Migration Dynamics*

Michele Moretto† Sergio Vergalli‡

August 28, 2005

Abstract

This paper tries to explain why most migration flows show some observable jumps in their processes, a phenomenon that seems to be sympathetic with the characteristic of irreversibility of migration. We present a real option model where the choice to migrate depends on both the differential wage between the host country and the country of origin, and on the probability of being fully integrated into the host country. The theoretical results show that the optimal migration decision of a single individual consists of waiting before migrating in a (coordinate) mass of individuals. The dimension of the migration flow depends on the behavioural characteristics of the ethnic groups: the more "sociable" they are, the larger the size of the wave and the lower the differential wage required. A second part of the paper is devoted to calibrating the model and simulating some migration flows to Italy in the last decade. The calibration is able to replicate the observable migration jumps in the short term. In particular, the calibrated model is able to conjecture the induced labour demand elasticity level of the host country and the *behavioural rationale* of the migrants.

JEL Classification Numbers: F22, J61, O15, R23.

Keywords: Migration, Real Option, Labour Market, Network Effect.

---

*We acknowledge the financial support from the University of Brescia under the 60% scheme and the two-years project, protocol N. 6413/2005.

†University of Brescia, Department of Economics, via S.Faustino 74/b, Brescia, Italy, moretto@eco.unibs.it

‡University of Brescia, Department of Economics, via S.Faustino 74/b, Brescia, Italy, vergalli@eco.unibs.it (corresponding author)

1 Introduction

The beginning of this century was characterised by a great mobility of money, products and people. Therefore, it is not surprising that a wide range of economic and sociological literature studies migration waves and tries to detect relations of cause and effect. However, by observing the migration phenomenon, it can be seen that its dynamics are in general characterised by some jumps in the migration flows, especially at the beginning of its process, followed by gradual waves. What could be the causes for these particular dynamics? In economic literature, the main variable that affects the decision to migrate is surely the wage differential between the host country and the country of origin (Todaro, 1969; Langley, 1974; Hart, 1975; Borjas, 1990, 1994). Nevertheless, even if wage differential is important, it is not sufficient to totally explain migrant behaviour: evidence seems to stress the focal role of community networks in the migrant's choice (Boyd, 1989; Bauer and Zimmermann, 1997; Moretti, 1998; Winters *et al.*, 2001; Bauer *et al.,* 2002; Coniglio, 2003; Munshi, 2001, 2003; Heitmueller, 2003). Moretti (1998), for example, with an alternative model to Todaro's, finds evidence that both the timing and the destination of migration could be explained by the presence of social networks in the host country.

Furthermore, the fact that migration decision is in many cases highly irreversible is a third element that may help to explain the presence of jumps in the migration flows. In this respect, Burda (1995), following a real option approach, implements Sjaastad's assumption (1962) that describes migration choice in terms of investment. Burda's results show that individuals prefer to wait before migrating, even if the present value of the wage differential is positive, because of the uncertainty and the sunk costs associated with migration [1]. Subsequently Khwaja (2002) and Anam *et al.*, (2004) developed Burda's approach by describing the role of uncertainty in the migration decision.

The aim of this paper is to merge in a unified framework the real option approach of investment decision and the works on network effects applied to the analysis of migration flows.

Assimilating the decision of each individual to migrate to a new country

[1] Investment is defined as the act of incurring an immediate cost in the expectation of future payoff. However, when the immediate cost is sunk (at least partially) and there is uncertainty over future rewards, the timing of the investment decision becomes crucial (Dixit and Pindyck, 1994, p.3).

as a decision on an irreversible investment, we investigate the role played by social networks to help the immigrants integrate in the host country, where an immigrant is completely integrated when his economic and social status is no different from the native one. We do this considering the opportunity that each immigrant becomes a member of a network (a community) of homogeneous individuals, located in the host country. The community helps the immigrants to obtain a higher wage or improve their labour condition if there are strong ties among the members ("positive network externalities"). The greater the size of the community, the higher the number of ties, the higher the flow of information on the job opportunities, and therefore the higher the probability of integrating.

Nevertheless, if the number of immigrants continues to increase, labour competition as well as higher alienation[2] among immigrants inside the community may reduce their net benefits ("negative network externalities").

The struggle between these two forces is captured by an inverted U-shaped benefit function which follows directly by modelling the probability of each immigrant being totally integrated in the host country *à la Bass* (1969). The Bass model[3] well describes the "behavioural rationale" of migration flows by focusing on the role played by two kinds of immigrants: the innovators or individualists, and the imitators. The innovators are those individuals that decide to migrate independently of the decisions of others. The imitators are those individuals influenced by the number of previous migrants: they share information among themselves and tend to establish a network. The weight of each different type of immigrant influences the timing of migration and then the size of the community.

Finally, we calibrate the model and simulate some migration flows to Italy in the last decade by using the ISTAT data. The results fit the theoretical approach and are able to replicate the observable migration jumps.

On the one hand, the higher the ties among the individuals, the larger the dimension of the wave will be. On the other hand, the presence of congestion in the community and/or strong competition among workers in the host country delays entry.

[2] This is the case in which the members of the incumbent population discontinue their attraction of immigrants (see Heitmueller (2003)).

[3] The Bass model was originally built to study the diffusion of new durable products and largely adopted in marketing literature.

2 Migration jumps

We analyse the four main foreign flows coming into Italy at the end of the last decade: Albanians, Chinese, Filipinos and Romanians[4]. Figure 1 below shows the migration flows and their growth rates in the period considered. All the data are taken from the ISTAT database. For the sake of completeness, Figure 2 also shows the wage differentials in the same period. These are obtained from the World Bank International Comparison Programme database and are deflated using the IRES[5] deflator.

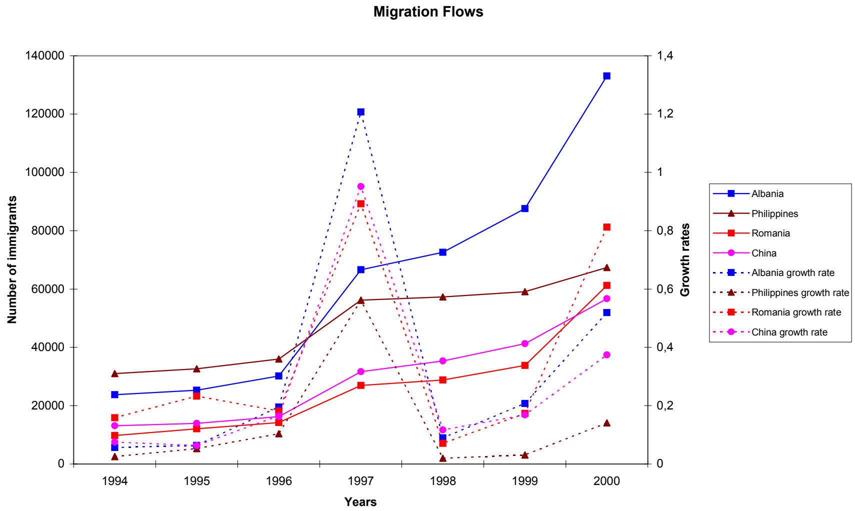

Figure 1: Migration jumps

[4]The same analysis is applied to Italian flows to the USA in the period between 1954 and 1984 in Vergalli S., "Migration Dynamics", Ph.D. thesis, 2005, University of Padua.

[5]Istituto di Ricerche Economiche e Sociali, www.ires.it.

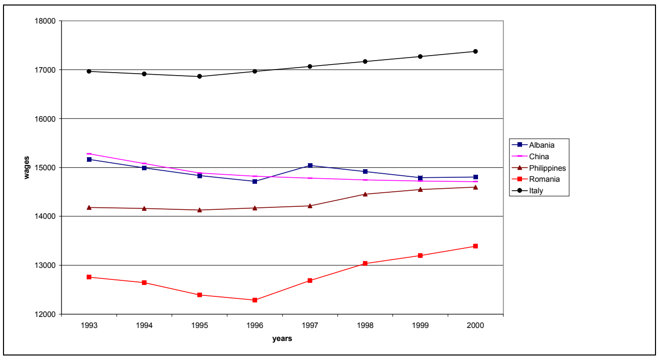

Figure 2: Wage levels

For all the four flows, it is evident that the migration process is not smooth. It shows jumps in its dynamics. In particular, this happens after a certain number of years characterised by low waves, as if a mass of individuals is waiting for something happen before deciding to migrate.

We can also observe that the wage differential cannot be considered the main variable driving migration flows. In all cases (except, partially, for Romania and Albania), the jumps do not occur in combination with a steady rise in the relative wage levels, as stressed by Moretti (1998).

Why do they wait before taking their decision to migrate? What are they waiting for? And why do they move in a mass? We try to answer these questions by examining whether the characteristic of investment of migration and the role of ethnic groups, behind any migration decision, can explain the migration jumps observed in Figure 1.

This paper is organised as follows. Section 3 presents the model and the basic assumptions. Section 4 develops the theoretical framework that combines real option theory and the network effects, namely the optimal migration strategy in the presence of positive and negative externalities. Section 5 calibrates the model and Section 6 makes some simulations which confirm the theoretical results. Finally, section 7 summarises the conclusions.

3 The Model

We assume that an individual that migrates to another country is completely integrated when his economic and social status is no different from the native one. Nevertheless, the timing of the migrant's integration suffers from a phenomenon of attrition because of the lack of information about the host country and its labour market. We also assume that in the host country a homogeneous group of people (a community/ a network) exists that can help each immigrant to increase his probability of integration. The larger the size of the community, the closer the ties among its members and then the higher the level of the integration probability. The number of ties also depends on idiosyncratic characteristics of the of immigrants, that we call "behavioural rationale" using the Bass terminology. That is, the more "sociable" an individual or a group of individuals, the stronger and the higher the number of ties they have.

