Neo-classical labour market dynamics and uniform expectations: chaos and the “resurrection” of the Phillips Curve.

by Luciano Fanti*

*Dipartimento di Scienze Economiche
Via Ridolfi 10,
I-56124 PISA, Italy
phone: ++39-050-2216369 (2216466)
fax: ++39-050-598040
e-mail: lfanti@ec.unipi.it

Abstract

In this paper we develop a simple, yet complete, model of the labour market in the neoclassical framework dating back to Friedman (1968) and Phelps (1968), among others. According to the existing literature wage expectations should be formed in a different way by firms and individuals in order temporary deviations from natural rate of employment to take place in the “expectations augmented” neoclassical labour market. On the contrary we are capable to show that not only temporary but long term regular fluctuations and chaotic behaviour of wages and employment emerge as a robust finding also when firms and individuals have uniform expectations. This suggests at least two noteworthy considerations: 1) the Walrasian equilibrium dynamics of the “expectations augmented” neoclassical labour market can cause long term unemployment; 2) a ‘reminiscence’ of the Phillips curve emerges in a neoclassical labour market context, by providing a new perspective to the long lasting controversial issue of the existence of the Phillips Curve.

J.E.L.: J0, E30, E24

Key words: neoclassical labour market, wage expectations, Phillips curve, business cycles, chaos

Introduction.

The relation between wage inflation and unemployment, extensively discussed since the early work of Phillips (1958) and Lipsey (1960), is a matter of renewed interest in the present days.[1] As argued by a vast literature, firms and individuals take their optimal decisions on the basis of an expected wage rather than the actual one.

The aim of this paper is 1) to develop a simple, yet complete, model of a neoclassical labour market with real wage expectations; 2) to study the dynamic properties of such a model. The analytical framework is similar to the one contained in the most part of the current text-books and its development, explicitly or implicitly, dates from Friedman (1968) and Phelps (1968), among others. Kierzkowski (1980), following a consolidated literature, stresses that different speeds of price expectations for workers and producers are a necessary condition for deviation from natural rate of employment to take place in his model. In fact "if price expectations are formed uniformly the economy should remain in equilibrium because the mistakes of various groups of economic agents will cancel each other." (p.198). Moreover a very remarkable fact is that "in this case, adaptive and rational expectations produce the same result: fluctuations of real variables are random". (p.198). Therefore, according this standard view, only if the expectations are different for different economic agents, and in particular only if worker's real wage expectations are more sluggish than those of producers, one will observe temporary increases in employment followed, after a while, from a reduction below its natural level, or in other words a possible cyclical adjustment towards the equilibrium. However, in any case, the possible cycle can be only temporary and the equilibrium condition will be always restored.

In contrast with this consolidated view, this paper shows that an appropriate dynamic analysis of the Friedman-Phelps model with wage expectations reveals both interesting and unexpected results. Indeed,

[1] See the Journal of Monetary Economics (1999, vol. 44, 2), special issue on: "The return of the Phillips curve".

irrespective of the numerous empirical studies as well as of its importance for policy-makers or the relative intense theoretical debate, the dynamic features of this model are often relegated to a corner: this paper aims to fill this gap.

A first conclusion of the present paper is straightforward and differs from the consolidated literature abovementioned: different price expectations for different economic agents are not necessary in order to be room for a temporary trade-off between employment and inflation[2].

But another part of the story is again more important: not only uniform price expectations are able to generate cycles, but in addition these cycles represent a long run equilibrium. This latter outcome restores the empirical fact of the Phillips curve in a somewhat new light: indeed we show that such a curve can appear as a long run equilibrium of a truly neoclassical economy. However the Phillips' curve emerging in our model cannot more interpreted as a set of different equilibrium points among which the policy maker could choose, as made by the traditional view of the curve, but only a dynamical outcome intrinsic to the neoclassical labour market.[3]

Finally, such cycles are chaotic so that the fluctuations can really look like the seemingly stochastically driven observed time-series of the unemployment rate: this means that we can observe a neoclassical endogenously determined deterministic real business cycle different from the stochastically driven one postulated by Real Business Cycle theory.

Notice that in the present model a robust chaotic behaviour may occur even if supply of labour is 'well-behaved', in contrast with the usual belief that only if supply of labour is 'strangely' negatively sloped and in addition steeper than demand for labour curve, instability of the neoclassical labour market occurs.[4]

[2] As noted by Kierzkowski (1980, p.198) "this trade-off is, however, very short-lived, since expectations are generally quickly revised across the economy".

[3] Indeed the points belonging to the Phillips curve emerged in the present model are just the realisation of a single trajectory of the underlying (fully deterministic) process.

[4] For instance, Kierzkowski (p.194), by considering the case of supply of labour with negative slope, states that "stability considerations require that demand curve be steeper than supply curve".

Other papers discovered chaotic behaviours in the Phillips curve (i.e. Soliman, 1996; Montoro et al., 1998) or chaotic dynamics in macro-models capable of mimicking a Phillips-type behaviour (Chichilnisky et al., 1995); however the former used an “empirically” formulation of the Phillips curve in discrete-time with somewhat ad-hoc non-linearities and the latter found complex dynamics combining a discontinuous production function with a special discrete-time formulation. Furthermore Fanti (2002a) shows that in a simple labour market represented by the traditional Friedman–Phelps “expectations-augmented Phillips curve” extended to consider both the compensation for “unexpected” inflation in the past contractual period and a price-rule according to an anticyclical mark-up, either regular or chaotic fluctuations of wages and unemployment appear as a robust result. Cycles and periodical long run unemployment also emerged in the case of forward-looking expectations and pro-cyclical mark-up (Fanti, 2002b).

We note that all above works showing chaotic outcomes have a framework at all different and quite special when compared with the present one.

The plan of the paper is as follows. In the second section we present the model. Some theoretical results and numerical simulations are reported in section three. Concluding remarks follow.

2. The model.

The analytical framework adopted is based on the model that, dating from Friedman (1968) and Phelps (1968), is presented in the most part of the current text-books.[5] We consider a one-good economy, with a single representative firm and a single representative worker-consumer. Let’s assume that capital is constant, so that it can be normalised to one, and so labour (*L*) is the only relevant input. We assume the standard neoclassical production function of the Cobb-Douglas type (where *D* is a parameter reflecting constant technological progress):

$$Y = DL^{a} \qquad 0 < a < 1, D > 0 \qquad (1)$$

Let Π and ω respectively define the total profit and the nominal wage rate. By defining p as the price of the output, the profit function is defined as

$$\Pi = pDL^a - \omega L \qquad (2)$$

Assuming perfect competition, profit-maximising producers will demand labour according to:

$$L^D = f_1(w) = \left(\frac{\omega}{paD}\right)^{\frac{1}{a-1}} \qquad (3)$$

where $w = \frac{\omega}{p}$ denotes the real wage (in what follows wage always means real wage).

