

The Role of Community in Migration Dynamics

May 23, 2006

Abstract

In this paper, we present a theoretical model that, implementing the pioneering work of Burda (1995), based on the Real Option Theory, investigates the roots of the migration dynamics. In the model the decision to migrate of each individual depends not only on the wage differential, but also on a U-shaped benefit function of a community of homogenous ethnic individuals, modelled according to the "*theory of clubs*". The theoretical results are able to give an explanation to the observable "jumps" in the migration flows and to describe how the trigger for entry can change depending on the dimension of the district. The analysis of the results also sheds light on the dynamics of the districts' development: some possible rigidities in the adjustment of the district dimension, as regards the optimal levels, could magnify the hysteresis process.

JEL Classification Numbers: F22, H49, O15, R23.

Keywords: Migration, Real Option, Theory of Clubs, Network Effect.

1 Introduction

Generally, in economic literature, migration depends on the wealth difference between two countries or two lands, because mainly "*people migrate in order to increase their welfare*"¹. This fact implies that migrants enter a new society when the expected value of their benefits minus their costs is greater than zero. After an individual enters a new society, she begins to build a group of people based on affinities, religions and the same way of life: this group is generally called "community". This aggregate of individuals that uses, like a family, the same goods, "*deriving mutual benefit sharing [...] production costs, the members' characteristics, or a good characterised by excludable benefits*", can be modelled by following economic theory of "club" (Sandler and Tschirhart, 1980; Buchanan, 1965; Berglas, 1976). The aim of this paper consists in studying how the community, modelled as a club, influences an individual's decision to migrate. To formulate the choice of the migrant, we use a real option approach as developed in recent literature by Burda (1995), Khwaja (2002) and Anam *et al.* (2004). We particularly want to stress how the decision to migrate, depending on wage differences and the benefits coming from a community, affects the dynamics of migrant flows into a country. The theoretical results are perfectly in line with the observable data and try to give an explanation to the observable jumps in the migration dynamics.

1.1 Some supporting evidence

Figure 1 below shows the four main migration nationalities in Italy and their growth rates in the period considered. The data are taken from the official national statistic database (ISTAT) for the years between 1994 and 2000 and from Caritas report for the period between 2001 and 2003. The migration flows are depleted from the two important regularizations for illegal immigrants introduced in Italy in 1996 and 1998, and registrated by the ISTAT database in the subsequent years. For the sake of completeness, Figure 2 shows also the the italian flow to Usa between 1960 and 1984, by using the Istat database. It is observable that, in both the cases, the migration process does not proceed in a smooth manner, but it shows some jumps in its dynamics, especially at the beginning of the phenomena, as if a mass of individuals is waiting for something to happen in order to decide to migrate.

¹ Khwaja, Y., "Should I Stay or Should I Go? Migration Under Uncertainty: A Real Option Approach", mimeo, March, 2002

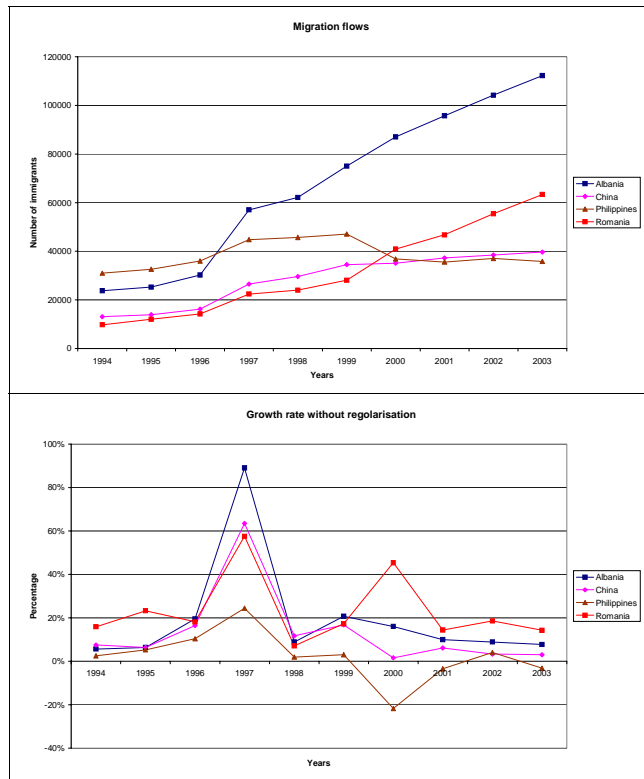


Figure 1: Migration Jumps

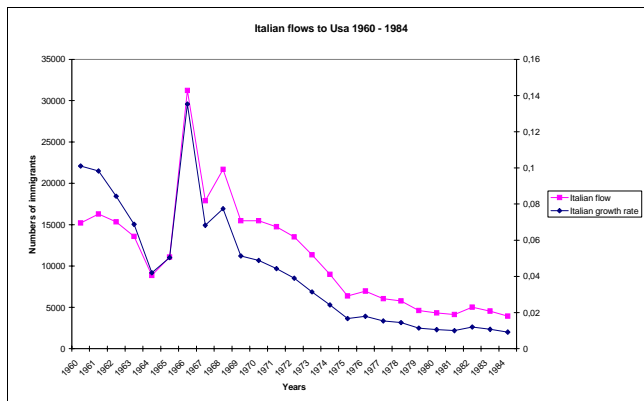


Figure 2: Italian Flows to Usa

Which is the reason why do they wait before taking their decision to migrate?

What are they waiting for? And why do they move in a mass? The aim of this paper consists in answering to these questions by verifying if the characteristic of investment of migration and the role of ethnic groups, behind any migration decision, can explain the migration jumps observed in Figure 1.

We proceed in the following manner: in Section 2, we explain the model. Sections 3 and 4 show the main results, namely the optimal migration strategy in the presence of positive and negative externalities. Finally, Section 5 summarises the conclusions.

2 Migration As Investment

Can migration decision be thought as an investment choice? In the light of the economic literature, it seems that the answer to this question is "Yes". In fact, according to Sjaastad (1962), "*people migrate if the discounted stream of earnings in the destination exceeds that of the origin by more than a fixed, one-time cost of migration*". The same statement is advisable in Bowles (1970): "*the decision process concerning migration may be viewed fruitfully as a comparison of the present value of the benefits and costs of moving*". In this regard, Burda (1995) and Khawaja (2002) apply the theory of real option, generally used to study investment decision, to the migration choice. In particular, which are the main variables affecting decision to migrate? In literature, the wage gap is generally considered the only variable that affects the decision to migrate (Burda, 1995; Todaro, 1969), but according to some recent works, it may not be sufficient to explain migration flows completely. In Moretti's paper (1998), the author sheds light on the specific role of the ethnic community on the migration waves. In particular, he proposes a model in which "*the probability of migrating to a country depends positively on the social networks that link the migrant to that country*". Another recent work (Bauer, Epstein and Gang, 2002) examines the relative importance and interaction of two alternative explanations of immigrant clustering: 1) network externalities and 2) herd behaviour. The same theme is studied in Epstein and Gang (2004), where the authors examine the roles "other people" play in influencing an individual's potential migration decision.

