Abstract

In this paper we assess higher education settings where the open access principle holds and where universities are (eventually) differentiated by the difficulty-standard of their courses. Standards are assumed to influence negatively the graduation probability and positively the human capital accumulation during studies. In this setting, secondary school graduates choose whether and where to enroll by considering their own talent, family willingness to finance education and the offered standard(s). We compare a homogeneous standard case (centralization) with a heterogeneous standard case (decentralization). If a homogeneous standard is set in order to maximize the enrollment rate, human capital accumulation is sub-optimal. The heterogeneous standard setting may perform worst in terms of human capital accumulation if moving costs are not completely financed and it always performs worst in terms of intergenerational mobility in education.

1 Introduction

There exists a huge literature on wage rate differences among graduates showing that the labour market rewards workers not only according to their educational level, field and grade, but also by the quality of the attended university. Recent results highlight that the return to college selectivity is sizeable, but, as underlined by [Dale and Krueger (2011)], adjusting “for
unobserved student ability by controlling for the average SAT score of the colleges that students applied to, our estimates of the return to college selectivity fall substantially and are generally indistinguishable from zero. There were notable exceptions for certain subgroups. ... for students who come from less-educated families... the estimates of the return to college selectivity remain large”.

The typical explanation of the wage premium differences between universities lays both on the selection of the enrolled, usually based on previous curricula and entry tests, and on the actual quality of the higher education institution attended. “Best” universities give their students a better learning environment (tutors, laboratories, better internships opportunities and so on) and therefore allows for a higher human capital accumulation during studies. Nevertheless, a good learning environment is costly and usually requires a higher expenditure per student and higher tuition fees.

In the theoretical model we present here, we assume that all universities ask for the same tuition fees and show the same efficiency in the learning process and that the (eventual) differences among them arise from the commitment required to student to pass the exams, from professors severity, from the whole policies of the university in helping weak students; all these universities characteristics will be defined university standards thereafter.

Furthermore, we consider institutional settings where the open-access policy holds and, once financial constraints have been taken into account, students can freely choose the university where to enroll to. We assume that they self-select in universities that matches their talent at the best, so that more talented students enroll in more difficult universities.

This hypothesis is hard to be empirically tested. Nevertheless, we should observe that students who actually choose the university where to enroll (movers) behave somewhat differently with respect to students that, mainly because of financial constraint, choose the university closer to their residence (stayers). By assuming that movers choose the university whose standard adapts better to their talent whereas stayers must accept the random standard of the closest university, we should expect that the drop-out rate of stayers depends negatively on their talent, whereas the drop-out rate of movers could be independent on talent.

In order to give some “naive” evidence of the above statement and given that Italian higher education is open-access, we analyse the survey “L’inserimento professionale dei diplomati”, provided by the Italian Institute of Statistics (ISTAT, 2007). The micro-data collected refer to 25512

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2 A different theoretical approach considers the signaling relying on college grades and selectivity, as in [Hershbein (2013)]

[Dillon and Smith (2015)] consider the effects of the interaction between student ability and college quality on academic outcomes and future earnings. They find “little evidence to support the “mismatch” hypothesis that college quality and ability interact in substantively important ways”.

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individuals who obtained their secondary school degree in 2004, interviewed 3 years later. Information on the (eventual) chosen university, on family background, on drop-out during studies are available. A proxy for individual talent can be built.  

Figure 1 shows the relationship between the drop-out rate and the proxy of individual talent for stayers and movers as a fitted quadratic plot with 95% confidence intervals. Figure 1 states that the negative relationship between the talent and the drop-out rate exists only for stayers. Movers are not significantly affected by talent in their drop-out decision.

How can this result be explained? The higher commitment once the moving decisions has been made explains the strong difference in the average drop out rate between stayers (22%) and movers (6%), but it cannot explain why talent does not significantly affect movers drop-out. Our explanation is the one proposed above: between movers, more talented students choose high standard universities, whereas less talented ones choose universities characterized by low standard.

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4Secondary school students obtain the final diploma by passing an exam (“esame di maturità”) set at Governmental level. In this exam, each student is evaluated by a numerical grade. Following Pigini and Staffolani, 2015, we build a proxy of the individual talent as the residual of an estimation where the grade is regressed on the interaction of the type of school and the province of residence during secondary school.

