appendix we compute the integral in 20. This allows us to express the equilibrium condition 21 as follows

$$\frac{1}{r+\rho-g_w-hs^{\alpha}} \left\{ \frac{\rho+g_q-hs^{\alpha}}{\rho+g_w+g_q-g_c} - 1 \right\} (1-\theta) = \frac{k}{\lambda} \tag{22}$$

Notice that, once one takes into account equation 8, the aggregate demand of financial wealth - i.e. the LHS of the above expression - increases with respect to the interest rate. Equation 22 may thus be thought of as determining the equilibrium interest rate.

Equilibrium in the commodity market

The market for the general purpose commodity clears if

$$\frac{C_t}{Y_t} + \frac{E_t}{Y_t} + \frac{H_t}{Y_t} = 1 \tag{23}$$

In this equation, C_t represents aggregate consumption. The expression may be thought of as determining the ratio E/Y and, ultimately, the rate of growth for the number of intermediate goods. However, to see how this equation determines g_N we need to express the three ratios in terms of growth rates.

Let us start with the ratio C/Y. Differentiate equation 20 and use the individual budget constraint 2:

$$\dot{A}_t = rA_t + w_t L_t - C_t \tag{24}$$

This expression is intuitive. Once transfers across cohorts are consolidated, the increase in aggregate wealth corresponds to aggregate savings. In turn, aggregate savings are given by the difference between aggregate income - from capital and from labour - and aggregate consumption. Divide both sides of equation 24 by A_t and use equations 21 and 10 to obtain:

$$\frac{C_t}{Y_t} = (1-\theta) - (g_Y - r)\frac{k}{\lambda} \tag{25}$$

Next, we focus on the ratio H/Y. Substitute the price $p_{i,t} = 1/\pi$ in equation 12 and recall that $H_t = N_t x_t$:

$$\frac{H_t}{Y_t} = \pi\theta \tag{26}$$

Equations 15, 25 and 26 provide the required expressions for the three ratios that appear in the equilibrium condition for the general commodity market. Substitute these expressions and derive the equation that determines g_N:

$$g_N = g_Y - r + \frac{\lambda}{k}\theta(1-\pi) \tag{27}$$

Equilibrium in the markets for intermediate goods

Aggregate demand and supply for the $i-th$ intermediate good are represented by equations 11 and 13 respectively. Combine these equations and use the pricing rule $p_{i,t} = \pi$:

$$\pi = \theta L_t^{1-\theta} N_t^b \left[\int_0^{N_t} x_t^{\pi} di\right]^{\frac{\theta}{\pi}-1} x_t^{\pi-1} \qquad x_t = H_t/N_t$$

This expression may be thought of as determining the amount of resources that are devoted to the production of intermediate goods and, ultimately, to the production of the aggregate amount of the commodity. Thus, its differential determines the rate g_Y:

$$g_Y = \sigma g_N + g_q \tag{28}$$

General Equilibrium

In general equilibrium all agents behave optimally and all markets clear. The optimal behaviour of individuals is represented by equations 7 and 8, the first determines the fraction of time devoted to education (s) while the second the rate of growth of individual consumption (g_c). Equilibrium in the four markets that compose the economy - labour, wealth, general commodity and intermediate goods - is represented by equations 19, 22, 27 and 28. These determine the growth of wages (g_w), the interest rate (r), the growth of varieties (g_N) and the growth of commodity output (g_Y).

We close the model by assuming an intergenerational externality for the initial level of skills. Thus, the rate of growth of initial skills across generations is equal to the rate of growth of skills within generations:

$$g_p = hs^{\alpha} \tag{29}$$

Once one uses this assumption and makes straightforward substitutions, the model turns out to be summarised by the following system:

$$
\begin{aligned}
y^2 + y\left[\frac{\lambda}{k}(1-\theta)+\delta\right] - \rho\,(\delta+\rho) &= 0 && (30)\\
g_N + y + \delta &= \frac{\lambda}{k}\theta(1-\pi) && (31)\\
\alpha h s^{\alpha-1} - \alpha h s^{\alpha} &= y + \delta + \rho && (32)\\
\\
\text{where } y &= r - g_Y - \delta
\end{aligned}
$$

For the sake of simplicity we have used y for the difference between the rate of interest net of the discount rate and the rate of growth. The first equation is the equilibrium condition in the market for financial wealth (equation 22) after substituting the education externality (equation 29) and the expressions for wage growth (equation 19) and for consumption growth (equation 8). The second equation is the equilibrium condition in the market for the general commodity (equation 27). Finally, the third equation is the condition that determines time devoted to education (equation 7) after substituting equations 19, 28 and 29.

