Nominal Wage Indexation and Real Wage Dynamics*

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Abstract

In this paper we analyse the local dynamic consequences for real wages generated by some non-linearities in the indexation scheme for nominal wages. Following the lines traced out by Dehez and Fitoussi (1986), we interpret those non-linearities as the possibility of multiple "quasi-equilibria" as originally defined by Hansen (1951). Specifically, by assuming that the share of inflation caught out by nominal wages is $\cap$-shaped with respect to the lagged value of the real wage, we show that in case of deflation (inflation) the "quasi-equilibrium" with full employment may be stable (is unstable), while the "quasi-equilibrium" with involuntary unemployment is unstable (may be stable). Moreover, we perform a non-parametric empirical analysis aimed to derive some insights on the actual shape of wage indexation in the US. Finally, we derive and simulate a stochastic version of the model by taking into account the effects of nominal and real shocks.

JEL Classification: E24, E31

Keywords: Nominal Wage Indexation, Linkage Function, "Quasi-Equilibria" and Real Wage Dynamics

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1 Introduction

The issue of real wage rigidities has been often addressed by referring to implicit contracts (Azariadis, 1975), efficiency wage theories (Solow, 1979 and Shapiro and Stiglitz, 1984) and insider-outsider relationships (Lindbeck and Snower, 1989). However, real wage rigidity can be achieved by exploiting a less cumbersome framework having a relatively autonomous life with respect to the explanations mentioned above. Specifically, if there exits a level of the real wage such that the *nominal* wage is fully indexed to the price level, such a level of the real wage will result in being constant.

The implications of a nominal wage that moves one-to-one with the price level was pioneered by Bent Hansen in *A Study in the Theory of Inflation* (1951). There (Chapters VII and VIII) he defined the so-called "quasi-equilibrium", that is, an equilibrium in which both the price level and the nominal wage rise (fall) without interruption by keeping constant their ratio. Moreover, in this situation some demand (supply) excesses are not zero. Therefore, when the economy reaches this kind of equilibrium, the wage-price ratio, the mentioned demand (supply) excesses and along with them the speed of the rise (fall) in the price level and in the nominal wage are constant. This is the dynamic sense in which the "quasi-equilibrium" can be interpreted as a proper equilibrium.

In this paper we analyse the local dynamics implications for real wages generated by some non-linearities in the indexation mechanism for nominal wages. Our theoretical analysis follows the lines traced out by Dehez and Fitoussi (1986) and it is aimed to interpret those non-linearities as the possibility of multiple "quasi-equilibria"[1]. Specifically, by exploiting a 2-period OLG model in which the real wage is the only determinant of employment, we analyse the dynamic properties of different "quasi-equilibria", each of them characterised by a different labour market tightness. Therefore, by resuming the Hansen's (1951) arguments, our relevant excess demand (or supply) will be uniquely the one of the labour market[2].

Moreover, we provide a non-parameteric empirical analysis addressed the US aimed to describe the profile of nominal wage indexation by using a sample of data starting from the beginning of the 60s. Specifically, we try to find evidence of non-linearities in the share of inflation caught out by nominal wages with respect to "historical" indicators of labour market tightness.

Finally, aiming to explore the dynamic effects generated by nominal and real shocks, we derive and simulate a stochastic version of the model in which both kind of disturbances

[1]The possibility of multiple "quasi-equilibria" is addressed also in Solow and Stiglitz (1968).

[2]All the other open markets will be assumed to clear.

are parametrised with actual data.

This paper is arranged as follows. Section 2 describes the model and it derives its dynamic properties. Section 3 is devoted to empirical analysis. Section 4 builds and simulates the stochastic model. Finally, Section 5 concludes.

2 The Model

We consider a 2-period OLG model in which the real wage is the only determinant of employment. Specifically, our model includes three markets: the labour market, the consumption goods market and the market for money.

Given the 2-period OLG structure, young and old agents live together. Only young agents are allowed to work and they supply inelastically a fixed amount of labour services that are normalised to unity. However, those labour services may be not always fully employed: the amount of employed services depends on the labour demand arising from the firms which, in turn, univocally depends from the prevailing level of the real wage. For sake of simplicity, old agents do not leave bequests.

In this model the price level is determined competitively. Therefore, the market for consumption goods clears all the times. On the other side, the nominal wage is determined by a "linkage" function which is fixed at the beginning of each period. This function states that the current nominal wage is linked to the inflation rate and to the past history of the labour market. Given particular hypotheses on the evolution of prices, the "linkage" function can be exploited to derive a law of motion for real wages. Moreover, assuming some non-linearities in the share of inflation caught out by nominal wages, this law of motion will be supposed to have a multiplicity of stationary points. Obviously, each of them will indicate a different "quasi-equilibrium" with a particular labour market tightness[3].

The consumers' side of model is quite standard. Specifically, young consumers distribute their labour income between consumption and money holdings. On the other hand, old consumers finance their consumptions through the proceeding on money holdings and savings. Given that the stock of money is assumed to grow (shrink) at a constant rate, money grants a fixed nominal return (loss)[4]. Moreover, savings are invested in produc-

[3]Even if the nominal wage and price level grow with the same speed by keeping constant their ratio, the "quasi-equilibria" described in our model are *structurally* different from the stationary equilibria defined by Hansen (1951). The underlying differences will be pointed out whenever a stationary solution in the law of motion of real wages will be found.

[4]This means that new printed (annulled) money is distributed (taken away) directly to consumers

tive firms and they earn the net-of-wage payments augmented by a fixed nominal return aligned to the earnings on money.

The productive sector of our model economy is simple. Firms behave competitively by taking prices and wages as given. Specifically, the representative firm produces a perishable-homogeneous consumption good by employing the labour supplied by young agents but taking into the consideration that its labour demand (given the fixed supply) may be rationed by the prevailing level of the real wage.

Finally, the theoretical analysis is closed by studying the local dynamic properties of the different "quasi-equilibria" described by the proposed (non-linear) indexation formula.

2.1 The Consumer's Side

We assume that there is a unique agent for each generation. Thereafter, by taking as given the current price of the consumption good (p_t), the nominal wage (w_t) and labour demand (l_t), the representative consumer solves the following maximisation problem:

$$\max_{c_t, c_{t+1}} U(c_t, c_{t+1}) = c_t^{\delta} c_{t+1}^{1-\delta} \qquad 0 < \delta < 1 \tag{1}$$

$$s.to$$

$$p_t c_t + m_t = w_t l_t \tag{2}$$

$$p_{t+1}^e c_{t+1} = \lambda_{t+1}^e m_t + \lambda_{t+1}^e \left[p_t f\left(l_t\right) - w_t l_t \right] \tag{3}$$

$$c_t \geqslant 0, c_{t+1} \geqslant 0, m_t \geqslant 0$$

where m_t are money holdings in nominal terms, p_{t+1}^e is the expected price for the next period, λ_{t+1}^e is (1 plus) the expected nominal interest rate and $f\left(l_t\right)$ is current production.