5 Movers are those secondary school graduates who moved to other districts even if a university offering the field of study they chose was located in their secondary school district (3556 obs). Among all the students who enroll without moving (11539 obs), stayers are a subsample (3556 obs) chosen considering the one-to-one best match in a propensity score matching model, where the regressors for the moving decision are the student’s talent, the type of school (5 classes), the region of residence (21 classes), the mother’s education (4 classes) and the father’s occupation (9 classes). All the estimates are available from the Authors.
In the described setting, differences among universities in the average human capital accumulation during studies depend on two reasons:

1. secondary school graduates select themselves into university according to their talent so that high-standard universities attract more talented students and, if talent and human capital accumulation during studies complement each other, high talented students accumulate more human capital.
   → human capital accumulation due to self-selection;

2. for any given talent, high standard universities require a stronger effort from their students.
   → human capital accumulation due to university standard.

To summarize, the model is based on the assumptions:

- tuition fees and “quality” are equal in all the universities;
- the open access principle holds so that the choice of the university where to enroll is in the students’ hands;
- university can offer differentiated standards.

Actually, standards act as “prices” of the risky investment in human capital: a higher standard (price) implies a higher risk of dropping-out (investment failure) and a higher wage (investment return). In this setting, we analyse theoretically:

- a “homogeneous standard case”, where all universities must comply with a given level of standard. We will also define this as centralized higher educational system. There, each student chooses between enrolling at the university located in his/her district or not enrolling;

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6Theoretical models that take account of the described setting are rare. An exception is Bratti et al. (2007).

7The model can apply to every choice where one among different costly investments can be made and the chosen investment increases long-life utility with a probability depending positively on individual talent and negatively on the return of the investment. For instance, illegal migration decision in the case that the migrant risks to be repatriated if discovered is higher in richer countries. Concerning education, it can apply to all levels of education but is more suitable for college choices. Public compulsory schools usually does not differ too much, at least in many European countries. Even in the secondary school system, once the type of school has taken into account, the quality of education should be similar because frequently all students should reach the same qualification through a uniform State-managed final exam. Furthermore, until secondary school students are supposed to live with their family, and hence the differences in the quality between schools are hardly taken into account, especially for individuals living in small town where the supply of education can be very limited.
• a “heterogeneous standard case”, where each university can choose the standard of the course and the commitment required from enrolled students. This is the case of a decentralized higher education system where each student can enroll in the university offering his/her preferred standard by sustaining moving costs, or enroll at the closest university (whose standard is random), or not enrol.

In the case of standards homogeneity, our theoretical model concludes that the level of the standard that maximizes human capital accumulation is higher than the level of the standard that maximizes the enrollment rate. Therefore policies oriented to raise enrollment risk to be detrimental to human capital accumulation.8

From the theoretical model we also obtain that the decentralized system performs better than the centralized one in term of human capital accumulation, but only if the choice of the university where to enroll is achievable for all students, so that only if their moving decisions are completely financed.9 If this is not the case and the share of students that cannot afford moving costs is sufficiently high, human capital accumulation can be higher in a centralized system whose standard is efficiently chosen. Given that moving toward the preferred university is more likely for students coming from richer families, a decentralized system would reduce inter-generational mobility in education.10 Nevertheless, the recommendations of the European Commission to the Member States goes in the opposite direction of reinforcing decentralization without thorough investigate the consequences.11

8The “Reform of the universities in the framework of the Lisbon strategy” suggests, among many other points, that “broaden access, support student commitment and raise the success rate thanks to greater program diversity and more mobility, improved guidance and counselling, flexible admission policies and cheaper fees”.

http://europa.eu/legislation_summaries/education_training_youth/lifelong_learning/c11078_en.htm In our view, the risk is that the goal of “broaden access and success rate” is mainly pursued by university lowering the standard of the courses.

9Note that the model considers how financial constraints affect both the enrollment and the moving decisions, but deepen the analysis of the latter constraint.

10Actually, richer students seem to not be influenced in their choices by the distance of the university: Denzler and Wolter (2011): “The results also show that distance does not influence study choices among students from the highest socioeconomic group, a finding that further indicates that distance to university is an expression of differences in the cost of a university education.”

11“Over-regulation and nationally defined courses hinder modernization and the effective management of universities in the EU. To reform their governance, European universities are calling for more autonomy in preparing their courses and in the management of their human resources and facilities. They also want to reinforce public responsibility for the strategic orientation of the whole system. Hence it is not a call for the withdrawal of the State but for a new allocation of tasks. The Commission invites the Member States to relax the regulatory framework so as to allow university leadership to undertake genuine change and pursue strategic priorities.”

2 The Model

We consider a cohort of students, endowed each with a given talent $\theta$, distributed according to $\phi(\theta)$, with $\theta \in (0, \bar{\theta}]$. They completed a secondary school cycle and should choose whether and where to enroll at university, eventually by moving to other geographical districts\textsuperscript{12}.