Therefore, if migration choice can also be thought as an investment choice, it is possible to use the Burda approach to study migration dynamics, even if, so far, the analysis has been focused on the decision to migrate of only one individual. What might happen if each individual chose to move to a host country with regards to other immigrants? What might happen if in the host country there existed a community of other homogeneous individuals that helped her to increase her benefits? This assumption is not far from recent economic literature (Moretti, 1998; Winters et al. , 2001; Coniglio, 2003; Munshi, 2003; Bauer et al., 2002; Epstein and Gang, 2004; Bauer and Zimmermann, 1997).

In the next steps we define a simple real option model and we combine this framework with the classical theory of clubs for stressing the role of community in the migration choice.

2.1 The model

This section presents a continuous-time model of migration where the differential benefits of migration, including the wage differential, evolves in a stochastic manner over time and there is ongoing uncertainty.

We can summarise our assumptions in the following manner:

1. There exist two countries: the country of origin where each potential migrant takes her decision and the host country.
2. At any time t each individual is free to decide to migrate to a new country. Individuals discount the future benefits at the interest rate ρ .
3. All immigrants are identical, are infinitely-lived, or choose vicariously for their descendants who will remain in the receiving country forever². Their size dn is infinitesimally small with respect to the total number of inhabitants.
4. Each individual enters a new country undertaking a single irreversible investment which requires an initial sunk cost K .
5. The wage differential for each migrant, called x , follows a geometric diffusion process:

$$dx = \alpha x dt + \sigma x dw \quad (1)$$

with $x_0 = x$ and $\alpha, \sigma > 0$. The component dw is a Weiner disturbance defined as $dw(t) = \varepsilon(t)\sqrt{dt}$, where $\varepsilon(t) \sim N(0, 1)$ is a white noise stochastic process (see Cox and Miller, 1965). The Weiner component dw is therefore normally distributed with zero expected value and variance equal to: $dw \sim N(0, dt)$. From these assumptions and from the (1) we know that $E[dw] = 0$; $E[dx] = \alpha x dt$.

6. In the host country there is a community of ethnically homogeneous individuals. Each individual becomes a member (finding a job) instantaneously when she enters the host country.
7. The community net benefit function for each member is U-shaped with regards to the number of members.

By these hypotheses and assuming $\theta_0 = \theta$ and $n_0 = n$, the value of migrating to the host country is:

$$V(x, n) = \max_{\tau_i} E_0 \left\{ \int_0^{\infty} e^{-\rho t} [x(t) + \theta u[n(t)]] dt - \sum_{\tau_i} J_{[\tau_i=t]} K \right\} \quad (2)$$

²It is possible to show that the "sudden death" formulation is a very natural generalisation of the infinite-life case (Dixit and Pindyck, 1993, p.205).

where $J_{[\tau_i=t]}$ is the indicator function that assumes the values one or zero depending on whether the argument is true or false, and the expectation is taken considering that the number of immigrants may change over time by new entry.

The next step consists in: i) defining function $u[n(t)]$ according to microeconomics theory; ii) solving (2) by using real option approach (see Appendix).

2.2 Ethnic community and the theory of clubs

We assume that an ethnic community can be modelised as a "club", in the light of theory of clubs' definitions. In fact, a community generally arises for mutual economic benefits and its members have generally the same characteristics, the same way of life and, sometimes, follow the same religion: all these affinities form strong ties that can push the individuals to help one another to share the cost of housing (as shown in the Ares2000 _Onlus report) or the costs of structures, like churches or cultural centres. This assertion is in line with McGuire (1972, 1974), Sadler and Tschirhart (1980), Bauer and Zimmermann (1997) and also with Locher (2001). Nevertheless, clubs involve sharing and this fact often leads to a partial rivalry of benefits as larger memberships crowd one another, causing a detraction in the quality of services received. This implies that a high number of members could induce increasing congestion costs, e.g. crowded houses or competition on the labour market³. The parallelism between community and theory of clubs is also confirmed by an extension of Buchanan and Goetz (1972) on the Tiebout model (1956)⁴. The trade-off between cost sharing and congestion is at the centre of collective good models that follow Buchanan (1965) and Tiebout (1956) and it guarantees a U-shaped average cost of provision and hence a unique minimum average cost, as shown by Edwards (1992).

Then, we present a model in which the immigrants undertake the decision to migrate with regards to three variables: wage differential between country of origin and the host country (as we can see in a great deal of literature on migration: Todaro, 1969; Harris-Todaro, 1970; Burda, 1995; Bencinverga-Smith, 1997); a fixed cost (travel costs and some psychological costs) and the net benefits stemming from the resident community in the host country.

2.2.1 The benefit function

Let us assume that the migrant is already in the host country: she belongs to her ethnic community in a district where there are different local public goods (G), such as churches, cultural centres and houses belonging to a group of homogeneous individuals. To describe the sum of buildings belonging to the community, as a public good, we follow the considerations of Edwards⁵.

³An idea of congestion costs in a host country is introduced by Coniglio (2003).

⁴According to these statements, if the total cost of using a common good is the sum of average cost plus congestion cost, when the number of users (i.e. the size of the community) increases, there is an initial fall in costs (an increase in net benefit) and a subsequent rise in integration costs when the congestion effect becomes greater.

⁵"there also exist collective solutions (...) among these are clubs, public provision and informal sharing arrangements (roommates)"

Let us assume for simplicity that the individual utility function is a quasi linear function, that is:

$$U(y, g(G, n)) = y + g(G, n) \quad (3)$$

Where y is the members consumption of the private good, G is her consumption of the club good, and n is the membership size. Since the utilisation rate of the club good is the same for all members, we have $g_i = G$ for all members, where g_i is the i^{th} member's utilisation rate of the club facility, and G is the size of the club facility.

Each member attempts to maximise utility subject to a resource constraint,

$$x = y + C(G, n) / n \quad (4)$$

where:

$$\begin{aligned} \partial g / \partial G &> 0; \partial g / \partial n < 0 \\ \partial C / \partial G &> 0; \partial C / \partial n > 0 \end{aligned}$$

x is the wage differential⁶ $x = x_h - x_o$, respectively between the wage of the host country (h) and the wage of the country of origin (o). Simplifying our analysis, we assume that x_o is equal to zero; the price of the private good is unity, and $C(\bullet)$ is the club's cost⁷.

It is possible to demonstrate that, for a given level of G (i.e. in the instant t), the migrant's utility function can be reduced to:

$$U(x, n) = x + \theta u(n) \quad (5)$$

where θ is a scale factor. The function $u(n)$ is twice continuously differentiable in n , and it is increasing over the interval $[0, \bar{n})$ and decreasing thereafter. That is, there exist positive externalities if the individual enters the community. After \bar{n} the costs increase more than the benefits because of the increasing congestion effect. We also assume that at zero and at some finite number of members N , the benefits fall to zero (i.e. $\theta u(n) = 0$, and $\theta u(N) = 0$)⁸.