Notice that the system has a recursive structure, the first equation determines y while the second and the third determine g_N and s as a function of y. Notice also that the first equation has two solutions - one positive and one negative - so we need a restriction to single out one of the two. For this purpose we observe that a necessary and sufficient condition for having positive aggregate wealth is

$$\frac{\rho + g_e - hs^{\alpha}}{\rho + g_w + g_e - g_c} - 1 > 0$$

This requires y to be positive so that the unique solution consistent with positive aggregate wealth is

$$y = \frac{-\left[\frac{\lambda}{k}(1-\theta)+\delta\right] + \sqrt{\left[\frac{\lambda}{k}(1-\theta)+\delta\right]^2 + 4\rho\,(\delta+\rho)}}{2}$$

Once substituted back in equations 31 and 32, the solution for y determines g_N and s. In turn, given g_N and s, one may retrieve all the endogenous of the model. Lastly, we are left to prove the conjecture stated in equation 6. For this purpose, notice that $y > 0$ jointly with the parameter restriction 5 guarantees that the conjecture is true in equilibrium.

4 Financial Imperfections

In this section we introduce two imperfections in financial relationships. The first imperfection relates to moral hazard in R&D investments. We assume that the innovator that borrows kY_t resources to open an R&D firm can appropriate a fraction $1-\zeta$ $(0 < \zeta \leq 1)$ of these resources without making any investment.

Thus, the innovator starts the firm only if she is awarded claims on future patent revenue with a current value V_t at least as large as $(1-\zeta)kY_t$:

$$V_t \geq (1-\zeta)kY_t$$

As a consequence, the value of claims left to investors is given by $\lambda N_t J_t - V_t$. Thus investors will lend the resources necessary to start the firm only if their share of future revenues is at least as large as the amount invested:

$$\lambda N_t J_t - V_t \geq kY_t$$

Competition among innovators and investors requires that these inequalities are both satisfied with the equal sign. This implies a new expression for the free entry condition in the R&D sector:

$$kY_t = \frac{\lambda N_t J_t}{2-\zeta} \tag{33}$$

Intuitively, the cost of setting up a new R&D firm is not equalised to the overall income from patents but only to the fraction $1/(2-\zeta)$ of income that accrues to investors. In addition, since $N_t J_t$ continues to represent the supply of financial wealth, the equilibrium condition in the market for wealth reads as follows:

$$A_t = \frac{k}{\lambda}(2-\zeta)Y_t \tag{34}$$

Once one looks at the flow of payments that involve R&D firms, a second consequence of moral hazard is that, for any patent, these firms earn a transfer given by

$$(1-\zeta)\, J_t/\,(2-\zeta) \tag{35}$$

Since this transfer is not captured by investors, it is not distributed to individuals in proportion of their financial wealth. For this reason, we assume that the transfer is uniformly rebated to all individuals independently from the size of their wealth.

The second imperfection relates to the annuity market. Here we follow Heijdra and Mieirau (2012) and assume that individuals are not able to perfectly commit over wealth repayments at their death so that, with a probability $1-z$ $(0 < z \leq 1)$, their wealth is not paid back to the mutual fund. This means that the financially fair interest rate paid by the fund drops from ρ to ρz. As for those resources that are not repaid to the fund we assume uniform redistribution to all individuals.

4.1 Consumption, schooling and wealth accumulation

The two imperfections modify the budget constraint of individuals. First, the friction in the annuity market reduces the total rate of return on financial wealth and generate a transfer that is independent from financial wealth. Second, the friction in R&D financing generates a further transfer independent from wealth. The modified budget constraint reads as follows:

$$\dot{a}_{\tau,t} = (r + \rho z)a_{\tau,t} + (1 - s_t)q_{\tau,t}w_t + \rho(1 - z)A_t + \dot{N}_t \frac{1 - \varsigma}{2 - \varsigma} J_t - c_{\tau,t} \tag{36}$$

The total rate of return on financial wealth decreases from $r+\rho$ to $r+\rho z$. The transfer due to the friction in the annuity markets is represented by $\rho(1 - z)A_t$ while the transfer due to moral hazard in R&D financing is given by $\dot{N}_t \frac{1-\varsigma}{2-\varsigma} J_t$. The term $\dot{N}_t$ is the number of transaction in the market for patents while the term $\frac{1-\varsigma}{2-\varsigma} J_t$ represents the transfer per patent as reported in equation 35.