We assume that their choice depends on the standards ($s > 0$) offered by the academic institutions. Standards are assumed to influence both the probability of graduation and the human capital accumulation during studies which, in turn, affects the expected wage rate on the labour market.

**Assumption 1** The probability of graduation $0 < p(s, \theta) < 1$ is increasing in talent and decreasing in the standard. It complies with the following:

\[
p'_s(s, \theta) < 0; \quad p'_\theta(s, \theta) > 0; \quad p''_{s\theta}(s, \theta) > 0; \quad p''_{ss}(s, \theta) > 0
\]

\[
\lim_{s \to 0} p(s, \theta) = 1 \quad \lim_{s \to \infty} p(s, \theta) = 0 \quad \lim_{\theta \to 0} p(s, \theta) = 0 \quad \lim_{\theta \to \bar{\theta}} p(s, \theta) < 1
\]

The human capital of a student endowed with talent $\theta$, who graduated in the university offering the standard $s$ is defined $\omega(s, \theta)$ that represents also the wage rate. The wage rate of undergraduates (both not-enrolled and dropped-out students) is linearly dependent on their human capital:

\[
\omega(0, \theta) = A \theta
\]

where $A$ is exogenously given.

**Assumption 2** The wage premium from graduation, $w(s, \theta) = \omega(s, \theta) - \omega(0, \theta)$, is an increasing, constant elasticity function of standard and talent. It complies with the following:

\[
w(0, \theta) = 0 \quad w'_s(s, \theta) > 0 \quad w'_{s\theta}(s, \theta) = 0 \quad w'_\theta(s, \theta) > 0 \quad w''_{s\theta}(s, \theta) < 0
\]

\[
\varepsilon_{ws}(s, \theta) = \beta \quad \varepsilon_{w\theta}(s, \theta) = 1
\]

where $\varepsilon$ indicate the elasticity.

Given the above assumptions and considering risk neutral individuals, the long-life utility associated to the non enrollment choice is given by:

\[
V^N = \frac{w(0, \theta)}{r} = \frac{A \theta}{r}
\]

\textsuperscript{12}The relationship between the ex-ante decision to starting university and the university outcomes is theoretically analysed in the seminal paper of Altonji (1993), that consider that the former decision is made under uncertainty. Oppedisano (2009) present a model of educational choices with uncertainty in countries with open admission policies at university entry. Our article add to the literature because it explicitly considers the differences in the standards offered by universities.
where $r$ is the discount rate.

The cost of enrollment is equal in all the universities; consumption during studies depends solely on family/State willingness to finance higher education and on the moving decision because of moving costs.

**Assumption 3** The family willingness to finance education $z$ depends on family wealth and, together with the student’s talent, defines consumption during studies, given by $z\theta$. Moving is costly and implies a lower value of $z$.

Therefore, the expected intertemporal utility associated with the enrollment choice is given by:

$$V^E = z\theta + \frac{1}{r(1+r)}[p(s,\theta)\omega(s,\theta) + (1-p(s,\theta))\omega(0,\theta)]$$

where the first addend represents utility during studies, and the second one is the ex-ante long-life utility after studies, given by the discounted sum of the wage rate of skilled and unskilled weighed by the probability of graduation.

Therefore the expected premium of enrolling is:

$$V^E - V^N = z\theta + \frac{1}{r(1+r)}[p(s,\theta)w(s,\theta) + A\theta] - \frac{A\theta}{r}$$

We define:

$$Z\theta = r(1+r)\left[\frac{A}{1+r} - z\right] \theta > 0$$

as the difference between utility in the case of non-enrolling discounted for one period and the well-being during studies. $Z$ represents the cost of failure, the cost of taking the enrolment decision if this decision end-up in drop out. It depends negatively on the family willingness to finance education, $z$ and it is therefore higher for poor and for movers (see assumption 3). If $Z < 0$ holds, then the student is always better off by enrolling. But, by assuming $Z > 0$ in the above equation, we remove this possibility.

The uni-periodal expected premium for enrolling function, that is the variation in expected uni-periodal utility coming from the enrolling decision, defined as $V(s,\theta,Z) = (V^E - V^N)r(1+r)$ becomes:

$$V(s,\theta,Z) \equiv p(s,\theta)w(s,\theta) - Z\theta$$

Where the first addend, $p(s,\theta)w(s,\theta)$ indicates the expected human capital accumulation during studies because it represents the expected increase in the wage rate due to graduation weighed by the graduation probability. The second addend is the cost of failure.

