Can Pension be the Worker Discipline Device as Efficiency Wage?
(Preliminary Version)
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Abstract
This study analyzes the pension’s effect on working incentives by introducing pensions into the labor shirking efficiency wage model by Shapiro and Stiglitz (1984), and reveals that pensions can serve as the worker discipline as two ways: by different conditions that the monitoring parameters satisfy pension will either enforce the efficiency wage effect on worker discipline, by affecting the wage level positively; or the increase of pensions will bring down the wage level, ie. pension and the wage level has a tradeoff effect, and pension is also an attraction of workers as well as higher wages.

Keywords: equilibrium unemployment, pensions, efficiency wage model, incentive
JEL: J13, J51, J65

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1. Introduction

This study analyzes the pension’s effect on working incentives by introducing pensions into the labor shirking efficiency wage model by Shapiro and Stiglitz (1984), and reveals that pensions can serve as the worker discipline as two ways: by different conditions that the monitoring parameters satisfy pension will either enforce the efficiency wage effect on worker discipline, by affecting the wage level positively; or the increase of pensions will bring down the wage level, ie. pension and the wage level has a tradeoff effect, and pension is also an attraction of workers as well as higher wages.

The remainder of this paper is organized as follows. Section 2 presents the model. Section 3 discusses the equilibrium. Section 4 describes a set of comparative statics analyzing the effects of pensions and child allowances, and presents the simulation. Section 5 concludes.

2. Model

In this section, we formulate a simple model which take unemployment as an incentive inducer.

2.1 Employers

We assume that in a closed economy, there are numbers of identical firms who use the technology of Cobb-Douglas, therefore the aggregate production function is

\[ Y = (eL)^\sigma \]

where \( e \) is the effective labor, or the effort. The profit maximization problem is

\[ \pi = Y - \omega L \]

\[ \frac{\partial \pi}{\partial L} = 0 \Rightarrow \frac{\partial Y}{\partial L} = \omega \Rightarrow \sigma e^\sigma L^{\sigma-1} = F'(L) \] (1)

Without any informational problems would write contracts to pay workers if they exert effort. However, the problem arises because firms cannot observe whether a worker has exerted effort or not, and cannot deduce it from output, since output is a
function of all workers’ efforts. Therefore, the firms use monitoring technology to
monitor employees’ efforts, and the shirking can be detected as a probability of \((\rho)\).

2.2 Workers

We discuss the lifetime activities of the workers in continuous time, and all agents
are infinitely lived. There exists \(N\) identical workers who dislikes exerting effort
but enjoy consuming goods. As Shapiro and Stiglitz (1984) has done, the utility
function in this study is assumed separable, and is written as

\[
U = \omega - e
\]

If the worker shirks in work time, \(e=0\), then the utility becomes \(\omega\). Workers are risk
neutral. The normal worker who does not shirk will retire at some age which is set
by law with the probability (or flow rate) of \(\delta\) per unit of time, and in the
employment period, they have the probability of \(\varepsilon\) per unit time to be separated
from his job due to employment shock or relocation. If the shirker is detected by
the managers, he will be dismissed, and no pension will be given to this shirker as
a punishment.

We define the expected lifetime utility of an employed non-shirker as \(U_N\), the
expected lifetime utility of an employed shirker as \(U_s\), and the expected lifetime
utility of an unemployed person as \(U_u\). The lifetime activities of different kind of
worker are summarized as follows.

<table>
<thead>
<tr>
<th>Employers</th>
<th>Unemployment</th>
<th>Retirement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-shirker</td>
<td>((\varepsilon, U_u))</td>
<td>((\delta, U_{r,N}))</td>
</tr>
<tr>
<td>Shirker</td>
<td>((\varepsilon + \rho, U_u))</td>
<td>((\delta + \rho, U_{r,s}))</td>
</tr>
</tbody>
</table>

Therefore, the fundamental asset equation for a non-shirker is

\[
r U_N = \omega(1 - \tau) - e + \varepsilon(U_u - U_N) + \delta(U_{r,N} - U_N)
\]  

(2)

While for a shirker,

\[
r U_s = \omega(1 - \tau) + (\varepsilon + \rho)(U_u - U_s) + (\delta + \rho)(U_{r,s} - U_s)
\]  

(3)