Let us rearrange figure 10.1 of Cornes R., Sandler T. (1986, page 169) in figure 3. Quadrant II shows the function $u(n)$ as the vertical difference between the gross benefit function and the costs⁹ per member: the resultant bold line is the net benefit per person associated with changing membership size, when the district size is fixed at G_1, G_2, G^* units.

⁶Dustmann (2003): "In a simple static model, migration increases with the wage differential between host- and home-country".

⁷Superscripts are dropped, from now on, whenever members are homogeneous.

⁸These theoretical results can be also explained by using a typical representation of theory of clubs taken by Sandler and Tschirhart (1980).

⁹The cost curves depict the cost per member when a facility of a given size is shared by a varying number of members.

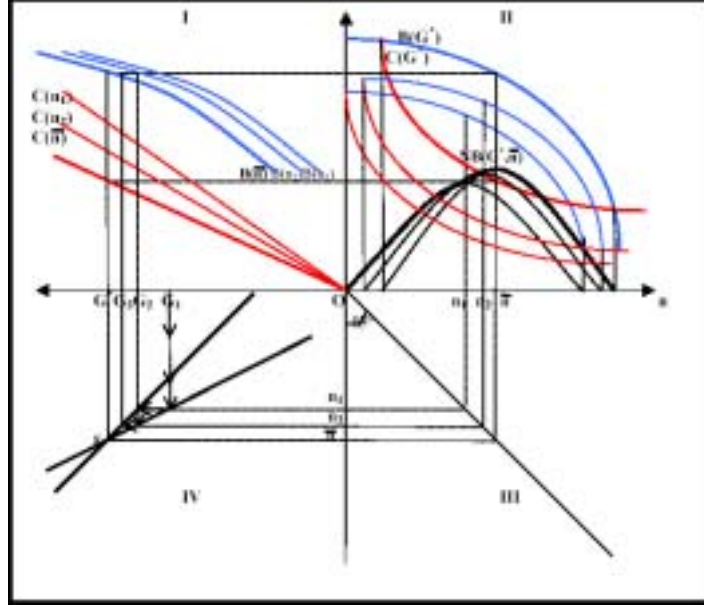


Figure 3: Benefit function

The shape of these curves indicates that *camaraderie* is eventually overpowered by crowding, and at that point the benefit per person begins to decline. Starting at the initial instant t of the migrant's choice, our assumption is that she knows the number of members of the community and the size of the district G_t . Her entry modifies the optimal couple (i.e. the dimension of the district and the optimal value of the number of the community's members): a new level of members needs a greater dimension of the club; this fact pushes the curves upwards and identifies a new optimal couple. This process continues until a new stable equilibrium is reached. Nevertheless, because of the instantaneity of the process, the only curve observable by the migrant is the envelope of the family of functions, i.e. the bold line in the quadrant II. The result is a U-shaped function¹⁰ which corresponds to the increment of benefits that each migrant could obtain if she entered the community¹¹.

3 Main Results

Result 1 *The optimal entry policy for each migrant, characterised by a mass of other migrants $n \geq \bar{n}$, is described by the upward-sloping curve (Figure*

¹⁰That is the envelope of the family of the U-shaped functions.

¹¹The same result can also be easily derived from the U-shaped cost curve used by McGuire (1974, Fig. 1b, page 118). The conclusion is that the graphic solution, displayed in figure 1.3, fits figure 1.2 perfectly.

4):

$$x^*(n) = \frac{\beta_1}{\beta_1 - 1} \cdot (\rho - \alpha) \cdot \left[K - \frac{\theta u(n)}{\rho} \right]; \text{ for } n \in [n^*, m] \text{ with } \frac{\beta_1}{\beta_1 - 1} > 1 \quad (6)$$

where $\rho > \alpha$ and $\beta_1 > 1$ is the positive root of the auxiliary quadratic equation $\Psi(\beta) = \frac{1}{2}\sigma^2\beta(\beta - 1) + \alpha\beta - \rho = 0$.

Proof. See the Appendix ■

Result 2 The candidate policy for a mass of individuals $n < \bar{n}$ is described by the following flat curve starting at $x^*(\bar{n})$ defined by (Figure 4):

$$x^*(\bar{n}) = \frac{\beta_1}{\beta_1 - 1} \cdot (\rho - \alpha) \cdot \left[K - \frac{\theta u(\bar{n})}{\rho} \right]; \text{ for } n \in [0, \bar{n}] \text{ with } \frac{\beta_1}{\beta_1 - 1} > 1 \quad (7)$$

Proof. See the Appendix ■

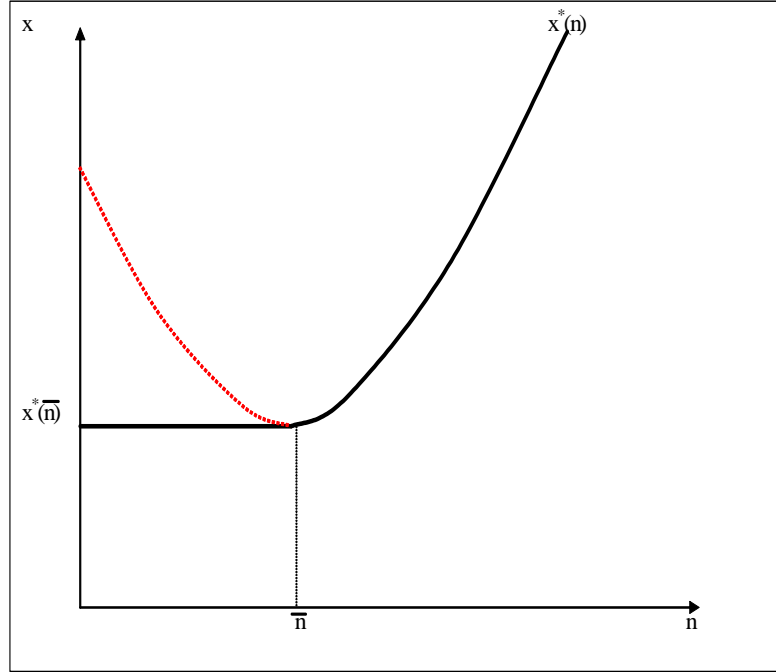


Figure 4: Optimal Policy

In fact, in the case of negative externalities, i.e. for $n \in [\bar{n}, N)$, if the differential wage x climbs to a trigger level x^* , migration will become feasible but, at the moment of entry, the total benefit declines along the function $u(n)$ due to congestion effects: we have a *reflecting barrier*. The differential wage

continues to move stochastically until a new entry episode occurs and the flow hits the optimal number \bar{n} . This case is a setting of competitive equilibrium in which every migrant is "totally myopic in the matter of other migrant's entry decision" (Dixit and Pindyck, 1993, p.291). In this way the "optimal competitive equilibrium policy need not take account of the effect of entry" (Moretto, 2003, p.8) and the wage level x that triggers entry by the single migrant in isolation, is the same as that of the migrant who correctly anticipates the other migrants' strategies x^{*12} .