The workers have the following optimal labor supply:

$$L^S = f_2(w) = S^\circ w^b \qquad (4)$$

where S° is a constant composite parameter including price and scale factors in the utility function and b represents the wage elasticity of the labour supply.

Let us now consider the dynamics of this economy. Let $e(w) = L^D - S^D$ be the excess demand for labour; following the dynamics given by the usual adjustment according to the laws of supply and demand, e.g. Chichilnisky et al. (1995), the wage is assumed to continuously adjust to the current excess demand for labour, $\dot{w} = \Phi[e(w)]$, and the function Φ is generally assumed to be linear:[6]

$$\dot{w} = c(L^D - L^S) \qquad (5)$$

where c is a positive constant.

Moreover the literature has argued that the expected value of wage rather than the current one matters as regards the decisions to demand for labour or to supply labour.

Following others, as Lucas-Rapping (1969), Kiezkowski (1980) has argued that the wage expectations should be different between firms and workers,

[5] This model is basically equal, among others, to the model developed by Kierzkowski, 1980.

[6] Equation (5) is the back bone of the Walrasian price adjustment theory. In this sense, following Chichilnisky et al. (1995), the model of the present paper can represent a Walrasian economy. It is worth to remind that such a neoclassical adjustment

remarking that only in that case (and, according to the traditional literature, obviously only temporarily) a fluctuation during the convergence towards the equilibrium could be observed.

In order to confute the belief that different speeds of price expectations for workers and producers are a necessary condition for deviation from natural rate of employment to take place in a neoclassical model, it is assumed that both firms and workers have the same wage expectations (w^e). These expectations, exactly as in Kierzkowski, are formed in a backward-looking way, following a vast academic literature as well as policy- oriented models of many central banks [7]. As known, the "perfect foresight – rational expectations" argument would consider as 'irrational' the backward looking behaviour, arguing for forward–looking expectations due to the assumption of a knowledge of the economic model by agents and then of the future values of the economic variables. The backward looking scheme is currently adopted in both Keynesian and "policy-oriented" models, while the forward-looking scheme is adopted in both the neoclassical-monetarist view and the so-called new-Keynesian Phillips curve implied by the rational expectations staggered-contracting models à la Taylor (1980). However we notice that also the scheme of adaptive expectations could be 'more' rational than that of rational expectations in the cases in which the model can generate a "chaotic" output (as will be shown for the present model in Section 3): in fact, since the chaotic dynamics result implies the rediscovery of stochastic behaviour of a purely deterministic system and in turn the consequent unpredictability of the outcome of the equilibrium dynamics resulting from a parametric shock, "the rational expectations argument loses much of its strength and non-optimising rules of behaviour – such as adaptive reaction mechanisms of the kind assumed in the backward-looking formation of wage expectations – might not be as irrational as they may seem at first sight" (Medio, 1992, p. 17). Furthermore a well-motivated defence of the

mechanism, according to Lipsey (1960), would be capable under opportune conditions to lead to the Phillips equation.

[7] As, for example, claimed by Hogan and Pichette (2000, p.1): "The short-run (or expectations-augmented) Phillips curve plays a key role in the conduct of monetary policy,

assumption of adaptive expectations in dynamic macroeconomics may be found in Flaschel-Franke-Semmler (1997).

The wage expectations process is assumed to obey

$$\dot{w}^e(t) = g(Z - w^e) \qquad (6a)$$

$$\dot{Z}(t) = g(w - Z) \qquad (6b)$$

which, as known, is equivalent to the following simple second-order[8] exponentially distributed lag function[9]

$$w^e(t) = \int_{-\infty}^{t} w(\tau) g^2 (t-\tau) e^{-g(t-\tau)} d\tau \qquad (7a)$$

$$Z(t) = \int_{-\infty}^{t} w(\tau) g e^{-g(t-\tau)} d\tau \qquad (7b)$$

This distribution is more general than the usual fixed lag assumption ($w^e = w_{t-1}$) and corresponds to the presence of numerous firms and individuals (implicit in the competitive market assumption) with different times of response, as argued by Invernizzi-Medio (1991). This distribution has a mean value of $2/g$, (and a variance of $2/g^2$), and g represents the speed with which the agents revise their wage expectations.

The final model takes the form:

$$\begin{aligned} &\dot{w} = c(L^D - L^S) = c\left(\left(\frac{w^e}{aD}\right)^{1/(a-1)} - S^\circ w^{e^b}\right) && c > 0 \\ &\dot{w}^e = g(Z - w^e) && (8) \\ &\dot{Z} = g(w - Z) && g > 0 \end{aligned}$$

3. Main theoretical and numerical results

The equilibria of the system are defined as the solutions of the equation (5), as the introduction of the expectations does not change the equilibrium emerging only from the (5).

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particularly in countries where the central bank has an explicit mandate to target the inflation rate".

[8] Although there few applications in economics of exponential lags greater than the first-order one, in his famous textbook Allen (1967) also considered the cases of second and third-order. Kierzkowski (1980), as the most part of the literature, specifies the expectation equation as a first-order exponentially distributed lag function.

[9] We recall that, via the so-called linear trick, the integro-differential humped memory is transformed into a couple of new variables both adjusting over time following adaptive mechanisms or, more precisely, we have a "recursion" of two adaptive mechanisms (see McDonald, 1978, and Fanti-Manfredi, 1998).

From

$$\dot{w} = c(L^D - L^S) = 0 \qquad (9)$$

we obtain

$$w^* = \left(\frac{H}{S^\circ}\right)^{\frac{1}{f}}$$

where $H = (aD)^{\frac{1}{1-a}}$; $\quad f = \dfrac{1+(1-a)b}{1-a}$

The solution $w^*=0$ is obviously not interesting.

The following proposition summarises our steady state analysis:

Proposition 1: the model (8) always admits a unique equilibrium point $E_1=(w^*, w^{e*}, Z^*)$, which is always economically meaningful (we omit the trivial proof).