If he is detected the shirking action, he will become unemployed, and no pension. The
return of retirement of non-shirk and shirk:

\[
r U_{r,N} = \theta
\]  

(4)
The asset function of the unemployed is

\[ rU_u = b + \phi(U_E - U_u) \]  (6)

where,

\[ U_E = \max \{U_N, U_s\} \]

From Equations (2) and (6),

\[ U_N = \frac{(r + \varepsilon + \phi)U_u + (\omega(1 - \tau) - e - b + \delta \theta)}{r + \varepsilon + \phi + \delta} \]  (7)

\[ U_N - U_u = \frac{-\delta U_u + (\omega(1 - \tau) - e - b + \delta \theta)}{r + \varepsilon + \phi + \delta} \]  (*)

From Equation (3) and (6),

\[
U_s = \frac{1}{r + \varepsilon + 2\rho + \delta} \left\{ \omega - b + (\delta + \rho)(1 - \rho)\theta - \frac{\phi(\omega(1 - \tau) - e - b + \delta \theta)}{r + \varepsilon + \phi + \delta} \right. \\
\left. + \frac{(r + \varepsilon + \rho)(r + \varepsilon + \phi + \delta) + \phi \delta}{r + \varepsilon + \phi + \delta} U_u \right\} \]  (8)

the worker will choose not to shirk if and only if \( U_N \geq U_s \), which is called Non-shirking condition. Therefore,

\[
(1 - \tau)\omega \geq \frac{\delta - r - \varepsilon - \phi}{2} U_u + e + b - \delta \theta + \frac{[e + \rho \theta(1 - \delta - \rho)](r + \varepsilon + \delta + \phi)}{2\rho} \]

\[
\omega \geq \delta - r - \varepsilon - \phi \]  \[ U_u \]

\[
\frac{e + \rho \theta(1 - \delta - \rho)](r + \varepsilon + \delta + \phi)}{2\rho} = \omega \]

From Equations (6) and (*)

\[ U_u = \frac{b(r + \varepsilon + \delta) + \phi(\omega(1 - \tau) - e + \delta \theta)}{r(r + \varepsilon + \phi + \delta) + \delta \phi} \]  (9)

2.3 Government

The government collect pension funding by unfunded pension- the defined benefit scheme. Therefore,

\[ \tau \omega L = \theta * \delta L \]

Equations (7), (8), (9) can be rewritten as

\[ U_N = \frac{(r + \varepsilon + \phi)U_u + (\omega - e - b)}{r + \varepsilon + \phi + \delta} \]  (7 *)
\[
U_s = \frac{1}{r + \epsilon + 2\rho + \delta} \left\{ \omega - b + \rho (1 - \rho - \delta) \theta - \frac{\varphi (\omega - e - b)}{r + \epsilon + \varphi + \delta} \right. \\
+ \frac{(r + \epsilon + \rho)(r + \epsilon + \varphi + \delta) + \varphi \delta}{r + \epsilon + \varphi + \delta} U_u \left. \right\}
\] 
(8 *)

\[
U_u = \frac{b(r + \epsilon + \delta) + \varphi (\omega - e)}{r(r + \epsilon + \varphi + \delta) + \delta \varphi}
\] 
(9 *)

Therefore, the non-shirking wage can be arranged as
\[
\tilde{\omega} = e + b + \frac{\delta - r - \epsilon - \varphi b(r + \epsilon + \delta) + \varphi (\omega - e)}{2} \frac{2}{r(r + \epsilon + \varphi + \delta) + \delta \varphi} \\
+ \left[ + \rho \theta (1 - \delta - \rho) \right] \frac{(r + \epsilon + \delta + \varphi)}{2\rho}
\] 
(10)

3 Equilibrium

In steady state, the flow into and out of the unemployment pool is equal, so
\[
\varphi (N - L) = (\epsilon + \delta) L
\]

Therefore,
\[
\frac{\varphi}{\epsilon + \delta} = \frac{1 - u}{u}
\] 
(11)

where \( u \) is the exogenous unemployment rate, and thus \( \epsilon + \delta + \varphi = \frac{\varphi}{1 - u} \).