If instead we consider the migrant's benefit function along the increasing part, that is $n \in [0, \bar{n})$, any potential entrant is subject to positive externalities, so their value of entering depends on the number of migrants already entered the community. Thus the timing of the decision is influenced by the decisions of the others: the single entrant cannot claim to be the last to enter the community¹³. Therefore, the higher the number of members the greater the benefits that the individual obtains if she enters. The network benefits make the individual face a choice between no entry and agreement. However, as all individuals are subject to the same stochastic shock, two equilibrium patterns are possible: either the community remains locked-in at the initial size, sustained by self-fulfilling pessimistic expectations (infinite delay), or a mass of individuals simultaneously rushes to enter. Excluding the former¹⁴, we have the following:

Proposition 1 *If the benefit function of belonging to an ethnic community is U-shaped, all the immigrants wait until the threshold level reaches the maximum. At $x^*(\bar{n})$ they co-ordinate migration together, causing a "jump" in the migration dynamic.*

Proof. See the Appendix B ■

Therefore, in aggregate,

Proposition 2 *the effect of a community is the reduction of the migration costs through the network system: this fact implies a lower threshold level that triggers the entry.*

¹²The myopic behaviour implies that:

1. the migrant is ignoring that future entry by other migrants will reduce her net benefits. Other things equal, this would make entry more attractive for the migrant that behaves myopically;
2. she ignores the fact that the prospect of future entry by other migrants reduces the option value of waiting. In fact, pretending to be the last to enter the host country, she thinks that she still has a valuable option to wait before making an irreversible decision. Other things equal, this makes the decision to enter less attractive. The two effects offset each other, allowing the migrant to act as she were in isolation.

¹³Leahy's results cannot be extended to this case.

¹⁴We exclude the former by using subgame-perfectness arguments (see Moretto (2003)).

4 Graphic Solution

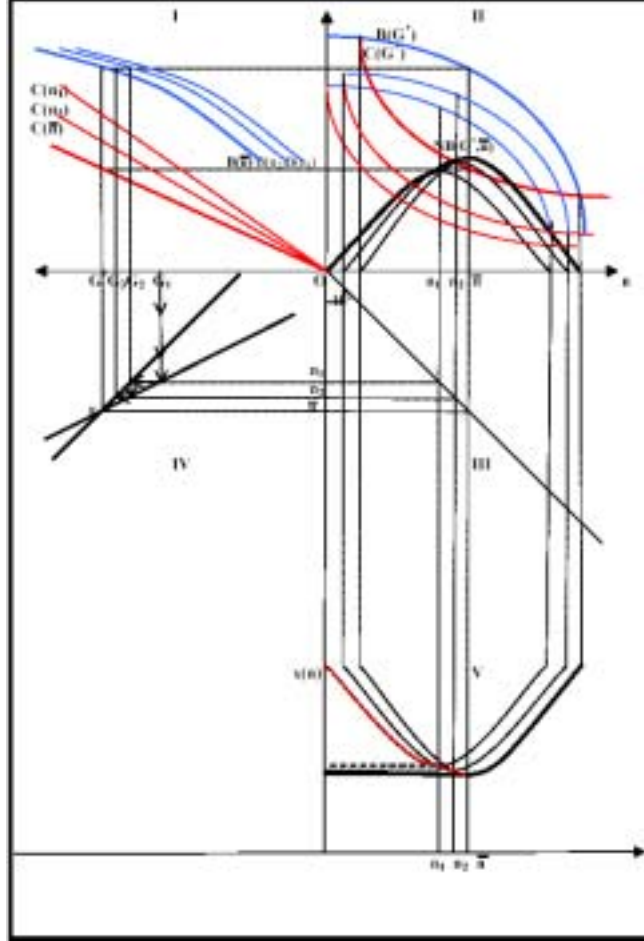


Figure 5: Optimal trigger

In figure 5 we extend figure 4, by adding the optimal trigger levels in quadrant V. Let us start in the instant t with a given dimension G_1 of the district. According to the theory of clubs¹⁵ the process of convergence until the optimal couple (G^*, \bar{n}) is reached instantaneously: this fact implies that the migrant's optimal policy moves along the envelope curve of the different threshold levels for different community dimensions, i.e. the lowest U-shaped curve (the bold black and red line¹⁶ in quadrant V). Nevertheless, when the network effect pre-

¹⁵In Cornes and Sandler: "the community [club] desires a membership n_1 when the dimension of the district is G_1 ; however, a larger district size G_2 is required to maximise average net benefits (in quadrant I) when membership is n_1 ".

¹⁶It is worth noting that the red line corresponds to the optimal policy of one migrant as if

vails (see result 2), the optimal policy consists of waiting until \bar{n} individuals are co-ordinated to enter: this implies that the optimal differential wage perceived by each migrant is the flat bold black line in quadrant V. How can change our model if the adjustment of the district size is not instantaneous? Some possible scenarios:

1. The dimension of the district changes very slowly when immigrants enter: in this case, if the equilibrium is far from the optimal couple, the new level of members requires an increase of the variable G . The migration dynamic should evolve according to the following path: an initial mass of entries, followed by an individual entry (due to the crowding effect and the myopic behaviour of the migrant). The subsequent increase of the district size continues the process of convergence until the optimal couple is reached. Therefore, the migrant entry process should follow the dotted line shown in *quadrant V* of the figure 4. This non-instantaneous process could imply two types of effects:
 - (a) the hysteresis phenomenon is amplified by the slowness of the development of the district: migrants wait until the number of members reaches the level \bar{n} ;
 - (b) immigrants are short-sighted and they are not able to correctly forecast the optimal couple (G^*, \bar{n}) . In this case, a group could start when the trigger reaches the level corresponding to the first horizontal dotted-line, because they do not forecast an increasing level of G . Subsequently, with the entry of this group of immigrants, the dimension of the district will increase at a higher level of G , reducing the threshold. The explicit migration dynamic should be represented by some jumps of lower and lower magnitude with a decreasing threshold;
2. The government is able to control the district size by imposing some limits to the urbanisation of a peculiar area. In this case, let us assume that the government thinks that a bound is required and publically declares that the dimension of the districts will be fixed at a given level G' . This action should reduce the number of migrants, because of a lower G respect to the optimal level. But is this policy credible? And do the potential immigrants really believe in this declaration? If immigrants believe the government, the entry dynamics will follow the dotted line in quadrant V. However, generally the government increases the permits for buildings if the migration inflows increase. In this case, if migrants perfectly forecast the optimal path (i.e. no bounds on the district size), the entry dynamic will be the bold black line¹⁷.

she were the last to enter the community, or as if there were a forced order for the entry.