As suggested above, (2) suggests that the young consumer distributes its labour income between consumption and cash-balances. On the other side, (3) states that the old consumer finances its consumption expenditure through the proceedings on money and the net-of-wage payments from the previous period[5].

Solving (3) for m_t and substituting in (2), allows us to derive an intertemporal budget constraint. Specifically,

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in proportion of their previous endowment of cash balances. Obviously, this generates a process of continuous inflation (deflation).

[5]This means that realised profits are distributed with a one-period lag.

$$c_t + \frac{1}{\theta^e_{t+1}} c_{t+1} = f\left(l_t\right) \tag{4}$$

where $\theta^e_{t+1} \equiv \frac{p_t \lambda^e_{t+1}}{p^e_{t+1}}$ is (1 plus) the expected real interest rate.

In what follows, we will treat interest and price expectations in a straightforward manner. Specifically, we will assume that the monetary authorities are carrying out a constant nominal interest rate policy. Hence,

$$\lambda^e_{t+1} = \lambda_t = \lambda > 0, \text{ all } t \tag{5}$$

Equation (5) suggests that the nominal stock of money grows at the constant rate $(\lambda - 1)$.

Moreover, we make the convenient assumption that

$$p^e_{t+1} = \lambda p_t, \text{ all } t \tag{6}$$

Given (6), the elasticity of price expectations is equal to unity[6]. Furthermore, given that the (expected) nominal interest rate results to be equal to the (expected) inflation rate, the (expected) real interest rate is always zero. Thereafter, (4) reduces to

$$c_t + c_{t+1} = f\left(l_t\right) \tag{4.a}$$

The maximisation of (1) subject to (4.a) leads to the following solutions:

$$c^*_t = \delta f\left(l_t\right) \quad \text{and} \quad c^*_{t+1} = (1-\delta) f\left(l_t\right) \tag{7}$$

2.2 The Production Side and the Labour Market

By symmetry, we assume that there is a single firm. Thereafter, taking as given the nominal wage and price level, the representative firm produces a perishable-homogeneous good by exploiting the labour supplied by the young consumer according to the following production function:

$$f\left(l_t\right) = \frac{1}{\beta} l_t^{\beta} \qquad 0 < \beta < 1 \tag{8.1}$$

As stated above, in our model economy the real wage $q_t \equiv \frac{w_t}{p_t}$ is the only determinant of employment. Thereafter, given that the (inelastic) labour supply is normalised to unity,

[6] In *Value and Capital* (1939), Hicks stressed the elasticity of price expectations as an important element for the stability of an economic system.

is also unitary the full employment real wage. Moreover - above and below unity - the labour market tightness may be well characterised. Specifically, when $q_t > 1$ the labour market experiences an excess of supply, that is, a positive (involuntary) unemployment rate u_t equal to $1 - f'^{-1}(q_t)$. By contrast, when $q_t < 1$ the labour market experiences an excess of demand so that actual employment is equal to unity[7].

From the arguments above, it follows that labour demand is equal to the following expression:

$$\frac{\partial f(l_t)}{\partial l_t} = \frac{1}{l_t^{1-\beta}} = q_t \Rightarrow l_t = q_t^{-\frac{1}{1-\beta}} \tag{8.a}$$

Therefore, (8.1) can be written as a function of the real wage only, that is

$$f[l_t(q_t)] = \frac{1}{\beta} q_t^{-\frac{\beta}{1-\beta}} \tag{8.b}$$

2.3 Short-Run Equilibrium

Now we consider the aggregate economy in a given time period t. If m_{t-1} is the stock of money inherited from the previous period while w_{t-1} and p_{t-1} are, respectively, the lagged value of the nominal wage and lagged value of the price level, a short-run equilibrium is defined by the triplet $\{p_t, w_t, l_t\} \in \Re^3_{++}$ such that

$$w_t = w_{t-1}[1 + g(q_{t-1}, \overline{q})(\frac{p_t}{p_{t-1}} - 1)] \tag{9}$$

$$c_t[f(l_t)] + \frac{\lambda m_{t-1}}{p_t} = f(l_t) \tag{10}$$

$$l_t = \min\left\{1, f'^{-1}(q_t)\right\} \tag{11}$$

where (9) is the "linkage" function, (10) is a manipulation of the equilibrium condition for the goods market[8] and (11) indicates actual employment.

The "linkage" function in (9) suggests that in each period the nominal wage catches up the fraction $g(q_{t-1}, \overline{q})$ of current inflation[9]. This share is a function of the previous

[7]In this case, the firm would be willing to employ an amount of labour that is higher than the fixed labour supply. Thereafter, when the real wage is below the full employment level, the firm is rationed in the labour market.

[8]See the Appendix.

[9]This kind of instantaneous adjustment has been deeply criticised by Jadresic (2002). Some insights on the consequences of nominal wage indexation to *lagged* inflation are given in Appendix.

stance of the labour market as summarised by the lagged value of the real wage. Notice that when $g(\cdot)$ is higher (lower) than unity, it occurs more (less) than-full-indexation. Moreover, $g(\cdot)$ is assumed to have the convenient property that $g(\overline{q}, \overline{q}) = 1$. This means that $\overline{q}$ defines the level of the real wage such that the nominal wage catches up the whole inflation. Therefore, $\overline{q}$ is a level of "quasi-equilibrium" for the real wage if we follow the Hansen's (1951) terminology[10]. In our theoretical analysis, we will consider the possibility that $\overline{q}$ could be not unique and different from unity (full employment).