The local stability analysis of the unique equilibrium of the system leads to the following main results (proofs in the Appendix):

Proposition 2: also if consumption and leisure are substitutes - and labour supply is a well-behaved increasing monotonic function of the wage - the equilibrium E_1 can be destabilised.

Hence in presence of backward-looking wage expectations in order to have instability is not required that both the supply of labour had a negative slope and the supply curve be steeper then the demand curve.

Proposition 3: 1) the equilibrium is always locally asymptotically stable (LAS) when $g>c\phi/2$; 2) on the contrary, when $g<c\phi/2$, instability occurs, where $\phi = \dfrac{1}{1-a} H\left(\dfrac{S_0}{H}\right)^{\frac{2-a}{1+(1-a)b}} + bS_0\left(\dfrac{S_0}{H}\right)^{\frac{(1-b)(1-a)}{1+(1-a)b}} > 0$.

The Hopf bifurcation locus is given by

$$g^2[2g-c\phi]=0 \qquad (10)$$

and it is easily proven that on the locus:

$$g_H=c\phi/2 \qquad (11)$$

a Hopf bifurcation occurs.

Inspection of the bifurcation curve g_H shows that the stability of the Walrasian equilibrium prevails for combinations of values of the speeds of adjustment of expected wages and of actual wages which lie above a critical line, given by the bifurcation curve.

Moreover the following remark holds:

Remark: the parameters affect the stability according to the following clear-cut roles[10]: 1) increases both in the speed of revision of the expectations g and in the technological index D work for stability; 2) increases both in the preferences parameter $S°$ and in the speed of wage adjustment c work for instability and chaos.

The previous findings on Hopf bifurcation, are summarised by the following (proofs in the Appendix):

Proposition 4: the unique equilibrium E_1 is locally asymptotically stable for $g>g_H=g_H(a,b,c,D,S°)>0$. When, starting from a parameter set in which E_1 is LAS, the parameter g decreases, the equilibrium point shows a Hopf bifurcation at the value $g_H=g_H(a,b,c,D,S°)>0$, with the appearance of local limit cycles (at least one) surrounding the E_1 equilibrium.

Viewed in terms of the speeds of adjustment g,c, the bifurcation locus g_H (given as the only admissible solution for g of the eq. (10)) it is a linear increasing function of c. The straight line $g=c\phi/2$ splits the plane into two regions. In the region above the line ($g>c\phi/2$) the system is always LAS. Conversely in the region below the line the system is unstable. The qualitative dynamics below the bifurcation locus (eq. 11) is shown in fig.1. The numerical simulations has revealed that a “crater” bifurcation occurs in the proximity of the bifurcation locus, generating a “bi-stable” system (see the next section for details): there is a coexistence both of a stable fixed point and of a stable limit cycle; subsequently for further reductions of g only one stable limit cycle exists, and this latter always exists until the ‘crisis’ line is reached: subsequent reductions of g generate chaotic behaviours. Finally further parametric reductions lead beyond an ‘exploding’ line where the trajectories of the system explode.

FIG. 1- A diagram showing the stability boundaries in (c, g) parameter space for the system (8) where the stability regions for the different

[10] The simple proof is in Appendix. As regards both the elasticity of labour supply and the returns of scale of the labour factor, for sake of brevity we have not investigated their role on the stability.

dynamic solutions are also indicated. Legend: A= stability region; B= bi – stability region; D= only one stable cycle region; C= chaotic region.

3.1 - Simulative evidence.

The Hopf bifurcation theorem used in the previous section is a local result which only permits to detect the existence of a (local) bifurcation of an equilibrium point in periodic orbits. Conversely, it nothing says about 1) uniqueness and 2) stability of the limit cycles emerging from the bifurcation. In particular, it nothing says on the question whether the bifurcation is supercritical or subcritical, i.e. whether the limit cycles which bifurcate from the stationary state are (at least) locally stable or not. Moreover predictions of the theorem are local also for what concerns the parameter space: "the Hopf bifurcation theorem is local in character and only makes predictions for regions of parameter and phase space of unspecified size" (Medio, 1992, ch.2). These last facts make interesting to resort to numerical simulations to investigate matters such as i) the stability properties of the involved periodic orbits, ii) their uniqueness, iii) the size and the shape of its (or their) basin of attractions, iv) the size and the shape of the parameter region in which limit cycles exist, v) the global rather than local behaviour of the system.

We will show that in addition to the local persistent periodic behaviour analytically discussed in the previous section, other, more complex, dynamic behaviours of the system are possible when the global behaviour of the model is numerically investigated.

In particular, two interesting dynamic behaviours are evidenced: i) the bi-stability case; ii) the chaotic case. The chosen bifurcation parameter is, in line with the main issue of the paper aiming at discussing the dynamic role of uniform expectations, the speed of adjustment of wage expectations, *g*. The simulations show how the structure of the attractors evolve as the bifurcation parameter is varied while all the other parameters are kept fixed. As an illustrative example, the following parameter constellation has been considered: $D=1$, $S°=0.5$, $a=0.51$, $b=0.42$, $c=4$. The corresponding equilibrium values are $w^*=w^{e*}=Z^*=0.76$.

The following table sums up the entire set of dynamic behaviours in terms of the parametric windows tuning the bifurcational scenarios occurring when g is smoothly decreased:

Table 1.

g>2.903	2.903>g >2.89	2.89> g>2.2	2.2>g>1.8	1.8>g
(Global) Stability	Bi-stability	Only one stable cycle	Chaotic cycles	Explosion of the trajectories

3.1.1 - The "bi-stability" case.

As known, while stable closed orbits in the supercritical case can be seen as stylized business cycles, the subcritical case, as pointed out by Benhabib and Miyao (1981), corresponds to the concept of corridor stability as developed by Leijonhufvud (1973) and Howitt (1978). The definition of corridor stability is straightforward: a fixed point is corridor stable if the system converges to its dynamic equilibrium for small perturbations, but shows no such tendency for larger shocks. The subcritical case means that the unstable orbits enclose a region of stability in which all orbits inside that region converge to the fixed point. It is known that the problem of analytically determining the *stability* of limit cycles and that of determining their *number* can be performed by using the nonlinear parts of an equation system, but this last issue, which is of critical relevance from the point of view of a substantive economic analysis, it is not tackled here analytically because it needs a huge amount of cumbersome algebra (see for instance Guckenheimer-Holmes, 1983), whose interpretation is generally economically meaningless.