3.1 The basic assumptions

Our main assumptions are the following:

1. At any time t a risk-neutral[6] individual is free to decide to migrate to a new country discounting future benefits (the wage differential between the host country and the country of origin) at the constant interest rate ρ.

2. When the migrant arrives in a host country, he receives only a percentage $\xi < 1$ of the host wage as first entry wage[7]. So defining w_i^o as the wage of his country of origin (where i is the country), we are able to write the differential wage as a percentage of the wage of the host country:

$$\xi w - w_i^o \equiv [\xi - w_i^o/w]w \equiv \phi_i' w$$

3. In the host country there is a community of ethnically homogeneous individuals that helps each member to integrate with the host labour market (or to obtain a legal job if he is working on the illegal market). When the immigrant is completely integrated, he gets the difference between the legal host current market wage w and the wage of his country of origin w_i^o, i.e.:

$$w - w_i^o \equiv [1 - w_i^o/w]w \equiv \phi_i w$$

[6] See Burda (1995), Khwaja (2002) and Locher (2002) for the use of this assumption.

[7] Empirical evidence shows that this is true whether the migrant finds a legal or an illegal job (see Chiswick,1978; Borjas, 1990; Massey, 1987).

4. For the sake of simplicity, we assume that the country-specific percentages ϕ_i' and ϕ_i ($\phi_i' \leq \phi_i$) are constant over time[8].

5. Each individual enters a new country undertaking a single irreversible investment which requires an initial sunk cost K.

6. The size of the immigrant flow dn is infinitesimally small compared to the total number of previous immigrants n.

7 Finally, the inverse labour demand for immigrants in the host country at time t is an isoelastic function of the total number of previous immigrants $n(t)$:

$$w(t) = \theta(t)\, n(t)^{\zeta} \tag{1}$$

where θ is a labour-demand-specific shock, $\zeta < 0$ is the elasticity and w is the average wage of the host country[9].

We introduce the uncertainty in the model by assuming that:

8. The labour-demand-specific shock θ follows a *Brownian motion*:

$$d\theta(t) = \alpha\theta(t)dt + \sigma\theta(t)dW(t) \tag{2}$$

with $\theta(t_0) = \theta$ and $\alpha,\ \sigma > 0$ are constant over time. The component $dW(t)$ is a Weiner disturbance defined as $dW(t) = \varepsilon(t)\sqrt{dt}$, where $\varepsilon(t) \sim N(0,1)$ is a white noise stochastic process (Cox and Miller, 1965).

9. The time taken to become perfectly integrated, say τ, is stochastic and depends on a distribution of probability defined as:

$$1 - F_\tau(t) \equiv \Pr(\tau > t \mid t > 0) \tag{3}$$

and its corresponding *hazard rate is*[10]:

$$p_\tau(t) \equiv \frac{f_\tau(t)}{1 - F_\tau(t)} \tag{4}$$

[8] We calibrate them as the loss in Purchasing Power Parity with respect to the initial year of our dataset. See section 5 below.

[9] There are two implicit assumptions beyond (1). Firstly, that all incumbent immigrants have a job and that all future immigrants seek a job. Secondly, that w refers to labour markets that are occupied mainly by immigrants so that we can ignore the role of native employees (Heitmueller, 2003).

[10] $p_\tau(t)$ is the migrant's conditioned probability of obtaining a better job at time $t + dt$, if he has worked at a low wage till t.

where $f_{\tau}(t)$ is the density function or *the likelihood of being perfectly integrated at t.*

Each immigrant decides when to enter a new country maximising his net benefit value defined as the expected discounted stream of wage differentials over the planning horizon (taken infinite for simplicity) minus the entry cost K.

By (1) and assumptions 3-8 the benefits he gets from being completely integrated at τ are given by[11]:

$$\begin{aligned} B(n(\tau),\theta(\tau)) &= E_{\tau}\left\{\int_{\tau}^{\infty} e^{-\rho(t-\tau)}\phi w(t)dt\right\} \\ &\equiv E_{\tau}\left\{\int_{\tau}^{\infty} e^{-\rho(t-\tau)}\phi\theta(t)\,n(t)^{\zeta}\,dt\right\} \end{aligned} \tag{5}$$

where $B(\bullet)$ accounts for the future evolution of the number of migrants $n(t)$, $t \geq \tau$. The expectation operator $E_{\tau}(\bullet)$ is taken with respect to the random variables τ and $\theta(t)$ (and then $n(t)$).

Next, taking into account the benefits the immigrant may gain before integrating, we end up with a total benefit value at the migration time zero as:

$$V(n,\theta) = E_0\left\{\int_0^{\tau} e^{-\rho t}\phi' w(t)dt + e^{-\rho\tau}B(n(\tau),\theta(\tau))\right\} \tag{6}$$

where $n(0) = n;\ \theta(0) = \theta$. By using an indicator function $J_{[\tau>t]}$ that assumes the value one or zero depending on whether the argument is true or false, we are able to write the (6) as:

$$V(n,\theta) = E_0\left\{\int_0^{\infty} e^{-\rho t}J_{[\tau>t]}\phi'\theta(t)n(t)^{\zeta}\,dt + e^{-\rho\tau}B(n(\tau),\theta(\tau))\right\} \tag{7}$$

[11] If all immigrants face the same instantaneous probability of death λdt, we can define $\rho = \widehat{\rho} + \lambda$, where $\widehat{\rho}$ is the market rate (Dixit and Pindyck, 1993, p.200).

Since $E\left[J_{[\tau>t]}\right]=1-F_\tau(t)$, we can plug the (3) in the (7) to obtain:

$$V(n,\theta) = E_0\left\{\int_0^\infty e^{-\rho t}\left[1-F_\tau(t)\right]\phi'\theta(t)n(t)^\zeta\,dt + \right. \\ \left. +\int_0^\infty e^{-\rho t}f_\tau(t)\,B\left(n(t),\theta(t)\right)dt\right\} \tag{8}$$

where the expectation is now taken only with respect to $\theta(t)$ (and $n(t)$).

If the benefit value function $V(\bullet)$ is known, the optimal migration policy implies that the return from migration must be at least equal to cost K at the entry point. In other words, we need to find the curve $\theta^*(n(t))$ (i.e. the value of the labour demand shock) at which the $n(t)$th migrant is indifferent between immediate entry or waiting another instant[12]:

$$V\left[n(t),\theta^*\left((n(t))\right]-K=0 \tag{9}$$

This is what we are going to do in the next section.

3.2 The entry time τ and the network effect

Before turning to the migrant's optimal policy, we need to model the probability of integrating (3). We define two different groups of migrants:

- the *innovators*: those individuals that decide to migrate independently of the decisions of other individuals in a social system. They are the pioneers or the individualists: their decision depends on their intrinsic characteristics.
- the *imitators*: those individuals influenced in the timing of migration by the number of previous migrants. In particular, we refer to the individuals who follow the innovators. Their particular behavioural characteristic is their sociality: they have strong ties among themselves and tend to establish a network[13].

[12] This condition is familiar in the real option theory with the name of matching value condition (see Dixit and Pindyck, 1994).

[13] A recent economic approach calls a similar phenomenon *herd behaviour,* i.e.: "I will go to where I have observed others go" (Bauer *et al.* (2002)).

Following Bass(1969), the probability that perfect integration occurs at t, given that no integration has yet occurred, is set as a linear function of the size of the community, i.e.:

$$p_\tau(t) = a + bF_\tau(t) \tag{10}$$

where $F_\tau(t)$ stands for the number of immigrants already entered; a is the coefficient of innovation, the influence on entry regardless of the number of previous members; b is the coefficient of imitation, the impact of previous members on the probability of entry at time t. By some algebraic operations (Bass, 1969, page 217), we get:

$$F_\tau(t) = \frac{m - n(t)}{m}, \tag{11}$$

and the fraction of the total immigrants integrating at time t is:

$$f_\tau(t) = a + \frac{(b-a)}{m} n(t) - \frac{b}{m^2} n^2(t) \tag{12}$$

where m is the (fixed) total number of immigrants over the planning horizon, which represents the critical "saturation" dimension of the community. Finally, we get $\lim_{n \to m} f_\tau(t) = 0$ and $f_\tau(t)$ is concave $iff \;\; b > a$.

By plugging (12) and (11) into (8), we simplify (8) as:

$$V(n, \theta) = E_0 \left\{ \int_0^\infty e^{-\rho t} n(t)^\zeta \left[\frac{m - n(t)}{m} \phi' + \frac{[a + \frac{b-a}{m} n(t) - \frac{b}{m^2} n(t)^2]}{\rho - \alpha} \phi \right] \theta(t)\, dt \right\} \tag{13}$$

3.3 The benefit function

Network migration theory suggests that benefit is a positive function in both wages and network size (Massey *et al.* 1993). However, by (13), suppressing the time for the sake of simplicity, we can write the benefit function per unit of time as:

$$\pi(n, \theta) \equiv u(n)\, \theta \tag{14}$$

where $u(n) \equiv n^\zeta \left[\frac{m-n}{m} \phi' + \frac{Bass(n)}{\rho - \alpha} \phi \right]$ and $Bass(n) \equiv [a + \frac{b-a}{m} n - \frac{b}{m^2} n^2]$.

