Migration, altruism and capital income taxation*

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Abstract

We tackle the issue of optimal dynamic taxation of capital income in an economy with migration and intra-family altruism. We show that the resulting disconnection among dynasties, properly accounted for by the government, makes room for hitting capital, both in the short and in the long run.

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1 Introduction

Starting from the seminal works by Judd [11] and Chamley [5], the issue of dynamic optimal capital income taxation has been analyzed by a number of researchers. In particular, Judd [12] has shown that the zero tax rate result stems from the fact that a tax on capital income is equivalent to a tax on future consumption: thus, capital income should not be taxed if the elasticity of consumption is constant over time. However, while in infinitely lived representative agent (ILRA) models[1] this condition is necessarily satisfied in

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[1] See Atkeson et al. [1] and Chari et al. [6].

the long run, along the transition path, instead, it holds only if the utility function is assumed to be (weakly) separable in consumption and leisure and homothetic in consumption[2].

Abandoning the standard ILRA framework in favour of Overlapping Generations models with life cycle (OLG-LC)[3] has delivered the important result of the violation of the zero tax rule also in the long run. This outcome can be understood by reckoning that in such a setup optimal consumption and labor, or, more precisely, the elasticity of consumption, are generally not constant over life and even at the steady state, due to life-cycle behavior.

In this work we extend the previous analysis by focusing on the role played by altruism and migration in determining the taxation of capital income: more precisely, we consider an economy with migration combined with overlapping "infinitely lived" dynasties, that is, formed by individuals who are altruistic only toward their own descendants[4]. The fiscal effects of immigration have already been studied under several perspectives[5] and its relevance as for the optimal policy of capital income taxation appears an interesting avenue for integrating this analysis.

The entrance of new dynasties in each period prevents the economy from behaving as a single, infinitely lived, representative individual and creates an overlapping dynasties mechanism. As in OLG models, the policymaker is thus assumed to maximize a social welfare function that is a weighted sum of the dynasties' utility functions, when choosing the optimal capital income taxation path.

The main result we find is that taxation can be non zero if the social weight attached to each dynasty by the government takes into account its demographic evolution, i.e. its relative size within the whole population. This element is obviously absent in ILRA models, but is also usually excluded in the OLG-LC framework, since up to now the social intergenerational weight has been taken as constant; such assumption can well be reasonable with finite lifetime horizon and invariant population (see, for instance, Erosa and Gervais [9]), since the share of each dynasty is typically constant through time. But this is not the case in the present model, where, due to a positive migration rate, the demographic weight of each dynasty decreases over

[2]De Bonis and Spataro [7] show that, if the individual and government intertemporal discount rates differ, another source of capital income taxation comes into play, due to "Pigouvian" arguments.

[3]See Atkinson and Sandmo [2] and Erosa and Gervais [9]; for a review see Renström [15] and Erosa and Gervais [8].

[4]For a presentation of this model in continuous time see Barro and Sala-i-Martin [4], chapter 9.

[5]See, for instance, Storesletten [16] for the impact on the welfare state balance.

time. Thus, while in the existing OLG-LC models the violation of the zero tax result depends crucially on the life-cycle behavior of consumption, in the present work the mere existence of the OLG mechanism (i.e. limited, intra-dynastic altruism) is sufficient for delivering it. In fact, we show that when agents are perpetually young, in the sense that the intertemporal allocation of consumption and leisure at each date does not depend on age, contrary to Erosa and Gervais [9] and Garriga [10], the non zero result applies, even in the case of the life-cycle motive being ruled out.

The intuition behind this result lies in the disconnection between residents and immigrants: in the absence of altruism towards migrants, in each instant new individuals get into the economy, whose welfare is not cared for by the existing generations; these do not consider that the new dynasties at each date provide extra resources for an intergenerational and intertemporal redistribution of the burden of taxation. On the other hand, by reckoning this possibility, the government reduces the oversaving of individuals by hitting future consumption proportionally to the immigration rate of the economy. The relevance of the "disconnectedness" of the economy has been firstly analyzed by Weil [17] in the context of the validity of the Ricardian equivalence proposition in a scenario with a continuous arrival in the economy of new individuals who are not linked to pre-existing cohorts via intergenerational transfers. He shows that finite horizons are not a necessary condition for the violation of the Ricardian equivalence; analogously, in this work we show that finite horizons, or, more precisely, life cycle behavior, is not a necessary condition for the violation of the zero capital income tax result in OLG models.