On the other hand, the modified equilibrium condition for the market for goods in (10) reminds that the price level is determined competitively. Moreover, by exploiting the expressions in (3) and (4), it is possible to show that (10) is equivalent to

$$p^e_{t+1} c_{t+1} \left[f\left(l_t \right) \right] = \lambda^2 m_{t-1} \tag{10.a}$$

Equation (10.a) is a modified equilibrium condition for the money market[11]. If we consider short-run equilibria in which expectations are always fulfilled, that is, $p^e_{t+1} = \lambda p_t$ for all t, the inflation rate outside the long-run "quasi-equilibrium" can be derived in a straightforward manner. Specifically, lagging by one period the expression in (10.a), we derive that

$$\frac{p_t}{p_{t-1}} = \frac{\lambda f\left(l_{t-1} \right)}{f\left(l_t \right)} \tag{12}$$

Given (8.a), the expression in (12) can be written as a function of the real wage only, that is

$$\frac{p_t}{p_{t-1}} = \lambda \left(\frac{q_t}{q_{t-1}} \right)^{\frac{\beta}{1-\beta}} \tag{13}$$

Notice that when the real wage achieves a stationary level, also the inflation rate achieves a stationary value that is equal to the nominal interest rate. As promised, this result suggests that in the stationary long-run "quasi-equilibrium" the expectations assumed in (5) and (6) are perfectly fulfilled.

Given (13), the "linkage" function in (9) can be exploited to derive the law of motion for the real wage, that is,

[10] Notice that in our model, the "quasi-equilibrium" is a long-run stationary equilibrium. By contrast, in Solow and Stiglitz (1968) "quasi-equilibria" are explicitly considered as short-run (stationary) equilibria.

[11] See the Appendix.

$$q_t = q_{t-1} g(q_{t-1}, \overline{q}) + \frac{q_{t-1}^{\frac{1}{1-\beta}} \left[1 - g(q_{t-1}, \overline{q})\right]}{\lambda q_t^{\frac{\beta}{1-\beta}}} \tag{14}$$

2.4 Local Dynamics

The linear expansion of (14) around the "quasi-equilibrium" $\overline{q}$ is given by

$$\left.\frac{dq_t}{dq_{t-1}}\right|_{q_t = q_{t-1} = \overline{q}} = \frac{\lambda - \overline{q} g'(\overline{q}, \overline{q})(1-\lambda)}{\lambda} \tag{15}$$

It not hard to see the local dynamics of the model crucially depends on λ and on the properties of the $g(\cdot)$ function.

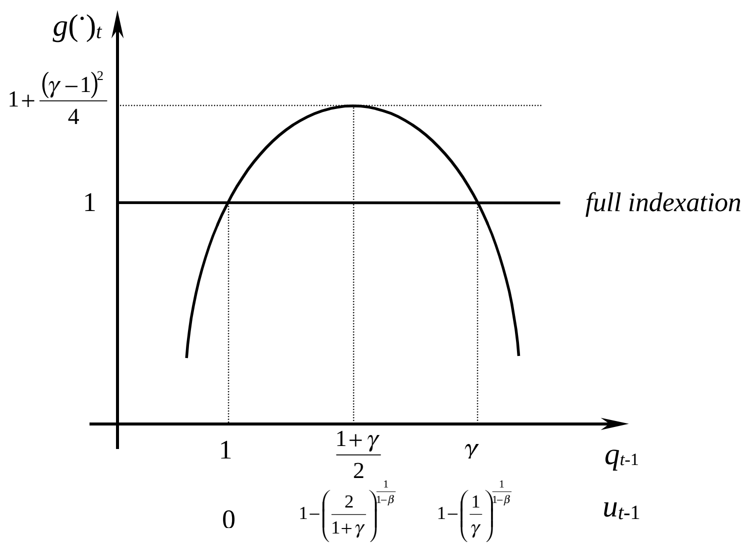

Figure 1: The $g(\cdot)$ function.

We will consider a particular specification for $g(\cdot)$. Specifically,

$$g(q_{t-1}, [1;\gamma]) \equiv 1 - (1 - q_{t-1})(\gamma - q_{t-1}) \tag{16.a}$$

or equivalently[12],

[12] The specifications in (16.a-b) would be actually identical in a deterministic economy with a fixed labour supply.

$$g(u_{t-1},\left[0;1-\gamma^{-\frac{1}{1-\beta}}\right])\equiv 1-\left[1-\left(\frac{1}{1-u_{t-1}}\right)^{1-\beta}\right]\left[\gamma-\left(\frac{1}{1-u_{t-1}}\right)^{1-\beta}\right] \tag{16.b}$$

where $\gamma > 1$.

The expressions in (16.a-b) suggest that $g(\cdot)$ is $\cap$-shaped with respect to q_{t-1} or u_{t-1}. Moreover, there are two "quasi-equilibria" that is, $\overline{q}_i = 1$ (full employment, that is, $\overline{u}_i = 0$) and $\overline{q}_{ii} = \gamma$ (involuntary unemployment, that is, $\overline{u}_{ii} = 1-\gamma^{-\frac{1}{1-\beta}}$). See figure 1.

If we think to the share of inflation caught out by nominal wages as a proxy for the bargaining position of unions and we assume that wage negotiations are conditioned by the lagged labour market tightness, the shape of $g(\cdot)$ can be rationalised as follows. At low levels of unemployment, the unions' strength is so high that the lagged value of the real wage and the share of inflation caught out by nominal wages are positively related. However, higher levels of the real wage lead to higher unemployment and this is likely to worse the unions' position[13]. Specifically, when u_{t-1} reaches the level $1-\left(\frac{2}{1+\gamma}\right)^{\frac{1}{1-\beta}}$, unions' strength starts to decrease and this allows to move towards the "quasi-equilibrium" with involuntary unemployment.

Consider $\overline{q} = \overline{q}_i$ so that $g'(\overline{q},\overline{q}) = \gamma - 1$. Thereafter, (15) becomes

$$\left.\frac{dq_t}{dq_{t-1}}\right|_{q_t=q_{t-1}=1} = 1+\frac{(\gamma-1)(\lambda-1)}{\lambda} \tag{15.a}$$

By contrast, consider $\overline{q} = \overline{q}_{ii}$ so that $g'(\overline{q},\overline{q}) = 1-\gamma$. Thereafter, (15) becomes

$$\left.\frac{dq_t}{dq_{t-1}}\right|_{q_t=q_{t-1}=\gamma} = 1+\frac{\gamma(1-\gamma)(\lambda-1)}{\lambda} \tag{15.b}$$

The first reading of (15.a-b) reveals that when the monetary authorities keep constant the stock of money, *i.e.* when $\lambda = 1$, the local dynamics of the real wage around the two "quasi-equilibria" is indeterminate[14]. Thereafter, the analysis of the magnitude of dq_t/dq_{t-1} has to distinguish between a deflationary and an inflationary scenario.

In case of deflation, *i.e.* when $0 < \lambda < 1$, the "quasi-equilibrium" with full employment is locally stable provided that λ and γ are not too far from unity while the "quasi-

[13]See Layard, Nickell and Jackman (2005).