The equilibrium wage and employment level are identified from Equations (1) and (10),
\[
e + b + \frac{\delta - r - \epsilon - \varphi b(r + \epsilon + \delta) + \varphi (\omega - e)}{2} \frac{2}{r(r + \epsilon + \varphi + \delta) + \delta \varphi} \\
+ \left[ + \rho \theta (1 - \delta - \rho) \right] \frac{(r + \epsilon + \delta + \varphi)}{2\rho} = F'(L)
\]

Substitute Equation (11) into the equation above, then
\[
e + b + \frac{\delta - r - \epsilon - \varphi b(r + \epsilon + \delta) + \varphi (\omega - e)}{2} \frac{2}{r \left( r + \frac{\varphi u}{1 - u} \right) + \delta \varphi} \\
+ \left[ + \rho \theta (1 - \delta - \rho) \right] \frac{r + \frac{\varphi}{1 - u}}{2\rho} = \sigma e^\sigma \left( (1 - u)N \right)^{\sigma - 1}
\]
\[
\begin{align*}
\frac{\delta - r - \varepsilon - \varphi}{2} \frac{\varphi(\omega - e)}{r \left( r + \frac{\varphi}{1 - u} \right) + \delta \varphi} &= \sigma e^\sigma ((1 - u)N)^{\sigma - 1} - e - b - \frac{[e + \rho \theta (1 - \delta - \rho)] \left( r + \frac{\varphi}{1 - u} \right)}{2 \rho} \\
- \frac{\delta - r - \varepsilon - \varphi}{2} \frac{b \left( r + \frac{\varphi u}{1 - u} \right)}{r \left( r + \frac{\varphi}{1 - u} \right) + \delta \varphi}
\end{align*}
\]

\[
\omega^* = \left[ \sigma e^\sigma ((1 - u)N)^{\sigma - 1} - e - b - \frac{[e + \rho \theta (1 - \delta - \rho)] \left( r + \frac{\varphi}{1 - u} \right)}{2 \rho} \\
- \frac{\delta - r - \varepsilon - \varphi}{2} \frac{b \left( r + \frac{\varphi u}{1 - u} \right)}{r \left( r + \frac{\varphi}{1 - u} \right) + \delta \varphi} \right] 2 \left[ r \left( r + \frac{\varphi}{1 - u} \right) + \delta \varphi \right] \frac{\varphi(\delta - r - \varepsilon - \varphi)}{\varphi(\delta - r - \varepsilon - \varphi)} + e \quad \text{(11)}
\]

\(\omega^*\) is the equilibrium wage.

4 Comparative Statics

In order to the relationship between pension and the wage, we use the comparative statics to identify.

\[
\frac{\partial \omega^*}{\partial \theta} = -\left( r + \frac{\varphi}{1 - u} \right) (1 - \delta - \rho) \left[ r \left( r + \frac{\varphi}{1 - u} \right) + \delta \varphi \right] \frac{\varphi(\delta - r - \varepsilon - \varphi)}{\varphi(\delta - r - \varepsilon - \varphi)}
\]

when \(- \frac{1 - \delta - \rho}{\varphi(\delta - r - \varepsilon - \varphi)} > 0\), \(\frac{\partial \omega^*}{\partial \theta} > 0\); when \(- \frac{1 - \delta - \rho}{\varphi(\delta - r - \varepsilon - \varphi)} < 0\), \(\frac{\partial \omega^*}{\partial \theta} < 0\).

**Proposition:**

Pensions can serve as the worker discipline as two ways: when the monitoring parameters satisfy that \(- \frac{1 - \delta - \rho}{\varphi(\delta - r - \varepsilon - \varphi)} < 0\), pension will enforce the efficiency wage effect on worker discipline, ie. the wage level will be affected by the pension level positively; however, when the monitoring parameters satisfy that \(- \frac{1 - \delta - \rho}{\varphi(\delta - r - \varepsilon - \varphi)} > 0\), the increase of pensions will bring down the wage level, ie. pension and the wage level has a tradeoff effect, and pension is also an attraction of workers as well as higher wages.
5 Conclusion

This study analyzes the pension’s effect on working incentives by introducing pensions into the labor shirking efficiency wage model by Shapiro and Stiglitz (1984), and reveals that pensions can serve as the worker discipline as two ways: by different conditions that the monitoring parameters satisfy pension will either enforce the efficiency wage effect on worker discipline, by affecting the wage level positively; or the increase of pensions will bring down the wage level, ie. pension and the wage level has a tradeoff effect, and pension is also an attraction of workers as well as higher wages.

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