Apart from the shock θ each immigrant shares the same "utility" $u\,(n)$. The overall shape of the $u(n)$ is ambiguous: it depends strongly on the *struggle* between the competitive effect (i.e. more immigrants reduces wages depending on the magnitude of the elasticity ζ) and the network effect (i.e. individuals gain "utility" by increasing the number of fellow countrymen which increases the probability of integration via the Bass function $Bass\,(n)$). According to the relative magnitude of these two effects, we can observe three shape of $u(n)$ as in Figure 3.

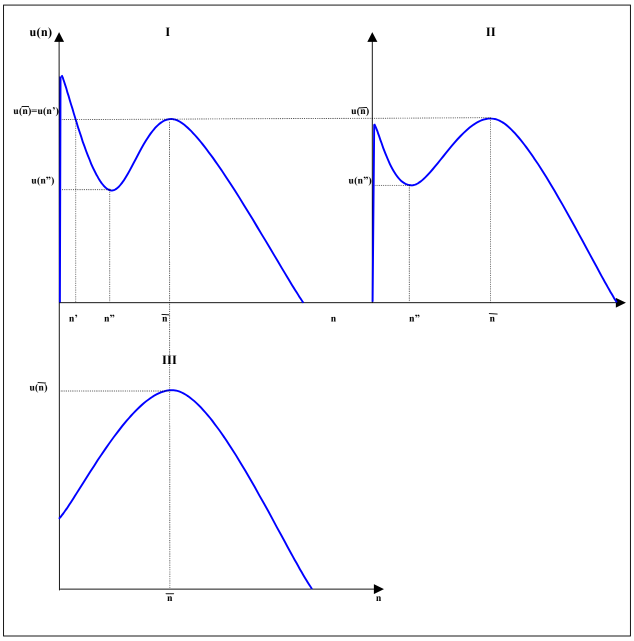

Figure 3: Peculiar shape of $u(n)$

Let's analyse Figure 3 from **quadrant I** to **quandrant III** for decreasing levels of elasticity, *ceteris paribus*:

quadrant I

This is the general case for a not very low level of elasticity ζ. A relative minimum in n" and a relative maximum in $\overline{n}$ exist that divide the function into three intervals:

1. In the interval $n \in (0, n")$, the competition effect prevails over the

network effect: a new entry reduces the benefit more than the gain caused by cooperation among members of the community.

2. In the interval $n \in (n", \overline{n})$ the network effect prevails: the benefit increases with n until the dimension of the network reaches the level $\overline{n}$.

3. In the interval $n \in (\overline{n}, m)$ the competition effect prevails: the benefit decreases with n until the dimension of the community hits the saturation level m. Competition is coupled with a phenomenon of congestion as n moves toward m.

As shown in Figure 3, within the interval $(0, n")$ a level n' exists such that $u(n') = u(\overline{n})$. Further, for $n > n'$ each immigrant earns benefit lower than $u(n')$ until the community size reaches the relative maximum $\overline{n}$. Then each immigrant receives a lower benefit if he enters with a community population $n \in (n', \overline{n})$.

Since the critical level of n' depends on the relative influence of the competition and network effects, for different levels of elasticity we can observe the following:

quadrant II

Even if for low value of n competition prevails over the network effect, the latter dominates any other effect as n increases. This implies that for each individual it is expedient to wait for the maximum benefit $u(\overline{n})$ before entering.

quadrant III

Since $\zeta \rightarrow 0$ implies that $n' \rightarrow 0$, for very low levels of elasticity, the benefit function simply assumes an inverse U-shape.

4 Migration dynamics

Applying Itô's Lemma to (14) and substituting (2) to eliminate $d\theta$, we get an expression for the rate of change of π in terms of the shock and the network size:

$$d\pi = \mu(n)\,\pi dn + \alpha\pi dt + \sigma\pi dw, \qquad with \;\; \pi_0 \equiv u(n_0)\,\theta_0 = \pi \tag{15}$$

In (15) the first term $\mu(n) \equiv u'(n)/u(n)$ captures the direct effect of migration flows. Migration influences the level of benefits through its effect on the labour market equilibrium depending on the dimension of the community. In particular, given any value of the shock θ, more immigrants imply a higher or lower equilibrium level of benefits depending on the presence of positive $\mu(n) > 0$ or negative $\mu(n) < 0$ network externalities respectively.

4.1 Optimal migration policy for $n > \overline{n}$ (and $< n'$).

If the initial size of the community is $n \geq \overline{n}$ (or $n \leqslant n'$), we expect migration to work in the following way. For any fixed n, the benefits per unit of time move according to the above stochastic process with $\mu(n)dn = 0$. If they climb to a certain level $\pi^* = u(n)\theta^*(n)$, migration becomes feasible, the network size increases from n to $n + dn$ and the benefits go downward along the function $u(n)$. Benefits will then continue to move stochastically without the term $\mu(n)dn$, until another episode of entry occurs[14]. This can be summarized by the following proposition:

Proposition 1 *If $n \geq \overline{n}$ (or $n \leqslant n'$), the optimal migration policy is described by the following upward-sloping curve (Figure 4):*

$$\theta^*(n) \equiv \frac{\beta_1}{\beta_1 - 1}(\rho - \alpha)\frac{K}{u(n)}, \qquad \textit{with } \frac{\beta_1}{\beta_1 - 1} > 1 \tag{16}$$

where $\rho > \alpha$ and $\beta_1 > 1$ is the positive root of the auxiliary quadratic equation $\Psi(\beta) = \frac{1}{2}\sigma^2\beta(\beta - 1) + \alpha\beta - \rho = 0$.

Proof. See Leahy (1993) and the Appendix ■

In the region above the curve, it is optimal to migrate: a wave of migrants will enter in a lump to move the benefits level immediately to the threshold curve. In the region below the curve the optimal policy is inaction: the individuals wait until the stochastic process θ moves it vertically to $\theta^*(n)$ and then again a flow of migrants will jump into the host country just enough not to cross the threshold.

The "utility" threshold that triggers migration by the single migrant in isolation is identical to that of the individual that correctly anticipates the

[14]In technical terms, the threshold π^* becomes an upper reflecting barrier on the benefit process (see Harrison,1985).

other immigrants' strategies. This remarkable property, first discovered by Leahy (1993), has an important operative implication: the optimal migration policy of each individual need not take account of the effect of rivals' entry. He can behave competitively as if he is the last to enter[15].

4.2 Optimal migration policy for $n' < n < \overline{n}$

For $n \in (n', \overline{n})$, the network benefit prevails over the competitive effect and then we expect the timing of an individual's entry is influenced by the entry decisions of others.

Intuition suggests that Leahy's result cannot be extended to cover this case. In other words, a migrant cannot continue to pretend to be the last to migrate in constructing his optimal entry policy. The gist of our argument is that there are positive externalities, so the higher the number of members in the community, the greater the advantage in terms of flow of benefits. This is evident in the case of an U-shaped benefit function (quadrant III in Figure 3) but it also works for the general case as $u(n') = u(\overline{n})$ and the "utility" is lower in within (quadrant I in Figure 3). Therefore, although entering may be profitable, it is more expensive to do it first than to enter later on, when others have already done so. This makes the trigger $\pi^* = u(n)\theta^*(n)$ no longer optimal: each migrant can do better by delaying entry[16]. Potentially conflicting preferences over appropriation of the positive *network benefits* make the immigrants face a choice between individual entry and agreement, that is between coordinate and non-coordinate entry.

However, as all individuals are subject to the same labour demand stochas-

[15] In other words, when an individual decides on entry, by pretending to be the last to migrate, he is ignoring two things: 1) He is thinking that his benefit flow is given by $u(n)\theta$, with n held fixed forever. Thus, as $u'(n) < 0$, he is ignoring that future entry by other members, in response to a higher value of θ, will reduce his "utility". All things being equal, this would make entry more attractive for the migrant that behaves myopically. 2) He is unaware that the prospect of future entry by competitors reduces his option value of waiting. That is, pretending to be the last to migrate, the individual also believes he still has a valuable option of waiting before making an irreversible decision. All things being equal, this makes the decision to enter less attractive. The two effects offset each other, allowing the migrant to act as if he were in isolation (see Dixit and Pindyck, 1994, p. 291).

[16] The decision problem involved here resembles one of *war of attrition* where each agent waits for his rivals to concede (Moretto, 2000).

tic shock, two equilibrium patterns are possible: either the community remains locked-in at the initial size $n' < n < \overline{n}$, sustained by self-fulfilling pessimistic expectations (infinite delay), or a mass of individuals simultaneously rushes to enter. Excluding the former[17], we expect entry to work in the following way: for a fixed size of the network, π moves according to the process (15) with $\mu(n)dn = 0$. If benefits climb to $\pi^{**} = u(n)\theta^{**}(n)$, it will trigger an entry of discrete size $\overline{n} - n$ that raises the dimension of the community instantaneously by a jump. The exact form of the trigger θ^{**} is given in the following proposition.

Proposition 2 *If $0 \leq n' < n < \overline{n}$, the optimal migration policy for a mass of individuals $\overline{n} - n$ is described by the following flat curve (Figure 4):*

$$\theta^{**}(n) = \theta^{*}(n') = \theta^{*}(\overline{n}) \equiv \frac{\beta_1}{\beta_1 - 1}(\rho - \alpha)\frac{K}{u(\overline{n})}, \tag{17}$$

Proof. See Moretto (2003) and the Appendix ■

Thus starting at n, if the initial shock is below the known trigger $\theta^{*}(\overline{n})$, all the migrants wait until θ rises to this level, and then *coordinate* their entry to bring the size to the optimal level n. Working back towards n', it is verified for every n, as long as $\theta^{*}(n')$ is equal to $\theta^{*}(\overline{n})$. In fact, if it were $\theta^{*}(n') > \theta^{*}(\overline{n})$, it could be convenient to delay entry until $\theta^{*}(\overline{n})$, because of a higher obtainable benefit. Once the optimal size is reached and to the right of $\overline{n}$, further decision to enter proceeds as explained in the previous section without externalities. Intuitively, starting at any $n' < n < \overline{n}$, Proposition 2 locates the optimal entry threshold so as to maximise the total benefits of the incremental number of members that enter $(\overline{n} - n)$. The shock value $\theta^{*}(\overline{n})$ that triggers this individual's *competitive run*[18] is the same threshold that justifies a further marginal entry under decreasing benefits.