The work proceeds as follows: in section 2 we present the model and derive the equilibrium conditions for the decentralized economy. Next, we characterize the Ramsey problem by adopting the primal approach. Finally, we present the results by focusing on the new ones. Concluding remarks and a technical appendix will end the work.

2 The model

We consider a neoclassical-production-closed economy in which there is a large number of agents and firms.

Private agents, who are identical in their preferences, differ as for their date of entry into the economy, s; natives are supposed to have entered the economy at some time $s < 0$, while migrants enter at a given rate α; both types of individuals have a constant rate of growth n: as a consequence, the population growth rate is equal to $(1+\gamma) \equiv (1+\alpha)(1+n)$.

In fact, the whole population at time t has cardinality:

$$N_t = N_0 \left(1+\gamma\right)^t,$$

and the size of each dynasty (started by the entry of the founder) is:

$$F_{s,t} = \alpha N_s (1+n)^{(t-s)},$$

with N_0 the size of population at time 0 and $s \leq t$.

Moreover, all individuals offer labor and capital services to firms by taking the net-of-tax factor prices, $\widetilde{w}_{s,t}$ and $\widetilde{r}_{s,t}$ as given. Firms, which are identical to each other, own a constant return to scale technology F satisfying the Inada conditions and which transforms factors into production-consumption units. Finally, the government can finance an exogenous stream of public expenditure G_t by issuing internal debt B_t and by raising proportional taxes both on interests and wages, referred to as $\tau^k_{s,t}$ and $\tau^l_{s,t}$ respectively. Notice that taxes can be conditioned on the date of birth of dynasties.

2.1 Private agents

Agents' preferences can be represented by the following instantaneous utility function:

$$U\left(c_{s,t}, l_{s,t}\right),$$

where $c_{s,t}$ and $l_{s,t}$ are instantaneous consumption and labor supply, respectively, of the dynasty founded in period s, as of instant t. Such a utility function is strictly increasing in consumption and decreasing in labor, strictly concave, and satisfies the standard Inada conditions. Since we assume that individuals care about the well being of their children, agents maximize the following utility function:

$$\max_{\{c(t),l(t)\}_s^\infty} \sum_{t=s}^{\infty} \left(\frac{1+n}{1+\beta}\right)^{t-s} U\left(c_{s,t}, l_{s,t}\right) \tag{1}$$

$$sub\ a_{s,t} = \frac{\left(1+\widetilde{r}_{s,t}\right)}{(1+n)} a_{s,t-1} + \widetilde{w}_{s,t} l_{s,t} - c_{s,t} \tag{2}$$

$$\lim_{t\to\infty} a_{s,t} \frac{(1+n)^{t-s}}{\prod\limits_{i=s+1}^{t}\left(1+\widetilde{r}_{s,i}\right)} = 0, \quad a_{s,s-1} = 0$$

where β is the intertemporal discount rate, a the agent's wealth, while $\widetilde{r}_{s,t} = r_t\left(1-\tau^k_{s,t}\right)$ and $\widetilde{w}_{s,t} = w_t\left(1-\tau^l_{s,t}\right)$ are the net-of-tax factor prices.

The FOCs of this problem imply:

$$\left(\frac{1+n}{1+\beta}\right)^{t-s} U_{c_{s,t}} = p_{s,t} \tag{3}$$

$$\left(\frac{1+n}{1+\beta}\right)^{t-s} U_{l_{s,t}} = -p_{s,t}\widetilde{w}_{s,t} \tag{4}$$

$$\frac{(1+\widetilde{r}_{s,t})}{(1+n)} p_{s,t+1} = p_{s,t}, \tag{5}$$

where the expression U_{i_t} is the partial derivative of the utility function with respect to argument $i = c, l$ at time t and $p_{s,t}$ is the shadow price of wealth. These conditions yield:

$$\frac{U_{c_{s,t}}}{U_{c_{s,t+1}}} = \frac{(1+\widetilde{r}_{s,t})}{1+\beta} \tag{6}$$

$$\frac{U_{l_{s,t}}}{U_{c_{s,t}}} = -\widetilde{w}_{s,t}. \tag{7}$$

2.2 Firms

Under the assumption of perfect competition, in each instant firms, supposed to be identical, hire capital and labor services according to their market prices (gross of taxes) and in order to maximize current period profits. This means that, for each firm i:

$$\frac{dF\left(K_{t-1}^i, L_t^i\right)}{dK_{t-1}^i} = r_t \tag{8}$$

$$\frac{dF\left(K_{t-1}^i, L_t^i\right)}{dL_t^i} = w_t \tag{9}$$

Note that capital is assumed to enter the production process with a one period lag. Assuming a CRS technology, such conditions can also be expressed for the economy as a whole, in per capita terms:

$$f_{k_{t-1}} = \frac{r_t}{(1+\gamma)} \tag{8'}$$

$$f_{l_t} = w_t, \tag{9'}$$

where $l_t \equiv \frac{L_t}{N_t} = \sum\limits_{s=-\infty}^{t} \nu_{s,t} l_{s,t}$, and $k_t \equiv \frac{K_t}{N_t} = \sum\limits_{s=-\infty}^{t} \nu_{s,t} k_{s,t}$, with $\nu_{s,t} \equiv \frac{F_{s,t}}{N_t} = \alpha(1+\alpha)^{s-t}$ the weight of dynasty s in the whole population at period t.

2.3 The government and market clearing conditions

The government is assumed to finance an amount of exogenous public expenditure by levying taxes on capital and labor income and by issuing debt. In order to rule out the problem of time inconsistency, we suppose that the government has access to a commitment technology that ties it to the announced path of distortionary tax rates whenever the possibility of lump sum taxation arises[6]. The only constraints on the possibility of debt issuing are the usual no-Ponzi game condition and the initial condition $B_0 = \overline{B}$. Thus, one obtains the usual equation for the dynamics of aggregate debt:

$$B_t = (1 + r)B_{t-1} + G_t - T_t, \tag{10}$$

where $T_t = \sum_{s=-\infty}^{t} \left(F_{s,t}\tau^l_{s,t} w_t l_{s,t} + \tau^k_{s,t} r_t a_{s,t-1} \right)$, which can also be written, in per capita terms:

$$b_t = \frac{(1 + r)}{(1 + \gamma)} b_{t-1} + g_t + \tau_t. \tag{11}$$

Finally, the market clearing condition implies that, at each date, the sum of capital and debt equals aggregate private wealth, that is:

$$A_t = K_{t-1} + B_t. \tag{12}$$

3 The Ramsey problem

Since we adopt the primal approach to the Ramsey [14] problem, a key point is restricting the set of allocations among which the benevolent government can choose, to those that can be decentralized as a competitive equilibrium[7]. Thus, in this paragraph we define a competitive equilibrium and the constraints that must be imposed to the policymaker problem, in order to achieve such a competitive outcome.

[6] In a dynamic setup, as far as capital income is concerned, there exists an incentive for the government to deviate from the announced (ex-ante) second best policy, upon achieving the instant in which it should be implemented; this is so because the stock of accumulated capital ex-post is perfectly rigid and now should be taxed more heavily than announced, since its taxation has a lump sum character. The commitment hypothesis implies also that the capital income tax at the beginning of the policy is given, that is, fixed exogenously at a level belonging to the $(0, 1)$ interval.

[7] See Atkinson and Stiglitz [3]; the alternative "dual" approach takes prices and tax rates as control variables (see Chamley [5] and Renström [15] for some examples).

The first constraint is the implementability constraint, i.e. the dynasty budget constraint with prices substituted for by the expression for them derived from the individual maximization problem FOCs (for the derivation see the Appendix):

$$\sum_{t=s}^{\infty}\left(\frac{1+n}{1+\beta}\right)^{t-s}\left(U_{c_{s,t}}c_{s,t}+U_{l_{s,t}}l_{s,t}\right)=0,\ \forall s \tag{13}$$

which is referred to as the "implementability constraint".

As for the second constraint, summing eq. (2) over the population to get aggregate wealth, subtracting eq. (10) and exploiting the market clearing condition, we get:

$$y_t \geqslant c_t + k_t - \frac{k_{t-1}}{(1+\gamma)} + g_t. \tag{14}$$

Such expression is usually referred to as the "feasibility constraint" (see the Appendix for a formal derivation).