[14]In this case, converegence becomes a matter of self-fulfilling expectations. See Benhabib and Farmer (1994).

equilibrium" with involuntary unemployment is irremediably unstable[15]. Specifically, the local dynamics of the real wage around unity is monotonically stable provided that the pair (γ, λ) belongs to the set M_D defined as follows:

$$M_D \equiv \left\{ (\gamma, \lambda) \in \Re^2 : \lambda > 1 - \frac{1}{\gamma} \text{ and } \gamma > 1 \right\} \tag{17.a}$$

On the other side, the local dynamics of the real wage around unity is stable with dumped oscillations provided that (γ, λ) belongs to the set D_D defined as follows:

$$D_D \equiv \left\{ (\gamma, \lambda) \in \Re^2 : \frac{\gamma - 1}{1 + \gamma} < \lambda < 1 - \frac{1}{\gamma} \text{ and } \gamma > 1 \right\} \tag{18.a}$$

By continuity, $\lambda = \frac{\gamma-1}{1+\gamma}$ and $\gamma > 1$ represent the pairs such that the unitary stationary solution is locally "flip" unstable. Moreover, for $\lambda < \frac{\gamma-1}{1+\gamma}$ and $\gamma > 1$ the local dynamics of the real wage around unity is unstable with divergent oscillations.

In case of inflation, *i.e.* when $\lambda > 1$, the "quasi-equilibrium" with full employment is locally unstable while the "quasi-equilibrium" with involuntary unemployment may be stable provided that λ and γ are not too far from unity. Specifically, the local dynamics of the real wage around γ is monotonically stable provided that the pair (γ, λ) belongs to the set M_I defined as follows:

$$M_I \equiv \left\{ (\gamma, \lambda) \in \Re^2 : \lambda < \frac{\gamma (1 - \gamma)}{1 + \gamma (1 - \gamma)} \text{ and } \gamma > 1 \right\} \tag{17.b}$$

On the other side, the local dynamics of the real wage around γ is stable with dumped oscillations provided that (γ, λ) belongs to the set D_I defined as follows:

$$D_I \equiv \left\{ (\gamma, \lambda) \in \Re^2 : \frac{\gamma (1 - \gamma)}{1 + \gamma (1 - \gamma)} < \lambda < \frac{\gamma (1 - \gamma)}{2 + \gamma (1 - \gamma)} \text{ and } \gamma > 1 \right\} \tag{18.a}$$

By continuity, $\lambda = \frac{\gamma(1-\gamma)}{2+\gamma(1-\gamma)}$ and $\gamma > 1$ represent the pairs such that the stationary solution γ is locally "flip" unstable. Moreover, for $\lambda > \frac{\gamma(1-\gamma)}{2+\gamma(1-\gamma)}$ and $\gamma > 1$ the local dynamics of the real wage around γ is unstable with divergent oscillations[16].

It is straightforward that if the shape of the parabola in figure 1 is reversed, is also reversed our dynamic analysis. In other words, if $g(\cdot)$ is $\cup$-shaped with respect to q_{t-1}

[15] Notice that in the framework proposed by Hansen (1951), a "quasi-equilibrium" with involuntary unemployment is characterised by an excess supply in the market for commodities and prices and wages that *fall* in order to keep constant their ratio (permanent deflation).

[16] Given the restrictions on γ and λ, local instability is never monotonic.

or u_{t-1}, in case of deflation (inflation) the involuntary unemployment (full employment) "quasi-equilibrium" may result in being locally stable, while the "quasi-equilibrium" with full employment (involuntary unemployment) is unstable[17]. However, if we interpret the share of inflation caught out by nominal wages as a proxy for the bargaining position of unions, the reversed shape entails an unusual positive relation between (lagged) unemployment and union pressure.

Finally, notice that the local dynamics of the real wage does not depend on the curvature of the production function: the magnitude of dq_t/dq_{t-1} is completely determined by λ and γ. As suggested by Dehez and Fitoussi (1986), "*this means that the process of indexation is such that inflation* ...[or deflation]... *drives the economy* (*not necessarily in a monotonic way*) *towards the long-run* ...[quasi-]...*equilibrium through the process of weakening or strengthening the bargaining position of the unions*".

3 Empirical Analysis

In this Section we perform an empirical analysis aimed to derive some insights on the actual shape of $g(\cdot)$. Specifically, this task is carried out by collecting US quarterly data provided by the Bureau of Labour Statistics (BLS) and exploring them with non-parametric techniques[18]. The exploration is aimed to find evidence of non-linearities in the share of inflation caught out by nominal wages with respect to "historical" labour market conditions as summarised by the lagged value of the real wage and the unemployment rate[19]. Therefore, we will provide a kernel estimation of the following expressions:

$$g\left(\cdot\right)_t = m\left(q_{t-1}\right) + \eta_t \tag{19.a}$$

$$g\left(\cdot\right)_t = n(u_{t-1}) + \xi_t \tag{19.b}$$

where η_t and ξ_t are erratic components while $m(\cdot)$ and $n\left(\cdot\right)$ are smooth functions estimated with a normal kernel[20].

[17]More in general, it will be possible to prove that in this model - in case of deflation (inflation) - the "quasi-equilibria" that result in being locally stable are preceded by less (more-) than-full-indexation and followed by (less-) more-than-full-indexation.

[18]The data (and the way in which they are treated) are briefly described in Appendix.

[19]All the estimations are performed with R 2.2.0. Moreover, the tests on linearity are carried out by exploiting the statistical package *sm*. See Bowman and Azzalini (1997).

[20]The bandwidth of the kernel has been obtained by assuming the normality of the probability density function of the regressor.

The specifications in (19.a-b) are aimed to check whether the occurrence of full-indexation may be related to different lagged values of the real wage and the unemployment rate.

Manipulating BLS quarterly data, we find that in the period $1964.2 - 2006.1$ the share of inflation caught out by (private) nominal wages followed the profile illustrated in figures 2. Furthermore, the paths of the real wage and the unemployment rate are given in figure 3.