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13 Actually, some families could finance the enrollment decision with an amount of money higher than the expected earnings of the student as unskilled, so that $Z < 0$. 

7
Two peculiarities of the premium for enrolling function should be highlighted: each secondary school graduates has a “preferred” standard if the \( V(s, \theta, Z) \) function has a maximum in \( s \); higher talented individuals are more willing to enrol than less talented ones if the \( V(s, \theta, z) \) function is increasing in \( \theta \).

The sign of the premium for enrolling function split secondary school graduates between enrolled and not-enrolled and it allows to compute the enrolment rate, the graduation probability and the human capital accumulation in the whole economic system: more precisely, for a given \( Z \), it defines the relationship among talent and standard that make convenient to enroll.

In order to study this relationship we differentiate equation 1 with respect to \( s \):

\[
\frac{\partial V}{\partial s} = p'(s, \theta)w(s, \theta) + w'(s, \theta)p(s, \theta)
\]

that, using elasticities, can be written\(^{14}\)

\[
\frac{\partial V}{\partial s} = \frac{p(s, \theta)w(s, \theta)}{s} (\varepsilon_{ps}(s, \theta) + \beta)
\]

(2)

The derivative of equation 1 with respect to \( \theta \), rearranged in term of elasticities, gives:

\[
\frac{\partial V}{\partial \theta} = \frac{p(s, \theta)w(s, \theta) [\varepsilon_{\theta}(s, \theta) + 1]}{\theta} - Z\theta
\]

(3)

where \( \varepsilon_{p\theta}(s, \theta) > 0 \) and \( \varepsilon_{w\theta} = 1 \) because of assumption 1 and 2.

Using equations 2 and 3 we define the slope of the isopremium for enrolling function:

\[
\frac{\partial \theta}{\partial s} = -\frac{\theta}{s p(s, \theta)w(s, \theta)} (\varepsilon_{p\theta}(s, \theta) + 1) - Z\theta
\]

(4)

that implies that different students endowed with different talents can obtain the same expected premium for enrolment by enrolling in universities offering different standards. Figure 2 shows the isopremium curves for different values of \( V \) (left graphs) and different values of \( Z \) (right graph).

**Proposition 1** Each secondary school graduate maximizes expected intertemporal utility and human capital accumulation by enrolling at the university offering his/her optimal standard \( s = s^*(\theta) \).

**Proof 1** In equation 4 the denominator is surely positive for all the enrolled because for them \( p(s, \theta)w(s, \theta) - Z\theta \geq 0 \) must hold and \( \varepsilon_{p\theta} > 0 \). The slope\(^{14}\) \( V(s, \theta, Z) \) is concave in \( s \) if the second order condition: \( 2p'w' + p''w + w'p > 0 \) is respected. We assume it is, because otherwise the optimal standard would have been defined by a corner solution, \( s = 0 \) or \( s \to \infty \).

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\(^{14}\)
of the isopremium function as defined in equation 4 depends on the sign of \( \varepsilon_{p,s}(s, \theta) + \beta \). Given that \( \varepsilon_{p,s}(0, \theta) = 0 \) we should demonstrate that \( \frac{\partial \varepsilon_{ps}}{\partial s} < 0 \).

The derivative gives \( \frac{\partial \varepsilon_{ps}}{\partial s} = p''^s s + p' (\frac{s}{p} p'_s) \) that can be written: \( \frac{\partial \varepsilon_{ps}}{\partial s} = \frac{1}{p} (sp''_s + p'_s (1 - \varepsilon_{ps})) \). Given \( \varepsilon_{ps} < 0 \) and \( p'_s < 0 \), a sufficient condition to have \( \frac{\partial \varepsilon_{ps}}{\partial s} < 0 \) is \( sp''_s + p'_s < 0 \) that must be respected because \( p'_s \) is a convex function. Isopremium curves show therefore a minimum if:

\[
\varepsilon_{ps}(s, \theta) + \beta = 0 \implies s^*(\theta)
\]

where \( s = s^*(\theta) \) is the standard that maximises the premium for enrolment function for an individual endowed with a talent \( \theta \), the one that solves \( \frac{\partial V}{\partial s} = 0 \) in equation 2.

**Proposition 2** The optimal standard depends positively on student’s talent and does not depend on the cost of failure.