¹⁷ Another hypothesis could be that migrants believe the government only once. When they realise that the government is a "liar" they will perfectly forecast the optimal path. According to this explanation, the dynamic decision will follow the dotted line until this touches the bold black line and it will follow the bold black line, thereafter.

Therefore, the final:

Proposition 3 *The static nature of the evolution of the district¹⁸ can strengthen the hysteresis phenomenon of migration choice.*

5 Conclusions

Real option theory suggests that migration may be delayed beyond the Marshallian trigger since the option value of waiting may be sufficiently positive in the face of uncertainty. Intuition, as is well known from the pioneering work of Dixit and Pindyck (1993), is that waiting may resolve uncertainty and thus enable avoidance of the downside risk of an irreversible investment. Burda (1995) was the first to use real option theory to explain slow rates of migration from East to West Germany despite a large wage differential. Subsequent works (Khawaja, 2002; Anam *et al.*, 2004) have developed this approach describing the role of uncertainty in the migration decision. In this work, we present a model where each individual can choose to migrate to a host country depending on the wage differential and an externality stemming from the community of individuals, in the light of recent literature showing that the role of the community is important for the migration decision (Moretti, 1998; Bauer *et al.*, 2004). In our model, the decision to migrate depends not only on the wage differential, but also on a U-shaped function modelled according to the "*theory of clubs*". By studying the Real Option Theory (Bartolini, 1995; Leahy, 1995; Moretto, 2003) in depth, it is possible to implement Burda's model with a possible co-ordination of migrants to enter the host country. The theoretical results are able to give an explanation to the observable "jumps" in the migration flows and to describe how the trigger of entry can change depending on the dimension of the district. In fact, given a particular shape of the community benefit function, the optimal entry policy consists in co-ordinating migration altogether, when the benefit received reaches the maximum level: this explains the observable mass of immigrants entering a host country. The analysis of the results also sheds light on the dynamics of the districts' development: some possible rigidities in the adjustment of the district dimension, as regards the optimal levels, could magnify the hysteresis process.

¹⁸that is fixed to a given G^* . This fact is an implicit consequence of assumption 7.

A Appendix

Starting within a time interval where no new immigrants enter, i.e. n is fixed, the Bellman equation is:

$$\rho V(x, n) dt = [x + \theta u(n)] dt + E[dV(x, n)] \quad (8)$$

Assuming $V(x, n)$ to be a twice-differentiable function as regards x and applying *Ito's Lemma* to expand $dV(x, n)$, and also taking into account that n is fixed, we obtain:

$$\frac{1}{2} V_{xx} \sigma^2 x^2 + V_x \alpha x - \rho V(x, n) + [x + \theta u(n)] = 0 \quad (9)$$

The solution of (9) is given by the sum of the general solution for the homogeneous equation and of a particular solution for the inhomogeneous equation. Therefore, a solution for the homogeneous equation must first be found:

$$\frac{1}{2} V_{xx} \sigma^2 x^2 + V_x \alpha x - \rho V(x, n) = 0 \quad (10)$$

Using a guess solution of the form:

$$V(x, n) = Ax^\beta$$

implies

$$\begin{aligned} V_x &= \beta Ax^{\beta-1} \\ V_{xx} &= \beta(\beta-1) Ax^{\beta-2} \end{aligned}$$

Substituting into the homogeneous equation and dividing by Ax^β leads to:

$$\frac{1}{2} \beta(\beta-1) \sigma^2 + \beta \alpha - \rho = 0 \quad (11)$$

The roots of the quadratic equation (11) are:

$$\beta_1 = \frac{(\alpha - \frac{1}{2}\sigma^2) + \sqrt{(\alpha - \frac{1}{2}\sigma^2)^2 + 2\sigma\rho}}{\sigma^2} > 0 \quad (12)$$

$$\beta_2 = \frac{(\alpha - \frac{1}{2}\sigma^2) - \sqrt{(\alpha - \frac{1}{2}\sigma^2)^2 + 2\sigma\rho}}{\sigma^2} < 0 \quad (13)$$

The general solution for the homogeneous equation (10) is:

$$V(x, n) = A(n)x^{\beta_1} + B(n)x^{\beta_2} \quad (14)$$

Then, the particular solution for the inhomogeneous equation (9) takes the form:

$$v(x, n) = E_0 \left[\int_0^\infty e^{-\rho t} [x - \theta u(n)] dt \right] = \frac{x}{\rho - \alpha} + \frac{\theta u(n)}{\rho} \quad (15)$$

because the boundary conditions require that $\lim_{x \rightarrow \infty} \{V(x, n) - v(x, n)\} = 0$, where the second term in the limit represents the discounted present value of the net benefits flows over an infinitive horizon starting from x (Harrison, 1985, p.44) and from (14) to keep $V(x, n)$ finite as x becomes small, i.e. $\lim_{x \rightarrow 0} V(x, n) = 0$, we discard the term in the negative power of x setting $B = 0$.

The general solution then reduces to (Dixit and Pindyck, 1993, pp.179-180):

$$V(x, n) = A(n)x^{\beta_1} + \frac{x}{\rho - \alpha} + \frac{\theta u(n)}{\rho} \quad (16)$$

The last two terms represent the value of the migrant net benefits in the absence of a new entrant, then $A(n)x^{\beta_1}$ is the correction due to a new entry and $A(n)$ must be negative. To determine this coefficient we need to impose some suitable boundary conditions. First of all, free entry requires the idle migrant to expect zero benefits on entry. Then, indicating by x^* the value at which the n^{th} migrant is indifferent between immediate entry or waiting another opportunity, the *matching value condition*:

$$V(x^*(n), n) = A(n)x^*(n)^{\beta_1} + \frac{x^*(n)}{\rho - \alpha} + \frac{\theta u(n)}{\rho} = K \quad (17)$$

It is worth noting that the number of individuals n affects $V(x, n)$ depending on x^* and $u(n)$. By totally differentiating as regards n we obtain:

$$\begin{aligned} \frac{dV(x^*(n), n)}{dn} &= V_n(x^*(n), n) + V_x(x^*(n), n) \cdot \frac{dx^*(n)}{dn} \\ &= A'(n)x^*(n)^{\beta_1} + \frac{\theta u'(n)}{\rho} + \left[A(n)\beta_1 x^*(n)^{\beta_1-1} + \frac{1}{\rho - \alpha} \right] \cdot \frac{dx^*(n)}{dn} = 0 \end{aligned} \quad (18)$$

Moreover, since each individual rationally forecasts the future development of the new entries by other migrants, at the optimal entry threshold, we obtain $V_n(x^*(n), n) = A'(n)x^*(n)^{\beta_1} + \frac{\theta u'(n)}{\rho} = 0$ (Bartolini, 1993; proposition 1). This modifies the above condition to:

$$V_x(x^*(n), n) \frac{dx^*(n)}{dn} = \left[A(n)\beta_1 x^*(n)^{\beta_1-1} + \frac{1}{\rho - \alpha} \right] \frac{dx^*(n)}{dn} = 0 \quad (19)$$

(19) and (17) explain that either each individual exercises her entry option at the level of x at which her value is tangent to the entry cost K , or the optimal x does not change with n . While the former case means that the value function is smooth at entry and the trigger is a continuous function of n , the latter

case says that if this condition is not satisfied, the individual would benefit from marginally anticipating or delaying her entry decision. In particular if $V_x(x^*(n), n) < 0$ (> 0), it means that the value function is expected to increase (decrease) if x drops (rises). In both situations (19) is satisfied by imposing $\frac{dx^*(n)}{dn} = 0$, then the same level of shock may either trigger entry by a positive mass of migrants or lock-in the entry at the initial level of individuals.