Due to the change in the rate of return on wealth, the modified solutions for consumption and education are the following[2]:

$$g_c = r - \delta - \rho(1 - z) \tag{37}$$

$$\alpha h s^{\alpha-1}(1 - s) = r + \rho z - g_w - h s^{\alpha} \tag{38}$$

The variations in flows of income affect consumption and wealth accumulation. For this reason we need to compute again the level of aggregate wealth A_t. In the appendix, we show that aggregate wealth is given by the following expression:

$$\begin{aligned} A_t &= \frac{1 - \theta}{(y + \delta + \rho z)\,(\rho + g_Y - g_c)\,/\,(g_c - g_Y) - \varphi} Y_t \qquad (39) \\ \varphi &\equiv \rho(1 - z) + g_N \frac{1 - \varsigma}{2 - \varsigma} \end{aligned}$$

In the appendix we also show that transfers due to the annuity friction affect individual wealth accumulation but do not affect aggregate wealth accumulation. In fact these transfers are made of already existing claims that flow from those who die to those that remain alive. By contrast, transfers that are caused by moral hazard in R&D financing represent genuine creation of new wealth. As a consequence, they affect the dynamics of A_t:

$$\dot{A} = rA_t + (1 - s)L_t w_t + g_N \frac{1 - \varsigma}{2 - \varsigma} A_t - C_t \tag{40}$$

[2] As for the frictionless economy, the solution for schooling holds under the conjecture that $r + \rho z - g_w - h > 0$. This conjecture can be shown to be true in equilibrium under the parameter restriction 5.

4.2 Equilibrium

Since we have introduced imperfections only in financial markets, the labour market and the market for intermediate goods are not affected. As a consequence, equation 19 continues to determine g_w while equation 28 continues to determine g_Y. The rate of growth of varieties g_N is determined in the market for the general commodity. Equilibrium in this market requires that the ratios of the three expenditure components - C/Y, H/Y and E/Y - sum up to one (equation 23). Ratios E/Y and H/Y continue to be given by equations 15 and 26 respectively. For C/Y combine 34 and 40:

$$\frac{C}{Y} = (1-\theta) + (r - g_Y)\frac{k}{\lambda}(2-\zeta) + g_N\frac{k}{\lambda}(1-\zeta) \tag{41}$$

Substitute these ratios in 23 and compute the new expression for g_N:

$$g_N = g_Y - r + \frac{\lambda}{k}\theta(1-\pi)\frac{1}{2-\zeta} \tag{42}$$

Lastly, we are left with the determination of the interest rate. For this reason, we now look at the market for financial wealth. Imposing equilibrium in this market amounts to substitute equation 39 in 34. Below we report the equilibrium condition after taking account of equations 37 and 42:

$$y^2 - y\left[\rho(1-z) - F(\zeta)\right] - \left[\rho(1-z)F(\zeta) + \rho\left(\delta+\rho\right)\left(2-\zeta\right)\right] = 0 \tag{43}$$

$$F(\zeta) \equiv \frac{\lambda}{k}\frac{1}{2-\zeta}\theta(1-\pi) + \frac{\lambda}{k}(1-\theta) + \delta$$

As in the frictionless economy, the positive root of 43 solves for y which, in turn, gives s and g_N once it is substituted in equations 38 and 42. All other endogenous follow straightforwardly.