**Proof 2** By computing:

\[
\frac{\partial s^*(\theta)}{\partial \theta} = -\frac{\frac{\partial \varepsilon_{ps}}{\partial s}}{\frac{\partial \varepsilon_{ps}}{\partial \theta}}
\]

where \( \frac{\partial \varepsilon_{ps}}{\partial s} \) is negative as shown in proof 1, so that \( \text{sign} \left( \frac{\partial s^*(\theta)}{\partial \theta} \right) = \text{sign} \left( \frac{\partial \varepsilon_{ps}}{\partial \theta} \right) \).

\[
\frac{\partial \varepsilon_{ps}(s^*(\theta), \theta)}{\partial \theta} = \left( p''^s \frac{s}{p} - \frac{s}{p} p'_s \right) \implies \frac{\partial \varepsilon_{ps}(s^*(\theta), \theta)}{\partial \theta} = \frac{1}{p} (sp''_s + \beta p'_s)
\]

where we substitute \( \varepsilon_{p,s}(s^*(\theta), \theta) = -\beta \) from equation 3. Note that \( p'_\theta > 0 \) and \( p''_s \theta > 0 \) by assumption 4. Therefore,

\[
\frac{\partial s^*(\theta)}{\partial \theta} > 0
\]

Therefore, more talented students choose universities offering higher standards. If \( s^*(\theta) \) is chosen, the dropping-out probability does not depend on the chosen standard if: \( \varepsilon_{s^*, \theta} = -\frac{\varepsilon_{p,s}}{\varepsilon_{p,s}} \), in other terms, if the elasticity of the optimal standard to talent is equal to the ratio between the elasticity of the probability of graduation to talent and standards. Indeed, there are no reason to think that the drop-out probability should be higher among the less talented students once the optimal standard has been chosen (as it seems to emerge from Figure 1, right graph).

Note that, alongside the isopremium curve, the following holds:

\[
\text{sign} \left( \frac{\partial \theta}{\partial s} \right) = -\text{sign} \left( \varepsilon_{ps}(s, \theta) + \beta \right)
\]
The isopremium curve of equation 4 computed for the marginal enrolled, the one whose wage premium is $V(s^*(\theta), \theta, Z) = 0$, becomes:

$$\frac{\partial \theta(s, Z)}{\partial s} \bigg|_{V=0} = -\frac{\theta(s, Z) \varepsilon_{ps}(s, \theta(s, Z)) + \beta}{\varepsilon_{ps}(s, \theta(s, Z))}$$

(9)

By solving equation (9) in $s$, we obtain the lower bound of the standard to which students would enrol, $s^*(\theta(Z))$, and the lowest talent among the enrolled at that standard, $\theta(Z)$, both displayed in the Figure 2 and depending on $Z$:

$$\varepsilon_{ps}(s, \theta(s, Z)) + \beta = 0 \implies s^*(\theta(Z)) \implies \theta(s^*(\theta(Z)), Z) = \theta(Z)$$

(10)

In what follows, we assume that the higher educational setting can be such that:

- all universities offer the same standard $s$. \implies homogeneous standard case;

- universities offer all the standards preferred by secondary school graduates, $s^*(\theta)$, with $\theta \in (0, \theta]$. \implies heterogeneous standard case.

Where, in the latter case, attaining the optimal standard generates moving costs.

\textsuperscript{15}Obviously, they must depend also on $\beta$ that here is treated as a completely exogenous parameter.
2.1 Homogeneous standards

If $s$ is given at some exogenous level, the *marginal* enrolled is the one whose talent solves $V(s, \theta, Z) = 0$:

$$V(s, \theta, Z) = 0 \implies \theta(s, Z)$$

(11)

that is dependent on the standard and on the cost of failure, $Z$. Equation [11] splits the population of secondary school graduates among the enrolled, those individuals that have a talent $\theta > \theta(s_0, Z)$, and the not-enrolled.

**Proposition 3** For any given standard, the talent required to find convenient the enrollment decision is increasing in the cost of failure and decreasing with family/State willingness to finance education.

**Proof 3** The relationship between $\theta(s, Z)$ and $Z$ can be shown by considering that, from equation [3], once condition [11] is considered, so that $p(s, \theta)w(s, \theta) = Z\theta$, we obtain:

$$\frac{\partial V}{\partial \theta(s, Z)} = Z\varepsilon p(s, \theta)$$

and, given $\frac{\partial V}{\partial Z} = -\theta$:

$$\frac{\partial \theta(s, Z)}{\partial Z} = \frac{\theta}{Z\varepsilon p(s, \theta)} > 0$$

(12)

A higher cost of failure $Z$ increases the talent of the marginal enrolled and, given equation [7], the standard at which he/she enrols.