5 Calibration

To simulate the optimal migration policy we need values for the variables and parameters in equation (14). We could then calculate (16) and (17) and then

[17] We exclude the former by using subgame-perfectness arguments (see Moretto (2003)).
[18] The term *competitive run* refers to Bartolini's definition (1993).

solve for n^*. To perform this calibration we use the migration flows for Albanians, Filipinos, Chinese and Romanians and the wage levels (deflated using the IRES deflator) obtained from the ISTAT database. As we show below, determining values for most of the model's inputs is reasonably straightforward. Estimating the coefficients of the labour demand's stochastic process θ and of the Bass probability of integration a and b is more complex as will be discussed below.

5.1 Basic inputs

The parameters to be calibrated are listed in Table 1: for the discount rate we have used a basic level $\rho_2 = 0.03$ (Nordhaus,1996) and a higher level $\rho_1 = 0.05$. We also add a mortality rate $\lambda = 0.001$ calculated by the Istituto Superiore della Sanità[19] on ISTAT data.

According to assumption 2 and 3, the differential wage is assumed to be a constant percentage of the wage of the host country and varies whether the immigrant is completely integrated in the host country or not: ϕ_i and ϕ_i' respectively. The percentage for complete integration ϕ_i has been calibrated considering the GDP per capita based on Purchasing Power Parity of initial year 1993, as listed in the International Comparison Programme database of the World Bank. If the immigrant is not integrate, he earns only a fraction ξ of the wage. We have calibrated ξ and then the corresponding percentage ϕ_i' referring to the works of Massey (1987), Borjas (1990) and Chiswick (1978)[20]. The resulting $\phi_i^{'}$ and ϕ_i are shown in Table 2.

5.2 Demand volatility

The lack of studies on Italy's demand function for immigrants[21] obliges us to refer to US works for the level of labour demand elasticity and standard deviation of the stochastic shock θ. For the US we have many papers that try to estimate the peculiar effect of entering immigrants on the labour wages. On this basis we use two representative values: $\zeta_1 = -0.2$ (Borjas, 1990) and

[19] www.iss.it

[20] In particular Massey estimates that the illegal wage is 63% of the legal wage. Borjas and Chiswick show that the entry wage for each immigrant is 79% or 85 % of the native one respectively.

[21] The only work we have found shows a positive elasticity (Venturini (1999)) that the author justifies as a short-term effect.

$\zeta_2 = -0.02$ (Borjas, 1994). These two values are in line with the literature[22]. To calculate an estimate of the variance of θ, we used the boot-strap method, obtaining two levels of variance σ_1 and σ_2 for each flow, corresponding respectively to $\zeta_1 = -0.2$ and $\zeta_1 = -0.02$. These values are reported in Table 3.

5.3 Bass parameters

Finally for the parameters of the Bass model (i.e. a, b, m), we applied a recursive method proposed by Bass (1969, p. 224) using the years 1996, 1997 and 1998 as initial conditions. The results are described in Table 3. Simple observations show that in all the cases the coefficient b is greater than a, which guarantees the concavity of the Bass function $Bass(n)$.

6 Results

To compare the different migration inflows we simulate the optimal trigger levels (16) and (17), for the four migration waves, in the case of elasticity levels -0.02 and -0.2. Because of the difficulty of perfectly quantifying the migration costs, we have normalised K to the same arbitrary level for all cases[23]. The principal results are shown in Figures 4 and 5 and are displayed in Table 4.

[22] Borjas (1994) reviewing the literature argues that the value of the elasticity should be between -0.01 and -0.06. Dos Santos (2000) affirms that "from an empirical point of view, many studies attempt to estimate the impact of immigration on wages. The elasticity of wages with respect to the number of immigrants is generally found to be between -0.01 and -0.02". Garson (1987), using 1985 data, coming from ISEE, finds a level of elasticity between -0.01 and -0.04. Borjas (1990), using data from the US census of 1990, shows a level around -0.2 and in a recent paper (2003) he obtains an elasticity around -0.33. Antonji and Card (1991), using data of the US census 1970-1980, finds a level of -0.3.

[23] This permits comparison of the timing and the "behavioural rationale" among the migration inflows.

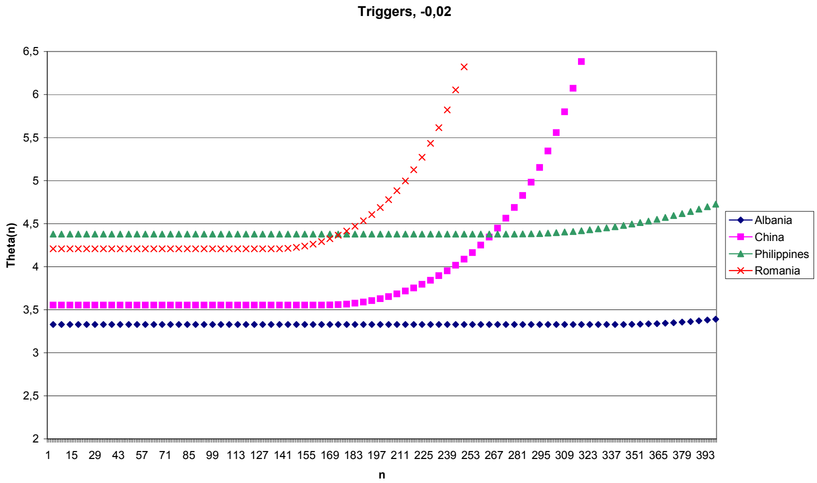

Figure 4: Optimal triggers level for $\zeta_2 = -0,02$

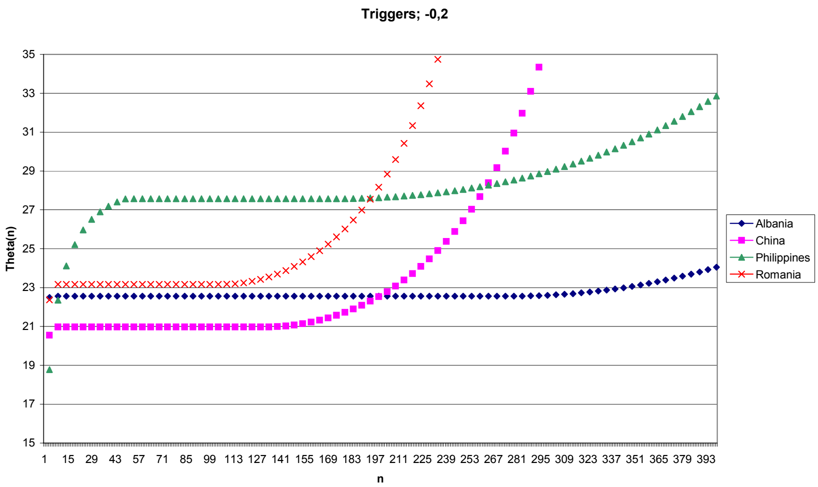

Figure 5: Optimal triggers level for $\zeta_1 = -0,2$

Some remarks are in order:

1. In all the ethnic groups, the wave starts when the network size, $(\frac{n^*}{m})$, reaches 30% or 40% of the critical saturation level m, for $\zeta_1 = -0.2$ and $\zeta_2 = -0.02$ respectively. Yet, the lower the elasticity level the

greater the wave dimension, that is, as market competition increases the network effect and the ties among immigrants reduce and they seem unable to perfectly coordinate entry.

2. The higher the value of the elasticity, the higher the threshold level θ^* and the lower the migration flow. This fact depends on the sum of two effects: (i) the *labour market competition*, increasing with the absolute level of ζ; (ii) the *network effect* that depends on the probability of being completely integrated (i.e. Bass function). The combined effect defines the magnitude of the benefit perceived by every migrant in the host country. On the one hand, a high number of incumbent immigrants increases the total benefit due to the network effect. On the other hand, however, low wave dimensions require a high shock to trigger entry.

3. A higher ρ magnifies the optimal trigger as expected.

4. The highest flows observed in the data are consistent with the predictions of the model (i.e. n^* with respect to n_1997 in Table 4): the real wave is between the upper and the lower simulated flow in every case studied.

5. The higher ϕ_i' or ϕ_i, the lower the entry trigger θ^* as expected.

In particular it emerges that the Albanian flow is the first to start in the case of low demand elasticity and the second in the case of high elasticity. This happens just behind the Chinese flow (the second and the first, respectively), with wide jump dimensions. Nevertheless, since the historical timing of the entries shows that the Chinese flow is more recent than the Albanian one, the level of elasticity on the labour market might be near to ζ_2[24].

The timing of the migration phenomenon depends also on the particular ethnic characteristics summarised in the Bass parameters: the higher the imitator's parameter b, the earlier the migration starts. This is due to a high network effect that offsets labour market competition with a larger wave dimension. Moreover, the higher the innovator's parameter a, the lower the ties among immigrants and the higher the number of first entries. This can explain the differences in behaviour among the four migration inflows

[24] We remember that, since the labour demand shock is depicted as a Brownian motion (2), the higher the threshold level, the longer the time elapsed.

observable in Figures 4 and 5. In fact, the Filipinos, characterised by a strong individualist behaviour, show a magnified first entry but a reduced jump size; vice versa, the Chinese, the Albanians and the Romanians are characterised by higher imitator parameters and a higher wave.