5 Comparative Statics

To compute the impact of the two financial imperfections we differentiate the positive root of 43 with respect to z and ξ. Below we present the derivatives computed for small departures from full efficiency. Yet, the sign of derivatives does not depend on how far is the economy from fully efficient financial markets:

$$\frac{dy}{dz} = -\rho\frac{y+F(1)}{2y+F(1)} < 0$$

$$\frac{dy}{d\zeta} = -y\frac{\frac{\lambda}{k}(1-\theta)+y+\delta}{2y+F(1)} < 0$$

These results imply that the difference between the interest rate and the rate of growth decreases as both imperfections loose momentum. To investigate the impact of imperfections on the rate of growth we need to look at changes in the schooling rate s (equation 38) and in the rate g_N (equation 42). We start with the impact on s by differentiating expression 38 with respect to z and ζ:

$$\frac{ds}{dz} = -\left[\alpha(1-\alpha)hs^{\alpha-2} + \alpha^2 hs^{\alpha-1}\right]^{-1} \rho \left[\frac{y}{2y+F(1)}\right] < 0$$

$$\frac{ds}{d\zeta} = -\left[\alpha(1-\alpha)hs^{\alpha-2} + \alpha^2 hs^{\alpha-1}\right]^{-1} \frac{dy}{d\zeta} > 0$$

The result is quite striking. The amount of time devoted to education increases if financial markets become more efficient in copying with moral hazard but decreases if the annuity market becomes more efficient in transferring wealth among individuals. Thus, if growth is driven only by human capital accumulation ($\sigma = 0$, $h > 0$), a more efficient financial market improves growth but a more efficient annuity market decreases growth.

As for the impact of imperfections on g_N we differentiate the expression 42:

$$\frac{dg_N}{dz} = -\frac{dy}{dz} > 0$$

$$\frac{dg_N}{d\zeta} = -\frac{dy}{d\zeta} + \frac{\lambda}{k}\theta(1-\pi) > 0$$

This implies that both imperfections are detrimental for the rate of growth of varieties. Thus, in an economy where growth is only sustained by variations in total factor productivity ($\sigma > 0$, $h = 0$), the rate of growth increases if both segments of financial markets become more efficient.

Financial frictions and industrial innovation

Why do the imperfections impact negatively on industrial innovation? The essence of the two imperfection is to insert two wedges between the actual interest rate for savers (r^s) and the internal rate of return on investments (r^i). The actual interest rate for savers corresponds to the rate of return from financial wealth net of the death rate. For the internal return on investments we intend the rate that equates the cost of the R&D project to the discounted flow of (private) returns.

The wedge due to the imperfection in the annuity market separates r^s from the risk-free interest rate:

$$r - r^s = \rho(1-z) \tag{44}$$

Rather intuitively, the size of the wedge is given by the product between the measure of imperfections and the death probability. If individuals were infinitely lived ($\rho = 0$), the two rates would coincide.

By contrast, the wedge due to moral hazard in R&D financing separates the risk-free interest rate from r^i. To compute this wedge applies the definition of r^i:

$$\lambda N_t \int_t^{\infty} e^{-r^i(\mu - t)} \frac{1-\pi}{\pi} x_\mu d\mu = kY_t \tag{45}$$

Then substitute equations 33 and 16 in the latter and solve the integrals:

$$r^i - r = \frac{1-\zeta}{2-\zeta} \frac{\lambda}{k} \theta (1-\pi) \tag{46}$$

This expression reveals that, apart from the friction itself, the wedge depends on the overall profitability of R&D activity. Profitability, in turn, is a composition of three factors. The first factor is the efficacy of primary resources invested in R&D (λ/k). The second factor is the relative size of the intermediate sector ($\theta\pi$). Finally, the third factor is the profit per unit of output in the intermediate sector ($(1-\pi)/\pi$).

The distance between $r^i - r^s$ is obtained by combining equations 44 and 46:

$$r^i - r^s = \rho(1-z) + \frac{1-\zeta}{2-\zeta} \frac{\lambda}{k} \theta (1-\pi) \tag{47}$$

This expression makes clear that the two rates coincide in the absence of frictions $[z = \zeta = 1]$. From this perspective, the negative impact of frictions on industrial innovation can be explained in two ways. First, for a given r^i, frictions decrease r^s and discourage wealth accumulation. Second, for a given r^s, frictions increase r^i which implies heavier discounting of profits and lower incentives for patent production.

Financial frictions and education

Why do the frictions impact in different ways on educational investments? The discussion conducted so far can be reinterpreted by saying that, for given current and future production, the two transfers reduce the flows of income accruing to financial wealth. Thus, everything else equal, the two imperfections reduce financial wealth. In turn, lower financial wealth is possible only if future aggregate profits $\left[\frac{1-\pi}{\pi} H_t\right]$ are discounted by means of an heavier adjusted discount rate. General equilibrium effects that reduce the rate of growth of H_t can not revert the direction of this mechanism. This explains the reason for y - i.e. the adjusted discount rate - to decrease as imperfections become less compelling.