We assume now that secondary school graduates population can be decomposed in two groups, defined as “poor”, whose cost of failure is $Z_2$, and “rich”, whose cost of failure is $Z_1 < Z_2$. Poor are a share $q_2$ of the whole population. The overall enrollment rate is:

$$E_U(s, q, Z_1, Z_2) = \sum_{i=1}^{2} q_i \int_{\theta(s, Z_i)} \phi(\theta)d\theta \quad \sum_{i} q_i = 1$$

(13)

**Proposition 4** It exists a homogeneous standard that maximizes enrollment; it depends positively on the share of poor in the population. If this standard is set, negative externalities between the share of rich in the population and the enrollment rate of poor students emerge (and vice-versa).

**Proof 4** For uniform $\phi$ distribution, $\sum_{i=1}^{2} q_i \frac{\partial \theta(s, Z_i)}{\partial s} = 0$ defines the maximum enrolment rate, where $\frac{\partial \theta(s, Z_i)}{\partial s}$ shows a minimum in $s$ (see equation [9]). Therefore there must exist a value of $s$ that gives:

$$-q_1 \frac{\partial \theta(s, Z_1)}{\partial s} = q_2 \frac{\partial \theta(s, Z_2)}{\partial s} \implies s = \sigma$$

(14)
where we defined with $\sigma$ the value of the homogeneous standard that maximizes the enrollment rate.

The value of $\sigma$ must be such that $s^*(\theta(Z_1)) < \sigma < s^*(\theta(Z_2))$ (see figure 3 and equation 10), so that the homogeneous standard is higher than the one that maximizes the rich enrollment rate and lower than the one that maximizes the poor enrollment rate. The higher the share of rich in the population, the lower the enrollment rate among the poor and vice-versa.

The graduation rate is:

$$G^U(s, q_i, Z_i) = \sum_i q_i \int_{\theta(s, Z_i)}^{\theta(s, Z_i+)} p(s, \theta) \phi(\theta) d\theta \quad (15)$$

From numerical simulations, it emerges that the graduation rate is maximized for a level of the homogeneous standard lower than the one that maximizes the enrollment rate.

Human capital accumulation is:

$$H^U(s, q_i, Z_i) = q_i \int_{\theta(s, Z_i)}^{\theta(s, Z_i+)} p(s, \theta) w(s, \theta) \phi(\theta) d\theta \quad (16)$$

**Proposition 5** The homogeneous standard that maximizes human capital accumulation is higher than the one that maximizes the enrollment rate.

**Proof 5** see appendix (A)

Figure 3 graphically presents the results of the above analysis.

### 2.2 Heterogeneous standards

Assume that all standards are offered by the academic institutions. It is worthwhile to note that, also in that case, some individuals prefer not to enroll. In fact, the premium for enrolling computed for the marginal individual, the one who is indifferent if enrolling or not at his/her preferred standard, is:

$$V(s^*(\theta), \theta, Z) = \frac{\partial}{\partial \theta} 0$$

where $\theta(Z) > 0$ because $V(s^*(0), 0, Z) < 0$ if $Z > 0$ (see equation 1) and $V(s^*(\theta), \theta, Z)$ is increasing in $\theta$ (see equation 3).

We assume here that poor can not afford moving costs so that they can only choose if enrolling at the “home” university, whose standard is randomly drawn, or not enrolling. Rich can move to the university offering their preferred standard. $q$ is the share of poor in the population. Therefore:

- all secondary school graduates coming from rich families whose talent $\theta$ is such that $\theta > \theta(Z_1)$ will enroll at the university offering their optimal standard. Their graduation probability is $p(s^*(\theta), \theta)$ and their human capital accumulation is $p(s^*(\theta), \theta)w(s^*(\theta), \theta)$. 


The overall enrollment rate is:

\[ p(s, \theta) = \frac{\phi}{\pi s^2}; \quad w(s, \theta) = \theta s^\beta; \quad \text{uniform } \phi \text{ distribution, } \theta = U[0, 2]; \quad \beta = 0.2; \quad Z = 0.58 \]

- all secondary school graduates coming from poor families whose talent \( \theta \) is such that \( \theta > \theta(s, Z_2) \) will enroll at the university offering the standard \( s \), where \( s \) is randomly drawn from the \( g(s) \) distribution\(^\text{16}\) with \( s(\theta(Z_1)) < s < \sigma \). Their graduation probability is \( p(s, \theta) \) and their human capital accumulation is \( p(s, \theta)w(s, \theta) \).