Resorting to the numerical simulations, we have ascertained the existence of the so-called "crater" bifurcation scenario, which, though rarely discussed in the economic literature, is important for its interesting economic interpretation. Indeed it enhances the description of Leijonhufvud's idea of corridor stability, allowing for large shocks do not lead to a totally unstable dynamics (as in the case of a subcritical bifurcation) but rather to a persistent cycle[11]. The economic importance of such a situation relies on the co-existence of two equilibria, the one a stable point, the other a stable oscillation, which can be also termed (in the words of Grasman, 1994) "bi-stability".

The "crater" bifurcation has been rarely discussed in an economic context, despite of its importance for economics, with the exception of Kind (1999) (who attributes such a definition to Lauwerier, 1986). Kind (1999) explains the emergence of the "bi-stability" phenomenon by means of this type of 'local' bifurcation. The existence of "bi-stability", at our knowledge, has been shown in economic models only in Semmler-Sieveking (1993), Grasman (1994) and Manfredi-Fanti (1999), but the interpretation of such phenomenon has been different: in fact while Kind has attributed the co-existence of two equilibria to the "local" "crater" bifurcation, Semmler-Sieveking have shown such a co-existence by means of a 'global' stability analysis while the other authors have attributed the phenomenon to a "relaxation" oscillation mechanism.

The main simulative results are as follows: E_1 is stable[12] for large g ($g>g_{\mathrm{Hsub}}=2.903$). Then it undergoes a subcritical Hopf bifurcation when g is equal to $g_{\mathrm{Hsub}}=2.903$, a value close to but beyond the critical threshold defined by (11) (given by g_H=2.89): this "catastrophic" bifurcation is a "crater" bifurcation. In this case for different starting points, the system shows different dynamics (see Fig. 2): i) in a region contained by an unstable limit cycle, the orbits converge to the fixed point; ii) outside this region the orbits form a closed cycle after some transients.

For $g<g_H=2.89$ trajectories starting sufficiently close to E_1 initially diverge, and subsequently converge to a stable limit cycle, which is unique: the Hopf bifurcation is supercritical.

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[11] Moreover phenomena like hysteresis loops and catastrophic transitions may all be described by this bifurcation scenario, as pointed out by Kind (1999).

[12] The conjecture emerging from our simulations is that E_1 is globally asymptotically stable.

3.1.2 - Chaotic evidence.

As is known, the existence of chaotic attractors can be proved analytically only in a few rather special cases, so that the presence of chaos is ascertained only numerically[13]. When g is further decreased ($g<g_C=2.2$) complex behaviours[14] arise[15]: this behaviour is very robust to further parametric changes and in fact is present until $g<g_{EX}=1.8$ when the trajectories explode (on yearly base this means that the chaos is present for an average lag in the period of formation of the wage expected approximately from ten to thirteen months).

The visual inspection reveals that the trajectories of the system wander erratically in a bounded region both of the phase plans w, w^e (fig. 3) and of the phase space w, w^e , Z (fig. 4).

Finally the figure 5, based on a plausible parameter set generating a stable long run cycle, illustrates that also in the long period dynamic regime, periods of full employment alternates with periods of unemployment, so that, on average, long term unemployment appears (in fact in the long run regime the employment oscillates from periods of full employment to periods in which the rate of unemployment shows a maximum value of about18%).

Finally the appearance of a long run Phillips curve can be shown by fig. 6, where, simulating the model with the chaotic parameter set

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[13] At present "complete (though approximate) information of the structure of orbits of continuous dynamic systems of dimension greater than two…can only be obtained by numerically integrating the equations of the systems…" (Medio, 1992, pp. 82).

[14] We recall the usual caveats with respect to the effects of computing approximation in continuous-time formulation, above all when detecting chaos, for which we refer to Lorenz (1993), Appendix 4, 276-281.). Our simulations have been performed with the package DMC, fourth-order Runge-Kutta method, fixed step 0.01 and 0.005.

[15] The use of techniques for the global analysis of the system (8) in order to obtain an analytical or geometrical detection of "chaos" is beyond the scope of this paper (see Wiggins (1990)). The emergence of a chaotic attractor may be detected through several measures: 1) by "eye"; 2) through bifurcation diagrams; 3) through numerical and statistical tests. Among these, we remark the computation of 1) a Poincarè map by numerical-graphical techniques which in the case of a simple bi-dimensional surface of section, permits to identify different types of dynamic behaviour, as limit cycle, subharmonic oscillations, quasiperiodic oscillations and the presence of a strange attractor; 2) the dominant first-order Lyapunov exponent (for a reconstructed attractor) which whether is positive gives a sign of existence of SDIC (Wolf at al, 1985); 3) the correlation dimension of the (reconstructed) attractor which whether is a non integer number indicates a fractal structure of the attractor (Grassberger – Procaccia, 1983). Such computations (for sake of brevity not reported here) have confirmed the presence of deterministic chaos in the system (8).

abovementioned, an evident Phillips relationship appear by plotting the time variation of wages (dw/dt) and the excess labour supply (L-S).

FIG. 2 - Trajectories in the phase plane (w,w^e) displayed for three different initial conditions of w.

FIG. 3 – Chaotic behaviour in the phase plane w, w^e.

FIG. 4 – Three-dimensional view of the chaotic behaviour in the phase space w, w^e , Z.

FIG. 5 – Time path of the rate of unemployment in a case of stable fluctuation.

FIG. 6 - A plot of the "Phillips" curve (restricted to the set (L-S)>0)

4. Conclusions

This paper has shown that regular fluctuations and chaotic behaviour of wages and employment may be a robust outcome of a 'well-behaved' neoclassical labour market, in which the importance of wage expectations in determining the level of employment and the equilibrium wage rate are recognised in line with the typical Friedman-Phelps version of the Phillips curve. As known the existing literature (e.g. Kierzkowski, 1980) has argued that wage expectations should be formed in a different way by firms and individuals in order temporary deviations from natural rate of employment to take place in the "expectations augmented" neoclassical labour market. In contrast with the existing literature, this paper has shown that not only temporary but long term regular fluctuations and chaotic behaviour of wages and employment emerge as a robust finding also when firms and individuals have uniform expectations. In particular when the unique equilibrium of this economy is destabilised, then the economic variables (wage and demand and supply of labour) evolve toward a stable attracting region within which their motion is chaotic. In this region a behaviour 'reminiscent' of the Phillips curve can be detected: this relationship is persistent in the long-run and it is not only a transitory disequilibrium phenomenon for a wage adjustment process converging to the equilibrium as postulated by the neoclassical interpretation of the Phillips curve[16].