6.1 Entry costs

So far, we have compared the different entry triggers based on normalised sunk costs K. This normalization allowed the Bass model to describe the migration behaviour of the flows, by defining the percentage of innovators and imitators in each flow, thus capturing the implicit signals that drive the waves.

We can now step back and, following the theory, "quantify" the entry costs faced by the four different ethnic groups by inverting (16) and (17) and evaluating them at their minimum level[25]. The results are in Table 5 from which we can conclude two main insights: (1) The geographical distance is not the focal element of the sunk costs, as generally stressed in economic literature. In fact, the Philippines and Romania face a similar K, like Albania and China. This fact implies that the sunk cost faced by the immigrant must be a wider basket of socio-economic elements; (2) it is important to stress that the sunk costs displayed correspond to the optimal threshold: in all the cases, since the migration occurred in the same year (i.e. 1997), the migrants entered a labour market with the same shock magnitude. This fact means that all the ethnic immigrants gained a similar labour market benefit but faced different costs: for the same level of wages some ethnic groups were able to face higher costs. Which element made the difference? The answer is in the "behavioural rationale": high cooperative behaviour helps each individual to face a higher cost. Therefore, the timing of the entry should be inversely related to the sunk cost in the optimum, i.e. Albania first, China, Romania and then Philippines. Comparing this rank with Figures 3 and 4, it appears that the true labour demand elasticity should be nearer to -0.02.

[25] In fact, if the jump starts when the trigger reaches the minimum level, we can take the value of the observed flows (see the 7^{th} column in the Table 4) and the level of the wage in the year of the peak and substitute these values in the following equation:

$$K^* = w^* \frac{\beta_1 - 1}{\beta_1 (\rho - \alpha)} \left[\frac{m - \overline{n}}{m} \phi_i' + \frac{Bass(\overline{n})}{(\rho - \alpha)} \phi_i \right]$$

6.2 Saturation level

Although the simulations appear to be consistent with the ISTAT data between 1994 and 2000, we wish to check whether the model is also consistent over time, by displaying the results of the 2004 CARITAS migration report in Table 6. According to our model, the Chinese community should be near to saturation level, but this fact does not correspond to the current data by CARITAS that shows an increase in Chinese immigration waves.

We suggest two explanations for this. First, our analysis uses the whole national migration flow as a single community, and this surely overestimates the alienation effect. We should consider single regional homogeneous ethnic groups and extend the model in this way. Secondly, due to the particular method used to calibrate the Bass parameters, the critical saturation level m is strongly time-dependent. To overcome this problem we have calibrated the Bass coefficients one and two steps ahead displaying the changes in "behavioural rationale" of this procedure.

6.2.1 "*One-Two-step-ahead*"

In the Bass methodology m depends on the years (initial conditions) used to calculate the parameters a and b. In particular we have used the years 1996, 1997 and 1998. We now repeat the analysis by using the years 1996, 1997 and 1999 and then 1996, 1997 and 2000. Values for a, b and m are reported in Table 7 and 8 respectively.

In Figure 6 we show three curves for the Albanian triggers. $theta98$ is the benchmark case calibrated with the years 1996, 1997 and 1998, $theta99$ with 1996, 1997 and 1999 and finally $theta00$ with 1996, 1997 and 2000.

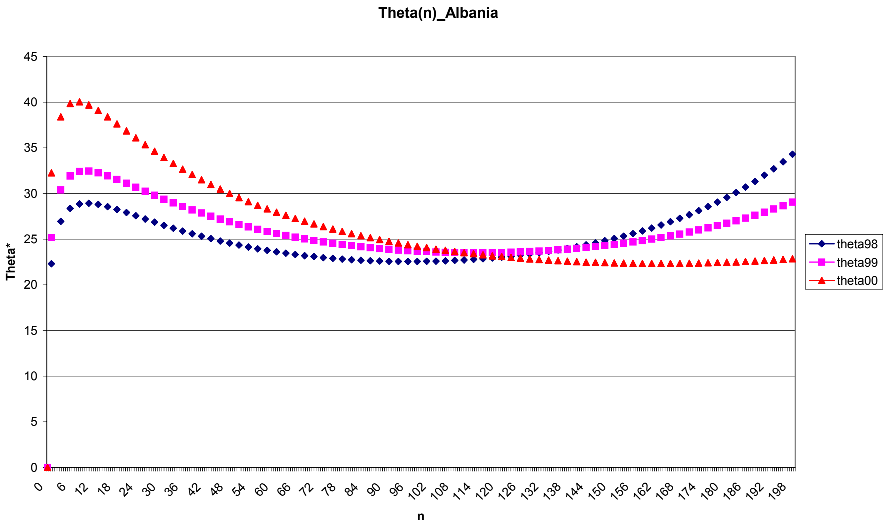

Figure 6

Moving *ahead* the last year in calibrating the Bass parameters results in a substantial change in the shape of the entry trigger functions. In particular, $m(t)$ increases from $theta98$ to $theta00$ which implies, *ceteris paribus,* an increase in the size of the jump. Yet, the network effect is magnified, diluting the innovators' weight (this is why θ^* increases for $n \rightarrow 0$).

The higher the imitators' coefficient, the greater the perceived saturation dimension will be [26]. Therefore, if an ethnic group has strong ties, its community will probably increase more than other groups, *ceteris paribus.*

Finally, comparing Tables 3, 7 and 8, we can highlight how the "behavioural rationale" changes for the four flows: the Chinese, Albanians and Romanians become more cooperative, whereas the Filipinos seem to remain more individualist. This can explain why some communities tend to explode and others increase at a constant rate.

7 Conclusions

This paper has tried to explain why migration flows are characterised by some observable jumps in their processes. Real option theory suggests that

[26] Comparing the Filipino flow to the Albanian flow we notice that the Albanian growth rate in the saturation level is higher than the Filipino one (see Tables 7 and 8).

migration may be delayed beyond the Marshallian trigger since the option value of waiting may be sufficiently positive in the face of uncertainty. Intuition is that waiting may resolve uncertainty and thus enable avoidance of the downside risk of an irreversible investment. Burda (1995) was the first to use real option theory to explain slow rates of migration from East to West Germany despite a large wage differential. Subsequent works (Khwaja, 2002; Anam et al., 2004) have developed this approach describing the role of uncertainty in the migration decision. Recent papers (Moretti, 1998; Bauer et al., 2004) show, however, that the role of the community is important in the migration decision. In this paper, we have presented a real option model where the choice to migrate depends on the differential wage and on the probability of being integrated into a host country. The corresponding integration probability is modelled following the Bass model (1969) where the "behavioural rationale" of the migration flows is captured by two kinds of immigrants: *innovators* or individualists and *imitators*. The weight of each different type influences the timing of migration and the size of the community. The closer the ties among the individuals, the higher the dimension of the wave and the higher the entry cost faced, *ceteris paribus*.

Furthermore, we observe two opposing forces that influence entry: on the labour market side, strong *competition* among workers in the host country delays entry; at the same time, the higher the numbers of immigrants, the higher the *network effect* that reduces the optimal threshold and anticipates the entry.

Simulations of some migration flows to Italy in the last century fit the theoretical approach and are able to replicate the observable migration jumps at least in the short-run. The model is able to conjecture the induced labour demand elasticity level of the host country and the "behavioural rationale" of the migrants. Nevertheless, the use of the national flows, as a proxy for the size of the communities, probably overestimates the results, suggesting future disaggregation of the ethnic flows.

A Appendix

This appendix is dedicated to proving propositions 1 and 2 in the text. To do this we rely on the works of Leahy (1993), Bartolini (1993), Dixit and Pindyck (1994) for the case of negative externalities (i.e. a competitive economy), and Moretto (2003) for the cooperative entry under positive externalities. The results presented by these authors can be applied with only minor modifications to the problem at hand. In particular, the special structure of the immigrant community considered leads to an important simplification of the analysis.

To determine the migrant's optimal entry policy, the first thing to do is to find his value of being perfectly integrated given each individual's optimal future entry policy. A solution of (8) can be obtained starting within a time interval where no entry occurs $(n, \theta < \theta^*)$. By the typical methodology of real options, we are able to obtain the general solution for (8) as (Dixit and Pindyck, 1994, p.181):

$$V(n,\theta) = A_1(n)\,\theta^{\beta_1} + A_2(n)\,\theta^{\beta_2} + v(n,\theta) \tag{18}$$

where $1 < \beta_1 < \rho/\alpha,\ \beta_2 < 0$ are, respectively, the positive and the negative root of the characteristic equation $\Psi(\beta) = \frac{1}{2}\sigma^2\beta(\beta-1) + \alpha\beta - \rho = 0$, and $A_1,\ A_2$ are two constants to be determined.