A.1 Proof of Result 1: optimal trigger value with congestion costs

Let us study the optimal trigger value in the decreasing part of the net benefit function (i.e. $n \geq \bar{n}$). We consider the value for a migrant starting at the point $(n, x < x^*)$, that would follow the optimal policy hereafter. Indicating by T the first time that x reaches the trigger x^* , $n_0 = n$ and $x_0 = x$, the optimal policy must then satisfy:

$$V(x, n) = \max_{x^*} E_0 \left[\int_0^T e^{-\rho t} [x_t + \theta u(n)] dt + \int_T^\infty e^{-\rho t} [x_t + \theta u(n)] dt \right] \quad (20)$$

Using (17), knowing also that $u(n)$ does not depend on the stochastic process, we can write:

$$\begin{aligned} V(x, n) &= \max_{x^*} E_0 \left[\int_0^T e^{-\rho t} [x_t] dt + \int_0^\infty e^{-\rho t} \theta u(n) dt + e^{-\rho T} \max_{x^*} \int_T^\infty e^{-\rho t} [x_t] dt \right] \\ &= \max_{x^*} E_0 \left[\int_0^T e^{-\rho t} [x_t] dt + \frac{\theta u(n)}{\rho} + e^{-\rho T} \left[V(x^*(n), n) - \frac{\theta u(n)}{\rho} \right] \right] \end{aligned}$$

and then:

$$V(x, n) = \max_{x^*} \left\{ E_0 \left[\int_0^T e^{-\rho t} [x_t] dt \right] + \frac{\theta u(n)}{\rho} + \left[K - \frac{\theta u(n)}{\rho} \right] \cdot E_0 [e^{-\rho T}] \right\} \quad (21)$$

The expected values above can be found using standard results in the theory of regulated stochastic processes¹⁹. Substituting the following expressions taken by Dixit-Pindyck (1993):

$$E_0 \left[\int_0^T e^{-\rho t} [x_t] dt \right] = \frac{x - x^{\beta_1} (x^*)^{1-\beta_1}}{\rho - \alpha} \quad (22)$$

¹⁹ see Karlin and Taylor (1974, ch.7); Harrison and Taksar (1983); Harrison (1985, ch.3), Dixit-Pindyck (1993, 315-316), Moretto (1995) and Moretto (2003).

$$E_0 [e^{-\rho T}] = \left(\frac{x}{x^*}\right)^{\beta_1} \quad (23)$$

we obtain:

$$V(x, n) = \max_{x^*} \left\{ \frac{x}{\rho - \alpha} + \frac{\theta u(n)}{\rho} - \left[\frac{x^*}{\rho - \alpha} - K + \frac{\theta u(n)}{\rho} \right] \cdot \left(\frac{x}{x^*}\right)^{\beta_1} \right\} \quad (24)$$

Now, to choose optimally x^* , the FOC is:

$$\frac{\partial V}{\partial x^*} = \left\{ (\beta_1 - 1) \cdot \frac{1}{\rho - \alpha} - \frac{\beta_1}{x^*} \cdot \left[K - \frac{\theta u(n)}{\rho} \right] \right\} \cdot \left(\frac{x}{x^*}\right)^{\beta_1} = 0$$

The optimal threshold function takes the form:

$$x^*(n) = \frac{\beta_1}{\beta_1 - 1} \cdot (\rho - \alpha) \cdot \left[K - \frac{\theta u(n)}{\rho} \right] \quad (25)$$

From (25) we can observe that the threshold value is inversely correlated with the U-shaped function of the net benefits of the community. Therefore, the greater the numbers of members in the community, the higher must be the threshold value in order that the migrant could migrate.

From (17) we obtain:

$$A(n) = \left[K - \frac{\theta u(n)}{\rho} - \frac{x^*(n)}{\rho - \alpha} \right] \cdot (x^*(n))^{-\beta_1} \quad (26)$$

Rearranging (25) and (26):

$$A(n) = \left[-\frac{K}{\beta_1 - 1} + \frac{\theta u(n)}{\rho(\beta_1 - 1)} \right] (x^*)^{-\beta_1} \equiv -\frac{(x^*)^{1-\beta_1}}{\beta_1(\rho - \alpha)} < 0 \quad (27)$$

Now we can observe that (27) is the solution of (19) and also, from Bartolini's assumptions, the solution of (18). Substituting (27) and (25) into (24), we obtain (16).

By (25), we can observe that the community function affects the threshold value, reducing it for a decreasing quantity in the interval $[\bar{n}, n]$. This fact implies that the congestion cost affects the migrant's decision and we have increasing competition among the members of community. If the individual claims to be unique or the last to enter the community, then $A'(n) = 0$ and the first order condition (18) reduces to $V_x(n, X^*(n)) = 0$.

A.2 Proof of Result 2: optimal trigger value with network externalities

Studying of the threshold value in the decreasing part of the U-shaped benefits function, we stress that, in the interval $[n, \bar{n}]$, the optimisation problem induces all immigrants to follow the same strategy: **waiting until the trigger value**

reaches $x^*(\bar{n})$. Assuming that all immigrants are equal and have the same characteristics, they will leave their country in the same instant involving a wave in the increasing part of the U-shaped function. A simple argument to explain the optimal choice of the migrant, takes into consideration the idea that she could claim to be the last to enter at $x = x^*$. By (15) her value is simply $V(x^*(n), n) = \frac{x}{\rho - \alpha} + \frac{\theta u(n)}{\rho}$. It then follows easily that:

$$V(x^*(n), n) - \lim_{x \rightarrow x^*(n)} V(x, n) = \frac{K}{\beta_1 - 1} - \frac{[\theta u(n)]}{(\beta_1 - 1)\rho} \quad (28)$$

From the (28) is easy to observe two facts:

1. In (28) the difference is greater than zero when $K > [\theta u(n) / \rho]$. This correction is due to a new entry into the community. This contradicts the *smooth pasting condition* (19), explaining that it isn't the optimality we are searching for. The only point in order that 28 is equal to zero, is the optimal one (we have in fact $A(n) = 0$);
2. The community effect modifies the benefit obtained by the migrant. From the *smooth pasting condition* we know that the optimality trigger is reached when $V_x(x^*(n), n) = 0$; therefore since the function is increasing with the number of migrants, the greater the number of community members the nearer x is to the optimal trigger.