Equipped with this interpretation we now consider the incentives to human capital accumulation. When deciding on time to devote to education, individuals compare the current cost in terms of foregone labour income to future returns in terms of an upward shift in the labour income path. In turn, since future returns are discounted at the same rate of future profits, heavier discounting implies a reduction in human capital incentives and, as a consequence, a reduction in time devoted to school.

Obviously the mechanism that has been discussed holds for both imperfections but, as for the annuity imperfection, a second mechanism is in place. What distinguishes financial wealth from human wealth is the fact of being fully tradable. Annuity markets, in particular, allow to trade financial wealth at the end of life in exchange of payments while still alive. By contrast, human wealth is simply destroyed at the end of life and does not allow for intertemporal transfers. Against this backdrop, the imperfection in the annuity market amounts to a deterioration in the terms of trade of financial wealth. Thus, the trade-off between financial and human wealth that is faced by individuals as they choose educational investments tilts in favour of the human wealth.

In the present economy, this second mechanism is more powerful than the first. As a consequence, the imperfection in the annuity market contributes to human capital investments and may increase the economy rate of growth if human capital is a more powerful driver than industrial innovation.

6 Concluding Remarks

In this paper we have studied how financial imperfections impact on human capital accumulation and industrial innovation. The key innovation of the paper consists in combining two types of imperfections - overpriced annuities and standard moral hazard of borrowers - and two engines of growth in a unified framework. Within this setting we reach two main conclusions. First, concerning industrial innovation, imperfect annuities have the same negative impact of standard moral hazard. Second, concerning human capital accumulation, moral hazard exerts a negative impact but imperfect annuities promote faster accumulation. The reason for this asymmetry lies in the lower insurance power of non-human wealth in a context of imperfect annuity markets.

Appendix

A Consumption and Schooling

Here we solve the dynamic problem stated in equations 1-4. The Hamiltonian of the problem is the following:

$$H = e^{-(\delta+\rho)(t-\tau)} \log(c_{\tau,t}) + \mu_{\tau,t} \left[(r+\rho)a_{\tau,t} + (1-s_t)q_{\tau,t}w_t - c_{\tau,t}\right] + \upsilon_{\tau,t} h s_{\tau,t}^{\alpha} e_{\tau,t}$$

Optimal conditions for consumption:

$$e^{-(\delta+\rho)(t-\tau)} c_{\tau,t}^{-1} = \mu_{\tau,t} \tag{48}$$

Optimal condition for schooling:

$$s_{\tau,t} \in (0,1) \qquad -\mu_{\tau,t} w_t + \alpha \upsilon_{\tau,t} h s_{\tau,t}^{\alpha-1} = 0 \tag{49}$$

$$s_{\tau,t} = 1 \qquad -\mu_{\tau,t} w_t + \alpha \upsilon_{\tau,t} h s_{\tau,t}^{\alpha-1} \geq 0 \tag{50}$$

$$s_{\tau,t} = 0 \qquad -\mu_{\tau,t} w_t + \alpha \upsilon_{\tau,t} h s_{\tau,t}^{\alpha-1} \leq 0 \tag{51}$$

Euler's conditions:

$$\mu_{\tau,t}(r+\rho) = -\dot{\mu}_{\tau,t} \tag{52}$$

$$\mu_{\tau,t} w_t (1 - s_{\tau,t}) + \upsilon_{\tau,t} h s_{\tau,t}^{\alpha} = -\dot{\upsilon}_{\tau,t} \tag{53}$$