We can now give the following definition, supposing that the policy is introduced in period t_0:

Definition 1 *A competitive equilibrium is: a) an infinite sequence of policies* $\pi=\left\{\tau^k_{s,t},\tau^l_{s,t},b_t\right\}^{\infty}_{t_0}$, *b) allocations* $\{c_{s,t},l_{s,t},k_t\}^{\infty}_{t_0}$ *and c) prices* $\{w_t,r_t\}^{\infty}_{t_0}$ *such that, at each instant* t*: b) satisfies eq.* (1) *subject to* (2)*, given a) and c); c) satisfies eq.* (8′) *and eq.* (9′)*; eqs.* (14) *and* (11) *are satisfied.*

Such allocations are often referred to as "implementable".

In the light of the definition given above, the following proposition holds:

Proposition 1 *An allocation is a competitive equilibrium if and only if it satisfies implementability and feasibility.*

Proof. The first part of the proposition is true by construction. The proof of the reverse (any allocation satisfying implementability and feasibility is a competitive equilibrium) is provided in the Appendix. ■

3.1 Solution

The policymaker's problem is the following:

$$\max_{\{c_{s,t},l_{s,t},k_t\}^{\infty}_{d}}\sum_{t=d}^{\infty}\sum_{s=-\infty}^{t}\mu_{s,t}\left(\frac{1+n}{1+\beta}\right)^{t-d}U_{s,t}$$

$$\sum_{t=d}^{\infty}\left(\frac{1+n}{1+\beta}\right)^{t-d}\left(U_{c_{s,t}}c_{s,t}+U_{l_{s,t}}l_{s,t}\right)=0,\forall s$$

and

$$y_t = c_t + k_t - \frac{k_{t-1}}{(1+\gamma)} + g_t, \ \forall t$$

$$\lim_{t\to\infty} \frac{k_t}{\prod\limits_{i=t_{0+1}}^{t} (1+f_{k_i})} = 0, \quad k_{t_0} = \overline{k}.$$

where $d = \max(s, t_0)$ and $\mu_{s,t}$ is the weight that the government attaches to dynasty s[8]. Note that we allow $\mu_{s,t}$ to vary with time.

The FOCs with respect to consumption and capital are, respectively:

$$\left(\frac{1+n}{1+\beta}\right)^{t-d} U_{c_{s,t}} \left[\mu_{_{s,t}} + \lambda_{d,t} \left(1 + H_{c_{s,t}}\right)\right] = \phi_t \nu_{_{s,t}} \tag{15}$$

$$\phi_t = \left[\frac{1}{(1+\gamma)} + f_{k_t}\right] \phi_{t+1} \tag{16}$$

where $\lambda_{d,t}$ is the multiplier associated to the implementability constraint, $H_{c_{s,t}} = \frac{\left(U_{cc_{s,t}} c_{s,t} + U_{lc_{s,t}} l_{s,t}\right)}{U_{c_{s,t}}}$, which is usually referred to as the "general equilibrium elasticity of consumption", and ϕ_t is the multiplier associated to the feasibility constraint.

By taking the first order condition relative to consumption as for period $t+1$ and dividing eq. (15) by it, we get[9]:

$$\frac{\left(\frac{1+n}{1+\beta}\right)^{t-d} U_c \left[\mu + \lambda \left(1 + H_c\right)\right]}{\left(\frac{1+n}{1+\beta}\right)^{t+1-d} U_c^{+1} \left[\mu^{+1} + \lambda \left(1 + H_c^{+1}\right)\right]} = \frac{\phi \nu}{\phi^{+1} \nu^{+1}} \tag{17}$$

and, by exploiting eqs. (6), (8') and (16) and by reckoning that $\frac{\nu}{\nu^{+1}} = (1+\alpha)$, we get:

$$\frac{1+\widetilde{r}^{+1}}{1+r^{+1}} = \frac{\mu^{+1} + \lambda\left(1 + H_c^{+1}\right)}{\mu + \lambda \left(1 + H_c\right)}. \tag{18}$$

which provides the implicit expression for the optimal capital income tax[10].

[8] We omit the government budget constraint since, by Walras' law, it is satisfied if the implementability and feasibility constraints hold.

[9] From now onward we omit the s and t indicators, whenever this does not cause ambiguity: hence, notation X^{+1} stands for $X_{s,t+1}$.