As the function is decreasing in the observed interval, the upward jump in benefits would decrease as more immigrants have already entered and it disappears when $n = \bar{n}$ where the value function is the known function (16). It follows that the optimal level of wage gap $x^*(\bar{n})$, that triggers entry, is where the total benefit flow is maximum for all the discrete sizes of entrants $(\bar{n} - n)$.

It is at the maximum level when (19) is checked and when $u'(n) = 0$, i.e. $n = \bar{n}$. This will be shown in the next part.

Let's assume that the migrant waits until the first time the process $x(n)$ rises to the myopic²⁰ trigger level $\kappa \equiv x^*(\xi)$, corresponding to an immediate increase of the community size to $\xi > n$ and let us also assume that the immigrant expects no more entry after ξ . Consequently, her expected payoff $V(x, \xi)$ from the time onwards equals the discounted stream of benefits fixed at $u(\xi)$, i.e. by (15):

$$V(x, \xi) = \frac{x}{\rho - \alpha} + \frac{\theta u(\xi)}{\rho} \quad (29)$$

Comparing (29) with (15) gives $A(\xi) = 0$

Now, to obtain the constant $A(n)$, subject to the claim that beyond ξ no other immigrant will enter the community, we substitute (15) into the condition $V_n(x^*(n), n) = 0$ resulting in:

²⁰ By Leahy (1993).

$$A'(n) = -\frac{\theta u'(n) x^*(n)^{-\beta_1}}{\rho} \quad (30)$$

integrating (30) between n and ξ we get:

$$\int_n^\xi A'(z) dz = -\frac{\theta x^*(n)^{-\beta_1}}{\rho} \int_n^\xi u'(z) dz$$

Knowing that $A(\xi) = 0$ we obtain the value of $A(n)$:

$$A(n) = \frac{\theta x^*(n)^{-\beta_1}}{\rho} [u(\xi) - u(n)] \quad (31)$$

As long as $u(\xi) > u(n)$ the first term in (31) is positive and it forecasts the advantage the immigrant would experience by the entry of $\xi - n$ immigrants when x hits x^* .

If the migrant could decide the optimal time for entry, she would choose when x reaches $x^*(\bar{n})$ and because all immigrants are equal, they coordinate a simultaneous entry into the host country and then into the economy.

B Coordination of Migrants

By using Moretto's methodology (2003), we show that the best entry strategy for each entrant pushes her to coordinate entering the country and the community. The entrants decide to enter in mass, leading to an observable jump, when the differential wage reaches the trigger level $x^*(\bar{n})$ at the maximum obtainable level of benefit.

Given a group of potential migrants n :

1. we consider a one-shot-discrete-time game between a generic i^{th} migrant and a pool of $(\bar{n} - n)_{-i} < N$ other migrants, called $-i^{th}$. The pool of migrants is considered like the other player;
2. the strategies of all immigrants are taken at fixed times: t_0 for $x^*(\bar{n})$ and t_1 for $x^*(n)$. From (6) and (7) is easy to observe that $x^*(\bar{n}) < x^*(n)$ and also that $t_1 > t_0$. When the differential wage reaches $x^*(n)$ for the first time, she has an action set that is $\{Entry[E], No\ Entry[NE]\}$. If she decides to enter, her set is $\{Stay\ in\}$ forever. If she doesn't decide to enter, she can wait for $x^*(n)$ to be reached before entering.

We use the following notation to describe the different actions that can occur:

1. $M(t_0)$ is the expected discounted value of each migrant if all invest together at t_0 ;
2. $L(t_0, t_1)$ is the (leader's) discounted value for the migrant that enters at t_0 , while all the others wait until t_1 ;

3. $F(t_0, t_1)$ is the (follower's) discounted value for the migrant that waits until t_1 before entering while the others make the move at t_0 ;
4. $MM(t_1)$ means that at t_1 it is always optimal to enter.

We have the following functions:

$$M(t_0) \equiv E_{t_0} \left[\int_{t_0}^{\infty} e^{-\rho t} (x_t + \theta u(\bar{n})) dt \mathbb{P} x_{t_0} = x^*(\bar{n}) \right] - 1 \quad (32)$$

$$= \frac{x^*(\bar{n})}{\rho - \alpha} + \frac{\theta u(\bar{n})}{\rho} - 1 \quad (33)$$

$$MM(t_1) \equiv E_{t_0} \left\{ e^{-\rho(t_1 - t_0)} \left[\int_{t_1}^{\infty} e^{-\rho t} (x_t + \theta u(\bar{n})) dt \mathbb{P} x_{t_1} = x^*(n) \right] - 1 \right\} \quad (34)$$

$$= \left(\frac{x^*(n)}{\rho - \alpha} + \frac{\theta u(\bar{n})}{\rho} - 1 \right) \left(\frac{x^*(\bar{n})}{x^*(n)} \right)^{\beta_1} \quad (35)$$

$$F(t_0, t_1) \equiv E_{t_0} \left\{ e^{-\rho \Delta t} \left[\int_{t_0 + \Delta t}^{\infty} e^{-\rho t} (x_t + \theta u(\bar{n})) dt \mathbb{P} x_{t_0} = x^*(\bar{n}) \right] - 1 \right\} \quad (36)$$

$$= \left(\frac{x^*(\bar{n}_{-i})}{\rho - \alpha} + \frac{\theta u(\bar{n})}{\rho} - 1 \right) \left(\frac{x^*(\bar{n})}{x^*(\bar{n}_{-i})} \right)^{\beta_1}$$

$$L(t_0, t_1) \equiv E_t \left\{ \int_{t_0}^{t_1 - \Delta t} e^{-\rho t} (x_t + \theta u(n_{+i})) dt + \left[\int_{t_1 - \Delta t}^{\infty} e^{-\rho t} (x_t + \theta u(\bar{n})) dt \mathbb{P} x_{t_0} = x^*(\bar{n}) \right] - 1 \right\} \quad (37)$$

$$= \left(\frac{x^*(\bar{n})}{\rho - \alpha} + \frac{\theta u(n_{+i})}{\rho} - 1 \right) + \left[\frac{\theta(u(\bar{n}) - u(n_{+i}))}{\rho} - \frac{x^*(n_{+i}) - x^*(\bar{n})}{\rho - \alpha} \right] \left(\frac{x^*(\bar{n})}{x^*(n_{+i})} \right)^{\beta_1}$$

It's possible to show and demonstrate that the payoffs of the four actions described follow this disequality:

$$L(t_0, t_1) < MM(t_1) < F(t_1) < M(t_0) \quad (38)$$

The (38) can be demonstrated considering that:

$$\left(\frac{\theta u(\bar{n})}{\rho} - 1 \right) \left(\frac{x^*(\bar{n})}{x^*(n)} \right)^{\beta_1} < \left(\frac{\theta u(\bar{n})}{\rho} - 1 \right) \left(\frac{x^*(\bar{n})}{x^*(\bar{n}_{-i})} \right)^{\beta_1} < \left(\frac{\theta u(\bar{n})}{\rho} - 1 \right)$$

because $\beta_1 > 1$ and $x^*(\bar{n}) < x^*(n)$ for every $n \neq \bar{n}$ and also because $x^*(\bar{n}) < x^*(\bar{n}_{-i}) < x^*(n_{+i}) < x^*(n)$.