Transversality conditions:

$$\lim_{t\to\infty} \mu_{\tau,t} a_{\tau,t} = 0 \tag{54}$$

$$\lim_{t\to\infty} \upsilon_{\tau,t} q_{\tau,t} = 0 \tag{55}$$

To compute the rate of growth of individual consumption combine 48 with 52:

$$g_c = r - \delta \tag{56}$$

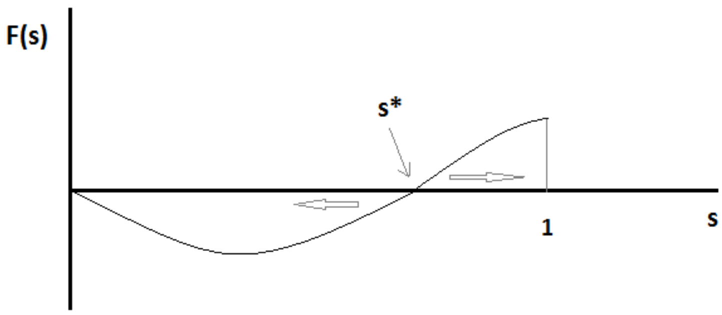
As for the choice of schooling, combine 49 with 53 to obtain the dynamics of $s \in (0,1)$:

$$\dot{s}_t = F(s_t) \qquad\qquad F(s_t) \equiv \frac{\alpha h s_t^{\alpha}(1-s_t) + h s_t^{\alpha+1} - (r+\rho-g_w)\, s_t}{(\alpha-1)}$$

The function $F(s)$ exhibits the following characteristics:

$$\begin{aligned}
\lim_{s\to 0} F(s) &= 0 \qquad \lim_{s\to 1} F(s) = \frac{r+\rho-h-g_w}{1-\alpha} > 0 \\
\lim_{s\to 0} F'(s) &= -\infty \qquad \lim_{s\to 1} F'(s) = \frac{r+\rho-h-g_w}{1-\alpha} > 0 \\
F''(s) &= \alpha^2 h s^{\alpha-1}\left(s^{-1} - \frac{1+\alpha}{\alpha}\right)
\end{aligned}$$

The sign of derivatives at the upper boundary of the interval depends on the conjecture 6. The behaviour of $F(s)$ is illustrated in figure 1 below:



The figure implies that the dynamics of s exhibits three steady states: I) $s=0$, II) $s=1$ and III) $F(s)/s=0$ with $s\in(0,1)$. The first two are stable while the third is unstable. In what follows we provide arguments that rule out paths leading to the two stable steady states. This implies that the dynamic system exhibits the saddle path property so that the solution for s is given by $F(s)/s=0$ as reported in the main text (equation 7).

Ruling out paths converging to $s=0$

To rule out these paths observe that, as $s\to 0$, the euler's condition 53 implies

$$\dot{\upsilon}_{\tau,t} = -\mu_{\tau,t} w_t$$

The fundamental solution of this differential equation is

$$\upsilon_{\tau,t} = \frac{\mu_{\tau,t} w_t}{r+\rho-g_w}.$$

Combine the optimality conditions that hold for $s \geq 0$ and substitute the expression that has been obtained for the shadow value of skills $\upsilon_{\tau,t}$:

$$-\mu_{\tau,t} w_t \left[1 - \frac{\alpha}{r + \rho - g_w} h s_{\tau,t}^{\alpha - 1} \right] \leq 0$$

It is obvious that this inequality does not hold for $s \to 0$ so that an inconsistency arises. Intuitively, as s approaches zero, the shadow value of skills as well as the marginal cost of time devoted to education become both proportional to $\mu_{\tau,t} w_t$. However, the marginal productivity of time devoted to education increases unboundedly so that the optimal level of s can not approach zero.

Ruling out paths converging to $s = 1$

To rule out these paths notice that the shape of F implies that $s = 1$ is reached at some finite time t^*. Thus, from t^* onwards, the costate variable ν decreases at rate h by the euler relationship 53. On the other hand, from t^* onwards, the skill endowment increases at rate h due to the technology of skill accumulation. This implies that the transversality condition 55 holds only if $\upsilon_{\tau,t^*} q_{\tau,t^*} = 0$. However, $q_{\tau,t^*}$is positive since the skill endowment starts from a positive value at time τ and can only increase afterwards. In addition, υ_{τ,t^*} is positive too since the optimal conditions for $s \in (0,1]$ imply $\upsilon_{\tau,t^*} \geq \mu_{\tau,t^*} w_{t^*} / \alpha h > 0$. Summing up, along paths that lead towards $s = 1$, the transversality conditions is not satisfied. This indicates that skill accumulation is suboptimal (excessive).