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\(^{16}\)Given that every \( s < s(\theta(Z_1)) \) would not be chosen by students coming from rich families, we assume that universities do not offer standard that would eventually be chosen only by poor students living in the district. Obviously, in order to analyze deeply the heterogeneous case, analysis on the university management optimal behaviour are needed.
\[ E(q, Z_1, Z_2) = (1 - q) \int_{\Theta(Z_1)}^{\Theta} \phi(\theta) d\theta + \]
\[ + q \int_{s(\Theta(Z_1))}^{\tilde{\Theta}} \int_{\Theta(s, Z_2)}^{\tilde{\Theta}} \phi(\theta) g(s) d\theta ds \]

The overall graduation rate is:

\[ G(q, Z_1, Z_2) = (1 - q) \int_{\Theta(Z_1)}^{\Theta} p(s^*(\theta), \theta) \phi(\theta) d\theta + \]
\[ + q \int_{s(\Theta(Z_1))}^{\tilde{\Theta}} \int_{\Theta(s, Z_2)}^{\tilde{\Theta}} p(s, \theta) \phi(\theta) g(s) d\theta ds \]

The overall human capital accumulation is:

\[ H(q, Z_1, Z_2) = (1 - q) \int_{\Theta(Z_1)}^{\Theta} p(s^*(\theta), \theta) \phi(\theta) d\theta + \]
\[ + q \int_{s(\Theta(Z_1))}^{\tilde{\Theta}} \int_{\Theta(s, Z_2)}^{\tilde{\Theta}} p(s, \theta) \phi(\theta) g(s) d\theta ds \]

3 Comparing the homogeneous standard and the differentiated standard case

By assuming that the standard in the homogeneous standard case is chosen in order to maximize human capital accumulation, by defining this standard with \( S \),

- rich accumulate more human capital in the differentiated standard case because they can choose their preferred standard, that is precisely the one that maximises human capital accumulation;

- poor accumulate more human capital in the differentiated standard case if:

\[ \int_{s(\Theta(Z_1))}^{\Theta} \int_{\Theta(s, Z_2)}^{\tilde{\Theta}} p(s, \theta) w(s, \theta) \phi(\theta) g(s) d\theta ds > \int_{\Theta(s, Z_2)}^{\tilde{\Theta}} \int_{\Theta(s, Z_2)}^{\tilde{\Theta}} p(S, \theta) w(S, \theta) \phi(\theta) d\theta \]

because, in the heterogeneous standard case, they enroll only in random standard offered in the district they live.
To be analytically demonstrated. By numerical simulation, the previous equation never holds.

Once demonstrated, in the heterogeneous standard case poor accumulate less human capital whereas rich accumulate more human capital. Higher the share of poor more likely is a higher human capital accumulation in the homogeneous standard case, that is the result emerging from numerical simulations.

**Proposition 6** Measuring efficiency as overall human capital accumulation during studies, if the share of secondary school graduates that can not afford moving costs is high, the centralized higher educational setting increase efficiency in the economic system.

**Proof 6** To be demonstrated

**Proposition 7** Measuring equity as intergenerational mobility in human capital accumulation, the centralized higher educational setting always increase equity in the economic system.

**Proof 7** To be demonstrated

4 Conclusions

To be done
References


Appendix (A): selected proofs

Proof of remark ?? — not a true proof....

We wonder if, in the centralised (homogeneous) standard case, the graduation rate is increasing or decreasing at the level of the standard which maximises the enrolment. The derivative of the graduation rate with respect to $s$ gives:

$$\frac{\partial G^U}{\partial s} = q \left[ -p(s, \bar{\theta}(s, Z_2)) \frac{\partial \theta(s, Z_2)}{\partial s} + \int_{\bar{\theta}(s, Z_2)}^{\bar{\theta}(s, Z)} \frac{\partial p(s, \theta)}{\partial s} \phi(\theta) d\theta \right] + (1-q) \left[ -p(s, \bar{\theta}(s, Z_1)) \frac{\partial \theta(s, Z_1)}{\partial s} + \int_{\bar{\theta}(s, Z_1)}^{\bar{\theta}(s, Z)} \frac{\partial p(s, \theta)}{\partial s} \phi(\theta) d\theta \right]$$

(17)

where the two integrals are negative because $\frac{\partial p(s, \theta)}{\partial s} < 0$. A sufficient condition to have $\frac{\partial G^U}{\partial s} < 0$ is therefore:

$$-qp(s, \bar{\theta}(s, Z_2)) \frac{\partial \theta(s, Z_2)}{\partial s} - (1-q)p(s, \bar{\theta}(s, Z_1)) \frac{\partial \theta(s, Z_1)}{\partial s} < 0$$

computed for $s = \sigma$. Using equation (14) the previous equation becomes:

$$-q \left[ p(s, \bar{\theta}(s, Z_2)) - p(s, \bar{\theta}(s, Z_1)) \right] \frac{\partial \theta(s, Z_2)}{\partial s}$$

where $\frac{\partial \theta(s, Z_2)}{\partial s} < 0$ and $p(s, \bar{\theta}(s, Z_2)) - p(s, \bar{\theta}(s, Z_1)) > 0$. Therefore, the sign of $\frac{\partial G^U}{\partial s}$ can not computed using the sufficient condition (unfortunately); it is nevertheless negative by numerical simulation.