We summarise these result in the following table:

i	-i		
		E	NE
	E	M,M	L, _—
	NE	F, _—	MM,MM

Table 1

It is worth noting that there are only two candidates for the symmetric Nash equilibria in pure strategies: "all E", "all NE", where "all E" is Pareto-dominant²¹. Therefore for the migrant, it is optimal to coordinate reaching the highest net benefits obtainable from the decision to migrate. When the migrant is on the increasing part of the benefit function, she waits till the wage hits the level $x^*(n)$ (i.e. the maximum benefit obtainable for a given dimension). With regard to this fact, the best thing she can do is to co-operate with other immigrants and to enter as a group at the threshold level $x^*(n)$.

²¹Moretto (2003) shows that the equilibrium is also maintained with mixed strategy equilibria.

References

- [1] Anam, M., S. H. Chiang and L. Hua, (2004), "Uncertainty and International Migration: an Option cum Portfolio Model", mimeo
- [2] Bartolini, L., (1993), "Competitive Runs: The Case of a Ceiling on Aggregate Investment", *European Economic Review*, Vol. 37, pp. 921-948
- [3] Bauer, T. G., Epstein and I. N. Gang, (2002), "Herd Effects or Migration Networks? The Location Choice of Mexican Immigrants in the U.S.", *IZA Discussion Paper*, N. 200
- [4] Bauer, T. and K. F. Zimmermann, (1997), "Network Migration of Ethnic Germans", *International Migration Review*, Vol. 1, 1, pp. 143-149
- [5] Bencinverga, V. R. and B. D. Smith, (1997), "Unemployment, Migration, and Growth", *The Journal of Political Economy*, Vol. 105, Issue 3, pp. 582-608
- [6] Berglas, E., (1976), "On the Theory of Clubs", *The American Economic Review*, Vol. 66, N. 2, pp.116-121
- [7] Buchanan and Goetz, (1972), "Efficiency Limits of Fiscal Mobility: An Assessment of the Tiebout Model", *Journal of Public Economy*, 1, pp. 25-43
- [8] Buchanan, J. M., (1965), "An Economic Theory of Clubs", *Economica*, Vol. 32, 125, pp. 1-14
- [9] Burda, M. C., (1995), "Migration and the Option Value of Waiting", *The Economic and Social Review*, 27, pp. 1-19
- [10] CARITAS, (2003), *Dossier Statistico Immigrazione*, Rome
- [11] CARITAS, (2003), *Contemporary Immigration in Italy*, Rome
- [12] Coniglio, N., D, (2003), "Migropolis: Migration Networks and Formation of Ethnic Clusters in Cities", *SNF Working Paper* , 46/03
- [13] Cornes, R. and T. Sandler, (1986), *Theory of Externalities, Public Goods and Club Goods*, Cambridge University Press, Cambridge, Mass
- [14] Cox, D. R. and H. D. Miller, (1965), *The Theory of Stochastic Process*, London, Chapman and Hall
- [15] Dixit, A. and R. S. Pindyck, (1993), *Investment under Uncertainty*, Princeton (NJ): Princeton University Press
- [16] Edwards, J. H. Y., (1992), "Indivisibility and Preference for Collective Provision", *Regional Science and Urban Economics*, 22, pp. 559-577

- [17] Epstein, G. S., and I. Gang, (2004), "Ethnic Networks and International Trade", *CEPR Discussion Paper*, 4616
- [18] Harris, J. R. and M. P. Todaro, (1970), "Migration, Unemployment and Development: A Two-Sector Analysis", *The American Economic Review*, Vol. 60, Issue 1, pp. 126-142
- [19] Harrison J. M., (1985), *Brownian Motion and Stochastic Flow Systems*, New York: John Wiley & Son
- [20] Istituto Nazionale di Statistica, (1965), *Sommario di Statistiche Storiche*, Rome
- [21] Khwaja, Y., (2002), "Should I Stay or Should I Go? Migration Under Uncertainty: A Real Option Approach", mimeo,
- [22] Leahy, J. P., (1993), "Investment in Competitive Equilibrium: the Optimality of Myopic Behavior", *Quarterly Journal of Economics*, 108, pp. 1105-1133
- [23] Locher, L. (2001), "Testing for the Option Value of Migration", *IZA Discussion Paper*, No. 405
- [24] McDonald, R. and D., Siegel, (1986), "The Value of Waiting to Invest", *The Quarterly Journal of Economics*, Vol. 101, Issue 4, pp. 707-728
- [25] Mc Guire, M., (1972), "Private Good Clubs and Public Good Clubs: Economic Models of Group Formation", *Swedish Journal of Economics*, 74, pp. 84-99
- [26] McGuire, M., (1974), "Group Segregation and Optimal Jurisdictions", *The Journal of Political Economy*, Vol. 82, 1, pp. 112-132
- [27] Moretti, E., (1999), "Social Networks and Migrations: Italy 1876-1913", *International Migration Review*, 33 (3), pp. 640-657
- [28] Moretto, M., (2003), "Competition and Irreversible Investments under Uncertainty", *Feem Nota di Lavoro* 32.03
- [29] Moretto, M., (1995), "Controllo ottimo stocastico, processi browniani regolati e optimal stopping", *International Review of Economics and Business*, 42, pp. 93-124
- [30] Munshi K., (2003), "Networks in the Modern Economy: Mexican Migrants in the U.S. Labour Market", *Quarterly Journal of Economics*, Vol 118 (2), pp. 549-599
- [31] Sandler, T. and J. T. Tschirhart, (1980), "The Economic Theory of Clubs: An Evaluative Survey", *Journal of Economic Literature*, Vol. 18, 4, pp. 1481-1521

- [32] Sjaastad, L. A., (1962), "The Costs and Returns of Human Migration", *The Journal of Political Economy*, Vol. 70, 5, Part 2, pp. 80-93
- [33] Tiebout, C., (1956), "A Pure Theory of Local Expenditures", *The Journal of Political Economy*, 64 (5), pp. 416-24
- [34] Todaro, P. M., (1969), "A Model of Labour Migration and Urban Unemployment in Less Developed Countries", *The American Economic Review*, Vol. 59, 1, pp.138-148
- [35] Urrutia, C., (2001), "On the Self-Selection of Immigrants", mimeo
- [36] Winters, P., A. de Janvry and E. Sadoulet, (2001), "Family and Community Networks in Mexico-U.S. Migration", *The Journal of Human Resources*, Vol. 36, 1, pp. 159-184