[10] Note that we do not have any condition ensuring that the tax rate will be in the $(0, 1)$ interval, while it is possible that such a rate keeps sticking at the upper limit for a (finite)

4 Discussion of the results

We now discuss the results concerning capital income taxation.

By inspection of the tax equation, it emerges that there are two independent forces determining the level of τ^k: 1) the dynamics of H_c, already discussed in the literature; 2) the dynamics of the social intergenerational weight, which is new.

We can now state the following proposition:

Proposition 2 *If the economy converges to a steady state, along the transition path, for $t > t_0$, and at the steady state the tax on capital income is different from zero unless* $\frac{\overset{+1}{\mu}+\lambda\left(1+\overset{+1}{H_c}\right)}{\mu+\lambda(1+H_c)} = 1$.

Proof. The proof is straightforward by inspection of eq. (18). ■

Factor 1) has been widely discussed in the literature: $H_c = \overset{+1}{H_c}$ obtains, for example, if one assumes that the utility function is homothetic in consumption and (weakly) separable in consumption and leisure. Otherwise, future consumption is taxed/subsidized if consumption demand is getting more/less inelastic. Moreover, as recalled above, this factor marks the difference between the ILRA and the OLG-LC models as for the steady state result: in fact, in OLG models H_c can vary with age even at the steady state. However, as shown in eq. (18), even in the absence of a life cycle, in the present model the non zero tax rule can still apply.

As for factor 2), its role can be isolated by supposing $H_c = \overset{+1}{H_c}$. Then, $\frac{1+\overset{+1}{\tilde{r}}}{1+\overset{+1}{r}} = \frac{\overset{+1}{\mu}+\lambda(1+H_c)}{\mu+\lambda(1+H_c)}$, which is different from one unless μ, the weight assigned to each dynasty by the government, is constant through time. This assumption is made in the existing OLG-LC models (see, in particular, Erosa and Gervais [9] and Garriga [10]): such element leads to zero taxation in the absence of life cycle, i.e. $H_c = \overset{+1}{H_c}$. However, this is typically not the case in this setup, where the weight of each cohort within the population decreases because of the entry of immigrants in each period. Let us therefore consider the situation in which the social weight of each dynasty is equal to its actual demographic weight within the population, i.e. $\mu_{s,t} = \nu_{s,t}$. Given our assumption of a constant rate of migration arrivals α, the relative size of each dynasty is decreasing through time, so that $\frac{\overset{+1}{\mu_g}}{\mu_g} = \frac{1}{(1+\alpha)}$ for each dynasty and, hence, the tax rate is positive.

period of time since the introduction of the policy. However, in the rest of the work we maintain the assumption of interiority of the equilibrium tax rates for $t > t_0$.

This new result clarifies that an independent source of taxation is represented by the "disconnection" of the economy, which is typical of OLG models: in fact, given the dynamics of μ, the government discriminates future consumption in favour of the present one; under a different perspective, one can note that at each date individuals tend to oversave relatively to what could maximize welfare, since they do not take into consideration the new arrivals into the economy. In fact these individuals at each date provide extra resources for redistributing (at least the burden of taxation) among the existing inhabitants. It is easy to show that, if individuals were altruistic towards migrants, the JC zero tax rule would be restored and that even in the presence of limited lifetime horizon this result still holds.

Note also that $\mu_{s,t} = \nu_{s,t}$ decreases with the dynasty's age and is zero for the oldest one when $t \to \infty$. This implies that the tax rate is also decreasing with age, being zero for the oldest. The economic intuition behind this result can be posed as follows: τ^k depends upon the ratio $\frac{\lambda}{\nu}$, i.e. the cost of resorting to distortionary taxation vis-à-vis the dynasty's weight within the population; when the relative size of the dynasty shrinks, the cost of the distortion deriving from hitting its capital income tends to outweigh the gain in terms of contribution to total revenues[11].

Again, we stress that in our model the age-dependency of the tax rates does not (necessarily) stem from the dynamics of H_c, as instead in Erosa and Gervais [9], but, rather, from that of the demographic weight. However, this characteristic of the solution disappears in the case of a logarithmic utility function, with $H_c = -1$. In fact, since the utility function displays a unitary intertemporal elasticity of substitution, the substitution and the income effect generated by an interest rate variation (due to taxation) cancel out. Hence, the change of future interest rates, that is, the change of the relative prices of future consumption, does not distort the individual consumption/saving intertemporal allocation.