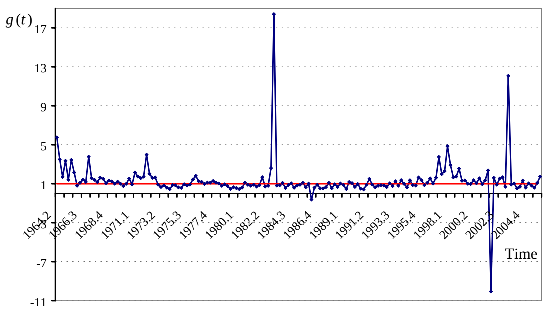

Figure 2: US Inflation caught out by nominal wage $(1964;2-2006;1)$

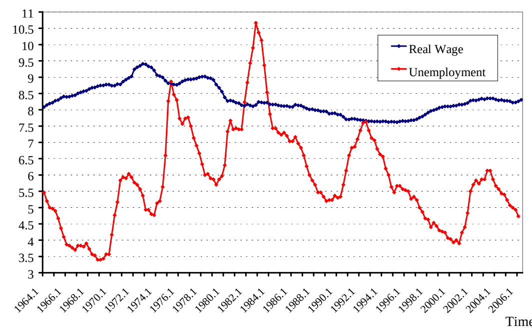

Figure 3: US real wage and unemployment $(1964;1-2006;1)$

The profile of the share of inflation caught out by nominal wages displays marked oscillations around unity. Moreover, some episodes of deflation lead $g(\cdot)_t$ to negative values[21]. On the other side, the path of the real wage shows pronounced oscillations during the 70s. Thereafter, the shape becomes quite flat. Finally, the profile of the unemployment rate displays evident fluctuations around a nearly-constant trend of about 6%.

The kernel estimations of (19.a-b) provide, respectively, the results illustrated in figures 4 and 5.

The smooth function in figure 4 reveals a $\cap$-shaped pattern but the test on linearity does *not* allow to reject the hypothesis that the estimated profile is linear[22]. Even the smooth function in figure 5 displays a $\cap$-shaped pattern, but in this case the test on linearity leads us to *reject* the hypothesis that the estimated profile is linear with a significance level of 10%. This apparent contradiction should be due (*inter alia*) to the

[21]These values, as well as values of $g(\cdot)$ higher than 6, are considered as outliers. Therefore, they are omitted in the non-parametric regressions.

[22]The significance of the test on the linear model is quite high (78.9%).

well-known business cycle regularity according to which real wages fluctuates less than employment and unemployment[23]. See figure 6.

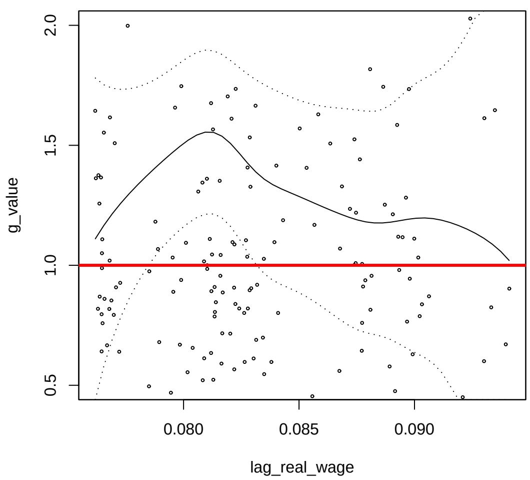

Figure 4: Kernel estimation, q_{t-1} *vs.* g_t

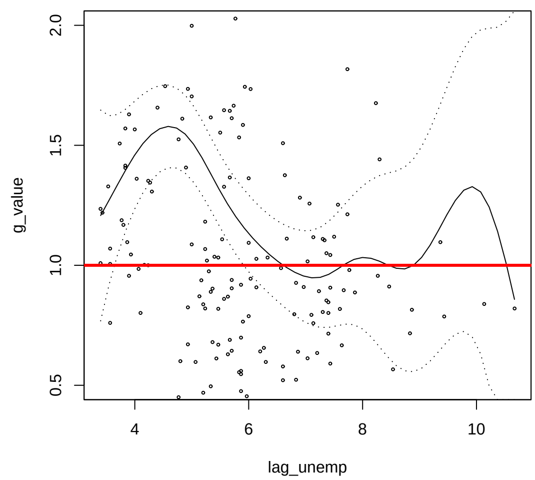

Figure 5: Kernel estimation, u_{t-1} *vs.* g_t

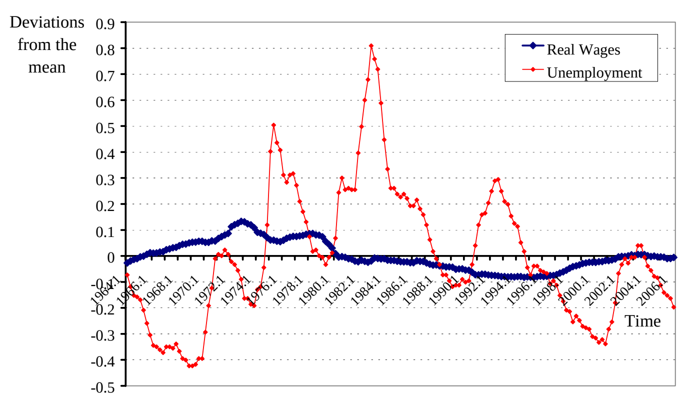

Figure 6: Real wage and unemployment fluctuations $(1964;1 - 2006;1)$

[23]Moreover, additional distortions should arise from the fact that real wages are assumed to embody productivity shocks.

4 A Stochastic Version of the Model

In a macroeconomic scenario characterised by mild inflation and moderate unemployment, the theoretical model described in Section 3 should predict a monotonic convergence towards the "quasi-equilibrium" with involuntary unemployment. However, a simple inspection of figures 2 and 3 suggests that movements along the curves derived with the kernel estimations failed to be monotonic. By contrast, we observed erratic fluctuations around the diagrams in figures 4 and 5.

In order to provide an explanation for this dynamic pattern, we derive a stochastic version of the model that allows to analyse the effects generated by nominal and real shocks. Specifically, each kind of disturbance is modeled by using a stochastic AR(1) process parametrised by exploiting actual data[24]. Thereafter, we use those results to perform a numerical simulation[25].

4.1 Nominal and Real Shocks

On the one hand, nominal shocks enter in the model by allowing λ_t to follow a stochastic AR(1) process, that is

$$\lambda_t = \kappa_\lambda + \rho_\lambda \lambda_{t-1} + \epsilon_t \tag{20}$$

where $\epsilon_t \sim N(0, \sigma_\lambda^2)$.

Using data on M1 provided by the Federal Reserve (FED), the OLS estimation of the parameters of the AR(1) process in (20) and the variance of the erratic term obtained with quarterly data are enclosed in table 1.