Proof of remark 5

The derivative of $H^U$ with respect to $s$ gives:

$$\frac{\partial H^U}{\partial s} = q \left[ -p(s, \bar{\theta}(s, Z_2)) w(s, \bar{\theta}(s, Z_2)) \frac{\partial \theta(s, Z_2)}{\partial s} + \int_{\bar{\theta}(s, Z_2)}^{\bar{\theta}(s, Z)} \frac{\partial [p(s, \theta) w(s, \theta)]}{\partial s} \phi(\theta) d\theta \right] + (1-q) \left[ -p(s, \bar{\theta}(s, Z_1)) w(s, \bar{\theta}(s, Z_1)) \frac{\partial \theta(s, Z_1)}{\partial s} + \int_{\bar{\theta}(s, Z_1)}^{\bar{\theta}(s, Z)} \frac{\partial [p(s, \theta) w(s, \theta)]}{\partial s} \phi(\theta) d\theta \right]$$

(18)

The previous equation can be computed for $s = \sigma$. In that case, the two integrals must be positive because $\frac{\partial p(s, \theta) w(s, \theta)}{\partial s} = 0$ for the marginal enrolled and positive for all the enrolled. The term:

$$-qp(s, \bar{\theta}(s, Z_2)) w(s, \bar{\theta}(s, Z_2)) \frac{\partial \theta(s, Z_2)}{\partial s}$$

$$-(1-q)p(s, \bar{\theta}(s, Z_1)) w(s, \bar{\theta}(s, Z_1)) \frac{\partial \theta(s, Z_1)}{\partial s}$$
can be computed for \( s = \sigma \) using equation 14:

\[
-q \left[ p(s, \theta(s, Z_2)) w(s, \theta(s, Z_2)) - p(s, \theta(s, Z_1)) w(s, \theta(s, Z_1)) \right] \frac{\partial \theta(s, Z_2)}{\partial s}
\]

where \( \frac{\partial \theta(s, Z_2)}{\partial s} < 0 \) and \( p(s, \theta(s, Z_2)) w(s, \theta(s, Z_2)) - p(s, \theta(s, Z_1)) w(s, \theta(s, Z_1)) > 0 \); therefore, \( \frac{\partial H_U}{\partial s} > 0 \) if \( s = \sigma \).
5 Appendix (B): Simulating the model

This section is based on the following specific functional forms of the equation \( p(h, s) \) and \( w(h, s) \):

\[
p(s, \theta) = \frac{\theta}{s + \theta} \quad w(s, \theta) = \theta s^\beta
\]

respecting all the conditions of assumption 1 and 2. We obtain:

\[
\varepsilon_{ps} = -\frac{s}{\theta + s}
\]

Given these functional forms, in the differentiated standard case, we obtain from equation ??:

\[
s^*(\theta) = \frac{\beta}{1 - \beta} \theta
\]

so that the standard’s optimal choice is linear in human capital and \( \frac{ds}{d\theta} > 0 \).

Note that:

\[
p(s^*(\theta), \theta) = 1 - \beta
\]

once each individual as chosen is preferred standard, the graduation probability does not depend on individual talent but on the elasticity of the wage rate to the standard alone, and:

\[
w(s^*(\theta), \theta) = \left( \frac{\beta}{1 - \beta} \right)^\beta \theta^{1 + \beta}
\]

The marginal enrolled satisfies \( M(s^*(\theta), \theta, Z) = 0 \) and, defining \( B = \beta^\beta (1 - \beta)^{1 - \beta} \) gives:

\[
\theta(Z) = \left( \frac{Z}{B} \right)^{\frac{1}{\beta}}
\]

so that:

\[
s(Z) = \frac{\beta}{1 - \beta} \left( \frac{Z}{B} \right)^{\frac{1}{\beta}} = \left( \frac{Z}{1 - \beta} \right)^{\frac{1}{\beta}} \quad \text{and} \quad \pi = \frac{\beta}{1 - \beta} \theta
\]

are the lower and upper level of the standards demanded by enrolled.

Unfinished