[16] In the "trapping" chaotic region: a) the Phillips curve necessarily re-emerges as a long term phenomenon; b) the Walrasian equilibrium, viewed as the "center of mass" around

Finally we note that our dynamic results can be exploited for policy purposes[17]. To sum up, our results differ substantially from the common wisdom of the economic theory mainly in the following points: 1) also in the case in which price expectations are formed uniformly the economy will show sensible fluctuations in employment; 2) more interestingly, the cycle can be permanent and the equilibrium condition will not be restored; 3) in the long run a Phillips-type behaviour occurs, but the points belonging to the Phillips' curve emerging in our model are just the realisation of a single trajectory of the underlying (fully deterministic) process, rather than a set of different equilibrium points among which the policy maker could choose as in the traditional view of the curve; 4) the permanent fluctuations are also of Chaotic type, so mimicking very well the seemingly random behaviour of the rate of unemployment and offering a new deterministic and endogenous explanation of the employment fluctuations: in other words, in a fully neoclassical economy, fluctuations of real variables are not necessarily stochastically-driven as so far argued by the prevailing view of business cycles.

References

Allen R.G.D. (1967), *Macro-economic theory*, Macmillan, London and Basingstoke.

W.J. Baumol (2000), Out of equilibrium, *Structural Change and Economic Dynamics*, 11, pp. 227-233.

Benhabib, J., Miyao, T. (1981), Some new results on the dynamics of the generalized Tobin model, *International Economic Review,* 22 (3), 589–596.

which the system evolves completely loose its "predictive ability". Hence another main finding of his paper is that the notion of Phillips curve appears to be a dynamic concept essentially related to some peculiar dynamic regime, namely the chaotic one, of a neoclassical labour market model with Walrasian adjustment rule.

[17] In fact as Baumol (2000) claimed if the analysis demonstrates " that a wage reduction, while it may sometimes stimulate employment, can in other circumstances exacerbate it and lead to dangerous oscillations, the result is surely substantive. It is a clear and unambiguous warning to be disregarded by policy designers at the economy's peril."(p.231).

Chichilnisky G., Heal G. and Lin Y. (1995), "Chaotic price dynamics, increasing returns and the Phillips curve", *Journal of Economic Behaviour and Organization*, 27, 279 – 291.

Fanti L. (2002a), The "expectations-augmented Phillips curve" with "price catch-up" and chaotic cycles, *Rivista Internazionale di Scienze Economiche e Commerciali – International Review of Business and Economics*, vol. IL, no. 3, September, 305-332.

Fanti L. (2002b), Chaos in the forward-looking "expectations-augmented Phillips curve" with "price catch-up", *Studi Economici,* no. 77, 2, 5-34.

Fanti L. – Manfredi P. (1998), A Goodwin-type Growth Cycle Model with Profit-Sharing, *Economic Notes*, vol. 27, no.3, pp. 371-402.

Flaschel P., Franke R., Semmler W. (1997), *Dynamic Macroeconomics,* MIT Press, Cambridge (Mass.).

Friedman M. (1968), The Role of Monetary Policy, *American Economic Review*, 58, pp. 1-17.

Grasman J. (1994), Co-existence of a limit cycle and an equilibrium in Kaldor's business cycle model, *Journal of Economic Behaviour and Organization*, 24, pp.369-377.

Guckenheimer J. - Holmes P. (1983), *Nonlinear Oscillations, Dynamic Systems and Bifurcation of Vector Fields,* Springer-Verlag, New York, Tokyio, Berlin.

Howitt P. (1978), The limits to stability of a full-employment equilibrium, *Swedish Journal of Economics* 80, 265–282.

Kind C. (1999), Remarks on the economic interpretation of Hopf bifurcations, *Economics Letters*, 62, pp. 147-154.

Grassberger P. and Procaccia I. (1983), "Measuring the Strangeness of Strange Attractors", *Physica 9 D*.

Hogan S. and Pichette L. (2000), *Some Explorations, Using Canadian Data, of the S-Variable in Akerlof, Dickens, and Perry (1996),* Working Paper 2000-6, Bank of Canada.

Kierzkowski H. (1980), Short-run Inflation- employment Trade-offs and the Natural level of employment, *Economica*, 47, 193-199.

Invernizzi S. - A. Medio (1991), On Lags and Chaos in Economic Dynamic Models, *Journal of Mathematical Economics*, 20, 521-550.

Lipsey R.G. (1960), The Relation Between Unemployment and the Rate o Change of Money Wage Rates in the U.K. 1861-1957: a Further Analysis, *Economica*, 27, 1960, 1-31.

Leijonhufvud A. (1973), On Keynesian Economics and the Economics of Keynes, *Swedish Journal of Economics, 75,* pp. 27-48.

Liu W.M. (1994), Criterion of Hopf bifurcation without using eigenvalues, *Journal of Mathematical Analysis and Applications,* 182, 250-256.

Lorenz H.W. (1993), *Nonlinear Economic Dynamics and Chaotic Motion*, Springer Verlag, New York, Tokio, Berlin.

Lauwerier H. (1986), Two-Dimensional iterative maps. In: Holden, A.V. (Ed.), *Chaos*, Manchester, pp. 58–95.

Lucas R. and Rapping L. (1969), Real wages, employment and inflation, *Journal of Political Economy*, 77, pp. 721-754.

Manfredi P. – Fanti L. (1999), Gestation Lags and Efficiency Wage Mechanisms in a Goodwin-type Growth Cycle Model, *Studi Economici*, n. 68, pp. 95-126.

McDonald N. (1978), *Time Lags in Biological Systems*, Lecture Notes Biomath. 29, Springer Verlag, Berlin.

Medio A. (1992), *Chaotic Dynamics*, Cambridge University Press, Cambridge, 1992.

Montoro J.D. – Paz J.V. – Roig M. (1998), Active Monetary policy and Instability in a Phillips Curve System, *Journal of Macroeconomics*, vol. 20, n° 4, Fall, pp.843-856.

Phelps E.S. (1968), Money-Wage Dynamics and Labor-market equilibrium, *Journal of Political Economy*, 76, pp. 678-711.