On the other hand, real shocks enter in the model as "neutral" shifts in the production function, that is,

$$f(l_t) = \alpha_t \frac{1}{\beta} l_t^\beta \qquad 0 < \beta < 1 \tag{8.2}$$

Keeping in mind (12), the relevant AR(1) stochastic process for the real wage dynamics is given by

$$\theta_t = \kappa_\theta + \rho_\theta \theta_{t-1} + \nu_t \tag{21}$$

where $\theta_t = \frac{\alpha_t}{\alpha_{t-1}}$ and $\nu_t \sim N(0, \sigma_\theta^2)$.

[24] Once again, data are briefly described in Appendix.

[25] In this work we used MATLAB 6.5. The code is available from the author.

Using data on the US real GDP provided by the Bureau of Economic Analysis (BEA), the OLS estimation of the parameters of the AR(1) process in (21) and the variance of the erratic term obtained with quarterly data are enclosed in table 1.

Parameter	OLS *Estimation*
κ_λ	0.3393
ρ_λ	0.6648
σ_λ	0.0085
κ_θ	0.0081
ρ_θ	0.9686
σ_θ	0.0002

Table 1: The parameters of nominal and real shocks

4.2 Numerical Simulations

Given (14), (16.a-b), (20) and (21), the trajectory of the real wage is generated by the following stochastic difference equation:

$$q_t = \frac{\theta_t q_{t-1}^{\frac{1}{1-\beta}}}{\lambda_t q_t^{\frac{\beta}{1-\beta}}} + \left[1 - \left(1 - q_{t-1}\right)\left(\gamma - q_{t-1}\right)\right]\left(q_{t-1} - \frac{\theta_t q_{t-1}^{\frac{1}{1-\beta}}}{\lambda_t q_t^{\frac{\beta}{1-\beta}}}\right) \tag{22}$$

The expression in (22) is calibrated by fixing $\beta = 0.6$, $\gamma = 1.03$ and imposing the initial conditions $q_0 = 1.001$ and $\lambda_0 = \theta_0 = 1$. This parametrisation seems to meet some of the regularities showed by the kernel regressions[26]. Moreover, the erratic components ϵ_t and ν_t will be generated by assuming that $cov\left(\epsilon_t, \nu_t\right) = 0$.

The results of a numerical simulation (400 replications) obtained by iterating (22) are illustrated in figures 7, 8 and 9.

The simulated profile of the share of inflation caught out by nominal wages displays very mild oscillations around unity. This result is quite at odds with respect to actual data, but it should be due to the fact that our theoretical model does not allow for (*i*) shifts in labour supply and (*ii*) variations in the "quasi-equilibrium" points.

[26] With $\beta = 0.6$, a value of γ equal to 1.03 delivers a "quasi-equilibrium" involuntary unemployment rate of 7.12%.

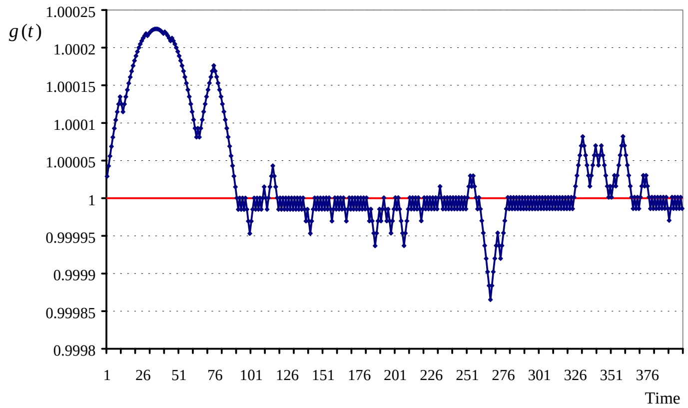

Figure 7: Simulated inflation caught out by nominal wages

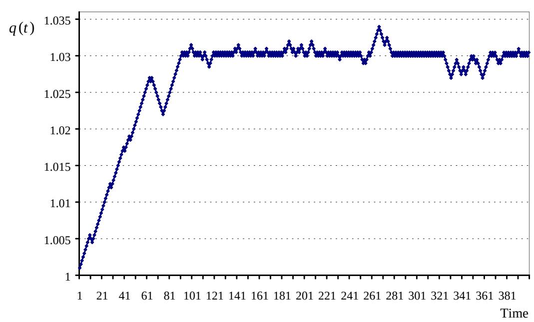

Figure 8: Simulated real wage

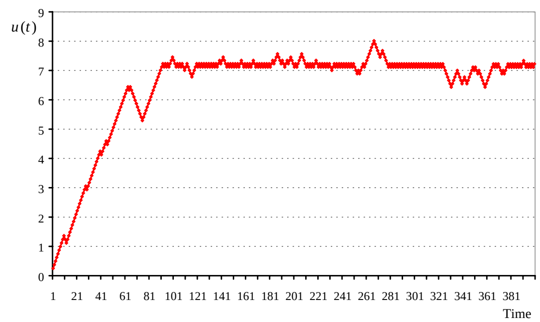

Figure 9: Simulated unemployment

On the other side, the simulated paths for real wage and unemployment show a nearly monotonic convergence to the their stationary values expected in an inflationary scenario. However, those stationary points are never reached: at the end of the period of convergence, the real wage and the unemployment rate start to track a cycle around their "quasi-equilibrium" values characterised by involuntary unemployment. As it happen in actual data, those cycles allow for erratic fluctuations around the $\cap$-shaped relationship describing the share of inflation caught out by nominal wages plotted against the lagged real wage or the lagged unemployment rate. See the kernel estimations of (19.a-b) obtained with simulated values in figures 10 and 11.

Finally, our simple model is also able to capture the business cycle regularity according to which real wage fluctuates less then unemployment. See figure 12.