To keep $V(n,\theta)$ finite as θ becomes small, i.e. $\lim_{\theta\to 0} V(n,\theta) = 0$, we discard the term in the negative power of θ setting $A_2 = 0$. Moreover, the boundary conditions also require $\lim_{\theta\to\infty}\{V(n,\theta) - v(n,\theta)\} = 0$, where the second term in the limit represents the discounted present value of the benefit flows over an infinite horizon starting from θ with n fixed. By (13) we get:

$$v(n,\theta) = \frac{m-n}{m}\frac{\phi'\theta n^{\zeta}}{\rho-\alpha} + [a + \frac{b-a}{m}n - \frac{b}{m^2}n^2]\frac{\phi\theta n^{\zeta}}{(\rho-\alpha)^2} \tag{19}$$

Recalling that $u(n) = n^{\zeta}\left[\frac{m-n}{m}\phi' + \frac{Bass}{\rho-\alpha}\phi\right]$ and $Bass(n) \equiv [a + \frac{b-a}{m}n - \frac{b}{m^2}n^2]$, the general solution of (18) becomes:

$$V(n,\theta) = A_1(n)\theta^{\beta_1} + \frac{\theta u(n)}{\rho-\alpha} \tag{20}$$

It is worth noting that for $a \leq b$ the function $u(n)$,is shaped according to the Figure 3. Since the last term represents the value of being in the community in the absence of new entry, then $A_1(n)\theta^{\beta_1}$ must be the correction due to the new entry, therefore $A_1(n)$ must be negative. To determine this coefficient for each n, we need to impose some suitable boundary conditions. First of all, free entry requires the (idle) migrant to expect zero benefits on entry. Then, indicating with $\theta^*(n)$ the value of the shock θ at which the n^{th} individual is indifferent to immediate entry or waiting for another opportunity, the condition (9) in the text (*matching value condition)* becomes:

$$V(n,\theta^*(n)) \equiv A_1(n)\theta^*(n)^{\beta_1} + \frac{u(n)\theta^*(n)}{\rho-\alpha} = K \tag{21}$$

Secondly, the number of migrants n affects $V(n,\theta)$ depending on the sign of $\theta^*(n)$. Since θ^{β_1} is always positive, any change in n either raises or lowers the whole function $V(n,\theta)$, depending on whether the coefficient $A_1(n)$ increases or decreases. Therefore, by totally differentiating (21) with respect to n we obtain:

$$\frac{dV(n,\theta^*(n))}{dn} = V_n(n,\theta^*(n)) + V_\theta(n,\theta^*(n))\frac{d\theta^*(n)}{dn} = 0 \tag{22}$$

Thirdly, since each individual rationally forecasts the future path of new entries by competitors, if θ moves to $\theta^*(n+dn)$, the value of a migrant conditional on n active immigrants must be equal to the value with $n+dn$ active, i.e. $V(n,\ \theta^*(n+dn)) = V(n+dn,\theta^*(n+dn))$. Taking the difference between these two, dividing by dn and taking the limit for $dn \to 0$, we get $V_n(n,\theta^*(n)) = 0$ (Bartolini, 1993, proposition 1)[27]. This reduces (22) to:

$$V_\theta(n,\theta^*(n))\frac{d\theta^*(n)}{dn} = 0 \tag{23}$$

In conjunction with the (21), the above *extended smooth pasting condition* says that either each migrant exercises his entry option at the level of θ at which his value is tangent to the entry cost, i.e. $V_\theta(n,\theta^*(n)) = 0$, or the optimal trigger $\theta^*(n)$ does not change with n. While the former means that the value function is smooth at entry and the trigger is a continuous function of n, [28] the latter case says that, if this condition is not satisfied,

[27] Note that this is a generalisation of the condition in Dixit (1993, p. 35). If the migrant pretends to be unique or the last entering the host country, then $u'(n) = A'(n) = 0$ and the first order condition reduces to $V_\theta(n,\theta^*(n)) = 0$

[28] Moreover, as we assumed that the individual's size is infinitesimal, then the trigger level $\theta^*(n)$ is also a continuous function in n.

a single individual would benefit from marginally anticipating or delaying his entry decision. In particular if $V_\theta(n, \theta^*(n)) < 0$ it means that the value of staying in the host country is expected to increase if θ falls (investing now will be expected to lead to almost certain benefits), on the contrary if $V_\theta(n, \theta^*(n)) > 0$ it means that a member would expect to make losses because of a decrease in θ. In both situations (23) is satisfied by imposing $\frac{d\theta^*(n)}{dn} = 0$, therefore the same level of shock may either trigger entry by a positive mass of migrants or lock-in the community at the initial level of members.[29]

The rest of the proof is devoted to showing that, for the $n \leq n'$ and for the $n \geq \overline{n}$, the *smooth pasting condition* reduces to the traditional one where $V_\theta(n, \theta^*(n)) = 0$ and $\theta^*(n)$ is increasing in n while, for $0 \leq n' < n < \overline{n}$, $V_\theta(n, \theta^*(n)) > 0$ which implies $\frac{d\theta^*(n)}{dn} = 0$.

It should be noted that using (20), (21) and (23), it is possible to find the optimal threshold function. The solution depends on the concavity of $u(n)$. As we have seen in the previous part, a generic representation of $u(n)$ distinguishes three intervals for the particular shape of the benefit function. Let's solve the model backwards.

A.1 Proof of Proposition 1

For the case of $n \leq n'$ or $n \geq \overline{n}$ we show two things: (i) the smooth pasting condition (23) reduces to $V_\theta(n, \theta^*(n)) = 0$; (ii) the optimal trigger $\theta^*(n)$ is equivalent to that of an individual in isolation, that is of a migrant pretending to be the last to migrate.

For (i), let's consider the value of a migrant being in the host country starting at the point $(n, \theta < \theta^*)$, and subject to the possibility of new entries when θ hits θ^*. Indicating with T the first time that θ reaches the trigger θ^*,

[29] If this condition does not hold, the expected benefit gain or loss at $\theta^*(n)$ would be infinite due to the infinite variation property of the stochastic process θ.

the optimal entry policy must then satisfy:

$$\begin{aligned}
V(n,\theta) &= \max_{\theta^*} E_0\left[\int_0^T e^{-\rho t}\left\{(1-F_\tau(t))\,\phi'\theta(t)n^\zeta + f_\tau(t)B\,(n,\theta(t))\right\}dt+\right. \qquad (24)\\
&\quad \left. + \int_T^\infty e^{-\rho T}\left\{(1-F_\tau(t))\,\phi'\theta(t)n(t)^\zeta + f_\tau(t)B\,(n(t),\theta(t))\right\}dt\right]\\
&= \max_{\theta^*} E_0\left[\int_0^T e^{-\rho t}\left\{\frac{m-n}{m}\phi'\theta(t)n^\zeta + [a+\frac{b-a}{m}n - \frac{b}{m^2}n^2]\phi\frac{\theta(t)n^\zeta}{\rho-\alpha}\right\}dt+\right.\\
&\quad \left. + e^{-\rho T}V\left(n,\theta^*\left(n\right)\right)\right]
\end{aligned}$$

where $V(n,\theta^*(n))$ represents the optimal continuation value of staying in the host country. Because, by (21), the present value of benefits at T is K, the above value can be written as:

$$V(n,\theta) = \max_{\theta^*}\left[u(n)E_0[\int_0^T e^{-\rho t}\theta(t)dt] + KE_0[e^{-\rho T}]\right]$$

or, after some simplifications:

$$V(n,\theta) = \max_{\theta^*}\left[\frac{u(n)\theta}{\rho-\alpha} - \left(\frac{u(n)\theta^*}{\rho-\alpha} - K\right)\left(\frac{\theta}{\theta^*}\right)^{\beta_1}\right] \qquad (25)$$

The value of being perfectly integrated (25) is the difference between the value of a migrant with a myopic strategy pretending to be the last to have to migrate $\frac{u(n)\theta}{\rho-\alpha}$ and the value of an idle individual pretending to be the last to migrate as expressed by $\left(\frac{u(n)\theta^*}{\rho-\alpha} - K\right)\left(\frac{\theta}{\theta^*}\right)^{\beta_1}$. To choose optimally θ^*, the first order condition is:

$$\frac{\partial V}{\partial \theta^*} = \left[(\beta_1 - 1)\frac{u(n)}{\rho-\alpha} - \beta_1\frac{K}{\theta^*}\right]\left(\frac{\theta}{\theta^*}\right)^{\beta_1} = 0 \qquad (26)$$

and the optimal threshold function takes the form:

$$\theta^*(n) \equiv \frac{\beta_1}{\beta_1 - 1}\left(\rho-\alpha\right)\frac{K}{u(n)}, \quad \text{with } \frac{\beta_1}{\beta_1-1} > 1 \qquad (27)$$

Since $u(n)$ is decreasing in the interval $[\overline{n}, m]$, $\theta^*(n)$ is increasing. Moreover, substituting (27) into (25) we can solve for $A(n)$ which is negative as required by (20):

$$A(n) = -\frac{[\theta^*(n)]^{1-\beta_1}}{\beta_1\left(\rho-\alpha\right)} < 0 \qquad (28)$$

Finally, substituting (28) into (25) and rearranging we obtain (20):

$$V(n,\theta) = A(n)\theta^{\beta_1} + \frac{u(n)\theta}{\rho-\alpha} \equiv -\frac{[\theta^*(n)]^{1-\beta_1}}{\beta_1(\rho-\alpha)}\theta^{\beta_1} + \frac{u(n)\theta}{\rho-\alpha} \tag{29}$$

from which it is easy to verify that $V_n(n,\theta) \neq 0$ within the interval $\theta < \theta^*(n)$ and zero at the boundary.

Now for (ii), let's suppose that all individuals have decided to enter at $\hat{\theta}$, with $\theta^* < \hat{\theta}$. This cannot be a (Nash) equilibrium because a single migrant can do better by entering at θ^*. In fact, the flow of benefits that each individual is able to obtain following the policy θ^* is the best that he can do, at least till T. However, by the principle of optimality, this choice is also optimal for the rest of the period as (24) shows: if the optimal policy of the single migrant calls for him to be active at $\hat{\theta}$ tomorrow, it immediately follows that the optimal policy today is to enter at θ^*. As (24) is a continuous function in θ^*, the limit as $\hat{\theta} \to \theta^*$ shows that θ^* is a Nash equilibrium (Leahy, 1993, proposition 1).