5 Conclusions

We reconsider the issue of optimal capital income taxation in an economy with dynastic altruism and migration applying the primal approach to the Ramsey problem.

[11] Since $U_{c_{s,t}} = U_{c_{s-1,t}}$, by exploiting eq. (15) for individual of dynasty $s-1$ as for time t, eq. (17) becomes $\frac{\left(\frac{1+n}{1+\beta}\right)^{t-s} U_{c_{s,t}}\left[\mu_{s,t}+\lambda(1+H_c)\right]}{\left(\frac{1+n}{1+\beta}\right)^{t-s+1} U_{c_{s-1,t}}\left[\mu_{s-1,t}+\lambda(1+H_c)\right]} = \frac{\nu}{\nu+1}$; since for the oldest dynasties $\lim_{t\to\infty} \nu(s,t) = \lim_{t\to\infty} \nu(s-1,t) = 0$, one gets that $F_K = \gamma$.

The thrust of the paper is that the Chamley-Judd rule comes out not to apply if the economy is disconnected and the policymaker consequently attaches to each dynasty a weight that corresponds to its actual share within the population: the optimal rule turns out to be a positive taxation of capital income proportionally to the rate of new migrants arrivals.

This factor, while being absent in both ILRA models and in OLG ones with constant population, plays a crucial role when, as it is likely to happen in the real world, the dynamics of the population is more complicated. We show that, in a OLG model with migration and limited (i.e. intradynastic) altruism, the demographic weight of dynasties decreases, so that a varying social intergenerational discount rate appears a sensible rather than an ad hoc assumption. When this is the case, the positive taxation of capital income is optimal because individuals, who are disconnected with respect to the immigrants, do not take into account the fact that the continuous arrival of new dynasties provides extra resources for an intertemporal/intergenerational redistribution of the burden of taxation. On the other hand, by reckoning this possibility, the government reduces the oversaving of individuals by hitting future consumption proportionally to the immigration rate.

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6 Appendix

6.1 Derivation of the implementability constraint

In order to obtain the implementability constraint, write eq. (2) in its intertemporal form:

$$\sum_{t=s}^{\infty}\frac{(c_{s,t}-w_{s,t})}{\prod\limits_{i=s+1}^{t}(1+\widetilde{r}_{s,i})}=0. \tag{19}$$

Since $\frac{U_{c_{s,t}}}{U_{c_{s,t+1}}}\frac{(1+\beta)}{(1+n)}=\frac{p_{s,t}}{p_{s,t+1}}=\frac{(1+\widetilde{r}_{t+1})}{(1+n)}$, we have

$$\frac{1}{(1+n)^{t-s}}\prod_{i=s+1}^{t}(1+\widetilde{r}_{s,i})=\frac{p_{s,s}}{p_{s,s+1}}\frac{p_{s,s+1}}{p_{s,s+2}}...\frac{p_{s,t-1}}{p_{s,t}}.$$

By substituting into eq. (19), we obtain

$$\sum_{t=s}^{\infty}\frac{(c_{s,t}-w_{s,t})\,p_{s,t}}{p_{s,s}}=0$$

and exploiting the FOCs from the individual maximization problem, we get

$$\sum_{t=s}^{\infty}\left(\frac{1+n}{1+\beta}\right)^{t-s}\left(U_{c_{s,t}}c_{s,t}+U_{l_{s,t}}l_{s,t}\right)=0,$$

which is eq. (13) in the text.

6.2 Derivation of the feasibility constraint

To derive the feasibility constraint, first aggregate eq. (2) over population at time t:

$$\sum_{s=-\infty}^{t}F_{s,t}a_{s,t}=$$

$$\sum_{s=-\infty}^{t}F_{s,t}\left[\frac{(1+r)}{(1+n)}a_{s,t-1}+w_tl_{s,t}-c_{s,t}-\tau^k_{s,t}r_ta_{s,t-1}-\tau^l_{s,t}w_tl_{s,t}\right] \tag{20}$$

and by recalling that $A_t\equiv\sum\limits_{s=-\infty}^{t}F_{s,t}a_{s,t}$, $F_{s,t}=F_{s,t-1}\,(1+n)$ and $F_{t,t-1}=0$, so that $\sum\limits_{s=-\infty}^{t}F_{s,t-1}a_{s,t-1}=A_{t-1}$, we can rewrite eq.(20) as follows

$$A_t=(1+r_t)A_{t-1}+w_tL_t-C_t-T_t.$$

Finally, by subtracting eq. (10) and exploiting the market clearing condition we obtain

$$K_t = (1 + r_t)K_{t-1} + w_tL_t - C_t - G_t$$

which, in per capita terms, becomes

$$k_t = \frac{(1 + r_t)}{(1 + \gamma)}k_{t-1} + w_tl_t - c_t - g_t$$

where $\frac{r_tk_{t-1}}{(1+\gamma)} + w_tl_t \equiv y_t$, due to CRS.