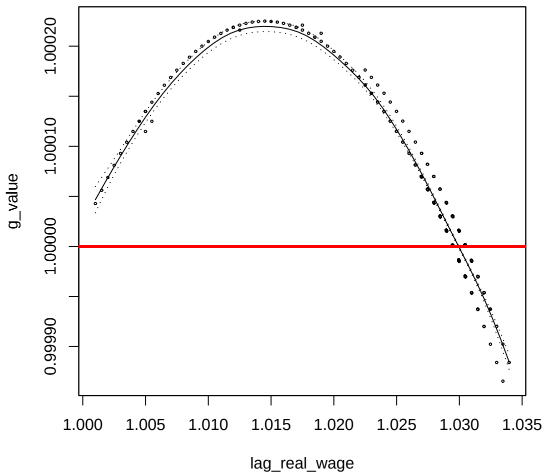

Figure 10: Kernel estimation, simulated q_{t-1} *vs.* g_t

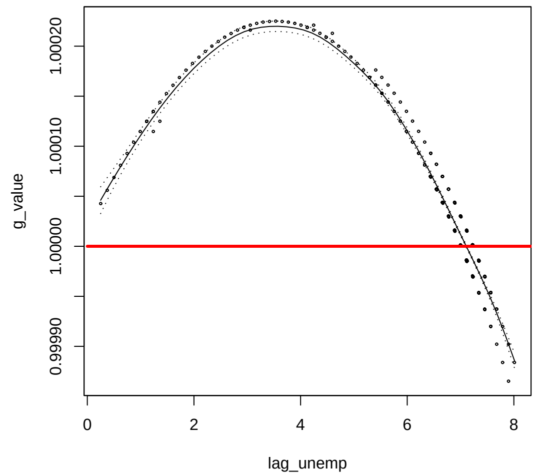

Figure 11: Kernel estimation, simulated u_{t-1} *vs.* g_t

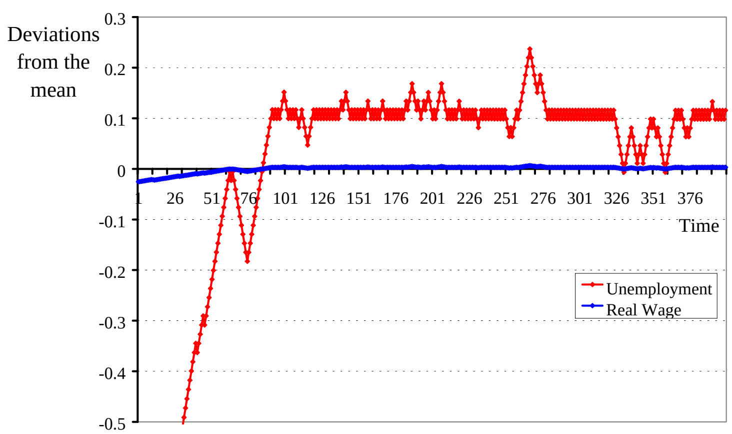

Figure 12: simulated real wage and unemployment fluctuations

5 Concluding Remarks

This paper aimed to derive the local dynamics implications for real wages generated by some non-linearities in the mechanism for nominal wages indexation. This task is carried out by following the lines traced out by Dehez and Fitoussi (1986) and interpreting those non-linearities as the possibility of multiple "quasi-equilibria" as originally defined by Hansen (1951). Specifically, by assuming that the share of inflation caught out by nominal wages is $\cap$-shaped with respect to the lagged value of the real wage, we demonstrated that in case of deflation (inflation) the "quasi-equilibrium" with full employment may be locally stable (is unstable) while the "quasi-equilibrium" with involuntary unemployment is unstable (may be stable).

Moreover, the theoretical dynamic model has been matched by an empirical analysis aimed to derive insights on the shape of indexation in the US context. Specifically, we performed some non-parametric regressions that linked the share of inflation caught out by nominal wages to an historical indicator of labour market tightness, that is, the lagged values of the real wage and the unemployment rate. Thereafter, we tested the linearity of the resulting relationships. Given some particular features of the business cycle, we found that the hypothesis of linearity can be strongly rejected only if we consider the lagged value of the unemployment rate.

Finally, in order to take into account the effects generated by nominal and real shocks we derived a stochastic version of the model. A simulation obtained by parametrising the shocks to actual data, suggested that the movement of the share of inflation caught out by nominal wages along the non-linear relationship assumed in the theoretical model - and some how confirmed in the empirical analysis - is not necessarily monotonic but it may occur through erratic fluctuations.

This work has to be extended in different directions. From a theoretical point of view, three points deserve certainly a wider development. First, the shape of the $g(\cdot)$ function should be endogenised through some bargaining setting so that the indexation scheme and the resulting local dynamics would become authentically endogenous. Second, productive capital should be added to the general framework by taking into account more articulated interest rate dynamics. Finally, a theory on labour supply fluctuations and wandering "quasi-equilibria" should be developed in order to match more carefully the business cycle regularities.

From an empirical point of view, the analysis carried out in Section 4 has to be thought as preliminary. In fact, in our non-parametric regressions we assumed that the only determinant of the share of inflation caught out by nominal wages were the lagged value of the real wage or the lagged unemployment rate. However, it is likely that this share could be explained by additional variables (*e.g.* union density, the rate of monetary expansion, etc.). Thereafter, our non-parametric regressions should be enriched with other regressors.

6 Appendixes

In this Section some of the equations used in the main text are explicitly derived. Thereafter, we provide a short description of the quarterly series used for the empirical analysis and the parametisation of stochastic shocks. Finally, we derive some insights on the consequence of wage indexation on lagged inflation.

6.1 An Application of Walras's Law

Consider a given time period t. On the one hand, the binding budget constraint for the young consumer is given by

$$c_t^Y\left[f\left(l_t\right)\right]+\frac{m_t}{p_t}=q_t l_t \tag{A.1}$$

where $q_t=\frac{w_t}{p_t}$.

On the other hand, the binding budget constraint for the old consumer is given by

$$c_t^O\left[f\left(l_{t-1}\right)\right]=\frac{\lambda_t m_{t-1}}{p_t}+\frac{\lambda_t}{p_t}\left[p_{t-1} f\left(l_{t-1}\right)-w_{t-1} l_{t-1}\right] \tag{A.2}$$

Applying the Euler's rule (A.2) becomes

$$c_t^O\left[f\left(l_{t-1}\right)\right]=\frac{\lambda_t m_{t-1}}{p_t}+f\left(l_t\right)-q_t l_t \tag{A.2.a}$$

Adding (A.1) and (A.2.a) leads to

$$c_t^Y\left[f\left(l_t\right)\right]+c_t^O\left[f\left(l_{t-1}\right)\right]-f\left(l_t\right)=\frac{\lambda_t m_{t-1}}{p_t}-\frac{m_t}{p_t} \tag{A3}$$

As a consequence of Walras's law, (A3) suggests that the excess demand on the market for goods has to be equal to excess supply on the market for money.