If the elasticity is not too low we obtain an interval $n \in (0, n')$ where the competitive effect prevails over the network effect. Therefore, by the above result, within the interval $(0, n')$ the optimal threshold is still given by (27) until n'. Finally, for $\zeta \to 0$, $n' \to 0$.

A.2 Proof of Proposition 2

For $0 \leq n' < n < \overline{n}$ we have to show three things: (i) that a single individual cannot pretend to be the last to migrate and, therefore, the optimal competitive trigger is no longer equivalent to that of a migrant in isolation; (ii) that the candidate policy, described in the proposition 2, satisfies the necessary and sufficient conditions of optimality; (iii) that it is a sub-game perfect equilibrium[30].

Let assume that $u(n)$ is U-shaped as in the quadrant III of figure 3. For (i) and (ii), let's begin with an idle individual that follows the optimal policy $\theta^*(n)$. Since $\theta^*(n)$ is decreasing in the interval $n < \overline{n}$: the higher the number of members in the community the greater his entry value. In other words, an idle migrant would maximise his entry option by pretending to be always the last to migrate. In fact a migrant that pretends to be the last to enter expects an inadmissible upward jump in benefits following the policy $\theta^*(n)$. To see

[30] See Moretto (2003) for a conjecture of how this can be proved.

this, consider an individual that pretends to have been the last to enter at $\theta = \theta^*(n)$; by (19) his value is simply $V(n, \theta^*(n)) \equiv v(n, \theta^*(n)) = \frac{u(n)\theta^*(n)}{\rho-\alpha}$. Then it is easy to check that:

$$V(n, \theta^*(n)) - \lim_{\theta \to \theta^*(n)} V(n, \theta) = \frac{\theta^*(n)}{\beta_1 (\rho - \alpha)} > 0 \tag{30}$$

In (30) the inequality holds since it represents the correction due to the new entry (i.e. $A(n)\theta^{\beta_1}$ in (20)). This contradicts the *smooth pasting condition* $V_\theta(n, \theta^*(n)) = 0$ and then the optimality of $\theta^*(n)$.

To verify that the necessary conditions are satisfied, let's calculate the value of an (incumbent) immigrant in the host country starting at the point (n, θ), that would follow a policy defined by two parameters: wait until the first instant T at which the process θ rises to a level $c > \theta$, corresponding to an immediate increase in the community size to $b > n$. Making use of (24) the expected payoff $V(n, \theta)$ from this policy is equal to:

$$\begin{aligned} V(n, \theta; b, c) &= E_0 \left[u(n) \int_0^T e^{-\rho t} \theta_t dt + e^{-\rho T} V(b, c) \right] \\ &= \frac{u(n)\theta}{\rho - \alpha} - \left[\frac{u(n)c}{\rho - \alpha} - V(b, c) \right] \left(\frac{\theta}{c} \right)^{\beta_1} \end{aligned} \tag{31}$$

If each individual were able to choose the best moment for the community's size as well as the dimension of the jump, the first order condition would be:

$$\frac{\partial V(n, \theta; b, c)}{\partial c} = \left[(\beta_1 - 1) \frac{u(n)}{\rho - \alpha} - \beta_1 \frac{V(b, c)}{c} + \frac{\partial V(b, c)}{\partial c} \right] \left(\frac{\theta}{c} \right)^{\beta_1} = 0$$

$$\frac{\partial V(n, \theta; b, c)}{\partial b} = \frac{\partial V(b, c)}{\partial b} \left(\frac{\theta}{c} \right)^{\beta_1} = 0$$

When b and c are chosen according to the candidate policy so that $b = \overline{n}$ and $c = \theta^*(\overline{n})$ the value function reduces to (20) and the *matching value condition* requires $V(b, c) = K$. These properties verify that the candidate policy satisfies the above first order conditions.

By processing (30) we can say more about the necessary conditions. Let the immigrant, as in (31), wait until the first time the process θ rises to the trigger level $c \equiv \theta^*(b)$, corresponding to an immediate increase of the network size to $b > n$, and assume also that he expects no more entry after b. Therefore his expected payoff $V(b, \theta)$ from this time onwards equals the discounted stream of benefits fixed at $u(b)$, i.e. by (19):

$$V(b, \theta) = \frac{u(b)\theta}{\rho - \alpha} \tag{32}$$

Comparing (32) with (20) gives $A_1(b) = 0$. Therefore to obtain the constant $A_1(n)$, subject to the claim that beyond b no other immigrants will enter, we substitute (20) into the condition $V_n(n, \theta^*(n)) = 0$ to get $A_1'(n)\theta^*(n)^{\beta_1} + \frac{u'(n)\theta^*(n)}{\rho-\alpha} = 0$ resulting in:

$$A_1'(n) = -\frac{\theta^*(n)^{1-\beta_1} u'(n)}{\rho - \alpha} \equiv -\frac{(\pi^*)^{1-\beta_1}}{\rho - \alpha} \frac{u'(n)}{u(n)^{1-\beta_1}} \tag{33}$$

Integrating (33) between n and b gives:

$$\int_n^b A_1'(x)dx = -\frac{(\pi^*)^{1-\beta_1}}{\rho - \alpha} \int_n^b \frac{u'(x)}{u(x)^{1-\beta_1}} dx$$

Taking account of the fact that $A_1(b) = 0$, the above integral gives the constant $A_1(n)$ as:

$$A_1(n) = \frac{(\pi^*)^{1-\beta_1}}{\beta_1(\rho - \alpha)} \left[u(b)^{\beta_1} - u(n)^{\beta_1}\right] \tag{34}$$

Substituting (34) into (20), which we rewrite to make explicit its dependence on the end size b, yields:

$$V(n, \theta; b, \theta^*(b)) = \frac{(\pi^*)^{1-\beta_1}}{\beta_1(\rho - \alpha)} \left[u(b)^{\beta_1} - u(n)^{\beta_1}\right] \theta^{\beta_1} + \frac{u(n)\theta}{\rho - \alpha} \tag{35}$$

As long as $u(b) > u(n)$ the first term in (35) is positive and it forecasts the advantage the immigrant would experience by the entry of $b - n$ new immigrants when θ hits $\theta^*(b)$. That is, if he were able to choose the optimal dimension of the jump, it would be $b \to \bar{n}$ which happens the first time that θ reaches $\theta^*(\bar{n})$. Thus, as opposed to before non-sequential entry are possibile, the necessary conditions would coordinate an optimal simultaneous entry by $\bar{n} - n$ new immigrants. If $u''(n) < 0$ the necessary conditions are also sufficients. Furthermore, substituting (35) into the *extended smooth pasting condition* (23) and letting $b \to \bar{n}$, we obtain:

$$\left[\frac{(\pi^*)^{1-\beta_1}}{\beta_1(\rho-\alpha)}\left[u(\bar{n})^{\beta_1}-u(n)^{\beta_1}\right]\beta_1 \theta^{*\beta_1}+\frac{u(n)}{\rho-\alpha}\right]\frac{d\theta^*}{dn}=0 \tag{36}$$

The term inside square brackets is always positive (i.e. there is no value $n^\circ \in (n, \bar{n})$ that makes it nil), and (36) holds with $\frac{d\theta^*}{dn} = 0$. That is, all immigrants in the range $(n, \bar{n})$ must enter at $\theta = \theta^*(\bar{n})$.

In other words, as the stochastic process θ is common knowledge, each immigrant can foresee the benefit from the entry of others and observing the realization of the state variable θ instantaneously considers when to enter by maximizing (35). Then, with simultanous entry, the immigrants' optimal strategies are easy to find: each individual enter as if he is the only to enter but with the expectation of earning all the network benefits, i.e. $\theta^*(\bar{n})$ is a (symmetric) Pareto-dominant Nash equilibrium for all $n < \bar{n}$ (see Moretto 2003). In addition, as the reaction lags are literally nonexistent, none has the incentive to deviate from the entry strategy $\theta \to \theta^*(\bar{n})$ and $b \to \bar{n}$ given that the others do not deviate. Finally, since θ is a Markov process in levels (Harrison, 1985, p.5-6), the conditional expectation (31) is in fact a function solely of the starting states so that, at each date $t > 0$, the immigrant's values resemble those described in (35) which makes the equilibrium subgame perfect.

Finally, it is easy to deal with the general case (quadrant I and II in Figure 3). If $n \in (n', n")$ we need first to find a network size n° such that $u(n) = u(n^\circ)$ with $u'(n^\circ) > 0$ and then to perform the same policy as in (35) or (36). That is, the optimal entry would be of $(\bar{n}-n^\circ)+(n^\circ-n)$ immigrants the first time that θ reaches $\theta^*(\bar{n})$.