B Derivation of equation 22

Since initial wealth is nil, the no-Ponzi condition imposes that the present value of income at birth must be equal to the present value of consumption:

$$\int_{\tau}^{\infty} e^{-(r+\rho)(t-\tau)} (1-s) q_{\tau,t} w_{\tau,t} dt = \int_{\tau}^{\infty} e^{-(r+\rho)(t-\tau)} c_{\tau,t} dt$$

This intertemporal budget constraint can be written as follows:

$$(1-s) q_{\tau,\tau} w_{\tau,\tau} \frac{1}{r + \rho - g_w - h s^{\alpha}} = c_{\tau,\tau} \frac{1}{r + \rho - g_c}$$

Thus, the initial level of consumption is

$$c_{\tau} = q_{\tau,\tau} w_{\tau} (1-s) \frac{r + \rho - g_c}{r + \rho - g_w - h s^{\alpha}}$$

Compute $a_{\tau,t}$ by integrating forward the individual budget constraint 2:

$$\begin{aligned} a_{\tau,t} &= \int_{\tau}^{t} e^{(r+\rho)(t-z)} \left[(1-s) q_{\tau,z} w_{\tau,z} - c_{\tau,z} \right] dz = \\ &= q_{\tau,\tau} w_{\tau} (1-s) \frac{e^{(r+\rho)(t-\tau)} - e^{(g_w + h s^{\alpha})(t-\tau)}}{r + \rho - g_w - h s^{\alpha}} - c_{\tau} \frac{e^{(r+\rho)(t-\tau)} - e^{g_c (t-\tau)}}{r + \rho - g_c} \end{aligned}$$

Substitute initial consumption:

$$a_{\tau,t} = \frac{(1-s)q_{\tau,\tau}w_\tau}{r+\rho-g_w-hs^\alpha}\left[e^{g_c(t-\tau)} - e^{(g_w+hs^\alpha)(t-\tau)}\right]$$

Use the expression for $a_{\tau,t}$ in the equation that defines aggregate financial wealth A_t:

$$\begin{aligned} A_t &= \int_{-\infty}^{t} \rho e^{-\rho(t-\tau)} a_{\tau,t} d\tau = \\ &= \rho\frac{(1-s)}{r+\rho-g_w-hs^\alpha}\left\{\frac{1}{\rho+g_w+g_q-g_c} - \frac{1}{\rho+g_q-hs^\alpha}\right\} q_{t,t}w_t \end{aligned}$$

Then, substitute 18 in this expression:

$$A_t = \frac{1}{r+\rho-g_w-hs^\alpha}\left\{\frac{\rho+g_q-hs^\alpha}{\rho+g_w+g_q-g_c} - 1\right\} L_t w_t$$

Finally, use 10 in the latter and substitute the resulting expression in 21 to obtain 22.

C Derivation of equation 39

Individual consumption

We have already computed the rate of growth of individual consumption in presence of frictions (equation 37). Here we compute the initial level of consumption. For this purpose, we substitute $A_t = N_t J_t$ in equation 36 and rewrite the individual budget equation as follows:

$$\begin{aligned} \dot{a}_{\tau,t} &= (r+\rho z)a_{\tau,t} + (1-s_t)q_{\tau,t}w_t + \varphi A_t - c_{\tau,t} \\ \varphi &\equiv \rho(1-z) + g_N\frac{1-\varsigma}{2-\varsigma} \end{aligned}$$

Since A_t increases at the same rate as Y_t (equation 43), this budget constraint implies the following level for initial consumption:

$$c_{\tau,\tau} = (1-s)q_{\tau,\tau}w_\tau\frac{r+\rho z-g_c}{r+\rho z-g_w-hs^\alpha} + \frac{r+\rho z-g_c}{r+\rho z-g_Y}\varphi A_\tau$$

Individual wealth accumulation

Since initial wealth is nil, $a_{\tau,t}$ is given by capitalised savings from time τ to time t:

$$a_{\tau,t} = \int_{\tau}^{t} e^{(r+\rho z)(t-\mu)}\left[\varphi A_\mu + (1-s)q_{\tau,\mu}w_{\tau,\mu} - c_{\tau,\mu}\right] d\mu$$

Integrate and substitute initial consumption:

$$a_{\tau,t} = \varphi A_\tau \frac{e^{g_c(t-\tau)} - e^{g_Y(t-\tau)}}{r+\rho z - g_Y} + q_{\tau,\tau} w_\tau (1-s) \frac{e^{g_c(t-\tau)} - e^{(g_w + hs^\alpha)(t-\tau)}}{r+\rho z - g_w - hs^\alpha}$$