Phillips A.W. (1958), The Relation Between Unemployment and the Rate of Change of Money Wage Rates in the U.K. 1861-1957, *Economica*, 25, 283-299.

Semmler W. - Sieveking M. (1993), Nonlinear liquidity-growth dynamics with corridor-stability, *Journal of Economic Behavior and Organization*, 22, 189–208.

Soliman A.S. (1996), Transition from Stable Equilibrium Points to Periodic Cycles to Chaos in a Phillips Curve System, *Journal of Macroeconomics*, Vol. 18, n° 1, Winter, pp. 139-153.
Taylor J.B. (1980), "Aggregate Dynamics and Staggered Contracts", *Journal of Political Economy*, 88, 1-23.
Wiggins S. (1990), *Introduction to Applied Nonlinear Dynamic Systems and Chaos*, New York-Berlin-Tokyo: Springer.
Wolf A., Swift J.B., Swinney H.L. and Vastano J. (1985), Determining Lyapunov exponents from a time series, *Physica*, 16D, 285-317.

APPENDIX: Proofs of the main results.

To investigate the stability of the Walrasian equilibrium E1=(w^*, w^{e*}, Z^*), we write the following jacobian, evaluated at E_1

$$J(w,w^e,Z)\Big|_{w=w^e=w^*}=\begin{bmatrix}0 & -c\phi & 0\\ 0 & -g & g\\ g & 0 & -g\end{bmatrix} \qquad \text{(A.1)}$$

The jacobian leads to a third order characteristic equation with coefficients a_i (i=0,1,2,3) defined as: $a_0=1;\quad a_1=2g;\quad a_2=g^2;\quad a_3=g^2c\phi$ (A.2)

We note that all the coefficients are always positive. Therefore the Routh-Hurwitz test gives the following stability condition for E_1:

$$a_1a_2-a_0a_3=2g^3-g^2c\phi>0 \qquad \text{(A.3)}$$

As $g>c\phi/2$ the last inequality is always true, showing that the E_1 equilibrium is always locally asymptotically stable. On the contrary, when $g<c\phi/2$, instability arises. This proves Proposition 2 and 3.

We claim that a Hopf bifurcation arises when the equality:

$$2g^3-g^2c\phi=0 \qquad \text{(A.4)}$$

holds. By solving (A.4) with respect to g we get[18]:

$$g_H=\frac{c\phi}{2}>0 \qquad \text{(A.5)}$$

[18] Obviously we disregards the zero (and the imaginary) solutions.

It is easy to check that only the solution g_H being positive is adequate to represent the desired bifurcation process. This shows that a bifurcation value always exists. It is of interest the shape of the bifurcation curve (A.5): the relation between *g,c,* i.e. the speeds of adjustment of the expected and of the current wage, is evident. Given the algebraic complexity of the function ϕ, it seems difficult to establish unambiguously the effects on cycles and stability of shocks on various parameters unless to resort to the calibration and numerical simulation. Fortunately we are able to analytically determine the relation between g_H and both the technological index D and the preferences parameter $S°$: 1) positive shocks on the level of technological efficiency are always stabilising; 2) positive shocks on the preferences parameter are always destabilising. The proof is simple: in fact g_H is a strictly increasing function of ϕ and then in order to determine the effect of the parameters on the stability locus it is sufficient to investigate the function ϕ, for instance with respect to the two parameters of interest D and $S°$. By rewriting the function ϕ as

$$\phi = (aD)^{\frac{-(1-b)}{1+(1-a)b}} S^{\circ\frac{2-a}{1+(1-a)b}} \frac{1+b(1-a)}{1-a}$$

it is straightforward to ascertain the role of D and $S°$ in the following way:

$$sign \quad \frac{\partial g_H}{\partial D} = sign \quad \frac{\partial \phi}{\partial D} < 0 \quad , \quad sign \quad \frac{\partial g_H}{\partial S°} = sign \quad \frac{\partial \phi}{\partial S°} > 0$$

To complete the proof of the appearance of a Hopf bifurcation of the E_1 equilibrium, let us now show that the pair of bifurcating eigenvalues cross the imaginary axis with nonzero speed. This is equivalent to show that (Liu, 1994):

$$\left(\frac{d}{dg}(a_1 a_2 - a_3)\right)_{g=g_H} \neq 0$$

$$\left(\frac{d}{dg}(a_1 a_2 - a_3)\right)_{g=g_H} \neq 0$$

We quickly have:

$$\left(\frac{d}{dg}(a_1 a_2 - a_3)\right)_{g=g_H} = \left(6g^2 - 2c\phi g\right)_{g=g_H} = 0.5c^2\phi^2 > 0$$

$$\left(\frac{d}{dg}\left(a_1a_2 - a_3\right)\right)_{g=g_H} = \left(6g^2 - 2c\phi g\right)_{g=g_H} = 0.5c^2\phi^2 > 0$$

which is always positive, thereby completing the proof of Proposition 4.

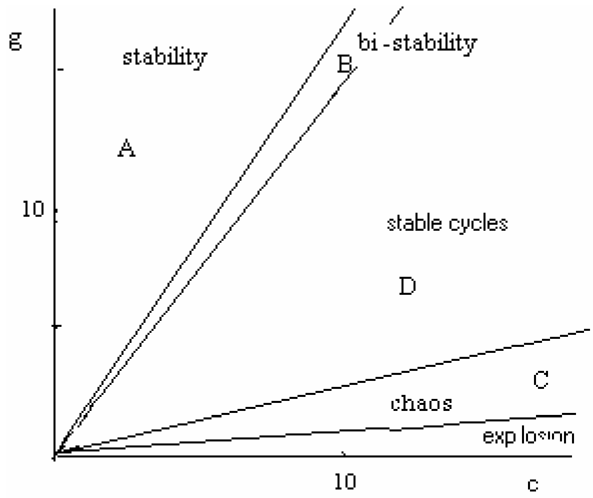
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FIG. 1