B Tables

Parameter	Description	Symbol	Source
Discount rate		ρ_1=0,03 ρ_1=0,05	Nordhaus (1996)
Elasticity	Labour demand elasticity	ζ_1=-0,2 ζ_2=-0,02	Borjas (1990) Borjas (1994)
Wage diferential	ϕ_i=(1-w°/w)	ϕ_i	World Bank
Wage differential	ϕ'_i=(ξ-w°/w)	ϕ'_i	World Bank
Entry salary	Average level	ξ	Chiswick (1978)
			Borjas (1990)
			Massey (1987)

Table 1: Parameters

Country	ϕ'_i	ϕ_i
Albania	0,639	29/33
China	0,649	8/9
Philippines	0,593	5/6
Romania	0,510	3/4

Table 2: ϕ_i and ϕ'_i calibration

	Albania	China	Philippines	Romania
σ_1	0,063	0,061	0,057	0,047
σ_2	0,055	0,055	0,056	0,054
b	0,973	0,850	0,648	0,828
a	0,110	0,117	0,141	0,123
m	0,274	0,138	0,256	0,115

Table 3: The Bass parameters respect to 1996, 1997 and 1998

Parameters	θ^*	n'/m	n'	n^*/m	n^*	n, 1997
			Albania			
0,05; -0,2	83,95	0,008	2275	0,33	91000	101634
0,05; -0,02	12,48	0,000	0	0,42	115150	
0,03; -0,2	22,56	0,004	1225	0,35	96600	
0,03; -0,02	3,33	0,000	0	0,43	118300	
			China			
0,05; -0,2	77,51	0,017	2375	0,30	42000	55352
0,05; -0,02	13,26	0,000	0	0,41	56000	
0,03; -0,2	20,97	0,008	1125	0,33	45500	
0,03; -0,02	3,55	0,000	0	0,42	58000	
			Philippines			
0,05; -0,2	98,81	0,000	0	0,10	25900	93837
0,05; -0,02	16,19	0,000	0	0,36	91700	
0,03; -0,2	27,58	0,066	16975	0,23	59850	
0,03; -0,02	4,38	0,000	0	0,38	96250	
			Romania			
0,05; -0,2	85,85	0,023	2625	0,29	34000	44413
0,05; -0,02	15,73	0,000	0	0,40	46250	
0,03; -0,2	23,17	0,012	1375	0,32	37000	
0,03; -0,02	4,21	0,000	0	0,41	47750	

Table 4: Main results

where:

- *parameters*: are respectively the discount factors (i.e. $\rho_1 = 0,05$; $\rho_2 = 0,03$) and the elasticity levels (i.e. $\zeta_1 = -0,2$; $\zeta_2 = -0,02$);
- θ^**:* represents the optimal trigger level at which the migration wave starts;
- n'/m: is the critical level that "*triggers*" the network effect as a percentage of the saturation dimension m;
- n^*/m: is the optimal dimension of the community in percentage of the theoretic maximum dimension m;
- n^*: is the level of the community that triggers the migration flow;
- n_year (i.e. n_1997) is the empirical jump observed in our data (see figure 1).

K^*	Albania	China	Philippines	Romania
$\rho_1 = 0{,}05$	1,29	1,21	1,00	1,01
$\rho_2 = 0{,}03$	1,30	1,22	1,00	1,02

Table 5: Different relative entry costs.

Country	Number of residents
Albania	0,234
China	0,100
Philippines	0,074
Romania	0,239

Table 6: Caritas report. Numbers of residents for millions of inhabitants, 2004.

t_1	Albania	China	Philippines	Romania
b	0,991	0,883	0,652	0,835
a	0,097	0,103	0,137	0,108
m	0,312	0,157	0,262	0,132

Table 7: The Bass parameters respect to 1996, 1997 and 1999

t_2	Albania	China	Philippines	Romania
b	1,194	1,077	0,667	0,952
a	0,073	0,086	0,122	0,059
m	0,412	0,189	0,294	0,242

Table 8: The Bass parameters respect to 1996, 1997 and 2000

References

[1] Anam, M., S. H. Chiang and L. Hua, (2004), "Uncertainty and International Migration: an Option cum Portfolio Model", mimeo

[2] Antonji, T. and D. Card, (1991), "The Effects of Immigration on the Labor Market Outcomes of Less-Skilled Natives", in John M. Abowd and Richard B. Freeman, eds. 1991, pp. 201-234

[3] Bartolini, L., (1993), "Competivive Runs: The Case of a Ceiling on Aggregate Investment", *European Economic Review*, Vol. 37, pp. 921-948

[4] Bass, F. M., (1969), "A New Product Growth for Model Consumer Durables", *Management Science*, 15/5, pp. 215-227

[5] Bauer, T. G., Epstein and I. N. Gang, (2002), "Herd Effects or Migration Networks? The Location Choice of Mexican Immigrants in the U.S.", *IZA Discussion Paper*, N. 200

[6] Bauer, T. and K. F. Zimmermann, (1997), "Network Migration of Ethnic Germans", *International Migration Review*, 1(1), pp. 143-149

[7] Borjas, G., J., (1990), "Friends and Strangers", Basic Books

[8] Borjas, G., J., (1994), "The Economics of Immigration", *The Journal of Economic Literature*, 32(4), pp. 1667-1717

[9] Borjas, G., J., (2003), "The Labor Demand Curve is Downward Sloping: Reexamining the Impact of Immigration on the Labor Market", *The Quarterly Journal of Economics,* 118 (4), pp. 1335-1374

[10] Boyd, M. (1989), "Family and Personal Networks in International Migration: Recent Developments and New Agendas", *International Migration Review*, 23(3), Special Silver Anniversary Issue: International Migration In Assessment for the 90's, pp. 638-670

[11] Burda, M. C., (1995), "Migration and the Option Value of Waiting", *The Economic and Social Review*, 27, pp. 1-19

[12] CARITAS, (2003), Dossier Statistico Immigrazione

[13] CARITAS, (2003), "Contemporary Immigration in Italy"

[14] Chiswick, B., R., (1978), "The Effect of Americanization on the Earnings of Foreign-Born Men", *Journal of Political Economy*, 86(5), pp. 897-921

[15] Coniglio, N., D, (2003), "Migropolis: Migration Networks and Formation of Ethnic Clusters in Cities", *SNF Working Paper* , 46/03

[16] Cox, D. R. and H. D. Miller, (1965), The Theory of Stochastic Process, London, Chapman and Hall

[17] Dixit, A. and R. S. Pindyck, (1993), "Investment under Uncertainty", Princeton (NJ): Princeton University Press

[18] Dos Santos, M. D., (2000), "Is the Unskilled Workers Immigration Necessarily Harmful for the Unskilled Natives?", mimeo

[19] Garson J. P., Moulier-Boutang Y., Silberman R., Magnac T., (1987), "La Substitution des Autochtones aux étrangers sur le Marché du Travail dans la CEE", CEE, Direzione Generale per l'occupazione, gli Affari Sociali e L'Educazione/Groupe de Recherche et d'Analyse des Migrations Internationales, mimeo

[20] Hart, R. A., (1975), "Interregional Economic Migration: Some Theoretical Considerations", *Journal of Regional Science*, 15(2), pp. 127-138

[21] Harrison J. M., (1985), "Brownian Motion and Stochastic Flow Systems", New York: John Wiley & Son

[22] Heitmueller, A., (2003) "Coordination Failures in Network Migration," IZA (Institute for the Study of Labor) Discussion Papers 770.

[23] Khwaja, Y., (2002), "Should I Stay or Should I Go? Migration Under Uncertainty: A Real Option Approach", mimeo.

[24] Istituto Nazionale di Statistica, (1965), "Sommario di Statistiche Storiche", Roma

[25] Istituto di Ricerche Economiche e Sociali, www.ires.it

[26] Istituto Superiore della Sanità, www.iss.it

[27] Langley, P.C., (1974), "The Spatial Allocation of Migrants in England and Wales", *Scottish Journal of Political Economy*, (21)3, pp. 259-77

[28] Leahy, J.P., (1993), "Investment in Competitive Equilibrium: the Optimality of Myopic Behavior", *Quarterly Journal of Economics*, 108, pp. 1105-1133

[29] Locher, L., (2002), "Testing for the Option Value of Migration", mimeo

[30] Massey, D., (1987), "Do Undocumented Migrants Earn Lower Wages than Legal Immigrants? New Evidence from Mexico", *International Migration Review*, 21(2), pp.236-274

[31] Massey, D., Arango, J., Hugo, G., Kouaouci, A., Pellegriono, A. and Taylor, J., (1993), "Theories of International Migration: A Review and Appraisal", *Population and Development Review,* 19, pp. 431-466.

[32] Moretti, E., (1999), "Social Networks and Migrations: Italy 1876-1913", *International Migration Review*, 33 (3), pp. 640-657.

[33] Moretto M., (2000), "Irreversible Investment with Uncertainty and Strategic Behavior", *Economic Modelling*, 17, 589-617.

[34] Moretto, M., (2003), "Competition and Irreversible Investments under Uncertainty", *Feem Nota di Lavoro* 32.03.

[35] Munshi, K., (2001), "Identification of Network Effects: Mexican Migrants in the U.S. Labor Market", paper presented at NEUDC conference, Boston, MA

[36] Munshi K., (2003), "Networks in the Modern Economy: Mexican Migrants in the U.S. Labour Market", *Quarterly Journal of Economics*, 118 (2), pp. 549-599

[37] Nordhaus, W. D. and Z. Yang, (1996), "A Regional Dynamic General-Equilibrium Model of Alternative Climate-Change Strategies", *American Economic Review*, 86(4), pp. 741-765.

[38] Sjaastad, L. A., (1962), "The Costs and Returns of Human Migration", *The Journal of Political Economy*, 70(5), Part 2, pp. 80-93.

[39] Todaro, P. M., (1969), "A Model of Labour Migration and Urban Unemployment in Less Developed Countries", *The American Economic Review*, 59 (1), pp.138-148.

[40] Winters, P., A. de Janvry and E. Sadoulet, (2001), "Family and Community Networks in Mexico-U.S. Migration", *The Journal of Human Resourches*, 36(1), pp. 159-184.

[41] World Bank, International Comparison Programme database.