Aggregate wealth accumulation
Define R_t as follows:

$$R_t \equiv \int_{-\infty}^{t} \rho e^{-\rho(t-\tau)} \left\{ \frac{(1-s) q_{\tau,\tau} w_\tau}{r+\rho z - g_w - hs^\alpha} \left[e^{g_c(t-\tau)} - e^{(g_w + hs^\alpha)(t-\tau)} \right] \right\} d\tau$$

Sum individual wealth over the population:

$$\begin{aligned} A_t &= \int_{-\infty}^{t} \rho e^{-\rho(t-\tau)} a_{\tau,t} d\tau = \\ &= \int_{-\infty}^{t} \rho e^{-\rho(t-\tau)} \varphi A_\tau \frac{e^{g_c(t-\tau)} - e^{g_A(t-\tau)}}{r+\rho z - g_Y} d\tau + R_t = \\ &= \frac{\rho\varphi}{r+\rho z - g_Y} A_t \left[\frac{1}{\rho + g_Y - g_c} - \frac{1}{\rho} \right] + R_t \end{aligned}$$

From this derive the following expression for aggregate wealth:

$$A_t = \frac{R_t}{1 - \frac{\rho\varphi}{r+\rho z - g_Y} \left[\frac{1}{\rho + g_Y - g_c} - \frac{1}{\rho} \right]}$$

Compute R_t:

$$\begin{aligned} R_t &= \int_{-\infty}^{t} \rho e^{-\rho(t-\tau)} \left\{ \frac{(1-s) q_{\tau,\tau} w_\tau}{r+\rho z - g_w - hs^\alpha} \left[e^{g_c(t-\tau)} - e^{(g_w + hs^\alpha)(t-\tau)} \right] \right\} d\tau = \\ &= \rho \frac{1}{r+\rho z - g_w - hs^\alpha} \left\{ \frac{1}{\rho + g_w + g_q - g_c} - \frac{1}{\rho + g_q - hs^\alpha} \right\} (1-s) q_{t,t} w_t \end{aligned}$$

Finally notice that R_t contains the factor $q_{t,t}(1-s)\rho / (\rho + g_q - hs^\alpha)$ and that this factor coincides with the supply of skills L_t (equation 18). In addition, notice that $L_t w_t$ represents a fraction $1-\theta$ of total commodity output (equation 10). This allows to derive the following expression for aggregate wealth:

$$A_t = \frac{\frac{1}{r+\rho z - g_w - hs^\alpha} \left\{ \frac{\rho + g_q - hs^\alpha}{\rho + g_w + g_q - g_c} - 1 \right\}}{1 - \frac{\varphi}{r+\rho z - g_Y} \left[\frac{\rho}{\rho + g_Y - g_c} - 1 \right]} (1-\theta) Y_t$$

The expression for A_t in the main text (equation 39) correspond to the latter once one takes into account the fact that $g_q = hs^\alpha$ (equation 29) and $g_Y = \sigma g_N + hs^\alpha$ (equation 28).

D Derivation of equation 40

Take the definition of aggregate wealth:

$$A_t = \int_{-\infty}^{t} \rho e^{-\rho(t-\tau)} a_{\tau,t} d\tau$$

Differentiate and recall that $a_{t,t} = 0$:

$$\begin{aligned}
\dot{A}_t &= \rho a_{t,t} - \int_{-\infty}^{t} \rho^2 e^{-\rho(t-\tau)} a_{\tau,t} d\tau + \int_{-\infty}^{t} \rho e^{-\rho(t-\tau)} \dot{a}_{\tau,t} d\tau = \\
&= -\rho A_t + \int_{-\infty}^{t} \rho e^{-\rho(t-\tau)} \left[(r+\rho z) a_{\tau,t} + (1-s) q_{\tau,t} w_t + \varphi A_t - c_{\tau,t}\right] d\tau = \\
&= -\rho A_t + (r+\rho z) A_t + \varphi A_t + (1-s) L_t w_t - C_t = \\
&= r A_t + (1-s) L_t w_t + g_N \frac{1-\varsigma}{2-\varsigma} A_t - C_t
\end{aligned}$$

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