6.3 Proof of Proposition 1

Proof. Since a competitive equilibrium (or implementable allocation) satisfies both the feasibility and the implementability constraints by construction, in this Appendix we demonstrate the reverse of Proposition 1: any feasible allocation satisfying implementability is a competitive equilibrium.

Suppose that an allocation satisfies the implementability and the feasibility constraints. Then, define a sequence of after tax prices as follows: $\widetilde{w}\,(s,t) = -\frac{U_{l(s,t)}}{U_{c(s,t)}}$, $\tilde{r}_{s,t} = \frac{p_{s,t}}{p_{s,t+1}}\,(1+n) - 1$, with $p_{s,t} = U_{c_{s,t}}\left(\frac{1+n}{1+\beta}\right)^{t-s}$, $\forall s$ and $\forall t$, and a sequence of before tax prices: $f_{k_{t-1}} = \frac{r_t}{(1+\gamma)}$, $f_{l_t} = w_t$. As a consequence, by construction such allocation satisfies both the consumers' and firms' optimality conditions.

The second step is to show that the allocation satisfies the consumer budget constraint. Take the implementability constraint and substitute $U_{c_{s,t}}$ and $U_{l_{s,t}}$ by using the expressions above:

$$\sum_{t=s}^{\infty}\left(p_{s,t}c_{s,t} - \widetilde{w}_{s,t}p_{s,t}l_{s,t}\right) = 0, \forall s.$$

Then, by recursively using the expression $p_{s,t} = p_{s,s}\frac{(1+n)^{t-s}}{\prod_{i=s+1}^{t}(1+\widetilde{r}_{s,i})}$, we get

$$\sum_{t=s}^{\infty}\frac{(1+n)^{t-s}}{\prod_{i=s+1}^{t}(1+\widetilde{r}_{s,i})}\left(p_{s,s}c_{s,t} - \widetilde{w}_{s,t}l_{s,t}\right) = 0.$$

Finally, by eliminating $p_{s,s}$ and defining $c_{s,t} - \widetilde{w}_{s,t}l_{s,t} = -(1+n)\,q_{s,t} + (1+\widetilde{r}_{s,t})\,q_{s,t-1}$, we get

$$\sum_{t=s}^{\infty} \frac{(1+n)^{t-s}}{\prod\limits_{i=s+1}^{t} (1+\widetilde{r}_{s,i})} \left[-(1+n)\, q_{s,t} + (1+\widetilde{r}_{s,t})\, q_{s,t-1}\right] = 0$$

that turns out to be:

$$-\lim_{t\to\infty} \frac{(1+n)^{t-s}}{\prod\limits_{i=s+1}^{t} (1+\widetilde{r}_{s,i})} q_{s,t} + q_{s,s} = 0$$

which holds if $q_{s,t} = a_{s,t}$ and $\lim\limits_{t\to\infty} \frac{(1+n)^{t-s}}{\prod\limits_{i=s+1}^{t} (1+\widetilde{r}_{s,i})} a_{s,t} = 0$.

As for the public sector budget constraint, by aggregating the individuals' budget constraints over population at time t and expressing them in per capita terms, we get

$$a_t = \frac{(1+r_t)}{(1+\gamma)} a_{t-1} + w_t l_t - c_t - \tau_t. \tag{21}$$

Finally, by subtracting the feasibility constraint

$$k_t = \frac{(1+r_t)}{(1+\gamma)} k_{t-1} + w_t l_t - c_t - g_t$$

and defining $b_t = -k_{t-1} + a_t$, we obtain

$$b_t = \frac{(1+r_t)}{(1+\gamma)} b_{t-1} + g_t - \tau_t$$

which is eq. (11) in the text. ■