If the stock of money grows at the constant rate $\lambda-1$, it holds that $\lambda_t=\lambda$ for all t. Moreover, a circular flow of cash implies that $m_t=\lambda m_{t-1}$. Therefore, the markets for money and goods always clear no matter which is the price level. In fact,

$$c_t^Y\left[f\left(l_t\right)\right]+c_t^O\left[f\left(l_{t-1}\right)\right]-f\left(l_t\right)=\frac{\lambda m_{t-1}}{p_t}-\frac{\lambda m_{t-1}}{p_t}=0 \tag{A.3.a}$$

In order to solve such an indeterminacy, we follow Dehez and Fitoussi (1986) and we impose the following equality:

$$c_t^O\left[f\left(l_{t-1}\right)\right]=\frac{\lambda m_{t-1}}{p_t} \tag{A.4}$$

The value of p_t that solves (A.4) allows to clear the market for goods and it is equivalent to

$$c_t^Y\left[f\left(l_t\right)\right]+\frac{\lambda m_{t-1}}{p_t}=f\left(l_t\right) \qquad \text{Q.E.D} \tag{A.5}$$

6.2 The Equilibrium Condition for the Money Market

Solving (2) for $\frac{m_t}{p_t}$ leads to

$$\frac{m_t}{p_t}=\frac{w_t}{p_t}l_t-c_t\left[f\left(l_t\right)\right] \tag{B.1}$$

Substituting for $c_t\left[f\left(l_t\right)\right]$ in (10) and solving for m_t yields

$$m_t=w_t l_t-p_t f\left(l_t\right)+\lambda m_{t-1} \tag{B.2}$$

Substituting in (3) by assuming that $\lambda_{t+1}^e=\lambda$ for all t leads to

$$p_{t+1}^e c_{t+1}\left[f\left(l_t\right)\right]=\lambda^2 m_{t-1} \qquad \text{Q.E.D} \tag{B.3}$$

6.3 Datasets

The quarterly data exploited in the empirical analysis in Section 4 were derived in the following way. Data on retributions aroused from the monthly seasonally adjusted series "*AVERAGE HOURLY EARNINGS OF PRODUCTION WORKERS, Super Sector: Total private, Industry: Total private*". This series is used as a measure of nominal wages. The inflation rate was calculated by using the monthly seasonally adjusted series "*Consumer Price Index - All Urban Consumers, Area: U.S. city average, Item: All items, Base Period: 1982-84=100*". Finally, the unemployment rate aroused from the seasonally adjusted series "*Unemployment Rate, Labor force status: Unemployment rate, Type of data: Percent, Age: 16 years and over*". These data are provided by the BLS (US Labor Department). Obviously, all these monthly series were averaged over a 3-months horizon in order to derive quarterly data.

The quarterly data exploited in Section 5 to model the process of nominal shocks arise from the *Money Stock Series* provided by the FED. Specifically, we exploited the monthly seasonally adjusted series of M1 (*currency, traveler's checks, demand deposits and other checkable deposits*). As above, this monthly series $(1959;1-2006;4)$ were averaged over a 3-months horizon in order to derive quarterly data.

Finally, the quarterly data exploited in Section 5 to model the process of real shocks arise from the seasonally adjusted "*Real Dollar*" GDP series provided by the BEA $(1947;1-2006;1)$. Specifically, the values of α_t have been derived by computing a non-linear trend in the GDP series extracted with the Hodrick-Prescott (HP) filter.

6.4 On Lagged Inflation

Here we follow the insights suggested by Jadresic (2002) and we examine the effect of wage indexation when the current nominal wage is "linked" to past inflation. In this case the "linkage" function is the following:

$$w_t = w_{t-1}[1 + g(q_{t-1}, \overline{q})(\frac{p_{t-1}}{p_{t-2}} - 1)] \tag{C.1}$$

Exploiting the result in (13), (C.1) can be used to derive a non-linear second-order difference equation for the real wage, that is

$$q_t = \frac{q_{t-1}}{\lambda} \left(\frac{q_{t-1}}{q_t} \right)^{\frac{\beta}{1-\beta}} \left\{ 1 + g(q_{t-1}, \overline{q}) \left[\lambda \left(\frac{q_{t-1}}{q_{t-2}} \right)^{\frac{\beta}{1-\beta}} - 1 \right] \right\} \tag{C.2}$$

Given the expression for $g(\cdot)$ in (16.a-b), unity and γ are stationary solution also for (C.2).The linear expansion of (C.2) around the generic stationary solution $\overline{q}$ is given by

$$dq_t + \frac{(1-\lambda)(1-\beta)\, g'(\overline{q}, \overline{q}) - (1+\beta)\lambda}{\lambda} dq_{t-1} + \beta dq_{t-2} = 0 \tag{C.3}$$

On the other side, the characteristic equation of (C.3) is the following:

$$f(x) = x^2 + \frac{(1-\lambda)(1-\beta)\, g'(\overline{q}, \overline{q}) - (1+\beta)\lambda}{\lambda} x + \beta = 0 \tag{C.4}$$

As general analytical results are difficult to derive, we resort to some numerical examples. Specifically, by setting $\beta = 0.6$ and $\gamma = 1.03$ and reminding that $g'(\overline{q}, \overline{q}) = (\gamma - 1)$ if $\overline{q} = 1$ while $g'(\overline{q}, \overline{q}) = (1 - \gamma)$ if $\overline{q} = \gamma > 1$, we calculate the characteristic roots of (C.4) for both stationary solutions by considering different values of λ. The results are given in table 2.

λ	*stationary solution*	x_1	x_2
0.95	full employment	0.9984	0.6010
0.95	involuntary unemployment	1.0016	0.5991
0.97	full employment	0.9991	0.6006
0.97	involuntary unemployment	1.0009	0.5994
0.99	full employment	0.9997	0.6002
0.99	involuntary unemployment	1.0003	0.5998
1	full employment	1	0.6
1	involuntary unemployment	1	0.6
1.01	full employment	1.0003	0.5998
1.01	involuntary unemployment	0.9997	0.6002
1.03	full employment	1.0009	0.5995
1.03	involuntary unemployment	0.9991	0.6005
1.05	full employment	1.0014	0.5991
1.05	involuntary unemployment	0.9986	0.6009

Table 2: The roots of the characteristic equation ($\beta = 0.6, \gamma = 1.03$)

The numerical results in table 2 suggest that in case of deflation (inflation), *i.e.* when $\lambda < 1$ ($\lambda > 1$), the stationary solution with full employment (involuntary unemployment) is locally stable while the stationary solution with involuntary unemployment (full employment) is a saddle point. On the other side, when the monetary authorities keep constant the stock of money, *i.e.* when $\lambda = 1$, the local dynamics around the two stationary solutions is *metastable*, that is, stable in the Liapunov's sense[27].

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[27]When the stationary solution is a saddle point there exists a unique converging trajectory while all the others diverges. By contrast, stability in the Liapunov's sense (*metastability*) means local convergence in the *neigbourhood* of the stationary solution.

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