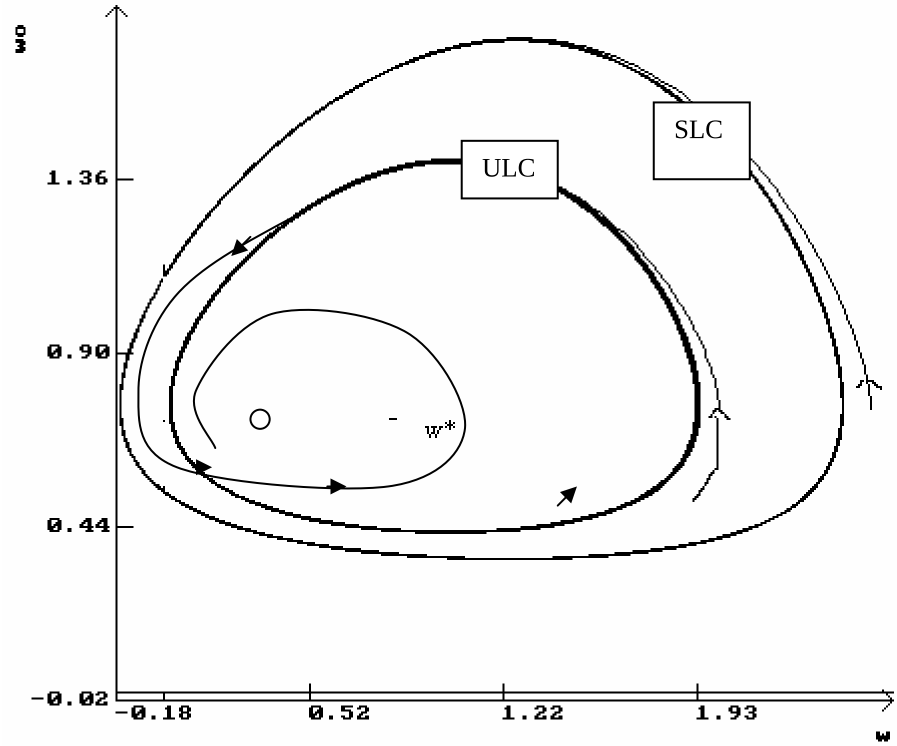

FIG 2. Trajectories in the phase plane (w,w°) when g =2.9 (parameter set: a=0.488, b=0.4239, S°=0.5, D=1, c=4) displayed for three different initial conditions of w (given $w^e(0)$=Z(0)=w*=0.76): w(0)=2.55, w(0)=2, w(0)=1.8. Legend: wo= w^e; w*= wage equilibrium value = 0.76; ULC= unstable limit cycle; SLC = stable limit cycle .

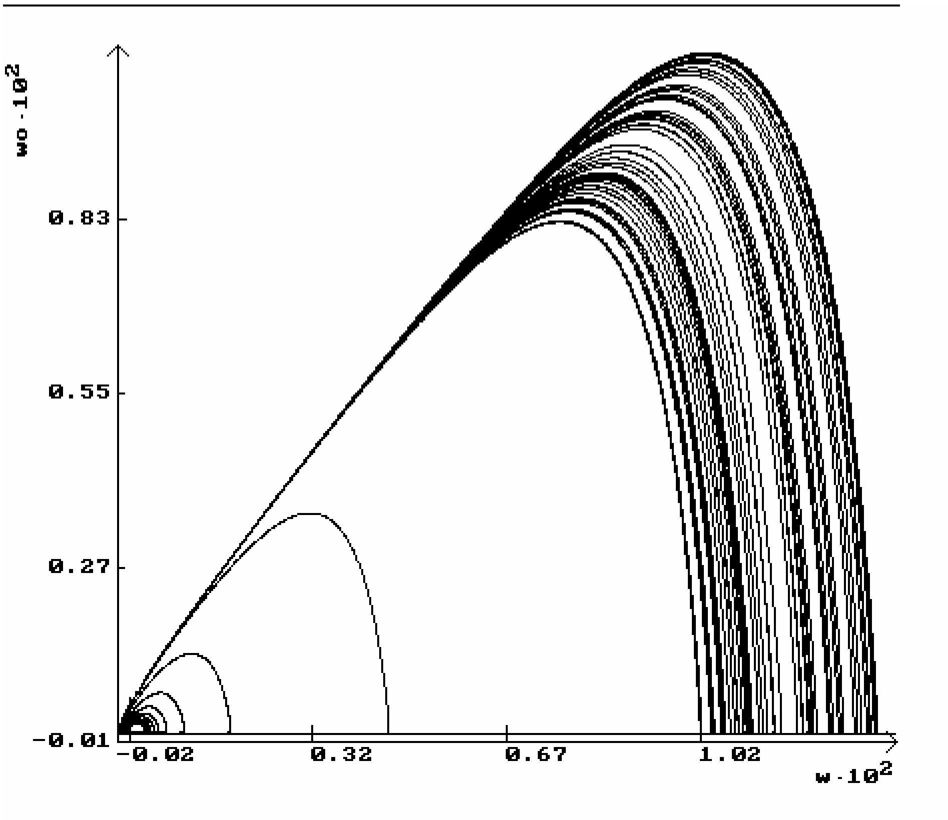

FIG.3. Chaotic trajectories in the phase plane (w.w°), when g=2.12 (other parameters as in fig.2) (I.C.: w(0)=2.5, w°(0)=w°°(0)=0.76)

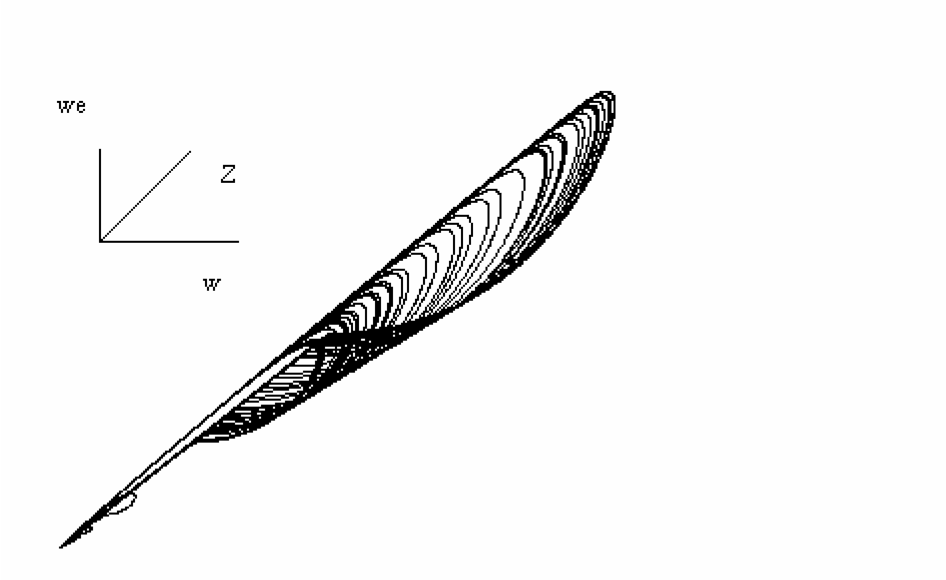

FIG. 4- Three-dimensional view of chaotic trajectories in the space (w,w^e,Z) when g=2.12 (other parameters as in fig.2) (I.C.: w(0)=2.5, $w^e(0)=Z(0)=0.76$)

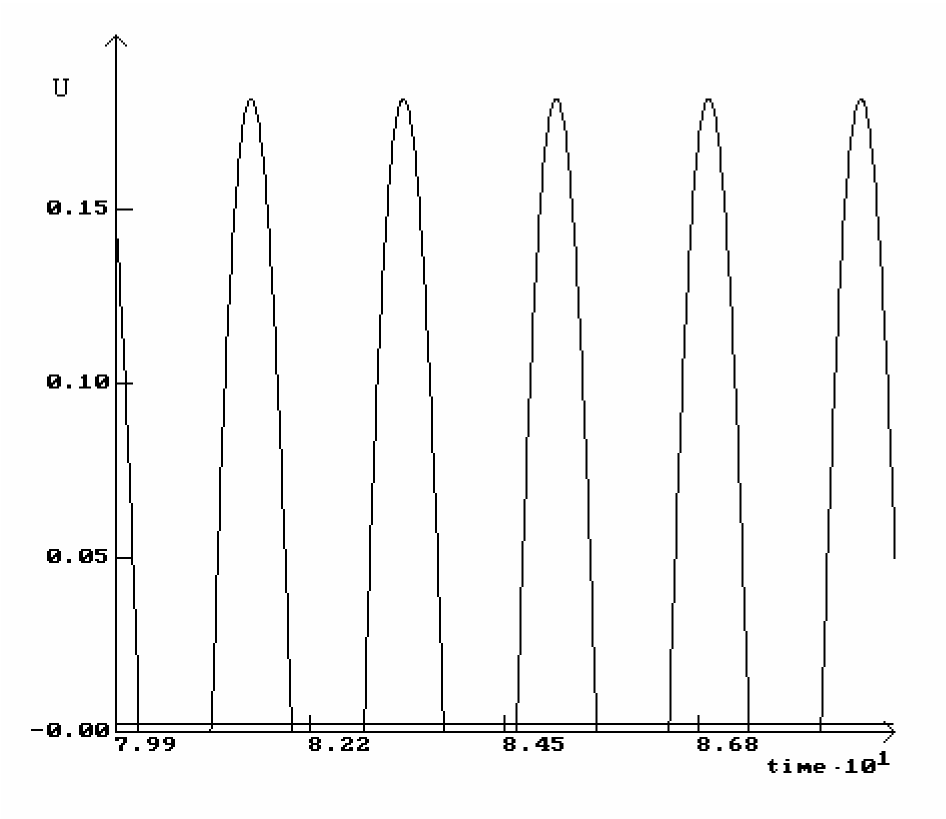

FIG. 5. Time path of the rate of unemployment U (U=1-L/S) (parameter set: a=0.5, b=0.2, S°=0.35, D=1, c=8.05, g=3.5; I.C.: $w^e(0)=Z(0)=w^*=0.85$), w(0)=1.035).

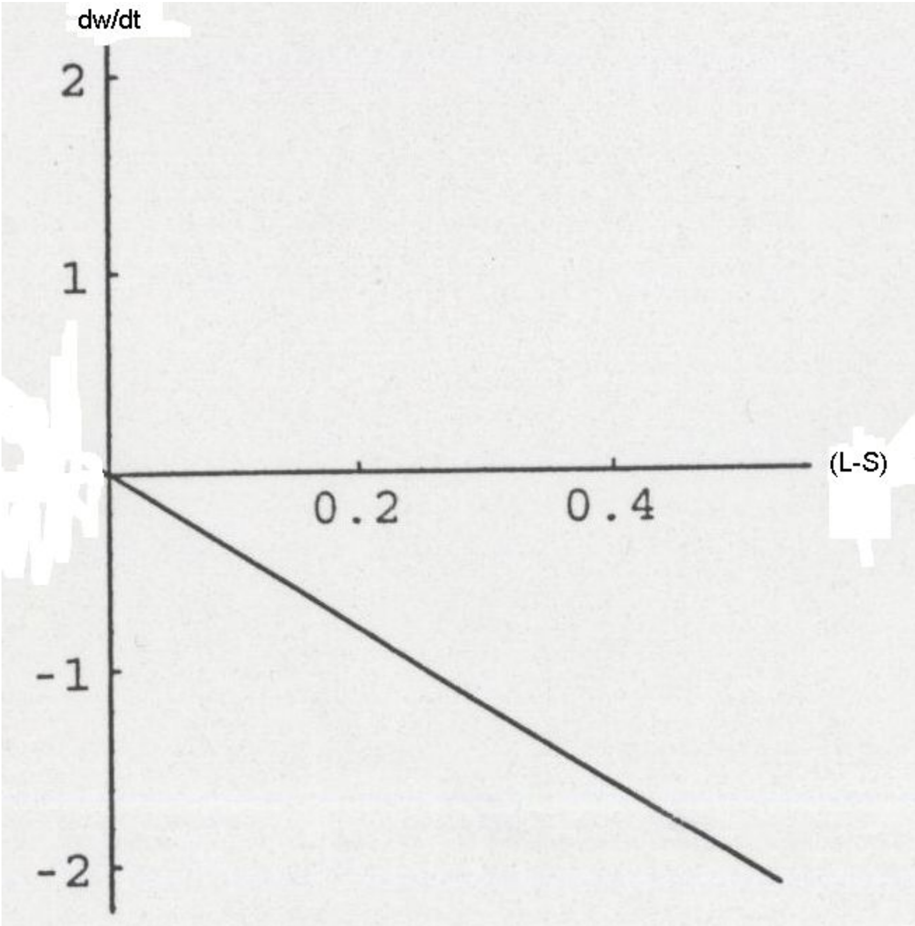

FIG. 6 - A plot of the “Pseudo-Phillips” curve (restricted to the set (L-S)>0) (parameter set as in fig. 3)